Central Perturbation-based Interval Type-2 Fuzzy C-Means Clustering for Image Segmentation

EN.525.770 – Fall 2022 – Course Project

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Presentation Overview

- **Image segmentation** is used to separate an image into regions of interest in order to identify objects or simplify the image for further analysis.
  - Popular image segmentation techniques include thresholding, edge detection, and clustering.
- **Fuzzy C-Means (FCM)** is a fuzzy variation of the popular k-means clustering algorithm and is used to group data points into a specified number of clusters. FCM allows each data point to belong to more than one cluster, with the degree of membership based upon the data point's distance from the cluster's center.
- **In this presentation**, we will:
  - Introduce the basic FCM algorithm and two of its variants: Interval Type-2 FCM and Central Perturbation-based Interval Type-2 FCM
  - Apply all 3 algorithm variations to 5 graphical datasets and 5 images from the Berkley Segmentation Dataset
  - Compare the results to those published by L. Rong et. al.

Fuzzy C-Means (FCM)

- FCM is a fuzzy variation of the popular k-means clustering algorithm and is used to group data points into a specified number of clusters.
- Doesn't perform as well on images with complex structures where data points do not clearly belong to a single cluster.

$$u_{ji} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ji}}{d_{ki}} \right)^{\frac{2}{m-1}}}$$

Membership grade calculation

$$v_j = \frac{\sum_{i=1}^{N} u_{ji}^{m} x_i}{\sum_{i=1}^{N} u_{ji}^{m}}$$

Cluster center calculation

Pseudo-code

- Initialize cluster centers
- While delta > threshold and iteration < maxIterations
  - Calculate distance from each data point to each cluster
  - Calculate new membership values of each point for each cluster
  - Update cluster centers

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Interval Type-2 Fuzzy C-Means (IT2FCM)

- IT2FCM is a variation of the FCM algorithm that accounts for uncertainty in the input data and more complex relationships between the inputs and clusters.
- The degree of memberships are represented by an interval instead of a single value.
- The complex interval calculations increase computational complexity and decrease efficiency.
- Traditional Euclidean distance does not account for correlation between features (e.g. RGB values in images).

- Upper and lower membership grade calculations

$$\underline{u}_j(x_i) = \min\left(1 \Big/ \sum_{k=1}^{c} \left(\frac{d_{ji}}{d_{ki}}\right)^{\frac{2}{(m_1-1)}}, 1 \Big/ \sum_{k=1}^{c} \left(\frac{d_{ji}}{d_{ki}}\right)^{\frac{2}{(m_2-1)}} \right)$$

$$\overline{u}_j(x_i) = \max\left(1 \Big/ \sum_{k=1}^{c} \left(\frac{d_{ji}}{d_{ki}}\right)^{\frac{2}{(m_1-1)}}, 1 \Big/ \sum_{k=1}^{c} \left(\frac{d_{ji}}{d_{ki}}\right)^{\frac{2}{(m_2-1)}} \right)$$

- Cluster center update calculation

$$\left[v_j^L, v_j^R\right] = \sum_{u(x_1)\in J_{x_1}} \cdots \sum_{u(x_N)\in J_{xN}} 1 \Big/ \frac{\sum_{i=1}^{N} x_i u(x_i)^m}{\sum_{i=1}^{N} u(x_i)^m}$$

- Cluster center defuzzification

$$v_j = \left(v_j^L + v_j^R\right)/2$$

Central Perturbation-based Interval Type-2 Fuzzy C-Means (CPIT2FCM)

- CPIT2FCM is a variation of IT2FCM where perturbations are added to the cluster centers to account for uncertainty in the cluster centers.
- The Mahalanobis distance is used in place of Euclidean distance to calculate distance between data points and cluster centers. This helps account for correlations between features.
- The same cluster center update rule and defuzzification strategy from the IT2FCM algorithm is utilized here.

