A Comparative Study of Target Motion Analysis (TMA) Techniques for Target Classification and Identification

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ABSTRACT

Non Co-Operative Identification (NCI) / Automatic Target Recognition (ATR) in Air, Ground and Maritime Applications is a key issue in the compilation process of tactical situation. Classification and identification is a process having various aspects. Classification and identification techniques using acoustic or electromagnetic signatures, imaging systems or other distinguishing paramaters are employed in different systems. Some techniques about Target Motion Analysis (TMA), which can improve the classification and identification process, are presented in this paper. The classical TMA problem is estimating the target motion parameters such as position, speed, acceleration and heading of target using acoustical bearings only. Although it is commonly associated with the acoustic underwater localization problems in conjunction with submarine warfare domain, the same problem problem appears in other target tracking systems which also requires passive localization of target and estimation of target motion. Once the target motion parameters are estimated, the speed can be used to distinguish a torpedo from a submarine in an underwater acoustic environment or similarly a missile from an airborne platform in an electromagnetic environment. More importantly, the motion patterns and kinematic characteristics of the target provide valuable information for classifying a target or even identifying the target with the utilization of an intelligence database. Correct determination of target position using multisensors, as in the TMA case, is also a crucial point for identification systems, allowing to correlate the other informations from those sensors. This study is focused on TMA algorithms using time difference of arrival (TDOA) information rather than the ones using only bearing information. The TDOA methods propose a better accuracy compared to bearing only methods because time accuracy of a receiver can be better than its bearing accuracy. This study compares performances of bearing only TMA methods with TDOA based algorithms and hybrid algorithms using both TDOA and bearing data in different scenarios. Applicability of these methods is also discussed at the end.

1.0 INTRODUCTION

Active sensors have some advantages and ease the solution of target tracking problems. On the other hand, operational concept, mission and platform characteristics may impose the utilization of passive sensors in some situations. [1]

A subproblem in target tracking is the target motion analysis (TMA) problem. The classical TMA problem involves estimation of target state including position, course and speed. Some applications are passive sonar, infrared (IR) and passive radar systems.

TMA in submarine warfare domain is perhaps one of the most important and challenging application areas of this problem. Traditionally, concealment is one of the most important features of the submarine platform therefore submarines rarely make active transmission [2]. The measurements, which are the noisy bearings from the radiating acoustic target, are processed to obtain an estimate of the target state in a classical TMA problem. The non-homogeneous medium and the reflections from different sources are the other causes which makes underwater acoustic tracking problems more difficult than the other tracking problems [3].

Airborne jamming scenarios is another application area of TMA where no range information can be extracted from the incoming signal but the engaging platform needs some range information which can be obtained from a TMA solution.

Airborne ESM applications are another domain of TMA and the problem resembles the passive sonar case.

TMA techniques are also applied in the field of missile guidance. Some modern anti-radiation missiles (ARM), e.g., exploit the radar transmissions for target state estimation in order to keep a lock-on in case the radar shuts down or operates intermittently for self-protection. Some other modern missiles are equipped with passive radar and/or IR receivers and estimate the target state in order to utilize optimal guidance procedures.

From the definition, passive target localization is a subset of TMA and involves the estimation of position only when the target is stationary. Conventional TMA, however, typically involves moving targets.

The TMA problem is characterized by the type of measurements extracted from the target signal. Different types such as bearing, frequency, time and signal strength introduce different estimation problems.

A peculiarity of passive tracking is the fact that the target may not be observable from the used measurement set. Fundamental ambiguities exist if no restrictions are imposed on the target motion. It turns out that for the considered types of measurement, target modeling is a prerequisite to ambiguity resolution. It is important to understand how target modeling affects TMA performance. Given the target model, the ambiguities can be resolved by suitable observer motions, which depend on the measurement set and the target model as well.

If the observer is free to move, then a further solution step is required. The objective of this step is to find an observer motion that maximizes estimation accuracy. Useful optimality criteria for the resulting optimal control problem can be derived from the Cramer-Rao Lower Bound (CRLB).

2.0 TMA PROBLEM DEFINITION

2.1 Measurements

Passive state estimation process is based on the information extracted from the signals radiated by the target. There are various types of informations that can be extracted from the incoming signal such as the angles from the observer to the target, the Doppler-shifted emitter frequencies, time delays, received signal strength. Some of those informations may have higher quality so the estimation process should rely on them more. Different accuracies of those parameters may be caused by the nature of the problem or some technical issues such as the receiver design. [1]

A basic requirement, however, for successful estimation is that the final measurement set contains information on the full emitter state, i.e., that the noise-free measurements can be uniquely assigned to a target state.

2.2 Target Motion

The motion of a submarine can be modeled similar to a plane. It performs either a linear motion with constant speed/acceleration or a coordinated turn with constant speed/acceleration and with constant angular rate/acceleration or a combination of those.

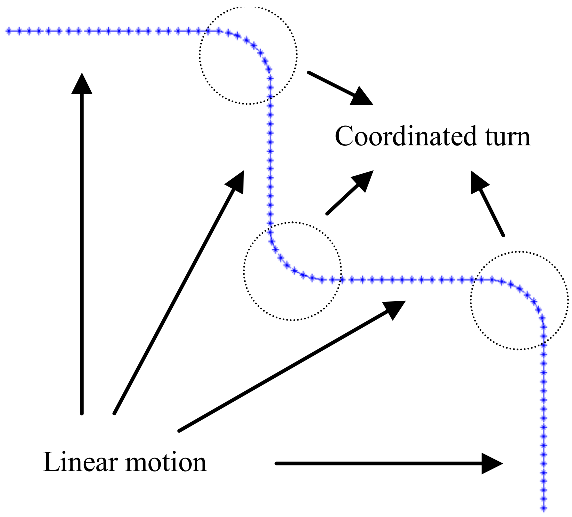


Figure 1 - Example Target Motion

In the most general sense the target motion in an appropriate window can be described as a combination of these parameters: [4]

Table 1 – Motion Models

Translational Motion	
Stationary Object Model	Initial position: $x(0)$ (meters) Velocity: $v(t)=0$ (meters/sec)
Constant Jerk Model or Continuous White Noise Acceleration (CWNA) Model or Second-Order Kinematic Model	Initial position: $x(0)$ (meters) Initial velocity: $v(0)$ (meters/sec) Acceleration: $a(t)=n(t)$ (meters/sec^2) = zero mean white Gaussian noise
White Noise Jerk Model or Continuous Wiener Process Acceleration (CWPA) Model	Initial position: $x(0)$ (meters) Initial velocity: $v(0)$ (meters/sec) Initial acceleration: $a(0)$ (meters/sec^2) Jerk: $j(t)=n(t)$ (meters/sec^3) = zero mean white Gaussian noise

Same models can be used for angular motion by replacing position x with angular position θ, velocity v with angular velocity ω, acceleration a with angular acceleration α, jerk j with angular jerk β. Also the turn radius can be considered to be time varying.

In the most general form: $x(t) = f(x(0), v(0), a(0), j(t), c(0), \omega(t))$

where

- Initial position: $x(0)$ (meters)
- Initial speed: $v(0)$ (meters/sec)
- Acceleration: $a(t)$ (meters/sec^2)
- Jerk: $j(t)$ (meters/sec^3)
- Initial course: $c(0)$ (degrees)
- Turn rate: $\omega(t)$ (degrees/sec)

As opposed to constant velocity or constant turn rate motions shown in the above figure, the target may:

- Changes its speed and accelerates
- Changes its acceleration and has a jerk
- Changes its turn rate
- Changes its turn radius
- Makes a combination of those
- The above cases may be valid for vertical motion also

2.3 Submarine Platform

An exampled of a submarine platform is presented briefly below corresponding to SSN-688. Other submarines have similar sensor configurations. [5]

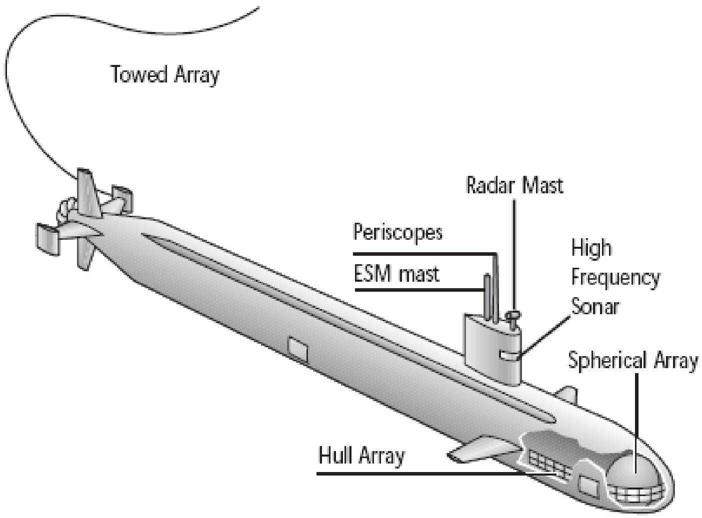


Figure 2 - Typical sensor suite of a SSN-688 submarine platform [5]

The submarine passive sonars are the primary information source for a TMA algorithm. The sonar suite of a submarine generally includes:

- **Spherical Array Sonar (SAS)**
    - Primary sonar of the submarine
    - Processes broadband signals better than other arrays
    - Provides frontal coverage
    - Provides vertical coverage. Large field of view up and down.
    - Mainly used for mid and high frequencies (750 Hz to 2.0 kHz for SSN-688)

- **Hull Array Sonar (HAS)**
  - Primary function is to estimate the bearing of the target
  - Detects low and mid frequency narrowband contacts (50 Hz to 1.0 kHz for SSN-688)
  - Provides side coverage for both starboard and port flanks

- **Passive Ranging Sonar (PRS)**
  - Primary function is to estimate the range of the target

- **Towed Array Sonar (TAS)**
  - Geometry is a linear array
  - Used for both broadband and narrowband tracking (10 Hz to 1.0 kHz for SSN-688)
  - TAS has an ambiguity problem which can be solved by maneuvering the ship
  - It is basically a long cable, up to 2000 m
  - Provides rear coverage
  - Used for triangulation

The usage of active sensors in the submarine is not preferable since they expose the submarine's presence, bearing, range and identification information to other platforms.

