



The Algorithmic Complexity of Landscapes

Fivos Papadimitriou

To cite this article: Fivos Papadimitriou (2012) The Algorithmic Complexity of Landscapes, Landscape Research, 37:5, 591-611, DOI: [10.1080/01426397.2011.650628](https://doi.org/10.1080/01426397.2011.650628)

To link to this article: <https://doi.org/10.1080/01426397.2011.650628>



Published online: 05 Apr 2012.



Submit your article to this journal [↗](#)



Article views: 237



View related articles [↗](#)



Citing articles: 27 View citing articles [↗](#)

The Algorithmic Complexity of Landscapes

FIVOS PAPADIMITRIOU

Hellenic Open University, Greece

ABSTRACT *A method to evaluate the algorithmic complexity of landscapes is developed here, based on the notion of Kolmogorov complexity (or K-complexity). The K-complexity of a landscape is calculated from a string x of symbols representing the landscape's features (e.g. land use), whereby each symbol belongs to an alphabet L , and can be defined as the size of the shortest string y that fully describes x . K-complexity presents several useful aspects as a measure of landscape complexity: a) it is a direct measure of complexity and not a surrogate measure, well supported by the literature of Informatics; b) it is easy to apply to landscapes of 'small' size c) it can be used to compare the complexity of two or more landscapes; d) it allows calculations of a landscape's changes in complexity with time; e) it can be a descriptor not only of the landscape's structural complexity, but also of its functional complexity; and f) it makes possible to distinguish two landscapes with the same diversity but with different complexity.*

KEY WORDS: Landscape complexity, K-complexity, Kolmogorov complexity, spatial complexity, Greece

1. Introduction

Geographers, landscape ecologists and land managers need to know whether a landscape is more complex than another and landscape complexity has been acknowledged as one of the key research areas within landscape ecology (Wu & Hobbs, 2002).

Understanding the complexity of landscapes (and being able to measure it), is crucial in order to proceed to assessments of landscape sustainability and landscape planning, which are practically important (Roe, 2000, 2007), and, in fact, also important in the implementation of integrated or legally binding approaches, such as the European Landscape Convention (Roe *et al.*, 2008).

In an earlier paper, Prouix (2007) acknowledged the missing link between complex systems theory and landscape analysis. But deciding whether a landscape is more 'complex' than another is a tricky issue, due to the obvious and visible (structural) relationships, as well as the sometimes invisible and implicitly assessed (e.g. functional) complex interrelationships within it. Despite these obvious difficulties, we also need to know whether a landscape's complexity increases or diminishes over a

certain time interval. Consequently, our practical needs relating to the management of landscape complexity do not match our knowledge about it, thus leading us to search for a measure of landscape complexity.

More than 30 years ago, Olson (1975) proposed to assess a map's structural complexity on the basis of spatial autocorrelation, but acknowledged the obstacles with that approach and eventually questioned whether there could ever be a general measure of map complexity at all. The problem of definition of 'map complexity' persisted and the growth of complexity theory in the 1990s complicated it even further, as, apart from map complexity (which has remained an enigma to geographers), further complications had to be accounted for in any study of 'landscape complexity': non-linear interactions, loops, distance from equilibrium, even emergence (Papadimitriou, 2010).

Ecological complexity, being a part of landscape complexity itself, was also problematic a notion, as Li (2004, p. 1) wrote that "the term ecological complexity could mean quite different things to different people". This being true for ecological complexity, we can figure out how much more difficult it would be to define landscape complexity, which is a broader concept than ecological complexity, due to the spatial extent of landscapes.

Even so, the entire spectrum of possible interpretations of the term 'landscape complexity' has probably failed to be perceived in all its extent. Thus, for instance, Snacken and Antrop (1983) used the term 'landscape complexity' to signify mainly geographic-photogeological characteristics of land units, while Steffen *et al.* (1996) embedded the notion of landscape complexity within that of biological complexity in its entirety. Both these approaches, and indeed most of the landscape-ecological approaches that followed, appear now inadequate to address landscape complexity, because it is neither map complexity only, nor ecosystem complexity either—it is both. To overcome this difficulty, in suggesting one of the first measures of landscape complexity, Papadimitriou (2002) used the terms "structural landscape complexity" and "functional landscape complexity", in accordance with the meanings of the words 'structure' and 'function' in the landscape ecological literature.

To date, with the major part of research in landscape complexity being undertaken by landscape ecologists, the landscapes' third physical dimension (topographic altitude) was usually left unconsidered. But altitude entails a geomorphological dimension of complexity, whose importance for complexity has also been stressed, because it involves significant landscape-related functional parameters such as soil erosion, slope stability, etc. (Fonstad, 2006; Murray & Fonstad, 2007; Werner, 1999).

Estimating landscape complexity has practical applications. It would help in ecological analyses, in ranking landscapes and hence in planning and landscape management. For instance, Moser *et al.* (2002) correlated the complexity of polygons of landscape patterns with species richness. Honnay *et al.* (2003) and Roschewitz *et al.* (2005) conceived landscape complexity as a predictor of plant species diversity and Papadimitriou (2009) used spatial landscape complexity as a potentially useful indicator for landscape conservation. In addition, as Poggio *et al.* (2010) showed, assessing landscape complexity can be useful in landscape management, because plant diversity correlates with the complexity of landscape corridors.

The lack of measures of landscape complexity has led researchers to employ already existing measures of landscape analysis as surrogate estimators of landscape complexity, with landscape diversity (measured with Shannon's formula) most prominent among them. Thus, for instance, Gabriel *et al.* (2005) used landscape diversity and patchiness as indicators of landscape complexity, Prouix and Parrott (2008) used diversity and spatial heterogeneity as indicators, while other researchers (Herzon & O'Hara, 2007) have used the term 'complexity' simply to signify heterogeneity. Also, diversity is intricately related to complexity, as in the 'stability-complexity' debate (May, 1973; Pielou, 1975) and in landscape connectivity analyses (Green & Sadedin, 2005).

Obviously, implicit assessments of landscape complexity from other landscape metrics (such as landscape diversity, landscape patchiness or any other), can only partially be epistemologically justified: it is not accepted by all researchers that measuring a certain physical magnitude (e.g. diversity) is enough to lead us to the assessment of another magnitude (e.g. complexity). For this reason, we need to search for direct measures of landscape complexity, derived from complexity theory. One example is the effort made by Papadimitriou (2009), who developed a measure of spatial landscape complexity, by using the Levenshtein algorithm.

Later, Papadimitriou (2010) classified landscape complexity in three distinct (yet interwoven) types:

- a) Spatial/structural (which can be measured from the landscape's maps);
- b) Functional/hierarchical (measurable from non-spatial variables and features, such as network connectivity, species movements, flows of food or energy over/in a landscape); and
- c) Qualitative (the type of landscape complexity, which corresponds to the landscape's meanings, values, aesthetics, etc.).

