Hierarchy in noise expressions for nullor-based amplifiers

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Abstract — **In the structured design of high-performance negative-feedback amplifiers the nullor is the basic block for the synthesis procedure. The nullor represents the ideal infinite gain, i.e. the plant to be controlled by the feedback network which is usually constituted by passive components. The paper establishes the hierarchical structure of the expressions for the equivalent noise source at the input for all four types of single-loop nullor-based amplifiers.**

1 Introduction

In nullor-based amplifiers, two main blocks can be distinguished, the feedback network and the active part. As a result, the design task can be divided into two main steps [1, 2, 3]: (i) the design of the feedback network, under the assumption that the nullor constitutes the active (ideal) part, and (ii) the synthesis of the nullor. The whole design procedure is accomplished with the aim of optimising three fundamental signal-processing quality aspects: noise, bandwidth and distortion.

This article expounds the expressions for the equivalent input noise source of the four types of nullor-based amplifiers and shows the inherent hierarchy of the expressions, which can be used as a key-factor for the design when optimising the amplifier vs noise.

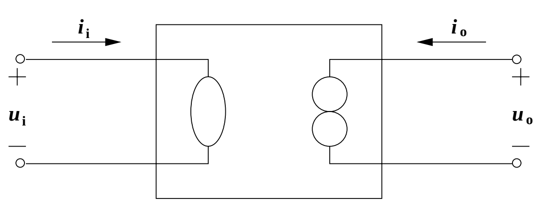

Figure 1: The nullor.

2 Nullor-based amplifiers

The nullor constitutes the active (ideal) block of the amplifier. In fact, the nullor is a two-port composed of two elements: **the nullator** connected at the input port and **the norator** connected at the output port as depicted in Figure 1.

On the one hand, the nullator has the relationships:

$$u_i = 0 \qquad i_i = 0$$

On the other hand, the norator has the relationships:

$$u_o = \text{arbitrary} \qquad i_o = \text{arbitrary}$$

i.e. current and voltage of the output are defined by the environment.

The transmission matrix of the nullor is given as [4, 5]:

$$\boldsymbol{K} = \begin{bmatrix} \frac{1}{\mu} & \frac{1}{\gamma} \\ \frac{1}{\zeta} & \frac{1}{\beta} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (1)$$

It clearly results that the nullor possesses infinite gains for all four transfer relationships, voltage (μ), current (α), trans-conductance (γ) and trans-impedance (ζ).

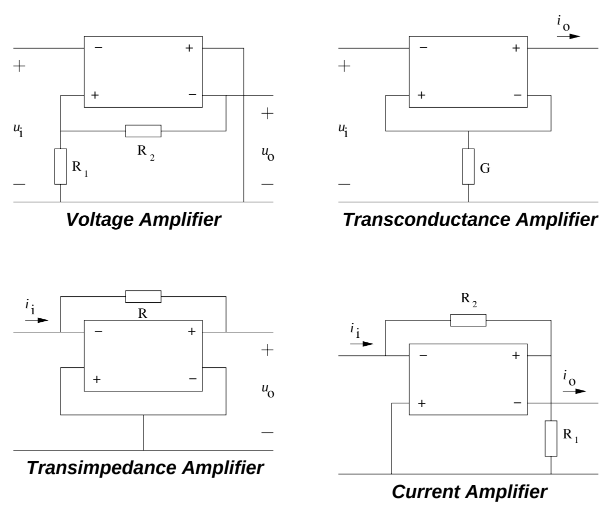

Figure 2: Negative-feedback amplifiers.

The four types of amplifiers can be obtained when a passive feedback network is connected with the nullor, in the negative-feedback configurations shown in Figure 2. For each amplifier the feedback network determines the transfer function, and then their ABCD matrix has only one non-zero element as shown in equations (2),(3),(4), (5)

$$\boldsymbol{K}_u = \begin{bmatrix} \frac{1}{1+R_1/R_2} & 0 \\ 0 & 0 \end{bmatrix} \tag{2}$$

$$\boldsymbol{K}_{g_m} = \begin{bmatrix} 0 & -\frac{1}{G} \\ 0 & 0 \end{bmatrix} \tag{3}$$

$$\boldsymbol{K}_{r_m} = \begin{bmatrix} 0 & 0 \\ \frac{-1}{R} & 0 \end{bmatrix} \tag{4}$$

$$\boldsymbol{K}_i = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{1+R_2/R_1} \end{bmatrix} \tag{5}$$

2.1 Noise in the amplifier

Noise can be regarded in terms of power spectral densities originated from the passive network and the synthesised nullor. Figure 3 shows the four basic configurations of the noisy amplifiers. Herein, $u_{n,n}$ and $i_{n,n}$ model completely the noise present in the nullor as in any two-port system [6, 1][1].