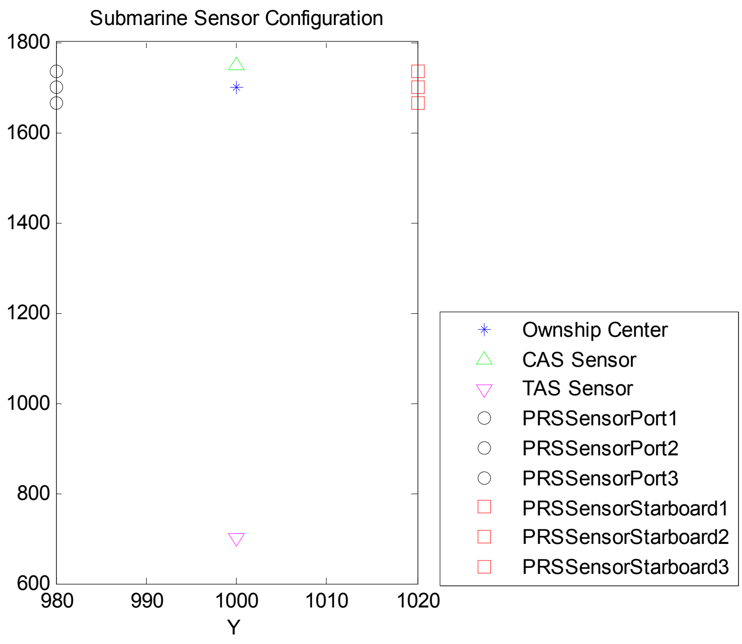


Figure 3 – Sensor locations relative to submarine

3.0 SOLUTION METHODS

Direction finding or bearing estimation or angle of arrival (AOA) estimation is a field that has been studied extensively in literature. These methods will not be described in this paper but a good summary of the subject can be found in [6, 7]

From an implementation viewpoint, previous solutions can be loosely classified into four basic categories: graphical, closed-form, recursive, and batch. The graphical solutions literally plot bearing lines along the observer's path and then find a region (range) were source motion along a straight line is divided into equal segments by the bearing lines. The closed-form solutions generally make some simplifying assumptions, such as zero range rate, and then produce an approximate target range solution, often based on some form of triangulation ranging. The recursive solutions are the predictor-corrector Kalman style algorithms and are based on a physical model and may or may not use a process noise model. The batch solutions also use physical models and may or may not use process noise models to obtain solutions from a "batch" of data. [8]

Graphical solutions, although slow if implemented manually, may be sufficiently accurate for favorable scenarios but can suffer greatly as conditions deteriorate. These methods were used before the computer aided TMA processing. The closed-form solutions can be sufficiently accurate under favorable conditions also, but can be quite biased under unfavorable scenarios. Also, the closed-form solutions generally lack a complete state solution (no velocity solution) and an error covariance estimate, although more recent methods have shown improvement. The recursive solutions are basically Kalman filters. Although these solutions can be very good, most suffer from a premature convergence of the covariance matrix on the first (unobservable) leg. The choice of modified polar coordinates correctly isolates the observable states from the unobservable states on the first leg and thereby effectively eliminates the premature convergence problem. However, as with all other recursive algorithms, there can be convergence problems on subsequent legs. The recursive algorithms can also track a maneuvering target and provide error covariance estimates. The batch algorithms are basically weighted least-squares fits to the data. These methods also use a process model and may or may not use a process noise model. Although not restricted to a batch method, the maximum likelihood estimate (MLE) is often implemented in this manner for this problem. As with the Kalman filters, the batch algorithms may provide error covariance estimates and track maneuvering targets. Generally, the batch solutions are iterative solutions and therefore computationally more demanding. However, they are also well suited for segmenting data for further analysis. Pseudolinear batch algorithms, which inherently do not require iterative solutions, have been developed. Early pseudolinear solutions could suffer from severe biases for certain geometries but subsequent improvements appear to have limited the bias problem. The performance of both the recursive and batch solutions can be sensitive to their initialization.

Although classical TMA problem is considered with single sensor bearing measurements, one must consider the usage of multiple sensors to improve TMA solution. The single sensor algorithms require the own ship to maneuver and increase observability. On the other hand, algorithms using multiple sensors can estimate the target position at each time instant independent of the maneuvers of the own ship. Therefore they can give a solution in a shorter time compared to single sensor algorithms. Also they don't have initialization problems. The multiple sensor algorithms has some limitations as well, they require some seperation between different sensors to work effectively. Considering the sensors of submarine including spherical array sonar (SAS) in front of the ship, passive ranging sonars (PRS) and flank array sonar (FAS) at each side of the ship and towed array sonar (TAS) at the back of the ship, the submarine platform seems a proper place to use multiple sensor algorithms.

3.1 Methods Using Single Sensor

Although multi sensor algorithms generally give better performance, they may not be applicable in every scenario. Considering a target in front of a submarine the SAS may be the only sensor to rely on because the aperatures of HAS/FAS and TAS are very small and their bearing accuracy may be very low. Another restriction is the frequency of the received signal which makes only single sensor algorithms being applicable. Also the sensitivity of a sensor in a frequency band can be better than others which also imposes the use of single sensor algorithms.

In the following section only maximum likelihood is summarized and some results are presented for comparison purposes in simulations. Other methods are briefly introduced but details of them can be found in the given references.

3.1.1 Maximum Likelihood Estimator

The model for a constant velocity target the maximum likelihood TMA problem can be formulated as follows [4]:

$$a \overset{\Delta}{=} \begin{bmatrix} a_1 & a_2 & a_3 & a_4 \end{bmatrix}^T \overset{\Delta}{=} \begin{bmatrix} x(0) & y(0) & \dot{x} & \dot{y} \end{bmatrix}^T$$

$$x(k) = x(0) + \dot{x}t_k$$

$$y(k) = y(0) + \dot{y}t_k$$

$$z(k) \overset{\Delta}{=} h(k, a) + w(k)$$

$w(k)$ is the measurement noise with zero mean Gaussian white sequence with variance σ.

$$E\{w(k)w(j)\} = \sigma\delta(k-j)$$

$$h(k,a) \overset{\Delta}{=} \tan^{-1}\left(\frac{y(k) - y_p(k)}{x(k) - x_p(k)}\right)$$

The likelihood function of the target parameter vector is:

$$\Lambda(a) = p(Z^n \mid a) = p[z(1),\ldots,z(n) \mid a] = \prod_{k=1}^{n} p[z(k) \mid a]$$

$$p[z(k) \mid a] = N[z(k); h(k,a), \sigma] = ce^{-\frac{1}{2\sigma}[z(k)-h(k,a)]^2}$$

The maximization of the likelihood function, which yields the maximum likelihood estimate (MLE), is equivalent to the following nonlinear least squares (NLS) problem:

$$\hat{a} = \arg\max_a \Lambda(a) = \arg\max_a \lambda(a)$$

where

$$\lambda(a) \stackrel{\Delta}{=} \frac{1}{2\sigma}\sum_{k=1}^{n}\left[z(k)-h(k,a)\right]^2$$

is the negative log-Iikelihood function with the irrelevant additive constants omitted. The above expression of the log-likelihood function is clearly that of an NLS criterion. The minimization of the log-likelihood function can be carried out via one of the many existing numerical optimization algorithms. Some of them are Newton-Raphson, quasi-Newton techniques, Nelder Mead Simplex, Non-linear least squares, Genetic Algorithms, Pattern Search, Levenberg – Marquardt Techniques.

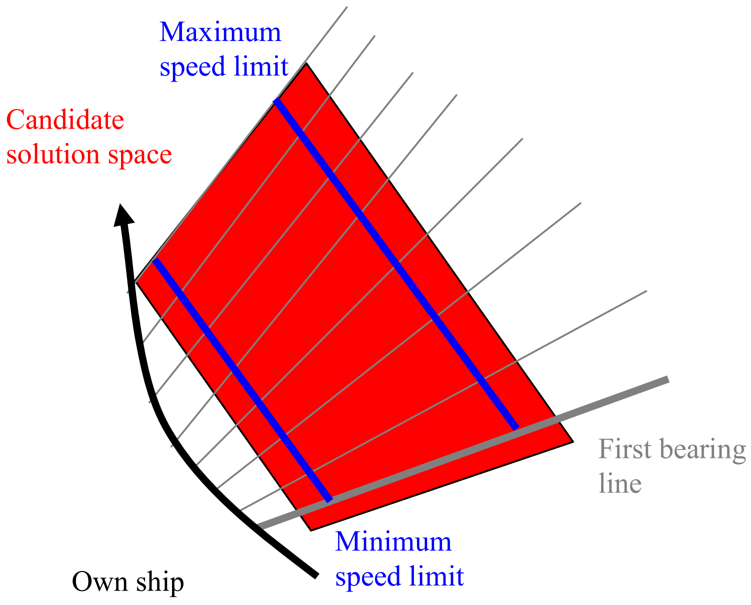
The MLE exhibits several characteristics which can be interpreted to mean that it is "asymptotically optimal". These characteristics include:

- The MLE is asymptotically unbiased, i.e., its bias tends to zero as the number of samples increases to infinity.
- The MLE is asymptotically efficient, i.e., it achieves the Cramér-Rao lower bound when the number of samples tends to infinity. This means that, asymptotically, no unbiased estimator has lower mean squared error than the MLE.
- The MLE is asymptotically normal. As the number of samples increases, the distribution of the MLE tends to the Gaussian distribution with mean θ and covariance matrix equal to the inverse of the Fisher information matrix.

