MULTIDATASET INDEPENDENT SUBSPACE ANALYSIS

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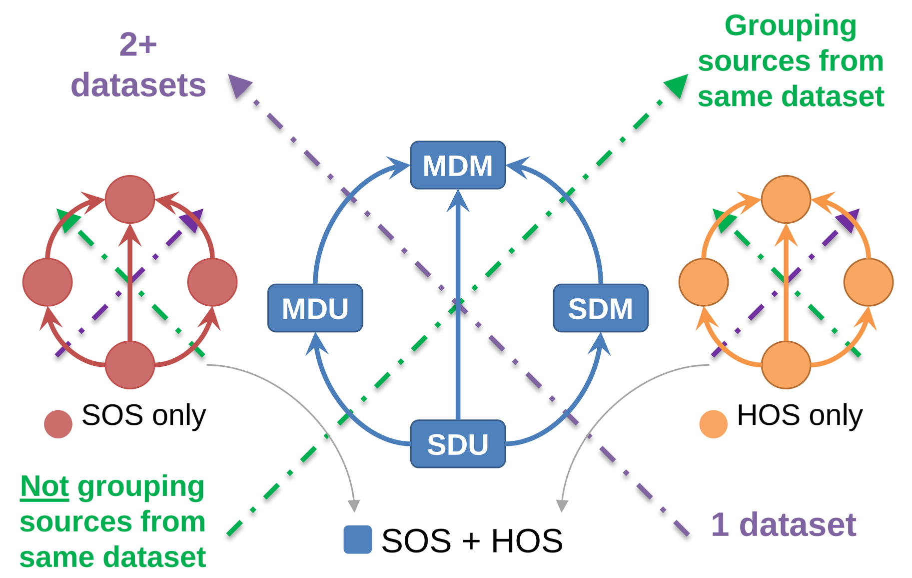
Hierarchy of Linear Factor Models [6]



- Single Dataset (**SD**) → Multidataset (**MD**)
- Unidimensional (**U**) → Multidimensional (**M**)
- **SOS** or **HOS** → **SOS** + **HOS**

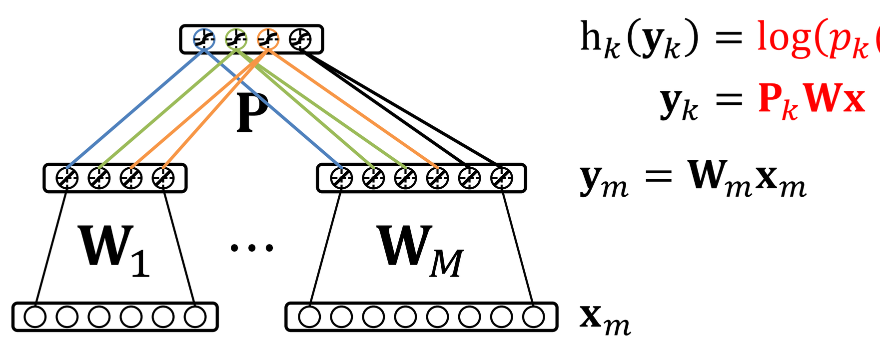
SOS: Second-Order Statistics, **HOS**: Higher-OS

- **Goal**: Solve MDM problems w/ SOS + HOS [5]

MISA: A Generalized Model for Independent Features

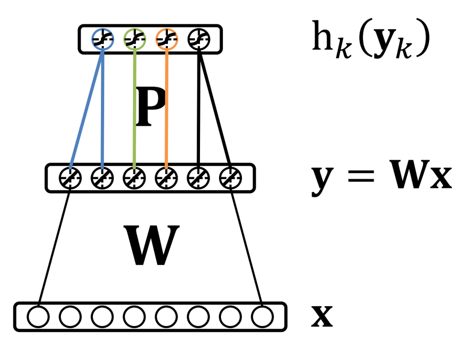
$$J(\mathbf{W}) = KL\left(p(\mathbf{y}), \prod_k^K p_k(\mathbf{y}_k|\mathbf{x})\right),\ \mathbf{W} = \text{blkdiag}(\mathbf{W}_m),\ \mathbf{x} = [\mathbf{x}_1^\top \ \cdots \ \mathbf{x}_M^\top]^\top$$

$$J(\mathbf{W}) = -\sum_m^M \sum_i^{C_m} \log|\sigma_{mi}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)] \qquad J(\mathbf{W}) = -\sum_i^C \log|\sigma_i| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)]$$



$$\mathrm{h}_k(\mathbf{y}_k) = \log(p_k(\mathbf{y}_k|\mathbf{x}))$$
$$\mathbf{y}_k = \mathbf{P}_k\mathbf{W}\mathbf{x}$$
$$\mathbf{y}_m = \mathbf{W}_m\mathbf{x}_m$$

