

# Practical integrator using operational amplifier

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# Contents

<b>1</b>	<b>Aim</b>	<b>1</b>
<b>2</b>	<b>Circuit diagram</b>	<b>2</b>
<b>3</b>	<b>Theory</b>	<b>3</b>
<b>4</b>	<b>Design</b>	<b>7</b>
<b>5</b>	<b>Simulation</b>	<b>8</b>
<b>6</b>	<b>Waveforms</b>	<b>9</b>
<b>7</b>	<b>Observations</b>	<b>10</b>
<b>8</b>	<b>Results &amp; Conclusions</b>	<b>12</b>

# 1 Aim

To design, implement and test a  $\mu\text{A}741$ -based voltage integrator.

## Components required

- $\mu\text{A}741$  op-amp
- Resistors  $R = 120\text{k}\Omega, 3.3\text{k}\Omega, 4.7\text{k}\Omega$
- Capacitor  $C = 0.01 \mu\text{F}$
- Signal generator
- Digital Storage Oscilloscope
- Breadboard
- Jumper wires

## 2 Circuit diagram

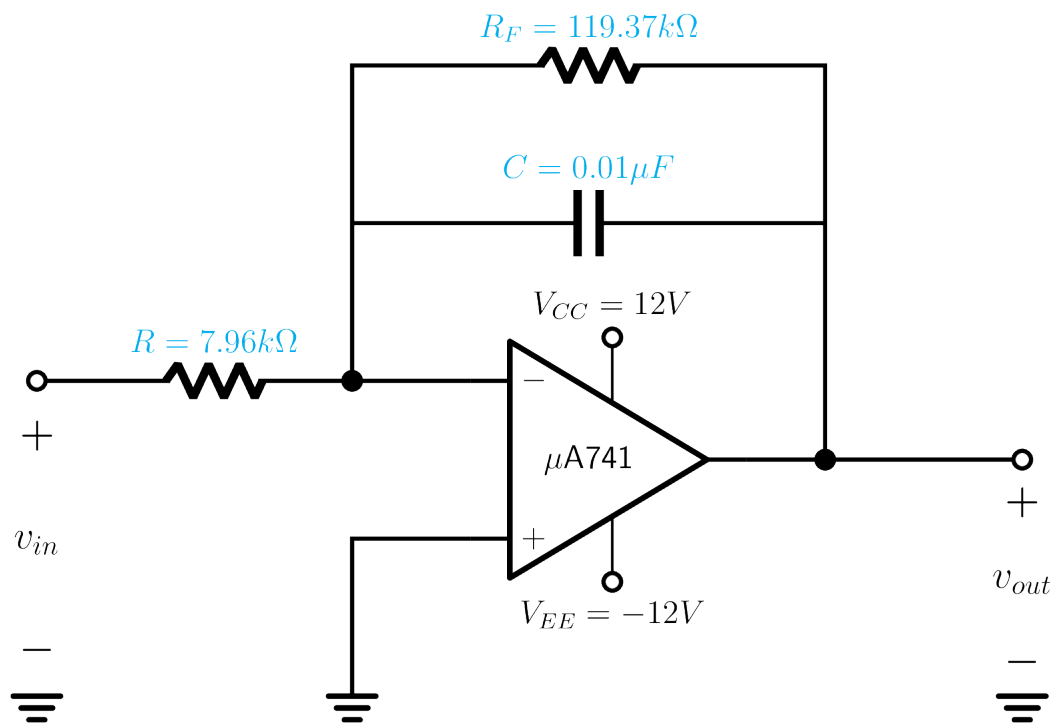


Figure 1: Final design of the practical integrator

### 3 Theory

The operational amplifier based integrator performs the mathematical operation of integration with respect to time, i.e. its output is proportional to the input voltage integrated over time. The integrator circuit is mostly used in analog computers, analog-to-digital converters and wave-shaping circuits such as charge amplifiers.

The response of an op-amp circuit with feedback reflects the characteristics of the feedback elements. Thus, in order to achieve integration, the feedback network is constructed using a capacitor.

An operational amplifier based integrator can ideally be constructed as shown in figure 2.

