Week 2: Compartmental Models & Expanded

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Week 2 Overview

- ~~Monday, August 2:~~
  - ~~Relating SIR models to epidemic parameters~~
  - ~~Estimating parameters in R~~
- Tuesday, August 3:
  - Guest lecture by Caroline Trotter
  - Modeling meningitis
  - Guided practice in R
- Thursday, August 5:
  - Using serological data for modeling
  - Guided practice in R

Objectives

- Learn how to include births and deaths in an SIR model
- Modify model structure to create an SEIR model

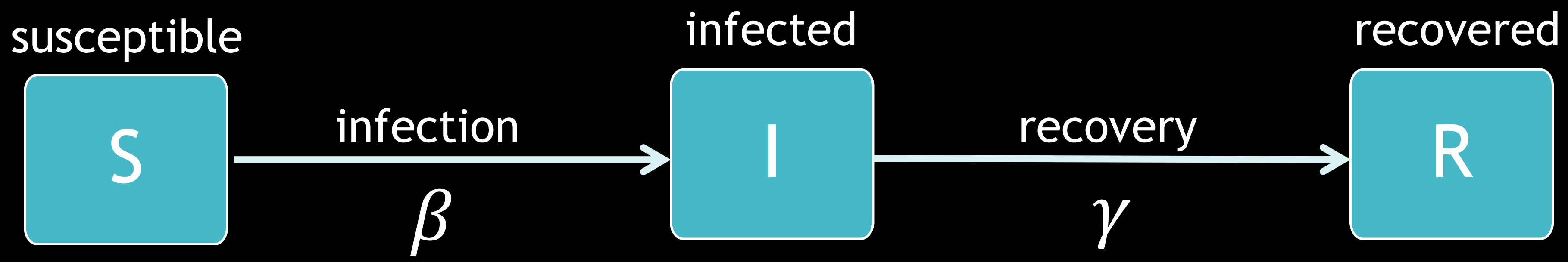
Post Questions in the Chat!

(or ask over microphone)

Workshop Schedule

Time	Topics
2:00—2:10 pm	~~Greetings~~
2:10—2:50 pm	Other Compartmental Models
2:50—3:00 pm	Break
3:00—3:50 pm	Guest Lecture: Modeling meningitis
3:50—4:00 pm	Break
4:00—5:00 pm	R Session

