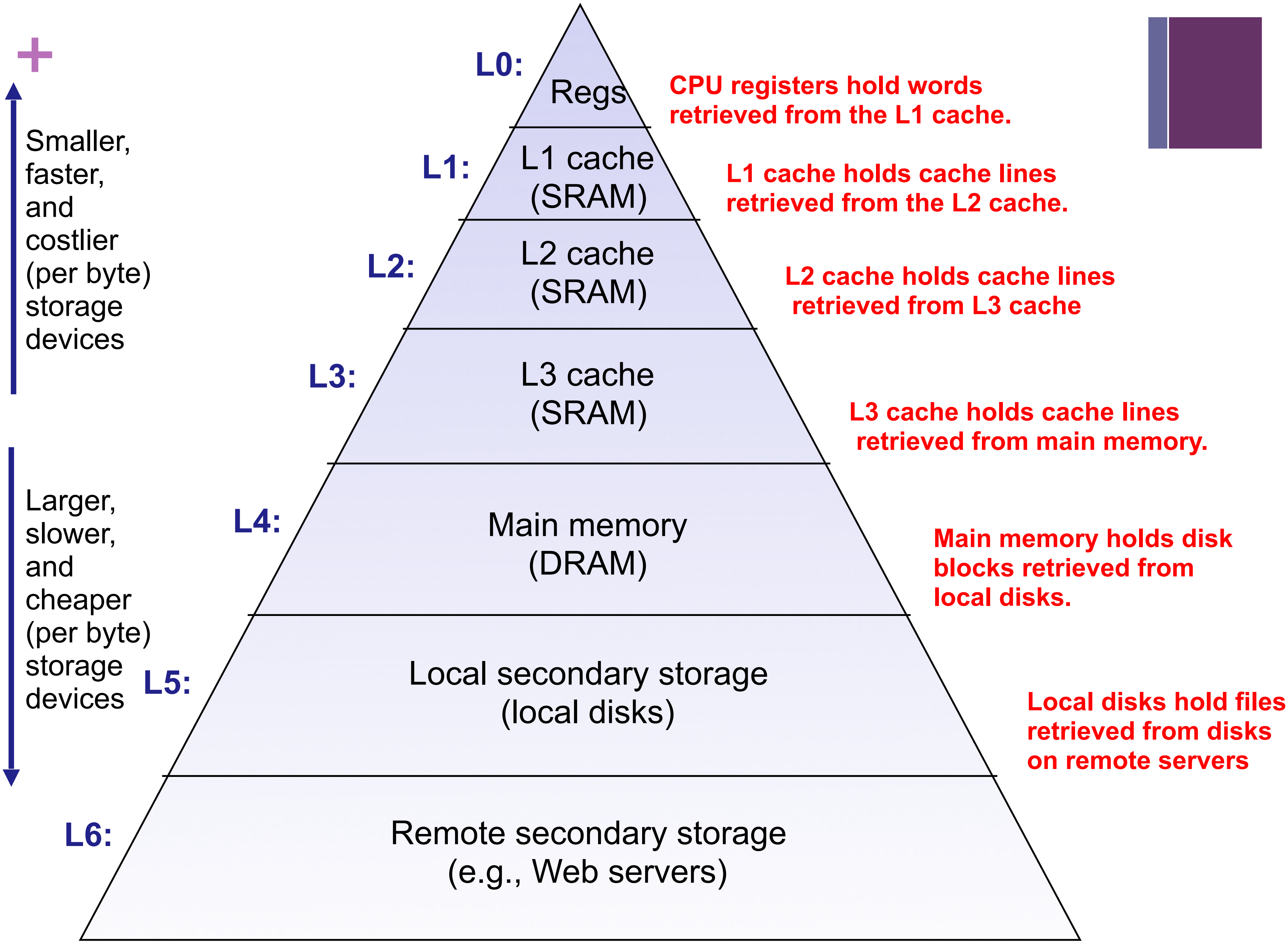
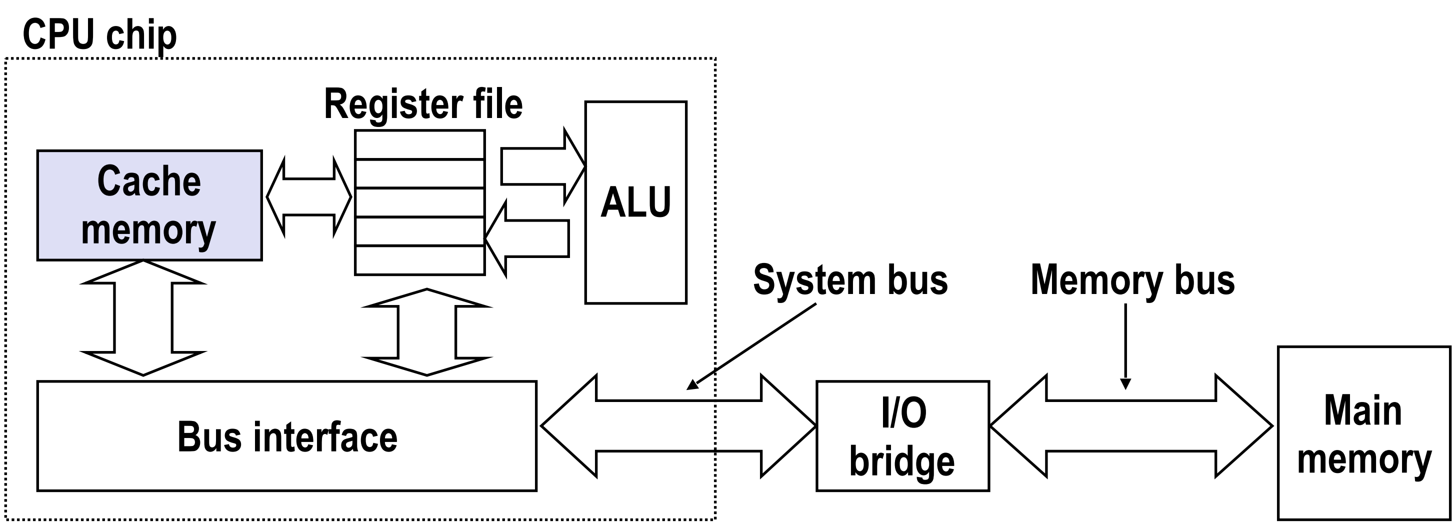
Cache Memories