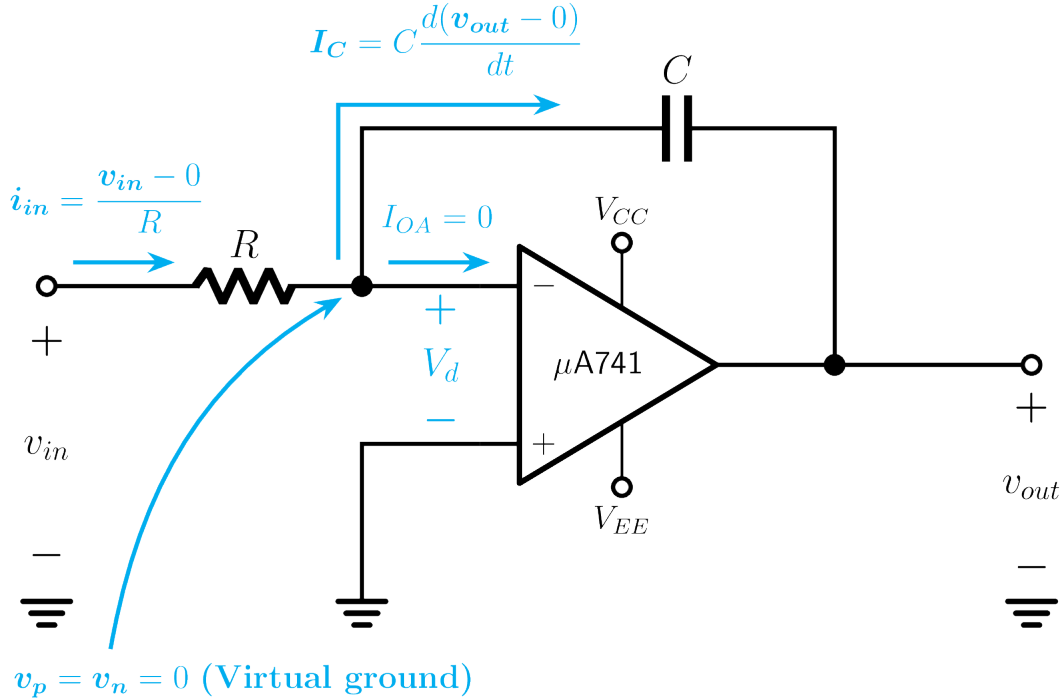


Figure 2: Ideal op-amp based integrator

The circuit can be analysed by applying Kirchhoff's current law at the node  $V_n$ , keeping ideal op-amp behaviour in mind.

$$i_{in} = I_C = \frac{V_{in}}{R} \quad (1)$$

The relationship between the capacitor's voltage and current is modelled as:

$$I_C = C \frac{dV_C}{dt} \quad (2)$$

Substituting eq. 1 in eq. 2:

$$\frac{v_{in} - 0}{R} = C \frac{d(0 - v_{out})}{dt} \implies \frac{v_{in}}{R} = -C \frac{dv_{out}}{dt}$$

Integrating both sides with respect to time:

$$\begin{aligned} \int_0^t \frac{v_{in}}{R} dt &= - \int_0^t \frac{dv_{out}}{dt} dt \\ v_{out} &= -\frac{1}{RC} \int_0^t \frac{dv_{out}}{dt} dt \end{aligned}$$

Thus the output of the circuit shown in figure 2 is the inverted, integrated input with a gain of  $\frac{1}{RC}$ .

The ideal integrator suffers from two main limitations. One comes from the fact that the output voltage of the op-amp cannot exceed the supply voltage. The output of the integrator is inversely proportional to the time constant  $\tau = RC$ . The larger the time constant  $\tau$ , the longer it takes to saturate the integrator. The second limitation is a consequence of the offset voltage present even for zero input. It may be only a few millivolts, but this gets integrated over time and it eventually drives the op-amp to saturation.

One can overcome the limitations of an ideal integrator by adding a resistor  $R_f$  in parallel with capacitor  $C$  as shown in figure 3.  $R_f$  prevents the op-amp from going into open loop configuration at low frequencies.

The resulting circuit is an inverting amplifier with  $R_f || \frac{1}{Cs}$  acting as the feedback resistance. The gain of such an amplifier is given by:

$$\begin{aligned} \frac{V_{out}(s)}{V_{in}(s)} &= -\frac{R_f || \frac{1}{Cs}}{R} \\ &= -\frac{R_f}{R} \frac{1}{(1 + R_f Cs)} \end{aligned}$$