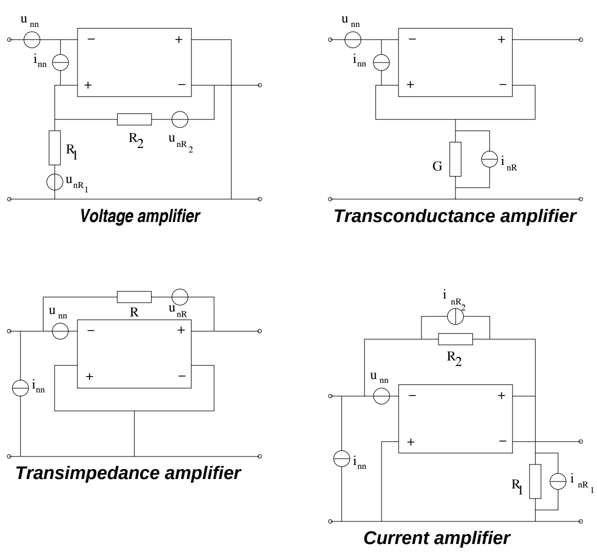

Figure 3: Noisy Negative-Feedback Amplifiers.

Because, it is necessary to obtain the total equivalent noise (either current or voltage) at the input, the noise sources present in the configurations from Figure 3 must be moved from their places to the input of the amplifier, and the rest of the amplifier becomes noiseless. This is accomplished by resorting to some simple transformations.

[1]Despite that the two-port system is a nullor, the two noisy sources are considered for the analysis because they appear later as a result of the nullor synthesis.

3 Source transformations

Several source movements or transformations are used in order to obtain the total noise equivalent at the input.

- *V-Shift Transformation*

  The V-shift on a voltage source u_n is accomplished by moving the source through the node 2 and creating two new instances the sources as shown in Figure 4. After the transformation is performed, the nodes 1 and 2 are clustered.

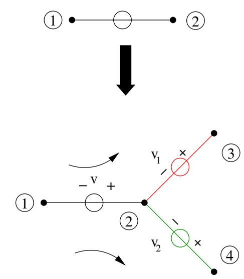

Figure 4: V-Shift Transformation.

- *I-Shift. Transformation*

  The I-shift transformation on a current source i is obtained by splitting it into two ocurrence of i_n, as shown if Figure 5.

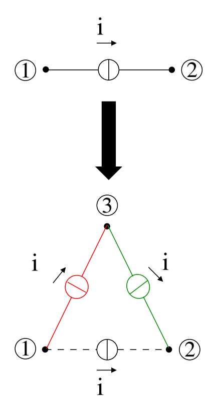

Figure 5: I-Shift Transformation.

- *Norton-Thevenin Transformation*

  The theorems of Thevenin and Norton are used to transform voltage sources into current sources and viceversa.

- *Two-port shift Transformation*

  Figure 6 illustrates the shift of sources from the output to the input of a two-port network characterised by its $\boldsymbol{K}$ matrix.

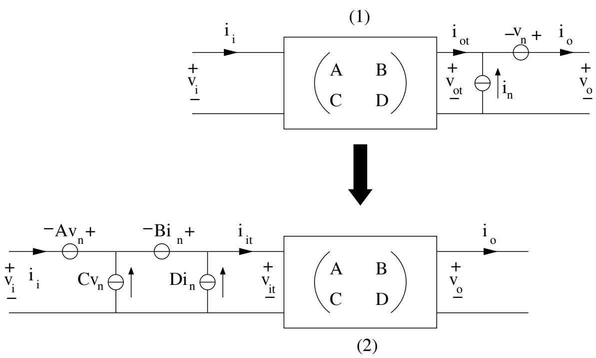

Figure 6: Two-Port Shift.