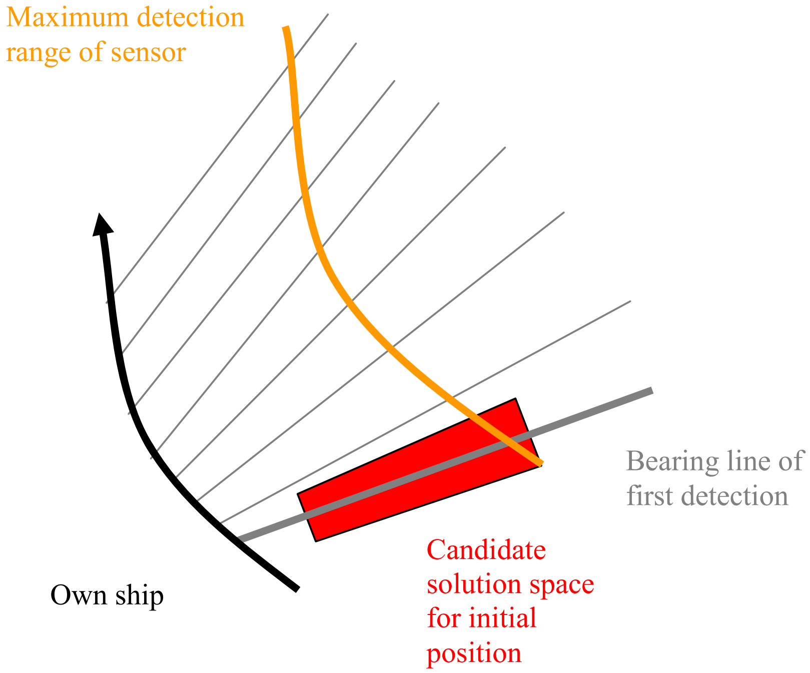
MLE is a batch method which requires the optimization to be performed for all samples when a new sample arrives. Therefore it is a computationally intensive algorithm compared to the recursive techniques.

Although MLE seems computationally intensive, some improvements can be performed imposing some practical constraints such as:

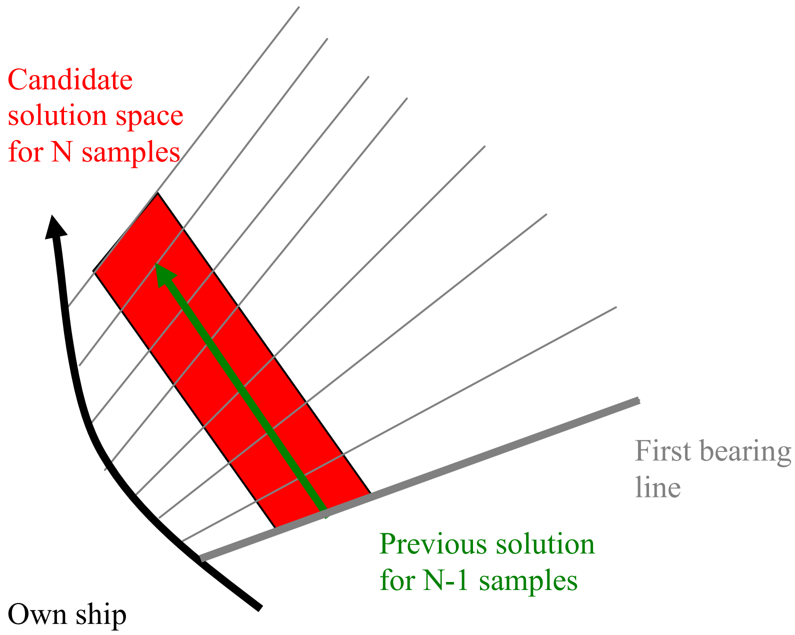
- Limiting the optimization problem between some values for $x(0)$, $y(0)$, $\dot{x}$, $\dot{y}$. The points close to the first bearing line is a good place to start optimization for $x(0)$ and $y(0)$. Also apriori knowledge about the minimum and maximum speeds for a target platform can be used to limit the optimization for $\dot{x}$ and $\dot{y}$.



- Detection ranges of the sensors can also be a constraint and an initial point to start optimization.

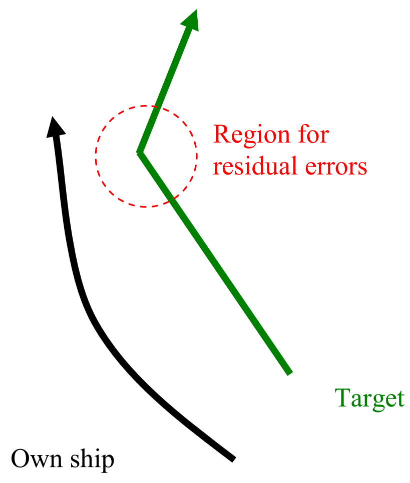


- Performing a prewindowing and limiting the optimization for the last M samples,
- Limiting the optimization problem for the N'th time instant to proximity of the previous solution of the (N-1)'th time instant.

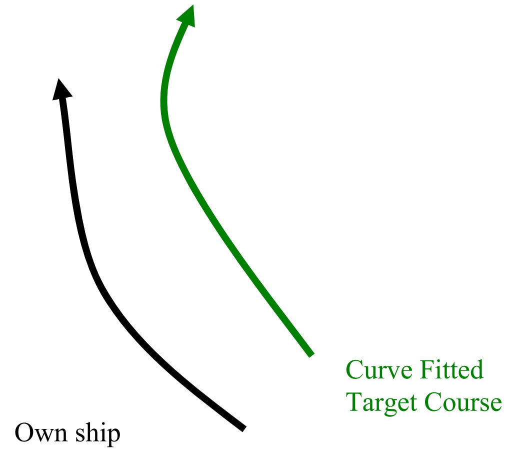


Application of MLE for Submarine Platform

Although MLE seems as if it is a demanding algorithm, its robustness is an advantage. Also the instantaneous residual errors between the measurements and the estimated model give an opportunity to detect target maneuvers.



After the MLE estimations the curve fitting can be performed for the regions with residual errors to estimate the angular motions and turns of the target. Sine-Cosine functions can be used for curve fitting because they offer better performance than polynomial fitting for angular motions and turns.



The MLE with these slight modifications can be employed in actual systems.

3.1.2 Recursive Methods

The recursive type algorithms have traditionally been based on the Kalman filter and generally show instability and filter divergance, particularly in highly nonlinear scenarios (e.g. close target-observer encounters). [9]

3.1.2.1 Nonmaneuvering Case

Bearings-only tracking of a nonmaneuvering target is a well-studied problem, with its research originating in the sonar tracking community. Many nonlinear filters such as the EKF in the Cartesian and modified polar (MP) coordinates, the pseudolinear estimator, maximum likelihood estimator, modified gain EKF and recently particle filters have been proposed as solutions.

3.1.2.2 Maneuvering Target Case

The algorithms for maneuvering targets can be classified as IMM-based algorithms and particle-filter-based schemes. These can be IMM-EKF, IMM-UKF, MMPF, AUX-MMPF and JMS-PF. Essentially these algorithms attempt to solve the jump Markov system (JMS) filtering problem.

Le Cadre and Tremois modeled the maneuvering target using the CV model with process noise and developed a tracking filter in the hidden Markov model framework.

3.2 Methods Using Multiple Sensors

Multiple sensor TMA algorithms are becoming popular and attractive for submarine platforms because they give solutions in a shorter time compared to the single sensor case. Also their results can be more accurate because they utilize the sensor suite baseline effectively. Although bearing triangulation (cross-bearing) algorithms and bearing-frequency algorithms are developed further [10, 11] and employed more effectively in recent systems, the TDOA based methods are still studied relatively less.

