

Schema Refinement Tutorial (part I)

Announcement

- Course assessment on AEFIS (10 minutes)

<https://assess.stevens.edu/>

Final Exam Logistics

- The final exam will be held in class next Tuesday (75 minutes)
- The exams will be held online in Canvas.
- The students still need to come to the class to take exams.
 - The seating chart will be signed during the exam

Cheat Sheet

- A one-sided, A4-size cheat sheet is allowed.
- The cheat sheet can be either handwritten or typed.
- The cheat sheet is NOT required to be submitted after the exam.

Type of Questions in Final Exam

- Mainly cover Functional Dependencies (FD)
 - True/false statement (cover the concepts in FD chapter)
 - Validation of correctness of SQL queries
 - Keys and normal forms (use FDs to find candidate keys, and check if the scheme satisfies 3NF and BCNF)
 - Lossless decomposition
 - 3NF decomposition

Make-up and Accommodations

- Make-up exams
 - The students who cannot attend the exams should inform the instructor at least one day BEFORE the exam.
 - The make-up exam request will be approved upon the validity of reasons.
 - The request will NOT be approved if the instructor is informed on the day of or after the exam.
- Accommodations
 - Student with accommodations send an email to the instructor for the arrangement.

Functional Dependencies (FDs)

- A functional dependency $X \rightarrow Y$ means

Given any two tuples in r , if the X values are the same, then the Y values must also be the same. (but not vice versa)

- Can read “ \rightarrow ” as “determines”

FD Inference Rules

- FD Inference: new FDs can be implied by old FDS
- **Armstrong's Axioms** (X, Y, Z are sets of attributes):
 - Reflexivity: If $X \supseteq Y$, then $X \rightarrow Y$
 - Augmentation: If $X \rightarrow Y$, then $XZ \rightarrow YZ$ for any Z
 - Transitivity: If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- Some additional rules (that follow from AA):
 - Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
 - Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$

FD Closure

- $F^+ = \text{closure of } F$ is the set of all FDs that are implied by F .
- Computing the closure of a set of FDs can be expensive. (Size of closure is exponential in # attrs!)
- Instead, we compute *attribute closure*
 - given FD $F: X \rightarrow Y$, $X^+ =$ Set of all attributes A such that $X \rightarrow A$ is in F^+

Attribute Closure

- Compute attribute closure of X (denoted X^+) wrt F .
 - $X^+ := X$
 - Repeat until no change: if there is an FD $U \rightarrow V$ in F such that U is in X^+ , then add V to X^+
- Approach can also be used to find the keys of a relation.
 - If $X^+ = \{\text{all attributes of the relation}\}$, then X is a superkey for R .

Exercise 1

- Given $FD = \{AB \rightarrow CD, B \rightarrow DE, C \rightarrow F, E \rightarrow G, A \rightarrow B\}$.
- Prove that $\{A \rightarrow FG\}$ is logically implied by FD , by using
 - (1) Armstrong's axiom
 - (2) Attribute closure

Solution 1: Armstrong's axiom

1	$A \rightarrow B$	Given
2	$A \rightarrow AB$	1, Augmentation
3	$AB \rightarrow CD$	Given
4	$A \rightarrow CD$	2, 3, Transitivity
5	$B \rightarrow DE$	Given
6	$A \rightarrow DE$	1, 5, Transitivity
7	$A \rightarrow ACD$	4, Augmentation
8	$ACD \rightarrow CDE$	6, Augmentation twice
9	$A \rightarrow CDE$	7, 8, Transitivity
10	$A \rightarrow CE$	9, Trivial dependency
11	$C \rightarrow F$	Given
12	$CE \rightarrow FE$	11, Augmentation
13	$E \rightarrow G$	Given
14	$FE \rightarrow FG$	13, Augmentation
15	$CE \rightarrow FG$	12, 14, Transitivity
16	$A \rightarrow FG$	10, 15, Transitivity

Solution 2: Attribute Closure

- Given $FD = \{AB \rightarrow CD, B \rightarrow DE, C \rightarrow F, E \rightarrow G, A \rightarrow B\}$.
- Prove that $\{A \rightarrow FG\}$ is logically implied by FD (by using attribute closure)
- Way of thinking
 - Prove that A^+ contains FG !
 - Initialize $A^+ = A$
 - Keep adding attributes into A^+ by following the algorithm of computing attribute closure.
 - $A^+ = ABCDEFG$

Using FDs to Find Keys

- A superkey K of a schema R is a set of attributes in R s.t. $K \rightarrow R$ holds
- A (candidate) key is a *minimal* superkey.

Exercise 2



- Consider the relation schema $R(A, B, C, D)$ with FDs: $A \rightarrow C$ and $B \rightarrow D$. Is $\{A, B\}$ a candidate key for R ?

Solution

- Step 1: check whether $\{A, B\}$ is a superkey:
 - Check whether $(AB)^+ = \{ABCD\}$;
- Step 2: check whether $\{A, B\}$ is minimal (i.e., Is any subset of $\{A, B\}$ is a superkey?):
 - Check whether $(A)^+ = \{ABCD\}$;
 - Check whether $(B)^+ = \{ABCD\}$;

Lossless and Dependency-preserving Decomposition

- Given a table R and a set of FDs F of R, the decomposition of R into X and Y is lossless with respect to F *if and only if*
$$X \cap Y \rightarrow X \text{ or } X \cap Y \rightarrow Y$$
 - i.e., the common attributes of X and Y is a superkey of either X or Y.
- Decomposition of R into X and Y is dependency preserving if $(F_X \cup F_Y)^+ = F^+$
 - i.e., consider only dependencies in the closure F^+ that can be checked in X without considering Y, and in Y without considering X, these imply all dependencies in F^+ .

Exercise 3

Consider a relation $R(A,B,C)$. It has FDs: $A \rightarrow B$, $B \rightarrow C$, and $C \rightarrow A$. Assume R is decomposed into $X=\{A,B\}$ and $Y=\{B,C\}$. Answer the following questions:

- (1) Is this decomposition lossless?
- (2) Is this decomposition dependency preserving?

Exercise 3 – Lossless Solution

- Step 1: find common attributes of X and Y.
 - $X=\{A,B\}$ and $Y=\{B,C\}$.
 - Common attribute: B
 - Step 2: Check if B is the superkey of X or Y
 - FDs hold on X: $A \rightarrow B$
 - FDs hold on Y: $B \rightarrow C$
 - For X: $B^+ = AB$: B is a superkey of X
 - For Y: $B^+ = BC$: B is a superkey of Y
- So the decomposition (X, Y) is lossless.

Exercise 3 – Dependency Preserving Solution

- $F^+ = \{A \rightarrow B, B \rightarrow C, C \rightarrow A, A \rightarrow C, B \rightarrow A, C \rightarrow B\}$
- $F_x = \{A \rightarrow B, B \rightarrow A\}$
- $F_y = \{B \rightarrow C, C \rightarrow B\}$
- $(F_x \cup F_y)^+ = \{A \rightarrow B, B \rightarrow C, B \rightarrow A, C \rightarrow B, A \rightarrow C, C \rightarrow A\}$
- $(F_x \cup F_y)^+ = F^+$, so it is dependency preserving.