Impact of Foot Curvature on the Stability and Efficiency of a Bipedal Walking Robot

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As robots develop the capability to perform more humanoid tasks, they will require the ability to efficiently walk on two legs without falling over. Efficiency in bipeds consists of trade-offs between energy required for active control/balance and for movement. Therefore, a biped's efficiency may be optimized by introducing semi-passive components (requiring less active control), meaning some joints are actuated using electrical power, while others operate due to the conversion between potential and kinetic energy. Although many factors affect the behavior of semi-passive bipeds, this study aims to improve existing bipedal locomotion by studying how foot curvature affects the stability and energy efficiency of a semi-passive bipedal walker. We designed and built a semi passive walker with a passive and actuated joint in each leg, attached to feet of varying curvatures. Efficiency was quantified by time taken to walk 1m, normalized by volume of foot. The results suggest a foot curvature of 0.200 cm^{-1} is the most efficient. Stability was defined by variation in y-position, and was not affected by curvature. This conclusion will be useful in increasing the efficiency of existing bipedal robots, which is an important step for developing humanoid robots that can perform more advanced tasks.

INTRODUCTION

Creating humanoid robots is an increasingly popular topic of interest among roboticists, primarily due to their ability to traverse adverse terrain[6]. Like humans, many of these robots walk on two legs, and are called bipedal walkers. Existing bipedal walkers with multiple actuators at each joint have shown smooth gait patterns that require precise control algorithms to determine how each joint should move [16]. However, an increased number of actuators requires more computational power and energy. To counteract these problems, completely passive walkers that operate without any computational power or electrical energy have been created[1,2,11]. According to Collins et al., passive dynamic walkers are significantly more energy efficient since they don't require energy for stabilisation[3]. Such passive walkers rely solely on converting potential energy to kinetic energy. While a passive approach is more energy efficient, such robots cannot walk on level ground. A more optimal approach would be a semi-passive walker that includes passive elements to conserve energy, while also adding some active elements allowing the robot to walk on flat ground and across more complicated terrain.

In nature, a large determinant of efficiency in locomotion is the variation in foot shapes. For example, endurance runners like horses have drastically different foot shapes compared to agile sprinters like cheetahs or hoppers like kangaroos[10]. Humans typically have similarly shaped feet. This led to questions about what foot shapes would lead to efficient bipedal walking [1].

The majority of the prior research on both passive walkers[2] and actuated walkers[3] examines the linkage between energy efficiency and control algorithms, or angle between leg and hip. This led to questions about what other aspects of a bipedal robot influence its efficiency. Asano et al highlight how curved feet can be effective in increasing energy efficiency for bipedal robots, describing their ability to act as extensions of ankle joints reducing energy required for locomotion[15]. While they showed curved feet can positively impact walking, not much research has focused on what the optimal curvature is. This prompted the following research question:

How does the curvature of the foot impact the stability and efficiency of a bipedal robot and what is that optimal curvature?

To evaluate efficiency and stability we constructed a robot with two active hip joints and two passive ankle joints. Due to the limited number of joints per leg, the robot's gait pattern more

[1] Either passive or active.
[2] Bipeds that do not have any motors and solely use gravitational potential energy converted to kinetic energy for locomotion.
[3] Bipeds that utilise motors for locomotion.

closely resembles that of a penguin rather than a human. Based on this gait and readings from McGeer, Asano and Collins[8,15,1,2,3] we hypothesize an inverse relationship between curvature of foot and energy efficiency. To ensure stability in the sideways directions, the feet of the robot face sideways to counteract the lateral energy that is created in waddling gaits such as that of a penguin[5]. The length of each foot remains constant, and a foot length approximately ⅓ the length of the leg produces most stable motion[7]. The interior angle of a pie cut from a cylinder, and the radius of the cylinder is used to determine the curvature of the foot, which is varied to determine its effect on the stability of the robot. We quantify stability and efficiency using the following metrics:

Stability: The range of the y coordinate of the biped's hip joint as it travels along a 1m track. We measure the y position of the hip joint with a computer vision algorithm.
Efficiency: We define efficiency as the time required for the biped to traverse the 1m track divided by foot volume.

While the objective is to find the most efficient foot shape with regard to lateral cross section, the robot foot is convex in a cross section perpendicular to the lateral view in order to automatically stabilize the robot sideways[8]. Convex foot shapes have been used both laterally and perpendicular to the lateral surface in robots that do not need active control for balancing[4,9]. This is especially effective in a penguin gait robot like this where the center of volume swings a lot sideways as it walks.

The effect of foot curvature on stability was tested both in simulation and using physical experiments. In simulation, the range of testable foot angles is narrowed by eliminating those that lead to a completely unstable system. The working range of internal foot angles is tested using a physical model to explore how foot angle affects stability of the bipedal robot. We find that increasing curvature of feet and efficiency of the robot are inversely correlated (except for really low curvatures), meaning larger curvatures lead to decreased energy efficiency. This paper will dive deeper into the precise methodology of the experimental and computational models, after which it will present the quantitative relationships between foot curvature and efficiency and stability. Finally, the impact of these findings on bipedal robotics will be discussed.

METHODS

This experiment is designed to evaluate the relationship between the curvature of the foot and the stability and efficiency of a bipedal walker. The experiment uses a robot built from a plywood base and pvc pipe legs, controlled by an atmega328p microcontroller. Each leg is made from pvc piping which is connected via a 3D printed universal joint to 3D printed feet of varying curvatures. Stability and efficiency are recorded using the metrics described above.