3.2.1 Triangulation Based Methods

Triangulation is the localization of an unknown point by using a triangle with two known points and the unknown point as the vertices. [12, 13]

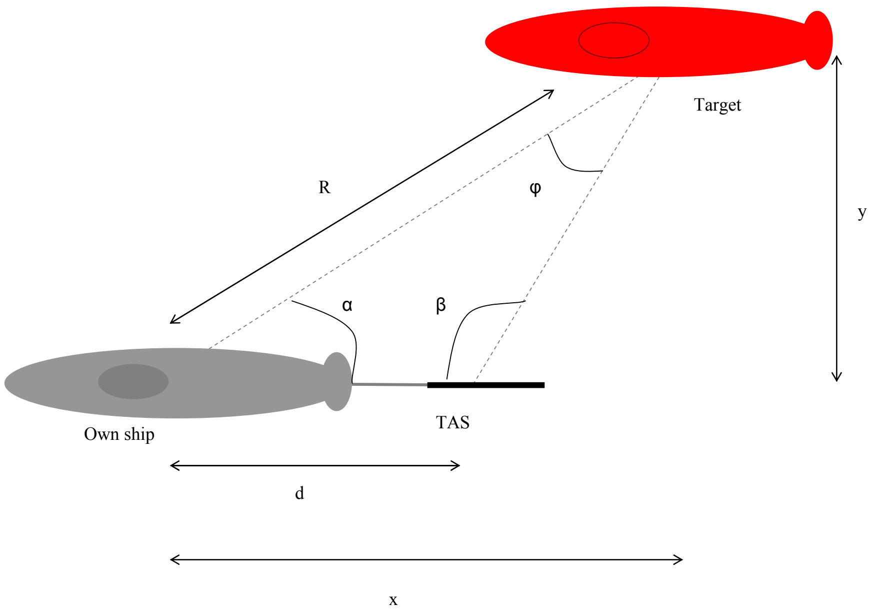


Figure 4 - Triangulation geometry

$$R = \frac{d.\sin\beta}{\sin\varphi}$$

$$x = R.\cos\alpha$$

$$y = R.\sin\alpha$$

If there are more than two bearing measurements, then there will be more than two intersection points. In this case, the centroid of the uncertainity area or the point having equal distance from all vertices can be determined as the estimate.

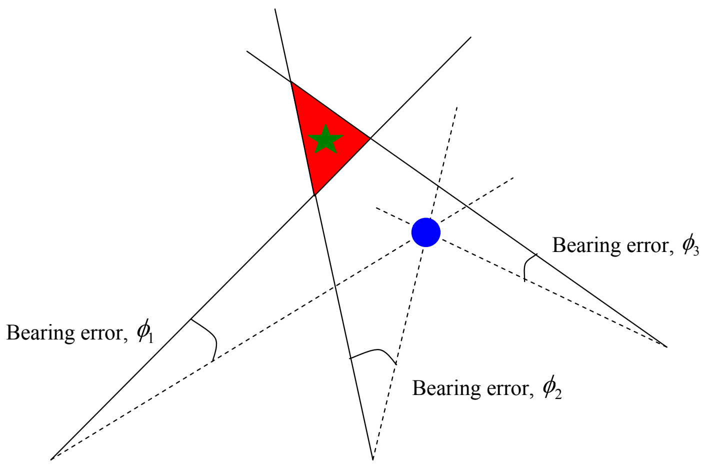


Figure 4 – Effects of bearing errors on triangulation

The dashed line indicates the true bearing of the target and blue circle indicates the true position of the target, while the solid lines correspond to the bearing measurements, the red area corresponds to the uncertainity region and the green star indicates the estimated location.

For the three dimensional case, if there are L bearings, the location estimation can be interpreted as an optimization problem for these L lines simulataneously minimizing the total distance between the estimated point and bearing lines:

$$p = \arg\max_p \sum_{i=1}^{L} d(l_i, p) \text{ ,where } p = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ is the estimated position and}$$

$d(l_i, p)$ is the distance between a point and a bearing line.

3.2.2 Horizontal Direct Passive Ranging (HDPR) Based Methods

Horizontal Direct Passive Ranging (HDPR) is a technique which uses the three arrays located at each flank of the submarine. It is a hybrid method using the time difference and bearing information together to estimate the range of the target.

The curvature of the wavefront of a signal from a distant target determined by measurements at three arrays can be used to estimate the range of the target. [12]

This algorithm has nice features such as it does not need a towed array to collect any measurements. But it is susceptible to noise when the target is in front or at the back of the submarine. Therefore, the submarine has to maneuver and increase the aperture to receive target signal from one side of the submarine for better performance.

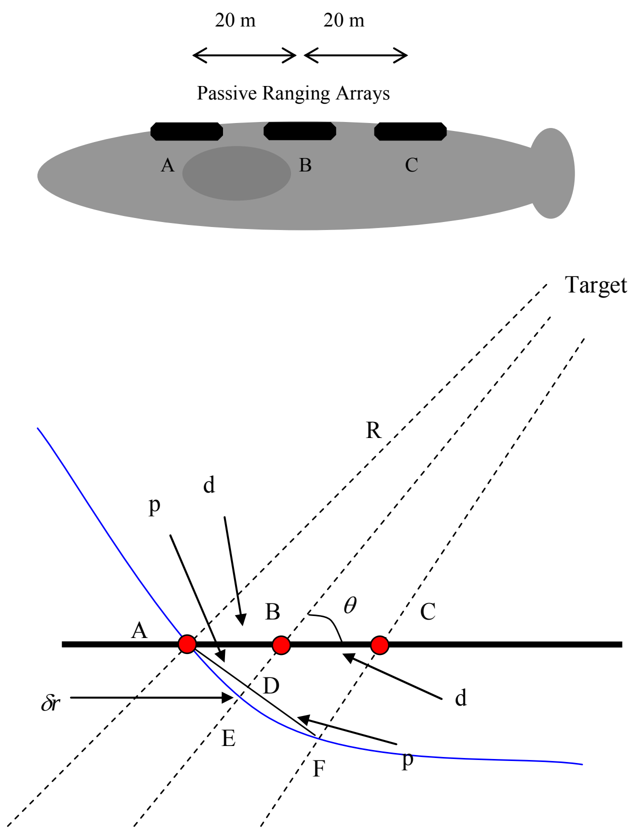


Figure 5 - Horizontal Direct Passive Ranging Geometry

Considering three collinear equispaced arrays A, B and C. The target is at range R and angle θ with the axis of the arrays:

$$R^2 = p^2 + (R - \delta r)^2$$

where δr is the additional path length due to curvature. We have $p = d \sin \theta$, therefore

$$R^2 = (d \sin \theta)^2 + (R - \delta r)^2$$

and when R is large compared with d, we have

$$R = \frac{(d \sin \theta)^2}{2\delta r}$$

To find the range, we therefore need to determine δr and θ. For a distant target, angle θ is almost the same for all three arrays. In practice, θ is determined by measuring and averaging the maximum response angles from the three arrays; where δr is given by

$$\delta r = BE - \frac{1}{2} CF$$

BE can be found by cross-correlating between arrays A and B, and CF can be found by cross-correlating between arrays A and C.

The range estimate relies upon the accurate determination of δr, which is dependent upon the precise measurement of the time differences between arrivals at the three arrays. Therefore the positions of the arrays themselves must be very accurately known in all three dimensipns. The range estimate is less sensitive to bearing errors, particularly at broadside where $\theta = 90°$.

The method is optimized by making δr - the difference in path lengths due to curvature only- as large as possible so that it can be accurately determined. This occurs where d is large and is close to $90°$. In practice, δr needs to be some significant proportion of wavelength, say at least $\lambda / 10$, for an accurate measurement, and for realistic values of d and array size, this limits the use of the technique to quite high frequencies.

3.2.3 Multilateration (TDOA Based Methods)

Multilateration or hyperbolic location systems, often called time difference of arrival (TDOA) systems, determine the position of the target based on the arrival times of the signals at three or more sensors. One sensor is determined as reference. Then the arrival times for any other sensor and the reference sensor are combined to constitute relative time information which is called as TDOA. The TDOA equation for this sensor pair defines a hyperboloid for the transmitter location. If the position is estimated in two dimensions, two hyperbolas or three sensors will intersect at two points. Then the location is determined using a priori constraints like some bearing information from sensors or introducing more sensors and more hyperbolas. At least four sensors are necessary to determine the target position for estimation in three dimensions. [7, 13]

In practice there are measurement errors or inaccuracies in the determination of TDOA values. So there will be some errors in location estimation. These errors can be reduced by introducing other sensors. If there are N sensors, there will be N-1 hyperboloids. Then this problem can be interpreted as an optimization problem which can be solved by a least squares method or with some other optimization methods.

One difficulty in practical systems is to move large datas from other sites to central processing site and then process this data set to accurately determine TDOA values. Time syncronization between sites is also a crucial issue affecting the system performance. Nevertheless, these issues are less complicated when the sensors reside on the same platform as in the case of a submarine.

TDOA Estimation

The methods for estimating TDOA is a key issue for the performance of the algorithm. The TDOA's between sensors are calculated simply from cross correlation of the received signals [14], maximum likelihood methods [15], orthogonal wavelet transform [16], or detecting the leading edge of pulsed waveforms [17].

Advantages

TDOA based methods also do not need array calibration which makes these methods appealing for most applications.

In bearing only target localization of multiple targets, there is a phenomenon known as "ghosting" in which some extra targets appear as the number of targets increase.

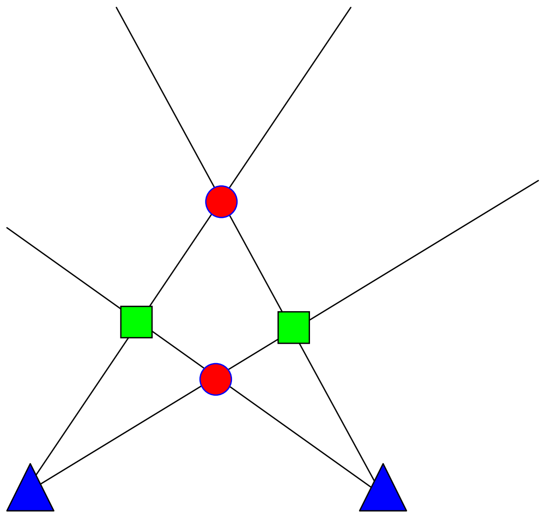


Figure 6 - Deghosting problem (blue triangles represent receivers, red circles represent targets, green squares represent ghost targets)

If there are more observers and targets together with sensor errors, the problem becomes more tedious as shown below.

