Week 3: Stochasticity

Dr. Rachel Sippy
University of Cambridge

Week 3 Overview

- Monday, August 9:
  - Guest lecture & R session by Megan O'Driscoll
  - Stochastic models
  - Guided practice in R
- Tuesday, August 10:
  - Step-by-step model building
  - Building a COVID-19 model
  - Guided practice in R
- Wednesday, August 11:
  - Comparing models to data & evaluating models
  - Guided practice in R

Post Questions in the Chat!

(or ask over microphone)

Workshop Schedule

Time	Topics
2:00–2:10 pm	~~Greetings~~
2:10–3:00 pm	Guest lecture: COVID-19 with Megan O'Driscoll
3:00–3:10 pm	Break
3:10–4:00 pm	Stochasticity
4:00–4:10 pm	Break
4:00–5:00 pm	R Session

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Stochasticity

Objectives

- Understand the difference between deterministic and stochastic models
- Understand options for adding stochasticity to a model

Modeling Methods

Deterministic

Stochastic

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- Describe what happens “on average” in a population
- Use average (single) value/transition rates in models

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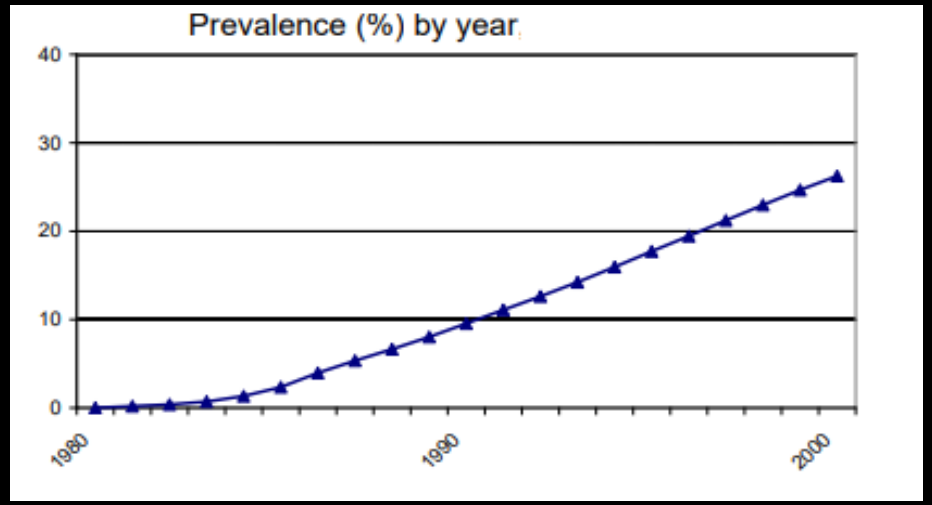

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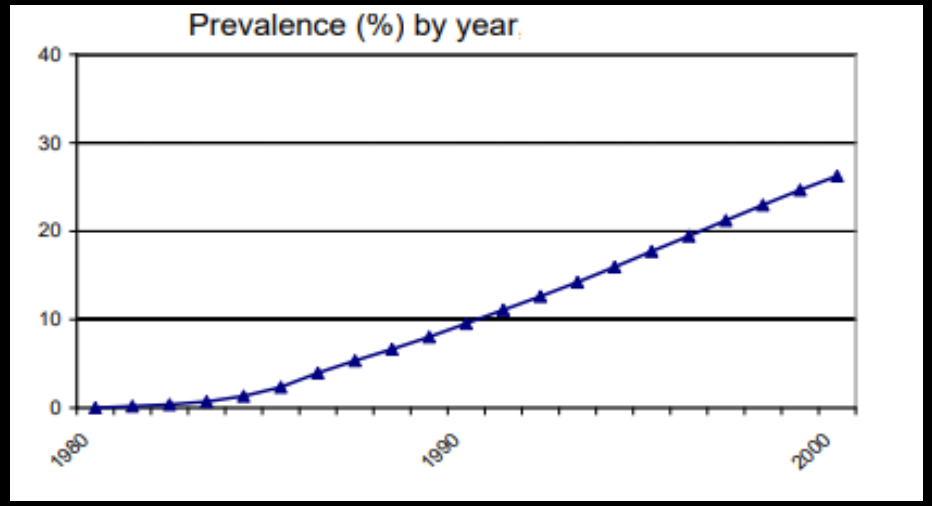

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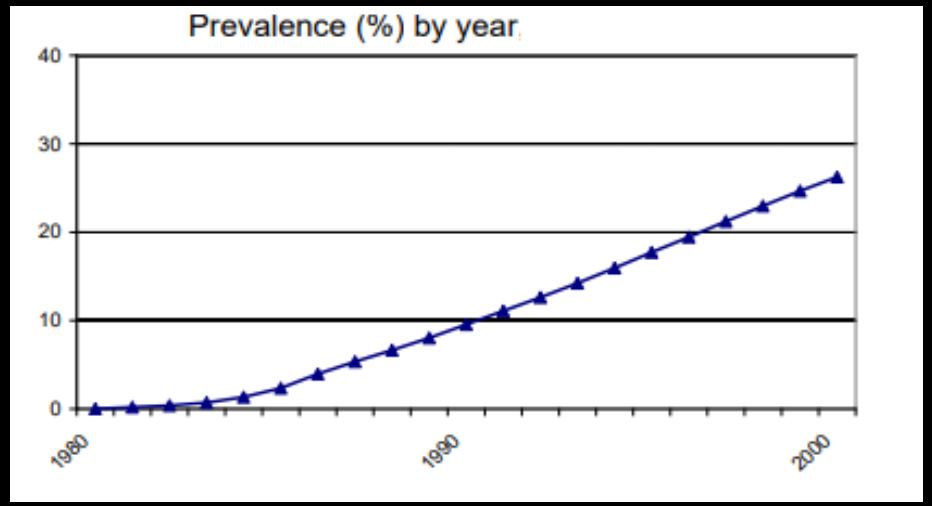

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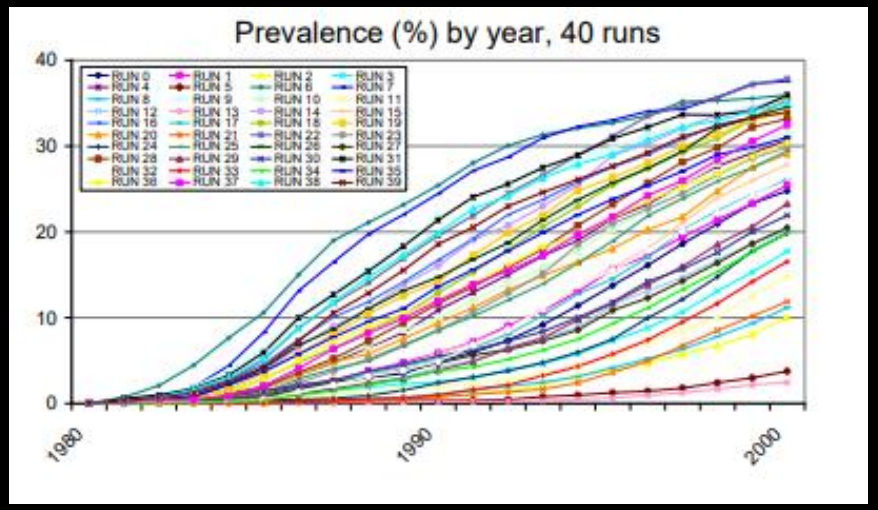

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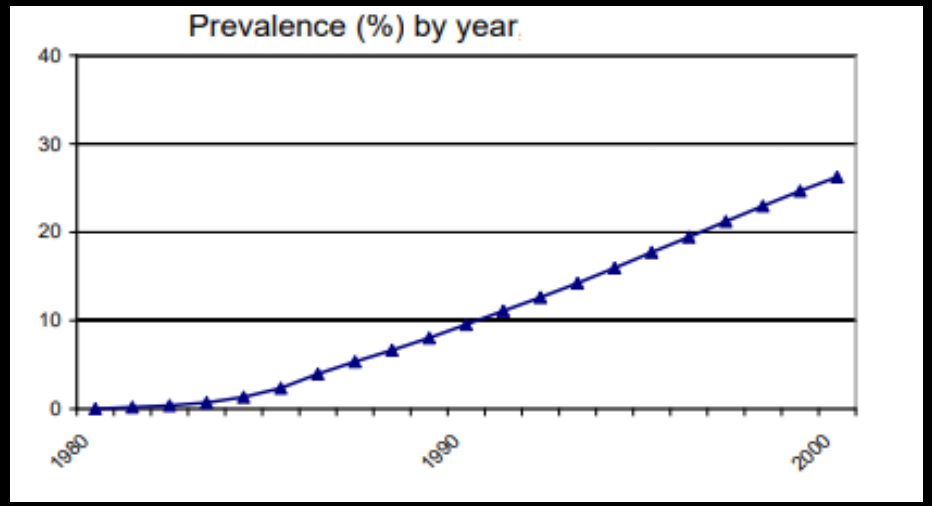

Stochastic – Random!

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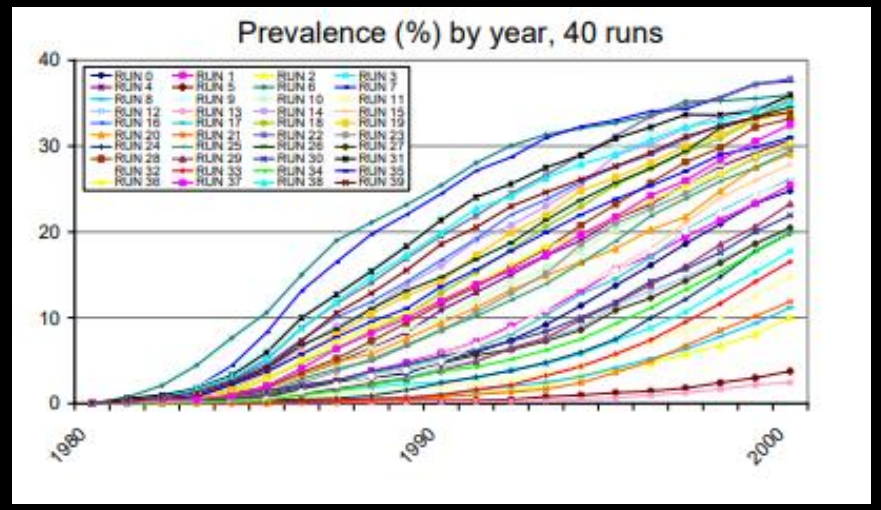