Parts

<u>Electronics:</u>

- 6V, 2A Power source
- Breadboard
- RobotGeek 180 Degree Servo Motors[1] (x2)
- ~1m insulated copper wire
- Arduino board - Atmega328p
- Computer with OpenCV and an IDE capable of programming an Arduino Atmega328p
- Camera

<u>Materials:</u>

- PVC - 0.5” diameter x 2’ long (enough for 2x 20cm legs)
- PLA + 3D printer
- Screws
- Aluminum Rails
- Epoxy: Araldite ARA-400005 Rapid Strong Adhesive
- ⅛” plywood for base
- Cotton cloth
- Wooden beams (2x 1.5m)

Build Procedure

The experimental set up for testing the hypothesis consists of a bipedal robot constructed from pvc piping, plywood, two servo motors and 3D printed parts, all controlled by an atmega328p. The schematic diagram for the robot can be found in figure 1. The biped was constructed in three stages. First the pvc legs were cut and motor horns were attached to one end of each leg, by threading paper clips through holes in the motorn horn attachment and the pvc pipe. A rectangular open-top box was constructed from laser cut plywood. The servo motors were attached to the plywood base using epoxy. Lastly the 3D printed universal joints and feet were inserted into the bottoms of the pvc pipe legs to complete the bipedal walker.

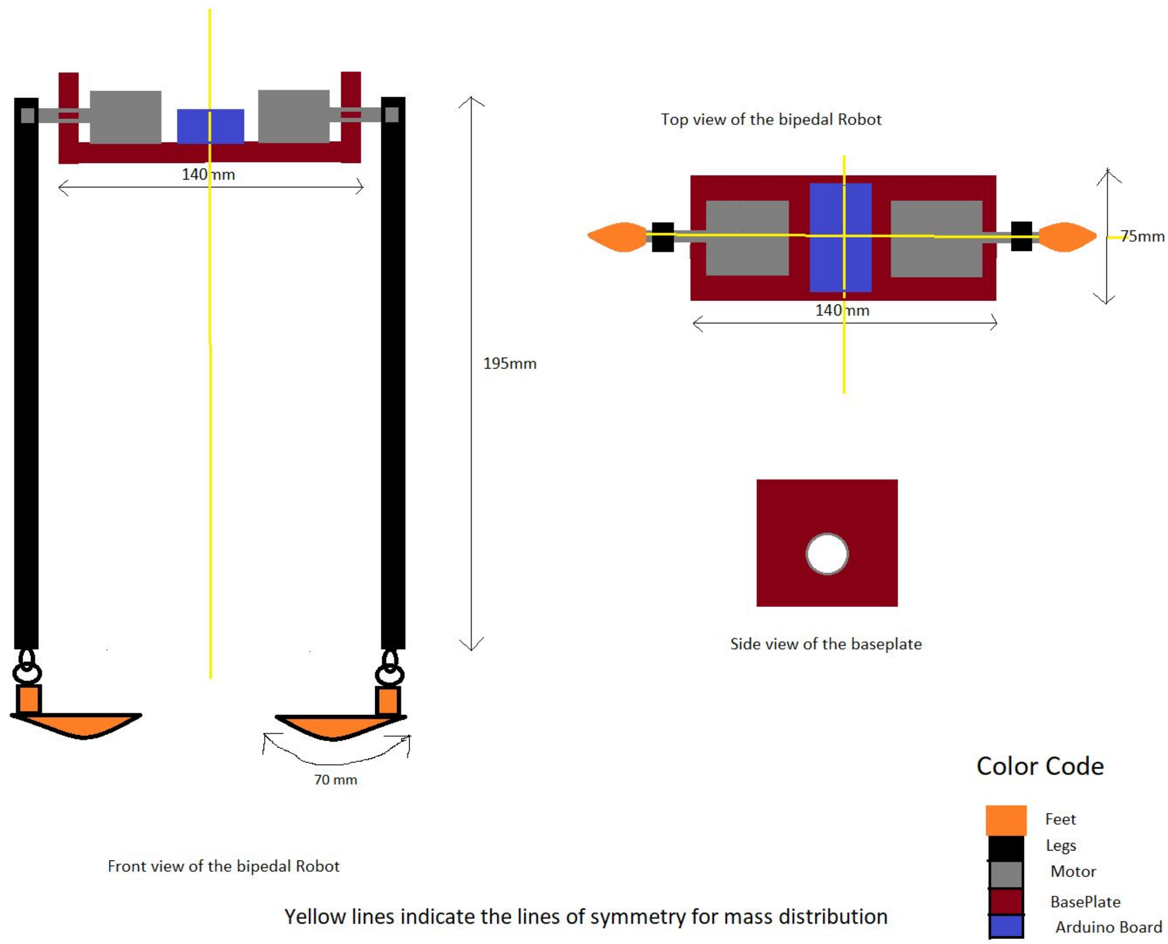

Figure 1. Front view of robot, top view of baseplate, side view of baseplate

Actuators

Two RobotGeek 180° servo motors were used at the hip joints, with the following specifications.

Table 1. RobotGeek Servo Motor Specifications[1]

RG-SRV180 Stats	
Operating Voltage	6v
Stall Torque	12 kg·cm 166.6oz·in
No-load Speed	43 RPM; .23 seconds/60°
Operating Angle	180°
Weight	60g
Size	30 x 45 x 51 mm
Stall Current	1600 mA
Standby/No Load Current	150 mA
Control Protocol	PWM
Cable Length	270mm
Material	Plastic Body and Metal Gears

Computational Resources

A 16MHz Arduino Atmega328p microcontroller was used to drive the hip motors. The angular positions of the two servo motors were controlled using two 8-bit timers to create pulse width modulated signals with a period of 16.3ms and duty cycle between 8.2% and 15.6% depending on the angular position of the motors.

