A Kalman Filter for RSSI Tracking

Robert S. Huston

July 5, 2020

Abstract

This article investigates the application of a Kalman filter for estimating RSSI signal levels acquired via a mobile device.

1 Introduction

The purpose of this article is to investigate the feasibility of applying a Kalman filter to smooth a series of acquired RSSI readings of a nearby Bluetooth device. The RSSI readings are captured using an iPhone running a simple iOS application that monitors Bluetooth signals in its environment.

We use a Gauss-Markov process to model the RSSI acquisition behavior of a Bluetooth Low Energy (BLE) device signal as seen by the mobile device. We use this model because its autocorrelation function seems to match what we have observed from the signal behavior of experimental RSSI data captured over time.

We then present two experiments that illustrate our findings.

2 The Gauss-Markov Process

A stationary Gaussian process that has an exponential autocorrelation function is called a *Gauss-Markov* process. If we designate $x(t)$ as a Gauss-Markov process, its autocorrelation function is of the form

$$R_x(\tau) = \sigma^2 e^{-\beta\,|\tau|} \tag{1}$$

where σ is the process mean-square value, and $\frac{1}{\beta}$ is the process time constant. The Gauss-Markov process is a popular model because it fits a large number of physical processes, and because it is has a simple mathematical description. It basically describes a process where its values are more correlated when the time spacing between successive measurements is small, and less correlated when the time spacing between successive measurements is large.

The Gauss-Markov process has a spectral density function of the form

$$S_x(s) = \frac{2\sigma^2\beta}{-s^2 + \beta^2} \tag{2}$$

From linear system theory, we know that if we can factorize a spectral density function into LHP and RHP forms of the same factor function $G(\cdot)$:

$$S_x(s) = G(s)G(-s) \tag{3}$$

then $G(s)$ is the shaping filter that shapes unity white noise into the spectral density $S_x(s)$. Thus, for the Gauss-Markov spectral density, we have

$$S_x(s) = \left(\frac{\sqrt{2\sigma^2\beta}}{s + \beta}\right)\left(\frac{\sqrt{2\sigma^2\beta}}{-s + \beta}\right) \tag{4}$$

and so the shaping filter is

$$G(s) = \frac{\sqrt{2\sigma^2\beta}}{s + \beta} \tag{5}$$

and the corresponding differential equation is

$$\dot{x} = -\beta x + \sqrt{2\sigma^2\beta}\, w(t) \tag{6}$$

where $w(t)$ is a unity white noise process.

The complementary solution is

$$x_c(t) = e^{-\beta t} \tag{7}$$

Hence, the discrete-time state transition function is

$$\phi_k = e^{-\beta\tau_k} \tag{8}$$

and the discrete-time process variance is

$$\begin{aligned} Q_k &= E[w_k^2] \\ &= \int_0^{\tau_k} \left(\sqrt{2\sigma^2\beta}\; e^{-v}\right)^2 dv \\ &= \sigma^2\beta\left(1 - e^{-2\tau_k}\right) \end{aligned} \tag{9}$$

3 An RSSI Kalman Filter

The Kalman filter implementation of a Gauss-Markov process model can be implemented very efficiently since it involves only scalar arithmetic.

For the kth time point, we obtain a measurement z_k at time t_k. We model our measurement error with a variance R_k. At each time point, the filter provides a best estimate, $\hat{x}_k$ and maintains a state estimate variance, P_k. We initialize the filter with $\hat{x}_0 = z_0$ and $P_0 = P0$, where $P0$ is a suitably chosen value based on empirical analysis.

Because our measurement is direct, the measurement transformation $H_k = 1$, and as such, it will not be denoted in our filter equations.