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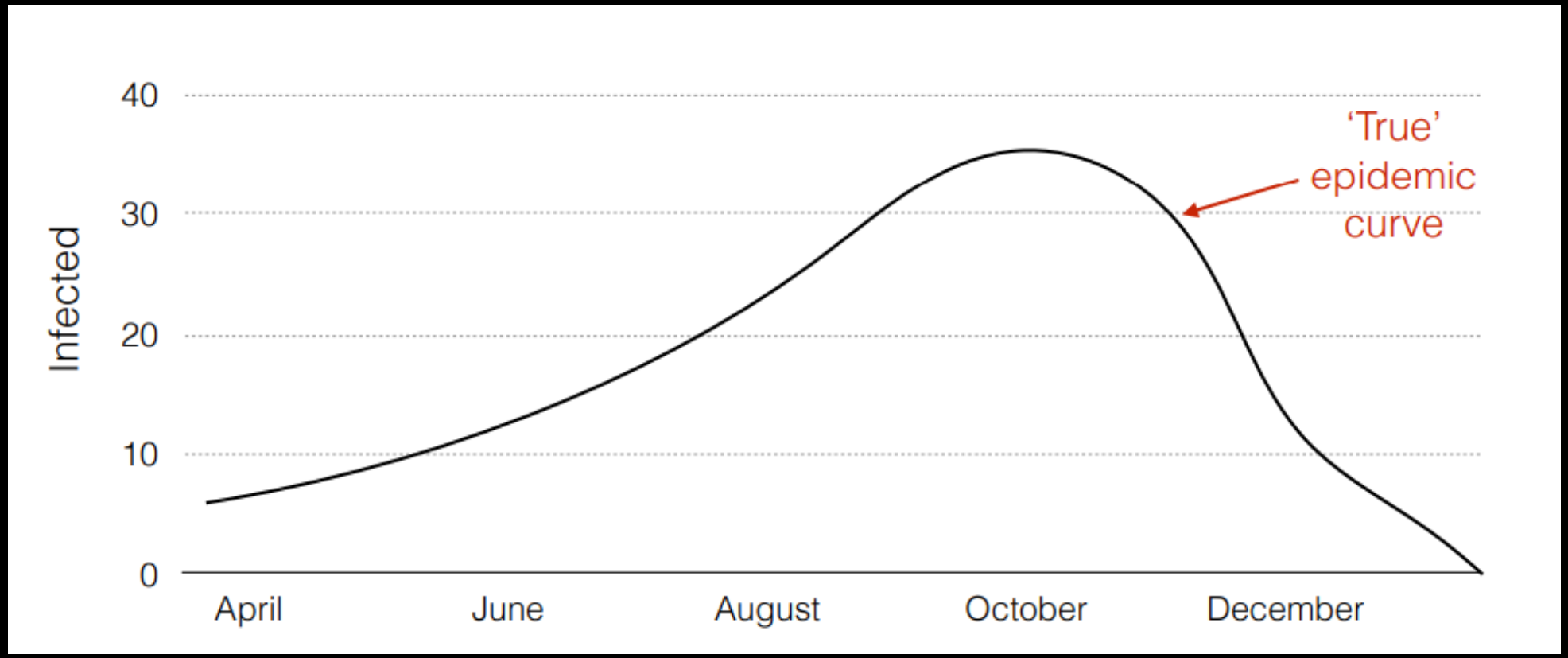
    - can have profound impact on epidemic dynamics

Observational Stochasticity



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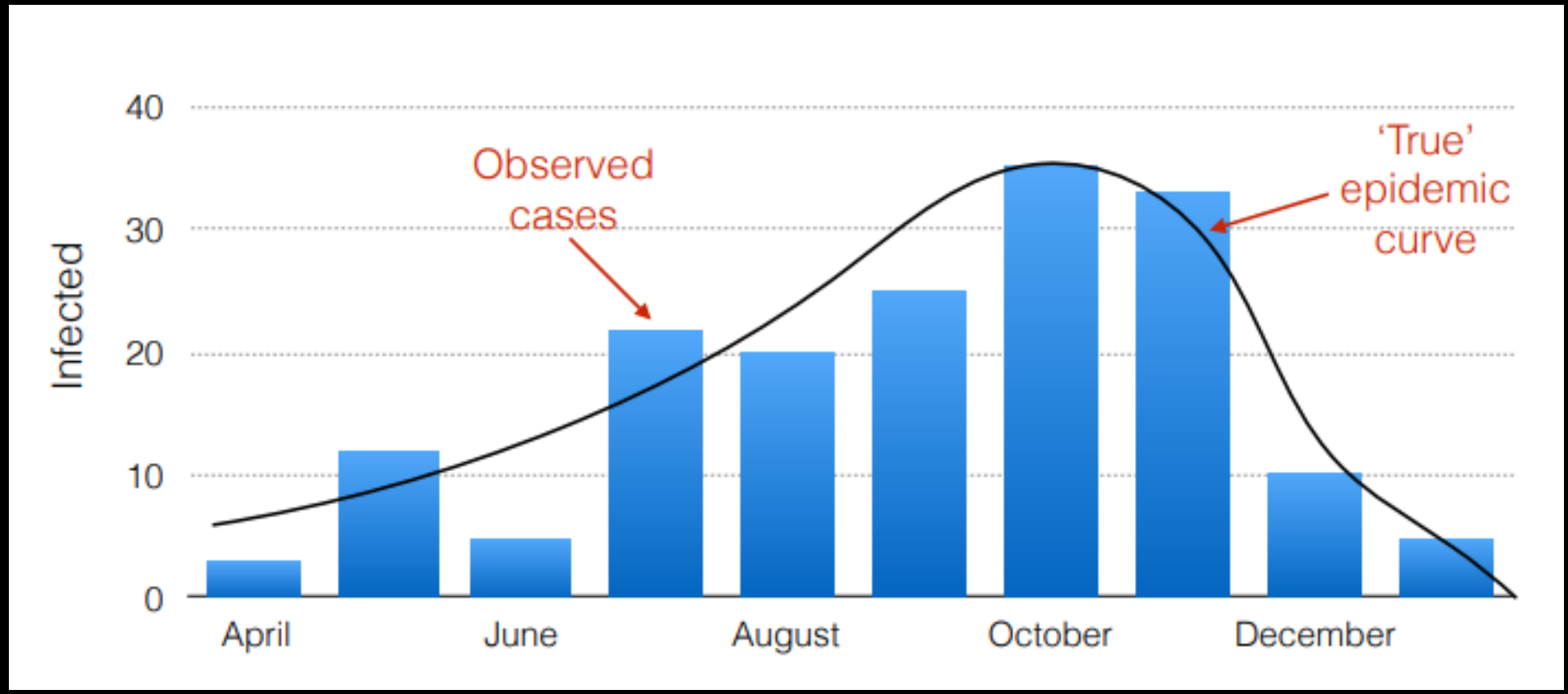

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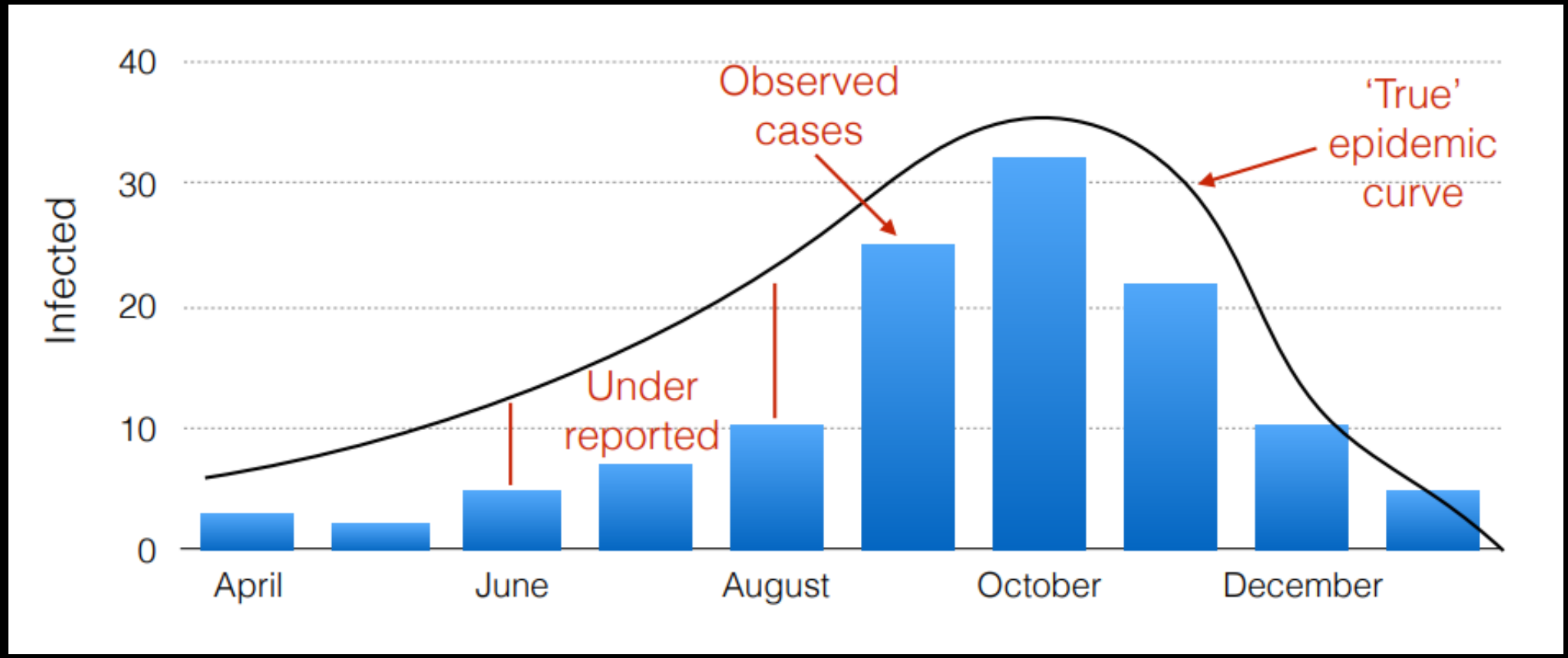
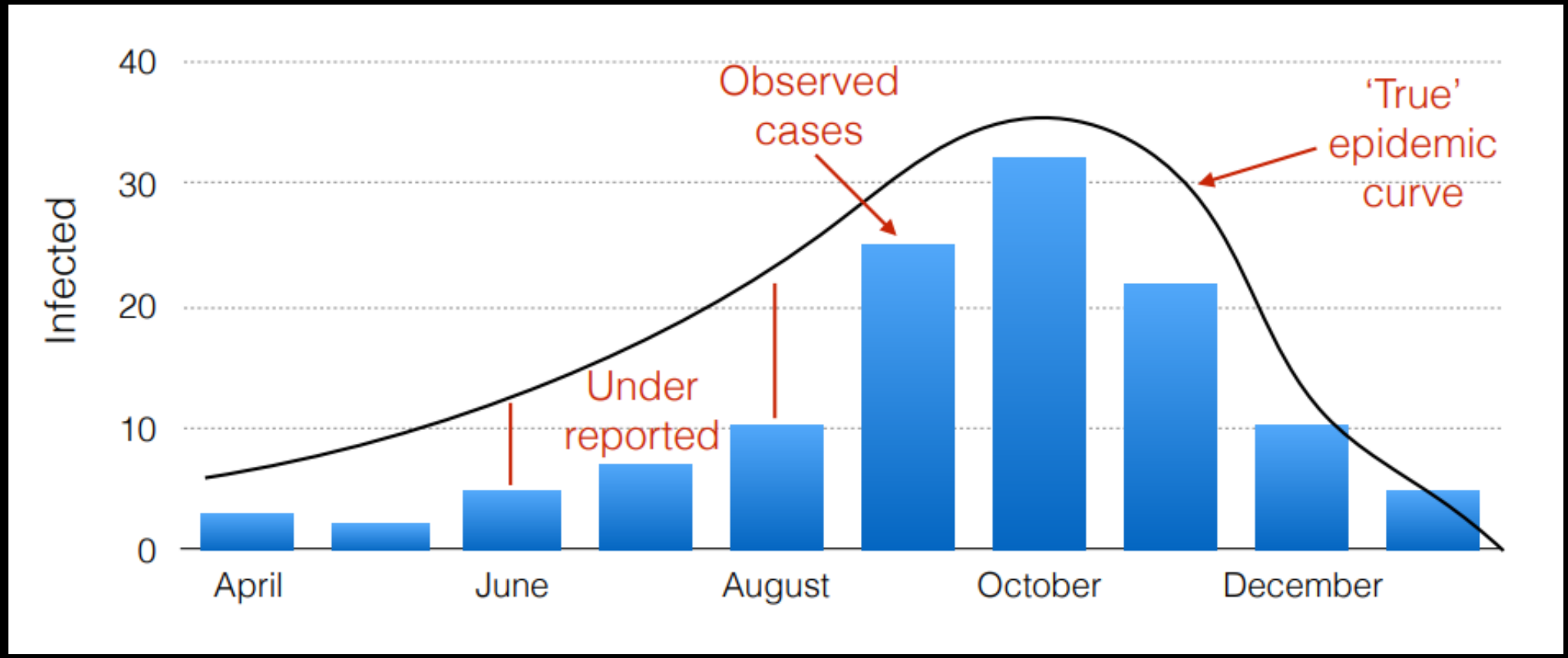


