

# Week 1: Modeling Goals, Parameters, & Structures

Dr. Rachel Sippy University of Cambridge

#### Objectives

- Add your first bullet point here
- Add your second bullet point here
- Add your third bullet point here

# Post Questions in the Chat!

(we will have breaks to answer these during the workshop)

#### Week 1 Instructors



Dr. Rachel Sippy



• Dr. Henrik Salje

#### Week 1 Overview

- Monday, July 26:
  - Introductory material, history of mathematical modeling
  - Introduction to R
- Tuesday, July 27:
  - Epidemic determinants & parameters
  - Guided practice in R
- Wednesday. July 28:
  - Model structures
  - Plots & compartmental models in R

# Workshop Schedule

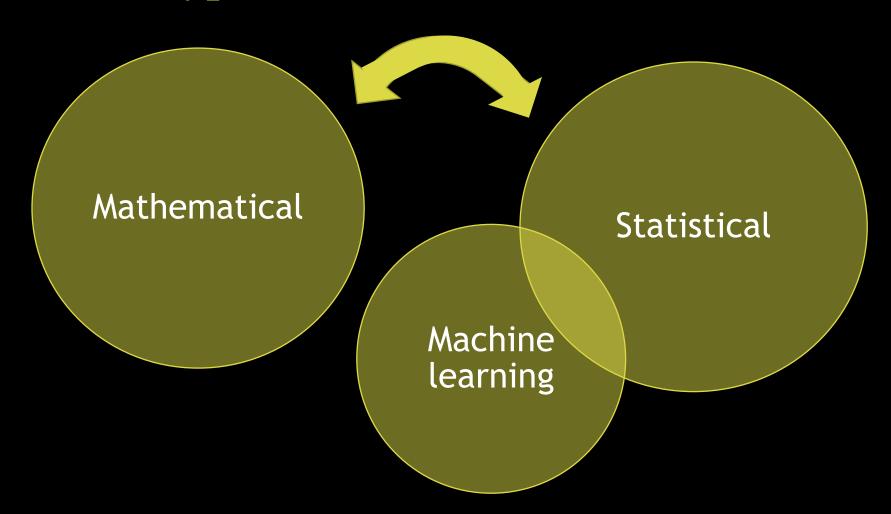
Time	Topics
2:00-2:10 pm	Outline & Introduction
2:10-3:00 pm	Defining Mathematical Models
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3:40-3:50 pm	Break
3:50-5:00 pm	Introduction to R

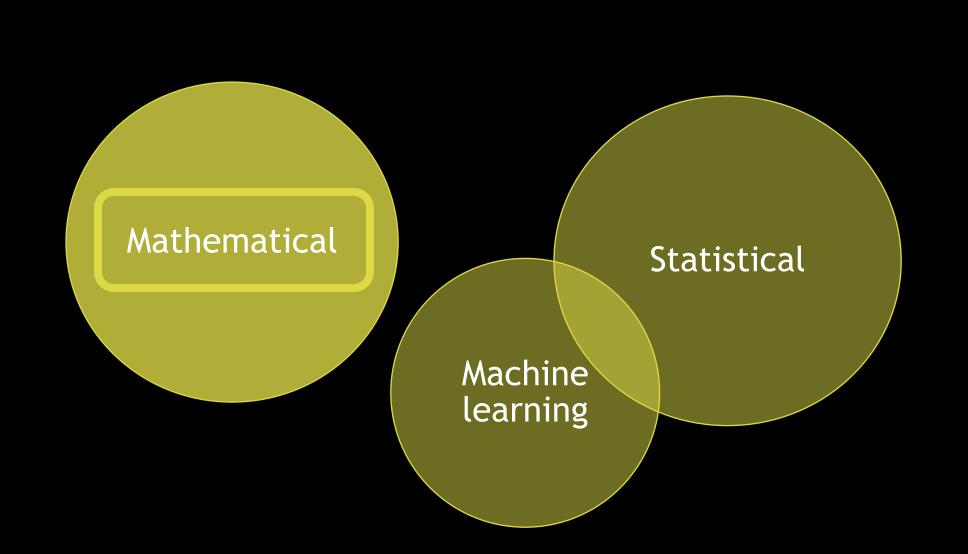
- Statistical models
  - Regression (many!), time series
- Mathematical models
  - Compartmental, mechanistic, agent-based
- Machine learning
  - Uses algorithms and statistical models