SIR Model



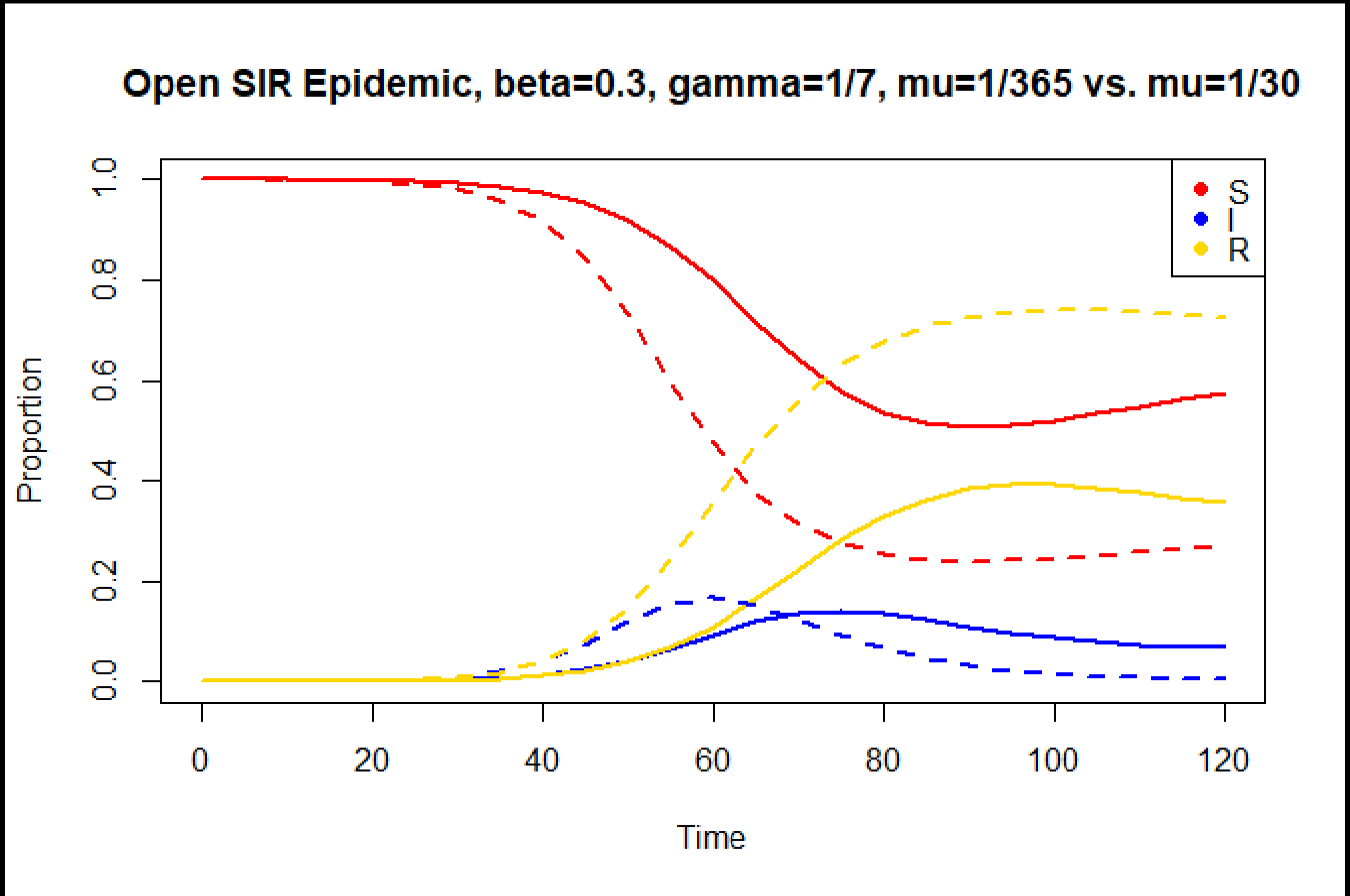
- this is a closed model
  - β is a transmission coefficient
  - γ is the recovery rate

$$\frac{dS(t)}{dt} = -\beta S(t) I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t) I(t) - \gamma I(t)$$