Observational Stochasticity

$$I_t = \rho C_t$$

I_t: number infected
ρ: reporting rate
C_t: case count

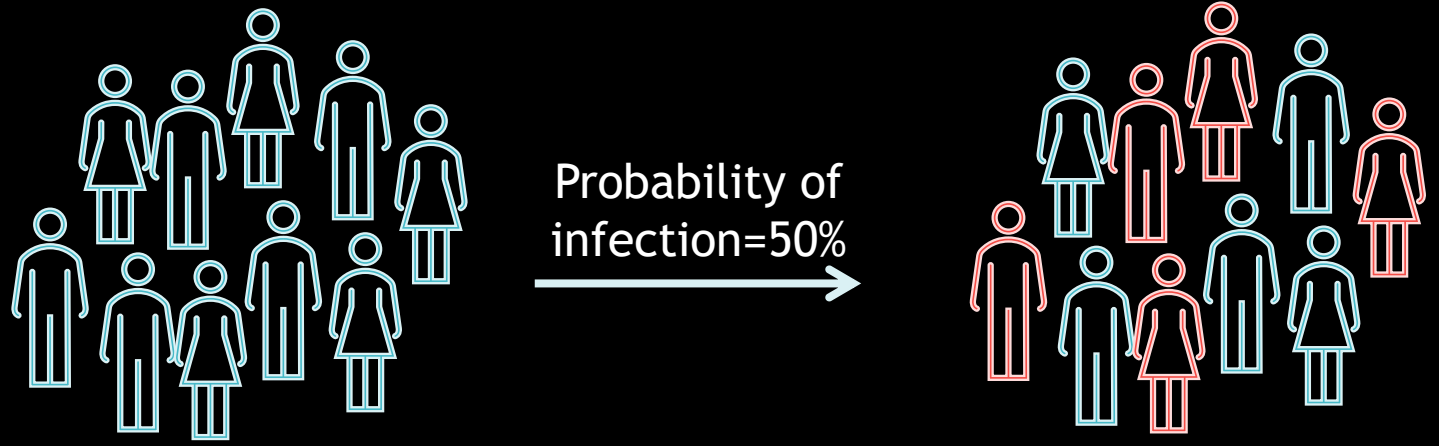


Process Stochasticity

- Infection events are random events!
  - number of individuals you are in contact with
  - whether disease is transmitted during a contact
- These events can have a major impact on an epidemic
  - we want to build this uncertainty into our model

Modeling Results

Deterministic



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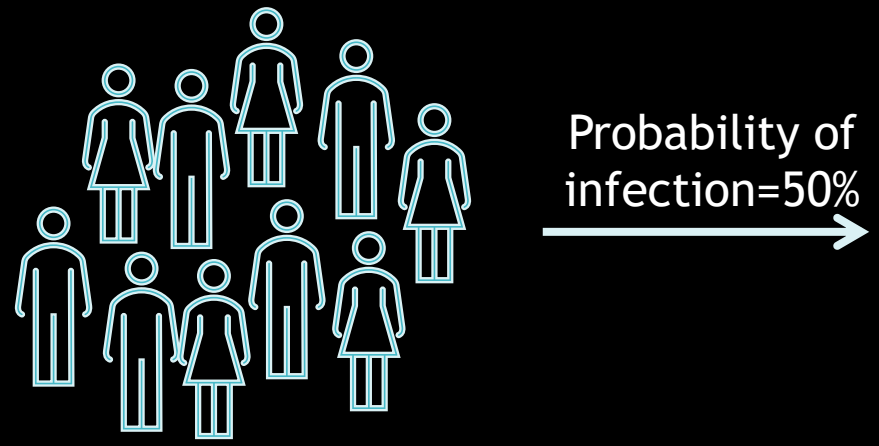

Starting population

Modeling Results

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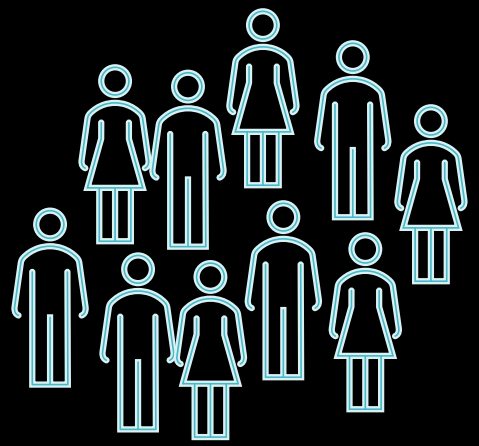
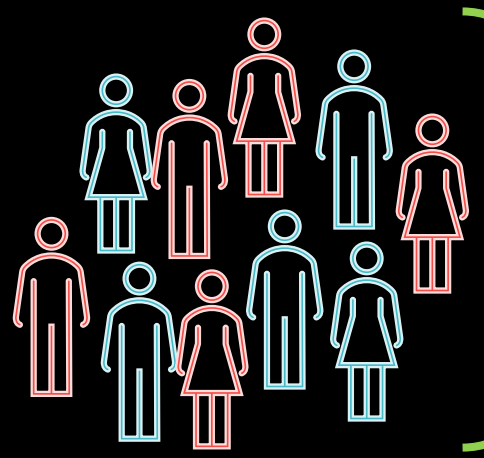
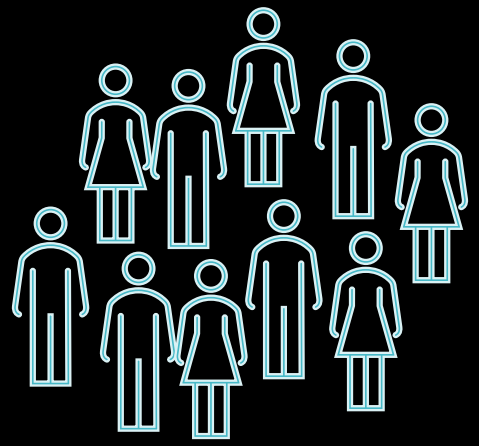


Probability of infection=50%



This is what the outcome will be no matter how many times we run the model!

Stochastic



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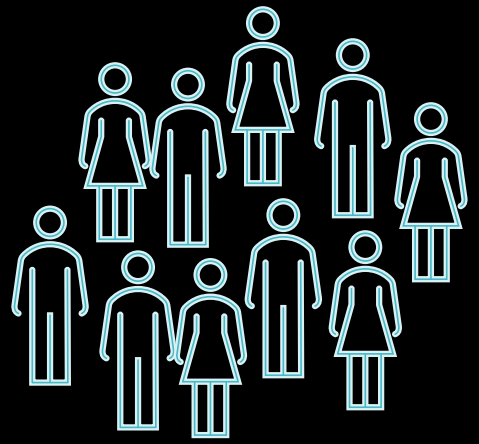

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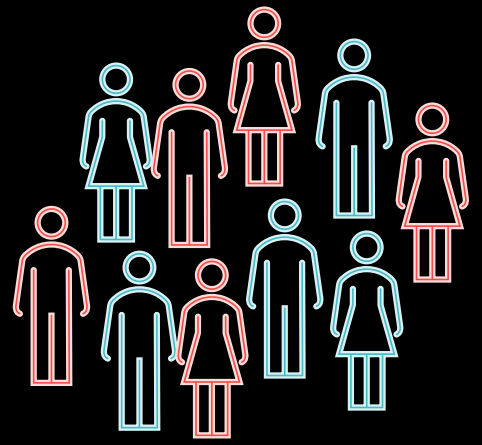

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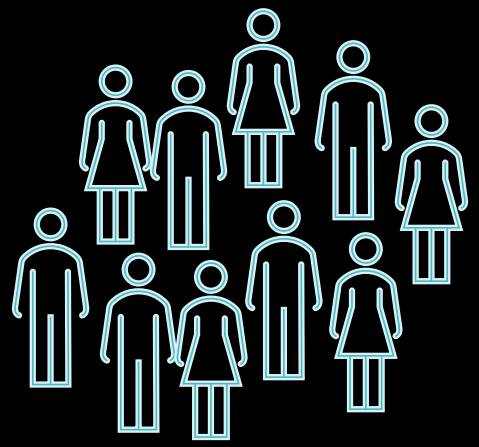

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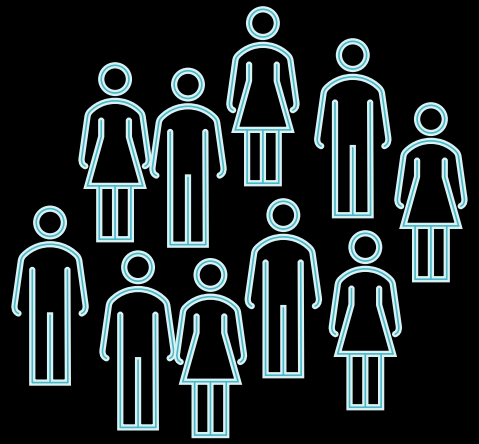

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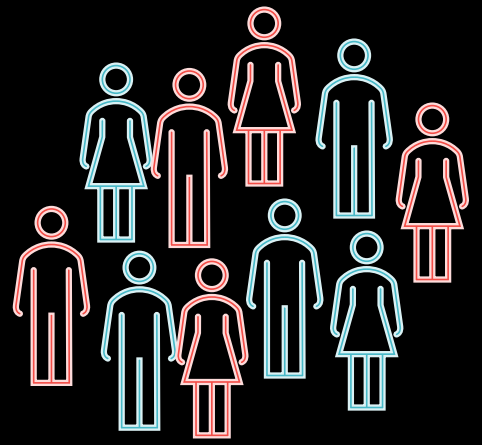

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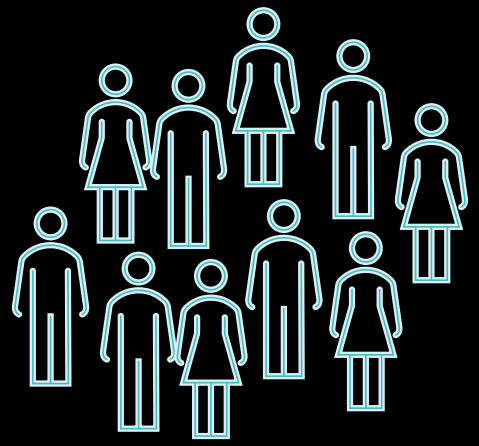

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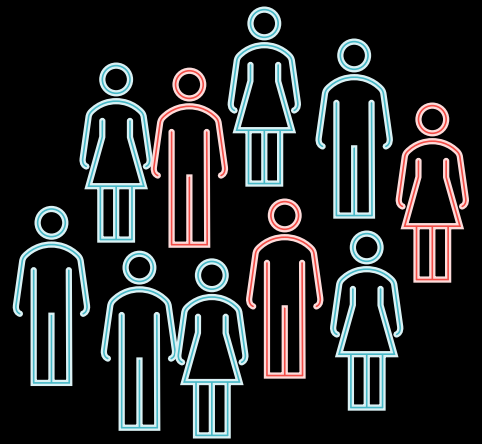

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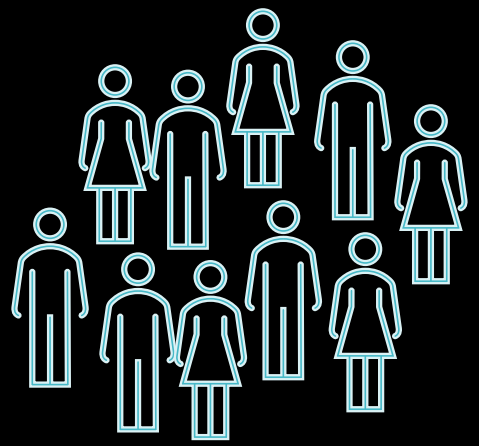