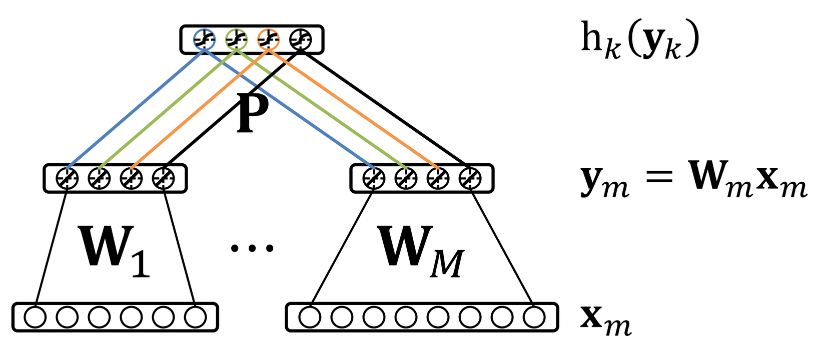
MISA



$$\mathrm{h}_k(\mathbf{y}_k)$$
$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

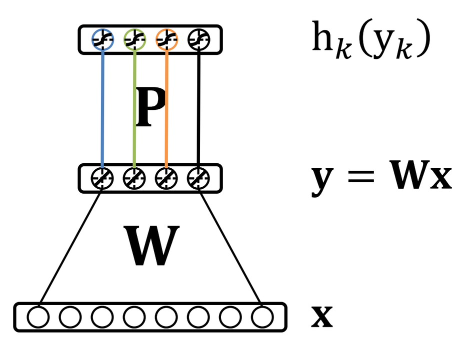
ISA

$$J(\mathbf{W}) = -\sum_m^M \sum_i^{C_m=K} \log|\sigma_{mi}| - \sum_k^K \mathbb{E}[\mathrm{h}_k(\mathbf{y}_k)] \qquad J(\mathbf{W}) = -\sum_i^{C=K} \log|\sigma_i| - \sum_k^K \mathbb{E}[\mathrm{h}_k(y_k)]$$



$$\mathrm{h}_k(\mathbf{y}_k)$$
$$\mathbf{y}_m = \mathbf{W}_m\mathbf{x}_m$$

IVA



$$\mathrm{h}_k(y_k)$$
$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

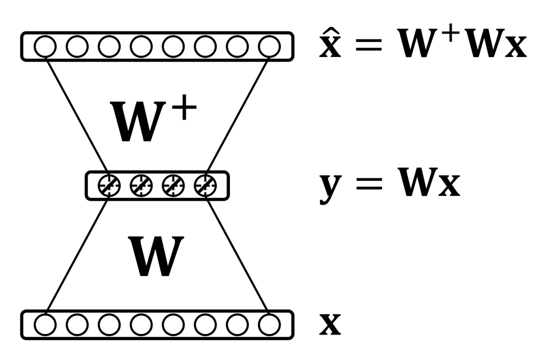
ICA

ICA: Independent Component Analysis ISA: Indep. Subspace Analysis [2,3] IVA: Independent Vector Analysis [4]

Regularization: Modified Linear Autoencoder

- Assuming white data $\mathbf{z}$, Le et al. [1] regularized ICA with a simple linear autoencoder $\hat{\mathbf{z}} = \mathbf{W}^\top\mathbf{W}\mathbf{z}$.
- Simultaneously estimate the reduced space while learning statistically independent hidden units.
- Modified linear autoencoder for non-white data: $\hat{\mathbf{x}} = \mathbf{W}^+\mathbf{W}\mathbf{x}$. Applied to all modalities as a **non-linear constraint**.