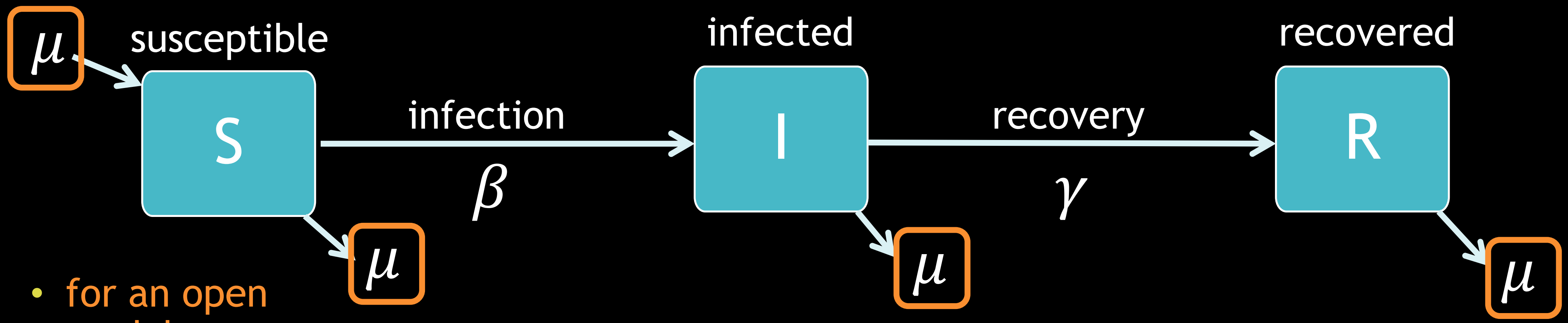
$$\frac{dR(t)}{dt} = \gamma I(t)$$

SIR Model Output



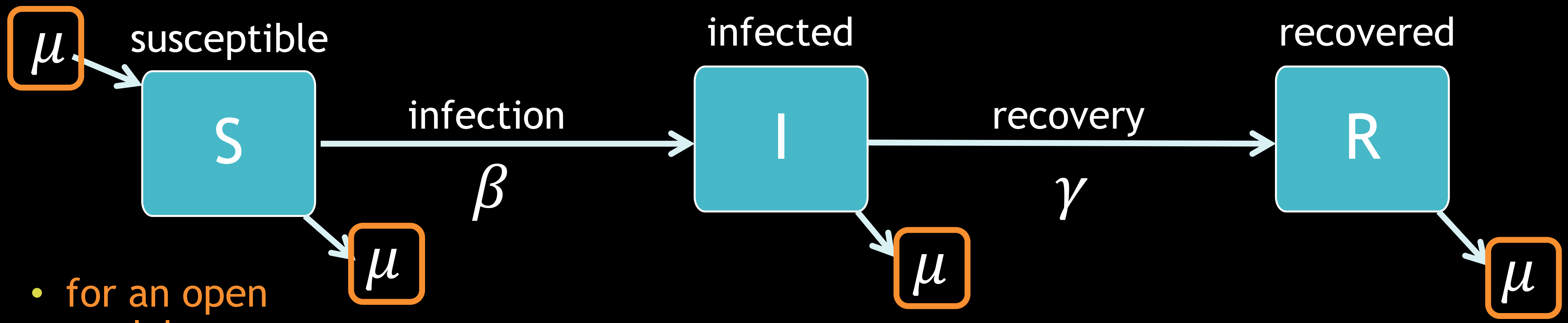
- Having an open population can greatly impact the epidemic dynamics!

SIR Model



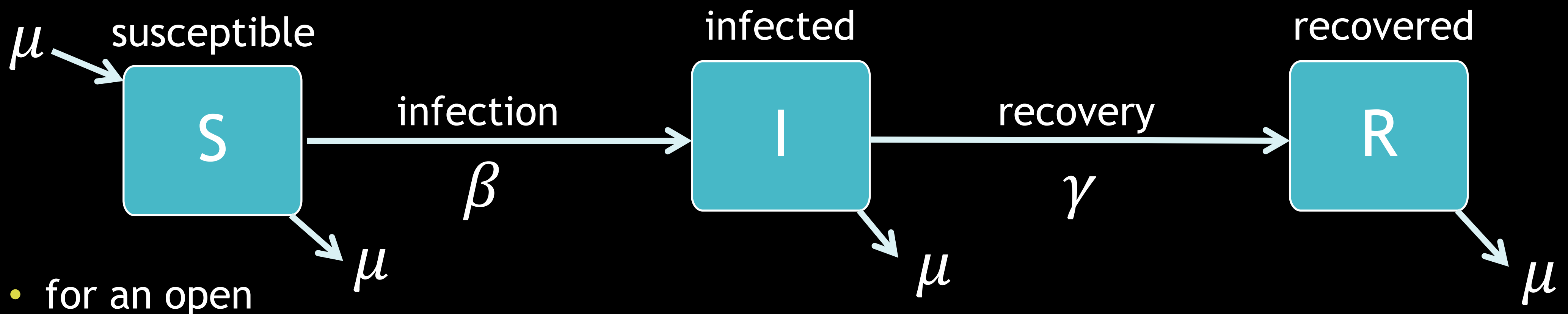
- for an open model:
  - μ is our birth/death rate
- What are our assumptions?