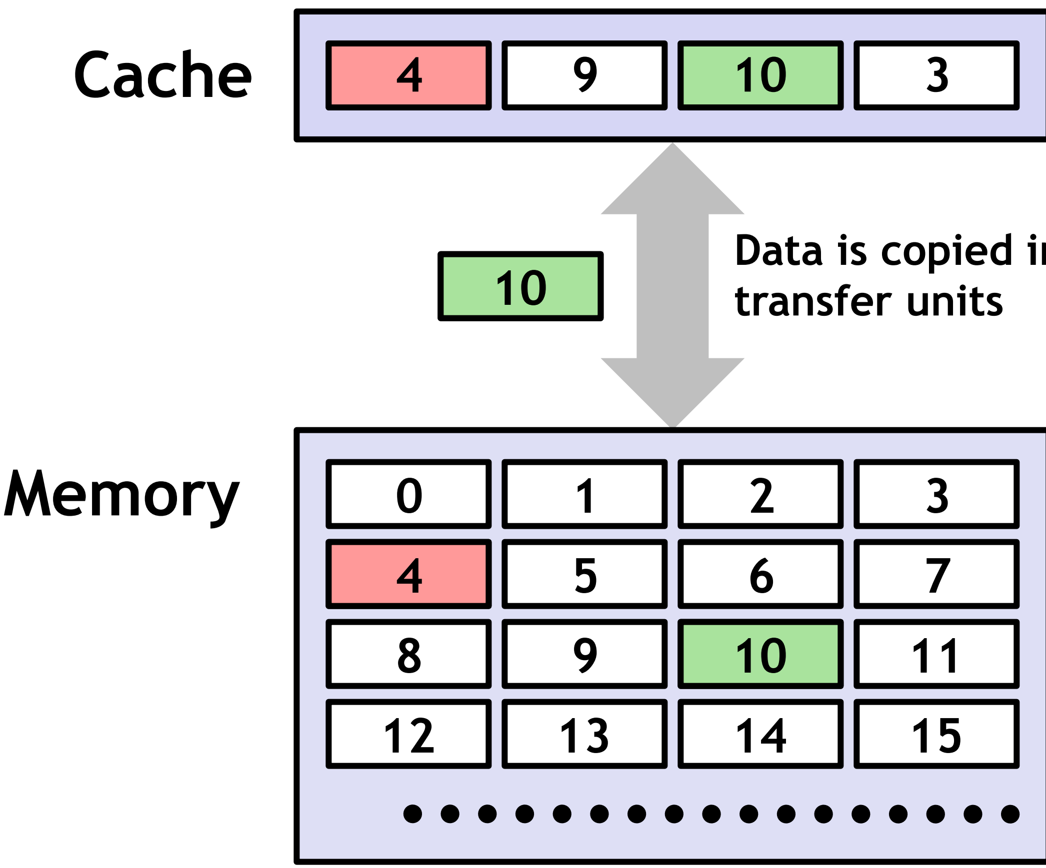
L0:
Regs
CPU registers hold words retrieved from the L1 cache.
L1:
L1 cache (SRAM)
L1 cache holds cache lines retrieved from the L2 cache.
L2:
L2 cache (SRAM)
L2 cache holds cache lines retrieved from L3 cache
L3:
L3 cache (SRAM)
L3 cache holds cache lines retrieved from main memory.
L4:
Main memory (DRAM)
Main memory holds disk blocks retrieved from local disks.
L5:
Local secondary storage (local disks)
Local disks hold files retrieved from disks on remote servers
L6:
Remote secondary storage (e.g., Web servers)
Smaller, faster, and costlier (per byte) storage devices
Larger, slower, and cheaper (per byte) storage devices

+ Cache Memories

- **Cache memories are small, fast SRAM-based memories managed automatically in hardware**
  - Hold frequently accessed blocks of main memory
- **CPU looks first for data in cache**
- **Typical system structure:**



+ General Cache Concept

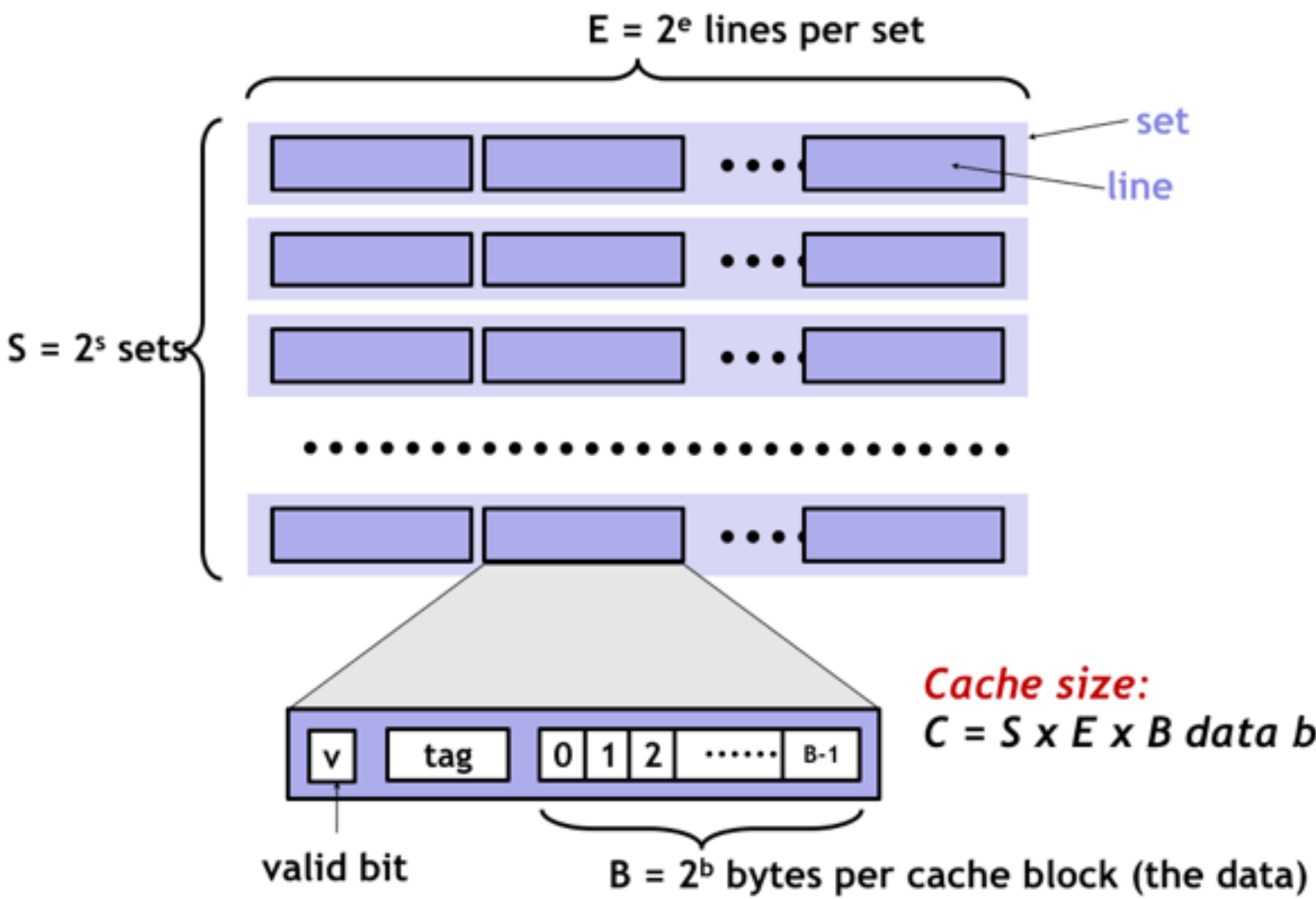


Smaller, faster, more expensive memory caches a subset of the blocks

Data is copied in block-sized transfer units

Larger, slower, cheaper memory viewed as partitioned into “blocks”