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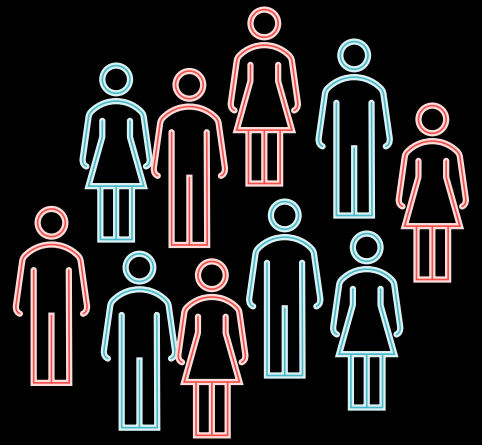

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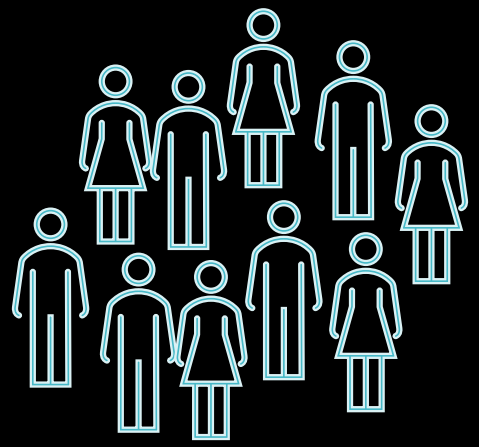

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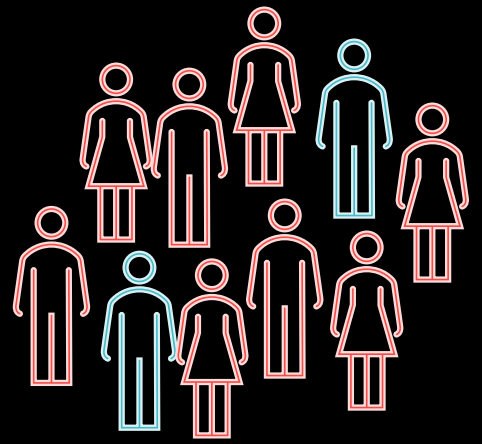

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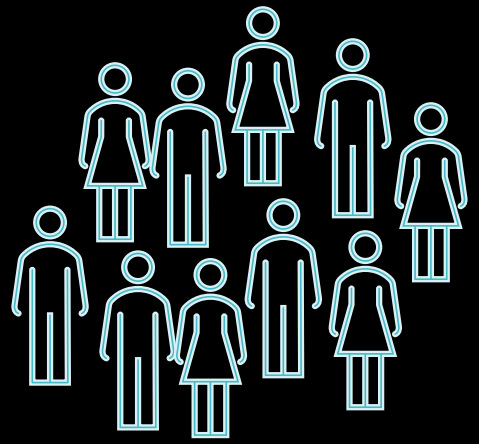

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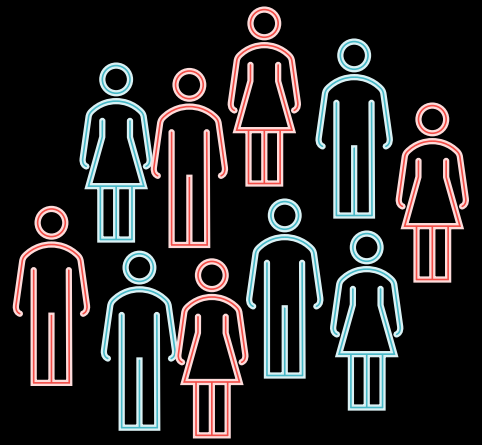

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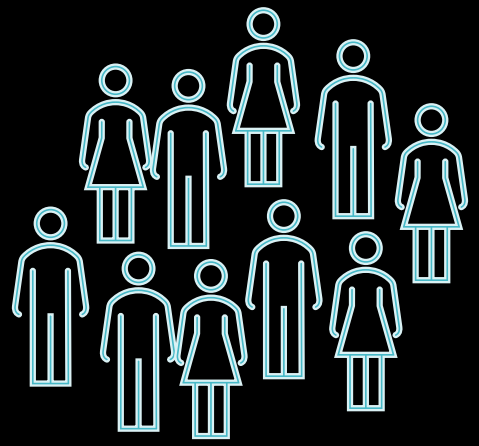

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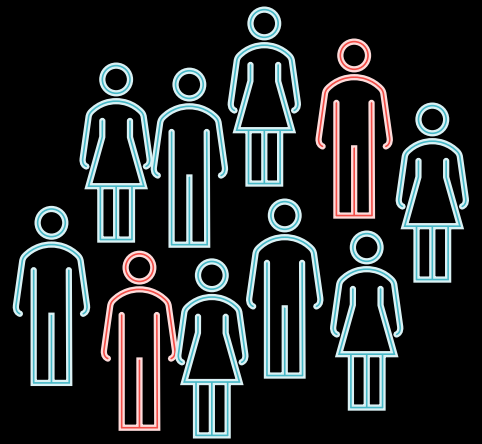

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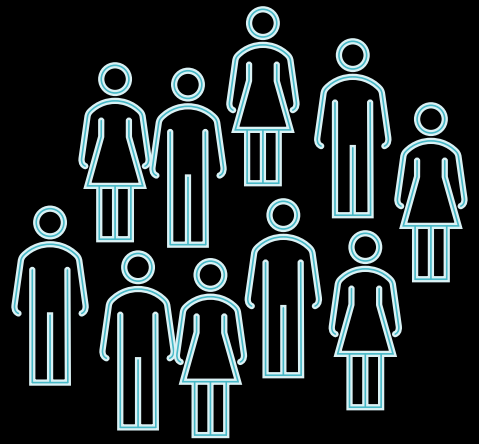

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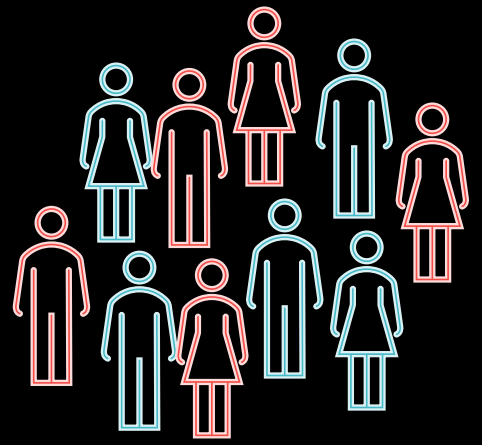

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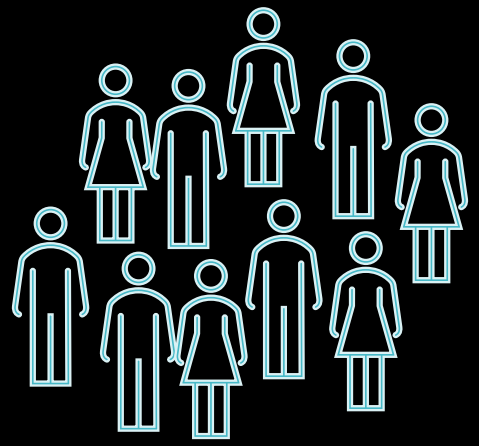

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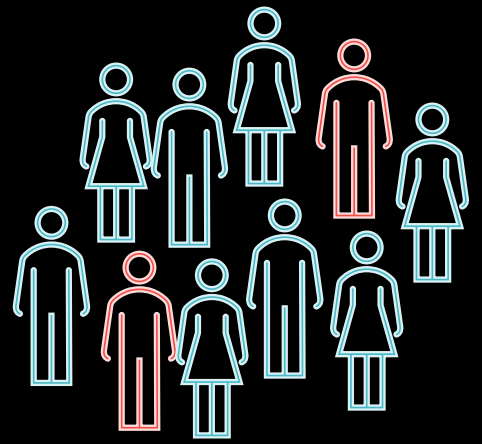

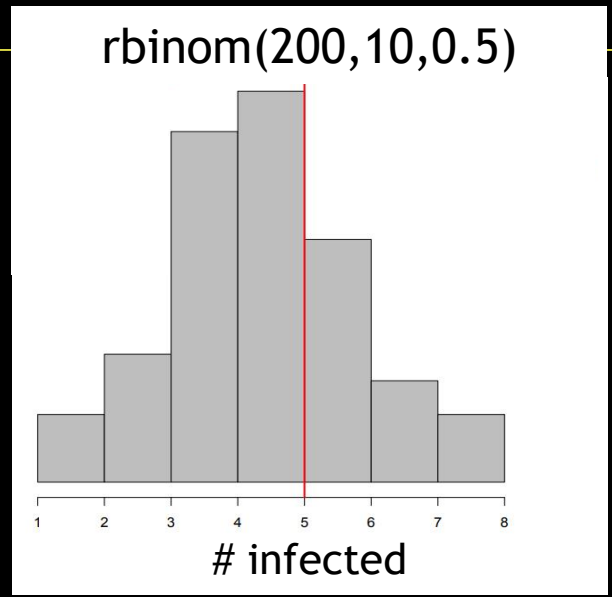

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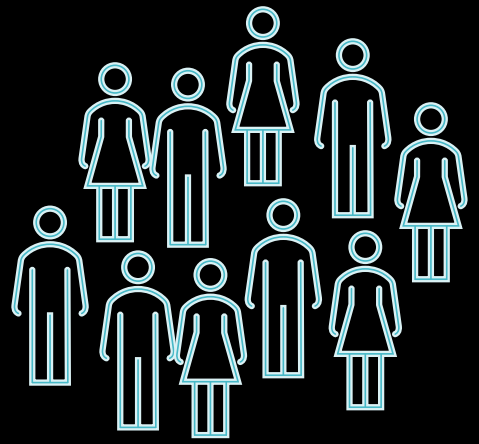

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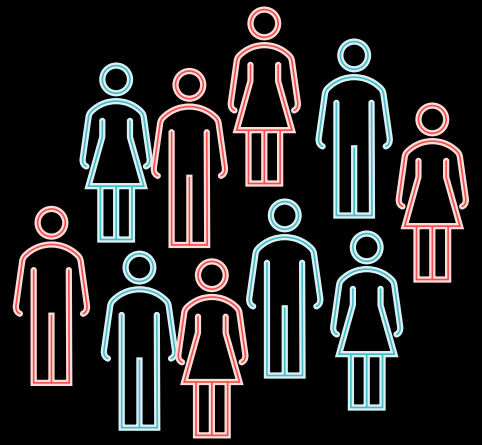

Average # infected will be 5

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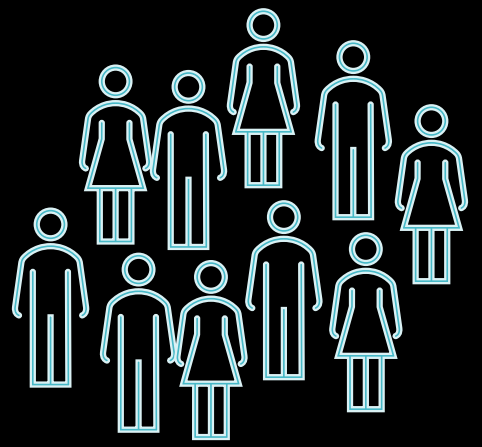

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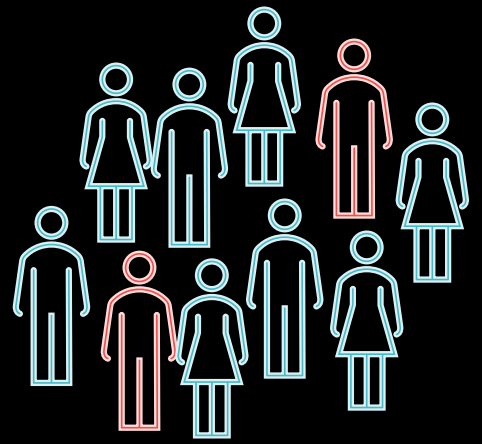