Evidently, computational approaches may safely be applied to the first two types of landscape complexity only.

Yet, it would not be advisable to embark in expressing judgements about a landscape's complexity without first examining the applicability of the standard measure of complexity established in Informatics (Kolmogorov, 1965), which is the 'Kolmogorov complexity', or 'K-complexity' (Kolmogorov, 1965).

Some refer to it as 'Kolmogorov-Chaitin complexity', due to the similar approach to complexity offered by Chaitin (1966). Kolmogorov complexity is different than 'Solomonoff complexity' (Solomonoff, 1964) and different from the 'Minimum Message Length' (MML) which was defined by Wallace and others (Wallace, 2005; Wallace & Dowe, 1999). The Minimum Message Length has already been used in ecology (Dale, 2001) and is based on Shannon's theory of information and on Bayes's rule. Solomonoff complexity is based on the theory of Turing machines and gives a probabilistic distribution of the set of binary strings as a model for the probability corresponding to the data of a string. This makes it somewhat cumbersome for applications, while the definition of Kolmogorov complexity is more straightforward and is based on the theory of Turing machines only.

The concept of Kolmogorov complexity is indeed very simple. In order to evaluate how complex a set of things is, we can start from the simplest case, which is when we put different things one next to the other. Consider, for instance, a simple example of a sequence of stationery items put on the table, beginning with staples, followed by a pencil, next a blue pen, next a red pen, and finally another pencil. Clearly, the more different things we have put there, the more difficult it will be to describe, to memorise, to explain what is there and how this sequence of items develops in space. This means that the more different things we have located along that line, the more 'complex' the sequence is. In computer science, we say that we have constructed a 'string' of items (a sequence, a line of items, one after the other). This string of things can be represented in an abstract way, by attributing a different symbol to each different item of the string: say for instance, *A* for the staples, *B* for the pencil, *C* for the blue pen, *D* for the red pen. Hence the sequence of the items previously mentioned can be represented by the 'string': *ABCDB*. This is a string of symbols representing our initial string of items. The same procedure can be followed with a landscape along a line, where in place of stationery items we have cells of different land cover/land use.

We can now introduce the notion of K-complexity formally: the K-complexity of a string of symbols *x* is the size of the shortest string *y* from which a 'Turing machine' can produce *x*. K-complexity refers to a single object (string) and measures the amount of information necessary to describe it. It measures the length of the shortest string *y*, from which a Turing machine produces *x* in *t(x)* steps. For further information about K-complexity, the reader is referred to the classic text of Li and Vitanyi (1997).

A string is said to be 'K-incompressible', if it cannot admit any shorter description. A K-incompressible string of length *n* cannot be printed by any program of size strictly less than *n* (e.g. by a program of size *n*−1), so given a portion of any K-incompressible string, it is impossible to compute the remaining part of the string. Otherwise put, K-complexity can give us a measure of simplicity of a string's description. A string *x* is called *incompressible*, if its *K(x)* is greater or equal to *x* itself. Obviously, a random string is incompressible.

Consider, for instance, the two following strings:

x: 0111011101110111
y: 0101111010001100

The string *x* clearly has a pattern: it is four times the block 0111, so if we multiply block 0111 four times, we get string *x* as a result. This string is therefore compressible; a characteristic that cannot be attributed to the *y*, which is incompressible, because there is no simpler description for it, no regularity and no pattern either.

But, is K-complexity suitable to describe landscape spatial/structural complexity? What are its advantages and disadvantages for this purpose? These are the questions this paper is trying to tackle: it aims to put the computation of landscape complexity in the context of K-complexity and, consequently, to shed light on the fields in which difficulties arise.

2. Methods and Data

2.1. Outline of Methods

In order to assess a landscape's structural complexity by means of K-complexity, it suffices to observe that a landscape's structure is adequately characterised by a string x of symbols, representing land cover or land use and the finite set of these symbols defines the language L of the landscape. It is precisely at this point that K-complexity should serve as the obvious method for the assessment of landscape complexity. Hence, the methods to derive calculations of landscape complexity unfold in four steps:

First step: Computation of structural landscape complexity of an example landscape with the use of K-complexity, so as to demonstrate the applicability (and limitations) of K-complexity as a measure of landscape complexity.

Second step: Calculation of functional K-complexity of another example landscape. At this step, we need to take into account that, from the computational point of view, the more physical-geographic descriptors (symbols) are added to the landscape (soil type, slope steepness, soil depth, etc.), the more the resulting 'alphabet' describing a string would grow in size. Hence, the description would encounter the obstacle of the problem of 'minimal length encoding', that is, the problem of finding minimal descriptions of the landscape, such that would fully describe the landscape.

Third step: Calculation of changes in K-complexity with time (structural and functional) on an example landscape.

Fourth step: Application of the methods developed previously to compute the structural K-complexity of a real landscape from Greece (the area of Pendeli, east of Athens, Greece).

The following conventions will apply henceforth:

$\{\dots\}$ will denote a string of symbols,

$U\{\dots\}$ will denote the calculation of the uncompressed length of the string (in symbols) and

$K\{\dots\}$ will stand for the K-complexity of the string y , resulting from the minimal length encoding of the string x .

So, in all cases, the general method suggested here is to follow these steps:

- a) convert the landscape's characteristics into a string x of symbols $\{\dots\}$,
- b) calculate the uncompressed length of that string of symbols: $U\{\dots\}$, by counting the total number of symbols contained in this string,
- c) create a shorter description y of the string x ,
- d) evaluate K-complexity of string y , by counting the total number of symbols contained in that string: $K\{y\}$.

2.2. Data

In order to test this method on a real landscape, a landscape from the northern outskirts of Athens was selected, in the mountainous area of Pendeli. This landscape is of area equal to 30 km² (dimensions 5 by 6 km) and was examined at a scale of 1:175 000. Two land cover maps have been constructed for this area: one for the year 1967 and another for 1988 (Figure 1).