For each acquisition event, k, we perform the following steps

1. Compute τ_k:

$$\tau_k = t_k - t_{k-1} \tag{10}$$

2. Compute ϕ_k and Q_k:

$$\phi_k = e^{-\beta\tau_k} \tag{11}$$

$$Q_k = \sigma^2\beta\left(1 - e^{-2\tau_k}\right) \tag{12}$$

3. Compute predicted state estimate:

$$\hat{x}_k^- = \phi_k\, \hat{x}_{k-1} \tag{13}$$

4. Compute predicted state estimate variance:

$$P_k^- = \phi_k \, P_{k-1} \, \phi_k + Q_k \tag{14}$$

5. Compute Kalman gain:

$$K_k = \frac{P_k^-}{P_k^- + R_k} \tag{15}$$

6. Compute updated state estimate:

$$\hat{x}_k^+ = \hat{x}_k^- + K_k \left(z_k - \hat{x}_k^- \right) \tag{16}$$

7. Compute updated state estimate variance:

$$P_k^+ = \left(1 - K_k \right) P_k^- \tag{17}$$

4 Experiments

A simple iOS application was developed that enables an iPhone to continuously scan for nearby Bluetooth devices, reading among other properties, their RSSI, UUID, and device name properties. Each reading gets timestamped and logged to a file that an be copied to a computer for further processing. A simple Python script was written to allow for the selective processing and plotting of data based on device UUID.

Two experiments were then performed. The first experiment involved walking the scanning iPhone (an iPhone Xs Max) away throughout the house (hallway and then dining room) from the development MacBook Pro computer (kitchen) and then moved back again. The second experiment involved taking both the scanning iPhone and a second iPhone outside and then walking the scanning iPhone from the corner of the back yard to the second iPhone and then away again. Figure 1 depicts a simple diagram of the walking routes taken (shown in red) for each experiment. The distance values are approximate.

In all cases, the RSSI Kalman filters were initialized with the following parameters:

$$P_0 = 5 \quad \sigma = 10 \quad \beta = 0.001 \quad R = 10$$

One particular note about the second experiment was that the second iPhone screen was active when the device was placed on a table on the deck before initiating the experiment, but, by the time the walk from the corner of the yard had reached the second iPhone, it was observed that the screen of the second iPhone had just gone to sleep. This can be seen in the rate of the acquired data, where the "awake" condition produced more Bluetooth scan