$$R(\mathbf{W}) = \|\mathbf{W}^+\mathbf{W}\mathbf{x} - \mathbf{x}\|_2^2$$

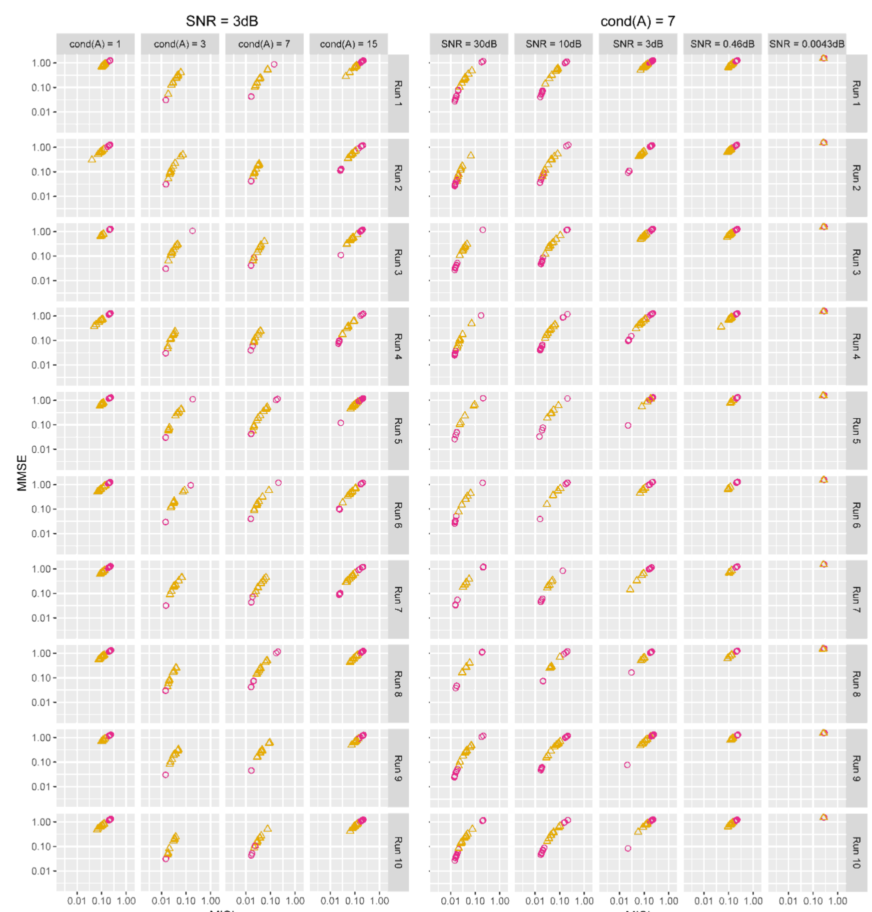


$$\hat{\mathbf{x}} = \mathbf{W}^+\mathbf{W}\mathbf{x}$$
$$\mathbf{y} = \mathbf{W}\mathbf{x}$$

