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Section 002

Lab 4: Application of Laplace Transforms: Operational Amplifier Filters

Task 1 —Design and Analysis of a 4th Order Butterworth Lowpass Filter

1. R_a and R_b are effectively 0, which is why they are not included in the circuit we built.
   The Gain is $K = 1 + R_a / R_b$, therefore the gain is going to be 1.
   This is called a voltage follower because it follows the voltage and doesn't change the gain.

Lab 4 :

3.) $$\frac{V_i(s)-V_1(s)}{R_1} + \frac{V_o(s)-V_1(s)}{1/(sC_1)} + \frac{V_o(s)-V_1(s)}{R_2} = 0$$

$$V_o(s) = \frac{1/(sC_2)}{R_2 + 1/(sC_2)} V_1(s) = \frac{1}{sC_2R_2+1} V_1(s)$$

$$V_1(s) = (sC_2R_2+1)V_o(s)$$

$$\frac{V_i(s) - (sC_2R_2+1)V_o(s)}{R_1} + \frac{\cancel{V_o(s)} - (sC_2R_2+1)\cancel{V_o(s)}}{1/(sC_1)} + \frac{\cancel{V_o(s)} - (sC_2\cancel{R_2}+1)\cancel{V_o(s)}}{\cancel{R_2}} = 0$$

$$\frac{V_i(s) - sC_2R_1V_o(s) - V_o(s)}{R_1} + -s^2C_1C_2R_2V_o(s) + -sC_2V_o(s) = 0$$

$$\frac{V_i(s)}{R_1} + \frac{(-sC_2R_2-1)V_o(s)\cancel{R_1}}{\cancel{R_1}} + -s^2C_1C_2R_2V_o(s)\,R_1 + -sC_2V_o(s)\,R_1 = 0$$

$$\frac{-V_i(s)\cancel{R_1}}{\cancel{R_1}}$$

$$(sC_2R_2+1+s^2C_1C_2R_1R_2+sC_2R_1)V_o(s) = V_i(s)$$

$$V_o(s) = V_i(s) / (sC_2R_2+1+s^2C_1C_2R_1R_2+sC_2R_1)$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{V_i(s)}{(sC_2R_2+1+s^2C_1C_2R_1R_2+sC_2R_1)}}{V_i(s)}$$

$$= \frac{1\,/C_1C_2R_1R_2}{\underbrace{s^2C_1C_2R_1R_2}_{C_1C_2R_1R_2} + \underbrace{sC_2(R_1+R_2)}_{C_1C_2R_1R_2}+\underbrace{1}_{C_1C_2R_1R_2}}$$

$$\boxed{H(s) = \frac{\frac{1}{C_1C_2R_1R_2}}{s^2 + \left(\frac{1}{R_1C_1}+\frac{1}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}}}$$

2.

3-4.

```
%%% Lab 4 MATLAB Code %%%
p = [2 4 6];
wc = 2*pi*3e3;
poles = [];
for i = p
   [B, A] =  butter(i, wc, 's');
   poles = [poles; roots(A); 0]; % the 0 in the vector separates each set of
poles
end
```

	P = 2	P = 4	P = 6
Poles	-13328.6488144751 + 13328.6488144751i -13328.6488144751 - 13328.6488144751i	-7213.41275861216 + 17414.7189128366i -7213.41275861216 - 17414.7189128366i -17414.7189128366 + 7213.41275861219i -17414.7189128366 - 7213.41275861219i	-4878.62406421921 + 18207.2728786944i -4878.62406421921 - 18207.2728786944i -13328.6488144751 + 13328.6488144751i -13328.6488144751 - 13328.6488144751i -18207.2728786944 + 4878.62406421915i -18207.2728786944 - 4878.62406421915i

4) $$H(s) = \frac{w_c^4}{(s - w_c e^{j5/8\pi})(s - w_c e^{-j5/8\pi})(s - w_c e^{j7/8\pi})(s - w_c e^{-j7/8\pi})}$$

$$H(s) = \underbrace{\frac{w_c^2}{(s - w_c e^{j5/8\pi})(s - w_c e^{-j5/8\pi})}}_{H_1(s)} \cdot \underbrace{\frac{w_c^2}{(s - w_c e^{j7/8\pi})(s - w_c e^{-j7/8\pi})}}_{H_2(s)}$$

$$H_1(s) = \frac{w_c^2}{(s^2 - s w_c e^{-j5/8\pi} - s w_c e^{j5/8\pi} + w_c^2)}$$

$$-2 s w_c e^{-j5/8\pi}$$

* Take the real part of e term and mult by 2 *

$$\boxed{H_1(s) = \frac{w_c^2}{(s^2 + 0.7654 w_c s + w_c^2)}}$$

$$H_2(s) = \frac{w_c^2}{s^2 - 2 s w_c e^{j7/8\pi} + w_c^2} \Rightarrow \boxed{H_2(s) = \frac{w_c^2}{(s^2 + 1.8478 w_c s + w_c^2)}}$$

5-6.