#### Three Major Model Types

(used in epidemiology)
(general modeling approaches)

## Model Types are Related





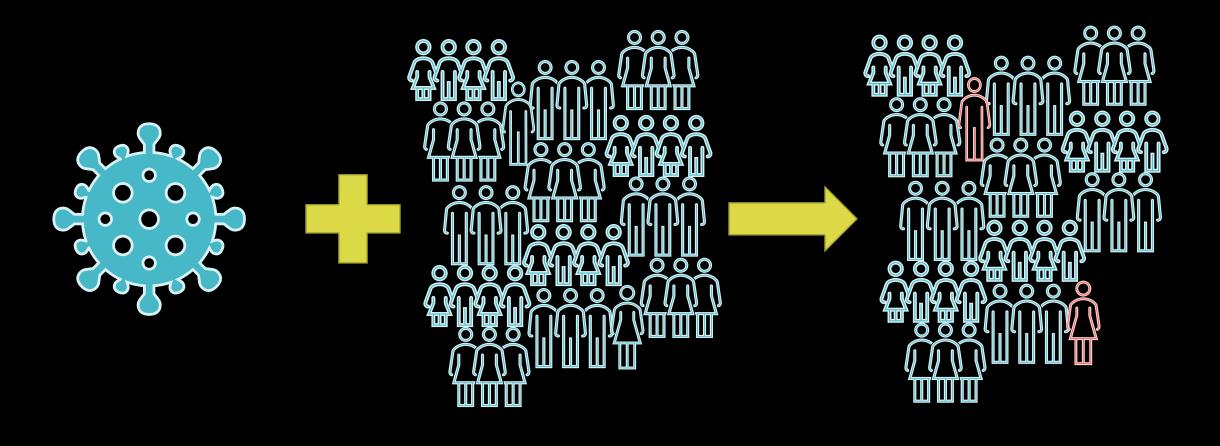
#### Mathematical Modeling

- Often called "mechanistic" modeling
- Used to assess explicit hypotheses about biological mechanisms of infection dynamics
- Model structure is critical first decision
- Models expressed as equations

#### Mathematical Modeling

- Used to estimate parameters/measures related to an epidemic
- Often, we use equations to create a scenario (set of conditions)
  - Example: population of city, level of interaction, certain disease, susceptibility of population
- If we can create a realistic model, then we can make changes to model and see what happens
  - Example: what happens when we vaccinate some people?
- This is similar to running an experiment and observing what happens
- Results from these observations can be compared to real data
  - Example: do model results match what happened in real life?

## **Epidemic Scenario**



#### Public Health Applications

#### Mathematical Modeling

- Estimate transmission parameters from data
- Construct and build mechanistic models
- Build more realistic models

#### **Public Health Questions**

- How big will the final epidemic be?
- What is the R0 value?
- How will interventions impact the epidemic?

#### Public Health Applications: Example

#### Scenario

- Emergence of H5N1 (2006) and swine flu (2009)
- Vaccines take many months to develop
- Vaccines may improve during the pandemic
- Manu countries have no or low stockpiles of antivirals

#### **Public Health Questions**

#### Public Health Applications: Example

#### Scenario

- Emergence of H5N1 (2006) and swine flu (2009)
- Vaccines take many months to develop
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#### **Public Health Questions**

- How should we distribute vaccines?
- Should everyone use the first available vaccine or wait for a better one?
- Do travel restrictions impact spread?
- What is the impact of closing schools?
- How many cases would occur if we run out of antivirals?
- Is it better to use antivirals as prophylaxis or to treat cases?