Control Algorithm

We based our control algorithm on a paper from Collins et al[18], who focused on building a control algorithm for a passive dynamic robot with curved feet. Our algorithm was implemented using two 8-bit timers: timer 0 and timer 2 on the Atmega328p microcontroller. They were used to create pulse width modulated signals to control the angular position of the hip motors. One leg is moved at a time either backward 10º or forward 10º. Figure 2 displays the pattern of the leg motion frame by frame. The right leg moves forward, after which the left leg moves backward to create an inverted letter "V" (see first position in Figure 2). Then the left motor rotates forward, after which the right one moves backward. Lastly the right leg moves forward again, and the pattern continues until the motors are no longer supplied with power.

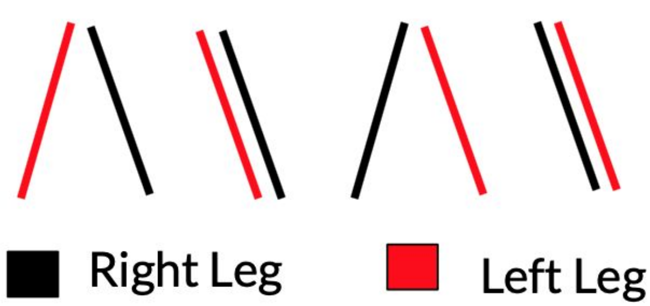

Figure 2. Robot gait pattern

Computational Model

To create a simple model of how foot curvature affects bipedal locomotion, an optimal foot shape can be found that allows for the most stable locomotion in a semi-passive bipedal walker. The biped is stable when the center of volume falls between the endpoints of the two feet positions[4]. Our control variable is the curvature of the foot (Figure 3), which can be seen in the physical model below. We quantify stability using the number of steps before the robot falls for the sake of the simulation, which is directly related to the time before the robot falls. Regardless of the foot shape, the robot moves its legs at the same angle for its steps, and the stride length is the same. So it is safe to assume that number of steps is also directly correlated with distance travelled- the actual metric we use in our physical experiments.

To predict how the physical model may behave, and to verify the experimental results, a computational simulation of the robot was made using Python version 3.2. The simulation uses Matplotlib to display the two legs as they rotate at the hip using our motor control algorithm, and show how the feet passively rotate about their ankle hinges using kinematic equations to model the foot movement about the ankle.

We modelled a biped with mass distribution and spatial features closely based on our actual robot. Then we used equations of circular motion and energy to simulate the sideways motion (sway) as the two feet, controlled by the microcontroller, move. The two circular feet were the pivot of the oscillatory motion, and the selection of feet as the pivot depended on which feet was in the stance phase.

The equations relevant to the simulation are: (See Resources [E] for equations)

Moment of Inertia of the system, where m and d are the volumes and positions(distances from the pivot) of the parts is

(Eq. 1) $$\mathbf{I = \Sigma md^2}$$

Kinetic energy of the system, which increases by an arbitrary value ‘a’ after every step, modeling instability at every step is given by

(Eq. 2) $$\mathbf{K = \tfrac{1}{2}\, I\, \omega^2 + a * steps}$$

Angular position of the system, constantly updated in the simulation and is calculated using

(Eq. 3) $$\mathbf{\Theta = \theta_0 + \omega \Delta t}$$

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[4] Base x position being defined as the x position of the C.O.M when the robot is in stance phase.

The code for the simulation can be found in the repository listed under the Resources section [A].

In real life, the kinetic energy does not consistently increase with every step. However, analyzing the motion of our biped made it obvious that the gait becomes more extreme as more steps are taken, and therefore the kinetic energy in our simulation models the intensity or the kinetic energy of the gait.
One thing hard to find in the simulation was the value of the constant a, as we do not know how fast the kinetic energy will increase with each step. As the workaround, we tried different values of ‘a’. Our simulation as real-time so we could visualize the motion of the robot real-time. We chose the value of ‘a’ by watching the simulation and picking the value that produced the closest match to the actual motion of the robot as seen in experimental procedures.

The point of falling will be when the center of gravity will cross the endpoints of the two feet [17], and the end points will be dependent on the curvature of the feet[3]. The more curved the feet is, the end points thresholds will be lower. Stability is measured for the number of steps the robot takes before falling over. Number of steps is also directly correlated with how long the robot stays up before falling over.