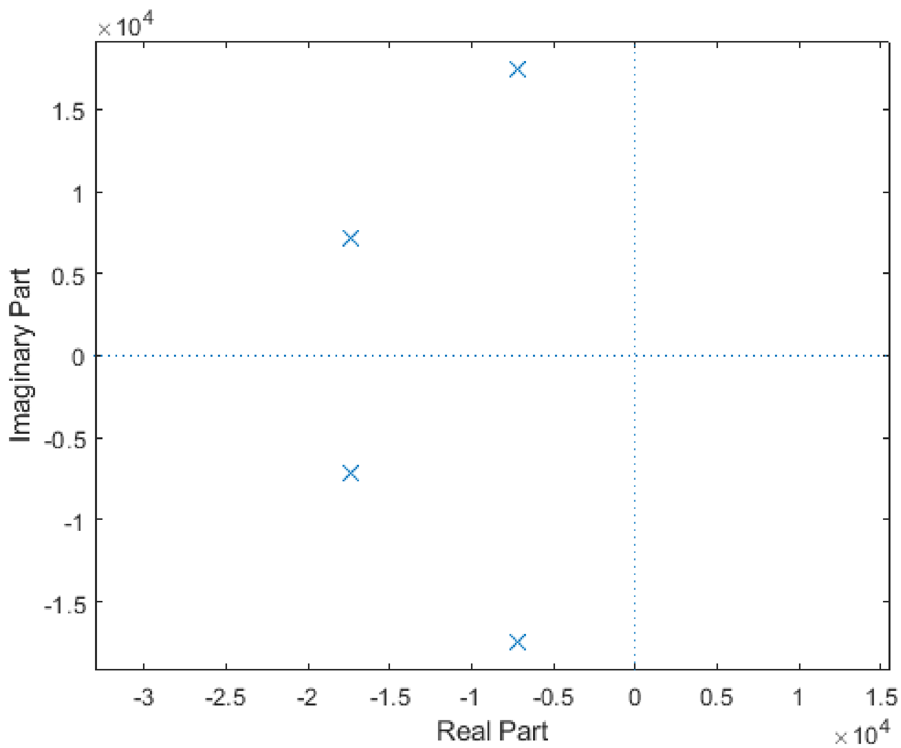
MATLAB code for tasks 1.5 and 1.6:

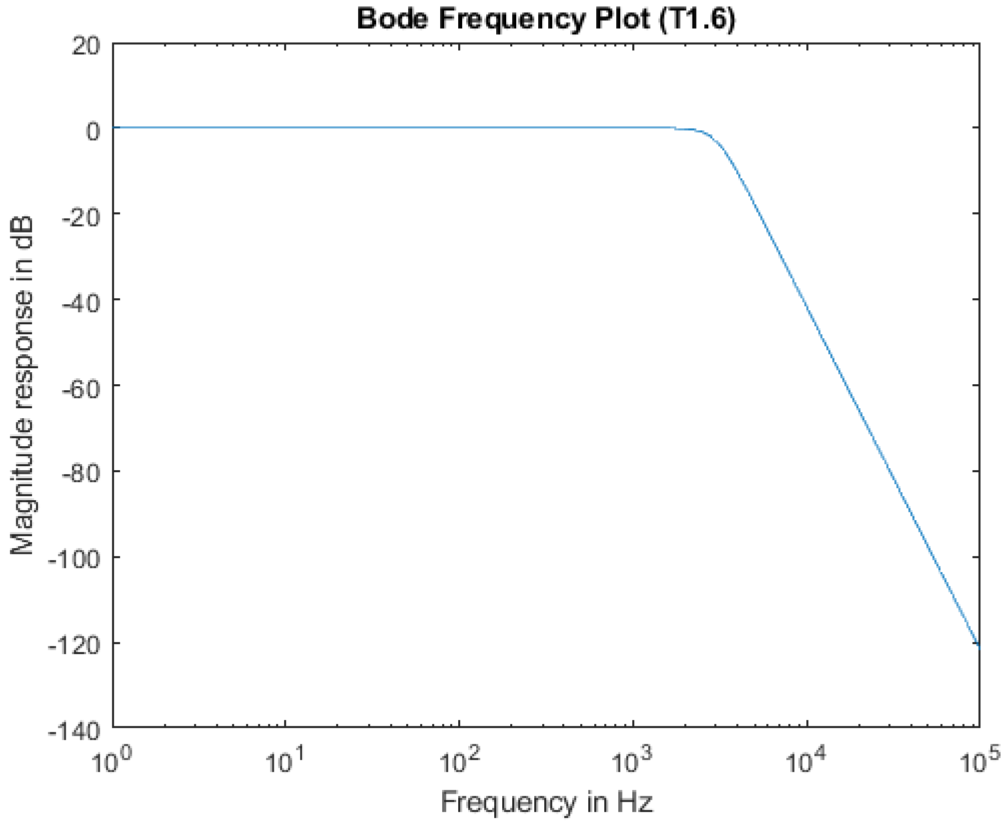
```
elseif task > 1.5
   p = 4;
   wc = 2*pi*3e3;
   [B,A] = butter(p,wc,'s');
   figure(1);
   zplane(B,A);
   B1 = [0,0,0,wc^2];
   A1 = [0,1,0.7654*wc,wc^2];
   B2 = B1;
   A2 = [0,1,1.8478*wc,wc^2];
   A_conv = conv(A1,A2);
   B_conv = conv(B1,B2);
   figure(2);
   zplane(B_conv(3:7),A_conv(3:7));
   if task == 1.6
       close all;
       figure(3);
       % this creates a row vector of equally spaced points from 10^param1
       % to 10^param2 with param3 number of points
       f = logspace(0, 5, 500);
       % this returns the frequency response vector H and w is the points
       % H is evaluated at on the graph in rad/s
```

```
        [H,w] = freqs(B,A,2*pi*f);
        % this is the function used to plot on a logarithmic scale
        semilogx(f, 20*log10(abs(H)));
        % label the plot axes
        title('Bode Frequency Plot (T1.6)');
        xlabel('Frequency in Hz');
        ylabel('Magnitude response in dB');
    end
end
```