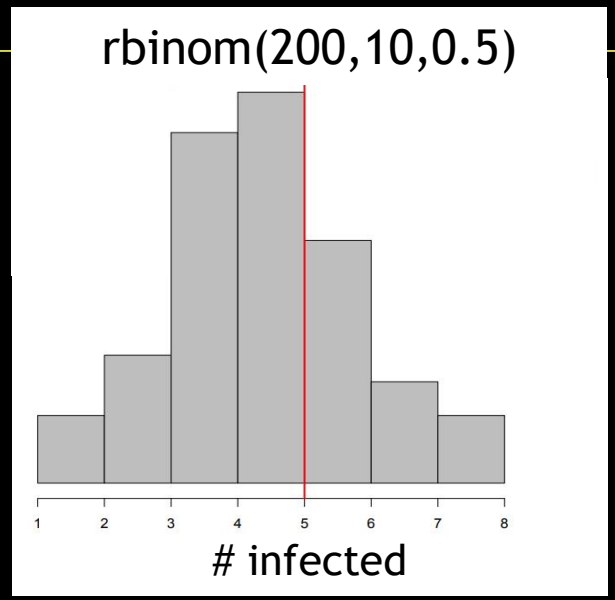

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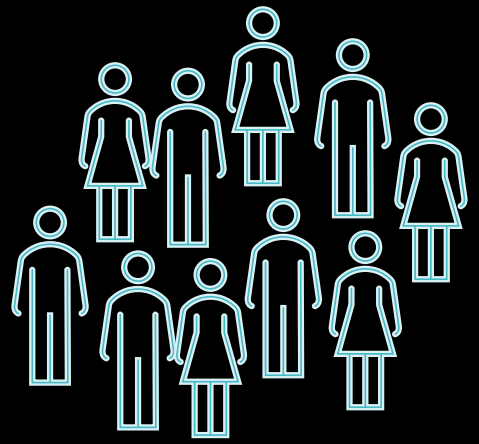

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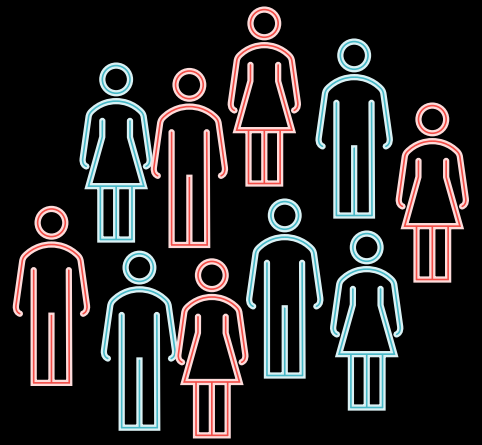

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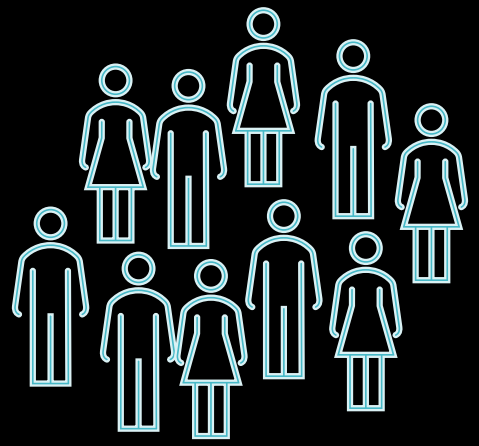

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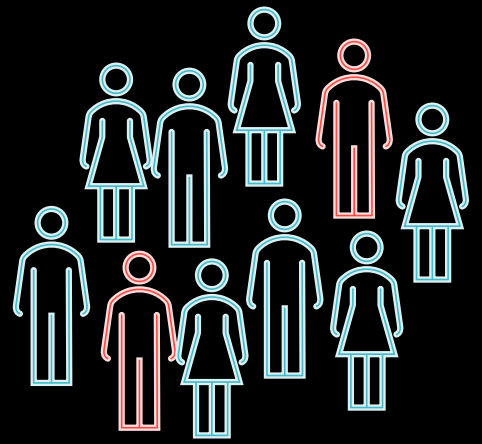

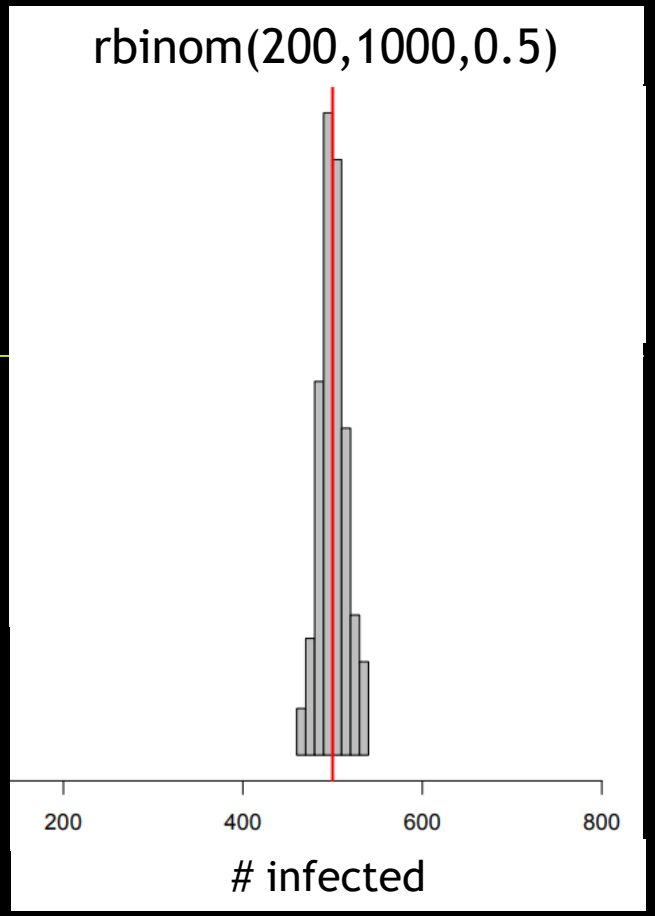

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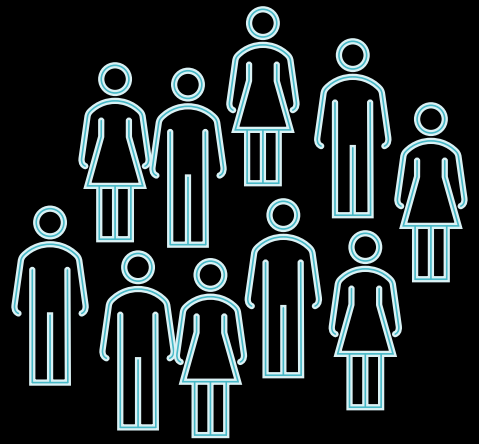

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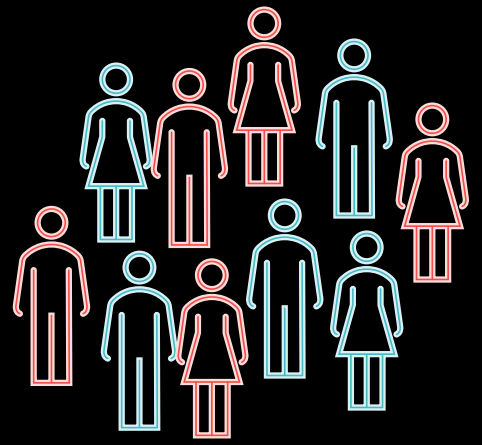

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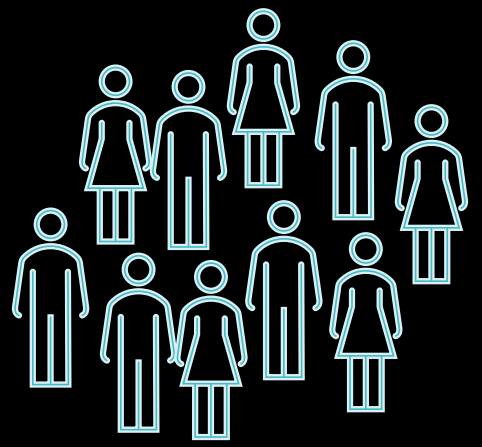

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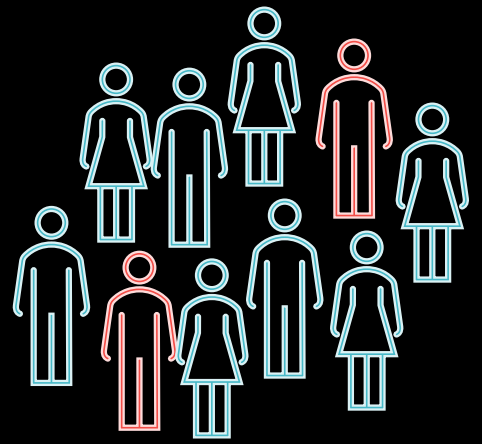
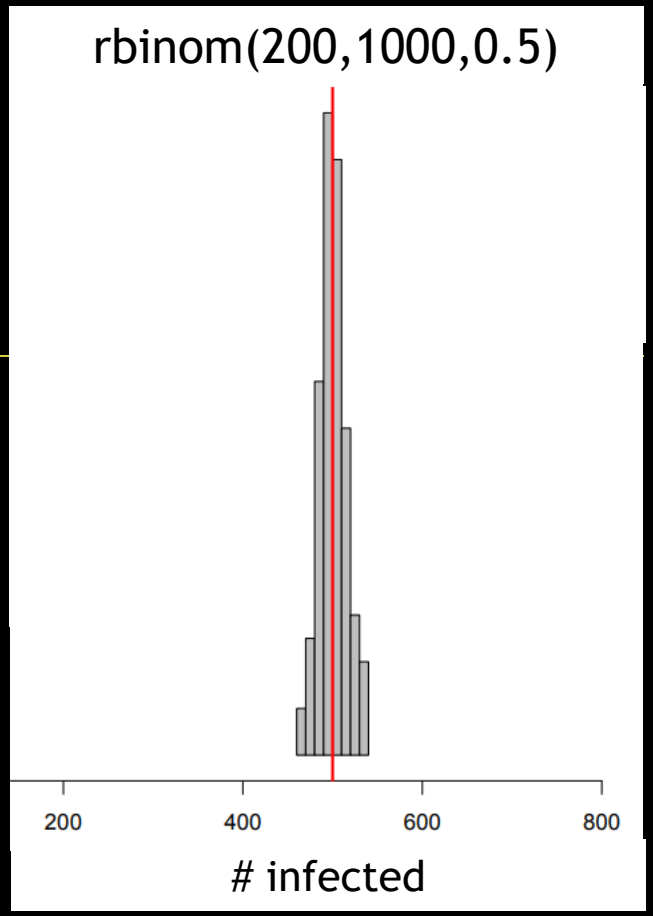
Flip a coin for each person
Probability of infection=50%



If we have many people, variance will be small – in this case deterministic predictions approximate reality



Process Stochasticity in Models

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Process Stochasticity in Models

- Depends on modeling methods being used, multiple ways to include in models
- Model able to produce different results – even without changing our input values
  - adding stochasticity causes this
- Why do we want to add stochasticity?
  - adds variability to each simulation/modeling run
  - demonstrate extremes of disease dynamics
  - more realistic, more robust

Stochasticity Through Distributions

- Instead of using a single value for our model parameters, we can use a distribution of values

