ORF467-HW3B

September 18, 2020

[236]:
```
import numpy as np
import pandas as pd
from matplotlib import pyplot as plt

df=pd.read_csv('distance_matrix.csv', sep=',',header=None)
df1 = pd.read_csv('flows.csv', sep=',',header=None).fillna(0)
header = df1.iloc[0]
df1 = df1.iloc[1:79]
df1.columns = header
df
```

[236]:

	0	1	2	3	4	5	\
0	1.800000	0.666667	1.498147	8.766472	10.063652	11.911945	
1	0.666667	1.039230	1.800000	9.346715	10.458548	12.300000	
2	1.498147	1.800000	0.600000	7.747259	8.658840	10.500000	
3	8.766472	9.346715	7.747259	9.429740	4.212390	5.447986	
4	10.063652	10.458548	8.658840	4.212390	1.587451	1.850813	
..	…	…	…	…	…	…	
73	17.621325	18.283326	17.447063	11.922093	15.679051	16.101242	
74	17.189379	17.757824	17.662540	15.289167	19.499636	20.528707	
75	17.796192	18.439631	17.844327	13.297190	17.298260	17.932373	
76	18.800030	19.446594	18.816216	14.029774	17.957325	18.506756	
77	19.235581	19.890277	19.164371	13.954029	17.758420	18.194728	

	6	7	8	9	…	68	69	\
0	13.204208	14.455084	16.408670	2.001111	…	16.921518	13.572562	
1	13.601471	14.850758	16.800000	2.473863	…	17.588127	14.100000	
2	11.801695	13.050862	15.000000	0.848528	…	16.551200	14.214429	
3	6.304342	7.378534	9.174853	6.930209	…	10.281219	13.274989	
4	3.142987	4.392909	6.345173	8.068533	…	13.764676	17.417145	
..	…	…	…	…	…	…	…	
73	16.208640	16.638660	17.532826	16.651727	…	2.415689	9.355747	
74	21.054579	21.817718	23.148225	17.071769	…	9.645856	4.008468	
75	18.179659	18.721779	19.759049	17.102631	…	4.972145	7.523962	
76	18.689302	19.173549	20.125854	18.064606	…	5.054921	8.497058	
77	18.297626	18.713515	19.570977	18.390300	…	4.378966	9.542700	

```
           70         71         72         73         74         75  \
0   15.609575  17.148294  16.942806  17.621325  17.189379  17.796192
1   16.180433  17.764277  17.590381  18.283326  17.757824  18.439631
2   16.080000  17.391795  16.956619  17.447063  17.662540  17.844327
3   13.899694  13.853433  12.339766  11.922093  15.289167  13.297190
4   18.111564  18.004544  16.347703  15.679051  19.499636  17.298260
..        …          …          …          …          …
73   7.405295   4.861022   2.244921   1.039230   7.594426   2.641969
74   1.582944   2.743657   5.363342   7.594426   2.400000   5.093654
75   5.201000   2.363690   0.958046   2.641969   5.093654   1.200000
76   6.078849   3.147147   1.860263   2.640076   5.767392   1.063015
77   7.196077   4.290924   2.474061   2.093591   6.940033   2.019437

           76         77
0   18.800030  19.235581
1   19.446594  19.890277
2   18.816216  19.164371
3   14.029774  13.954029
4   17.957325  17.758420
..        …          …
73   2.640076   2.093591
74   5.767392   6.940033
75   1.063015   2.019437
76   1.469694   1.176063
77   1.176063   1.697056

[78 rows x 78 columns]
```

[237]:
```
#initial populaiton of where people live and how many are in each zone
P = vec.iloc[:,0]
P = pd.to_numeric(P)
P = pd.Series(P).to_numpy()
```

[238]:
```
#Where people work and how many per zone
A = vec.iloc[:,1]
A = pd.to_numeric(A)
A = pd.Series(A).to_numpy()
```

[239]:
```
#np.size(A)
print(np.sum(P))
print(np.sum(A))
```

```
375072
375073
```

[240]:
```
#distance matrix? with diagonals representing the average distance traveled to␣
 ↪get to zone center
```

```
D = DM
```