SIR Model



- for an open model:
  - μ is our birth/death rate
  - births are all susceptible
  - anyone can die

SIR Model



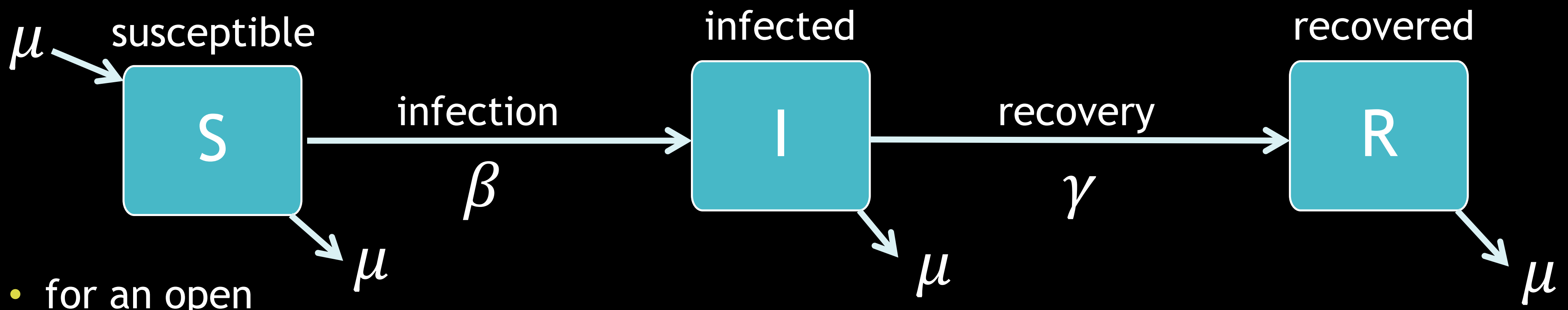
- for an open model:
  - μ is our birth/death rate
  - births are all susceptible but anyone can give birth
  - anyone can die

$$\frac{dS(t)}{dt} = \mu(S(t) + I(t) + R(t)) - \mu S(t) - \beta S(t)I(t)$$

$$\frac{dI(t)}{dt} = \beta S(t)I(t) - \gamma I(t) - \mu I(t)$$

$$\frac{dR(t)}{dt} = \gamma I(t) - \mu R(t)$$

SIR Model



- for an open model:
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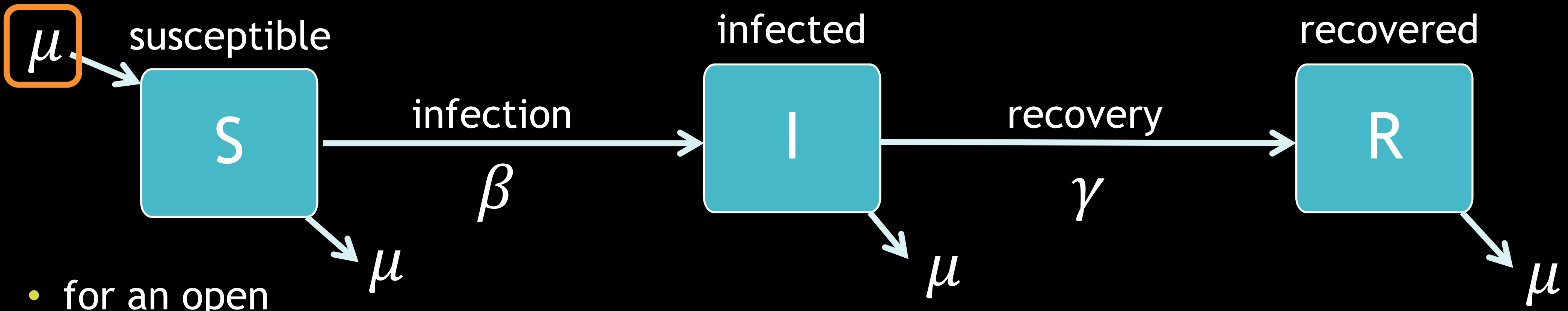
$$\frac{dS(t)}{dt} = \mu(S(t) + I(t) + R(t)) - \mu S(t) \underline{- \beta S(t) I(t)}$$