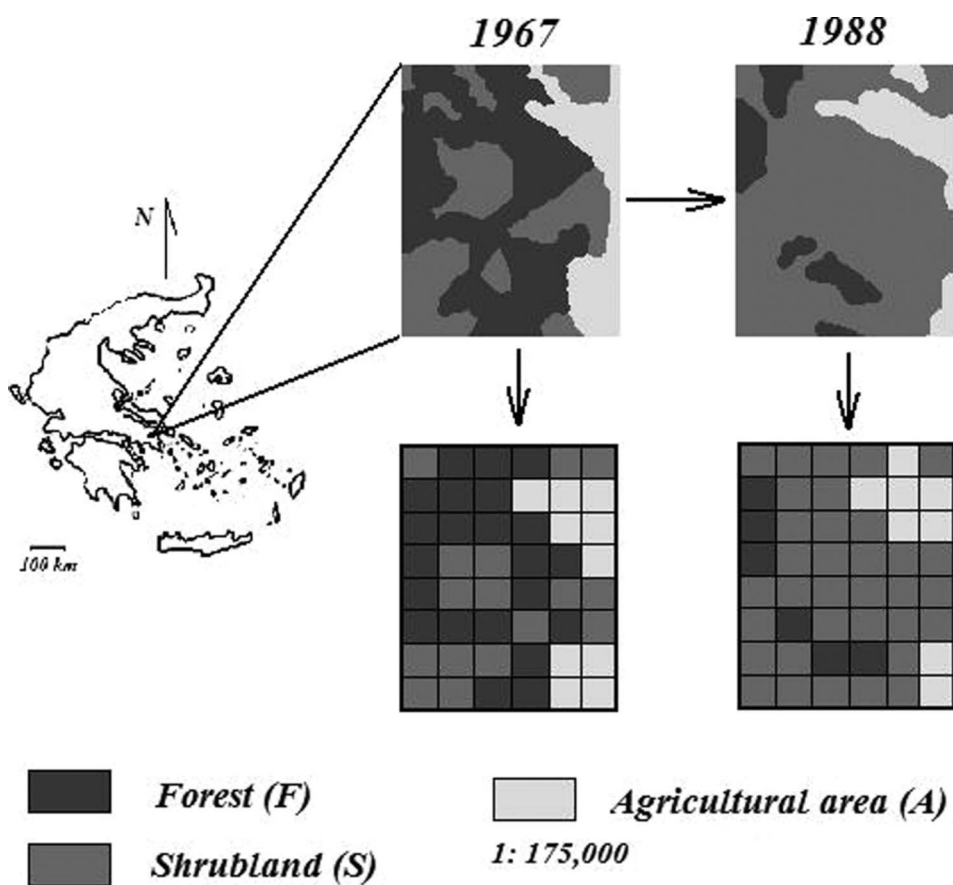


Figure 1. The landscape of Pendeli in 1967 and in 1988 and its location within the country. The landscape maps of 1967 and 1988 have been generalised below to the rasterised 6 x 8 cells maps.

The former was created on the basis of photo-interpretation of aerial photographs taken in that year and shows three types of land cover: Forest (F), Shrubland (S) and Agriculture (A). The latter map presents the same land cover types and was created from supervised classification of LandsatTM satellite imagery and corrected after field mapping. Both maps were processed on a geographical information system (GIS) and were subsequently divided by a 6 by 8 cells grid. The prevailing land cover was assigned to each cell of the grid (cell area 0.625 km²), so that each cell had only one land cover type. Consequently, it was possible to derive a map of land cover transformations from year 1967 to 1988 for subsequent analysis with the aid of the GIS Idrisi.

The area selected is ecologically interesting because of its severe land degradation which has been well documented (Domenikiotis *et al.*, 2002; Keramitsoglou *et al.*, 2004; Papadimitriou, 1995; Vakalis *et al.*, 2004). A prominent feature of this nexus of ongoing land degradation processes is the transformation of large parts of landscape

from the original Mediterranean *Quercus ilex* and *Pinus halepensis* forest (F) to degraded vegetation of *Quercus coccifera*, *Euphorbia* sp., *Sarcopoterium* sp. and other shrubby vegetation species (S) of lower eco-physiological complexity. These are land degradation processes typically encountered in Southern European landscapes (Papadimitriou & Mairota, 1996). The same succession sequences of expanding shrubby vegetation are encountered over the time interval considered on the region's agricultural (A) lands also. The main cause of these land degradation processes is the forest fires, which have repeatedly ravaged the Pendeli mountains during recent decades, with the last ones in 2007 and 2009.

3. Computation and Results

3.1. Computation of Structural Landscape K-complexity

The K-complexity of a landscape's structure describable by a string or sentence x of symbols (whereby each symbol belongs to a language L) is the size of the shortest string y that fully describes x . Hence, the assessment of the landscape's algorithmic complexity as per the landscape's structural components is equivalent to the evaluation of y from the landscape's land use/cover map.

Consider, for instance, the example landscape shown in Figure 2, covered by four distinct land cover types (A,B,C,D), so the alphabet representing this landscape consists in four symbols only (A,B,C,D). This alphabet could also be extended to contain numerals (for instance, powers of symbols: A^n , when a symbol is repeated n -times consecutively), as well as any other symbol: %, / #@, etc.

Then, the corresponding string x of the example landscape, representing these symbols expressed in L , taken from left to right, cell by cell, row by row, one row after the other, is the following:

$$x = \{A C B A C B A^2 B C D A C B C D B C D C B A D B C B A D A^2 D A\}$$

As previously stated, the string's length $U\{x\}$ is equal to the total number of symbols it contains (characters, parentheses, numerals, etc.), so $U\{x\} = 32$.

But its K-complexity may be less than 32. To verify this, we need to search for any regularities and patterns in the apparitions of certain blocks of symbols in the

A	C	B	A	C	B	A	A
B	C	D	A	C	B	C	D
B	C	D	C	B	A	D	B
C	B	A	D	A	A	D	A

Figure 2. Example landscape for the calculation of structural K-complexity.

uncompressed string. In this example landscape, we observe that two blocks of symbols (CBA and BCD) recurrently appear throughout the string x :

A C B A C B A A

B C D A C B C D

B C D C B A D B

C B A D A A D A

In order to determine the K-complexity of string x then, we need to calculate the string y with the ‘minimum length encoding’ of string x . That is we simply need to represent these frequently occurring blocks of symbols with other symbols. Thus, we may use the symbol λ for the block CBA and the symbol μ for BCD and we may now substitute λ and μ in the original string x . In this way, the original string x becomes:

$$x = \{A \lambda^2 A \mu A C \mu^2 \lambda D B \lambda D A^2 D A\}$$

The length of this string is 18, but the new total string, including the strings denoting these abbreviations, is the string y sought:

$$y = \{A \lambda \lambda A \mu A C \mu \mu \lambda D B \lambda D A^2 D A \lambda = CBA \mu = BCD\}$$

This string has $U\{y\} = 28$ and the K-complexity of the initial string x is, as previously, the cardinality of the string y :

$$K\{x\} = U\{y\} = 28$$

therefore concluding that ‘the string x has a K-complexity equal to 28’, or, equivalently, ‘the structural K-complexity of the landscape whose structure is represented by the string x is 28 letters’.

Certainly, a simplification of the original string x is meaningful only then, when the string y has cardinality strictly less than that of x . Otherwise the simplification is trivial and the original string x is treated as a random string which admits no simplification:

If $U\{y\} < U\{x\}$, then the K-complexity of x equals to $K\{y\}$

If $U\{y\} = U\{x\}$, then the K-complexity of x equals to $K\{x\}$

We now observe that since a landscape’s land cover map is a two-dimensional object, its complexity might be computed by taking the landscape’s cells column by column instead of row by row. Hence, from the landscape of Figure 2 again, we derive the string x' :

$$x' = \{A B B C C C B D A A A C D C C B A B B A A A C D D A D B A\}$$

The minimal length encoding y' of this string is derived from the substitutions:

$$A A A C D = \kappa \text{ and } C C B = v$$

$$y' = \{A B B C C \kappa B D D \kappa v B B \kappa D A D B A \kappa = A A A C D \ v = C C B\}$$

Since $U\{y'\} = 31$, we observe that the same landscape has two different minimal descriptions: one taken row by row resulting with $U\{y\} = 28$ and another taken column by column, with $U\{y'\} = 31$. This forces us to always need to compute two K-complexities, that is two minimal encodings (row by row and column by column respectively) for each landscape and consequently select the encoding with the least value as the final evaluation of its spatial complexity. Using the formalism adopted so far, we can therefore derive a measure of the landscape's spatial K-complexity K_S as the overall minimum of its descriptions taken horizontally (strings x, y) and vertically (strings x', y'):

$$K_S = \min(U\{x\}, U\{x'\}, U\{y\}, U\{y'\}).$$

Hence, for the landscape of Figure 2, we have:

$$K_S = \min(U\{x\}, U\{x'\}, U\{y\}, U\{y'\}) = U\{y\} = 28$$

3.2. Computation of Functional Landscape K-complexity

A landscape's 'chorological' and 'topological' characters can be studied as well, by means of K-complexity. It suffices to observe that, as landscape ecologists and geomorphologists know, certain functional characteristics of a landscape can be studied together with the spatial units to which they relate.

Take, for instance, a landscape with three kinds of land cover: F =forest, S =shrubland, B =bare ground/rock (Figure 3).

These land cover types are found either upslope or downslope over a rectangular partition on this landscape. Depending on their relative position on the landscape,

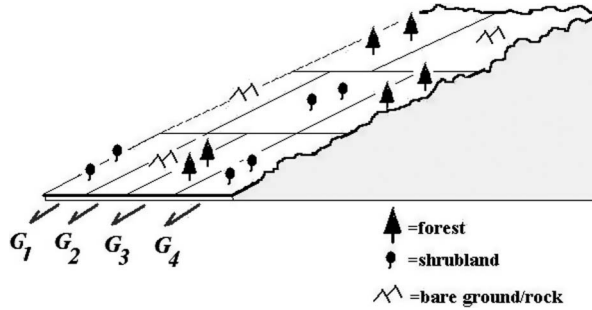


Figure 3. An example landscape with three types of land cover: F =forest, S =shrubland, B =bare ground/rock (to which correspond the landscape functions shown in Table 1). The land use patches together with the landscape functions of Table 2 give the strings G_1, G_2, G_3 and G_4 of structural-functional characteristics of this landscape.

they pertain to particular landscape functions, with code numbers shown in Table 1. (Certainly, this table is only indicative of the multiplicity of landscape functions that can be encountered and it is given here for illustration purposes only.)

Therefore, a landscape such as that shown in Figure 3 will be described by a series of four strings, G_1 to G_4 . The list of strings G_i describing the landscape of Figure 3 is given in Table 2.

The string G_1 describes the landscape from the first land cover encountered upslope (this being F) through B , down to the last land cover downslope (this being S), passing through the functions code-numbered 3,5,7. Similarly, the string G_4 describes the landscape from B (upslope) through F to S (downslope), passing through the functions 3,5,7 and 1,3,4,8 (Figure 4).

Therefore, an arbitrary transect on a landscape outcropping the strings G_2 , G_3 , G_3 , G_4 with this order (Figure 4) will have the following functional K-complexity K_F :

$$\begin{aligned} U_F \{G_2 G_3 G_3 G_4\} &= \\ &= U_F \{F 2 6 S 3 7 B B 3 5 8 F 2 6 S B 3 5 8 F 2 6 S B 3 5 8 F 2 6 S B 3 5 7 S 1 3 4 8 F\} \\ &= U_F \{F 2 6 S 3 7 B (B 3 5 8 F 2 6 S)^3 B 3 5 7 S 1 3 4 8 F\} \Rightarrow K_F = 28. \end{aligned}$$

Table 1. Functional characteristics of a landscape with three types of land cover: F =forest, S =shrubland, B =bare ground/rock

Code number of function	Brief description of function (or process)
1	Eco-physiological degradation of the forest and expansion of shrublands in a forest
2	Ecological succession and expansion of forest on shrublands
3	Surface runoff increased due to erosion
4	Soil erosion and transport of eroded materials from shrubland upslope to forest downslope
5	Rockslides from bare rocks upslope to downslope
6	Soil erosion and transport of eroded materials from forest upslope to a shrublands area downslope
7	Shrub colonisation on bare ground
8	Movement of animal species from bare ground to forest

Table 2. Strings of functional characteristics of the landscape of Figure 3 with kinds of land cover: F =forest, S =shrubland, B =bare ground/rock and functions shown in Table 1

String code	String ($< =$ upslope downslope $= >$)
G_1	{F 3 B 5 7 S}
G_2	{F 2 6 S 3 7 B}
G_3	{B 3 5 8 F 2 6 S}
G_4	{B 3 5 7 S 1 3 4 8 F}

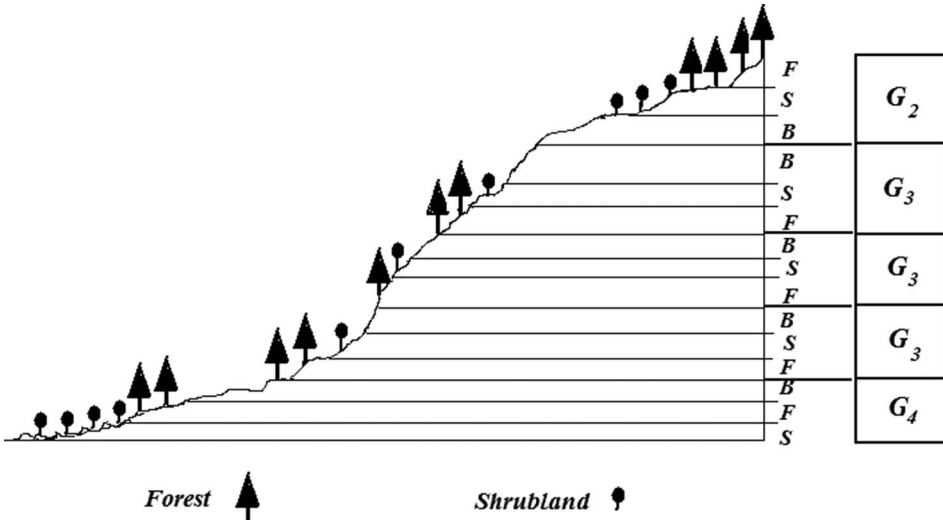


Figure 4. An example landscape for the calculation of functional K-complexity (for explanations of G_1 to G_4 and symbols see Figure 3).

3.3. Computation of Changes in K-complexity of Landscapes with Time

a) Changes in structural K-complexity (Ks)

Let a landscape A be considered at two time instants (t_1 and t_2), with string representations of its structure as follows (F =forest, S =shrubland, B =bare ground/rock):

From:

$$A_{t_1} = \{F B S F B S F S B S F B S B B S F B S F\}$$

To:

$$A_{t_2} = \{S S F F F S B B S F F F S B S F F F S F\}$$

In order to compute the landscape complexity changes, we need to assign a code number to each one of the landscape's structural transformations during the time period considered. A simple encoding that attributes code letters to land use/cover transformations is the following string of symbols:

<u>From:</u>	<u>F</u>	<u>S</u>	<u>B</u>
<u>F</u>	γ	δ	ε
<u>S</u>	ζ	π	σ
<u>B</u>	τ	ψ	ω

where δ means a transformation of forest (F) to shrubland (S) from time t_1 to time t_2 , σ means a transformation from shrubland (S) to bare ground (B) and so on.

Consequently, the landscape transformation map of landscape A t_1 to landscape A t_2 (as shown in Table 3) will be represented by the following string:

$$x = \{\delta \psi \zeta \gamma \tau \pi \varepsilon \sigma \psi \zeta \gamma \tau \pi \omega \psi \zeta \gamma \tau \pi \gamma\} \Rightarrow U(x) = 20$$

and substituting the abbreviation $\xi = \psi \zeta \gamma \tau \pi$, we obtain a shorter encoding \underline{y} :

$$K_S(A_{t_2} - t_1) = K_S(y) = K_S\{\delta \xi \varepsilon \sigma \xi \omega \xi \gamma, \xi = \psi \zeta \gamma \tau \pi\} = 16$$

b) Changes in functional K-complexity (K_F)

The changes in functional K-complexity (K_F) can be calculated in the same way as with structural K-complexity (K_S). Take, for instance, a landscape B at time t_1 , with string B $t_1 = \{G_2 G_1 G_1 G_4\}$ and at time t_2 , with string B $t_2 = \{G_2 G_3 G_1 G_4\}$.

We notice that the only change over the time interval is from G_1 to G_3 . The changes in landscape functions are therefore:

$$U_F\{B t_1\} = U\{G_2 G_1 G_1 G_4\} \Rightarrow K_F\{F26S37B (F3B57S)^2 B357S1348F\} = 26$$

$$U_F\{B t_2\} = U\{G_2 G_3 G_1 G_4\} \Rightarrow K_F\{F26S37B B358F26S F3B57S B357S1348F\} = 31$$

Consequently,

$$K_F\{B t_2\} - K_F\{B t_1\} = 5$$

Table 3. Coding of landscape transformations of the landscape at time t_1 (A_{t_1} , left column) to time t_2 (A_{t_2} , middle column). *F* = forest, *S* = shrubland, *B* = bare ground/rock. Greek symbols in the right column are the code symbols of the landscape transformations (e.g. δ means transformation from *F* to *S*)

Landscape A t_1	Landscape A t_2	Landscape Transformation Symbol:
F	S	δ
B	S	ψ
S	F	ζ
F	F	γ
B	F	τ
S	S	π
F	B	ε
S	B	σ
B	S	ψ
S	F	ζ
F	F	γ
B	F	τ
S	S	π
B	B	ω
B	S	ψ
S	F	ζ
F	F	γ
B	F	τ
S	S	π
F	F	γ

Yet the complexity of landscape changes must be measured from a landscape transformation map and it should not be calculated by subtracting one value of K from the other (it is not just the difference of the two complexities). In fact, it is:

$$K_F \{B \ t_2 - t_1\} = K \{F3B57S/B358F26S\} = 15$$

where the symbol '/' denotes the landscape transformation from G_1 to G_3 , which means that $K_F\{B \ t_2\} - K_F \{B \ t_1\} \neq K_F\{B \ t_2 - t_1\}$; a result that will be discussed later.

3.4. K-complexity of the Pendeli Landscape

Dividing the two landscape maps of Pendeli (Figure 1) by a 6 x 8 grid, we can calculate the structural complexity for the years 1967 and 1988, as well as the changes in structural K-complexity. The resulting strings can be calculated horizontally and vertically for both land cover maps for this real landscape (denoted P for notational convenience here).

- i) Land cover map of 1967
- horizontally (h) :

$$x\{Ph_{1967}\} = \{SF^3 S^2 F^3 A^3 F^4 A^2 FS^2 F^2 AFS^2 FS^4 F SFS^4 FA^2 S^2 F^2 A^2\}$$

hence $U[x\{Ph_{1967}\}] = 39$,

which admits a simplification with the replacement $S^2F=\mu$ and therefore:
 $y\{Ph_{1967}\} = \{SF^3 \mu F^2 A^3 F^4 A^2 F\mu FAF\mu S^2 \mu SFS^2 \mu A^2 \mu FA^2, \mu = S^2 F\} = 38$
 whence $U[y\{Ph_{1967}\}] = 38$

- vertically (v):

$$x\{Pv_{1967}\} = \{SF^4 S^3 F^3 S^5 F^3 S^2 FSF^2 AF^3 SF^2 SA^2 FSFA^2 SA^3 S^2 A^2\}$$

which has $U[x\{Pv_{1967}\}] = 38$,

which admits a simplification with the replacement $FSF=\Theta$:

$$y\{Pv_{1967}\} = \{SF \Theta^3 S^5 F^3 S^2 \Theta F AF^2 \Theta FSA^2 \Theta A^2 SA^3 S^2 A^2, \Theta = FSF\}$$

and therefore $U[x\{Pv_{1967}\}] = 36$

Hence, $K_S\{Ph_{1967}\} = U[y\{Ph_{1967}\}] = 38$ and $K_S\{Pv_{1967}\} = U[y\{Pv_{1967}\}] = 36$,
 from which we deduce that:

$$K_S\{P_{1967}\} = \min (K_S\{ Ph_{1967}\}, K_S\{ Pv_{1967}\}) = \min (38, 36) = 36$$

- ii) Land cover map of 1988
- horizontally (h):

$x\{Ph_{1988}\} = \{S^4 ASFS^2 A^3 FS^3 A^2 FS^{12} F S^6 F^2 SAS^5 A\}$, which admits no simplification.

- vertically (v):

$x\{Pv_{1988}\} = \{SF^3 S^9 FS^8 FS^2 AS^4 FSA^3 S^6 A^2 S^3 A^2\}$, which admits no simpler description.

Hence,

$$K_S \{Ph_{1988}\} = U[x\{Ph_{1988}\}] = 28,$$

$$K_S \{Pv_{1988}\} = U[x\{Pv_{1988}\}] = 26$$

and therefore,

$$K_S \{P_{1988}\} = \min (K_S \{Ph_{1988}\}, K_S \{Pv_{1988}\}) = 26$$

These results allow us to conclude that the Pendeli landscape's K-complexity has diminished from 1967 to 1988 from 36 to 26. The decrease of complexity in the Pendeli area is mainly due to the expansion of shrublands in that landscape. As happens in many other Mediterranean ecosystems, expansion of shrublands there present an environmentally undesirable evolution, because these are degraded Mediterranean-type shrublands (associations of *Quercus coccifera*) indicative of species with reduced eco-physiological complexity and increased resistance to drought. So in this case, the reduction in landscape complexity took place simultaneously with environmental and ecological degradation.