[241]:
```
# Gravity model - given A, we want A^* to be used so sum_i T_ij = A_j
# A_j is the total trip attraction at zone j
#P_i = total trip production at zone i
#T_ij is the trips produced at I and attracted to j
#F_ij is the calibration term for interchange ij also known as the friction␣
 ↪factor or tracel time factor

dims = A.shape[0]
A = A.reshape(dims, 1)
P = P.reshape(dims, 1)

def calculate_T(A, P, D):
    # invert and square for disutility
    F = np.power(D,-2)
    T = np.zeros_like(D)

    for i in range(T.shape[0]):
        for j in range(T.shape[1]):
            denom = F[i,:] @ A
            T[i,j] = P[i]*F[i,j]*A[j]/denom

    return T

def calculate_A_star(A, P, D):
    A_old = A.copy()
    C = np.ones_like(A)
    eps = 1e1
    n = A.shape[0]

    while (np.linalg.norm(C-A) > eps):
        T = calculate_T(A_old, P, D)
        C = T.sum(axis=0).reshape(n,1)

        for j in range(n):
            if C[j] != 0:
                A_old[j] = A[j]*A_old[j] / C[j]

    return A_old

A_star = calculate_A_star(A, P, D)
T = calculate_T(A_star, P, D)

print(T.sum(axis=0)) # should equal A
print(T.sum(axis=1)) # should equal P
```

```
[    0.             0.             0.         20000.83072619
     0.          3691.87559059     0.             0.
     0.             0.             0.             0.
     0.          5537.35652514     0.             0.
     0.             0.             0.             0.
     0.          5537.15781195  7384.96046057     0.
     0.             0.             0.          5537.59221025
     0.             0.             0.         59594.41434532
     0.             0.             0.             0.
     0.             0.             0.             0.
     0.             0.         59599.88952283     0.
     0.         52141.53494795     0.             0.
     0.             0.             0.             0.
     0.             0.          5538.32259401     0.
 52145.43651462  3691.73878001     0.             0.
     0.          9230.4393026      0.             0.
     0.             0.             0.          9230.5347726
     0.             0.          9229.98264313     0.
     0.             0.             0.          7384.81068396
     0.         59595.12256829]
[16758.     0.     0.     0. 18035.     0.     0.     0.     0.     0.
     0.     0. 21758.     0.     0. 14897.     0.     0.     0.     0.
     0.     0.     0.     0. 18621.     0.     0.     0.     0. 11172.
     0.     0. 14311.     0.     0.     0.     0. 36656.     0.     0.
     0.     0.     0.     0.     0.     0. 72622.     0.     0.     0.
     0.     0.     0. 16172.     0.     0.     0.     0.     0.     0.
     0.     0.     0. 57725.     0.     0. 16758.     0.     0.     0.
     0. 59587.     0.     0.     0.     0.     0.     0.]
```

[242]:
```
print(T)
print(np.shape(T))
np.sum(T)
```

```
[[  0.           0.           0.         …  22.92451799   0.
  350.36323139]
 [  0.           0.           0.         …   0.           0.
    0.        ]
 [  0.           0.           0.         …   0.           0.
    0.        ]
 …
 [  0.           0.           0.         …   0.           0.
    0.        ]
 [  0.           0.           0.         …   0.           0.
    0.        ]
 [  0.           0.           0.         …   0.           0.
    0.        ]]
(78, 78)
```

[242]: 375072.0

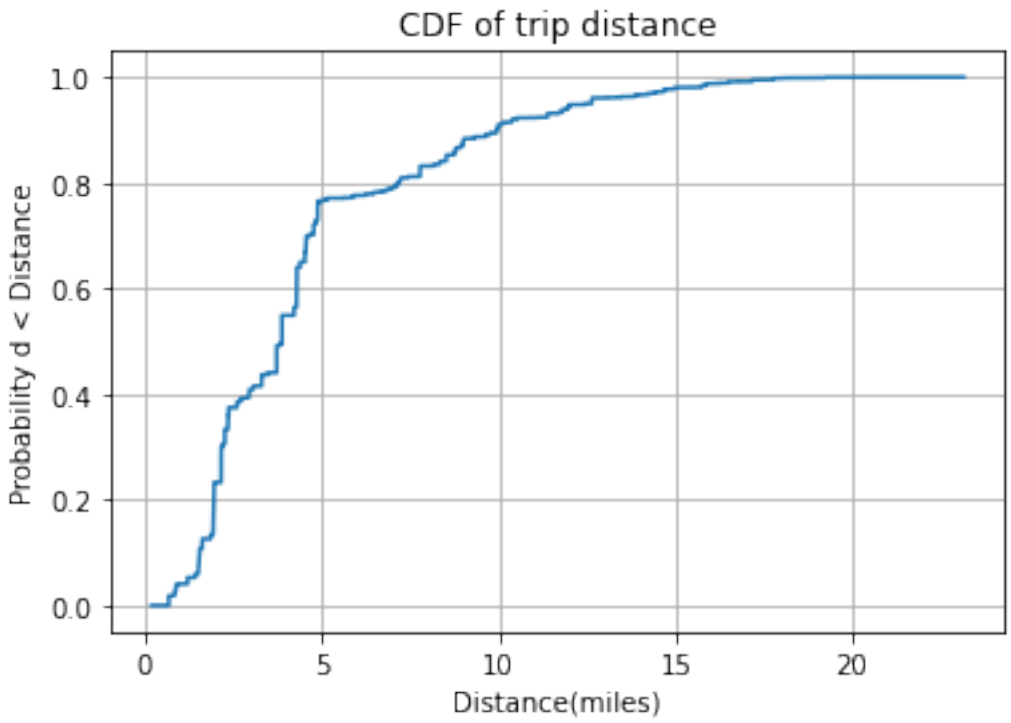
[243]:
```
N = np.unravel_index(np.argsort(D.ravel()), np.shape(D))
print(N)
```

```
(array([64, 62, 16, …, 11, 74,  8]), array([62, 64, 17, …, 74,  8, 74]))
```