$$\frac{dI(t)}{dt} = \underline{\beta S(t) I(t) - \gamma I(t)} - \mu I(t)$$

$$\frac{dR(t)}{dt} = \underline{\gamma I(t)} - \mu R(t)$$

- we still have the same parts from the closed model

SIR Model



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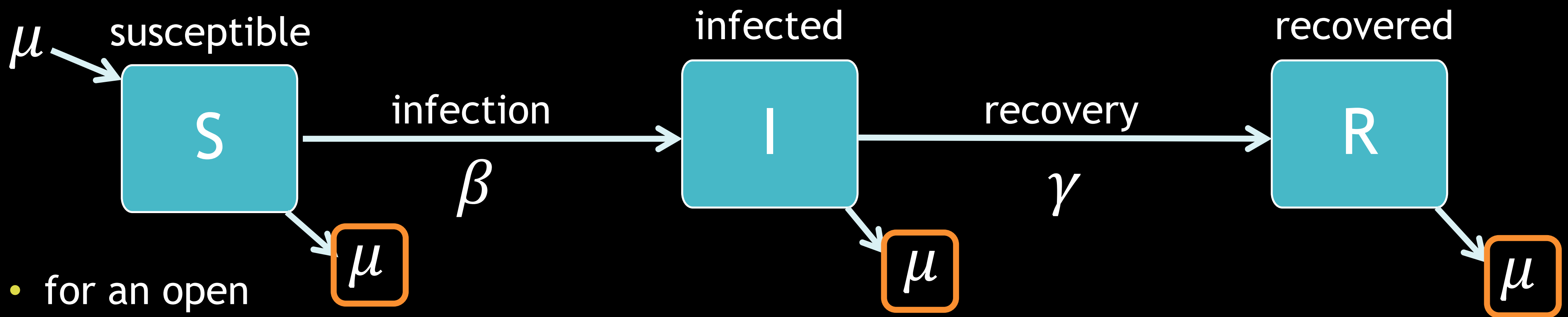
SIR Model



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R_0 & Model Structure

- The calculation of R0 depends on the structure of the model being used

- for our closed SIR model:
  - $R_0 = \frac{\beta}{\gamma}$

- for an open SIR model:
  - $R_0 = \frac{\beta}{\gamma+\mu}$

R_0 & Model Structure

- The calculation of R0 depends on the structure of the model being used

- for our closed SIR model:
  - $R_0 = \frac{\beta}{\gamma}$
- for duration of infection:
  - $\frac{1}{\gamma}$

- for an open SIR model:
  - $R_0 = \frac{\beta}{\gamma+\mu}$
- for duration of infection:
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R_0 & Model Structure