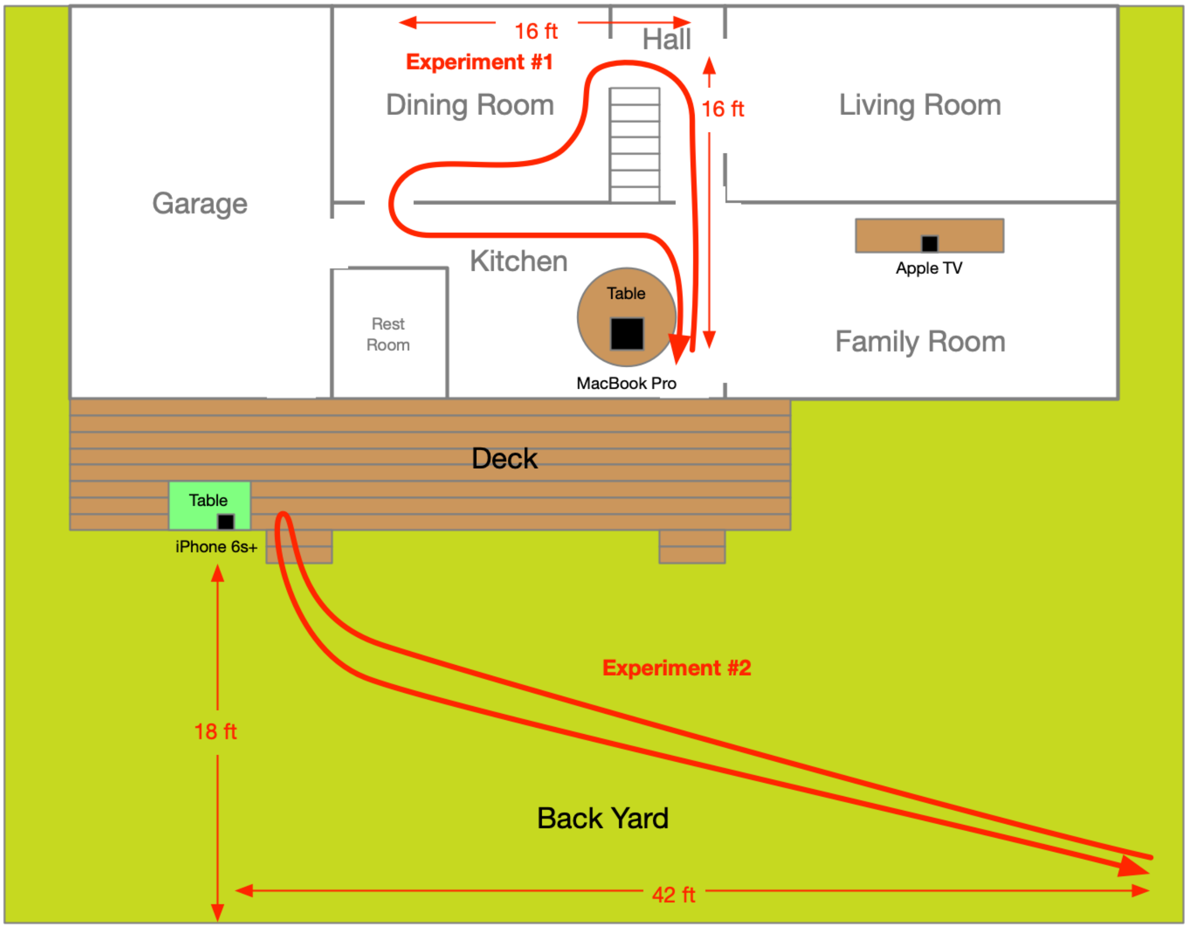

Figure 1: Diagram of Experiment Walk Routes (Shown in Red)

responses than the "sleep" condition. This was a fortunate event, because it provided a perfect data set to illustrate why a Gauss-Markov process model is suitable.

For the first experiment, the UUID of the development MacBook Pro (Red Green) was monitored. The scanning iPhone was walked away from and then back to the computer. The results can be seen in Figure 2. The RSSI measurements are blue, and the filtered RSSI values are in red.

For the second experiment, both the scanning iPhone and the second stationary iPhone (an iPhone 6s+) were outside in the back yard. The scanning iPhone was walked from the corner of the yard to the stationary iPhone and back again. As was stated previously, the stationary iPhone screen went to sleep when the scanning iPhone reached the stationary iPhone location. The scanning iPhone was then walked away back to the corner of the yard. The results for the second iPhone can be seen in Figure 3.

Also available from the second experiment was the scanned Bluetooth signal data from both