As previously stated, the calculation of the complexity of landscape change is different, as it must be calculated from a landscape transformation map resulting from overlaying the map of the landscape of the year 1988 on the map of the year 1967. For this calculation, we need to codify the landscape transformations of the Pendeli landscape P in some way, for example as follows:

	To (1988):		
From			
(1967):	<u>F</u>	<u>S</u>	<u>A</u>
<u>F</u>		U	
<u>S</u>	Z		V
<u>A</u>		W	

$K_S \{Ph_{1988-1967}\} = 42$ and $K_S \{Pv_{1988-1967}\} = 40$,
therefore concluding that $K_S \{P_{1988-1967}\} = \min(42, 40) = 40$.
This result verifies that:

$$K_S \{P_{1988-1967}\} \neq K_S \{P_{1988}\} - K_S \{P_{1967}\}$$

3.5. Effects of Granularity and Reclassification on K-complexity

The maps of the Pendeli landscape may also be used to reveal the effect of granularity on K-complexity. Calculating K-complexity for increasingly larger blocks of cells, that is, blocks of four cells, of six, 24, and eventually 48 cells (which is the area of the entire landscape), we see that K-complexity decreases with increasing quadrat size (Table 4). Yet, it decreases differently in the case of the maps of 1967 and 1988.

Table 4. The effect of granularity on K-complexity: the algorithmic complexity diminishes with increasing quadrat size, but differently so in the maps of 1967 and 1988

Number of cells in the grid	Minimal string 1967	Minimal string 1988	K-complexity 1967	K-complexity 1988
1	$SF\Theta^3S^5F^3S^2\Theta$ $FAF^2\Theta FSA^2\Theta A^2$ $SA^3S^2A^2, \Theta = FSF$	$SF^3S^9FS^8$ $FS^2AS^4FSA^3$ $S^6A^2S^3A^2$	36	26
4	F^3SF^4SASA	$SFS^6A^2S^2$	9	8
6	F^3SAFSA	SAS^6	7	4
24	FS	S^2	2	2
48	F	S	1	1

Expectedly, any reclassification of the land use/land cover classes will result in a different map of landscape structure, and hence, in a different assessment of K-complexity. Hence, if we reclassify the areas of forests F and shrublands S of the Pendeli landscape as N (natural vegetation areas), then our landscape has only two land use types: N and A (agriculture).

In this case, the calculations for the year 1967 yield the following strings:

horizontally: $K\{N^9 A^3 N^4 A^2 N^5 AN^{16} A^2 N^4 A^2\} = 19$
and vertically: $K\{N^{25} AN^7 A^2 N^3 A^2 NA^3 N^2 A^2\} = 19$
and therefore $K_{1967} = 19$

In the same way, for 1988 we have:

horizontally: $K\{N^4 AN^3 A^3 N^4 A^2 N^{23} AN^5 A\} = 18$
and vertically: $K\{N^{25} AN^6 A^3 N^6 A^2 N^3 A^2\} = 16$
and therefore $K_{1988} = 16$

Evidently, K-complexity decreases as we unify land cover/land use categories and it is expected to increase as we subdivide land cover/land use categories (or, at least, to remain the same).

4. Discussion

The calculation of a landscape's structural complexity as the length of y is uncomplicated for 'short' strings. Yet, there is a certain degree of incongruence between information-theoretic conventions and the reality of landscape analysis: while in informatics it is legitimate for the calculation of K-complexity of a string to place all rows (or all columns) one after the other to calculate the spatial complexity of the resulting total string x , in landscape analysis this calculation might present a problem of interpretation: there may be no ecological justification why a landscape's string at its 'upper right corner' should be 'followed' by the next row down, because the last pixel at the upper right corner of the landscape may have nothing in common (historical evolution, functions, etc.) with the first pixel at the left side of the second

row. From the landscape of Figure 2, for instance, the upper right corner's cell with land cover A may have not much in common with the second row's first cell with land cover B (in terms of historical evolution or ecosystem species composition), apart from the fact that they are constituent parts of the same landscape. This also applies to the column-by-column calculation of the landscape's complexity, when the string x' is used. Consequently, it might be argued that calculating the landscape's spatial complexity from a single unified string connecting the entire landscape would be meaningful if the landscape as a whole presents a low spatial variability or if we restrict our analysis to landscape transects only.

Despite this, the adoption of an effective information measure providing us an answer to the problem of landscape complexity should not restrain us from using K-complexity, because we should not lose sight from the fact that our primary goal here is to evaluate landscape complexity itself (and this is what K-complexity actually does), and not to interpret the ecological variability of a landscape across columns and rows by using a complexity measure and K-complexity provides us precisely with this overall complexity measure.

Aside from Kolmogorov's 'K-complexity', there are two other algorithmic approaches to complexity, closely related to K-complexity, which need to be discussed here: 'computational complexity' and 'resource-bounded complexity'.

The former refers to a set of objects, measuring the computational resources necessary to recognise the elements of a set. Computational complexity is also concerned with the problem of whether a given string belongs to a set, but there is no standard definition of K-complexity for sets. Besides, it does not appear workable to analyse a landscape with terms of set theory. Therefore, the notion of computational complexity does not seem promising for the measurement of landscape complexity as K-complexity does.

As for resource-bounded complexity, in alternative approaches, Hartmanis (1983), Kolmogorov (1986) and Sipser (1983) provided definitions of time-resource bounded complexity, which measures the computation time required for the calculation of a system's complexity. Hartmanis's definition allows for more detailed studies of complexity than that of Kolmogorov's, for according to his alternative definition, if two alternative spaces $S_1(n)$ and $S_2(n)$ are required to represent the complexity of a string of length n by a Turing machine and it happens that $S_2(n)$ grows faster than $S_1(n)$:

$$\liminf_{n \rightarrow \infty} \left(\frac{S_1(n)}{S_2(n)} \right) = 0,$$

then the problem of complexity evaluation of that string is solvable in space S_2 , but not in space S_1 (and a similar argument holds for time bounds instead of space bounds). Hartmanis's definition offers the advantage of the possibility to study separately the size of a string's description and the time required to build that string, but this is beyond the scope of the usual needs of landscape analysis, simply because the size of data to be analysed usually does not require long computation times. Besides, complete and adequate landscape complexity analyses (string representation, calculations, etc.) such as those presented here can be carried out without automated means. With even Hartmanis's approach not opening any more

promising avenues towards the computation of landscape complexity, K-complexity remains the unique possible *algorithmic* complexity measure for computing landscape complexity directly.