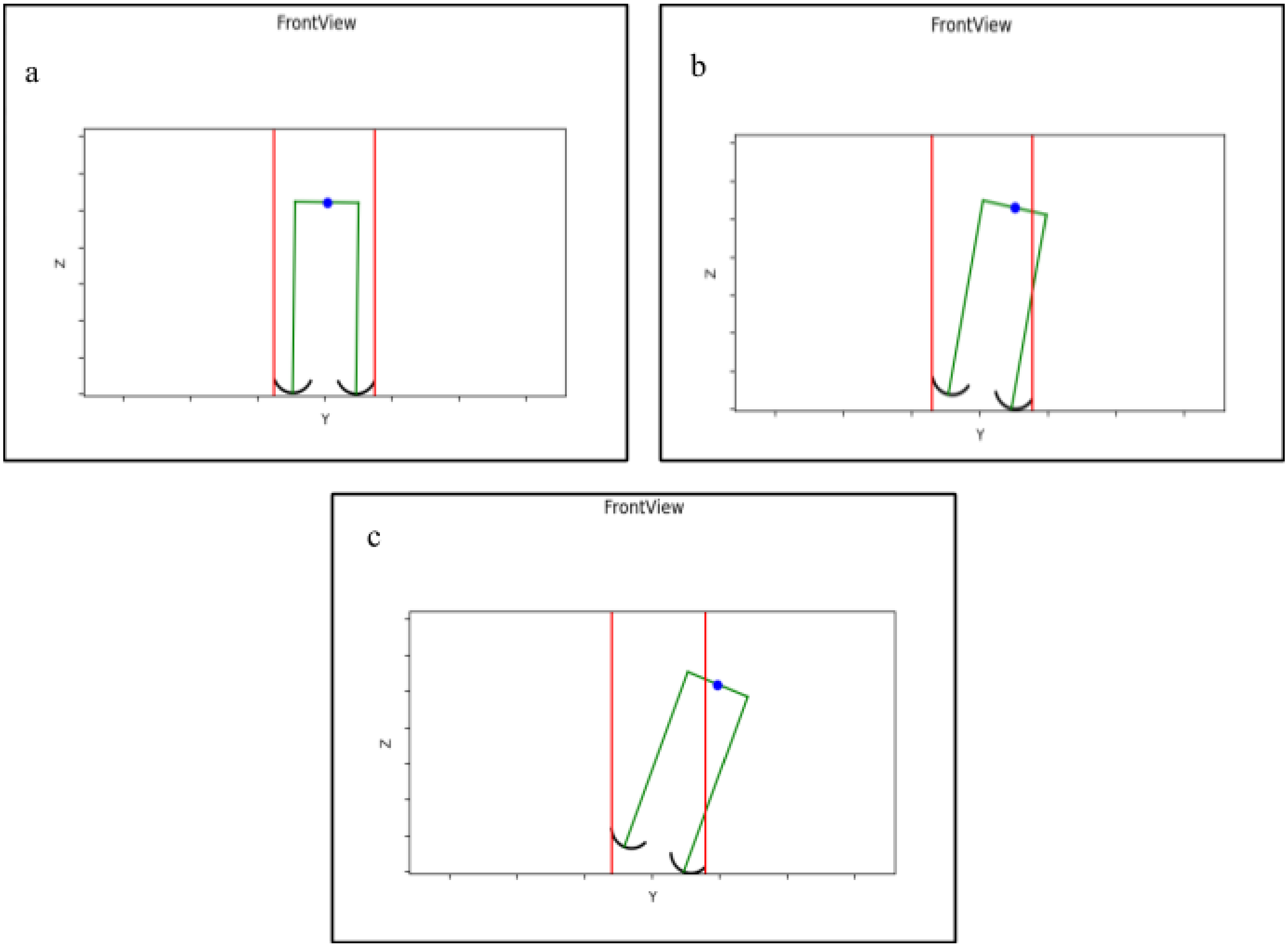

Figure 3. a) Robot in its initial position when K.E. is 0. b) Robot when K.E. has increased and the biped is in oscillatory motion. c) Robot when K.E. has increased to the point of falling. This is the falling condition as the Center of Gravity is outside the area within two feet [17] (between the red lines)

Physical Model

Freecad, an open source CAD software, was used to design the feet. Each foot consists of a solid extruded arc (Figure 4) of length 7 cm. The curvature of the foot is varied by varying the radius r of the cylinder it is cut from, and the angle cut out α, keeping the ratio of r/α constant so that the length of the arc is kept constant at 7 cm (approximately ⅓ of the length of the legs [13]). The top of the foot has a cylindrical attachment point for a universal joint [B] which serves as the passive ankle joint. The CAD files for the foot can be found on the resources section [C].

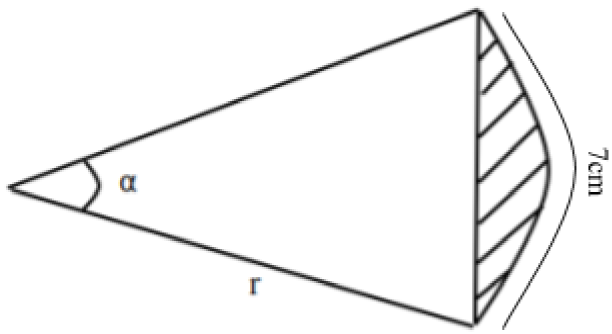

Figure 4. The shaded area represents the curved part of the foot with curvature 1/r., where r and α are varied to keep the ratio r/α = 7 cm, while changing the curvature

Once the models were prepared in FreeCAD (Figure 5), they were sliced using Ultimaker Cura 4.0 with the print settings in Table 2. The models were printed vertically, that is, with the cross section laying flat on the base plate. The feet were printed with a Prusa i3 mk3 using single extrusion.

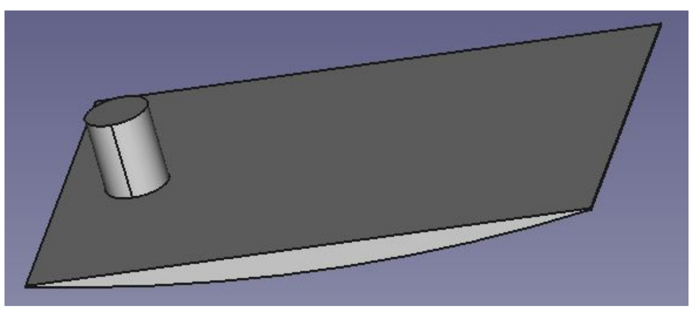

Figure 5. CAD model of a foot with curvature 1/20 cm^{-1} showing the attachment point on the top

The settings that were used to print the CAD models of the feet can be seen in Table 2.