Synthetic Data

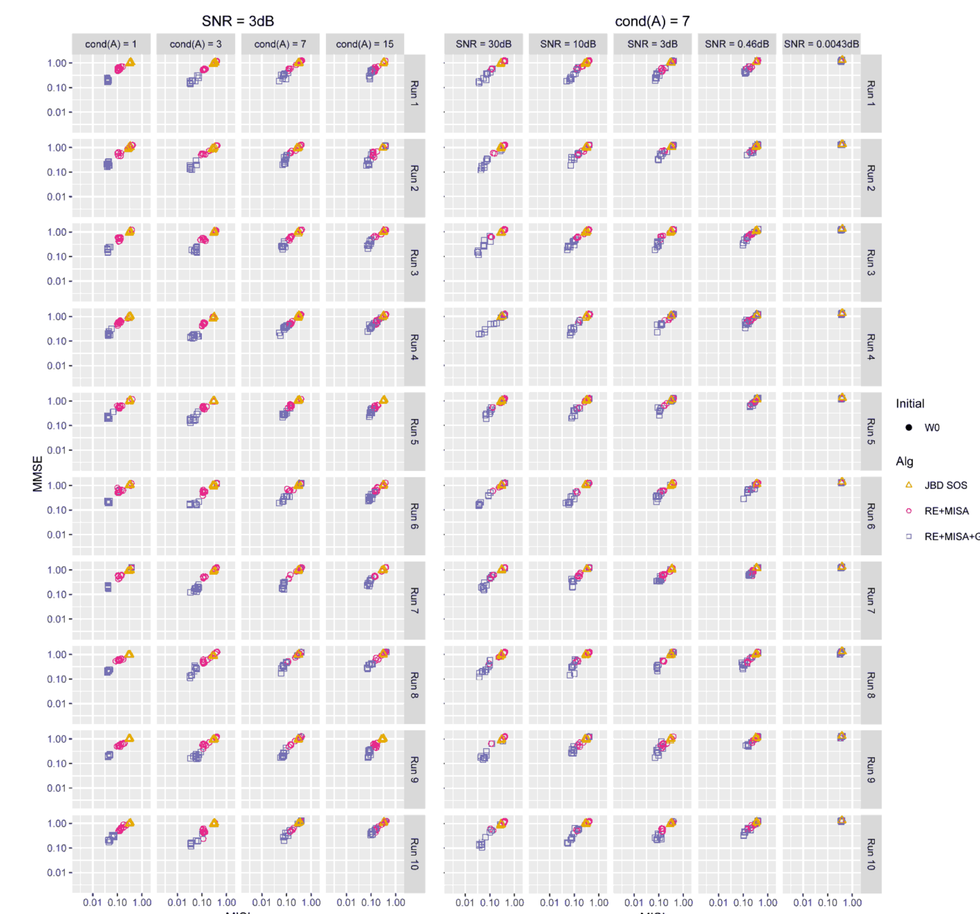
ICA

- M = 1 dataset
- C = 75 hidden units into K = 75 one-dimensional subspaces
- N = 3500 examples sampled from a Laplace distribution
- Each run a new, unique (V × C) rectangular mixing matrix A (V = 8000)
- Initialized with ten different random row-orthogonal W0



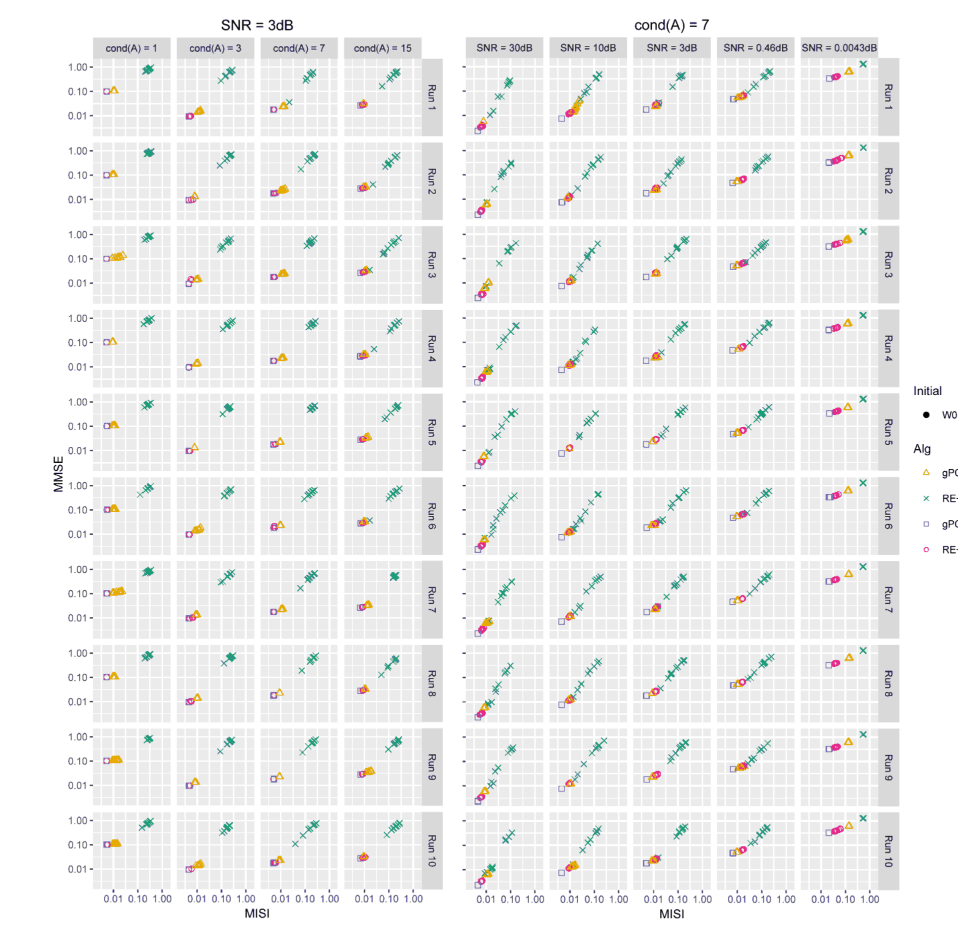
ISA

- M = 1 Dataset
- C = 51 hidden units
- K = 18 d_k-dimensional subspaces, d_k = [1:5; 5:1; 1:5; 2; 2; 2]
- N = 5250 examples sampled from a Multivariate Laplace Distribution
- Each run a new, unique (V × C) rectangular mixing matrix A (V = 8000)
- Initialized with ten different random row-orthogonal W0



