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Section 002

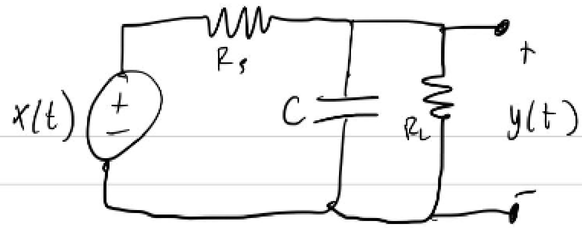
Lab 2: Continuous-Time LTI Impulse Response and Convolution

Task 1 - Calculate an RC Impulse Response

1) For part 1 of task 1, we started by determining the corner frequency we wanted by taking the 3.0kHz and dividing by 2π to get 477.5Hz.
2) Using knowledge from ECEN 240 found the thevenin resistance. We found RS // RL and found that we needed a resistance of 46.15 Ω. Then we calculated the Capacitance C needed as 1.15 uF.

Lab 2

Task 1



1.) Design a lowpass filter with a −3 dB corner. Wc near 3 kHz

$$\frac{3000\ Hz}{2\pi} = 477.5\ Hz$$

2.) $W_c = \frac{1}{R_T C}$ $\quad R_T = R_S // R_L$ Find Parallel Combination of RS//RL

$R_S = 50\Omega$

$R_L = 600\Omega$

$$\frac{R_S R_L}{R_S + R_L} = \frac{(50)(600)}{650} = \boxed{46.15\Omega}$$

$$3000 = \frac{1}{(46.15)C} = 7.222 \times 10^{-6}\ F \text{ or } \frac{7.22\ \mu F}{2\pi} = \boxed{1.15\ \mu F}$$

3) The Equation for part three is as follows.

$$C\frac{dy(t)}{dt} + \frac{1}{R_L}y(t) = \frac{1}{R_s}\left(x(t) - y(t)\right)$$

4) In step 4 we solved for the impulse response

4.) $C\frac{dh(t)}{dt} + \frac{1}{R_L}h(t) = \frac{1}{R_s}\delta(t)$

$P(D) = \frac{1}{CR_s}$

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$$\left(CD + \frac{1}{R_L} + \frac{1}{R_s}\right)y(t) = \frac{\frac{1}{R_s}}{C}x(t) - \cancel{\frac{1}{R_s}y(t)}$$

$$\frac{1}{CR_s}x(t)$$

$$\lambda + \frac{R_s+R_L}{CR_sR_L} = 0 \qquad \lambda = \frac{-(R_s+R_L)}{CR_sR_L}$$

$$\lambda = \frac{-1}{R_L}$$

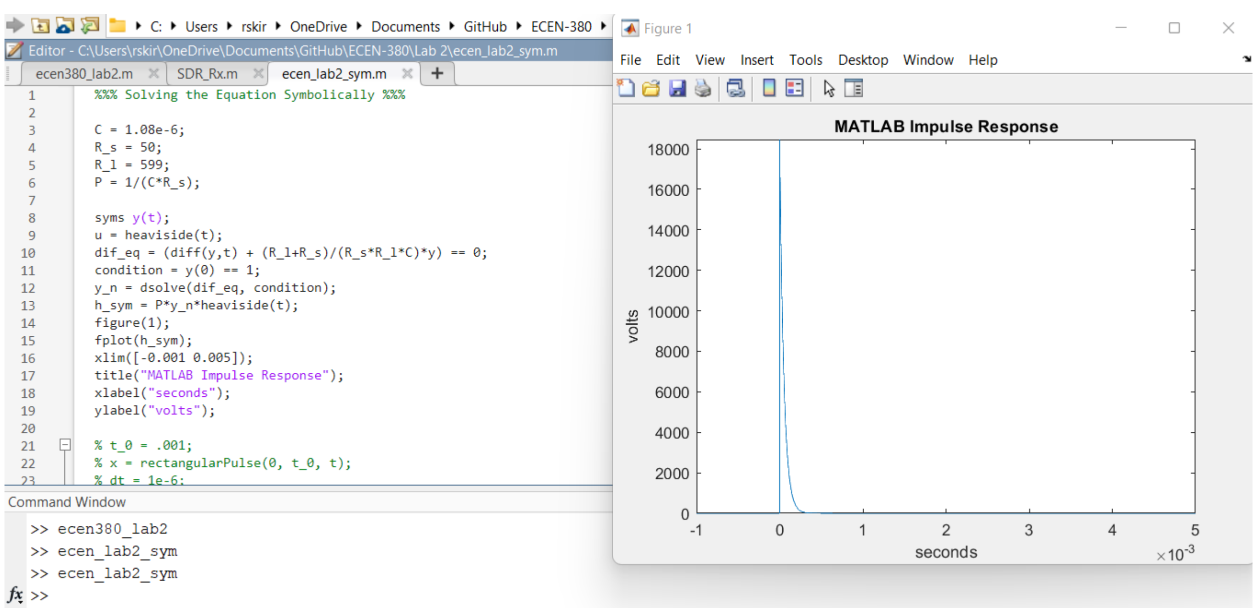
$$y_n(t) = C_1 e^{\frac{-(R_s+R_L)t}{CR_sR_L}} \qquad y'_n(t) = \frac{-(R_s+R_L)}{CR_sR_L}e^{\frac{-(R_s+R_L)}{CR_sR_L}}$$

$C_1 = 1$

$$\boxed{h(t) = \left(\frac{1}{CR_s}e^{\frac{-(R_s+R_L)t}{CR_sR_L}}\right)u(t)}$$

5)

MatLab Code & Graph:



After analyzing the circuit design, we see we have a capacitor and resistor in parallel. This is literally the definition of a RC circuit. We would expect that our results would look a lot like the natural response of a RC circuit because that is what we built. Outside noise could have impacted our results to make it slightly non-ideal, as well as the different resistor and physical capacitor values. Often the capacitors that we have are not exactly as they are rated which would slightly change our results from the natural response.