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# History of Mathematical Modeling of Infectious Disease

Early efforts & recent advances

 1700
 1800
 1900
 2000



1700 1800 1900 2000

Daniel Bernoulli, 1760

It is, then, only the risk which is attributed to inoculation which should keep us undecided... 'What would be the state of the human race if, at the price of a certain number of victims, we could procure for it freedom from natural smallpox?' [5, p. 284]



1700 1800 1900 2000

Bernoulli

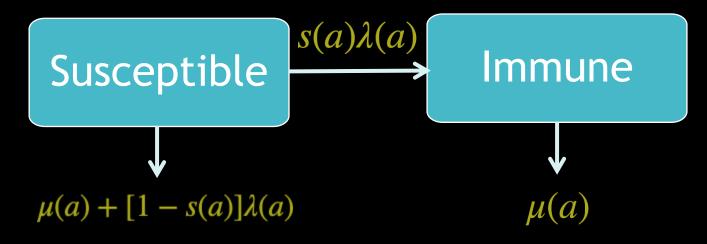
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 What is the impact of inoculation against smallpox?



 1700
 1800
 1900
 2000

Bernoulli

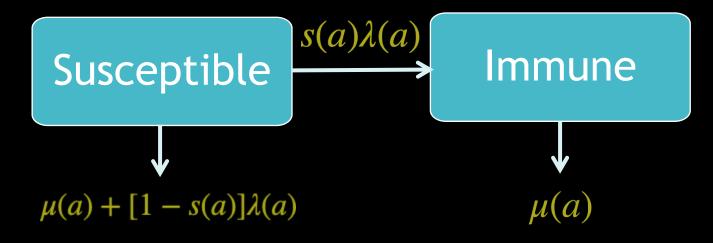


- s(a)=probability of surviving infection
- $\lambda(a)$ =force of infection
- $\mu(a)$ =death rate



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 1900
 2000

Bernoulli



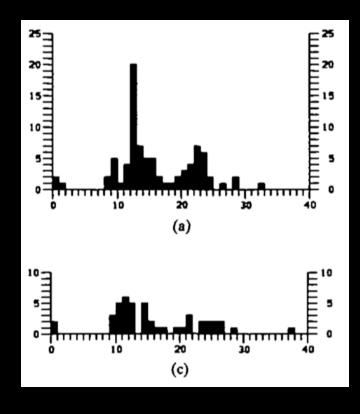
 Inoculation would increase life expectancy at birth by ~3 years



1700 1800 1900 2000

Bernoulli

- Pyotr En'ko, 1889
- How does contact between susceptible and infected people impact a measles epidemic?



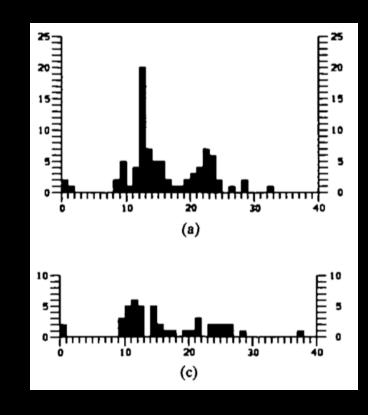
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Bernoulli

En'ko

 How does contact between susceptible and infected people impact a measles epidemic?

The number of contacts depends on the way of life. For a solitary way of life—one's home, one's servants, a selected circle of acquaintances—the number of contacts is less. In schools the patients are isolated at the first signs of the disease and can infect their schoolmates only as long as they appear completely healthy; therefore it is assumed that a patient has contact with the same number of individuals as a healthy individual.



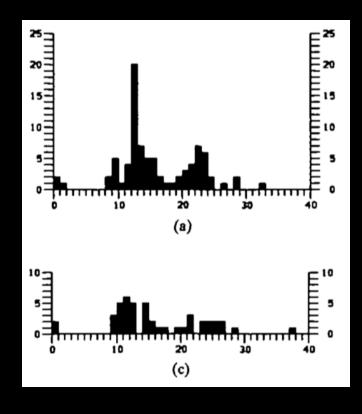
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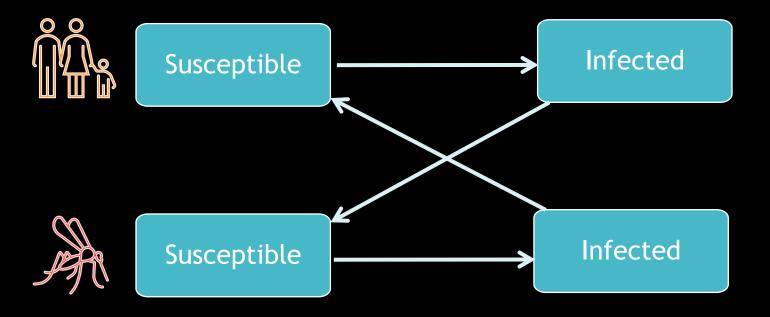
En'ko