IVA

- M = 16 datasets
- C = 75 hidden units into K = 75 sixteen-dimensional subspaces
- N = 66000 examples sampled from a Multivariate Laplace Distribution
- Each run a new, unique (V × C) rectangular mixing matrix A (V = 250)
- Initialized with ten different random row-orthogonal W0



MISA + modified autoencoder (RE) yields good performance despite small training sets.

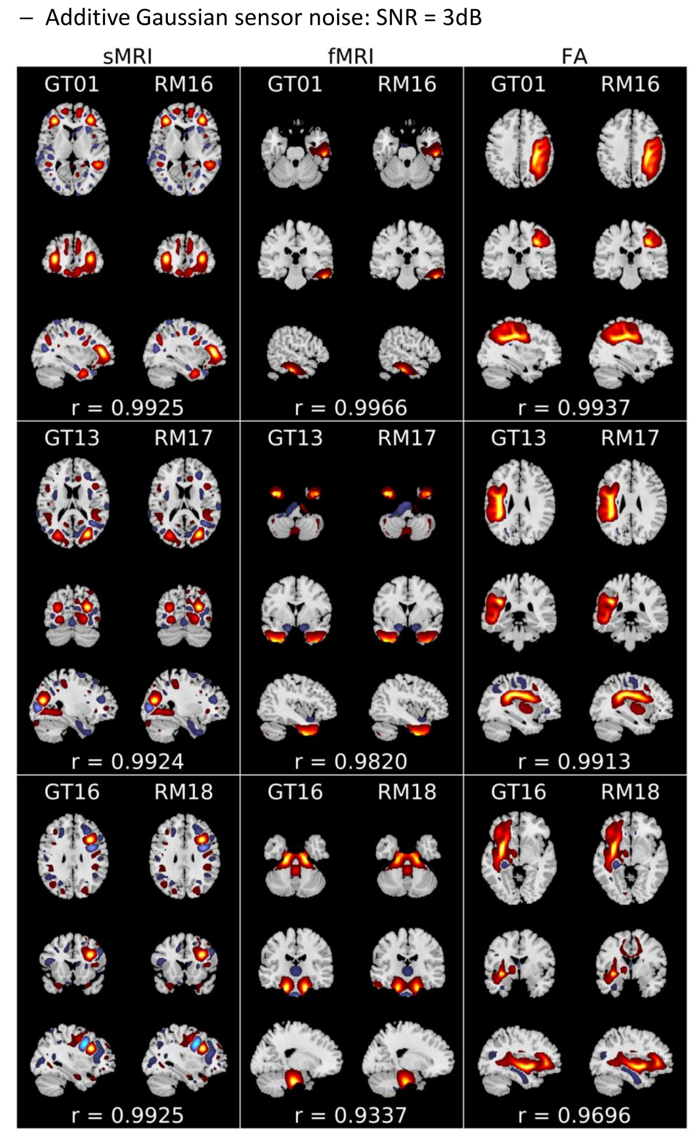
ISA & MISA + GP: Greedy combinatorial optimizer significantly improves performance.