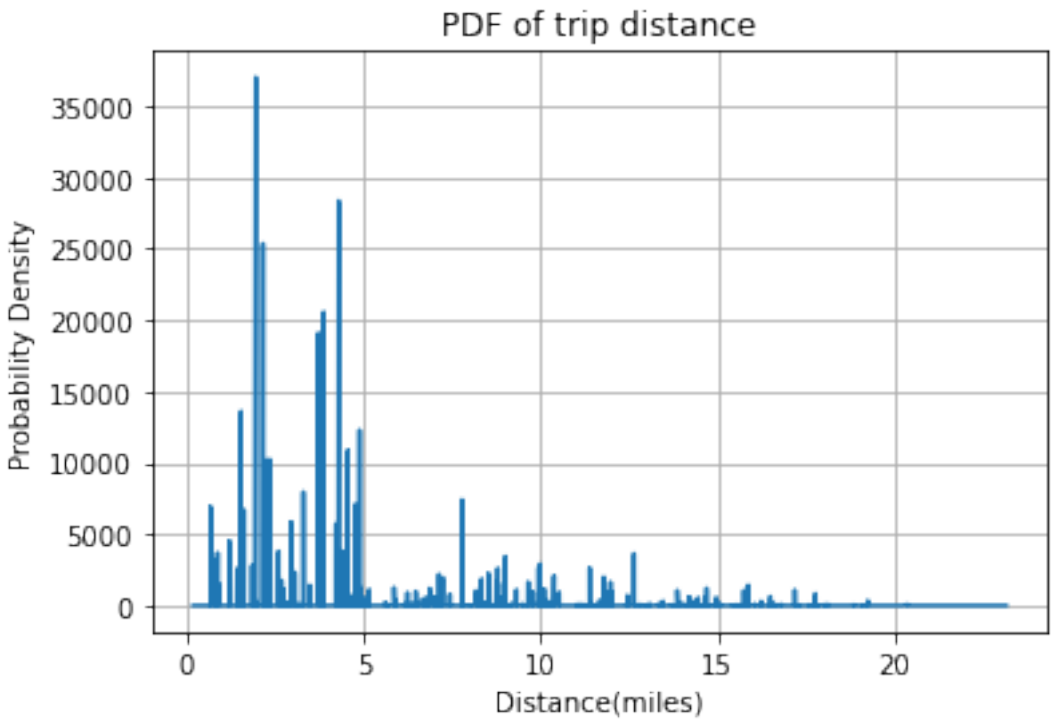
[244]:
```
y = np.zeros(np.size(T))
y_dens = np.zeros(np.size(T))
x = np.zeros(np.size(T))
for i in range(np.size(T)):
    if i == 0:
        y[i] = T[N[0][i], N[1][i]]
    else:
        y[i] = y[i-1] + T[N[0][i], N[1][i]]
        y_dens[i] = T[N[0][i], N[1][i]]
    x[i] = D[N[0][i], N[1][i]]
print(y)
print(x)
```

```
[     0.      0.      0. … 375072. 375072. 375072.]
[ 0.18294641  0.18294641  0.56568542 … 22.24613028 23.14822482
 23.14822482]
```

[245]:
```
plt.plot(x,y/max(y))
plt.xlabel('Distance(miles)')
plt.ylabel('Probability d < Distance')
plt.title("CDF of trip distance")
plt.grid()
```



[234]:
```
plt.plot(x,y_dens)
plt.xlabel('Distance(miles)')
plt.ylabel('Probability Density')
plt.title("PDF of trip distance")
plt.grid()
```


PDF of trip distance
Probability Density
35000
30000
25000
20000
15000
10000
5000
0
0
5
10
15
20
Distance(miles)

[ ]: