**Reviewer 1**

The authors propose a new method to recognize regularity patterns in point

clouds by assigning to each point a quantitative index that represents the degree

of disorder of that point’s neighborhood. Given a point cloud, the core idea is

to select two close points and compare their neighborhoods by computing an

optimal matching between the points of the two neighborhoods and averaging

the matching distances. The Index of Disorder (IoD) assigned to each unit of the

point cloud is the average of the aforementioned quantity over all the pairings

of the point itself and a point in its neighborhood.

The proposed method is applied to simulated data and to different real-

world scenarios: patterns of reforestation areas, organization of an orchard and

ordering of the buildings in a city neighborhood.

The work is well written, and all the steps of the algorithms are clearly

explained. The simulated examples and the real-world cases are meaningful

and the results of the analysis are interesting.

Thank you. Please find below our responses to reviewer’s suggestions.

However, more formal justifications of some steps is lacking.

Additional information to the methodology to clarify the selection and implementation of steps were included.

A more detailed analysis of the model parameters or the possibility to automatically optimize

them using, for example, information theoretic criteria, is not properly treated.

Thank you for the suggestion. Information theoretic criteria algorithms, such as Akaike information criteria (Akaike, 1974) and minimum description length (Schwarz, 1978) are commonly used to select statistical models based on relative comparison between model alternatives based on a penalty function. The penalty function is used to quantify how far/close a candidate solution is to a problem-specific objective (observed data, minimum number of input parameters, etc). In this study, during the theoretical evaluation, no reference (observed) information is available to develop a penalty function. During the applied evaluation, a basic sensitivity analysis to determine the optimal input parameters was performed and identified the optimal input values for radius and Km. We feel this approach is appropriate since only two input parameters needed to be optimized.

H. Akaike, “A new look at the statistical model identification,” IEEE Trans. Automat. Contr., vol. 19, pp. 716-723, 1974.

G. Schwarz, “Estimating the dimension of a model,” Ann. Star., vol. 6 , no. 2 , pp. 461-464, 1978.

I think using ITC/AIC goes beyond the scope of what we’re doing here…. Also to my understanding ITC is about selecting the optimal model, not optimizing a model. So I’m not sure how to handle this

Moreover, the literature review does not stress enough the importance and the

concrete purpose of the proposed solution.

Additional text (see below) stressing the novelty and purpose of the IoD to the introduction section ().

“In disciplines like spatial statistics, spatial point patterns are quantified using a variety of methods. Intensity methods, such as quadrant count and kernel estimation provide a global idea of sampling intensity but they are sensitive to spatial scale and therefore disregard point spatial pattern as scales smaller than the spatial scale selected (Bailey and Gatrell, 1995 and Lloyd, 2010). Alternatively, methods based on nearest-neighbor distances, such as even-event, point-event and reduced second moment measure (K function), can be used to compare observed with random point pattern, but they still provide only a global measure of the point pattern (Bailey and Gatrell, 1995) in comparison with pure random pattern.”

and

“Up until this point, such a quantitative measure of geometric disorder in point sets has not been described for use in geospatial contexts, and thus the IoD provides a way to enrich the analysis of such point sets by creating a novel predictor for each individual point which is spatial variable.”

The following are some specific comments for the authors to improve the manuscript:

the possibility of using optimal transport (OT) algorithms for matching the sub point clouds of the two neighborhoods is not mentioned, though some OT algorithms would remove the problem of non matching points by finding a hybrid matching. Setting the problem of neighborhoods matching in the OT framework could make the solution more theoretically justified.

The problem of optimally matching neighborhoods is an interesting one, and there are many possible solutions. The Hungarian method is a classic solution to the assignment problem which is why we chose to use it. This study was meant to demonstrate one possible implementation of the IoD; though there may be more optimal implementations, we feel testing them goes beyond the scope of we what were intending to demonstrate. However, we have added a note in section 2.1.1 (Point Pair Assignment) about the possibility of using other solutions to the assignment problem.