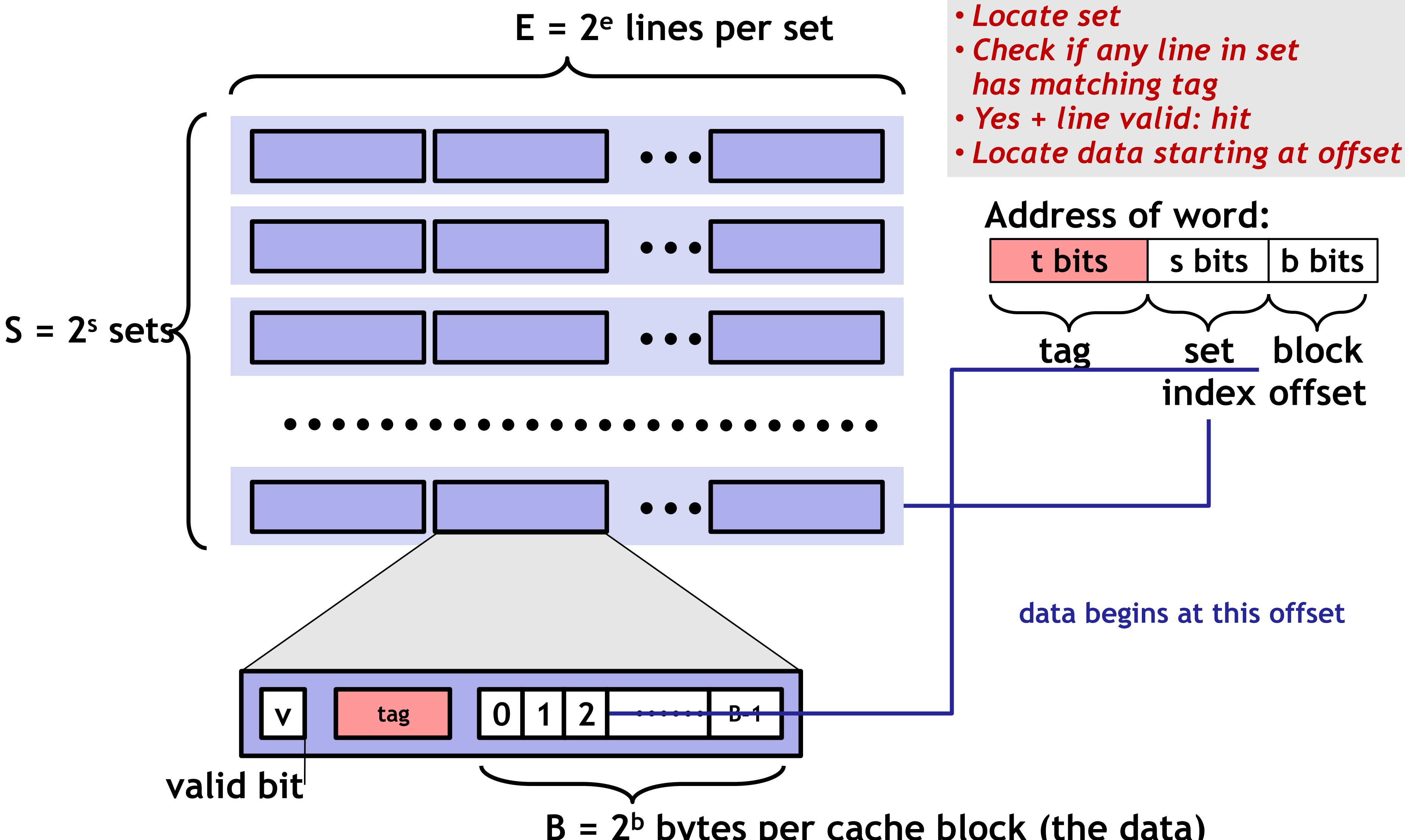
General Cache Organization



Cache size:

C = S x E x B data bytes

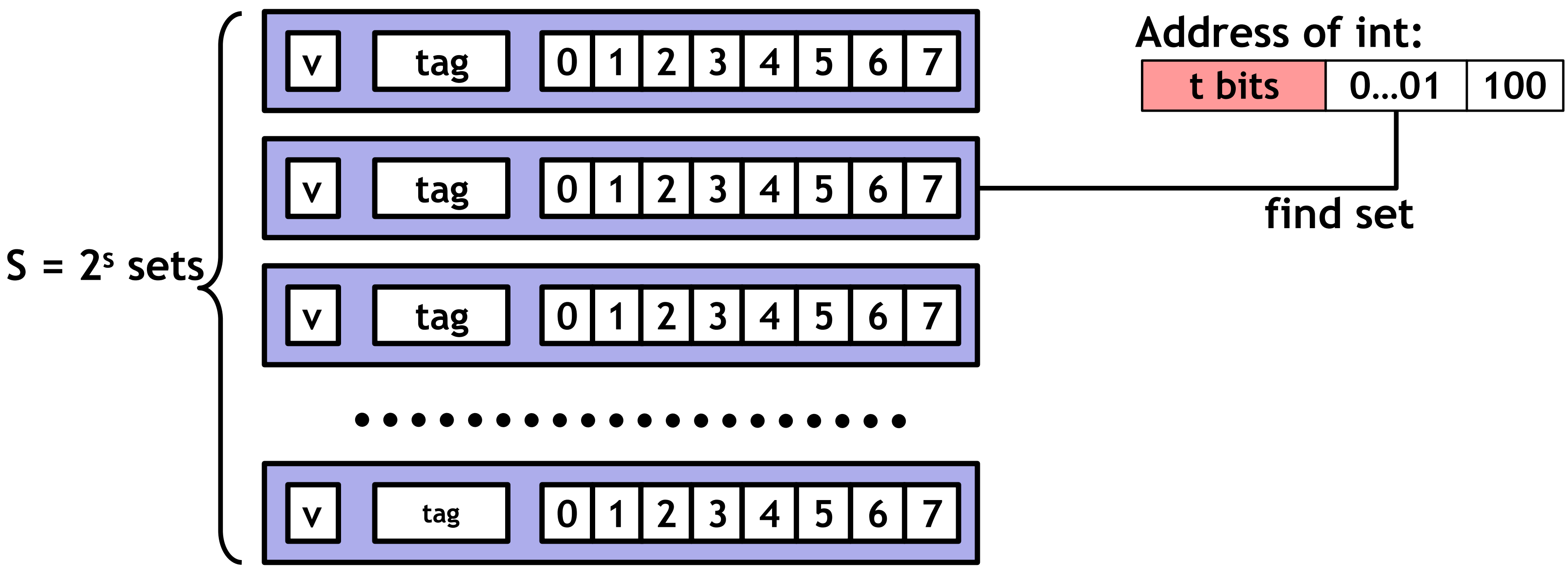
Cache Read



- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*

+ Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size 8 bytes



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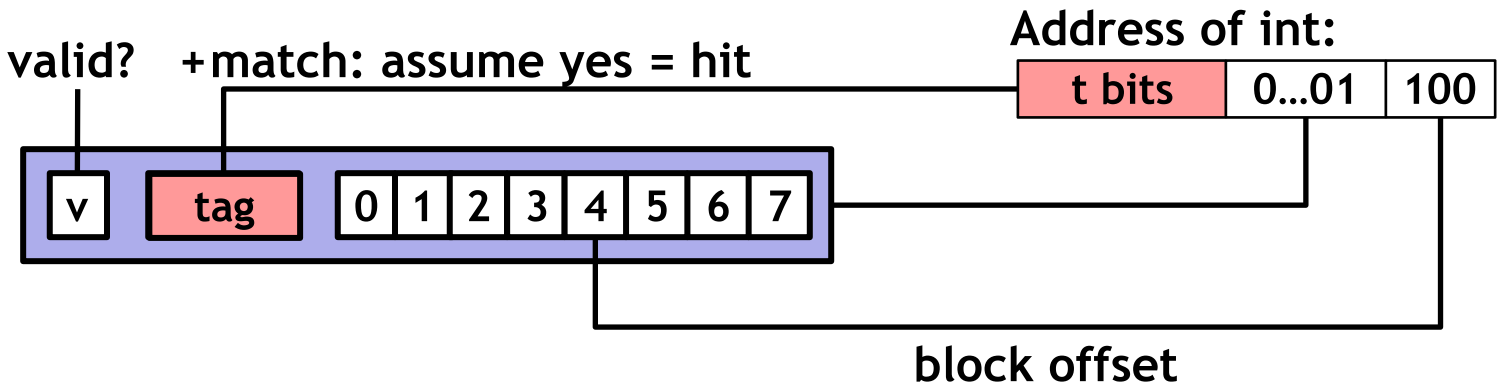

Direct Mapped Cache (E = 1)

Direct mapped: One line per set
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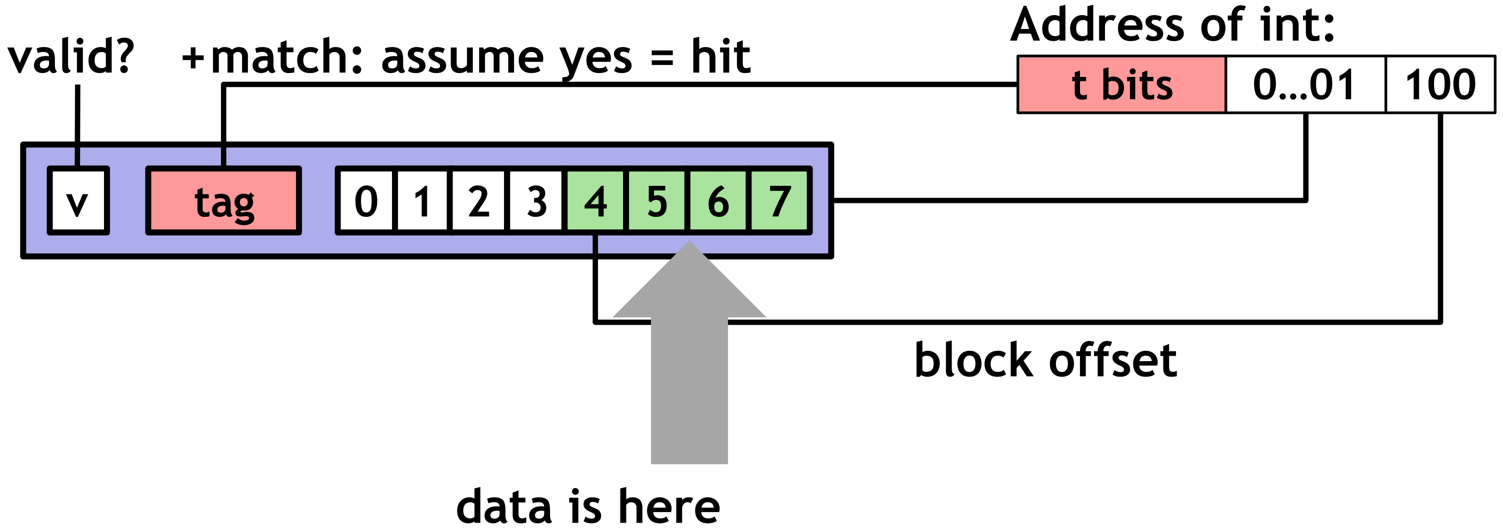


If tag doesn't match: old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 bytes (4-bit addresses), B=2 bytes/block, S=4 sets, E=1 line/set

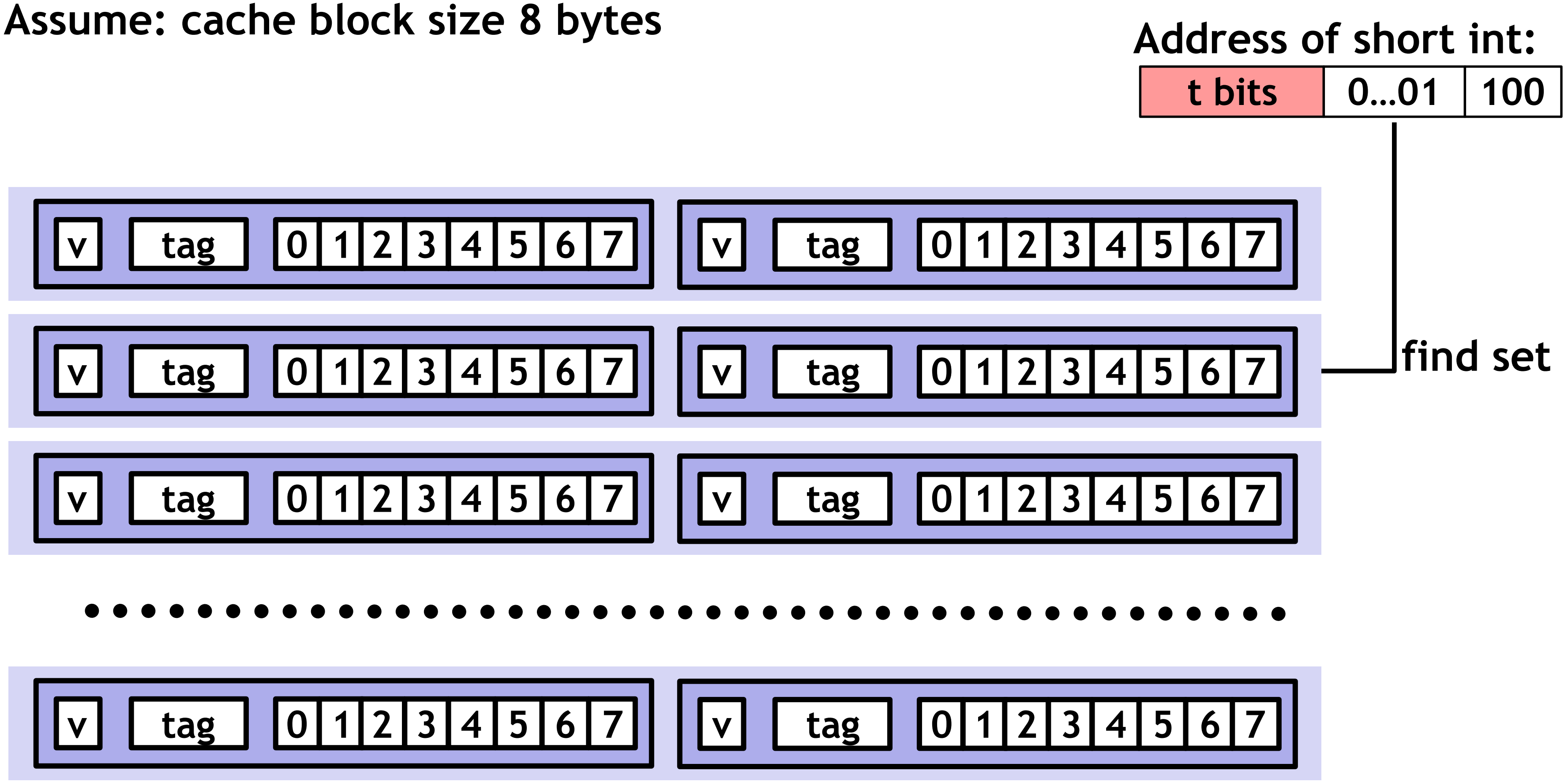
Address trace (reads, one byte per read):