- Upper and lower membership grade calculations

$$\bar{u}_j(x_i) = \min\left(1 \Big/ \sum_{k=1}^{c} \left(\frac{\overline{D}_{ji}}{\overline{D}_{ki}} \right)^{\frac{2}{m-1}}, 1 \Big/ \sum_{k=1}^{c} \left(\frac{\underline{D}_{ji}}{\underline{D}_{ki}} \right)^{\frac{2}{m-1}} \right)$$

$$\underline{u}_j(x_i) = \max\left(1 \Big/ \sum_{k=1}^{c} \left(\frac{\overline{D}_{ji}}{\overline{D}_{ki}} \right)^{\frac{2}{m-1}}, 1 \Big/ \sum_{k=1}^{c} \left(\frac{\underline{D}_{ji}}{\underline{D}_{ki}} \right)^{\frac{2}{m-1}} \right)$$

- Mahalanobis distance calculation

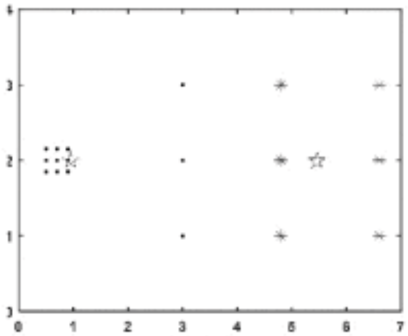
$$\left(\overline{D}_{ji}\right)^2 = \left(x_i - \bar{v}_j\right)^T \Sigma^{-1} \left(x_i - \bar{v}_j\right)$$

$$\left(\underline{D}_{ji}\right)^2 = \left(x_i - \underline{v}_j\right)^T \Sigma^{-1} \left(x_i - \underline{v}_j\right)$$

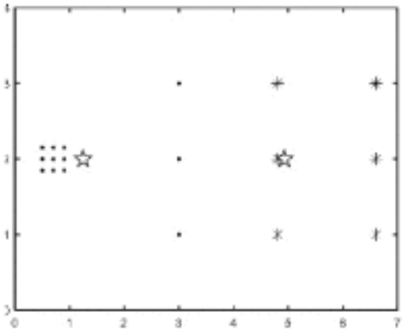
- Cluster center perturbation calculation