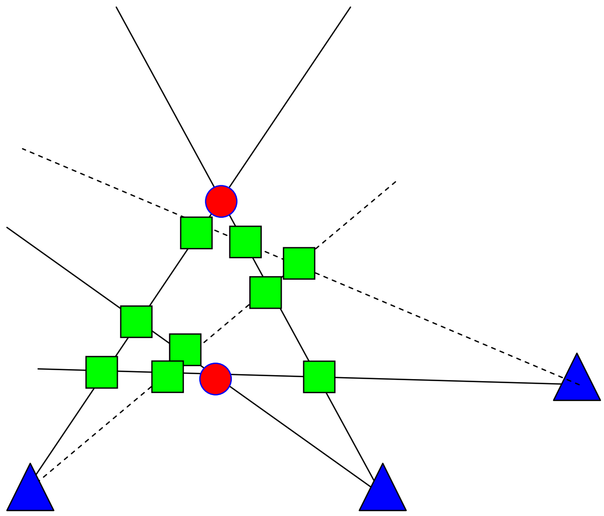


Figure 7 - Deghosting problem with more sensors and sensor errors (blue triangles represent receivers, red circles represent targets, green squares represent ghost targets, dashed lines represent measurements with errors)

There are some methods for ghost suppression but they use typically knowledge about allowed position, maximal or minimal velocity, maximal acceleration, direction of movements and others. There are also other strategies for deghosting but their effectiveness can be limited in some scenarios. [18]. On the other hand, the utilization of TDOA based location estimation methods provides an efficient signal seperation. The TDOA's and locations can be determined uniquely, since the received signals have different characteristics allowing them to be resolved by TDOA estimation algorithms.

Solution Methods

The formulated problem can be solved using maximum likelihood methods, Taylor series expansion [13], spherical interpolation [19] or a least squares solution with the addition of new variable [20]. The least sqares solution technique is used in the simulations which introduces the distance between the first sensor and the target as a new variable to linearize the equations:

$$d^2 = x_1^2 + y_1^2 + z_1^2 - 2x_1x_T - 2y_1y_T - 2z_1z_T + x_T^2 + y_T^2 + z_T^2$$

$$d^2 + 2dc\tau_2 + c^2\tau_2^2 = x_2^2 + y_2^2 + z_2^2 - 2x_2x_T - 2y_2y_T - 2z_2z_T + x_T^2 + y_T^2 + z_T^2$$

$$d^2 + 2dc\tau_3 + c^2\tau_3^2 = x_3^2 + y_3^2 + z_3^2 - 2x_3x_T - 2y_3y_T - 2z_3z_T + x_T^2 + y_T^2 + z_T^2$$

$$-2dc\tau_2 - c^2\tau_2^2 = x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 - 2(x_2 - x_1)x_T - 2(y_2 - y_1)y_T - 2(z_2 - z_1)z_T$$

$$-2dc\tau_3 - c^2\tau_3^2 = x_1^2 + y_1^2 + z_1^2 - x_3^2 - y_3^2 - z_3^2 - 2(x_3 - x_1)x_T - 2(y_3 - y_1)y_T - 2(z_3 - z_1)z_T$$

$$\begin{bmatrix} c^2\tau_2 + x_1^2 + y_1^2 + z_1^2 - x_2^2 - y_2^2 - z_2^2 \\ c^2\tau_3 + x_1^2 + y_1^2 + z_1^2 - x_3^2 - y_3^2 - z_3^2 \\ . \\ . \\ . \\ c^2\tau_N + x_1^2 + y_1^2 + z_1^2 - x_N^2 - y_N^2 - z_N^2 \end{bmatrix} = 2\begin{bmatrix} x_1 - x_2 & y_1 - y_2 & z_1 - z_2 & -c\tau_2 \\ x_1 - x_3 & y_1 - y_3 & z_1 - z_3 & -c\tau_3 \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ x_1 - x_N & y_1 - y_N & z_1 - z_N & -c\tau_N \end{bmatrix}\begin{bmatrix} x_T \\ y_T \\ z_T \\ d \end{bmatrix}$$

$$a = Cb$$

The solution is:

$\hat{b} = C^* a = (C^H C)^{-1} C^H a$ where C^* is the pseudoinverse of C.

Multilateration Accuracy

Multilateration can be more accurate for locating an object, than techniques such as triangulation (as it may be easier to measure time accurately than it is to form a very narrow beam in some situations). This makes it a good alternative to increase TMA accuracy. The accuracy of multilateration is a function of several variables, including:

- The geometry of the receiver(s) and transmitter(s)
- The timing accuracy of the receiver system
- The accuracy of the synchronisation of the receiving sensors
- The bandwidth of the emitted pulse(s)
- Uncertainties in the locations of the receivers

There are studies ongoing for optimizing the sensor geometry to improve the performance. [21]

Application of Multilateration for Submarine Platform

At least 3 sensors should be available for the determination of target positions in 2D and at least 4 sensors should be available for the determination of target positions in 3D. Considering the SSN-688 submarine platform these can be the whole array or subarrays of the following sensors:

- For frequencies between 50 Hz and 1.0 kHz: HAS/PRS and TAS
- For frequencies between 750 Hz and 1.0 kHz: SAS, HAS/PRS and TAS

3.2.3.1 Simulation Results

Scenario 1

Initial position of target is x = 1200 m and y = 2800 m.

Initial position of ownship is x = 1900 m and y = 1700 m.

The absolute bearing of target is around 75 degrees. So for the intervals 0 – 25 and 50 – 75 the target is closer to the front side of the ownship and for the intervals 25 – 50 and 75 – 100 the target is closer to the right flank side of the ownship.

Both ownship and target has 10 m/s speeds.

If the bearing sensors have 0.36 degree rms error and time sensors have 0.5 microsecond rms error, the following results are obtained.

CAS sensor is used for bearing only methods.

CAS and TAS sensors are used for triangulation methods.

CAS and 3 PRS sensors are used for TDOA methods.