0	**[$0\underline{00}0_2$],**	miss
1	**[$0\underline{00}1_2$],**	hit
7	**[$0\underline{11}1_2$],**	miss
8	**[$1\underline{00}0_2$],**	miss
0	**[$0\underline{00}0_2$]**	miss

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

E-way Set Associative Cache (E = 2)

E = 2: Two lines per set
Assume: cache block size 8 bytes



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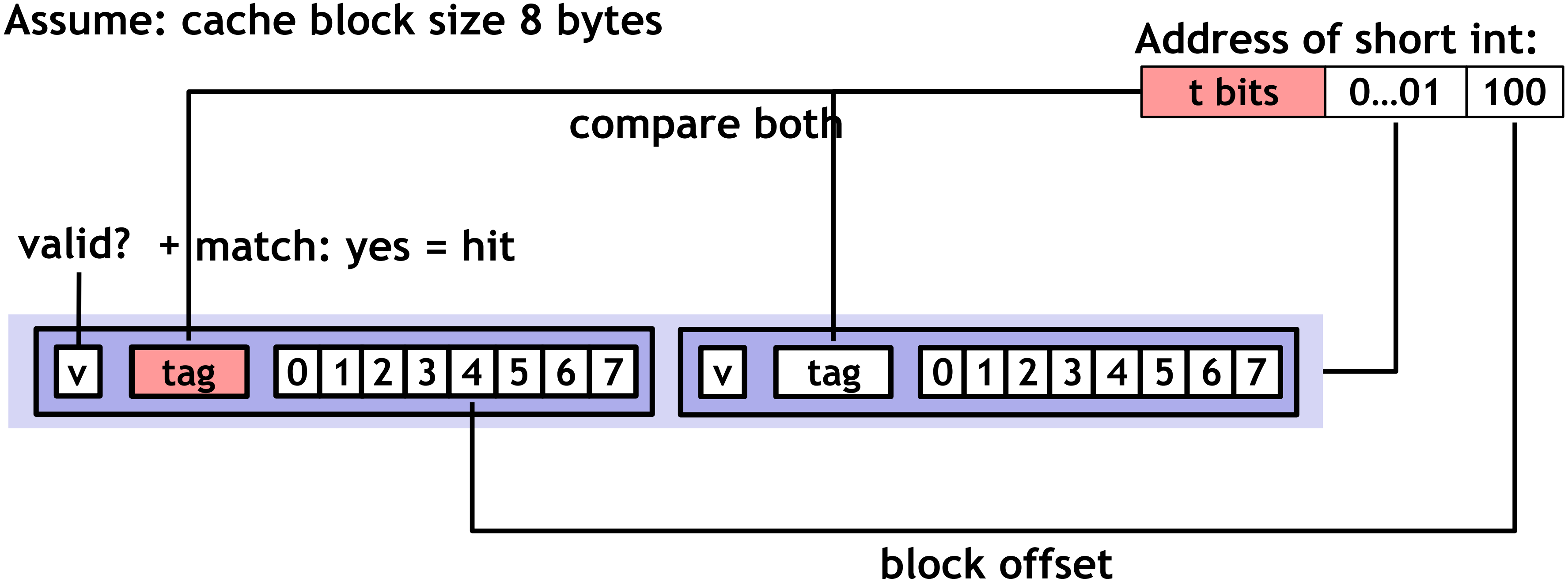
E = 2: Two lines per set
Assume: cache block size 8 bytes



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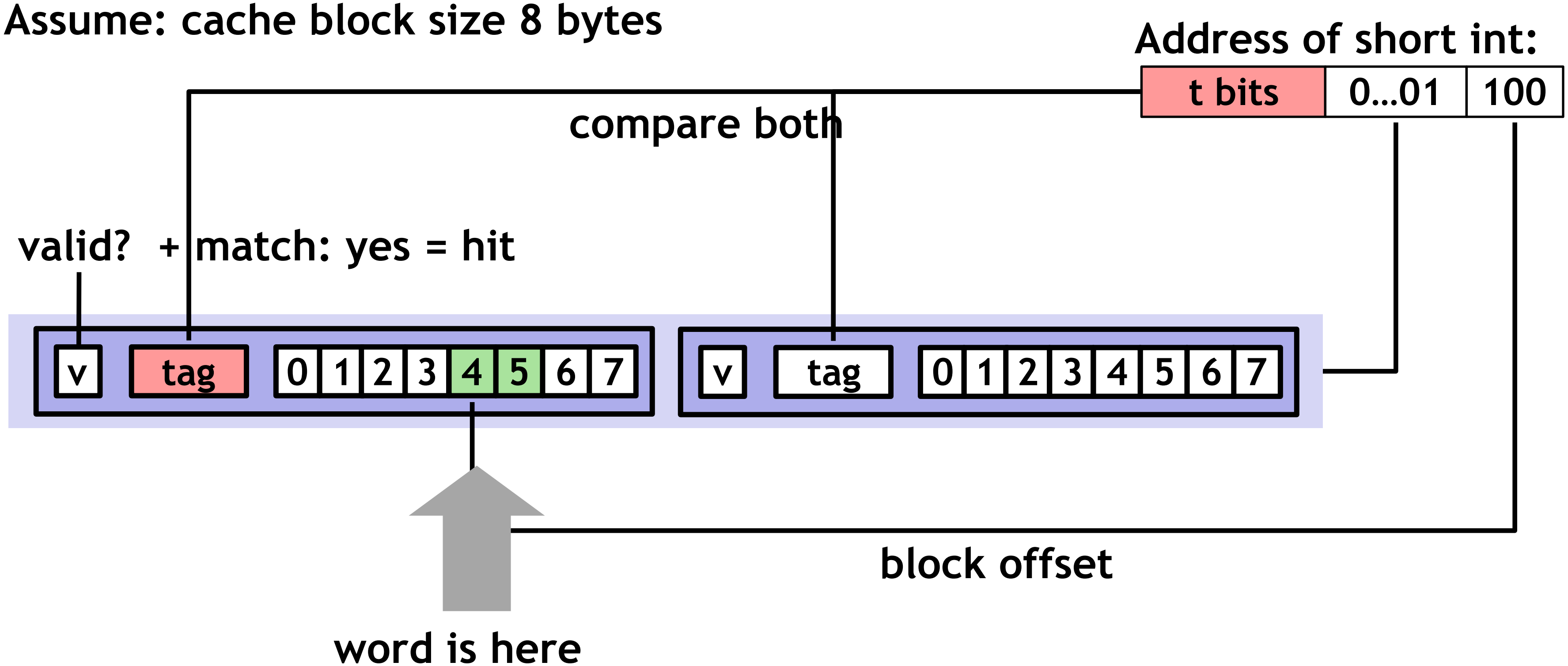
Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