$$\bar{v}_j = v_j - \delta, \quad \underline{v}_j = v_j + \delta$$

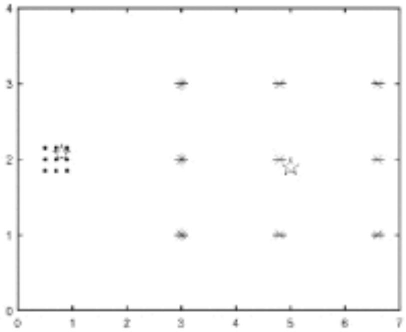
Graphical Results: Squares



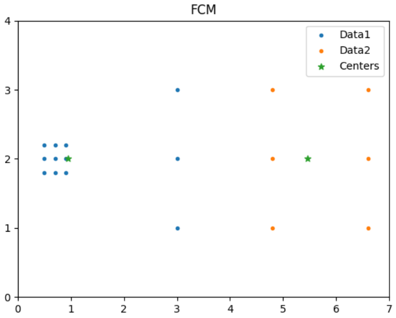
(b) FCM

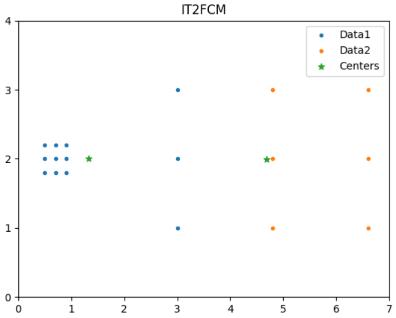


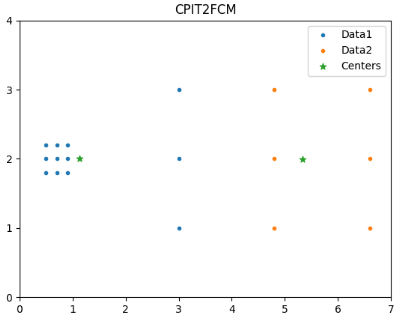
(c) IT2FCM



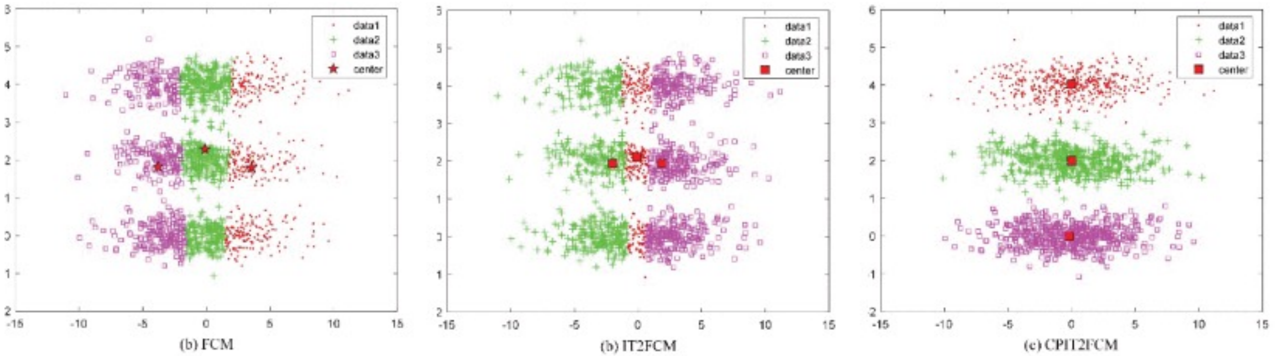
(d) CPIT2FCM

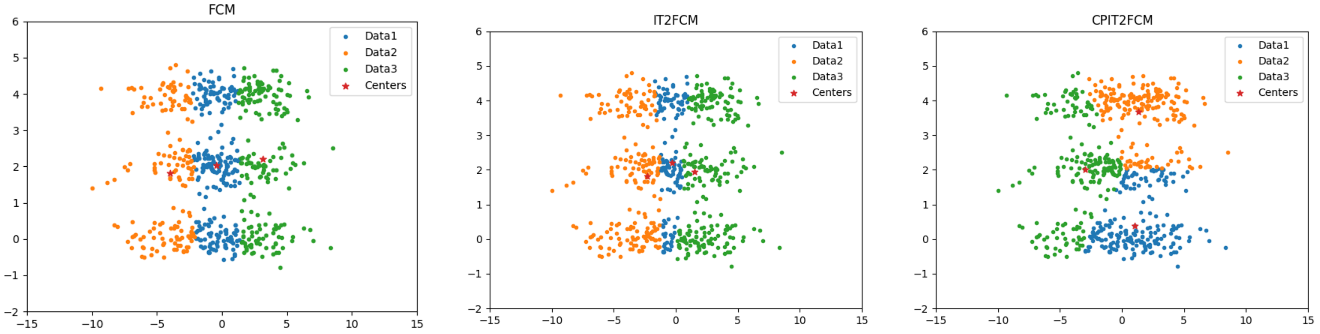




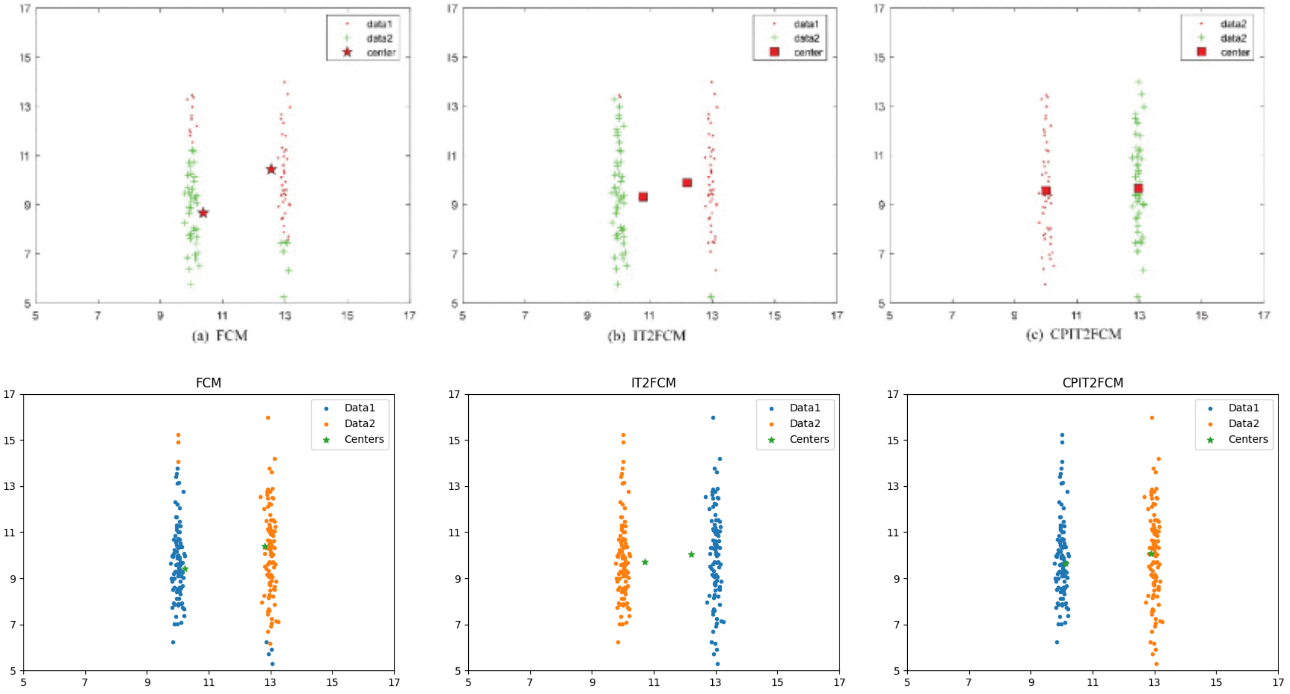


Graphical Results: Spheres

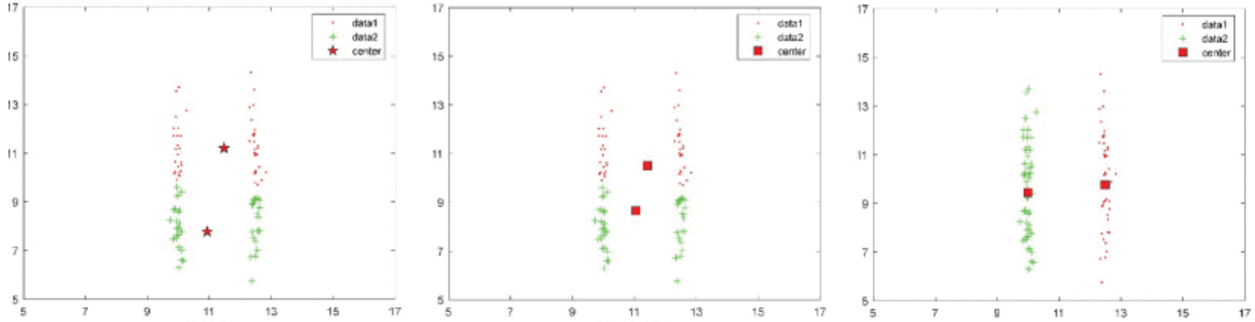




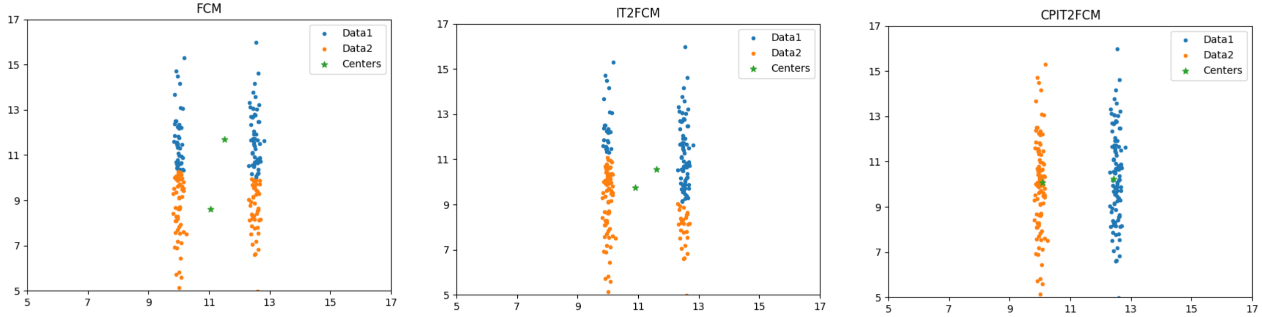
Graphical Results: Rods-3



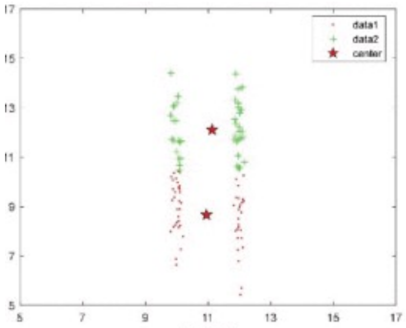
Graphical Results: Rods-2.5



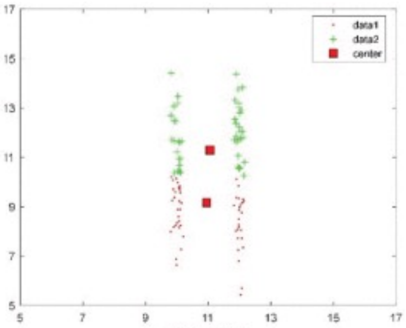
(a) FCM (b) IT2FCM (c) CPIT2FCM



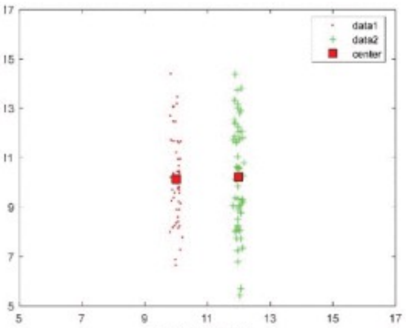
Graphical Results: Rods-2



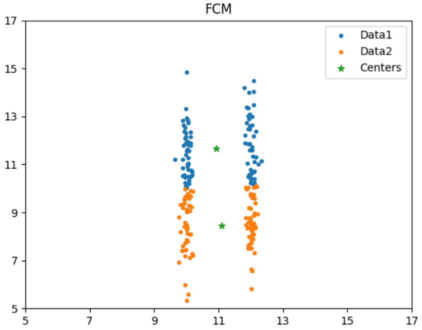
(a) FCM

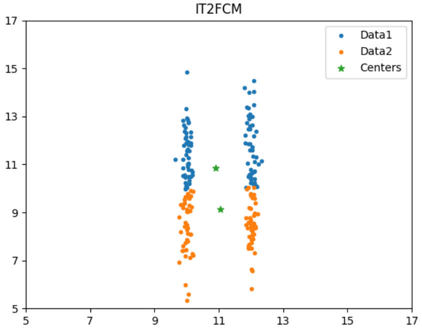


(b) IT2FCM



(c) CPIT2FCM





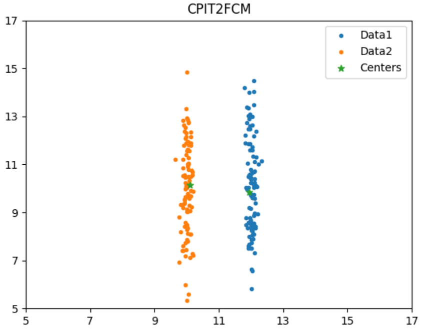


Image Segmentation Results: #10075

(a) Images (b) Ground truth (c) FCM (d) IT2FCM (e) FRFCM (f) A-PSO-IT2IFCM (g) MMGR-AFCF (h) CPIT2FCM

Original Image

Ground Truth

FCM

IT2FCM

CPIT2FCM

Image Segmentation Results: # 3096

(a) Images | (b) Ground truth | (c) FCM | (d) IT2FCM | (e) FRFCM | (f) A-PSO-IT2IFCM | (g) MMGR-AFCF | (h) CPIT2FCM