Papadimitriou (2009) suggested a measure of landscape complexity by making use of the Levenshtein algorithm, which is an algebraic-algorithmic measure and valid only for *spatial* landscape complexity, while, as we see in the present paper, K-complexity is purely algorithmic and is suitable for evaluating *functional* landscape complexity as well.

An additional advantage of K-complexity is that it allows landscape ecologists, ecologists and geographers to clearly distinguish between a landscape's complexity and its diversity. Consider, for instance, the example strings shown in Table 5. They all correspond to the same land covers (*A* and *B*), they all have the same length of encoding (18 symbols) and the same cardinality of land cover symbols (equal number of *A* and *B* cells per landscape: nine each). Since there are nine cells of each land cover type out of 18 in total, this means that $P = \frac{1}{2}$ in the (Shannon's) landscape diversity formula, which thus gives:

$$H = - \sum_{i=1}^n P_i \log P_i = - \sum_{i=1}^2 \frac{1}{2} \log \left(\frac{1}{2} \right) = 1$$

for all these landscapes' strings.

Consequently, they all have the same diversity, but their complexities are altogether different (Figure 5 and Table 5).

We thus see that it is impossible to distinguish between the spatially simple landscape T1 and the spatially complex landscape T4, on the basis of Shannon's 'landscape diversity' formula (as they both have the same Shannon diversity), but this distinction becomes possible only with the use of K-complexity. Furthermore, K-complexity increases with increasing local differentiation within each string, from T1 to T4.

As concerns real landscapes, significant changes in land use diversity over the period 1967–1988 have been documented by Papadimitriou and Mairota (1998) for all Euro-Mediterranean countries. The case study of the Pendeli landscape, gives

Table 5. Four example landscapes (T1 to T4) showing the inadequacy of landscape diversity to describe landscape complexity: they all have the same length of encoding (18 symbols) and the same Shannon diversity (1), but different values of K-complexity. Furthermore, K-complexity increases with increasing local differentiation within each string, from T1 to T4. Landscapes T1 to T4 are shown in Figure 5

Landscape	Initial string x	Initial length	Minimal description y of string x	K-complexity (length of string y)
T1	AAAAAAAAABBBBBBBBB	18	$A^9 B^9$	4
T2	ABABABABABABABAB	18	$(AB)^9$	5
T3	ABBAABABBAABABBA	18	$k = ABBAAB, k^3$	11
T4	ABBBAAABAAAABBBBB	18	No minimal description possible ($y = x$)	18

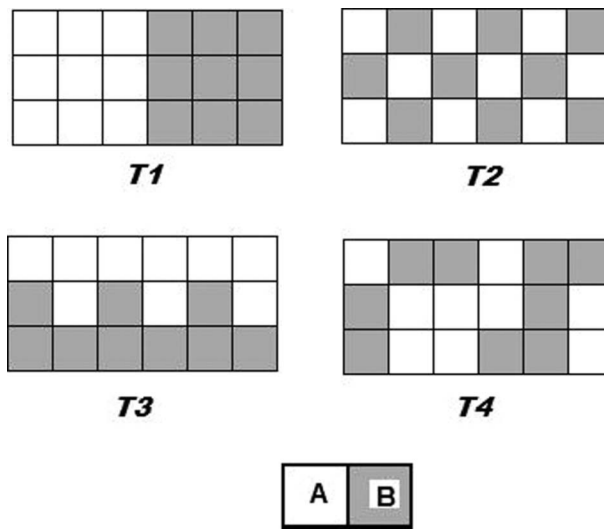


Figure 5. Four example landscapes: all have the same Shannon diversity (equal to $H = 1$), but each one of them has different values of K-complexity: $K\{T1\} = 4$, $K\{T2\} = 5$, $K\{T3\} = 11$, $K\{T4\} = 18$. For calculations see Table 5.

hints that diversity and K-complexity changes may in certain cases be time-correlated, since the diversity values for that landscape were $H_{1967} = 1.46$ and $H_{1988} = 1.09$, meaning that a decrease (by 25.3%) in diversity coincided with a decrease (by 38.46%) in structural K-complexity also; a fact that may be attributed to the reduction in patchiness over the same time interval.

As for the significance of the third spatial dimension in landscape analysis, the assessment of K-complexity in this study showed a method for encoding the landscape's third spatial dimension (altitude) within a landscape's computation of functional complexity, and these calculations can be made on both static and dynamic landscape data. Specifically, for calculations of landscape dynamics by means of K-complexity, it is noteworthy to observe that the result formally derived here:

$$K_F\{B_{t_2}\} - K_F\{B_{t_1}\} \neq K_F\{B_{t_2} - t_1\}$$

corroborates earlier results on landscape complexity analyses (Papadimitriou, 2002, 2009), showing that the 'complexity of landscape changes' is generally different than 'changes in landscape complexity'.

By proposing the application of K-complexity to the evaluation of landscape complexity, it is important to notice that the computation of landscape complexity remains no more difficult than the computation of complexity of any other systems. Any restrictions applicable to landscape analysis, are, as a matter of fact, also applicable to the computability of other complex systems. For this reason, it might be more modest to resort to the term 'evaluation' of a landscape's complexity rather

than ‘computation’. This evaluation however, can be effectuated with the specific computational procedures that were developed here.

5. Conclusion

The need to evaluate landscape complexity appears increasingly significant, both theoretically and for practical reasons. In this paper, it is suggested that K-complexity can be used to compute structural and functional landscape complexity and that it can be an adequate descriptor of a landscape’s temporal complexity changes.

K-complexity, being the preferred tool of theorising about complexity in information science, mathematics and physics, offers the advantage of being a well-studied, well-documented and reliable measure of complexity. So any technical limitations to the computation of a landscape’s complexity are expected more likely to emerge from the increasing complexity of the ways by which each particular landscape is being analysed than from the capability of K-complexity as a tool for evaluating complexity.

Moreover, K-complexity presents significant advantages as a measure of landscape complexity, besides being the most widely used measure of complexity: a) it can be useful to compute structural as well as functional landscape complexity both statically and dynamically; b) it allows us to discern complexity differences between landscapes of the same diversity; and c) its use is relatively uncomplicated for a wide class of simplified (raster) landscape data.