+ 2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

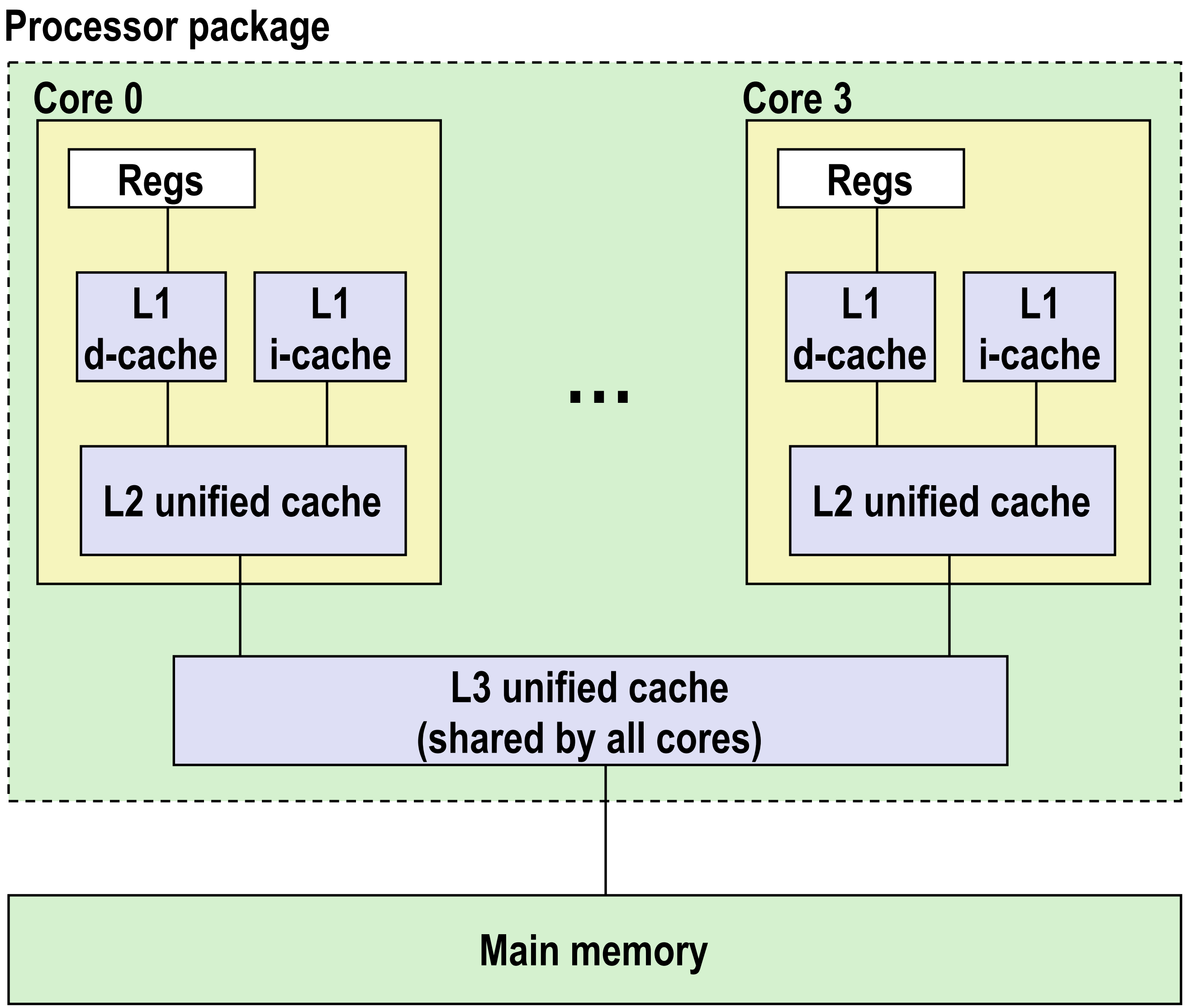
0	**$[00\underline{0}0_2]$,**	miss
1	**$[00\underline{0}1_2]$,**	hit
7	**$[01\underline{1}1_2]$,**	miss
8	**$[10\underline{0}0_2]$,**	miss
0	**$[00\underline{0}0_2]$**	hit

	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

+ What about writes?

- **Multiple copies of data exist:**
  - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
  - ***Write-through*** (write immediately to memory)
  - ***Write-back*** (defer write to memory until replacement of line)
    - Need a dirty bit (line different from memory or not)
- **What to do on a write-miss?**
  - ***Write-allocate*** (load into cache, update line in cache)
    - Good if more writes to the location follow
  - ***No-write-allocate*** (writes straight to memory, does not load into cache)
- **Typical**
  - Write-back + Write-allocate

Intel Core i7 Cache Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

+ Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache (misses / accesses) = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - < 1% for L2, depending on size, etc.
- **Hit Time**
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 4 clock cycle for L1
    - 10 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory

+ Let's think about those numbers

- **Huge difference between a hit and a miss**
  - Could be 100x, if just L1 and main memory

- **Would you believe 99% hits is twice as good as 97%?**
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles

  - Average access time:

  97% hits: 1 cycle + 0.03 * 100 cycles = **4 cycles**

  99% hits: 1 cycle + 0.01 * 100 cycles = **2 cycles**

- **This is why "miss rate" is used instead of "hit rate"**

+ Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions

- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Matrix Multiplication Example