$$1 - ((N-1-x)/(N-1))^A$$

- N=population size
- x=number of infected
- A=number of contacts



• Ronald Ross, 1911





Bernoulli

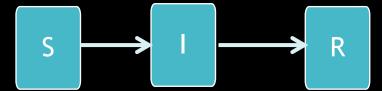
En'ko Ross

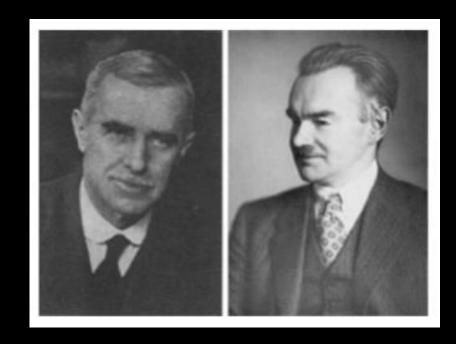
- How does vector control impact a malaria epidemic?
- If you increase mosquito mortality from 5% to 45% you could eradicate malaria in Africa
- Argued for establishment of new field of study, co-authored with Hilda Hudson



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• Kermack and McKendrick, 1926





 1700
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 2000

Bernoulli

- Evaluated plague and cholera epidemics
- Formalization of mechanistic models for epidemics

En'ko Ross

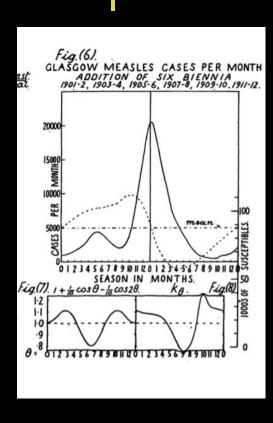
Kermack & McKendrick

$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t)$$

Bernoulli



En'ko Ross

Kermack & McKendrick
Soper

 Evaluated seasonality of measles epidemics

- Use of simulation to understand epidemics
- Presentation & TV show
- Formally published by Helen Abbey
- Basis for modern epidemic modeling

Kermack & McKendrick
Soper
Reed-Frost

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- Extended Ross malaria models for use on computers
- Ross-MacDonald models, formalized by Smith & McKenzie, Aron & May, and Anderson & May

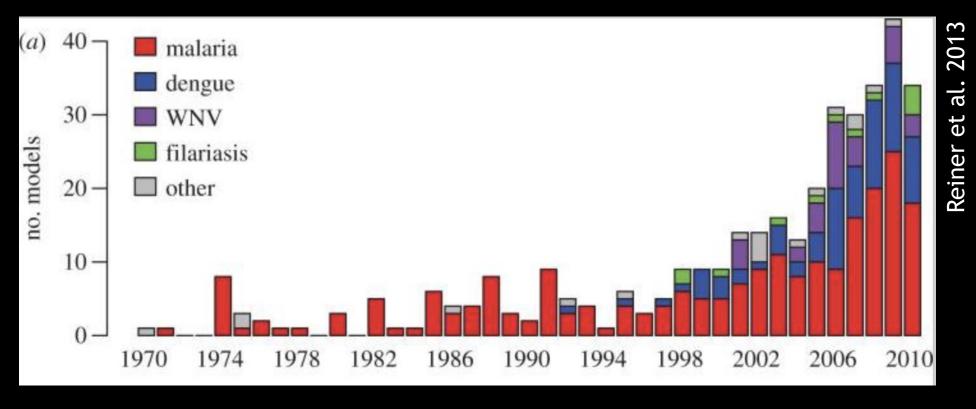
Kermack & McKendrick
Soper
Reed-Frost
MacDonald