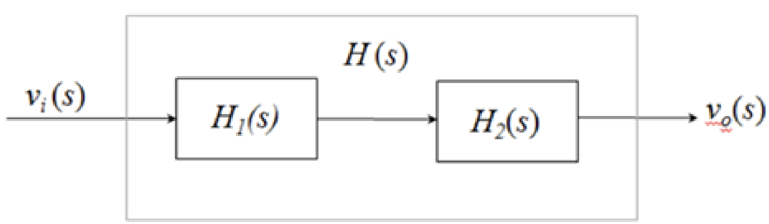
Figures 1 and 2 are the same showing that they produced the same answer.

Plots:





7.



We know that multiplying in the frequency domain is the same convolution in the time domain. This allows us to get These properties can be seen in table 4.2 from the textbook.

Time convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$

8.

8.) $H_1(s) = \dfrac{w_c^2}{s^2 + 0.7654\, w_c s + w_c^2}$ (8)

$$H(s) = \frac{\frac{1}{R_1R_2C_1C_2}}{s^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1}\right)s + \frac{1}{R_1R_2C_1C_2}} = \frac{B(s)}{A(s)} \quad (4)$$

Filter 1

$$H(s) = \frac{\frac{1}{(1k)(1k)C_1C_2}}{s^2 + \left(\frac{1}{(1k)C_1} + \frac{1}{(1k)C_1}\right)s + \frac{1}{(1k)(1k)C_1C_2}}$$

$w_c = 2\pi(3000)$ $\quad \left(2\pi(3000)\right)^2 = \dfrac{1}{(10^6)\,C_1C_2}$ $\quad \boxed{C_2 = 20.3\text{nF}}$

138.6nF

$$0.7654\left(2\pi(3000)\right) = \frac{1}{10^3 C_1} + \frac{1}{10^3 C_1}$$

$$14427.45 = \frac{1}{10^3 C_1} + \frac{1}{10^3 C_1} \qquad \boxed{C_1 = 138.6\text{ nF}}$$

$H_2(s) = \dfrac{w_c^2}{s^2 + 1.8478 w_c s + w_c^2}$

Filter 2

$$H(s) = \frac{\frac{1}{(1k)(1k)C_1C_2}}{s^2 + \left(\frac{1}{(1k)C_1} + \frac{1}{(1k)C_1}\right)s + \frac{1}{(1k)(1k)C_1C_2}}$$