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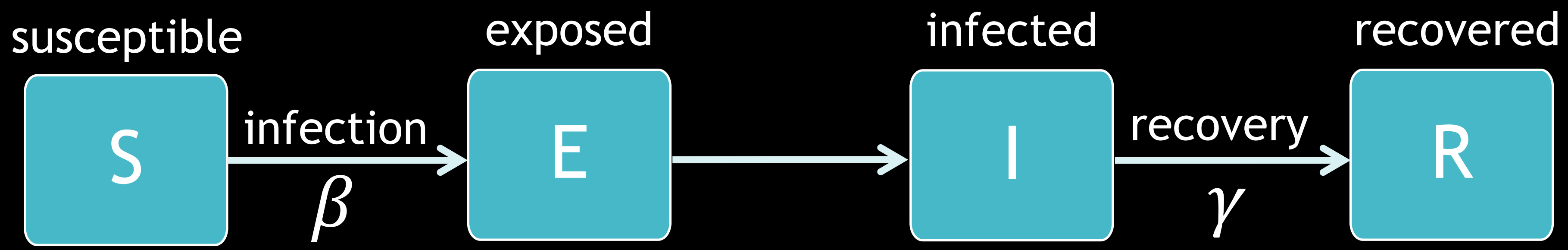
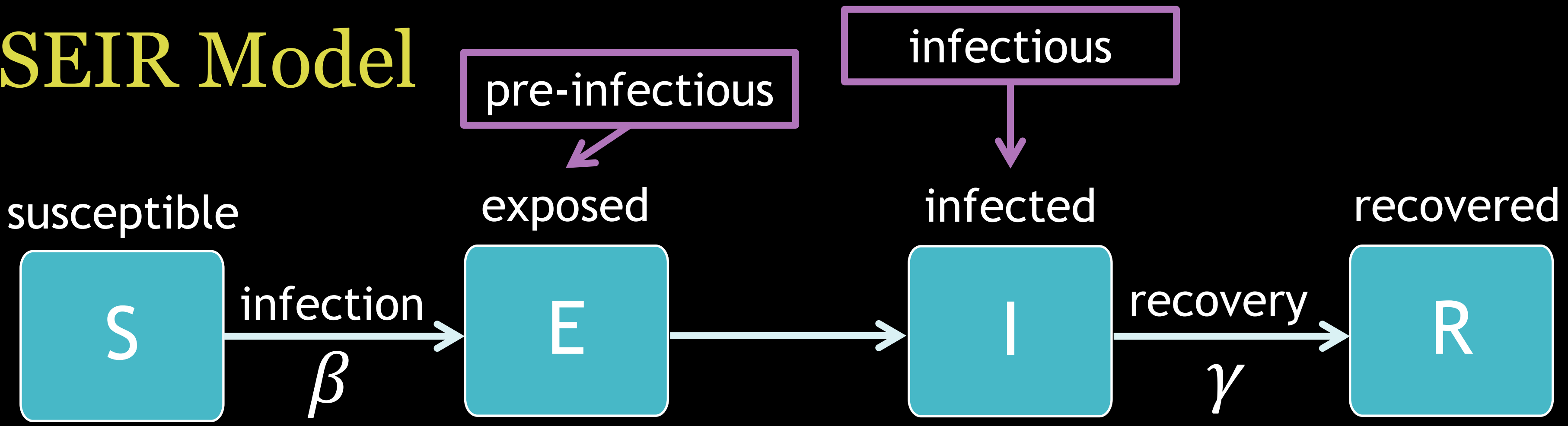
- for our closed SIR model:
  - $R_0 = \frac{\beta}{\gamma}$
- for duration of infection:
  - $\frac{1}{\gamma}$
- for an open SIR model:
  - $R_0 = \frac{\beta}{\gamma+\mu}$
- for duration of infection:
  - $\frac{1}{\gamma+\mu}$
- But usually $\gamma \gg \mu$ so:
  - $R_0 = \frac{\beta}{\gamma} \approx R_0 = \frac{\beta}{\gamma+\mu}$