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# Modern Modeling

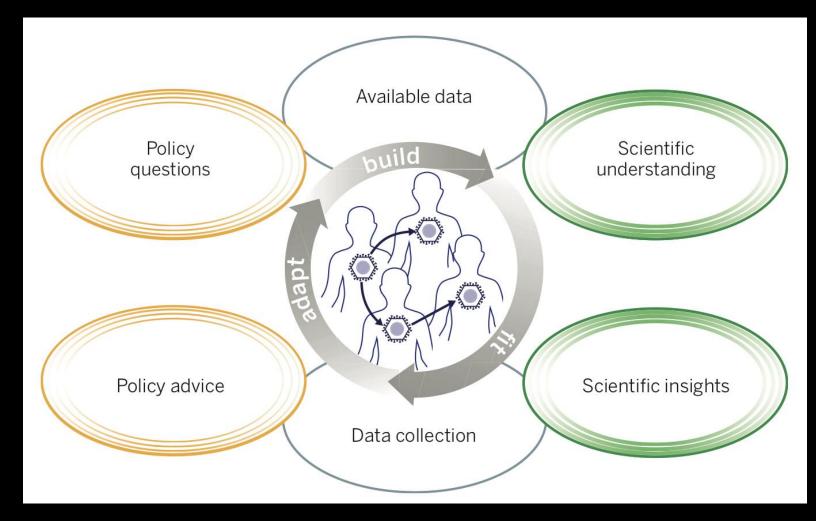
#### Modern Modeling



• Early models have been expanded and developed for a wide variety of scenarios and applications

## Modern Modeling

 Modeling is used to fit and answer policy questions



#### D **Policy questions** Policy advice Introduce only when minimum Should rubella vaccination be coverage is achieved, which may introduced? depend on birth rate. If so, who should be targeted? Transfer from targeting only girls to When should large age-range including into routine vaccination if campaigns be considered? coverage sufficiently high. Consider vaccine heterogeneity Model design E Available data **Data collection** Case/age surveillance following vaccine introduction Adapt cases<15 . . . . . . . . . . . . . . . . . . 1200 ···· **a** cases≥15 → Build 1980 1985 1990 1995 Scientific understanding Scientific insights 9.416.£(3.8)12.010.7 9.8 9.0 8.4 8.0 7.6 S age group 1 .713.7 11.4 9.8 8.7 7.9 7.3 6.8 6.3 6.0 <u>.</u>⊆ 5.512.0 9.9 8.5 7.6 6.8 6.3 5.8 5.4 5.1 bigs of the state age group 2 2.910.0 8.2 7.0 6.2 5.6 5.1 4.7 4.4 1 9.3 7.6 6.5 5.8 5.2 4.7 4.4 4.1 12 16 20 25 29 33 37 42 46 50 Births per 1000

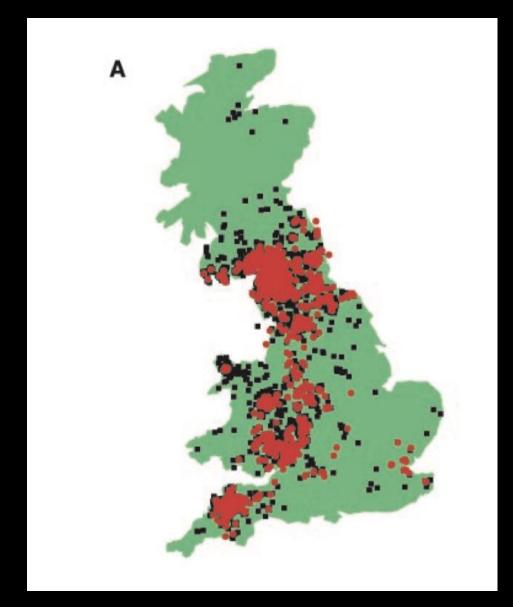
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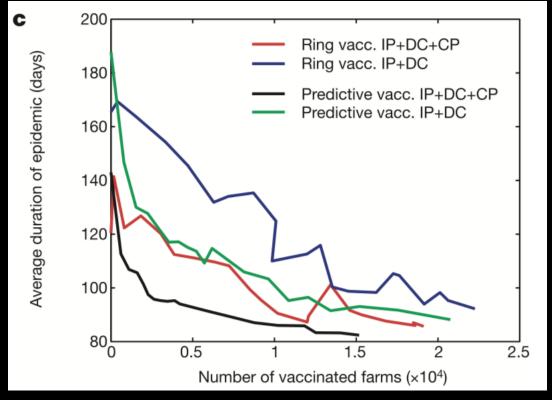
al. et Heesterbeek