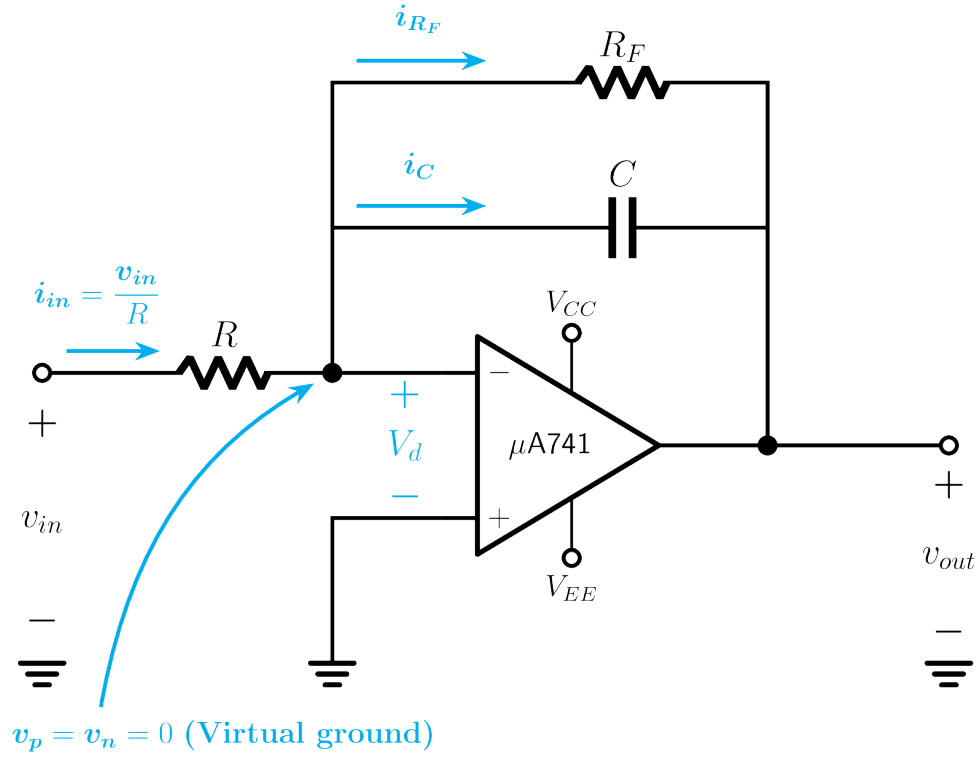


Figure 3: Practical integrator circuit

Thus, the frequency response of the practical integrator is given by,

$$H(s) = \frac{-R_F || \frac{1}{Cs}}{R} \quad (3)$$

$$H(j\omega) = \frac{-R_F}{R} \left[ \frac{1}{1 + R_F C j\omega} \right] \quad (4)$$

The magnitude and phase response are given by,

$$|H(j\omega)| = \frac{R_F}{R} \frac{1}{\sqrt{1 + \omega^2 R_F^2 C^2}}$$

$$\angle H(j\omega) = \pi - \tan^{-1}(\omega R_F C)$$

DC gain is obtained by putting  $\omega = 0$  in equation 4.

$$\text{DC gain} = H(j0) = -\frac{R_f}{R} \equiv -20 \log_{10} \left( \frac{R_F}{R} \right) \text{ dB} \quad (5)$$

Phase shift  $\Delta\phi$  between the input and output signals is given by

$$\Delta\phi = \pi - \tan^{-1}(\omega R_F C) \quad (6)$$

The practical integrator acts as a first order filter with corner frequency (also cut-off or pole frequency) at  $f = -1/R_F C$ . Thus,  $f_{-3dB}$  or cutoff-frequency is given by

$$f_{-3dB} = \frac{1}{2\pi R_F C} \quad (7)$$

Unity gain bandwidth  $\omega_u$  is obtained from  $|H(j\omega)| = 1$ :

$$\begin{aligned} H(j\omega) = 1 &= \frac{R_F}{R} \frac{1}{\sqrt{1 + \omega^2 R_F^2 C^2}} \\ \left(\frac{R_F}{R}\right)^2 &= 1 + (\omega R_F C)^2 \\ \left(\frac{R_F}{R}\right)^2 &\approx (\omega R_F C)^2 \quad \left(\text{for } \frac{R_F}{R} \gg 1\right) \\ \Rightarrow \omega_u &= \frac{1}{RC} \text{ or } f_u = \frac{1}{2\pi RC} \end{aligned} \quad (8)$$

Roll-off rate of the frequency response is theoretically -20 dB/decade.