Original Image

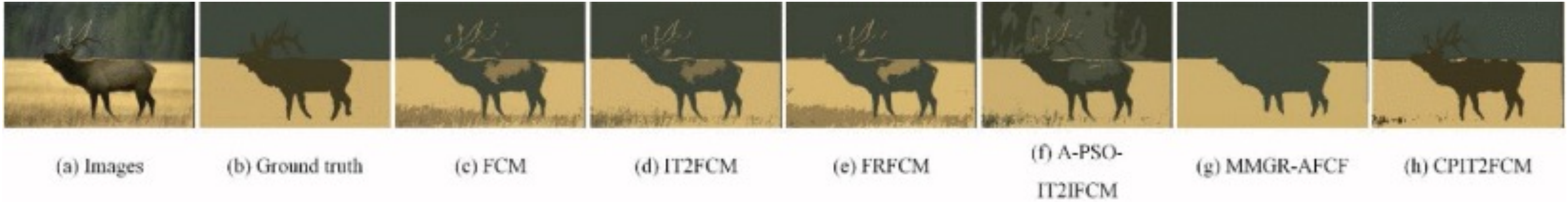
FCM

CPIT2FCM

Ground Truth

IT2FCM

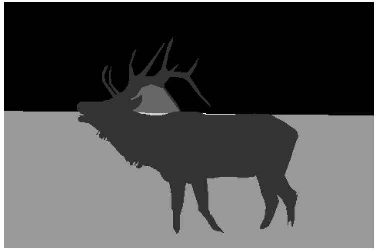
Image Segmentation Results: # 41004




Original Image

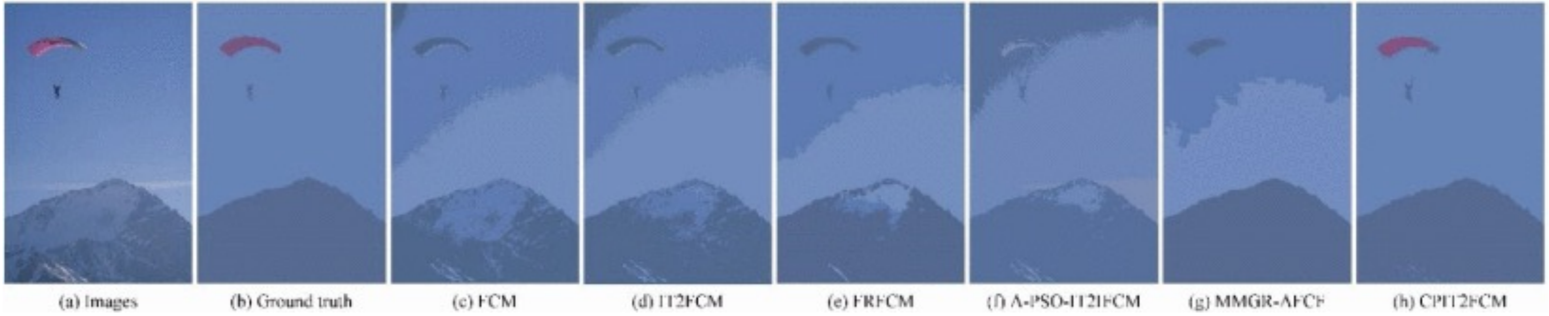

FCM

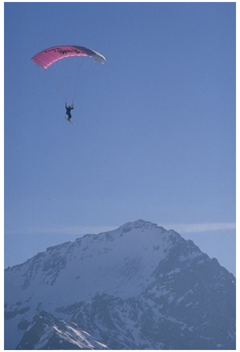

CPIT2FCM


Ground Truth


IT2FCM

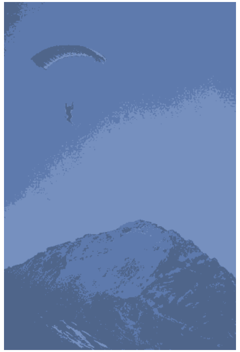
Image Segmentation Results: # 60079





Original Image

Ground Truth



FCM

IT2FCM

CPIT2FCM

Image Segmentation Results: # 10080

(a) Images (b) Ground truth (c) FCM (d) IT2FCM (e) FRFCM (f) A-PSO-IT2IFCM (g) MMGR-AFCF (h) CPIT2FCM