Greedy Combinatorial Optimizer

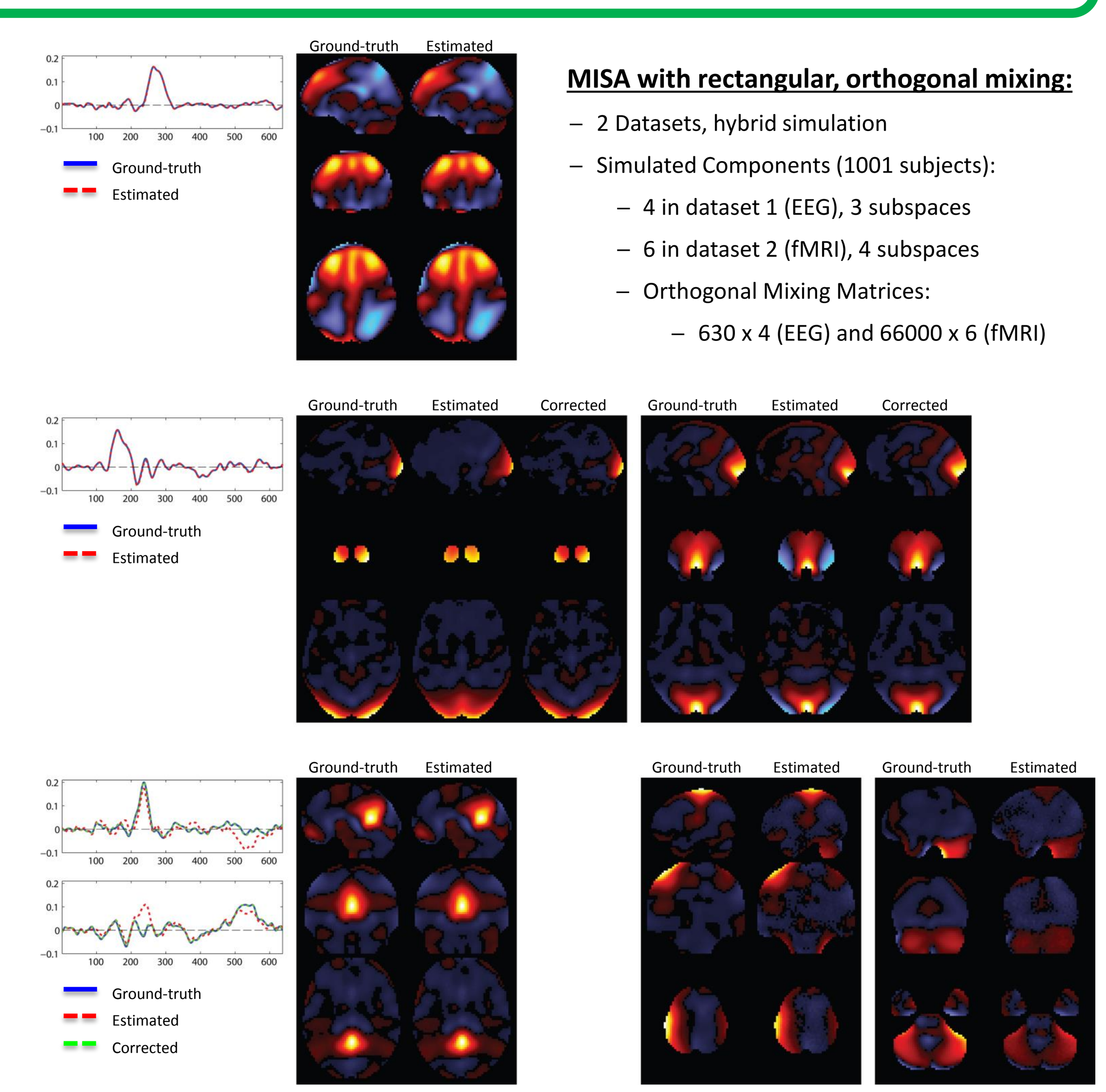
- ISA & MISA: local minima due to incorrect assignment of hidden units.
- Greedy combinatorial optimizer moves the solution out of the local minimum.

Multimodal Hybrid Data

- M = 16 datasets
  - sMRI: V ≈ 300K voxels, fMRI: V ≈ 67K voxels, FA: V ≈ 15K voxels
- Mixing matrices $\mathbf{A}_m$ are the "real" part of the datasets
  - condition numbers: 1.52, 4.59, 1.63
- C = 20 hidden units into K = 20 three-dimensional subspaces
- N = 600 examples sampled from a Gaussian copula
- Additive Gaussian sensor noise: SNR = 3dB



IVA



MISA with rectangular, orthogonal mixing:

- 2 Datasets, hybrid simulation
- Simulated Components (1001 subjects):
  - 4 in dataset 1 (EEG), 3 subspaces
  - 6 in dataset 2 (fMRI), 4 subspaces
  - Orthogonal Mixing Matrices:
    - 630 x 4 (EEG) and 66000 x 6 (fMRI)

Excellent performance on realistic data settings.

What's Next?

- Explore deep non-linear models before subspace formation to identify optimal modality-specific depth.
- Learn the strictly sparse Subspace Assignment Matrix $\mathbf{P}$ automatically.
- Modality-specific architectures such as RNNs with hidden independent subspaces for modalities with sequential data

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