## 4 Design

Q. Design a  $\mu A741$  based voltage integrator with unity gain and  $f_{-3dB} = f_{in}/15$  for a sinusoidal input  $v_{in} = 2\sin(4000\pi t)$ , keeping the phase error below 5%.

$$\text{Let } C = 0.01\mu F,$$

$$f_{in} = 2000Hz$$

$$f_{-3dB} = \frac{f_{in}}{15} = 133.33Hz$$

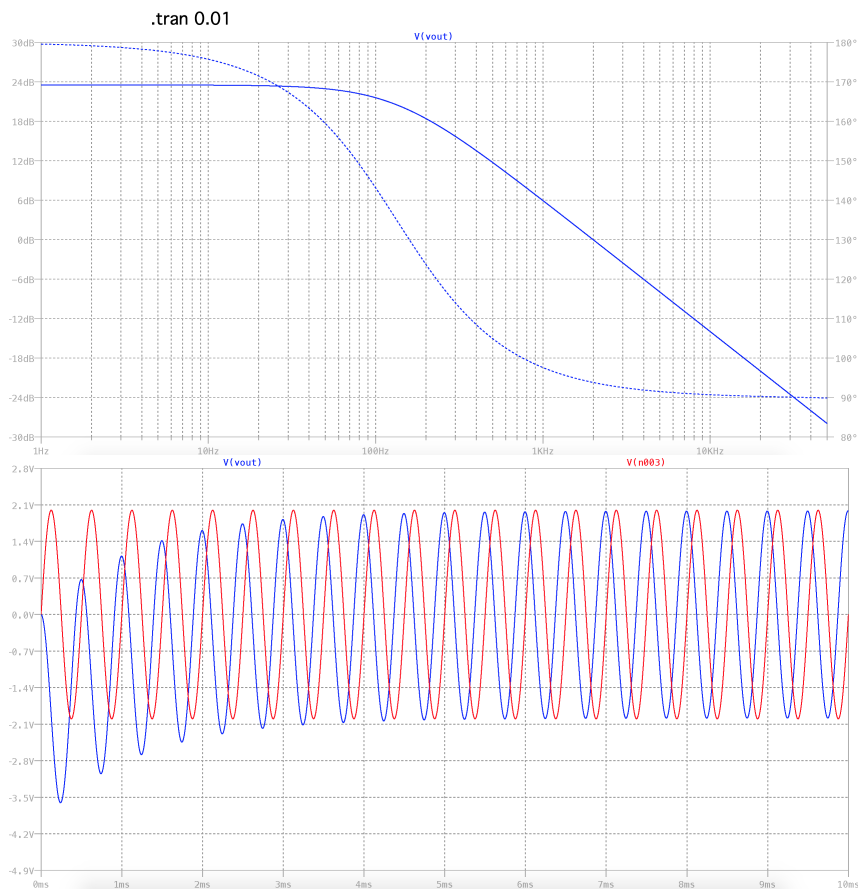
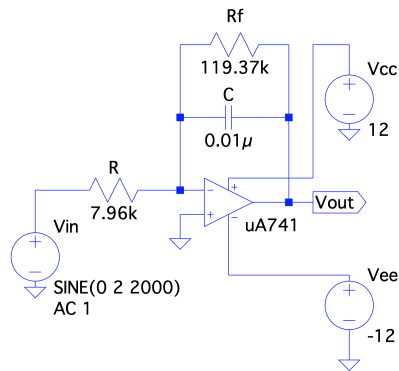
$$\frac{1}{2\pi R_F C} = 133.33$$

$$\frac{1}{2\pi RC} = 2000$$

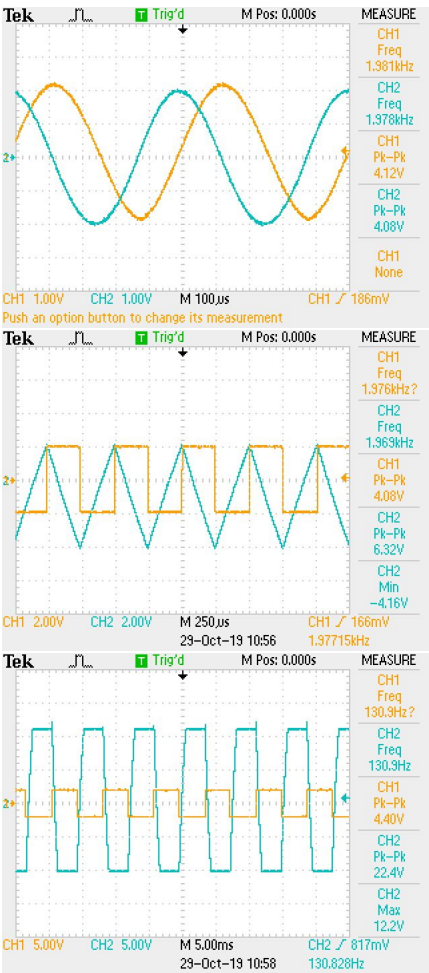
$$R = 7.96k\Omega$$

$$R_F = 119.37k\Omega$$

## 5 Simulation



# 6 Waveforms



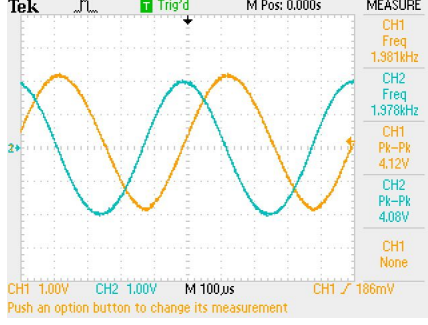


Figure 4: Result of passing a sinusoidal input through the integrator;  $V_{out}$  &  $V_{in}$