Original Image

Ground Truth

FCM

IT2FCM

CPIT2FCM

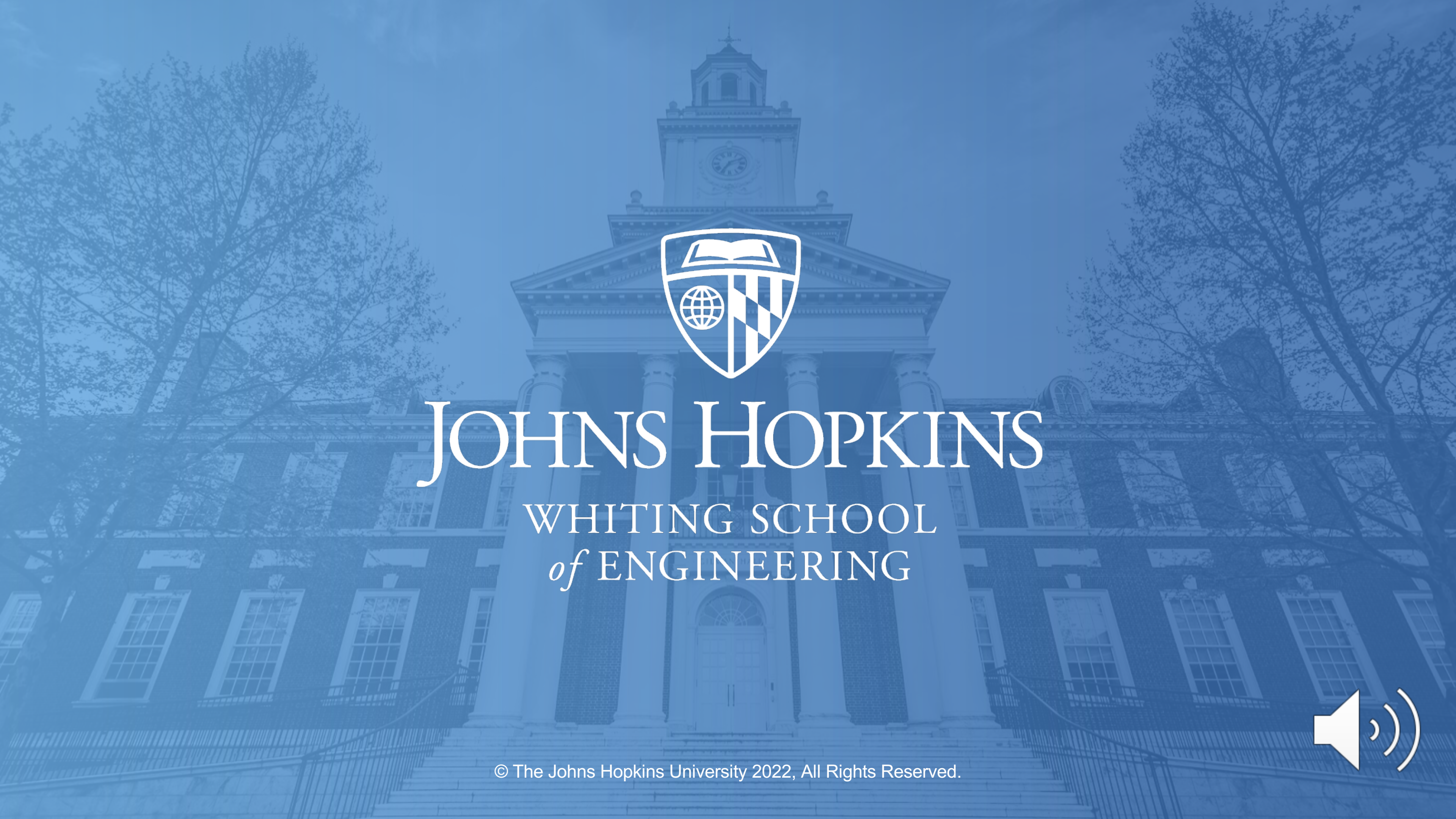
Image Segmentation Results Quantified

- Signal Accuracy (SA)
  - $\frac{Number\ of\ correctly\ classified\ pixels}{Total\ number\ of\ pixels}$
- Normalized Mutual Information (NMI)
- Peak Signal to Noise Ratio (PSNR)
  - $10log_{10}(\frac{R^2}{MSE})$
  - MSE = mean squared error
  - R = maximum possible pixel value (e.g. 255 for 8-bit unsigned integers)