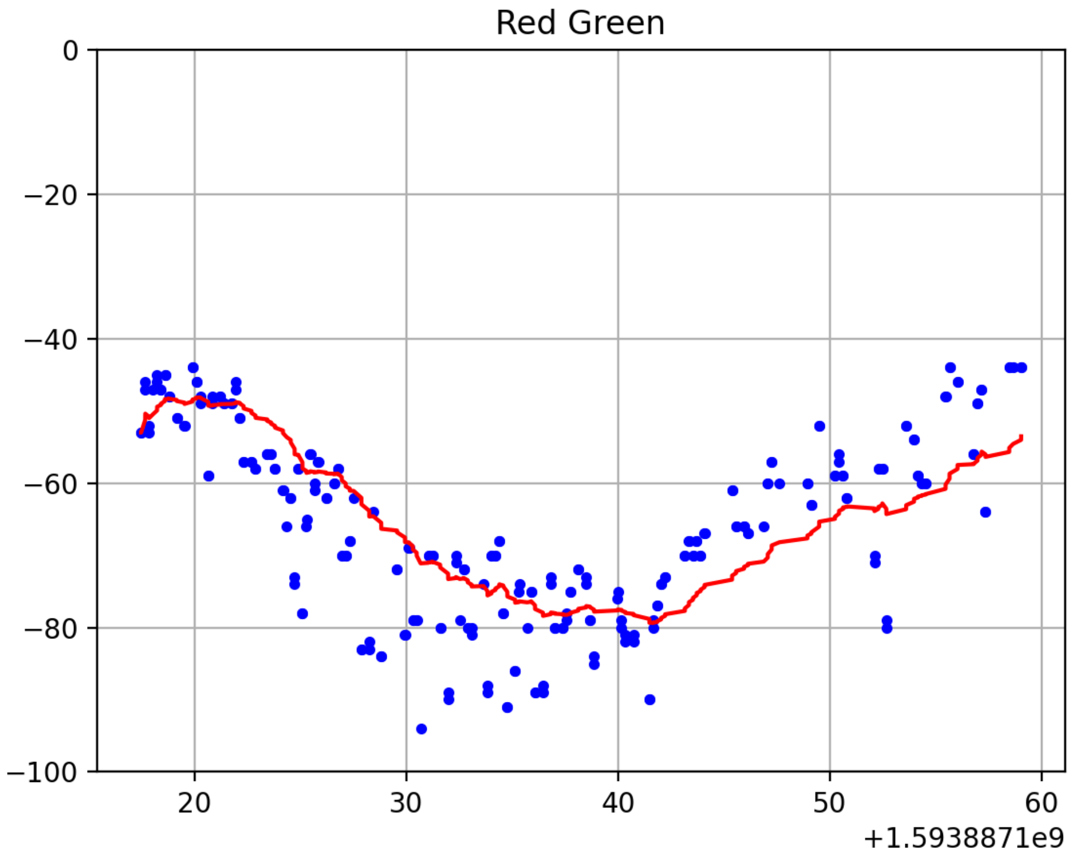

Figure 2: First Experiment - Scanning MacBook Pro from Indoors

the development MacBook Pro computer in the kitchen, and an Apple TV in the family room. As shown in Figure 1, both devices are in rooms that face the back yard, and their distances from the scanning iPhone are comparable. The results for the MacBook Pro can be seen in Figure 4, and the results for the Apple TV can be seen in Figure 5.