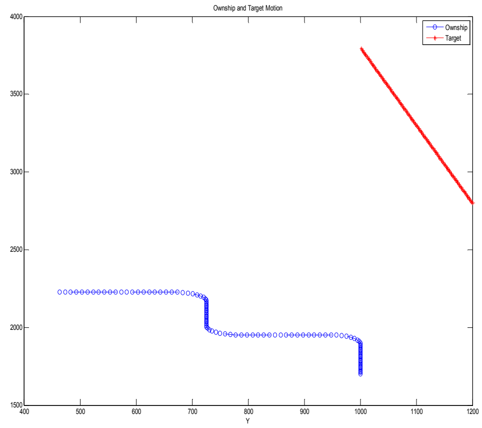


Figure 8 - Ownship and Target Courses

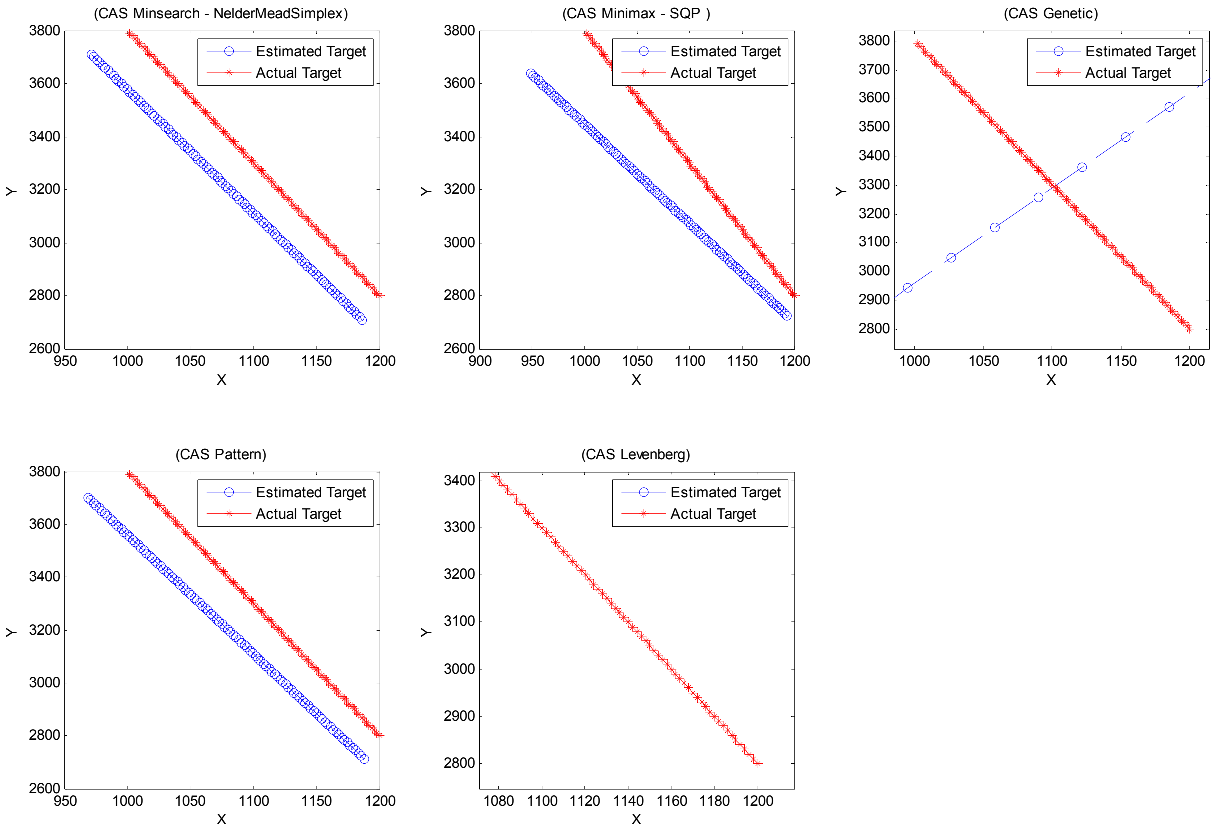


Figure 9 - Single Sensor ML Performance

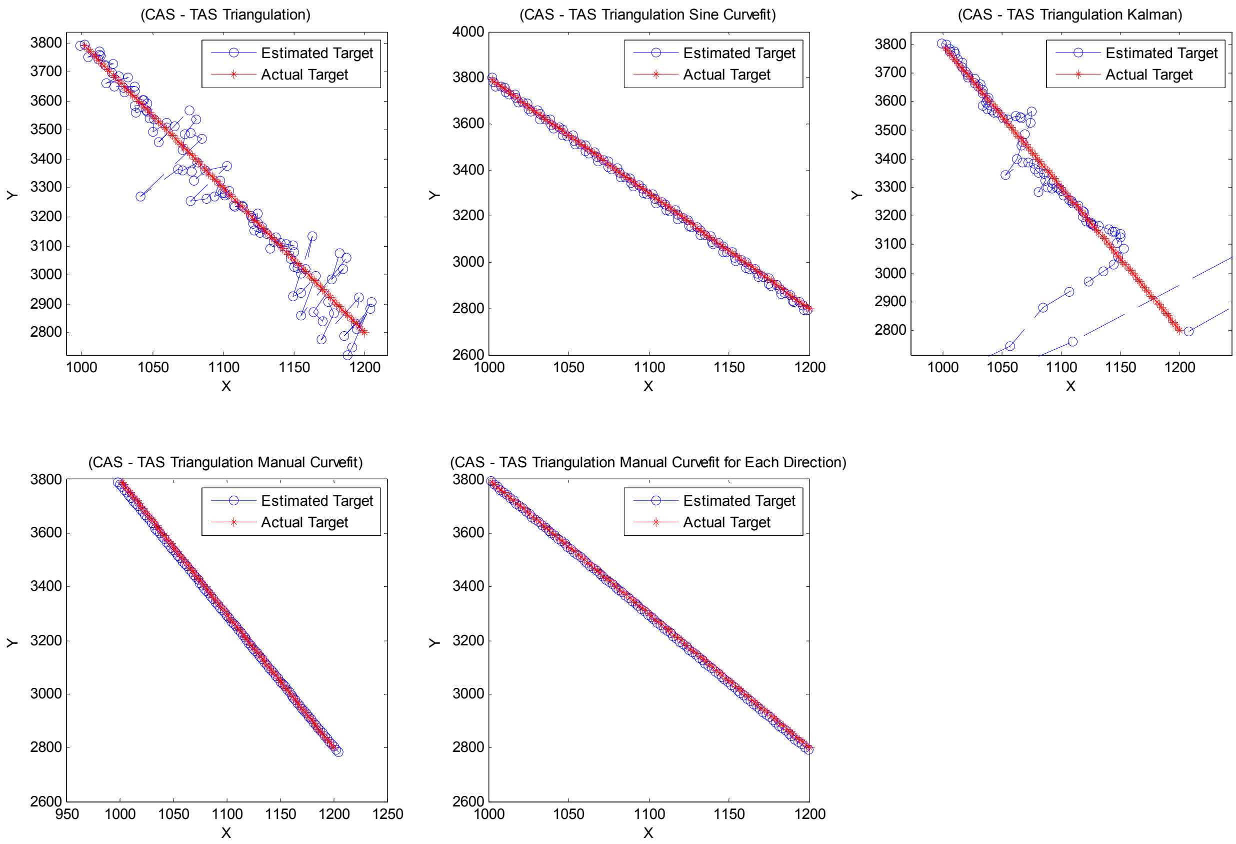


Figure 10 - Multi Sensor Triangulation Performance

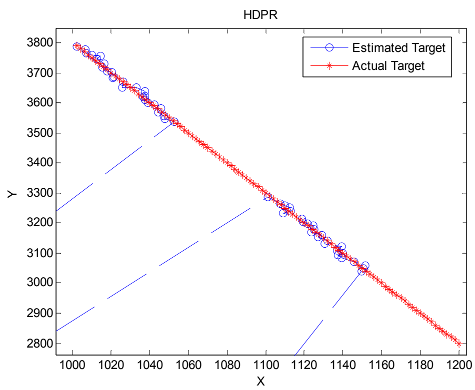


Figure 11 - Multi Sensor HDPR Performance

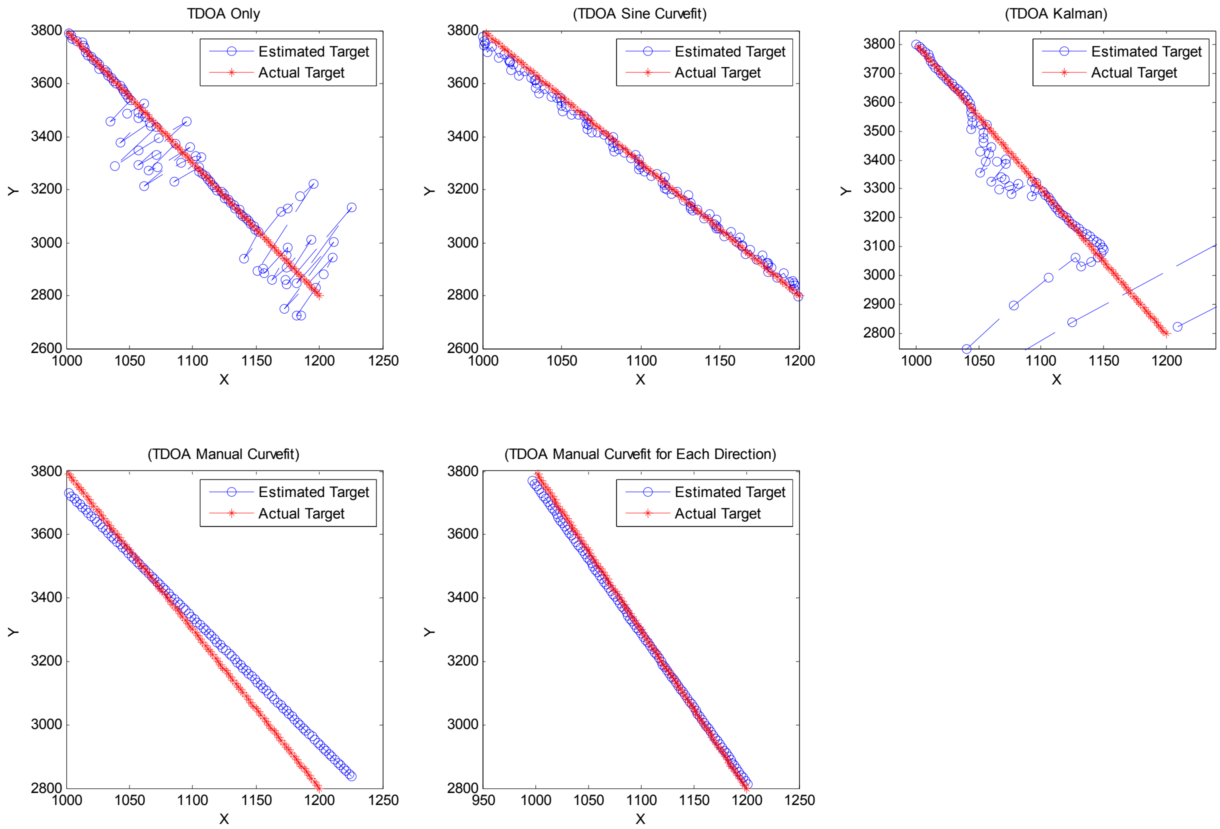


Figure 12 - Multi Sensor TDOA Performance

For the whole scenario

Single sensor bearing only algorithms have RMS errors between 50 – 120 m.

Multi sensor triangulation algorithms have RMS errors between 5 – 25 m.

Multi sensor TDOA algorithms have RMS errors between 12 - 37 m

Multi sensor HDPR algorithm is only accurate when the target is closer to the flank side of the ownship; but it makes large errors when the target gets closer to the front side of the ownship, so it is not effective for the whole scenario.

But for the time intervals when the target is in the flanks of the submarine

Single sensor bearing only algorithm has RMS errors of 447.7 m

Multi sensor triangulation only algorithm has RMS errors of 64.8 m.

Multi sensor TDOA only algorithm has RMS errors of 11.1 m

Multi sensor HDPR algorithm has RMS errors of 19.5 m.