References

- Chaitin, G. (1966) On the lengths of programs for computing binary sequences, *Journal of the Association of Computing Machinery*, 13, pp. 547–569.
- Dale, M. D. (2001) Minimum message length clustering, environmental heterogeneity and the variable Poisson model, *Community Ecology*, 2(2), pp. 171–180.
- Domenikiotis, C., Dalezios, N. R., Loukas, A. & Karteris, M. (2002) Agreement assessment of NOAA/AVHRR NDVI with Landsat TM NDVI for mapping burned forested areas, *International Journal of Remote Sensing*, 23(20), pp. 4235–4246.
- Fonstad, M. (2006) Cellular automata as analysis and synthesis engines at the geomorphology–ecology interface, *Geomorphology*, 7(7), pp. 217–234.
- Gabriel, D., Thies, C. & Tschardtke, T. (2005) Local diversity of arable weeds increases with landscape complexity, *Perspectives in Plant Ecology, Evolution and Systematics*, 7(2), pp. 85–93.
- Green, D. G. & Sadedin, S. (2005) Interactions matter: Complexity in landscapes and ecosystems, *Ecological Complexity*, 2, pp. 117–130.
- Hartmanis, J. (1983) Generalized Kolmogorov complexity and the structure of feasible computations. Proc. 24th IEEE Symposium on the Foundations of Computer Science, pp. 439–445.
- Herzon, I. & O’Hara, R. B. (2007) Effects of landscape complexity on farmland birds in the Baltic states, *Agriculture, Ecosystems and Environment*, 118(1–4), pp. 297–306.
- Honnay, O., Piessens, K., Van Landuyt, W., Hermans, M. & Gulinckx, H. (2003) Satellite based land use and landscape complexity indices as predictors for regional plant species diversity, *Landscape and Urban Planning*, 63, pp. 241–250.
- Keramitsoglou, I., Kiranoudis, C., Sarimveis, H. & Sifakis, N. (2004) A multidisciplinary decision support system for forest fire crisis management: DSS for forest crisis management, *Environmental Management*, 33(2), pp. 212–225.
- Kolmogorov, A. (1965) Three approaches to the quantitative definition of information, *Problems of Information Transmission*, 1, pp. 1–17.

- Kolmogorov, A. (1986) On the notion of infinite pseudorandom sequences, *Theoretical Computer Science*, 39, pp. 9–33.
- Li, B. L. (2004) Editorial, *Ecological Complexity*, 1, pp. 1–2.
- Li, M. & Vitanyi, P. (1997) *An Introduction to Kolmogorov Complexity and its Applications* (New York: Springer-Verlag).
- May, R. M. (1973) *Stability and Complexity in Model Ecosystems* (Princeton, NJ: Princeton University Press).
- Moser, D., Zechmeister, H. G., Plutzer, C., Sauberer, N., Wrba, T. & Grabherr, G. (2002) Landscape patch shape complexity as an effective measure for plant species richness in rural landscapes, *Landscape Ecology*, 17(7), pp. 657–669.
- Murray, B. & Fonstad, M. (2007) Preface: Complexity (and simplicity) in landscapes, *Geomorphology*, 91(3–4), pp. 173–177.
- Olson, J. (1975) Autocorrelation and visual map complexity, *Annals of the Association of American Geographers*, 65(2), pp. 189–204.
- Papadimitriou, F. (1995) Socioeconomic and political factors generating land degradation in rural Attica, Greece. Proceedings IGU International Conference on Mediterranean Erosion and Desertification, University of Aveiro, 14–18 June, pp. 429–432.
- Papadimitriou, F. (2002) Modelling indicators and indices of landscape complexity: An approach using GIS, *Ecological Indicators*, 2, pp. 17–25.
- Papadimitriou, F. (2009) Modelling spatial landscape complexity using the Levenshtein algorithm, *Ecological Informatics*, 4(1), pp. 48–55.
- Papadimitriou, F. (2010) Conceptual modelling of landscape complexity, *Landscape Research*, 35(5), pp. 563–570.
- Papadimitriou, F. & Mairota, P. (1996) Spatial-scale dependent policy planning for land management in Southern Europe, *Environmental Monitoring and Assessment*, 39(1–3), pp. 47–57.
- Papadimitriou, F. & Mairota, P. (1998) Land use diversity changes, in: P. Mairota, J. Thornes & N. Geeson (Eds) *Atlas of Mediterranean Environments: The Desertification Context*, pp. 89–90. Chichester: Wiley.
- Pielou, E. C. (1975) *Ecological Diversity* (New York: Wiley Interscience).
- Poggio, S., Chaneton, E. J. & Ghersea, C. M. (2010) Biological, landscape complexity differentially affects alpha beta and gamma diversities of plants occurring in fencerows and crop fields, *Biological Conservation*, 143(11), pp. 2477–2486.
- Proulx, R. (2007) Ecological complexity for unifying ecological theory across scales: A field ecologist's perspective, *Ecological Complexity*, 4(3), pp. 85–92.
- Proulx, R. & Parrott, L. (2008) Measures of structural complexity in digital images for monitoring the ecological signature of an old-growth forest ecosystem, *Ecological Indicators*, 8(3), pp. 270–284.
- Roe, M. H. (2000) Landscape planning for sustainability: Community participation in estuary management plans, *Landscape Research*, 25, pp. 157–181.
- Roe, M. H. (2007) The scale and scope of landscape and sustainability, in: J. F. Benson & M. H. Roe (Eds) *Landscape and Sustainability*, 2nd edn pp. 1–15 (London: Routledge).
- Roe, M. H., Jones, C. J. & Mell, I. C. (2008) *Research to Support the Implementation of the European Landscape Convention in England* (No. PYT02/10/1.16), Research Report for Natural England (available at: <http://www.ccnetwork.org.uk/elc/research>).
- Roschewitz, I., Gabriel, D. & Tschardtke, T. (2005) The effects of landscape complexity on arable weed species diversity in organic and conventional farming, *Journal of Applied Ecology*, 42(5), pp. 873–882.
- Sipser, M. (1983) A complexity-theoretic approach to randomness. Proc. 15th ACM Symposium Theory of Computing, pp. 330–335.
- Snacken, F. & Antrop, M. (1983) Structure and dynamics of landscape systems, in: J. Drdos (Ed.) *Landscape Synthesis: Geoecological Foundations of the Complex Landscape Management*, pp. 10–30 (Bratislava: Veda Publishing House of the Slovak Academy of Sciences).
- Solomonoff, R. J. (1964) A formal theory of inductive inference I, II, *Information and Control*, 7(2), pp. 224–254.
- Steffen, W. L., Chapin III, F. S. & Sala, O. (1996) Global climate change and ecological complexity: An international research agenda, *Trees*, 11(4), pp. 186–192.
- Vakalis, D., Sarimveis, H., Kiranoudis, C. T., Alexandridis, A. & Bafas, G. (2004) A GIS based operational system for wildland fire crisis management II. System architecture and case studies, *Applied Mathematical Modelling*, 28(4), pp. 411–425.

- Wallace, C. S. (2005) *Statistical and inductive inference by minimum message length* (New York: Springer Science & Business Media, Inc.).
- Wallace, C. S. & Dowe, D. L. (1999) Minimum length and Kolmogorov complexity, *The Computer Journal*, 42(4), pp. 270–283.
- Werner, B. T. (1999) Complexity in natural landform patterns, *Science*, 284(5411), pp. 102–104.
- Wu, J. & Hobbs, R. (2002) Key issues and research priorities in landscape ecology: An idiosyncratic synthesis, *Landscape Ecology*, 17, pp. 355–365.