SEIR Model


susceptible
exposed
infected
recovered
S
infection
β
E
I
recovery
γ
R

SEIR Model


pre-infectious
infectious
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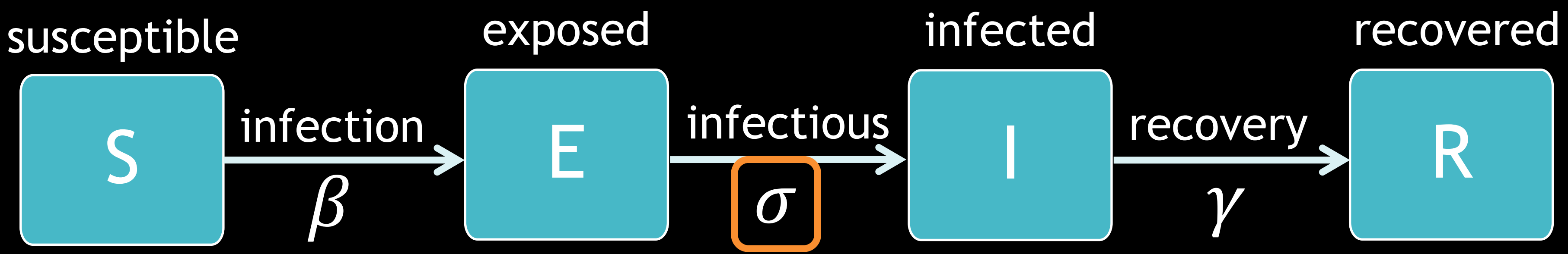
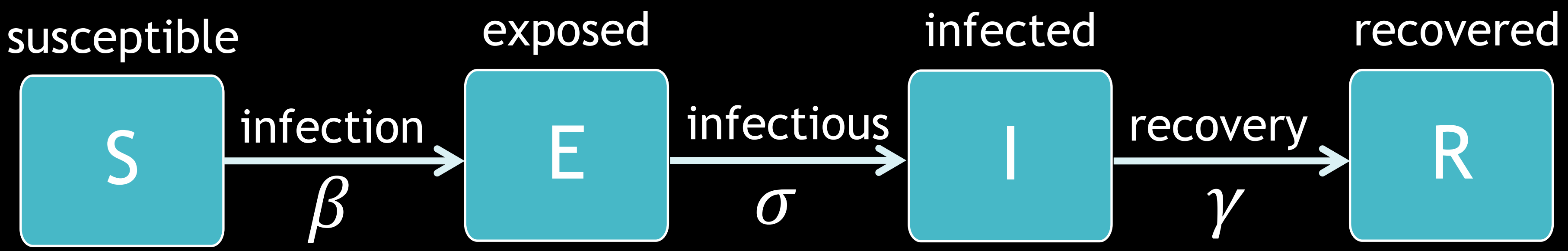
SEIR Model



- for an SEIR model:
  - σ is the rate at which people change from exposed to infectious
  - 1/σ is the latent period

SEIR Model



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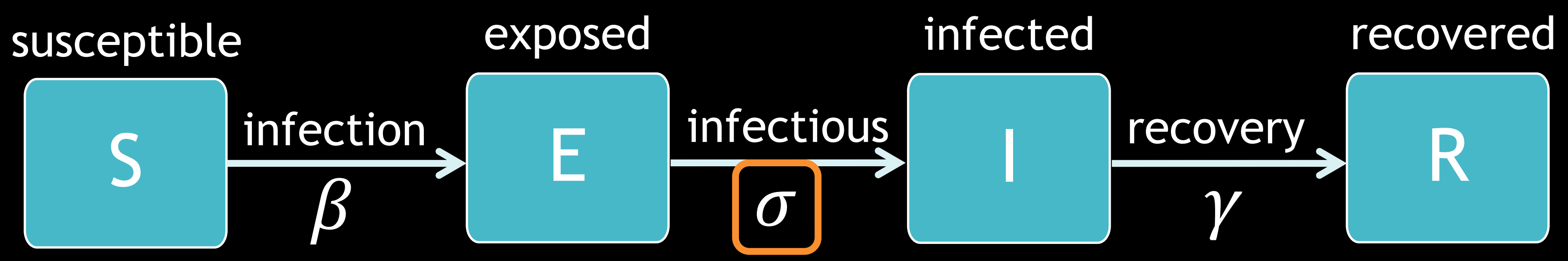
$$\frac{dS(t)}{dt} = -\beta S(t)I(t)$$