$w_c = 2\pi(3000)$

Eq 1 $\quad 2\pi(3000)^2 = \dfrac{1}{10^6 C_1C_2}$

57.4nF

$\boxed{C_2 = 49.03\text{ nF}}$

Eq 2

$$1.8478\left(2\pi(3000)\right) = \frac{1}{10^3 C_1} + \frac{1}{10^3 C_1}$$

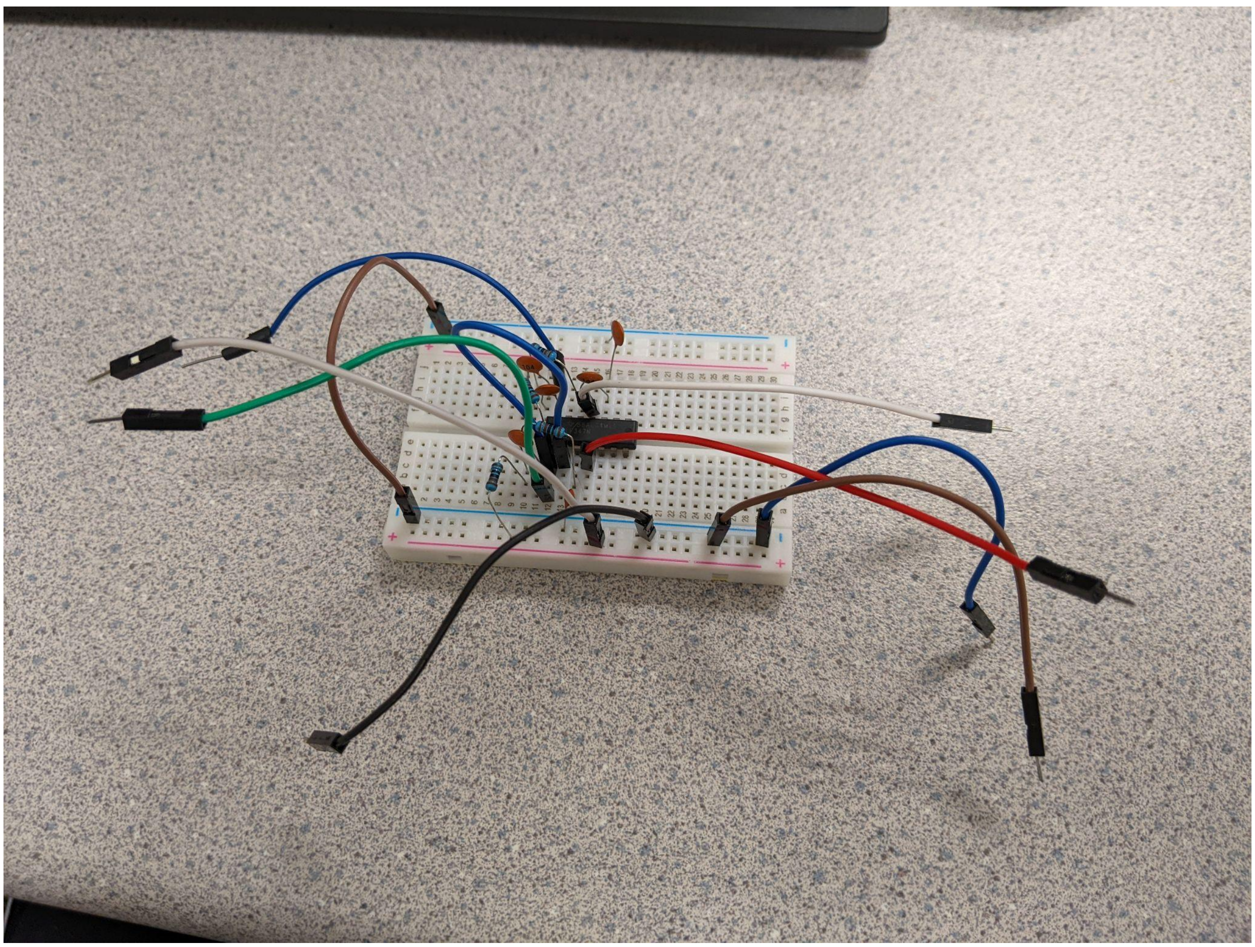
$$34830.21 = \frac{1}{10^3 C_1} + \frac{1}{10^3 C_1}$$