The transformations above, when applied on the configurations in Figure 3, generate the following expressions for equivalent noise at the input:

Voltage amplifier

$$u_{n,eq,in} = u_{n_{source}} + u_{n_{nullor}} + (R_s + \frac{R_1 R_2}{R_1+R_2}) i_{n_{nullor}} + \frac{\frac{R_1 R_2}{R_1+R_2}}{R_1} u_{n_{R_1}} + \frac{\frac{R_1 R_2}{R_1+R_2}}{R_2} u_{n_{R_2}} \tag{6}$$

Transconductance amplifier

$$u_{n,eq,in} = u_{n_{source}} + u_{n_{nullor}} + (R_s + R) i_{n_{nullor}} + R i_{n_R} \tag{7}$$

Transimpedance amplifier

$$i_{n,eq,in} = i_{n_{source}} + i_{n_{nullor}} + (\frac{1}{R_s} + \frac{1}{R}) u_{n_{nullor}} + \frac{1}{R} u_{n_R} \tag{8}$$

Current amplifier

$$i_{n,eq,in} = i_{n_{source}} + i_{n_{nullor}} + (\frac{1}{R_s} + \frac{1}{R_1+R_2}) u_{n_{nullor}} + \frac{R2}{R_1+R_2} i_{n_{R_2}} + \frac{R_1}{R_1+R_2} i_{n_{R_1}} \tag{9}$$

4 Hierarchy of the expressions

The expressions (6–9) for the equivalent input noise for all amplifiers show a very uniform format.

The variable in which the noise equivalent is expressed (the left hand side of the expressions), depends on the type of input variable of the amplifier. For instance, for the case of a transconductance amplifier ($i_{out} = g_m u_{in}$) the input variable is the voltage (u_{in}), and so is the variable for noise equivalent ($u_{n,eq,in}$).

As a result, the terms of the right hand side of the expressions must also have the same units of the input noise equivalent variable. Several noise contributions may be identified on these terms:

- The noise from the source.

  This contribution is of the same type of the input variable of the amplifier. This is a data usually defined by the specs of the source resistor, and therefore it is a figure that cannot be modified. It is denoted either as $u_{n_{source}}$ or as $i_{n_{source}}$.

- The nullor noise of the proper-variable.

  This is the noise of the synthesised nullor expressed in the same type of noise source of the input variable. It is either $u_{n_{nullor}}$ or $i_{n_{nullor}}$. $u_{n_{nullor}}$ is connected in series and $i_{n_{nullor}}$ in parallel.

- The nullor noise of the disjoint-variable.

  The noise contribution of the synthesised nullor expressed as an Ohm's Law relationship:

$$u = (R_s + R_{eq}) i_{n_{nullor}}$$

or:

$$i = (G_s + G_{eq})u_{n_{nullor}}$$

with R_s the source resistance, and $G_s = 1/R_s$. Besides $G_{eq} = 1/R_{eq}$. The expressions for R_{eq} are given as:

Amplifier	Req
Voltage	$R_1 \| \| R_2$
Transconductance	R
Transimpedance	R
Current	$R_1 + R_2$

It clearly results that this contribution depends on the value of the source resistance and the values of the linear resistors implementing the feedback network.

- The noise from the passive network.

  This is a noise contribution from the resistors that constitute the gain of the amplifier. For the voltage and current amplifiers, the gain involves two resistors (R_1 and R_2), and thus two terms in the sum arise. For the other amplifiers, the gain is determined by a single resistor, therefore only one term appears in the sum.

5 Acknowledgments

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6 Conclusions

The expressions of the equivalent input noise of the single-loop configurations of nullor-based amplifiers have been obtained. These are obtained by using simple transformations on the noise sources present in the circuit. The hierarchy of the expressions has been highlighted. One important noise contribution comes from the source resistance. The active network (the synthesised nullor) contributes with noise in both forms, voltage and current. As expected, the noise of the active network is closely related to the resistors of the configuration. Finally, the feedback network adds noise to the expression. The uniform structure of the equivalent input noise expressions allows the designer to focuss the attention on the various sources of noise present in the nullor-based configurations.

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