Table 2. Printer settings for the Prusa i3 mk3

Property	**Value**
Material	PLA
Layer Height	0.15 mm

Infill Density	0 %
Printing Temperature	200°C
Build Plate Temperature	60°C
Print Speed	60 mm/s

Experimental Procedure

To determine the effect of foot curvature on the stability and efficiency of a semi-passive bipedal walker, different feet with a range of curvatures were attached to the universal joint at the ankle of the biped. The curvature is defined as 1/radius of the cylinder it is cut from, and the radii of the cylinders range from 5 cm to 40 cm. The same walking control algorithm will be used for each trial.

A 1m long track along which the robot walks was built out of wooden beams and a 1.5m long piece of cloth (Figure 6). The beams were just high enough to allow the feet to comfortably touch the ground and the path was slightly inclined to allow the semi-passive biped to move down a slope [8]. The biped was placed on the right end of the track and the time taken for it to walk 1m was measured. Five trials were carried out for each foot curvature. The feet were then exchanged for a different set and the experiment repeated.

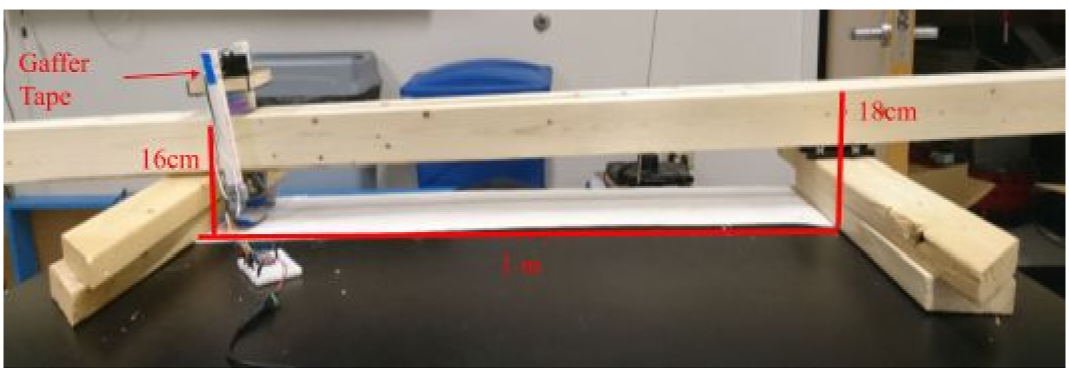

Figure 6. Experimental setup consisting of a 1m long track made from wooden beams, slightly inclined using metal rods on one end. Cloth is placed along the track to increase friction between the feet and the surface. Blue piece of tape used to track the position of the robot visible on the hip.

The x and y positions of the hip of the biped were tracked as the robot progressed along the beam in each trial. A blue piece of Gaffers tape was placed on one side of the hip (Figure 6) and computer vision was used to track the tape using OpenCV for Python 2.7 with the camera on a 2015 13” Macbook Pro. The algorithm tracked both the x and y positions of the blue tape, and the elapsed time, as our robot walked across the 1 meter long track. The spatial and temporal data were saved in a CSV file, and analyzed and visualized later using matplotlib in Python 3.7.

RESULTS

The goal of this study was to determine the optimal foot curvature that allowed for the most stable and efficient walking in a semi-passive biped. To quantify efficiency, we used the following metric:

Efficiency = (time to travel 1m) / (volume of foot)

It was important to normalize our stability metric by the volume of the foot so that our results may be scaled and compared to the results of repeated trials of this experiment with robots and feet of different sizes.

It is important to note that we were unable to build a completely self balancing biped, and as a result we made adjustments to our testing setup. Initially we envisioned measuring how far our robot was able to walk before falling over; however, since it always fell over almost immediately, we decided to build guide rails underneath the robot to enable the robot to at least walk. As a result instead of measuring distance to fall we pivoted and measured the time it took the robot to travel along this 1m guided rail. The metric still acts as a proxy for efficiency as more efficient robots would be able to cover the same distance in a shorter period of time, and our results support this.

Foot Design

Initial versions of our foot design included wedge-like feet; however, we realized that this foot shape would result in our biped stubbing its toe every time it tried to take a step. After examining the foot shape in existing robots and altering our design over several iterations, the feet used in this experiment are curved as shown in Figure 5.

This curved shape allows for motion in the frontal plane, which enables the biped to swing its leg forward in the sagittal plane without stubbing its toe. The lateral motion raises the swinging foot (the foot that is moving next) above the ground, allowing space for forward motion without obstruction. A foot of length $l = \frac{1}{3} L$, where L is the length of the leg and l is the length of the foot, is considered to be the optimal foot length [15]. To keep foot length constant while varying curvature, the angle $\boldsymbol{\alpha}$, sliced from the cylinder defining the arc of the foot was varied (Figure 4). Curvature is defined as 1/r where r is the radius of the circle.

Table 3. The radii, curvatures and volumes of the feet used in our experiments

Radius (cm)	Curvature (cm^{-1})	Volume (cm^3)
5	0.200	27.5
10	0.100	16.5
20	0.050	7.0
30	0.033	5.0
40	0.025	3.5

The passive ankle joint was created using a universal joint constrained to 1 D.O.F about the z-axis. The size of the universal joint was determined by the diameter of the PVC tube used for the legs of the biped. Consequently, the size of the attachment point on the feet was determined by the size of the ankle joint. The attachment point for the feet in this experiment consists of a cylinder of radius 3.66 x 10^{-3} m that fits into the universal joint.

Simulation Results

The results for curvature against stability for our Computational Model are given in figure 7 below.