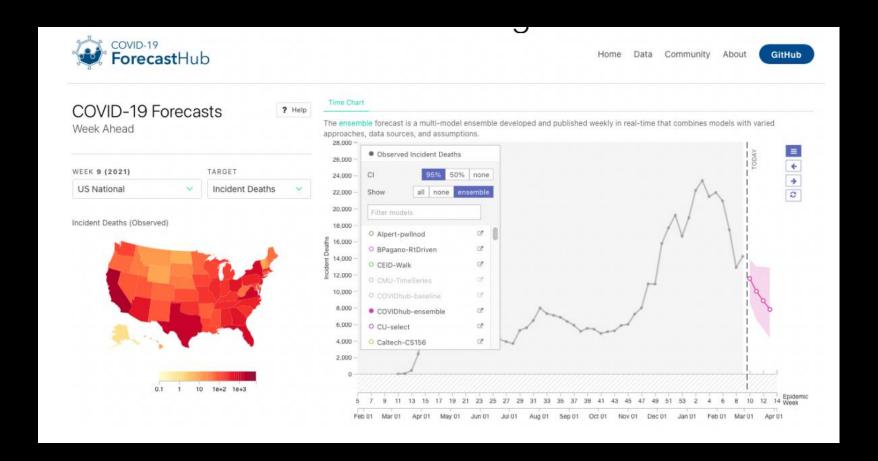


#### Foot & Mouth Disease, UK 2001

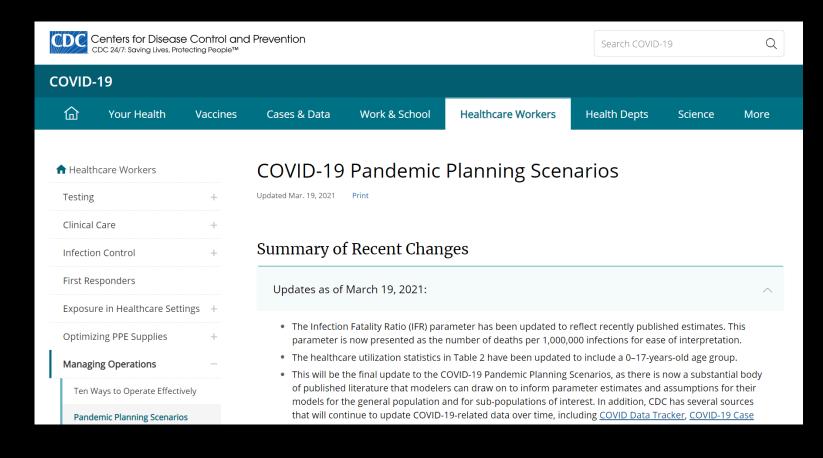
- Series of outbreaks among farms
- Questions around severity of control and which controls to implement
- Test the impacts of different control scenarios