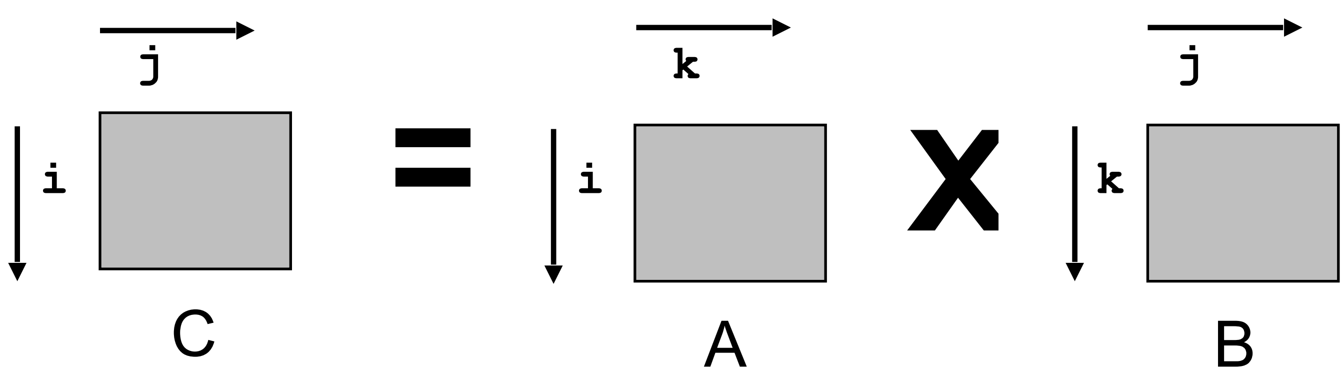
- **Description:**
  - Multiply N x N matrices
  - Matrix elements are doubles (8 bytes)
  - $O(N^3)$ total operations
  - N reads per source element
  - N values summed per destination

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Variable *sum* held in register

+ Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Block size = 32B (big enough for 4 doubles)
  - Matrix dimension (N) is "very large"
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



+ Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations
- **Stepping through columns in one row:**
  - for (i = 0; i < N; i++)

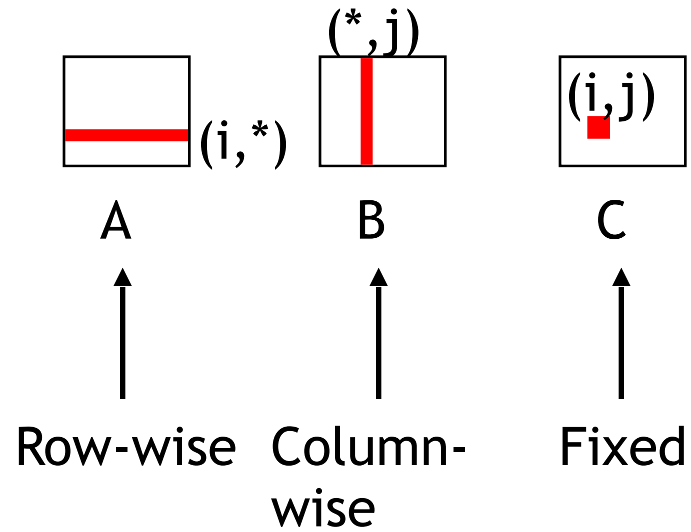
    sum += a[0][i];
  - accesses successive, contiguous elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- **Stepping through rows in one column:**
  - for (i = 0; i < n; i++)

    sum += a[i][0];
  - accesses distant elements (stride-rowsize pattern)
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

+ Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

Inner loop:



double[2][2]

```
sum += A[0][0] * B[0][0]
sum += A[0][1] * B[1][0]
sum += A[0][0] * B[0][1]
sum += A[0][1] * B[1][1]
sum += A[1][0] * B[0][0]
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sum += A[1][0] * B[0][1]
sum += A[1][1] * B[1][1]
```

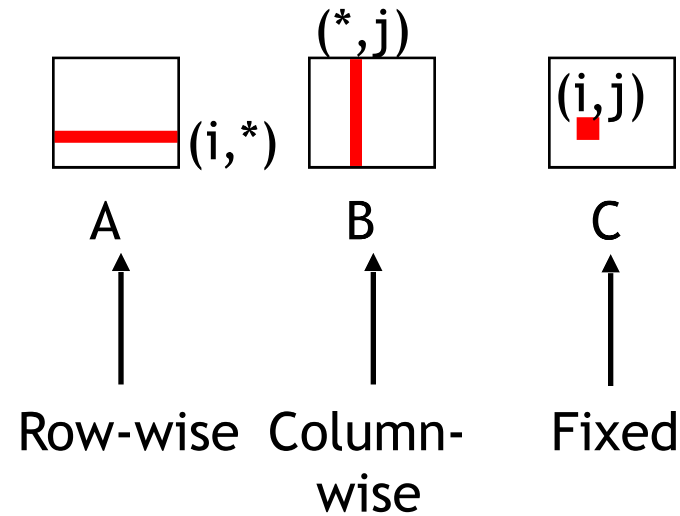
<u>Misses per inner loop iteration:</u>

A	B	C
0.25	1.0	0.0

+ Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:



Misses per inner loop iteration:

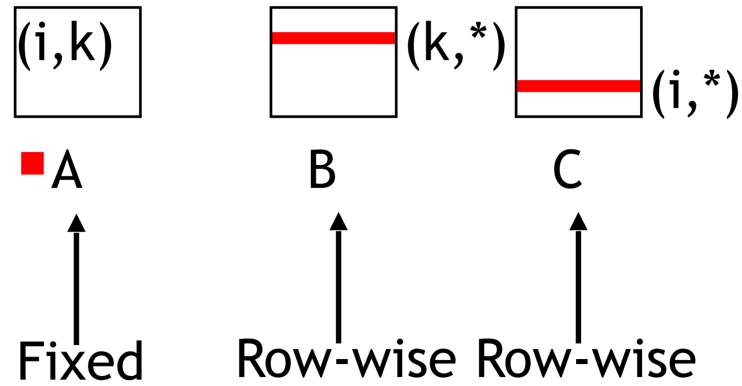
A	B	C
0.25	1.0	0.0

Effectively same as ijk

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



<u>Misses per inner loop iteration:</u>

<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

double[2][2]

```
C[0][0] += r * B[0][0]
C[0][1] += r * B[0][1]
C[1][0] += r * B[0][0]
C[1][1] += r * B[0][1]
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```

+ Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
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    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```
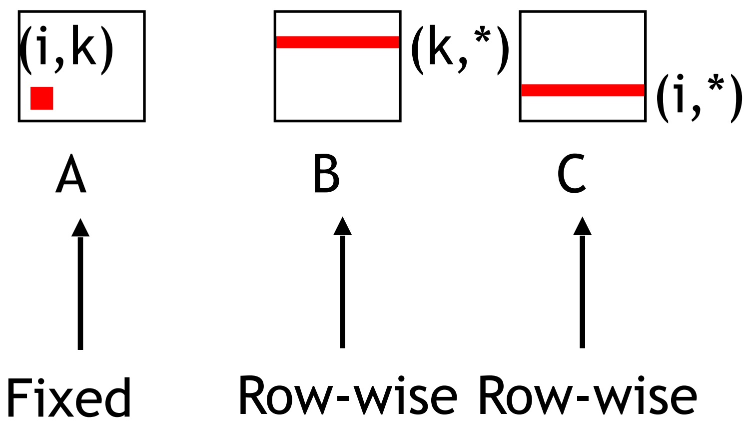

Inner loop:



<u>Misses per inner loop iteration:</u>

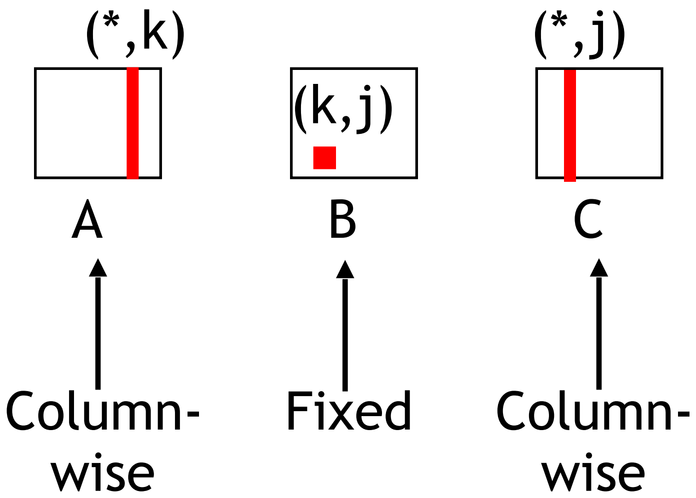
<u>A</u>	<u>B</u>	<u>C</u>
0.0	0.25	0.25

Effectively same as kij

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

double[2][2]

```
C[0][0] += A[0][0] * r
C[1][0] += A[1][0] * r
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  }
}
```
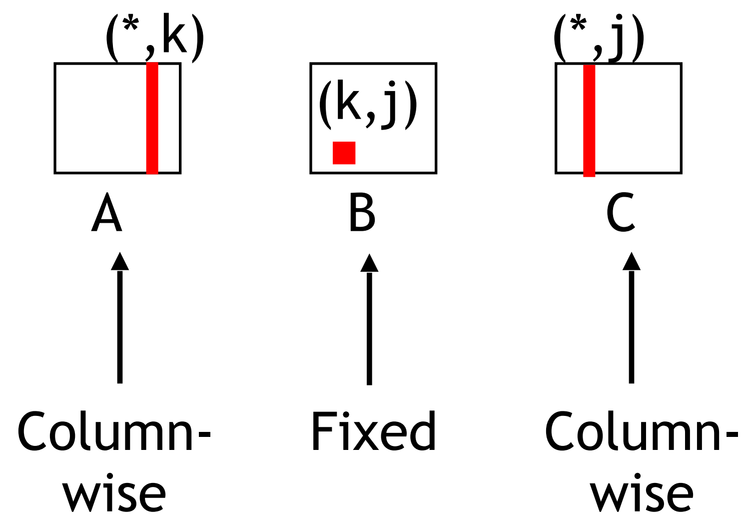

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Effectively same as jki

+ Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

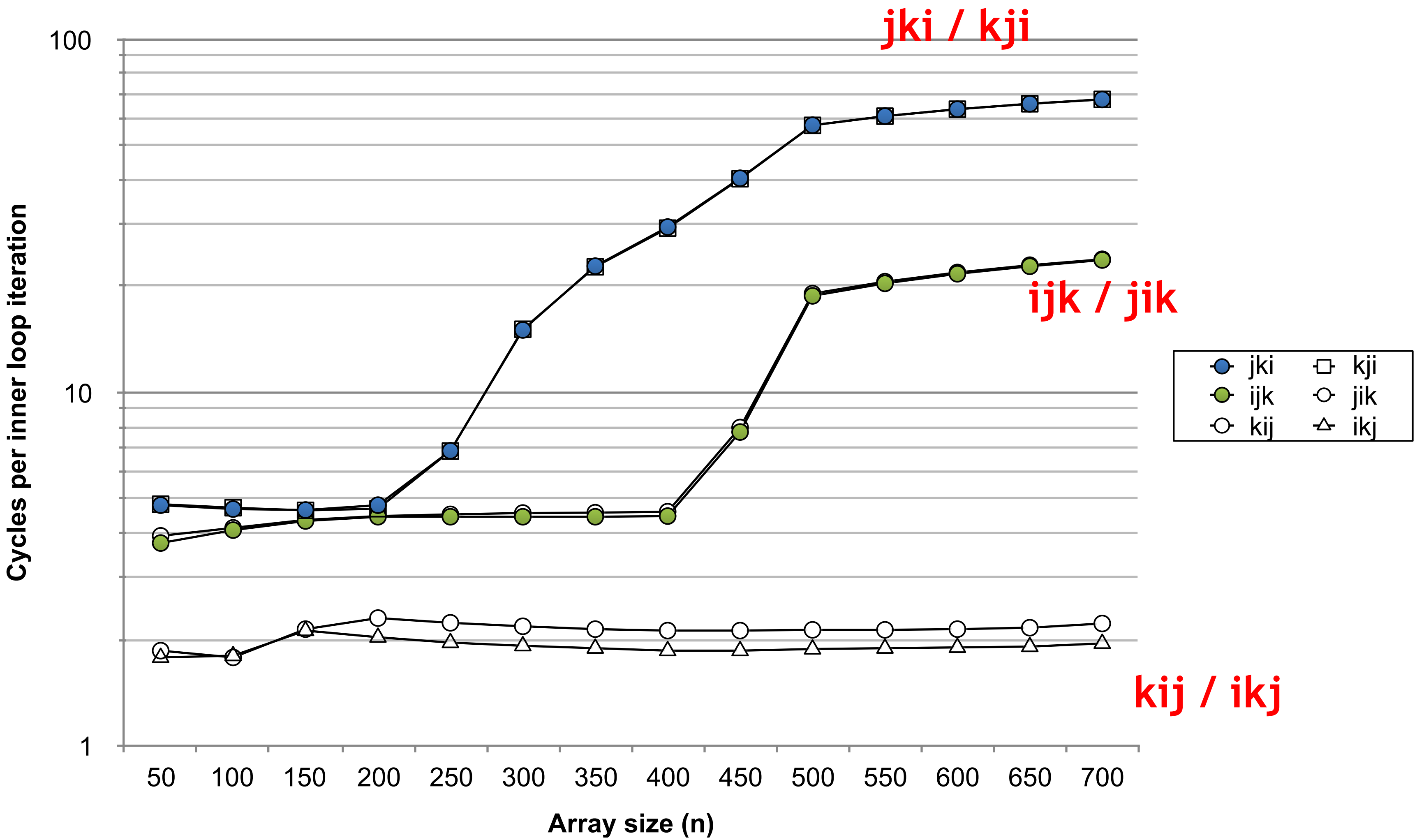
```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

+ Core i7 Matrix Multiply Performance



Cache Summary

- **Cache memories can have significant performance impact**

- **You can write your programs to exploit this!**
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.