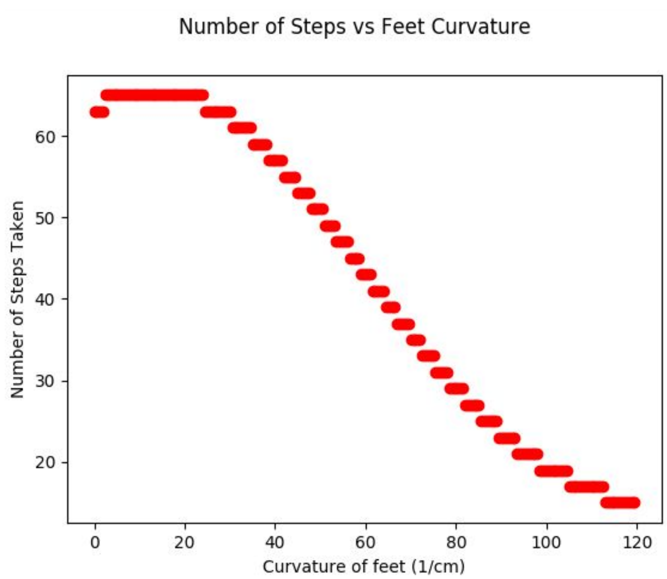

Figure 7. Predictions of Feet Curvature vs Number of Steps before falling (Stability indicator for simulation) for our computational model

Generally, stability decreases with increasing curvature. However, for really low curvatures, the inverse is true. Low curvature should result in a more stable robot, but completely flat feet results in decreased stability as well.
The trend observed from the simulation is consistent with our experimental observations, and indicates a negative correlation between foot shape and stability, as predicted by our hypothesis, except for feet with curvatures.

Experimental Results
The results indicate that as the curvature increases, the efficiency, which is directly correlated to the time taken to travel the 1m across the beam, decreases. From the efficiency metric defined earlier, efficiency is inversely related to the time taken to traverse the beam. Using this efficiency metric, the results support our original hypothesis that efficiency decreases with increased foot curvature.

Table 4. Table summarizing the time taken to traverse the 1m beam for 5 trials with each foot curvature.

Curvature (cm^{-1})	Total Time (s)						
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Standard Deviation
0.025	29.7	28.9	28.7	28.2	29.9	29.1	0.634
0.033	42.4	34.4	25.7	36.7	25.1	32.9	6.63
0.050	37.4	31.3	28.3	39.9	28.9	33.16	5.21
0.100	44.6	30.0	28.9	29.6	35.6	33.7	5.93
0.200	39.6	41.0	54.2	50.6	35.9	44.3	6.95

The experimental results indicate a decaying exponential relationship between foot curvature and efficiency, where efficiency is quantified as distance traveled/volume of foot (Figure 8).

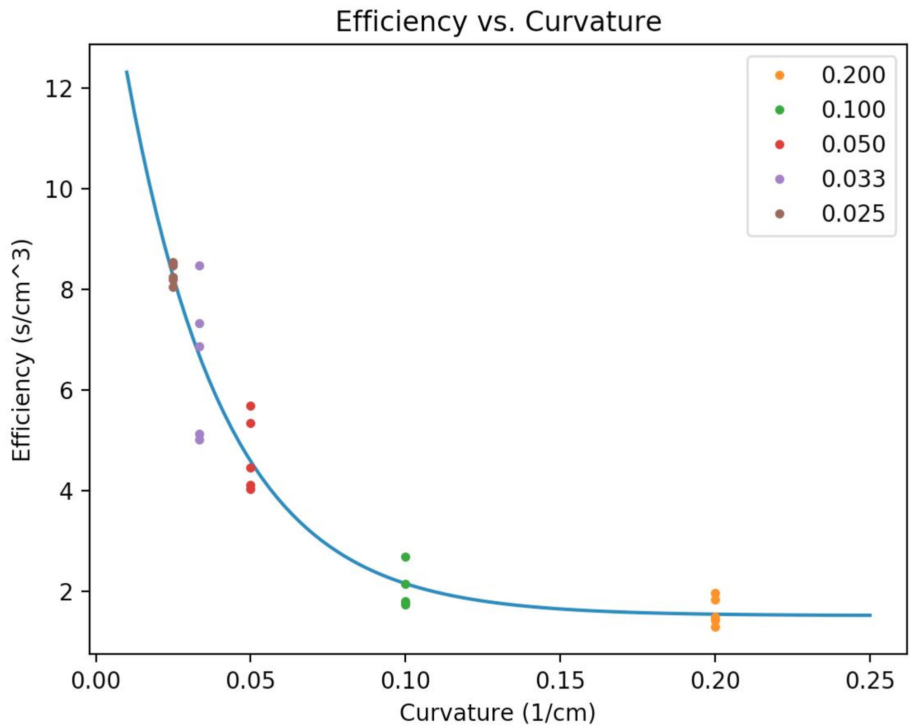

Figure 8. Scatterplot showing the exponentially decaying relationship between foot curvature and efficiency of the biped, where efficiency = $14.8 * e^{-31.5(\text{curvature})} + 1.5$

The peak efficiency can be seen at a foot curvature of 0.200 cm^{-1}. Efficiency begins to decline for curvatures greater than this peak. This supports our hypothesis that an optimal foot curvature is attainable and creates the most efficient locomotion in a semi-passive biped.

In addition to calculating efficiency using the time required for the robot to travel 1m with differing foot curvatures, the y position of the hip was also tracked as the robot moved along the track. This was done using a computer vision program and openCV and tracking the location of a blue piece of Gaffer tape attached to one side of the hip (Figure 6). This data was visualized by plotting y-position cs. time, which was used to quantify stability. Measuring the y position allows for observations that describe how much the robot bounces up and down, where more stable robots would bounce less.