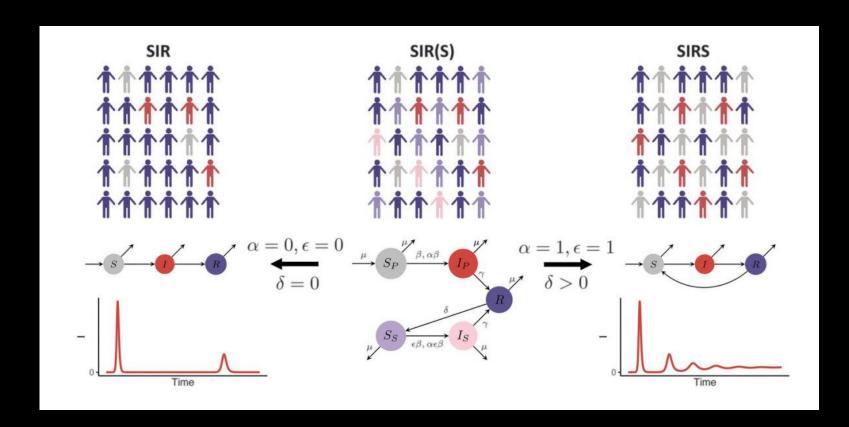




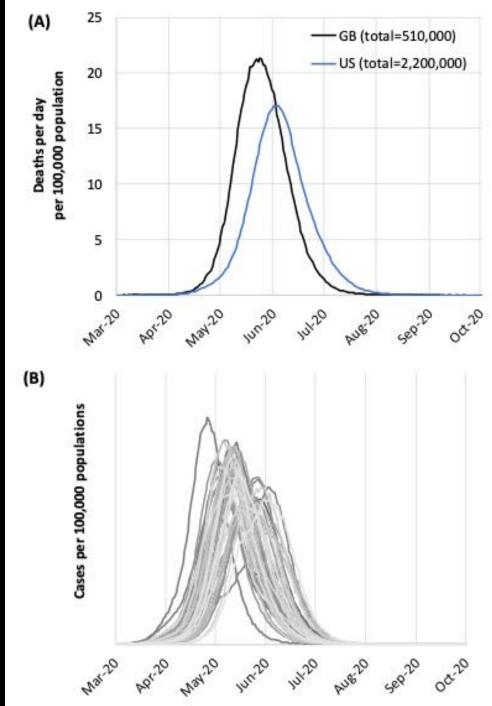
 Efforts to forecast disease incidence and deaths for different locations on a weekly basis



- Scenario planning: estimates of outcomes using best-case, worst-case, and most-likely parameters for disease transmission and severity
- Based on biological and epidemiologic characteristics of SARS-CoV-2



 Understanding immunity, vaccination, and long-term dynamics



- Questions around severity of control and which controls to implement
- Test the impacts of different control scenarios

Figure 1: Unmitigated epidemic scenarios for GB and the US.

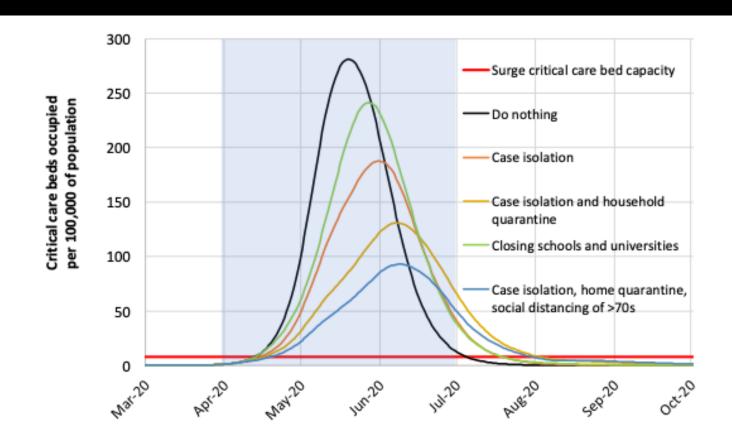


Figure 2: Mitigation strategy scenarios for GB showing critical care (ICU) bed requirements. The black line shows the unmitigated epidemic. The green line shows a mitigation strategy incorporating closure of schools and universities; orange line shows case isolation; yellow line shows case isolation and household quarantine; and the blue line shows case isolation, home quarantine and social distancing of those aged over 70. The blue shading shows the 3-month period in which these interventions are assumed to remain in place.

- Questions around severity of control and which controls to implement
- Test the impacts of different control scenarios

# "All models are wrong, but some are useful."

-George Box

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