Stochasticity Through Distributions

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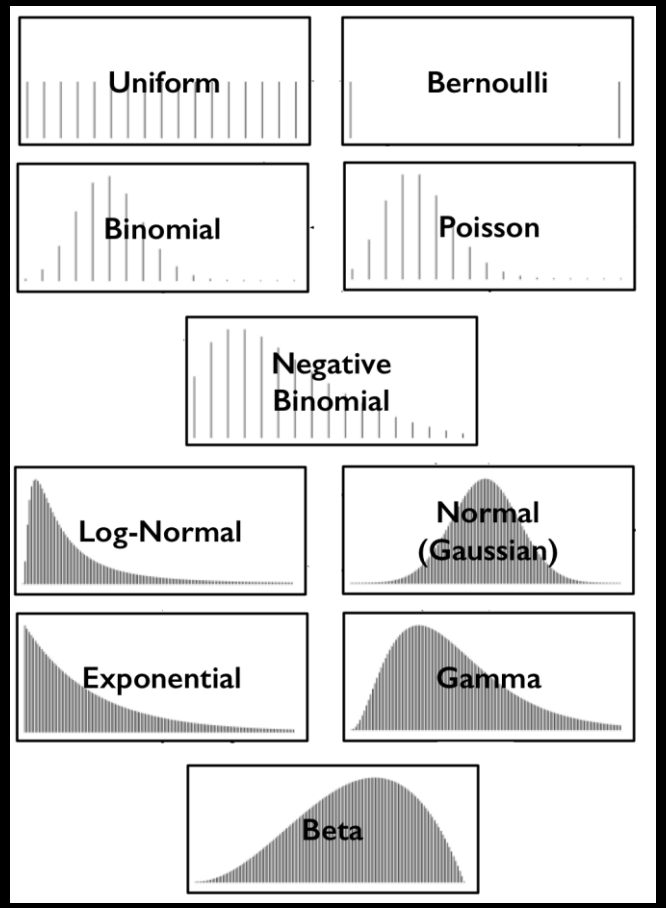
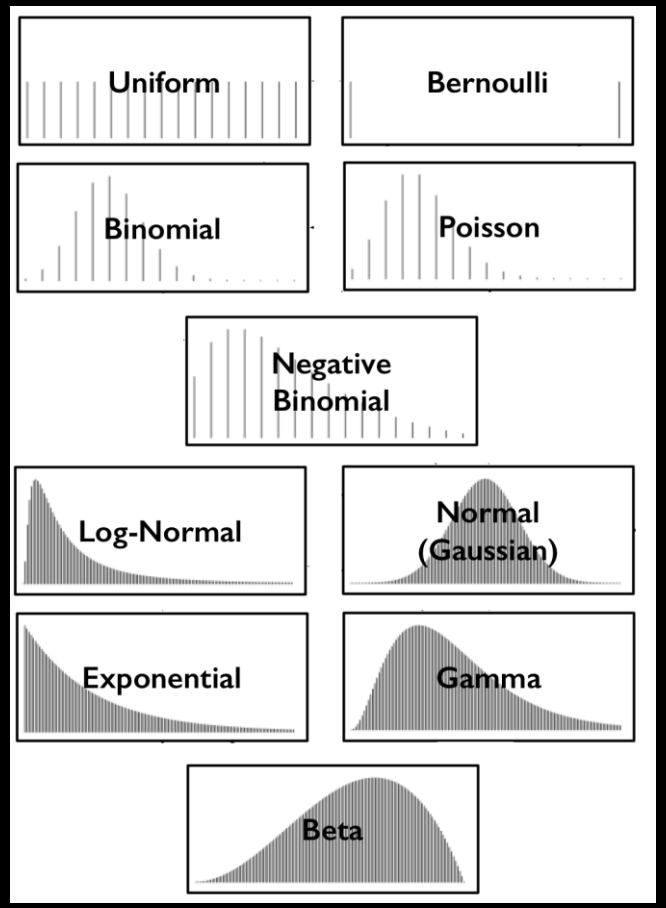
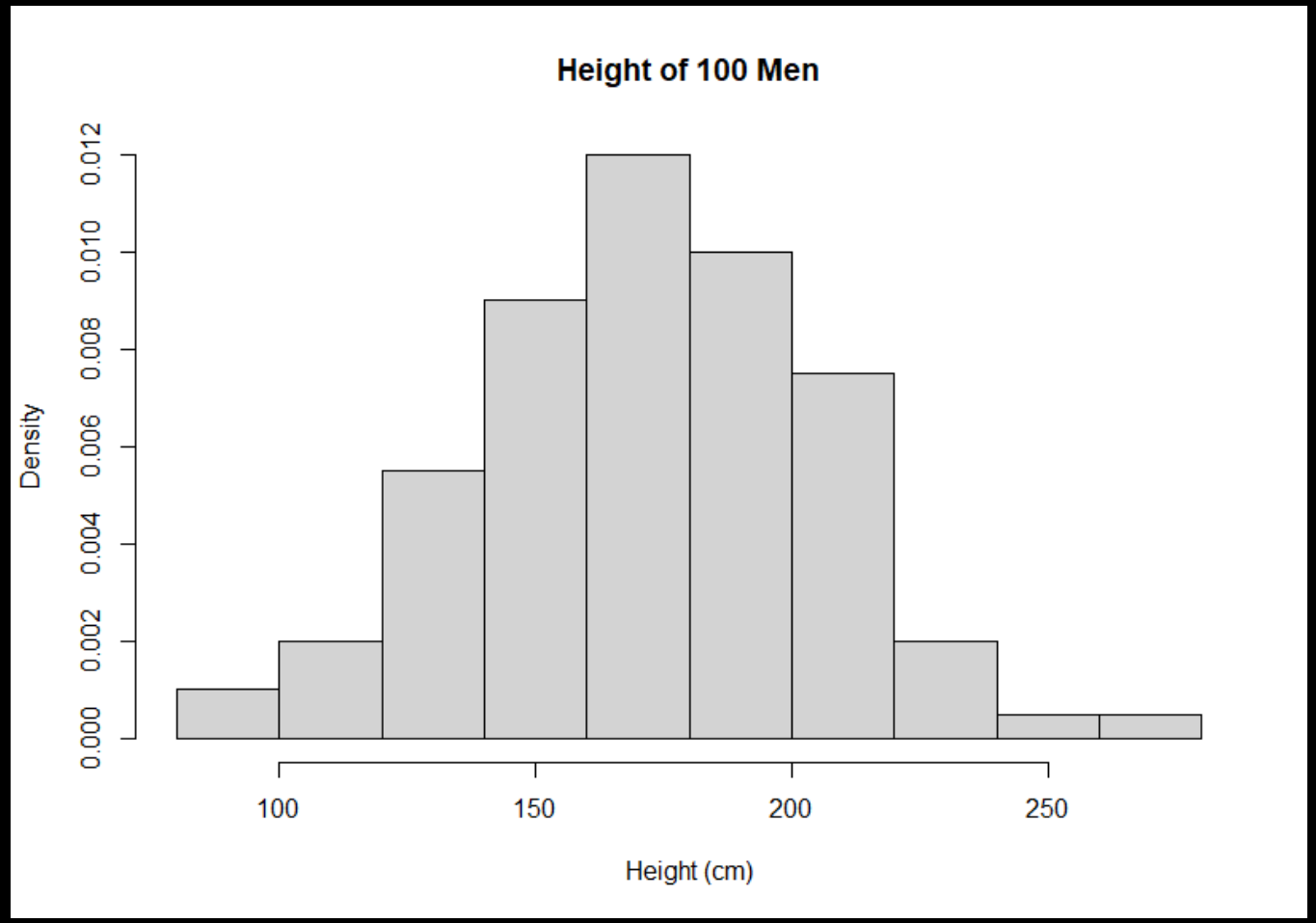
- What are distributions?



Stochasticity Through Distributions

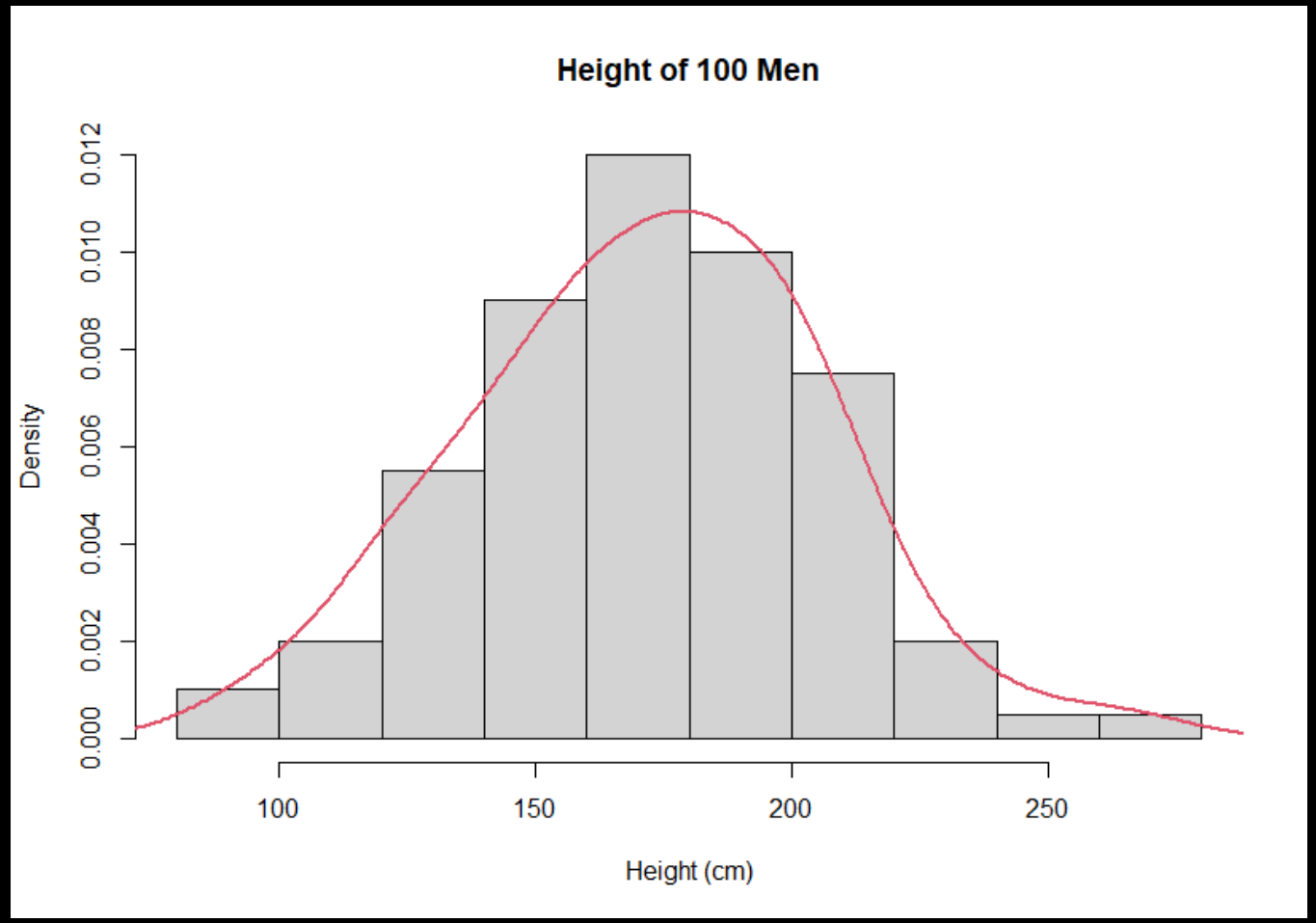
- Instead of using a single value for our model parameters, we can use a distribution of values
- What are distributions?
  - represents probability that different outcomes will occur





