Solution 1: FFT/inverse FFT

The first solution method to derive the “pure” signal of the luminescence emitted from the fast scintillator is using Fourier/inverse Fourier transform.

Fourier transform decomposes an arbitrary function of time $y(t)$ into the corresponding function $Y(\omega)$ of frequencies ω which form $y(t)$. The useful hint in our case is that in the space of frequencies ω a convolution operation is replaced by simple multiplication. Hence the following transformation has to be made:

$y(t) = f(t) * g(t) \rightarrow Y(\omega) = F(\omega) \cdot G(\omega)$, where y, f, g chars correspond to output signal, luminescence and excitation impulse, respectively.

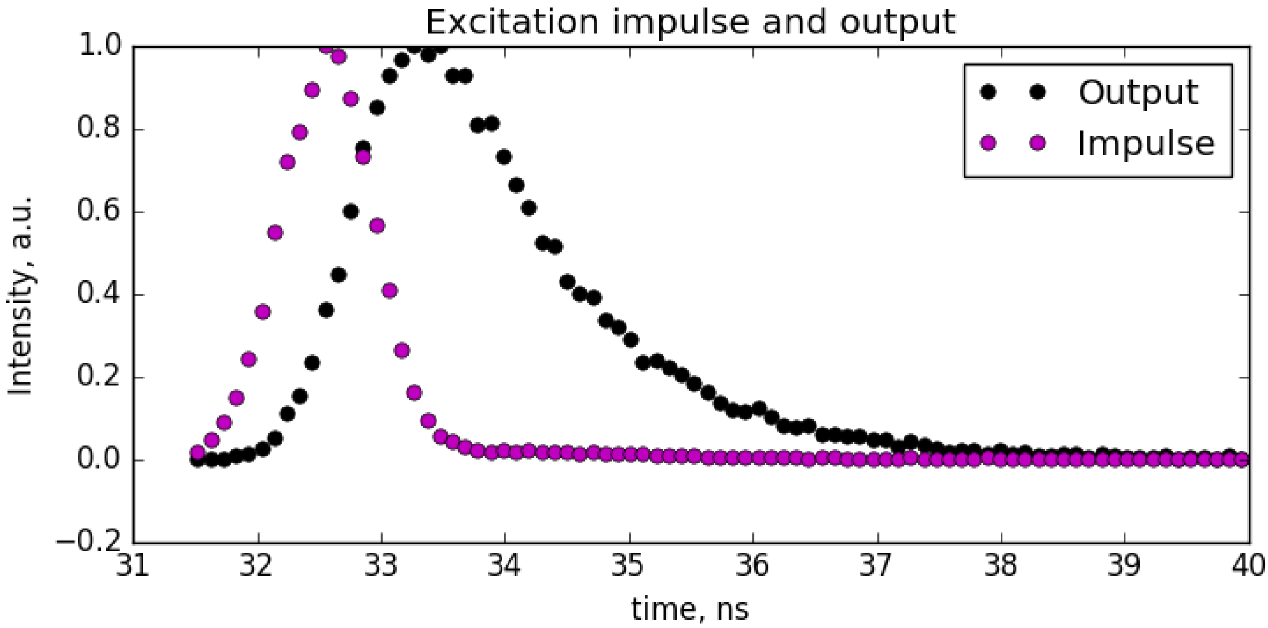
From this equation, $F(\omega) = Y(\omega)/G(\omega)$. Therefore by performing the inverse operation called “inverse Fourier transform”, one can obtain luminescence $f(t)$.