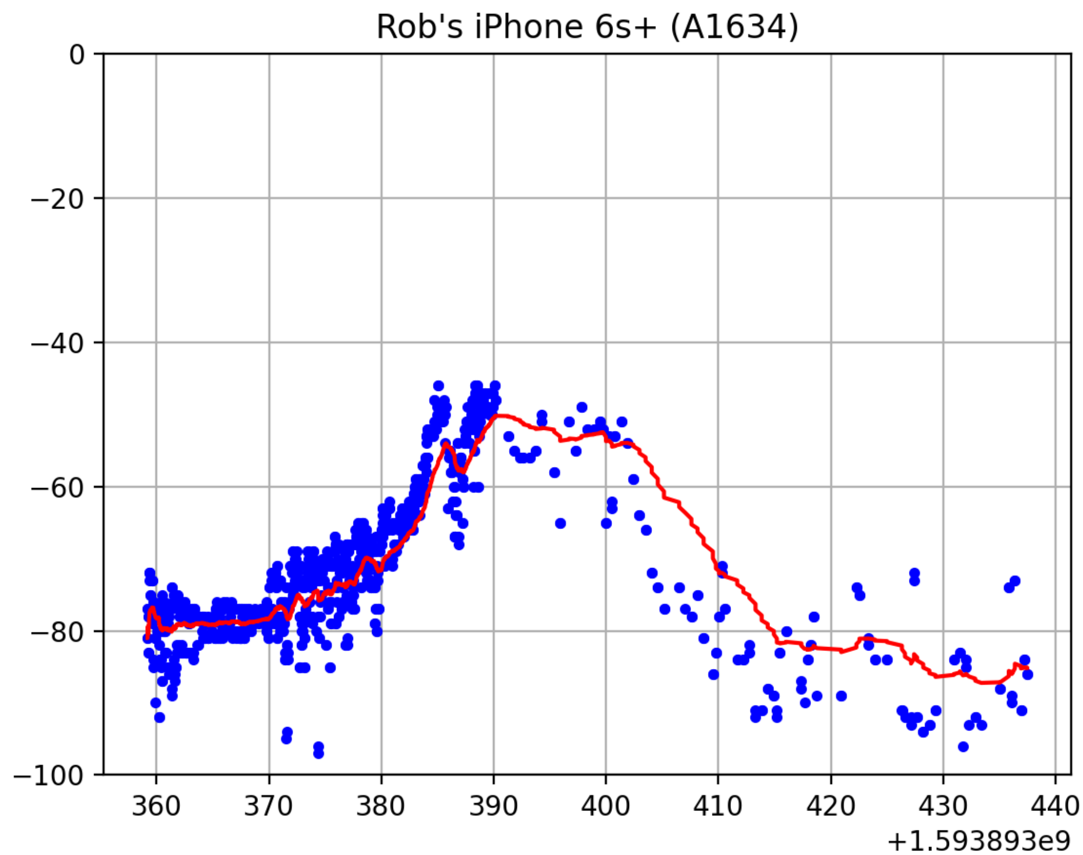

Figure 3: Second Experiment - Scanning Second iPhone from Back Yard

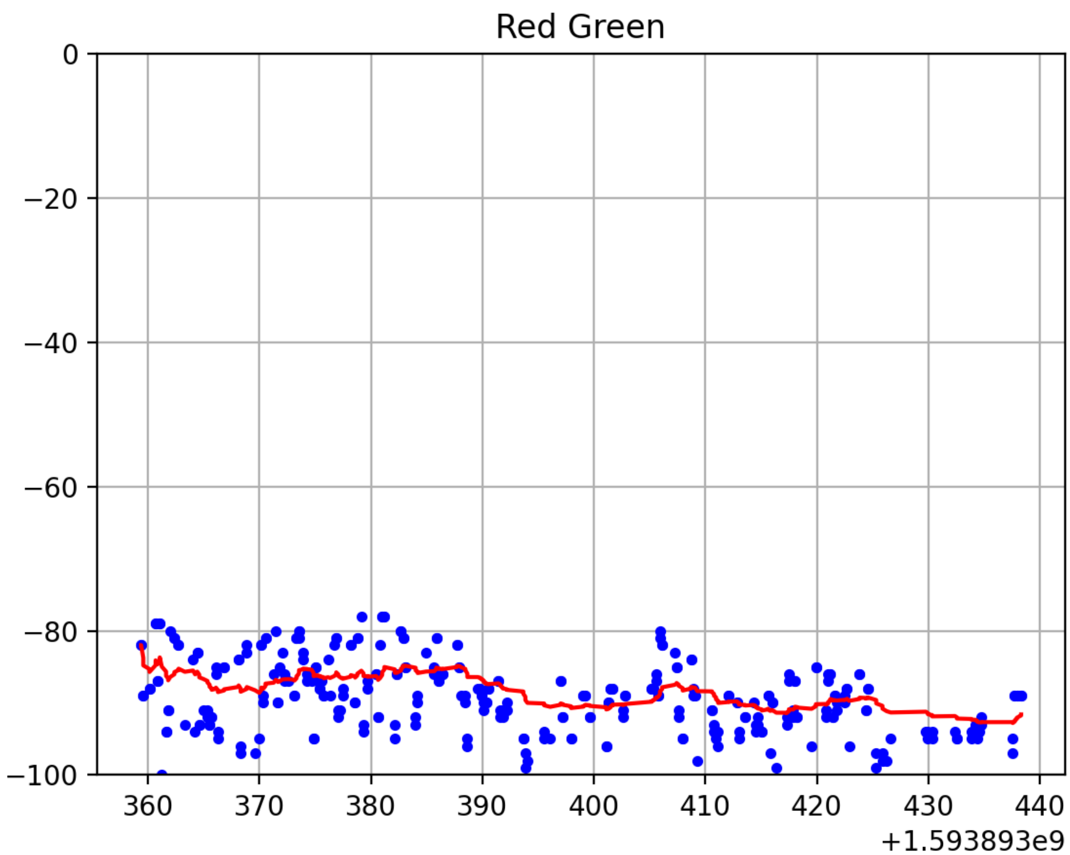

Figure 4: Second Experiment - Scanning MacBook Pro from Back Yard

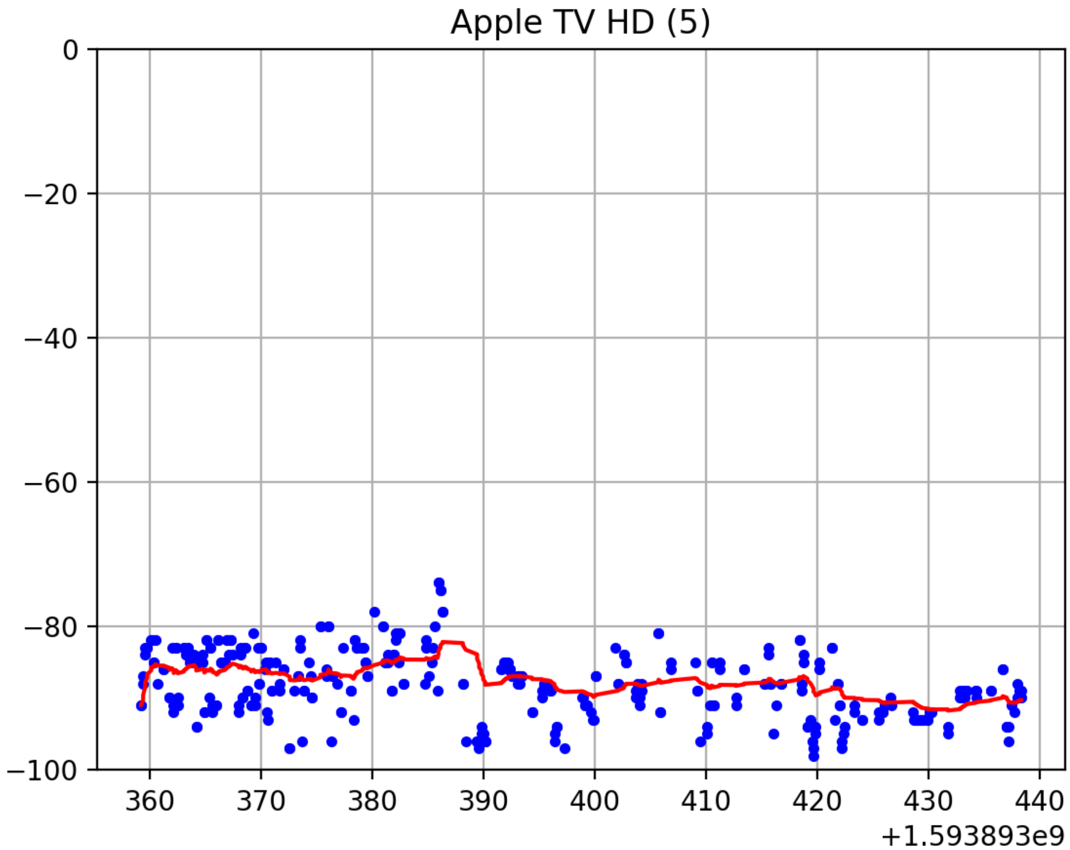

Figure 5: Second Experiment - Scanning Apple TV from Back Yard

5 Summary

Although this was a simple exercise, it can be easily seen that, by suitably filtering the raw RSSI measurements, one can obtain a more reliable value that has less jitter than the raw measurement.

There certainly exist other models that may better lend themselves to RSSI tracking. With suitable characterization data, one can incorporate this data into the Kalman filter process model, which would improve the performance of the filter.

While additional refining of tuning the filter parameters is needed before producing a production-worthy filter, it can be concluded that the feasibility of using a Kalman filter for tracking RSSI data is justified.

General References

[1] Brown, R. G., 1983,
Introduction to Random Signal Analysis and Kalman Filtering, John Wiley & Sons, Inc., New York, NY.

[2] Sorenson, H. W. (Ed.), 1985,
Kalman Filtering: Theory and Application, IEEE Press, New York, NY.

[3] Chui, C. K. and Chen, G., 1987,
Kalman Filtering with Real-Time Applications, Springer-Verlag, New York, NY.

[4] Grewal, M. S., and Andrews, A. P., 1993,
Kalman Filtering: Theory and Practice, Prentice-Hall, Englewood Cliffs, NJ.