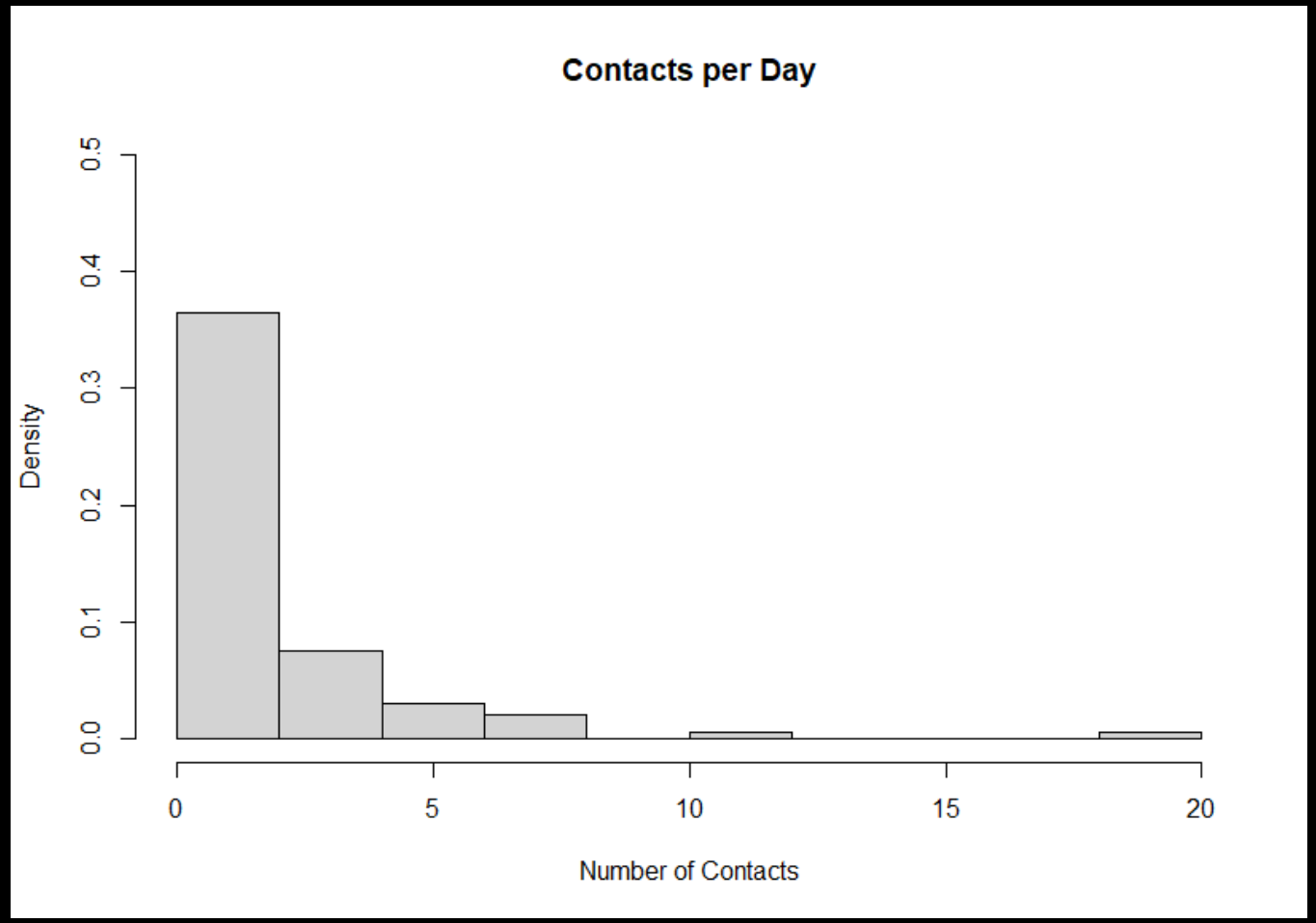
Distributions Can Represent Real-life Phenomena

- Histogram of heights of 100 men



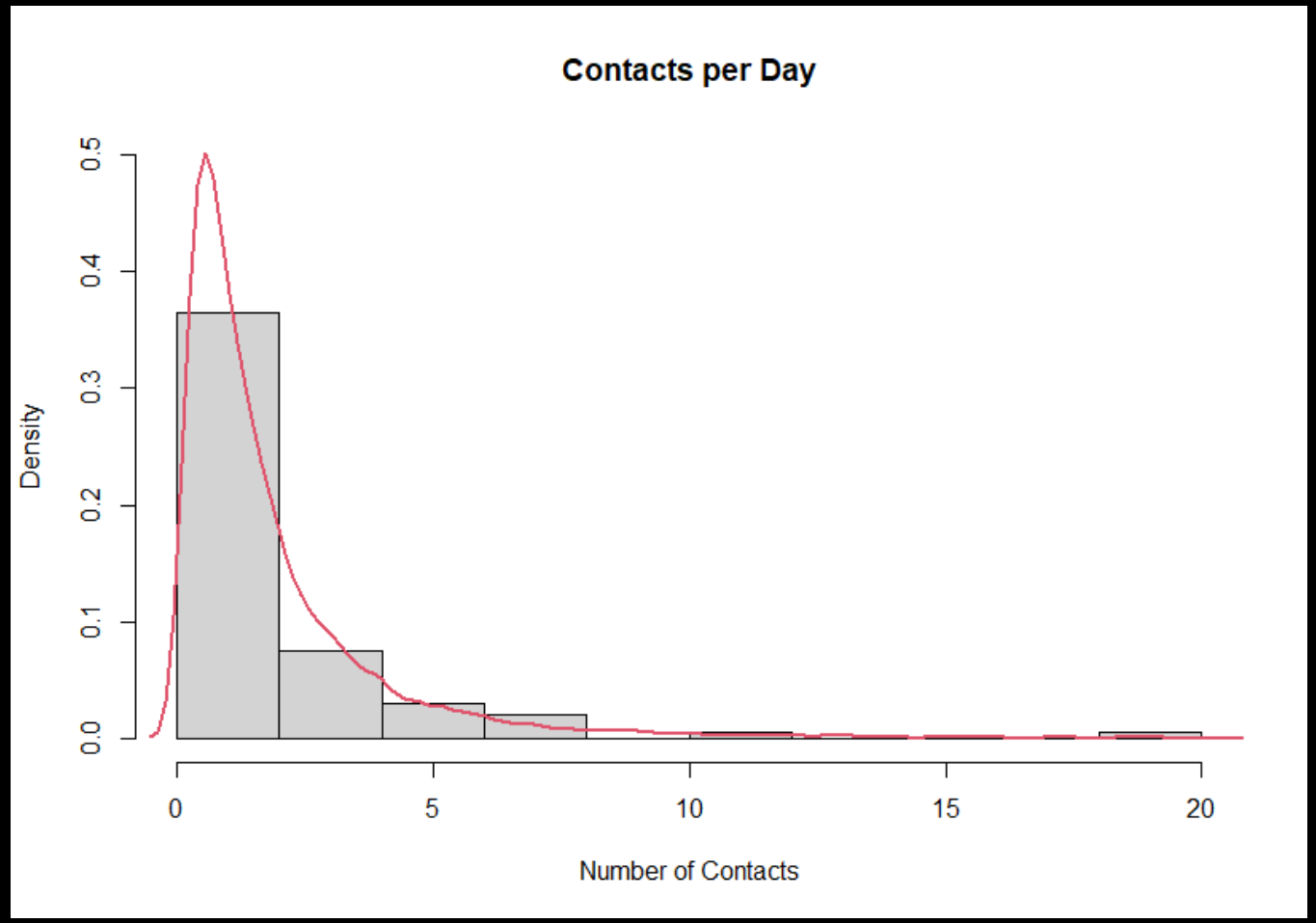
Distributions Can Represent Real-life Phenomena

- Histogram of heights of 100 men
- Approximately normal!
- $Height \sim N(170, 30)$



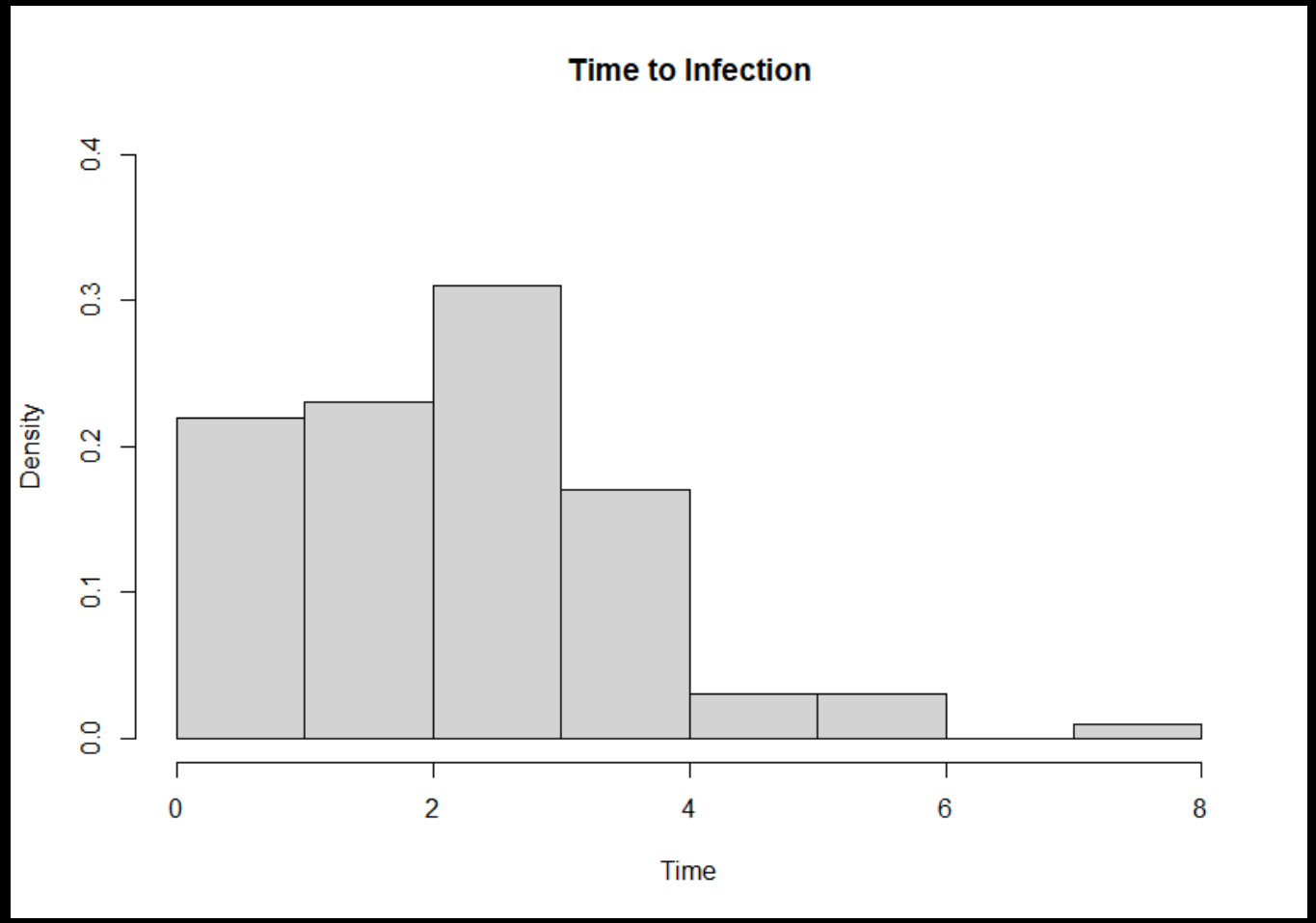
Distributions Can Represent Real-life Phenomena