“It is important to note that the Hungarian method was selected and explained here to solve the assignment problem because it is well-accepted and a ubiquitous solution, however, others algorithms could had been used instead.”

It is not clear how to select a good IoD threshold. A more formal definition of the meaning of a threshold on the IoD would improve the quality of the manuscript and the practical applicability of the methodology proposed.

The selection of an IoD threshold is meant to separate individual points into ordered and disordered (converting from continuous values into discrete values - binary). This is user-defined and site-specific based on quantitative comparison with reference information. In this study, a value of 0.8, 0.75, and 0.7 were selected Crab Orchard, TN, Mooresville, NC, and Nashville, TN, respectively.

Additional information was added to section 2.3

“Continuous IoD values were classified into two classes of “ordered” or “disordered” by defining a IoD threshold value in which individual points would be assigned to class “disordered” if their IoD was at or above that threshold, and ordered otherwise. A IoD threshold value was selected for each study site based on quantitative comparison with manually generated reference datasets. Once the IoD threshold was selected, a sensitivity analysis was then conducted to determine the optimal neighborhood radius *r* and *Km* for each study site-threshold value combination.”

the advantages of a quantitative index of disorder instead of a pattern detection algorithm are not clearly explained and underlined. In the three real-world examples considered in the manuscript, a threshold is applied to the IoD to obtain an ordered/non-ordered classification of the points:

what is the advantage of the proposed algorithm with respect to classical pattern-detection algorithms such as those described in the introduction?

Additional information was added to the Conclusions section.

“Classic pattern recognition algorithms are able to recognize discrete repeating patterns, but do not provide a quantitative measure of disorder. Standard spatial statistics methods a global measure of the point pattern in comparison with random pattern, and therefore a limited in demonstrating changes in pattern in space. Thus, these methods are not suitable for quantifying individual point deviation from order needed for thresholding operations or for creating additional predictor variables to be used in further analyses.”

• comparison with similar algorithms on the same three use cases is lacking. In the real-world examples, where the IoD is used to detect patterns, comparison with classical pattern-detection algorithms should be made.

The IoD provides a quantitative measure of disorder at each point within a set of points. Thus, it cannot be directly compared to the classical pattern detection algorithms described in the paper, as these algorithms delineate repeating motifs rather than quantify disorder at each coordinate location. These algorithms are not suitable for thresholding ordered vs disordered points. The IoD also cannot be compared to the raster-based operators described in the passage, as the IoD evaluates disorder in point sets, not raster grids.

The IoD could theoretically be compared to the algorithm described in Antuono et al. (2014). However, that algorithm was designed for use in simulated fluid particle systems and makes assumptions of an underlying grid structure. While such a comparison could be interesting, it is beyond the scope of this paper.

• the meaning of the comment in the sentence at lines 368 to 370 about the backward use of the IoD is not clear. A more detailed explanation or an example may help clarify it.

Thank you for the suggestion. The sentence was revised from:

“However, this limitation allows the IoD to be used backwards: if there is a priori knowledge of the classification labels of points in a system then an optimization algorithm can be applied to the IoD in order to estimate the scale of the pattern and magnitude of the pattern deviation.”

To

“However, if reference information (priori knowledge of the point classification) is available, then a sensitivity analysis and/or an optimization search algorithm can be used to determine the site-specific scale of the pattern and magnitude of the pattern deviation, which are represented by *r* and *Km* respectively.”

• The authors should check carefully the English throughout the manuscript,

here are some typos:

\* Line 21 “Different set of methods” → “Different sets of methods”

The words “Different set of methods” was replaced with “Different methods”

\* Line 92: “such that that average” → “such that average”

The extra word “that” was removed.

\* Line 138: “... when assigning a neighborhood with few points to another with many points what could lead to a small IoD” → “... assigning a neighborhood with few points to another with many points, that could lead to a small IoD”

The sentence was revised as:

“The simplest option is to ignore unpaired points when calculating the mean assignment cost. An obvious drawback is that assigning a neighborhood with few points to another with many points could lead to a small IoD despite the fact that no true point correspondence necessarily exists.”