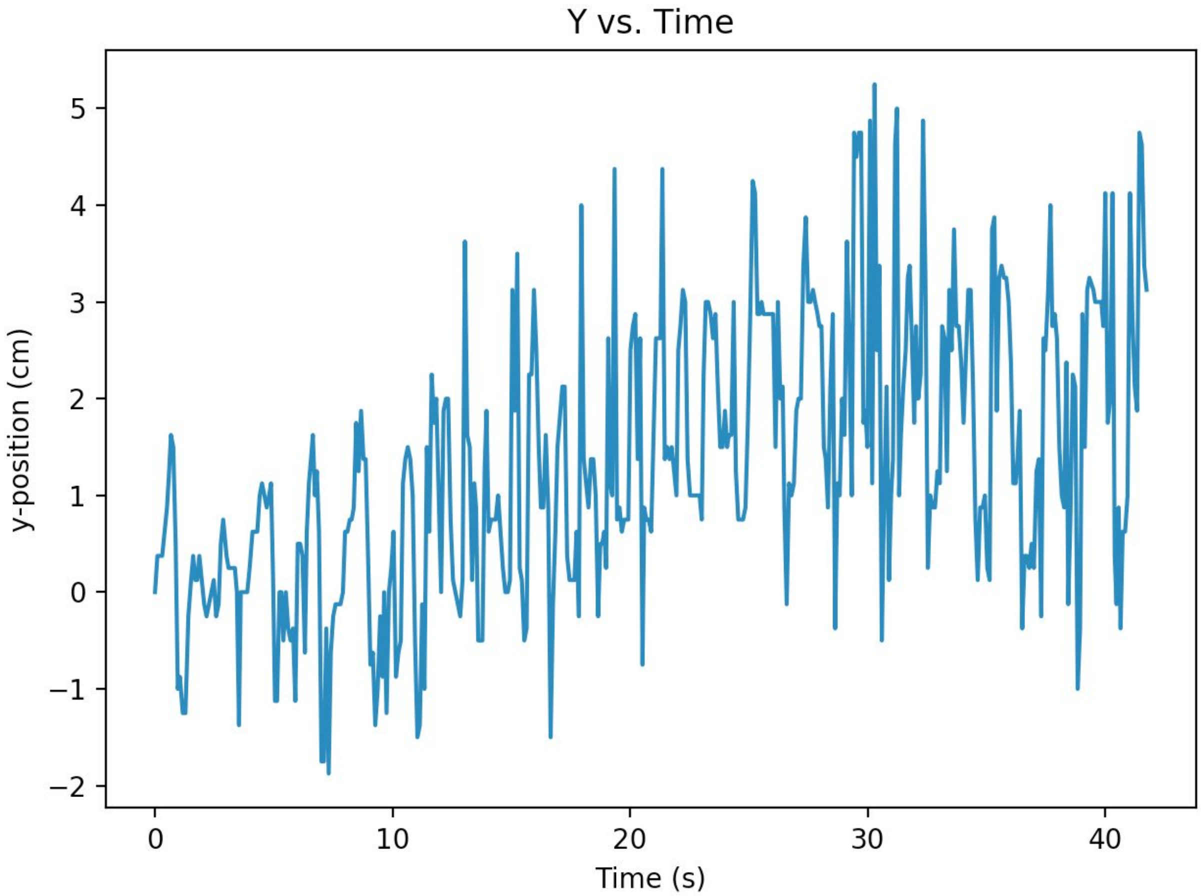

Figure 9. Scatterplot showing variation of y position of the hip motor during trial 2 with the foot with a curvature of 0.200 cm^{-1}.

The figure above displays the y position of the hip motor as the robot walks along the 1m track; however, since there is a high degree of variability in the data, we attempted to fit a sinusoidal wave through the data points, seen in figure 10. Fitting a sinusoid through the data would allow for comparisons of the amplitudes and frequencies between each foot curvature to another, thereby enabling us to draw better conclusions about stability differences among the foot shapes. However, it was not possible to fit these sinusoids for reasons stated below.