- Histogram of daily number of contacts



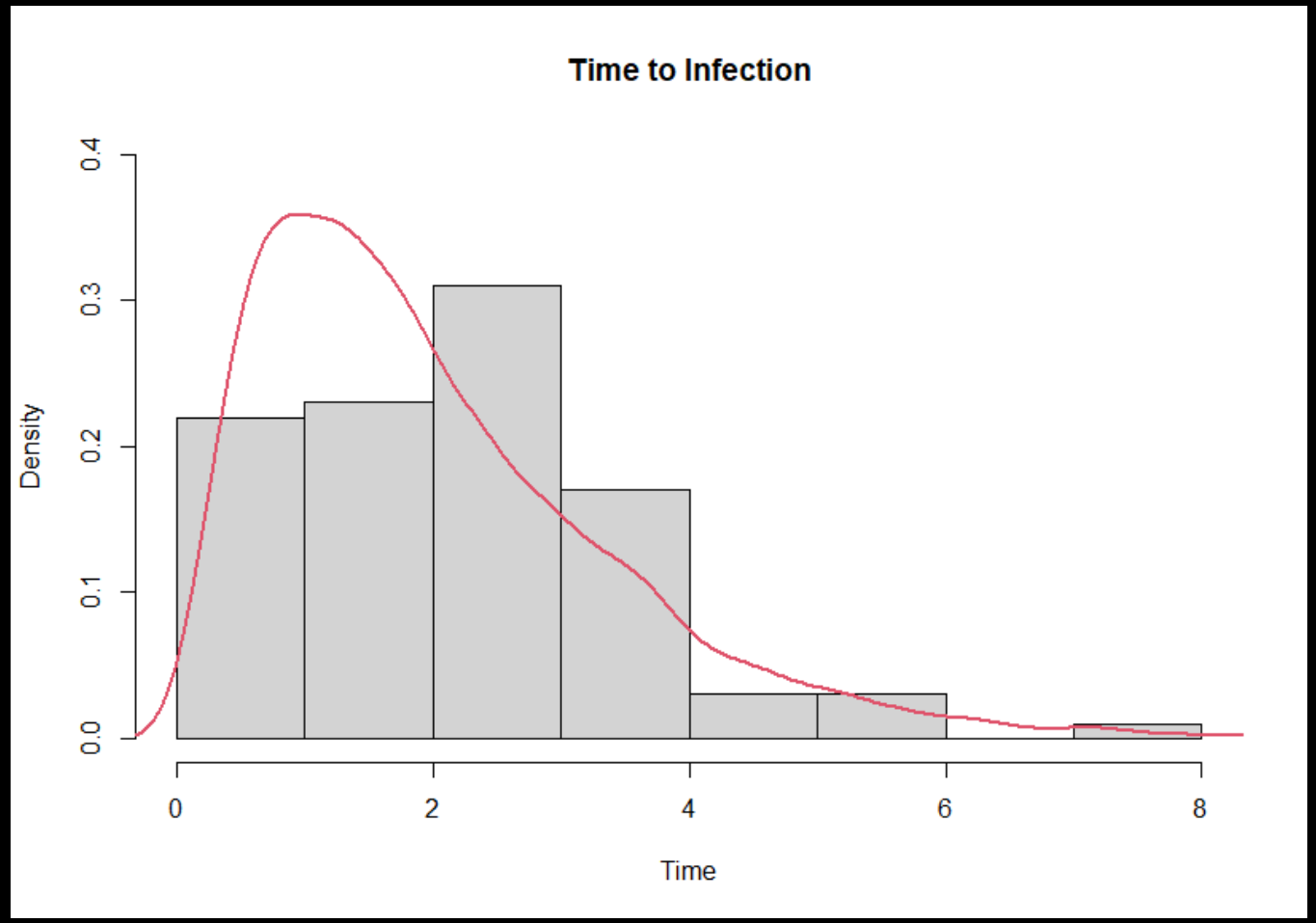
Distributions Can Represent Real-life Phenomena

- Histogram of daily number of contacts
- Lognormal distribution!
- $C \sim LogNormal(0.2)$



Distributions Can Represent Real-life Phenomena

- Histogram of time to infection

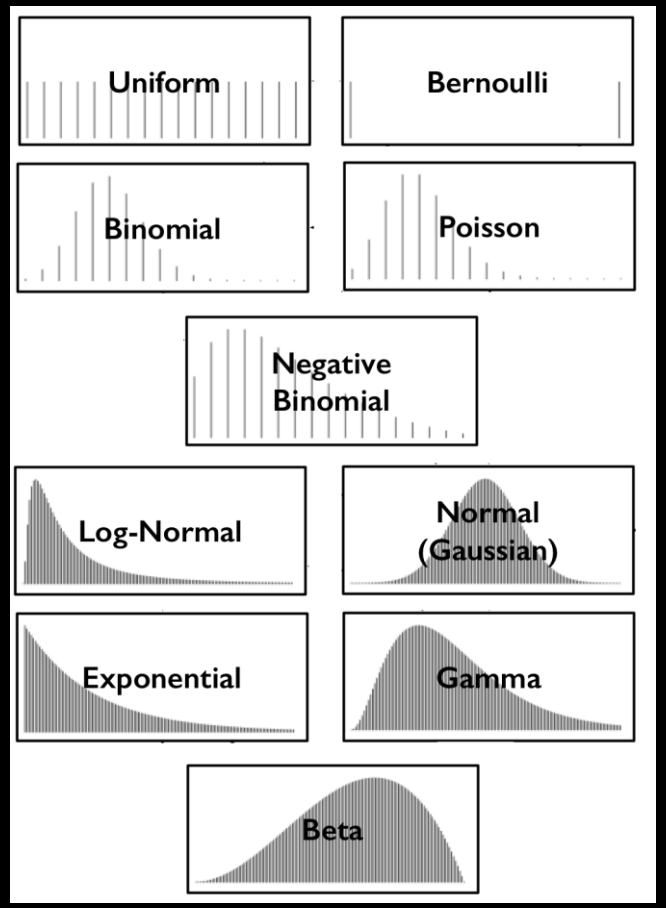


Distributions Can Represent Real-life Phenomena

- Histogram of time to infection
- Gamma distribution!
- $tInf \sim Gamma(2, rate = 1)$

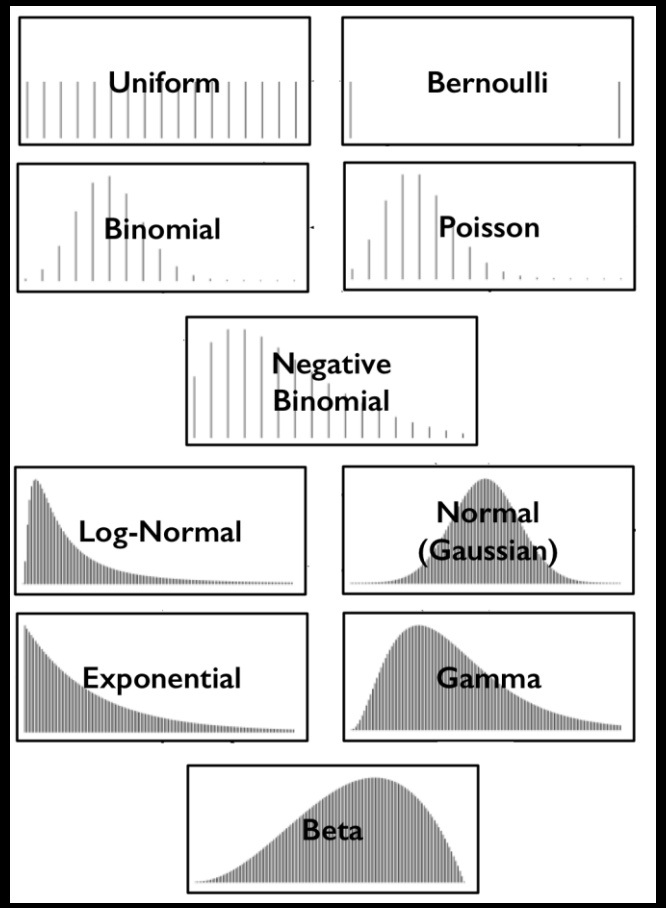
Stochasticity Through Distributions

- Instead of using a single value for our model parameters, we can use a distribution of values
- What are distributions?
  - represents probability that different outcomes will occur
- We can tell the model to select one of the values from the distribution for each run/simulation
- We can include a distribution when making calculations/estimates



Stochasticity Through Distributions

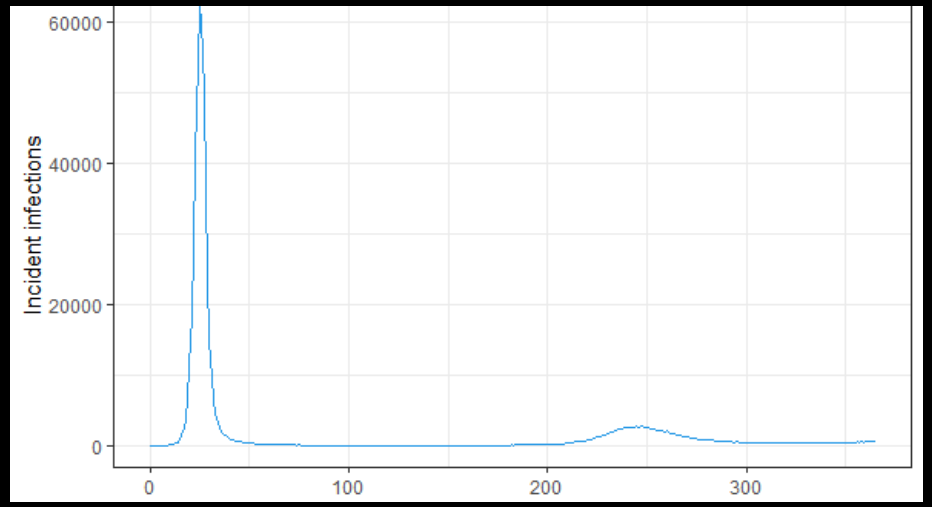
- Example: modeling force of infection (λ)
  - $\lambda(t) = \beta S(t) I(t)$
  - $I(t+1) = \sim NegBin(\lambda(t), S(t))$



Comparison

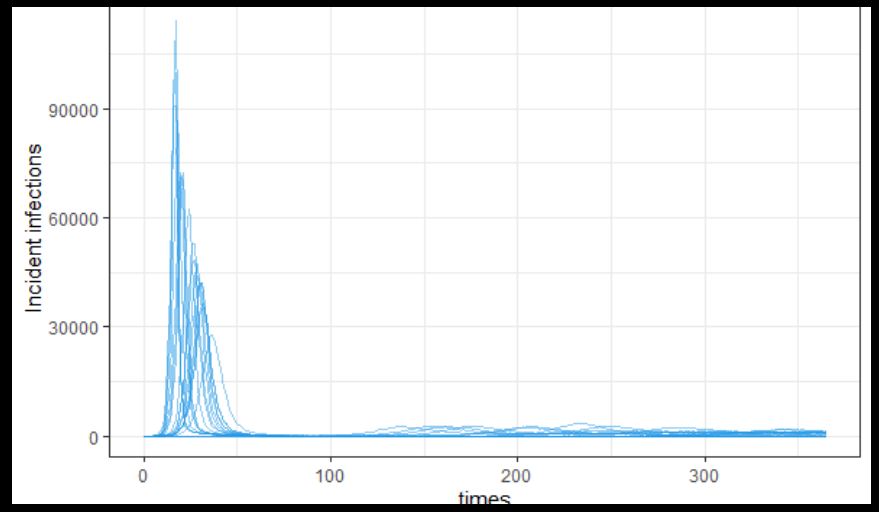
Deterministic

- Use average (single) transition rates in models
  - $\beta = 1$



Stochastic - Random!

- Use many (distribution) transition rates in models
  - $\beta \sim N(1, 0.25)$



Modeling with Stochasticity

- Important part of infectious disease dynamics
- Can impact trajectory of disease
- Many ways to integrate stochasticity
  - directly add chance to your equations

Questions?

10 minute break

Workshop Schedule

Time	Topics
2:00–2:10 pm	~~Greetings~~
2:10–3:00 pm	~~Guest lecture: COVID-19 with Megan O'Driscol~~
3:00–3:10 pm	~~Break~~
3:10–4:00 pm	~~Stochasticity~~
4:00–4:10 pm	Break
4:00–5:00 pm	R Session