\* Line 369: “then then an optimization” → “then an optimization”

This sentence was revised and this

\* Caption of Figure 2: “The process then repeated” → “The process is then repeated”

The word “is” was added.

**Reviewer 2**

This article presents a metric (IoD - Index of Disorder) to find patterns in sets of points, i.e., ordered points and disordered points. The metric consists of quantifying the distance (or similarity) between two subsets of points (two neighborhoods), with or without a previous alignment of the points in the two neighborhoods. The authors argue that there are cases where alignment improves results and in others it worsens. The experiments use datasets generated by the authors and three real datasets. The three case studies are interesting and the proposed method has good results in two of them.

The authors make simple choices when applying the IoD metric, but they seem to have strong implications in the results. For example, the authors decide 'to punish unpaired points if and only if they are within the convex-hull of the subset of points that have an assignment'. This choice is well suited when the contour enclosing the points to be analyzed is approximately convex (a rectangle or a circle, as in the proposed examples), but it may not be the case when that contour is irregular, e.g., if there are large convexities or empty regions (holes). It also does not solve issues like scaling as referred to in the article. In my opinion, the authors should work on these issues to make the overall solution more robust and to achieve good results in a wider range of use cases.

Issues with pattern scaling may be obviated by the application of Iterative Closest Point (or similar) reorientation algorithm, which can include scaling.

The concerns raised regarding the deficiencies of using the convex hull to make a decision on whether to punish unpaired points are valid, and the text has been updated to reflect this in section 2.1.4. Future work should include investigation on how to handle irregularly contoured patterns as this could be the subject of an entire new study.

“This was the option adopted for this paper, thought it should be noted that this solution may not perform well for highly irregular patterns with large convexities or topological holes.”

+ + + Detailed comments

+ L81..84: the second coordinate transformation refers to the alignment of two subsets (neighbourhoods)? The meaning of the sentence "The IoD sub-score is not the IoD itself, but rather an intermediate parameter used …" is difficult to understand. Please, explain it better, including the meaning of the intermediate parameter.

Thank you for the suggestion. The sentence

“The IoD sub-score is not the IoD itself, but rather an intermediate parameter used in the calculation of the IoD for a given point.”

Was revised as

“The IoD sub-score is an intermediate parameter used in the calculation of the IoD for a given point. Once the IoD sub-score is calculated for every child point within the parent point’s neighborhood, the final IoD of the parent point is calculated as the mean of these IoD sub-scores.”

+ L115..117: In my opinion, Figure 4 does not illustrates well issues such as translation, rotation, skewing, etc. of two point sets, but these are well-known concepts in point set registration algorithms and so it is not necessary to illustrate them. Suggestion: remove the cross-reference to the figure.

We removed the cross reference to the figure.

+ L166..170. There are choices in 'The values adopted for…' hard to understand, e.g., 1) what is the meaning of 'punishment of points outside of the convex hull set to false'?

Thank you for your suggestion.

The sentence

“The values adopted for the theoretical evaluation were a neighborhood radius of 15 units, deviation value at which sigmoidal scoring function assigns a score of 0.5 (*Km*) of 3 units, degree of cooperativity of the sigmoidal scoring function of 0.5, assignment score given to unpaired points of 1, punishment of points outside of the convex hull set to false, and  Euclidian distance selected as cost function.”

Was revised as:

“The parameterization of the IoD algorithm for the theoretical evaluation used the following values. The neighborhood search radius of the IoD was set at 15 units. The value at which the sigmoidal scoring function assigns a score of 0.5 (*Km*) was to set to 3 units. The degree of cooperativity of the sigmoidal scoring function was set to 5. The assignment score (punishment) given to unpaired points was set to 1, but was only applied to unpaired points falling within the convex hull of the set of paired points.”