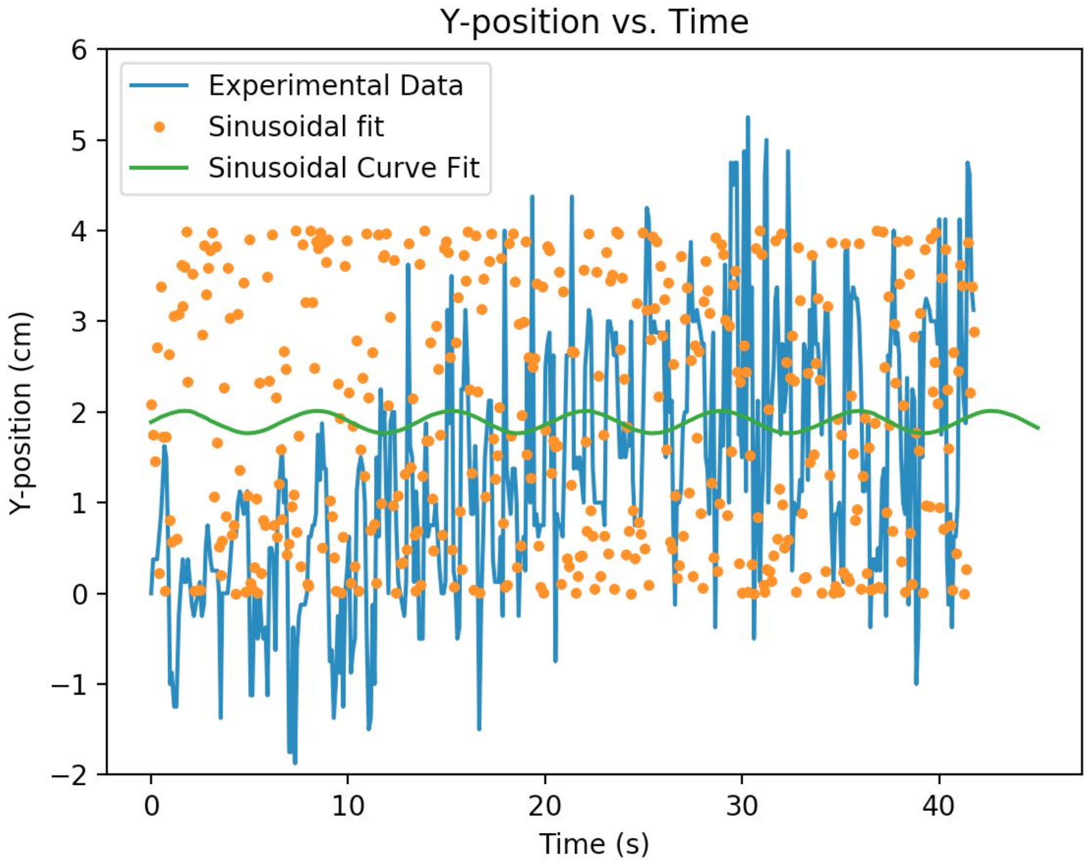

Figure 10. Sinusoidal fit for Y-position vs time for the movement of the biped during trial 2 with the foot with a curvature of 0.200 cm^{-1}.

The sinusoidal fit points seen in Figure 10 were calculated using the time values of the original data points and plugging them into the equation for a sinusoid with amplitude a, period b, and y-offset c: $y = a*\sin(bt) + c$, where t represents the time inputs. The amplitude was estimated by looking at the range of experimental data, and the period b was varied until the root mean-squared error of the sinusoidal fit was minimized. These calculated fit points seem like they could potentially be used to create a smooth sinusoidal curve to approximate the experimental data; however, when scipy was used to fit a sinusoidal curve to the sinusoidal data points, the curve did not accurately fit the data. Instead of using this poor sinusoidal fit, the range of y-positions was used to quantify stability. A higher range is indicates a less stable robot. This is because it is expected that a more stable robot would maintain a constant stride length without too much vertical motion, by avoiding large steps that could cause imbalance. Table 5 displays the range of y-positions obtained in each trial of data collection for each foot curvature.

Table 5. Table summarizing the range of y-positions obtained by tracking the y-position of the hip using OpenCV as the robot progressed along the track.

Curvature (cm^{-1})	Y-Position Range (cm)						
	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Average	Standard Deviation
0.025	5.25	5.625	10.125	5.125	4.875	6.2	2.21
0.033	5.25	8.5	4.75	7.0	5.75	6.25	1.51
0.050	8.75	15.5	17.625	18.125	9.125	13.825	4.57
0.100	10.875	5.5	5.125	5.25	5.75	6.5	2.46
0.200	16.625	7.125	5.5	14.0	6.875	10.025	4.95

To visualize the relationship between stability and foot curvature, these two variables were plotted (Figure 11).

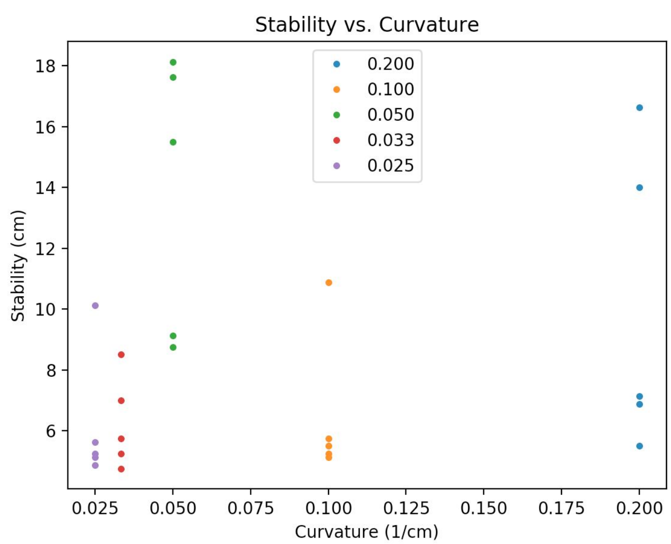

Figure 11. Stability vs. Curvature derived from experimental results. No trend is observed in the data.

The figure above shows that there is no correlation between stability and curvature, hence suggesting that foot shape does not impact the stability of our biped.

CONCLUSION

This paper proposed an optimal foot curvature that leads to a stable and efficient robot, based on our proposed metrics to measure stability and efficiency. We find that there are slight improvements in efficiency with a slight curvature, but in general increasing the foot curvature decreases the stability of a semi-passive walking biped. Hence we conclude that slightly curved feet are beneficial to bipedal walking while very high foot curvatures become detrimental to walking. For future work, we anticipate quantifying the energy efficiency with varying foot curvature even further and possibly looking at feet of different shapes and sizes.

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RESOURCES

[A] Simulation code: https://github.com/ananta399/bipedRobotSimulation

[B] The universal joint can be downloaded from https://www.thingiverse.com/thing:264017

[C] CAD files can be accessed at https://drive.google.com/open?id=1RS35jElAaFeUFGgpLM9qNVfSl7AyyOi1

[D] Data collection and analysis code: https://github.com/rskarp/biped

[E] Circular motion equations: http://www.freestudy.co.uk/dynamics/moment%20of%20inertia.pdf