Scenario 2

In this simulation both triangulation and TDOA methods use CAS and 3 PRS sensors

If the bearing sensors have 0.0073 degree rms error and time sensors have 2.5 microsecond rms error, the following results are obtained.

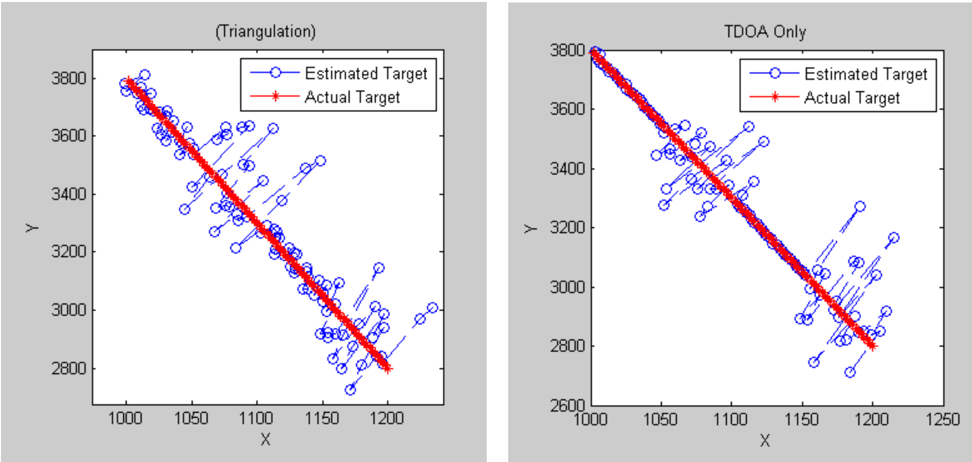


Figure 13 – Triangulation vs. TDOA Comparison

Again, the TDOA based algorithm performs better than the triangulation based algorithm, when the target is in the flanks of the submarine.

Results

From these simulations, it was observed that:

- Both triangulation and TDOA methods outperform the bearing only methods.
- Multi sensor triangulation algorithms are more favorable when the target gets closer to front of the submarine. Performance of TDOA based algorithms degrade significantly in these situations.
- Multi sensor TDOA algorithms are more favorable when the target is in the flanks of the submarine. Besides, TDOA algorithms can perform better than triangulation algorithms without using TAS sensor. The performance of the TDOA algorihtms will certainly increase further with the addition of the TAS sensor.
- Curve fitting techniques may perform better than Kalman filter in some situations.
- Single sensor algorithms sometimes diverge, so they should be used carefully.

3.3 Relationship of TMA with Classification and Identification

Although many classification and identification systems and techniques were developed in recent years, many of them still rely on the operator interpretation of the sensor data based on his/her experience. Therefore the first aim of these systems should be narrowing the candidates to be presented to the operator. This process can only be performed efficiently with the help of an intelligence database.

3.3.1 Feature Extraction

Before starting the classification and/or identification process, the TMA and target tracking results should be abstracted somehow. This can be done by selecting a suitable time window and segmentation method to perform analysis. Then the platform motion will be estimated as the best fitting model among the simpified models such as linear motion, coordinated turn and other predefined models.

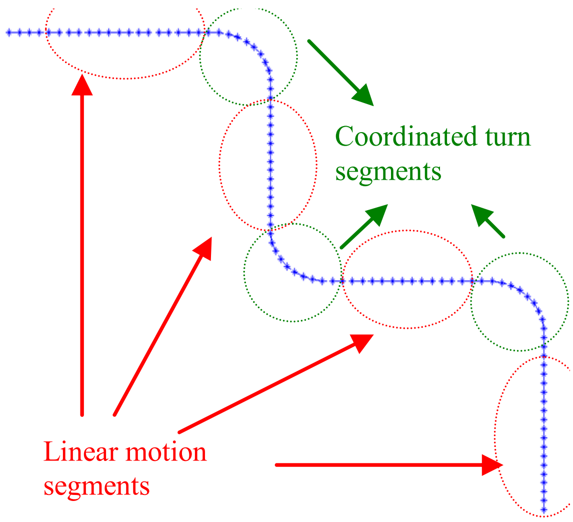


Figure 14 - Segmentation of target course

As an example consider the following scenario with the target manuevering between time interval 95 – 105. The Kalman filtered triangulations results in the following graph:

Target starts at x = 1200 and y = 2800.

Ownship starts at x = 2500 and y = 1700.

Both ownship and target has 10 m/s speeds.

Time sensors have 0.5 microsecond rms error.

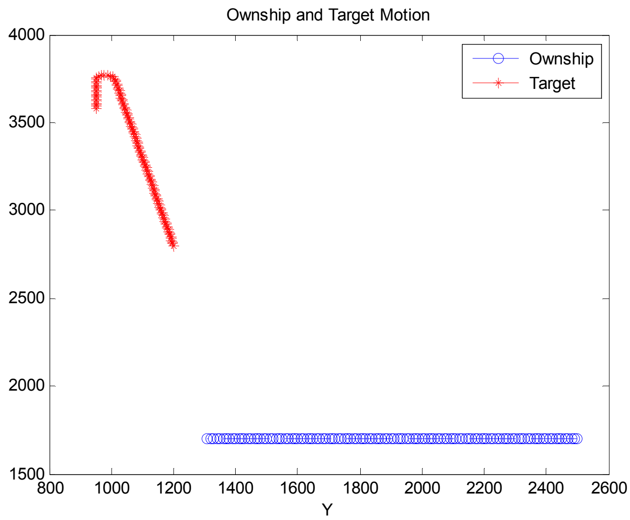


Figure 15 – Simulation Scenario

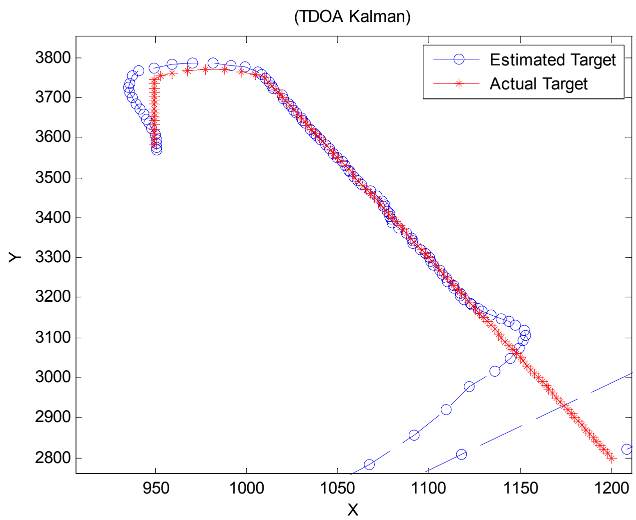


Figure 16 – TDOA/Kalman Filtering Results

A 20 sample gate is used for least squares fit for this example to detect the target maneuvers.

For x dimension the least squares equations for a linear constant velocity model are

$$Ax = B \text{, where } A = \begin{bmatrix} 0 & 1 \\ T & 1 \\ 2T & 1 \\ \vdots & 1 \\ \vdots & 1 \\ (N-1)T & 1 \end{bmatrix},\ x = \begin{bmatrix} vx \\ x0 \end{bmatrix},\ B = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ \vdots \\ x(N-1) \end{bmatrix}$$

The solution is:

$\hat{x} = A^* B = (A^H A)^{-1} A^H B$ where A^* is the pseudoinverse of A.

$$Errorx = \left\| (A\hat{x} - B)^2 \right\|$$

The same will be calculated for y dimension.

The least squares curve fit error residual is the sum of errors in x and y dimensions.

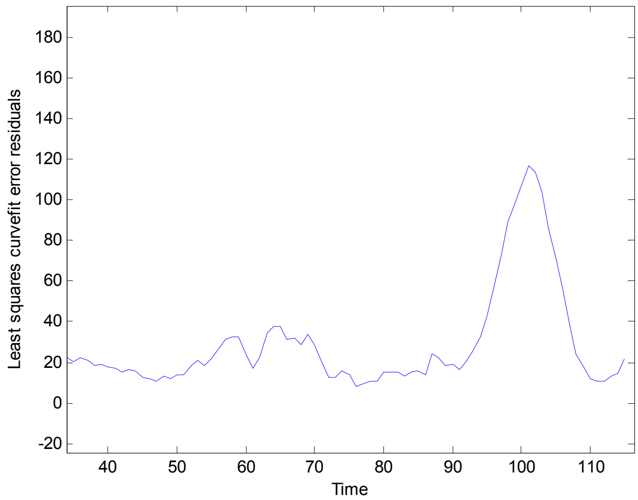


Figure 17 - Least squares curve fit error residuals

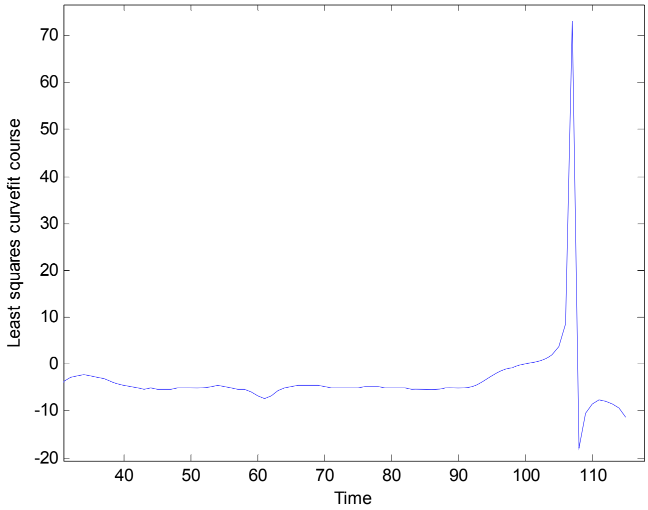


Figure 18 - Least squares curve fit course

Excluding the time interval 0 – 25 which corresponds to the transient behaviour of Kalman filter, we may observe the increase errors that in the interval 95 – 105 which shows that the target is deviating from the linear motion model. Also, the change in the slope of the fitted curves after time instant 95 gives some strong evidence about the target manuever. So, a different model should be used in that region. For that region, the circular motion model can be employed. If the circular motion model doesn't fit the measured data set, other motion models from the motion model library should be applied for best fit.