Image	Index	FCM	IT2FCM	FRFCM	A-PSO-IT2IFCM	MMGR-AFCF	CPIT2FCM
#100075	SA	0.7760	0.5839	0.6896	0.7686	0.9019	**0.9326**
	NMI	0.1635	0.0056	0.0700	0.1883	0.5121	**0.8849**
	PSNR	3.2419	3.1688	8.6485	3.5366	10.5639	**21.3698**
#3096	SA	0.9859	0.8118	0.9889	0.9620	0.9920	**0.9946**
	NMI	0.7231	0.1974	0.7865	0.5013	0.8304	**0.8715**
	PANR	10.4522	6.3591	15.6389	13.5696	14.6314	**22.4220**
#41004	SA	0.7473	0.7950	0.8794	0.6682	0.9547	**0.9605**
	NMI	0.5585	0.5979	0.6635	0.5334	0.8181	**0.8368**
	PSNR	2.6548	2.9440	4.6356	2.1671	11.3544	**13.4517**
#60079	SA	0.6230	0.7832	0.8002	0.6398	0.8563	**0.9526**
	NMI	0.2889	0.3369	0.3945	0.2574	0.4700	**0.9034**
	PSNR	2.6395	3.0031	4.3646	2.9635	7.3246	**20.1986**
#100080	SA	0.5436	0.6017	0.8520	0.6220	0.8882	**0.9237**
	NMI	0.3494	0.3584	0.6219	0.4309	0.7506	**0.8136**
	PSNR	3.6998	3.6355	5.6989	4.0236	6.3656	**15.6398**

Summary and Conclusions

- In the authors' experiments, the CPIT2FCM algorithm outperformed all other FCM variations tested
- FCM and IT2FCM results were reproduced on all datasets
  - The fact that the authors used MATLAB R2019b on Windows 10 and I used Python 3.9.6 on MacOS Monterey 12.6 did not seem significant
- CPIT2FCM results were reproduced on rod datasets only possibly due to lack of detail from the authors regarding how they applied their perturbations
  - For the BSDS images, the reproduced CPIT2FCM algorithm seemed to perform better on darker images (#10075, #41004, #100080)
- Quantitative image results could not be reproduced due to lack of detail from the authors regarding how they created the ground truth images
- Overall, variations in FCM algorithms provide significantly different results in image segmentation problems, and the selected algorithm depends on how the segmented results will be used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Appendix A: Code Description

- **Datasets (class):** contains functions to generate the graphical datasets
- **FCM (class):** contains functions to run the basic Fuzzy C-Means clustering algorithm
- **IT2FCM (class):** contains functions to run the Interval Type-2 Fuzzy C-Means clustering algorithm
- **CPIT2FCM (class):** contains functions to run the Central Perturbation-based Interval Type-2 Fuzzy C-Means clustering algorithm
- **ImageSegmentation (class):** contains functions to apply all 3 variations of FCM algorithms to in image from the BSDS dataset
- **run_graphical_experiments (function):** runs all 3 FCM algorithms on all 5 graphical datasets
- **run_single_image (function):** runs all 3 FCM algorithms on the given image from BSDS
- **run_image_experiments (function):** runs all 3 FCM algorithms the 5 selected BSDS images
- **run_all_experiments (function):** runs all graphical and image experiments

Appendix B: References

- L. Rong, Q. Na, H. Tianlong, Z. Feng, Y. Haiyan and Z. Lu, "Central Perturbation-based Interval Type-2 Fuzzy C-Means Clustering for Image Segmentation," *2022 4th International Conference on Natural Language Processing (ICNLP)*, 2022, pp. 100-108, doi: 10.1109/ICNLP55136.2022.00025.
- C. Hwang and F. C. -H. Rhee, "Uncertain Fuzzy Clustering: Interval Type-2 Fuzzy Approach to C-Means," in *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 1, pp. 107-120, Feb. 2007, doi: 10.1109/TFUZZ.2006.889763.
- D. Martin, C. Fowlkes, D. Tal and J. Malik, "A database of human segmented natural images and its application to evaluating segmentation algorithms and measuring ecological statistics," *Proceedings Eighth IEEE International Conference on Computer Vision. ICCV 2001*, 2001, pp. 416-423 vol.2, doi: 10.1109/ICCV.2001.937655.