$$\boxed{C_1 = 57.4\text{ nF}}$$

	R1(Ω)	R2(Ω)	C1(F)	C2(F)
Filter 1	1k	1k	138.6n	20.3n
Filter 2	1k	1k	57.4n	49.03n

Task 2 - Build and Test your Active Anti-Aliasing Lowpass Filter

1. Build the circuit

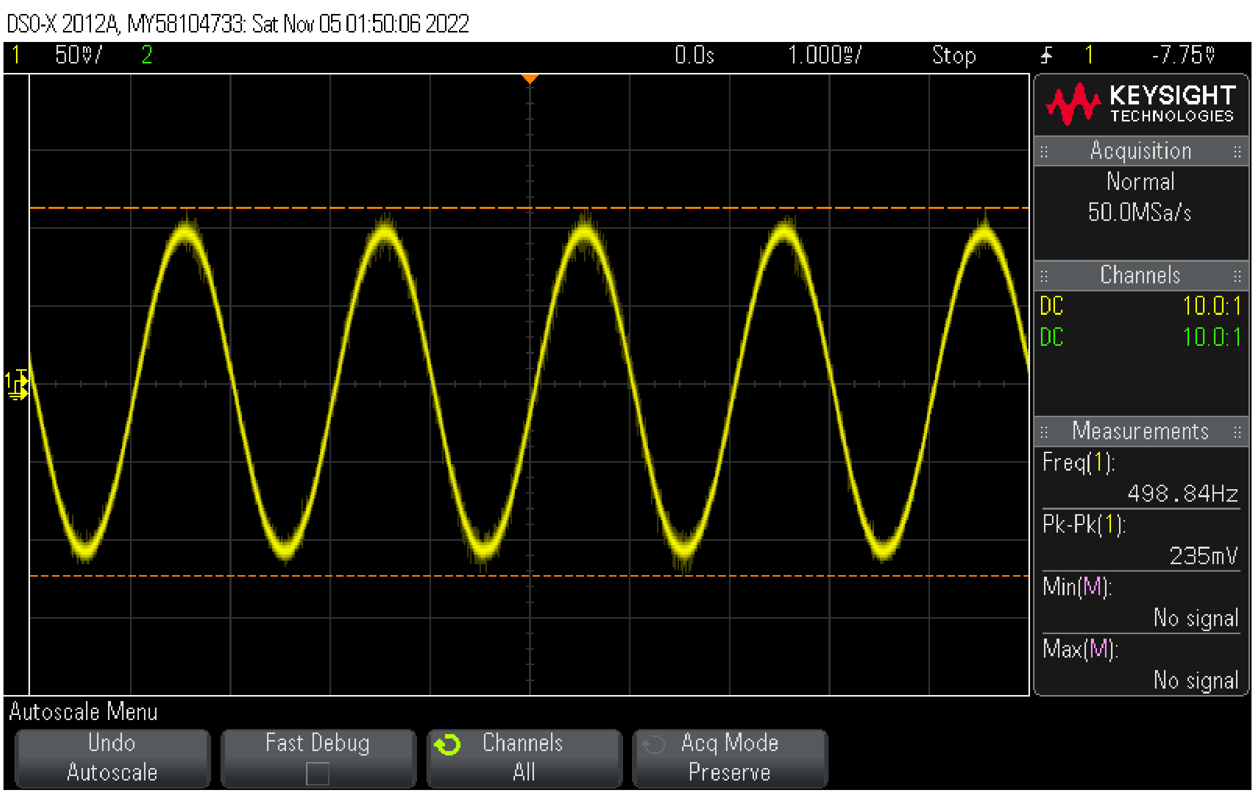


2. As we swept through frequencies from 20Hz to 20kHz we were still able to hear frequencies until around 17Hz. We're unsure if this was because of loss of power or if we were physically unable to hear the sound.
3. We used the Oscilloscope to measure the output of each stage for varying frequencies. We found our corner frequency to be right around 3.2kHz which is very close to the 3kHz we were aiming for.

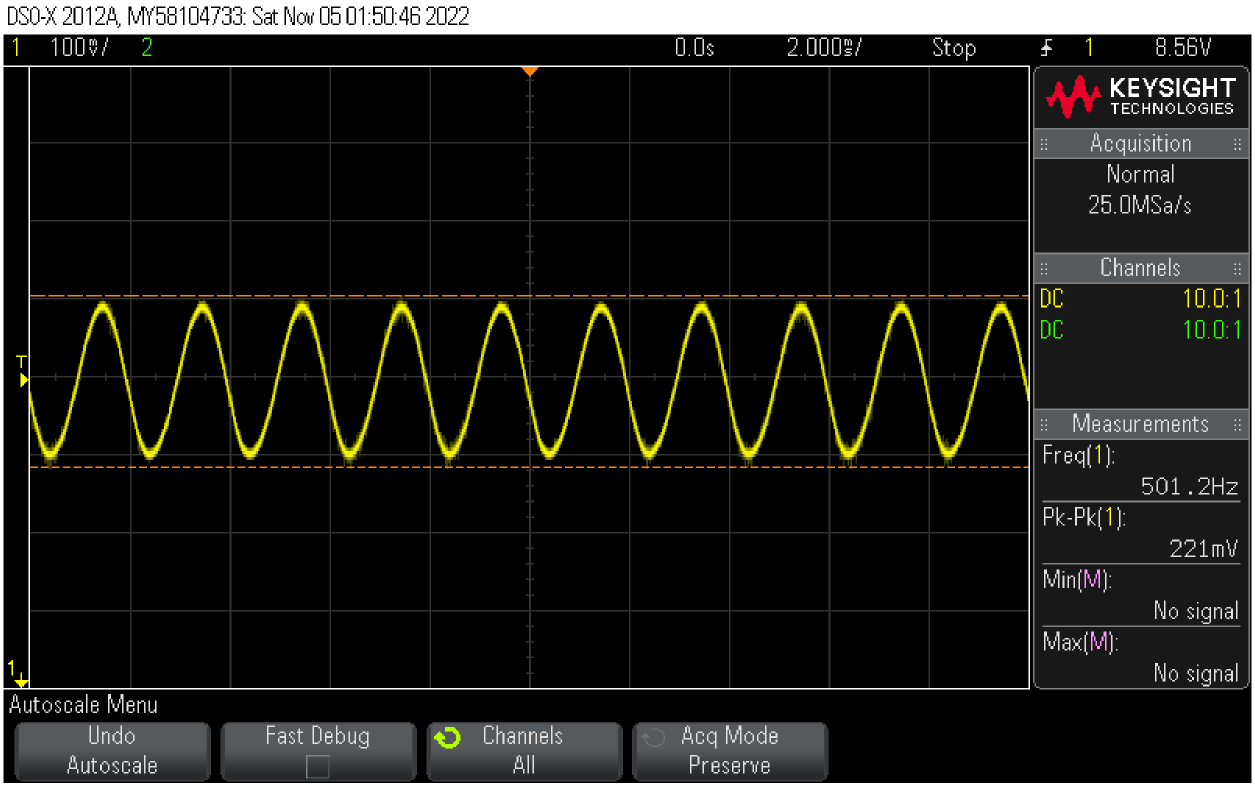
4. The recording we listened to was still audible, but the quality was very poor but we could understand the words.
5. Below are the oscilloscope captures of stage 1 and both stages cascaded for frequencies of 500Hz, 3kHz and 7kHz. Looking at the Matlab plot we had from earlier, this matches very well. As mentioned earlier, we measured our corner frequency to be 3.2kHz with the filters cascaded. The first stage had a higher corner frequency and still allowed some higher frequencies to pass through, but the second filter brought the overall corner frequency to exactly where we wanted it. It has some differences of course, likely because the resistance and capacitance of our components are not exactly what we calculated, but were very close. Additionally, these components are not ideal, which always causes differences between what we calculate and what we measure.