+ L196..198, L208..212 and L228..230 are basically the same (repeated). This is a comment on aesthetic only.

This is true. We have elected to retain this as it may help orient readers to our methods who may not be reading the paper in its entirety or in order

+ L281. Why, what does it means and how do you ascertain that 'points become disordered'?

Because the optimal parameterization of Km in this context is 5m, then points may deviate roughly 5m from the general neighborhood pattern before they are harshly penalized by the IoD. Points that have a neighbor-pair within 5m are thus considered roughly “ordered” by the IoD, while those with larger deviances are considered “disordered”. To clarify this point in the text, the sentence:

“Thus, the characteristic scale of the orchard in Site 1 is between 70 to 80 meters, and the threshold of the pattern deviation before points become disordered is approximately 5 meters.”

Was revised as:

“Thus, the characteristic scale of the orchard pattern in Site 1 is between 70 to 80 meters, and the characteristic deviation of the trees in the orchard from the planting pattern is approximately 5m.”

+ L284..285. Explain how the result obtained for a subset of the data (scale independence is assumed?) can be generalized to other cases (different scales).

Thank you for noting this. The sentences

“Because of this scale dependence, the IoD is not appropriate when there is no prior knowledge of scale of pattern deviance or when trying to quantify patterns with multiple scales. In those cases, it is recommended using the IoD the characterize pattern scales on a subset of data before applying it more broadly.”

Were revised as:

“These site-specific IoD characteristics were determined based on prior knowledge of planting pattern in the form of reference information on tree crowns labeled as either orchard or natural trees. In cases when the prior knowledge of the point pattern is not available, the IoD can still be calculated but the interpretation of scale magnitude and scale pattern deviance is limited. In those cases, it is recommended using the IoD the study and characterize pattern scales on a subset of data before applying it more broadly.”

+ I did not understood Figure 7. Can you improve the explanation?

Thank you for bringing this up.

The caption for Figure 7 was updated and the following additional information was added to section 3.3.2:

*“*Some types of patterns, such as repeated stamps (red circles and orange triangles in Figure7), show slightly increased discrimination between ordered and disordered points when reorientation is applied. The realignment procedure depresses the IoD of the unperturbed, ordered points slightly more than it depresses the IoD of the disordered points beyond the perturbation distance. More homogenous patterns, such as a square grid (dark blue circles and light blue triangles in Figure 7), show reduced discrimination when reorientation is applied. The IoD is most effective as a discriminatory metric when the difference between the mean IoD of the ordered points and mean IoD of the disordered points is maximized.”

**Reviewer 3**

I liked reading this paper which offers a new index to quantify a particular concept of geometric disorder. I have several comments that try to make things more clear and highlight some aspects of the approach that are not explicitly developed.

The method is explained in Section 2. I had to read this section three times to get the idea of the new index. This section is poorly written and right now it brings lots of hidden or badly explained aspects of the method. Indeed, the paper novelty should be described here, and right now the reader gets into an overall confusion about the strategy followed.

Fair.

Having said this, the papers comes often into mixing point patterns and continuous geostatistical patterns. Clearly, the presented method applies over points, but in some comments, and datasets it seems as if the points were sampled from a continuous field yielding a geostatistical pattern. This should be made clear.

The method was developed for and evaluated using discrete point sets. Some of the points sets were generated using random grid-like structures (theoretical evaluation) while others used GIS layers (LiDAR, aerial photograph, and polygons) to aid in the manual identification of tree crowns and buildings center location.

In addition, there are a number of tunning parameters playing important roles in the building of the index. How the radii are selected to setup the neighbourhoods. Do they depend on the spatial structure of the point pattern? I believe they should.

Yes. The selection of the radius *r* is the characteristic scale of the suspected pattern and Km is the expected intra-pattern noise. The IoD can still be calculated, but when the prior knowledge of the point pattern is not available, the IoD can still be calculated but the interpretation of scale magnitude and scale pattern deviance is limited. The following information was added to section 3.3.1. to clarify this.