- the first equation is exactly the same

SEIR Model



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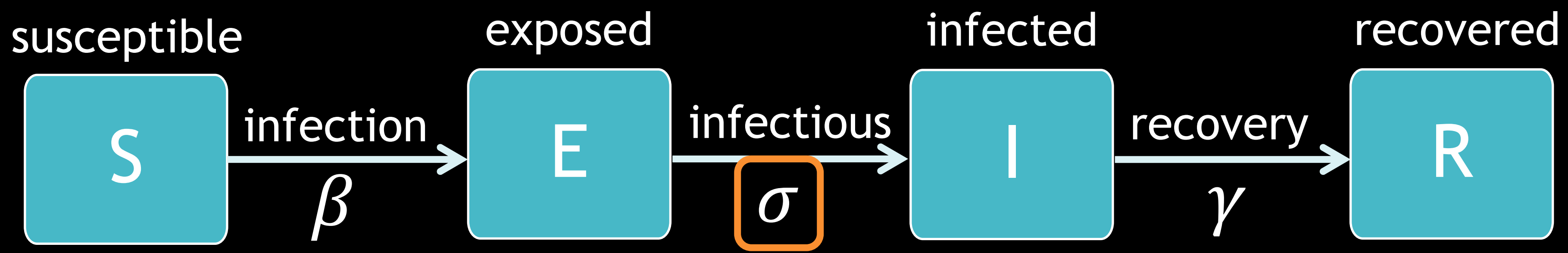
SEIR Model



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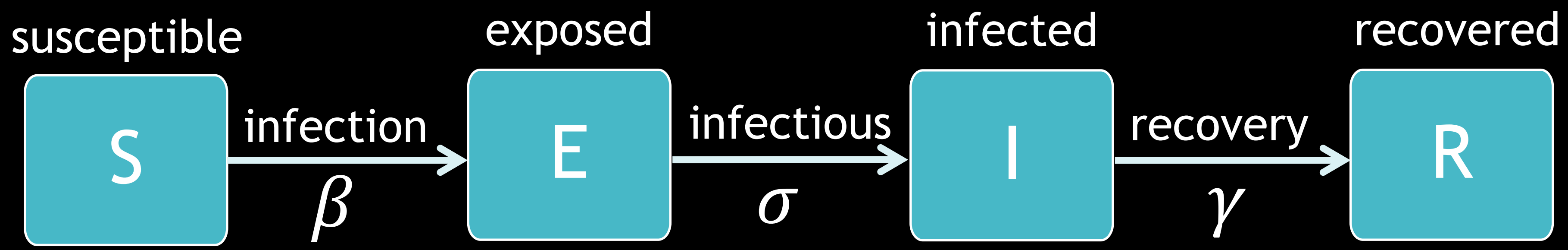
SEIR Model



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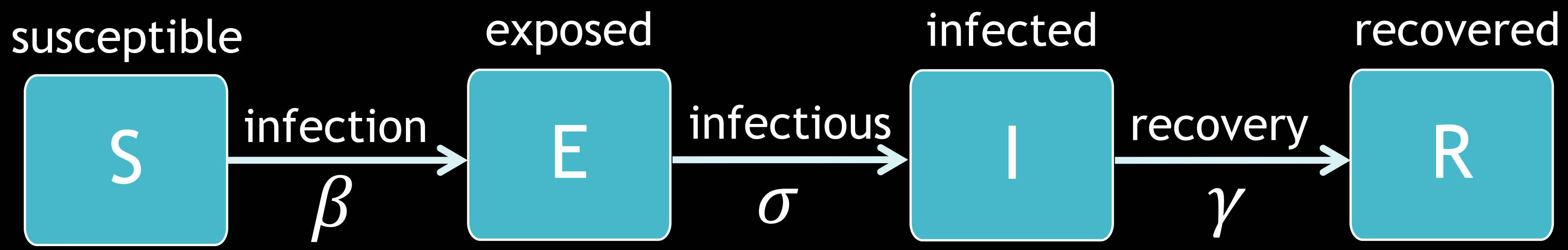
$$\frac{dR(t)}{dt} = \gamma I(t)$$

- the last equation is exactly the same

SEIR Model



- We could also make the SEIR (or a model with any other structure) be open (include birth/death/migration)
  - we can also make different death rates for each compartment
  - models are extremely flexible!

Questions?

10 minute break

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