A few models are presented in the following table:

Table 2 - Example motion models

<table>
<tr><td>Accelerated linear motion model</td><td>$Ax = B$
where
$$A = \begin{bmatrix} 0 & 0 & 1 \\ T^2 & T & 1 \\ 4T^2 & 2T & 1 \\ \vdots & \vdots & 1 \\ \vdots & \vdots & 1 \\ (N-1)^2T^2 & (N-1)T & 1 \end{bmatrix},\ x = \begin{bmatrix} ax \\ vx \\ x0 \end{bmatrix},\ B = \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ \vdots \\ x(N-1) \end{bmatrix}$$
The solution is:
$\hat{x} = A^* B = (A^H A)^{-1} A^H B$ where A^* is the pseudoinverse of A.</td></tr>
</table>

Constant speed circular motion model	$\hat{x}(t) = x(0) + \hat{A}.\cos(2\pi\hat{w}t - \phi) - \hat{A}\cos\phi$ $\hat{y}(t) = y(0) + \hat{A}.\sin(2\pi\hat{w}t - \phi) - \hat{A}\sin\phi$ ϕ is the heading of the ownship, before turn maneuver begins. This becomes another optimization problem with 2 variables which gives turn rate ω and turn radius A. $\left[\hat{A}, \hat{\omega}\right] = \arg\min_{A,\omega} \sum_{t=0}^{N-1} \left((\hat{x}(t) - x(t))^2 + (\hat{y}(t) - y(t))^2 \right)$
Accelerated circular motion model	$\hat{x}(t) = x(0) + (\hat{B}.t + \hat{A}).\cos(2\pi\hat{w}t - \phi) - \hat{A}\cos\phi$ $\hat{y}(t) = y(0) + (\hat{B}.t + \hat{A})\sin(2\pi\hat{w}t - \phi) - \hat{A}\sin\phi$ $\left[\hat{A}, \hat{B}, \hat{\omega}\right] = \arg\min_{A,B,\omega} \sum_{t=0}^{N-1} \left((\hat{x}(t) - x(t))^2 + (\hat{y}(t) - y(t))^2 \right)$ This becomes another optimization problem with 3 variables which gives turn rate ω, turn radius A and acceleration B.

After finding the target motion characteristic parameters, a feature based classification can be executed to use these distinguishing parameters in classification and/or identification. These parameters are stated below:

- Position and course of the target disclosing the operational area of the platform,
- Speed of the target disclosing the speed profile (average value and limits) and cruising speed of the platform,
- Acceleration of the target disclosing the acceleration profile (average value and limits),
- Jerk of the target disclosing the jerk profile (average value and limits),
- Depth of the target disclosing the operational depth of the platform,
- Depth change rate of the target disclosing the depth change profile (average value and limits),
- Turn rate of the target disclosing the maneuver profile,
- Turn radius of the target disclosing the maneuver profile,
- Tactical maneuvers and tactics of the target (can be assisted by a neural network to determine the matching patterns from library but the final decision remains to the operator).

3.3.2 Cluster Analysis Based Classification/Identification

The parameters that are extracted in feature extraction step can be compared with the values in the intelligence database to propose the operator the candidates for feature based classification and/or identification. Rather than treating these parameters separately, a multidimensional cluster analysis can be performed to sort the candidates according to the defined distance measure, which will determine how the similarity of two elements is calculated. The confidence levels for the candidates should also be presented to the operator at the end of the analysis.

Most commonly used distance measures are: [22]

- The Euclidean distance (2-norm distance)
- The Manhattan distance (1-norm distance)
- The maximum distance (max-norm distance)
- The Minkowski distance
- The average distance
- The Mahalanobis distance (corrects data for different scales and correlations in the variables)
- The cosine similarity distance (angle between two vectors)
- The Hamming distance (minimum number of substitutions required to change one member into another)

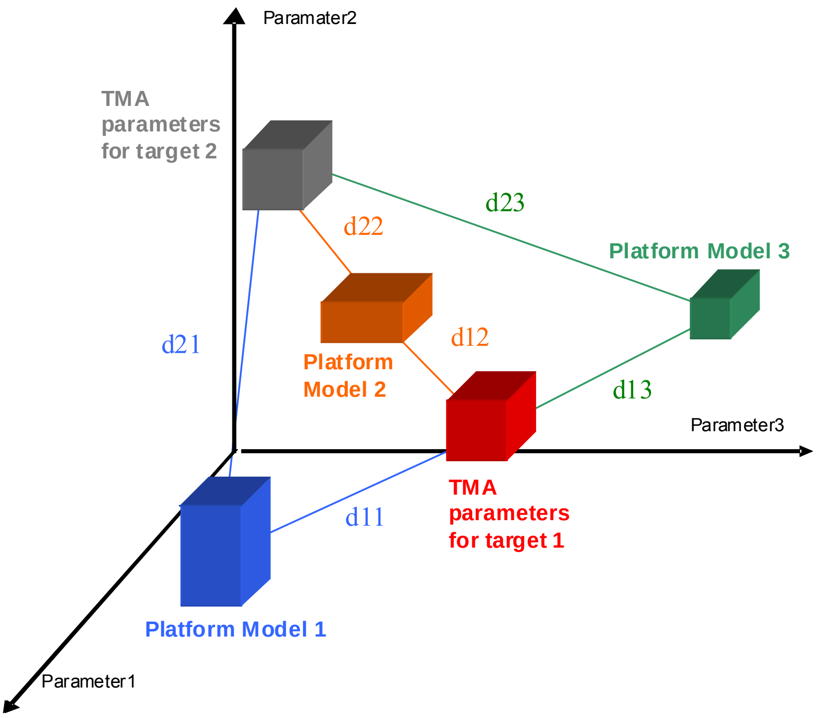


Figure 19 - Cluster analysis for three parameters and two targets

Classification/identification is based on calculation of distance between the technical description of the TMA estimation and the different technical descriptions of platform models in intelligence library.

During the construction of intelligence database, a multidimensional cluster analysis can be helpful to determine platform model classes or clusters.

There are numerous clustering techniques which can be used in cluster analysis. The most effective method can only be determined after the analysis of collected and recorded mission data. The most commonly used clustering techniques are: [23]

- Hierarchical clustering (produces nested clusters)
  - Single link
  - Complete link
- Partitional clustering (produces only one cluster)
  - Square error (k-means and its variations)
  - Graph theoretic
  - Mixture resolving
  - Mode seeking

There are also some cross-cutting issues that broaden the family of clustering methods:

  - Hard vs. fuzzy
  - Agglomerative vs. divisive
  - Monothetic vs. polythetic
  - Deterministic vs. stochastic
  - Incremental vs. non-incremental

For a reliable classification/identification framework, TMA classification/identification results should be combined with other distinguishing parameters (if available) during cluster analysis to improve the overall classification/identification performance. These parameters can be:

- Sonar analysis: Frequency, amplitude, pulse width (if received signal is a pulsed waveform) of signal and number of propeller blades (if determined by DEMON analysis)
- Radar: Radar cross section.
- ESM: Bearing, frequency, amplitude, polarization, pulse width, radar mode (single frequency, CW, stagger, jitter, etc.)
- Periscope: Image classification results.

4.0 CONCLUSION

During the design process of algorithms of a submarine, the platform features should be considered. The submarine platforms' dimensions facilitate the large sensor array spacings and baselines which allows techniques other than single sensor TMA methods possible to use. The information gathered should be merged considering the platforms' sensor characteristics. Single sensor and multi sensor methods should be merged efficiently, possibly in a multi model algorithm to improve results.

Application of TDOA methods should be considered as important alternatives to bearing only methods or HDPR methods. TDOA methods offer better performance if designing a receiver having a better time accuracy is more feasible compared to designing a receiver having a better bearing accuracy. The application of multilateration is also possible without array calibration which is an important advantage of the algorithm.

TMA is a well known and extensively studied subject due to the challenging nature of the problem. But the relationship between TMA results and classification/identification is relatively overlooked. After the TMA parameters are estimated, cluster analysis techniques can be used as a decision support tool to classify and/or identify the target.

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7.0 ABBREVIATIONS

CV	Constant Velocity
DEMON	Demodulated noise
DOA	Direction of Arrival
EKF	Extended Kalman Filter
HAS	Hull Array Sonar
HDPR	Horizontal Direct Passive Ranging
IMM	Interactive Multiple Model
JMS	Jump Markov System
MMPF	Multiple Model Particle Filter
PF	Particle Filter
PRS	Passive Ranging Sonar
SAS	Spherical Array Sonar
TAS	Towed Array Sonar
TDOA	Time Difference of Arrival
TMA	Target Motion Analysis
UKF	Unscented Kalman Filter