“These site-specific IoD characteristics were determined based on prior knowledge of planting pattern in the form of reference information on tree crowns labeled as either orchard or natural trees. In cases when the prior knowledge of the point pattern is not available, the IoD can still be calculated but the interpretation of scale magnitude and scale pattern deviance is limited. In those cases, it is recommended using the IoD the study and characterize pattern scales on a subset of data before applying it more broadly.”

I will add a section describing how to select the correct parameters. This is partially described in the results sections wherein it is proposed that an optimal *r* is the characteristic scale of the suspected pattern and *Km* is the expected intra-pattern noise. However, it is not explicitly states that the algorithm’s ought to be tuned based in this information.

And talking about spatual structures, all I can see is different arrangements of points that are independent of each other, so the authors are working with inhomogeneous Poisson patterns. Is this right or am I missing something? because the method should be also tested against several structures and see how spatial interactions act on the behaviour of the index.

The theoretical evaluation was performed by generating different patterns. Some patterns a considering each event has equal probability of occurring in any position within the study space and that the position of each event is independent of each other. This could be considered homogeneous. Others, were pre-determined by geometry. However, each of the pattern was then modified with random noise increasing in magnitude from the center towards the edges, therefore becoming inhomogeneous. Real world examples are by definition inhomogeneous because they reflect anthropogenic patterns.

We are not proposing to contrast the existing distribution with expected (theoretical => complete spatial randomness) distribution nor to assess their statistical uncertainty. We are also not trying to develop statistical models representing these phenomena. Rather, we are calculating a **relative** metric for each event in the set based on the spatial arrangement of its surrounding neighbors. Therefore, the statistical distribution does not affect our calculations. We did evaluate the IoD metric using a wide range of different spatial structures, artificial and natural occurring.

There are no connections between the new index and the concept of order/disorder given by complexity measures. There is an important line of research to which this paper should be linked. Complexity measures and Information theory seem to provide similar information in terms of order. Also some connection to fractality indexes should be considered. Stochastic geometry goes close to this aspect.

Thank you for the suggestion. We did a literature review on information theory and complexity measures specifically applied to geospatial datasets.

Traditional disciplines looking at spatial point patterns, proposed global metrics to evaluate point patterns – one value for the entire point pattern. The following information was added to the introduction section.

“In disciplines like spatial statistics, spatial point patterns are quantified using a variety of methods. Intensity methods, such as quadrant count and kernel estimation provide a global idea of sampling intensity but they are sensitive to spatial scale and therefore disregard point spatial pattern as scales smaller than the spatial scale selected (Bailey and Gatrell, 1995 and Lloyd, 2010). Alternatively, methods based on nearest-neighbor distances, such as even-event, point-event and reduced second moment measure (K function), can be used to compare observed with random point pattern, but they still provide only a global measure of the point pattern in comparison with complete spatial randomness (Bailey and Gatrell, 1995).”

Information theory methods have been developed to quantify complexity in a wide range of disciplines, including spatial sciences. Batty et al. (2014) proposed a measure of entropy for spatial information by extending the work of Shannon (1948). In this work, a global entropy value is calculated based on probabilities but do not account for the spatial location of the data (Altieri et al., 2020). Similarly, Schilcher et al., (2017) evaluated the effects of rotation and translation to point patters processes of different inhomogeneity measures. For each point pattern process, an overall inhomogeneity value was calculated by using a grid-based analysis to estimate the local deviation of the number of points within each grid cell to the expected number of points. These proposed spatial entropy and inhomogeneity measures refers to a single global value of the entire point pattern process and they were designed to quantify how much the system deviates from equilibrium (Angule and Batty et al., 2014).

1. I am not sure what is meant by complexity measures or how it relates to the concept of geometric disorder
2. Dong (2000) uses lacunarity, a concept intricately related to fractality, as an order-sensing measure. This is discussed in the introduction.
3. In general I’m not quite sure what to make of this comment