

3NF Decomposition

R&G Chapter 19

Features of a Good Decomposition

- **A good decomposition is**
 - Lossless
 - Can obtained by BCNF decomposition
 - Dependency preserving

Problem #2 of Decomposition: Dependency preservation

A	B	C
1	2	3
4	5	6
7	2	8

A	C
1	3
4	6
7	8

B	C
2	3
5	6
2	8

$A \rightarrow B; C \rightarrow B$

$X=\{A, C\}, Y=\{B, C\}, X \cap Y = \{C\}, C \rightarrow \{B, C\}$
Lossless decomposition!

A	C
1	3
4	6
7	8

Join

B	C
2	3
5	6
2	8

=

A	B	C
1	2	3
4	5	6
7	2	8

But, now we can't check $A \rightarrow B$ without doing a join!
(Problem #2 of decomposition!)

BCNF and 3NF Decomposition

- **Some rules to remember**

	BCNF	3NF
Data redundancy	NONE	May still have some
Lossless-join decomposition	Guaranteed	Guaranteed
dependency-preserving decomposition	Not guaranteed	Guaranteed

Task #2

- **How to decompose the table so that dependency checking does not need joins?**

Dependency Preserving Decomposition

Projection of set of FDs F

If R is decomposed into X and Y , then the projection of F on X (denoted F_X) is the set of FDs $U \rightarrow V$ in F^+ such that all of the attributes U, V are in X . (*similarly for F_Y*)

- E.g., $F^+ = \{A \rightarrow B, A \rightarrow C, D \rightarrow E\}$
- Two tables: $X = \{A, B, C\}$, $Y = \{C, D, E\}$
- What are F_X and F_Y ?

Dependency Preserving Decomposition

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- E.g., $F^+ = \{A \rightarrow B, A \rightarrow C, D \rightarrow E\}$
- Two tables: $X = \{A, B, C\}$, $Y = \{C, D, E\}$
- What are F_X and F_Y ?
 - $F_X = \{A \rightarrow B, A \rightarrow C\}$,
 - $F_Y = \{D \rightarrow E\}$.

Dependency Preserving Decompositions (Contd.)

- **Decomposition of R into X and Y is dependency preserving if $(F_X \cup F_Y)^+ = F^+$**
 - i.e., consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+ .

Decomposition into 3NF

- The algorithm for BCNF decomposition does not ensure dependency preservation (it only assures lossless join)
- To ensure dependency preservation (and lossless-join)
 - Instead of the given set of FDs F , use a *minimal cover for F* .

Minimal Cover for a Set of FDs

- Given a set of FDs F , G is the *minimal cover* of F if G satisfies the following three conditions:
 1. Right hand side (RHS) of each FD in G is a single attribute;
 2. $G^+ = F^+$; and
 3. G is minimal: if we modify G by deleting an FD in G , G^+ changes.
- Intuitively, every FD in G is needed, and “*as small as possible*” in order to get the same closure as F .

Finding Minimal Cover

- **Step 1: Minimize right-hand-side (RHS) of FDs so that they only contain single attributes**
 - Change $X \rightarrow YZ$ to be $X \rightarrow Y$ and $X \rightarrow Z$
- **Step 2: Minimize left-hand-side (LHS)**
 - If there are two FDs: $A \rightarrow B$ and $ABX \rightarrow Z$, replace $ABX \rightarrow Z$ with $AX \rightarrow Z$
 - For example: if $AB \rightarrow C$ and $A \rightarrow B$, then replace $AB \rightarrow C$ with $A \rightarrow C$.
- **Step 3: Remove redundant FDs**
 - If $X \rightarrow Y$ can be inferred from other FDs, remove $X \rightarrow Y$



Example

- **$R(ABCDE)$**
- **$F = \{A \rightarrow D, BC \rightarrow AD, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$**
- **What is the minimal cover of F ?**



Example - Solution

- **R(ABCDE)**
- **$F = \{A \rightarrow D, BC \rightarrow AD, C \rightarrow B, E \rightarrow A, E \rightarrow D\}$**
- **What is the minimal cover of F?**
- **Way of thinking:**
 - Step 1: Minimize RHS to be single attribute:
 - Split $BC \rightarrow AD$ to $BC \rightarrow A$, $BC \rightarrow D$
 - Step 2: Minimize LHS:
 - Since $C \rightarrow B$, replace $BC \rightarrow A$ with $C \rightarrow A$, and $BC \rightarrow D$ with $C \rightarrow D$
 - Step 3: Remove redundant FDs
 - $C \rightarrow A, A \rightarrow D \Rightarrow C \rightarrow D$, so remove $C \rightarrow D$
 - $E \rightarrow A, A \rightarrow D \Rightarrow E \rightarrow D$, so remove $E \rightarrow D$
 - Minimal cover: $\{A \rightarrow D, C \rightarrow A, C \rightarrow B, E \rightarrow A\}$

3NF Decomposition

Input: a relation R with FDs F.

- Step 1: Find the **minimal cover F'** of F.
- Step 2: Generate a BCNF decomposition $\{R_1, \dots, R_n\}$ of R w.r.t. **F'**
- Step 3: Identify the dependencies N in **F'** that is not preserved by BCNF decomposition $\{R_1, \dots, R_n\}$
 - i.e., those dependencies in F' whose attributes are not included in any table in $\{R_1, \dots, R_n\}$
- Step 4: For each $X \rightarrow Y$ in N, create a relation schema XY and add it to $\{R_1, \dots, R_n\}$

Notes:

- Step 2 guarantees it is a lossless decomposition
- Step 3 and 4 ensure it is a dependency-preserving decomposition
 - Both steps ensure $F_{R_1} \cup \dots \cup F_{R_n} = F'$
 - Thus it must be true $(F_{R_1} \cup \dots \cup F_{R_n})^+ = F'^+$
- 3NF decomposition is guaranteed to be lossless and dependency-preserving

3NF Decomposition Example



- $R(ABCDE)$
- $F = \{A \rightarrow D, BC \rightarrow AD, C \rightarrow B, E \rightarrow B, E \rightarrow D\}$
- Decompose R in a lossless-join, dependency-preserving way (i.e., decompose R into 3NF tables!)

3NF Decomposition Example (cont.)



- **Solution:**

- Step 1: Find minimal cover F' of F
 - $F' = \{A \rightarrow D, C \rightarrow A, C \rightarrow B, E \rightarrow B, E \rightarrow D\}$
- Step 2: create a lossless-join BCNF decomposition D of R
 - Step 2.1. find candidate key(s): CE
 - Step 2.2. construct BCNF decomposition based on F' : $D = \{BC, CE, AC, AD\}$ (following order of $A \rightarrow D, C \rightarrow A, C \rightarrow B$)
- Step 3: Identify the dependencies N in F' that is not preserved by D
 - $N = \{E \rightarrow D, E \rightarrow B\}$
- Step 4: For each $X \rightarrow Y$ in N , create a relation schema XY and add it to D
 - $D = \{BC, CE, AC, AD, ED, EB\}$

BCNF Versus 3NF Decomposition

	BCNF	3NF
Redundancy	NONE	May still have some
Lossless-join decomposition	Guaranteed	Guaranteed
dependency-preserving decomposition	Not guaranteed	Guaranteed

- **For any given R, there is always a 3NF decomposition.**
 - But it may not have a BCNF decomposition
 - Remember, BCNF is stricter than 3NF
 - A BCNF decomposition is always a 3NF decomposition.