   The response of the whole sysem cascade was clearly better. First, it had a corner frequency that was closer to our desired corner frequency. This meant that attenuated the signals we didn't want better. Second, the signal was clearer through the cascaded filters. It's hard to tell from the screenshots, but there seemed to be much less noise on the signal through both filters.

500Hz Stage 1

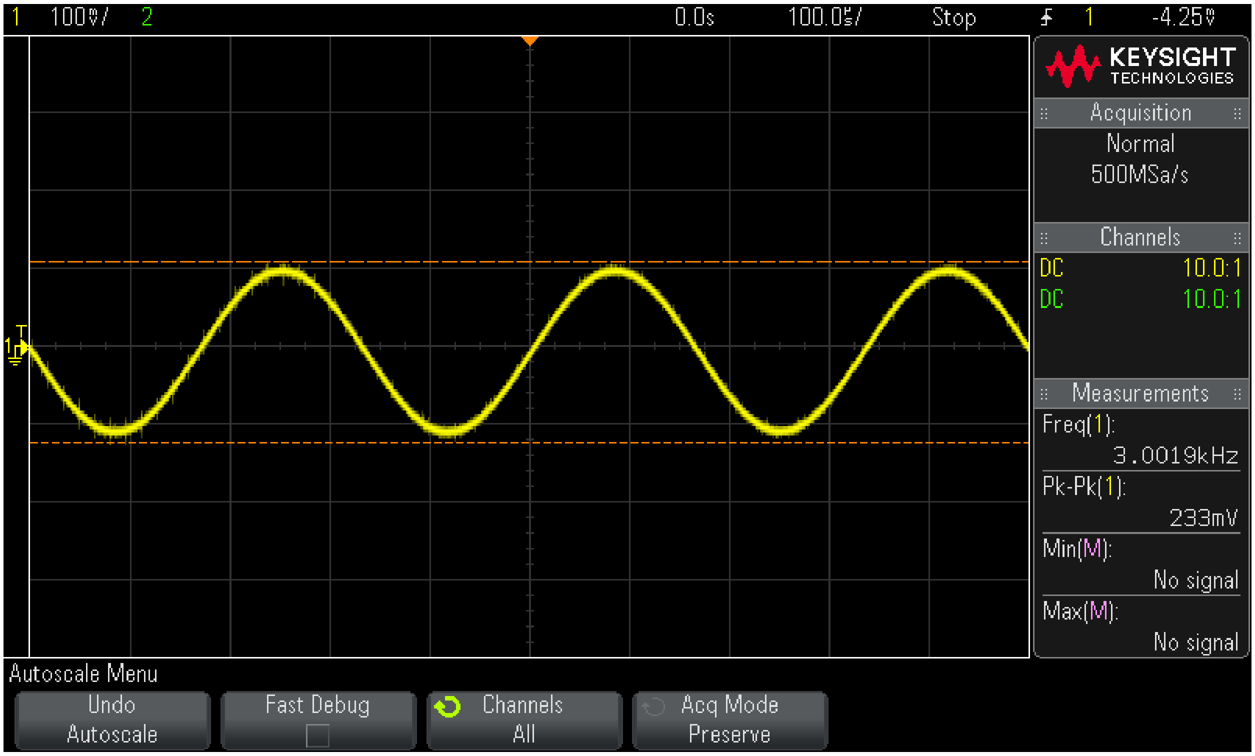


500 Hz Stage 2



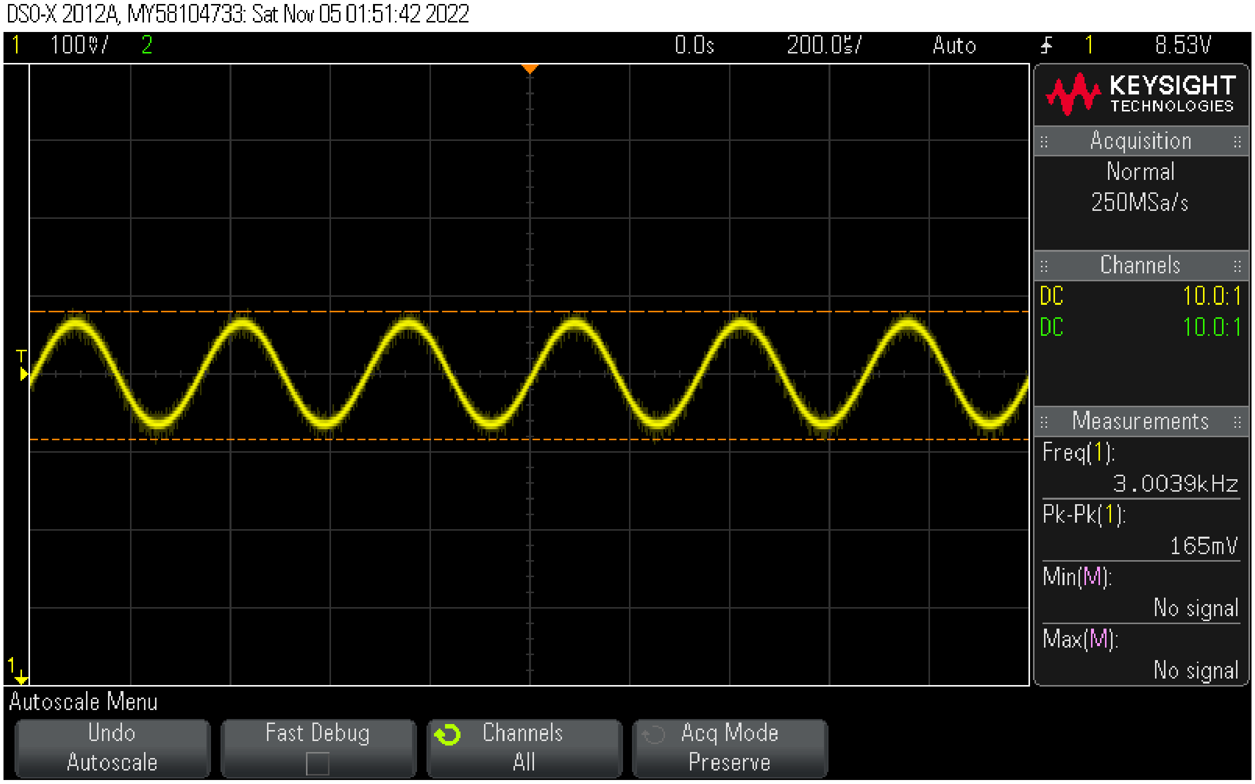
3kHz Stage 1

DSO-X 2012A, MY58104733: Sat Nov 05 01:55:34 2022

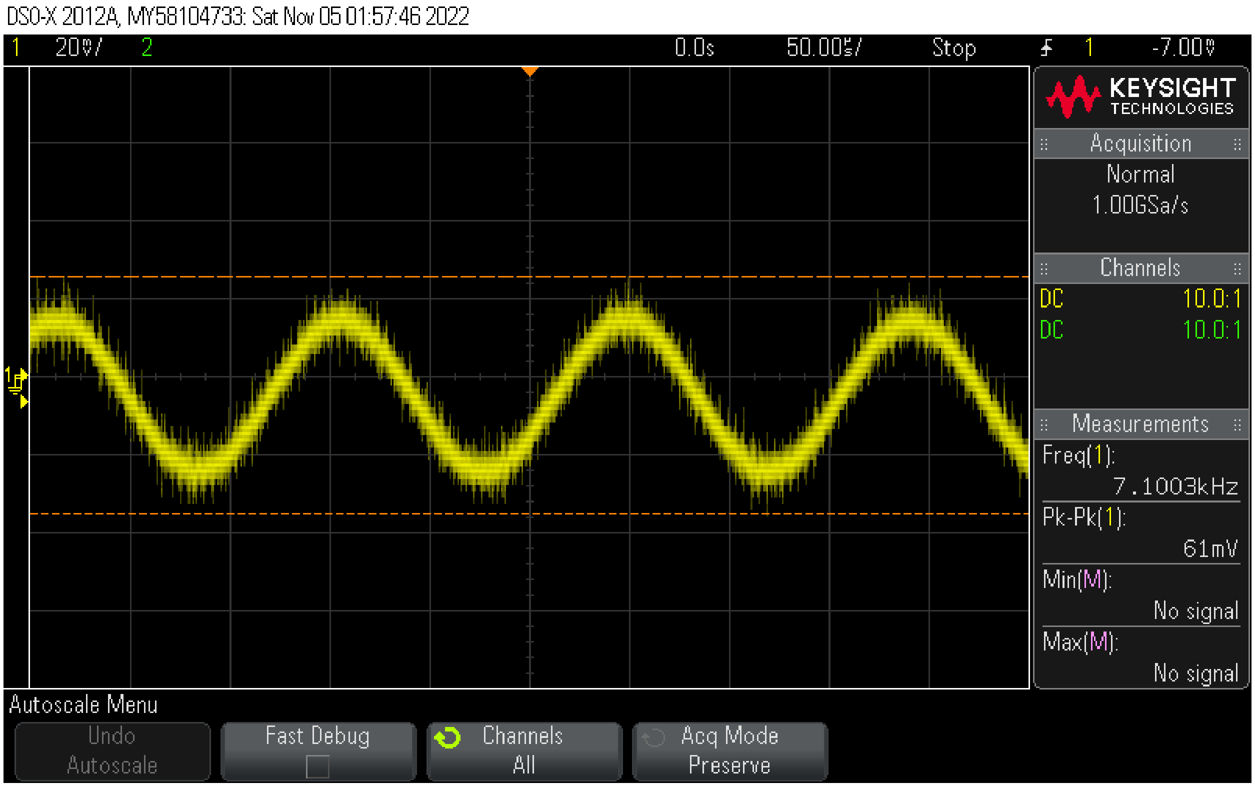


3kHz Stage 2

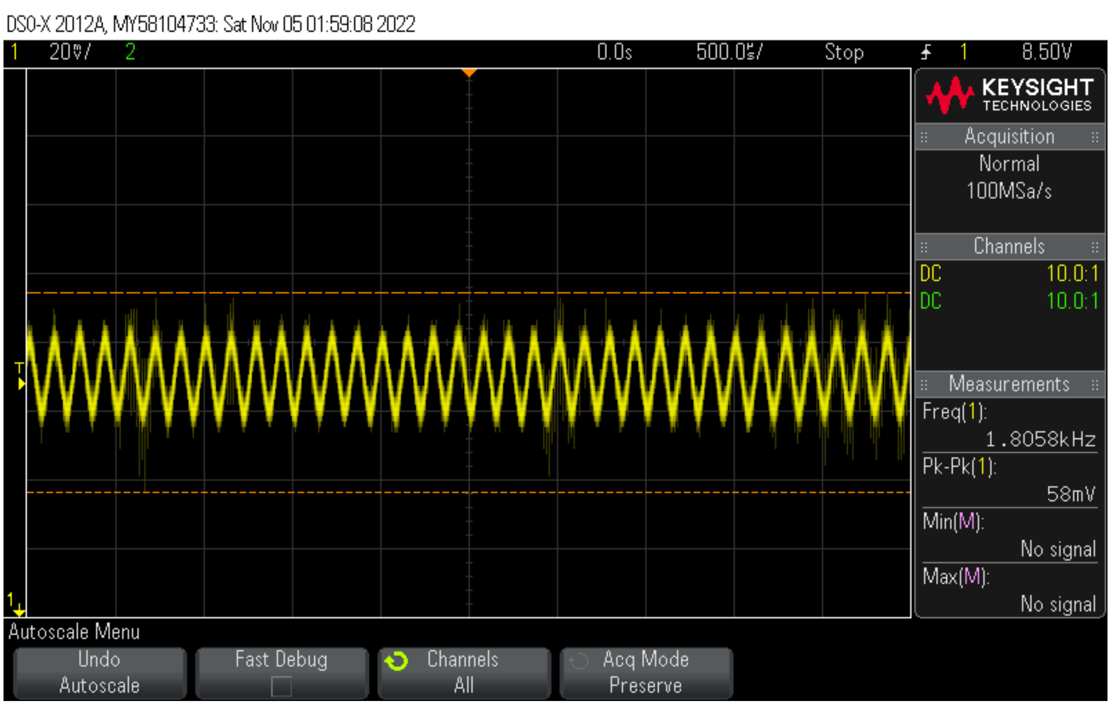
DSO-X 2012A, MY58104733: Sat Nov 05 01:51:42 2022



7kHz Stage 1



7kHz Stage 2



Conclusion:

In this lab we were able to learn a lot about low pass filters. We learned about the math behind them and how to derive the equations as well as how to build a low pass filter and test it. We had the opportunity to use the circuit that we built to play audio through. We used the audio black box and our circuit to hear different frequencies through a set of headphones. It was really interesting to change the frequency of the signal generator and hear the difference. We were not able to hear any sound past 17 kHz, which was interesting. These very high frequencies could have just been too high for our ears, or maybe there was a problem with the circuitry or the power that didn't allow us to hear those high frequencies. We were also able to use the Oscilloscope and measure the output of our filter at different frequencies. We measured our output at each stage and saw that our low pass filter was working properly. Low signals were passing through just fine, but as we got to the higher frequencies our Peak to Peak measurement dropped drastically and we noticed that the signal was very small and very noisy. This noise could have been because it was such a small signal the noise signals seemed larger, or possibly there was more interference and noise while we measured our stages for the high frequency signals. It was really cool to be able to actually build and test a high quality filter, and it was nice to physically see the results of what we have been talking about in our Junior core classes.