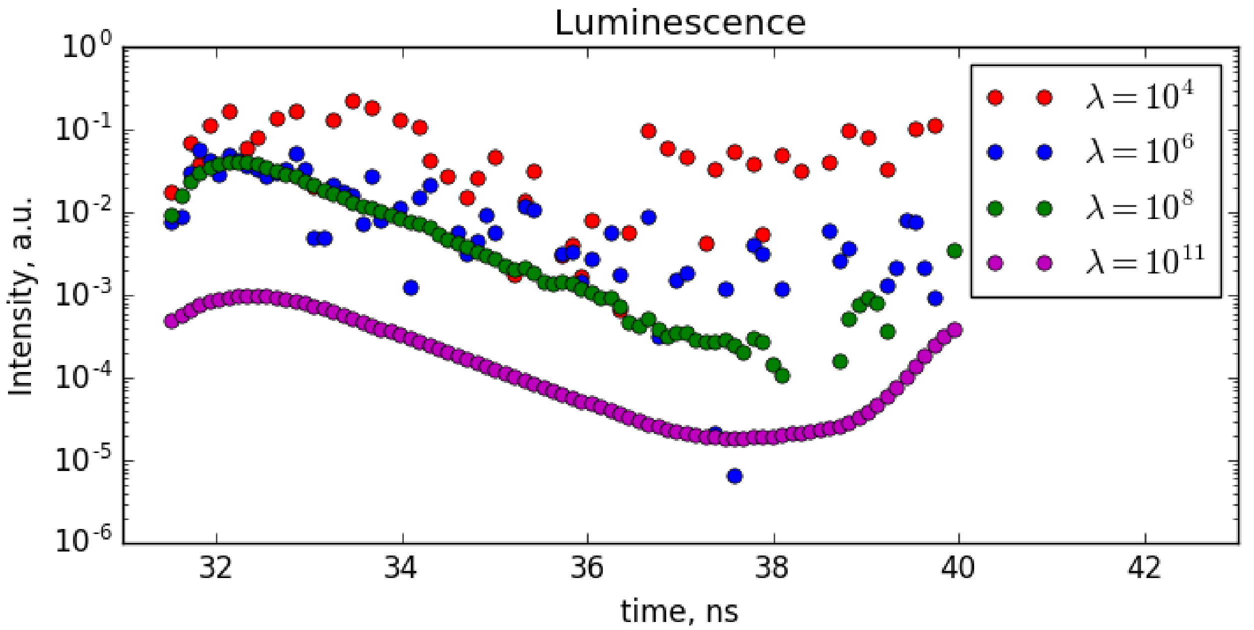
However, real measurements imply errors, so direct calculation will eventually provide inappropriate noise-like results. This is also caused by applying simplified **fast Fourier transform (FFT)** and its inverse analogue.

One of the possible solution to the problem is regularization. Here a modified Tikhonov regularization model has been used. The function $F(\omega)$ is multiplied and divided by conjugate $G^{*}(\omega)$: $F(\omega) = \frac{Y(\omega) \cdot G^{*}(\omega)}{G(\omega) \cdot G^{*}(\omega)}$. Later a small parameter λ (regularization parameter) is added in order to remove noise:

$F(\omega) = \frac{Y(\omega) \cdot G^{*}(\omega)}{G(\omega) \cdot G^{*}(\omega) + \lambda}$.

By exploring different values of λ, one can obtain a value, good enough to obtain $f(t)$. From the curves in the next figure it is clear, that the best value is $\lambda = 10^{8}$.





The simplest model for the luminescence decay is $f(t) = I_0 \exp(-t/t_0)$. Log-scaled intensity behaves linearly in the $33 \div 37$ nm span. Therefore, just performing linear fitting (regression), the decay kinetic constant t_0 could be obtained.

The input data file “BaF2_78nm.dat” contains of three columns: time t (ns), output y and excitation impulse g. The code file “FFT_regularization.py” performs FFT/inverse FFT transform using Tikhonov regularization on the arbitrary file **FFT_regularization.py <file name>** to obtain unknown integral luminescence decay function for polystyrene:BaF_2 composition (BaF_2 nanoparticles have mean size of 78 nm). It prints out four different functions

$f(t)$ to file “Output_BaF2_78nm.dat” for different regularization steps λ. The program also performs linear regression in order to calculate the decay kinetic constant t_0 (printed to stdout), using log-scaled data from $f(t)$.