## 7 Observations

A. The practical integrator is first designed based on the following constraints: Unity gain,  $f_{-3dB} = f_{in}/15$  for a sinusoidal input  $v_{in} = 2\sin(4000\pi t)$  and a phase error below 5%.

A leading phase sinusoidal wave appears at the output of the integrator as shown in figure 4. The DC gain, -3dB frequency  $f_{3dB}$ , unity gain frequency  $f_u$ , roll-off rate and phase shift are noted from the waveforms obtained on the DSO and the phase error is verified to be below 5%.

DC gain = 14.89

-3dB frequency  $f_{-3dB} = 132$  Hz

Unity gain frequency  $f_u = 2.07$  kHz

Roll-off rate = -18.35 dB/decade

Phase shift  $\phi = 1.608^\circ$  or  $92.13^\circ$ , an error of 2.4%

B. The feedback resistor  $R_F$  is then removed and the effect on the op-amp integrator configuration is observed.

The output is given by  $V_{out} = \pm V_{sat}$  due to the capacitor acting as an open circuit at low frequencies as shown in figure 5. This sends the op-amp into an open loop configuration.

C. The sinusoidal input is replaced by a square wave of  $4V_{pk-pk}$  amplitude and a frequency of 2kHz to observe the integration operation of the configuration more clearly.

A triangular waveform is obtained as a result of the square wave being integrated, as shown in figure 6.

$$C\Delta V = I\Delta t$$

Substituting  $C = 0.01\mu F$ ,  $I = \frac{2}{8k}$ ,  $\Delta t = \frac{0.5}{2000}$ ,

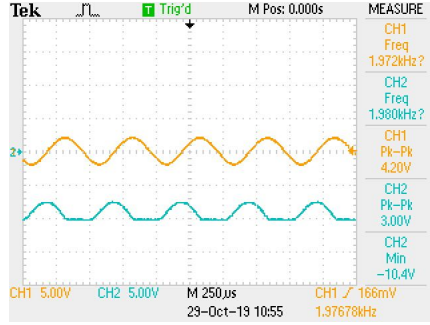


Figure 5: The output of the integrator circuit without  $R_F$

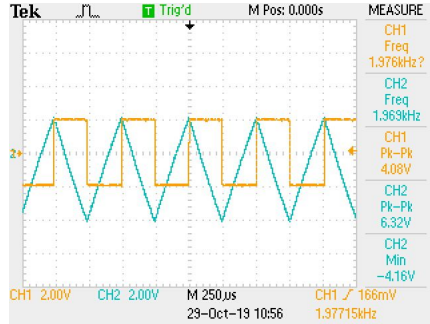


Figure 6: Result of integrating a square wave  $V_{out}$  &  $V_{in}$

$$\Delta V = 6.25V$$

The output triangular waveform has a peak to peak voltage of around 6.25 V.

D. The frequency of the input is lowered to 130Hz and the output is observed again on the DSO.

$$C\Delta V = I\Delta t$$

$$\text{Substituting } C = 0.01\mu F, I = \frac{2}{8k}, \Delta t = \frac{0.5}{130},$$

$$\Delta V = 96.15V$$

Since  $\Delta V > 2V_{sat}$ ,  $V_{out}$  gets clipped as shown in figure 7.

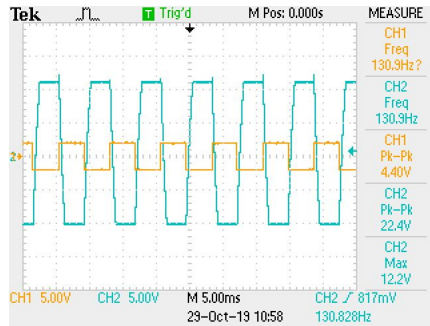


Figure 7: Effect of lowering the input frequency on the output  $V_{out}$  &  $V_{in}$

## 8 Results & Conclusions

The  $\mu$ A741 based voltage integrator was designed and implemented successfully.