Task 2 - Measure an RCImpulse Reponse

For Task 2 we built the RC circuit on our breadboard and measured it using the oscilloscope.

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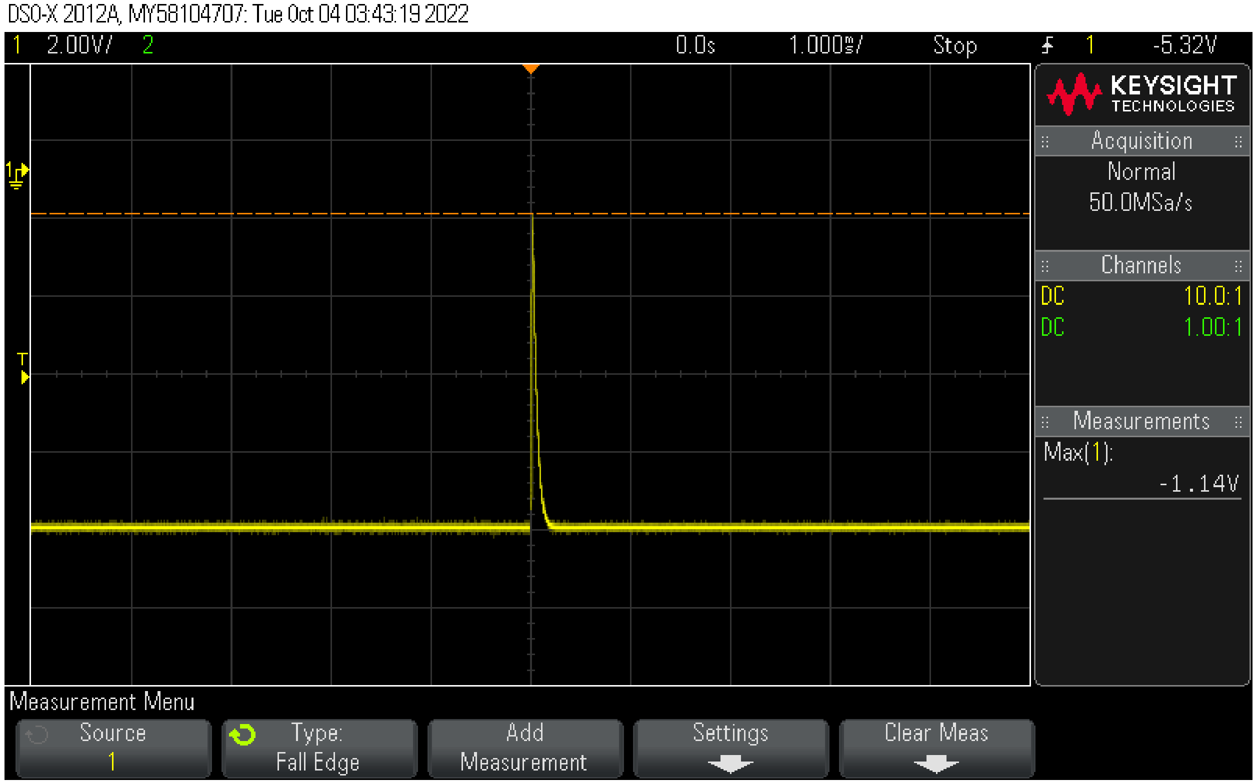


The impulse response that we measured has the same shape as the calculated impulse response. The different values are the result of non-ideal circuit components. Since we cannot expect the components to support extremely high voltages, the impulse response cannot go to near infinity. So that is partly why our measured response is so much lower.

Task 3 - RC Output and Convolution

In task 3 we calculated the convolution by hand and used matlab to do the convolution and graph the results. We also adjusted our rectangular pulse on the function generator, then measured the output using the oscilloscope.

Task 3:

$$y(t) = rect(t,t_0) * h(t) = \int_{-\infty}^{\infty} rect(\tau, t_0)\, h(t-\tau)\, d\tau = \int_{0}^{\min(t,t_0)} h(t-\tau)\, d\tau$$

$$h(t) = \left[\frac{1}{CR_s} e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)t}\right] u(t)$$

$$t > t_0: \quad \int_0^{t_0} \frac{1}{CR_s} e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)(t-\tau)} = \frac{-R_L}{R_s+R_L} e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)(t-\tau)}\Bigg|_0^{t_0}$$

$$= \frac{-R_L}{R_s+R_L}\left[e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)(t-t_0)} - e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)t}\right] u(t-t_0)$$

$$t < t_0: \quad \int_0^{t} \frac{1}{CR_s} e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)(t-\tau)} = \frac{-R_L}{R_s+R_L} e^{\left(\frac{-(R_s+R_L)}{CR_sR_L}\right)(t-\tau)}\Bigg|_0^{t}$$

$$= \frac{-R_L}{R_s+R_L}\left[1 - e^{-\frac{(R_s+R_L)}{CR_sR_L}t}\right] u(t)\, u(t-t_0)$$

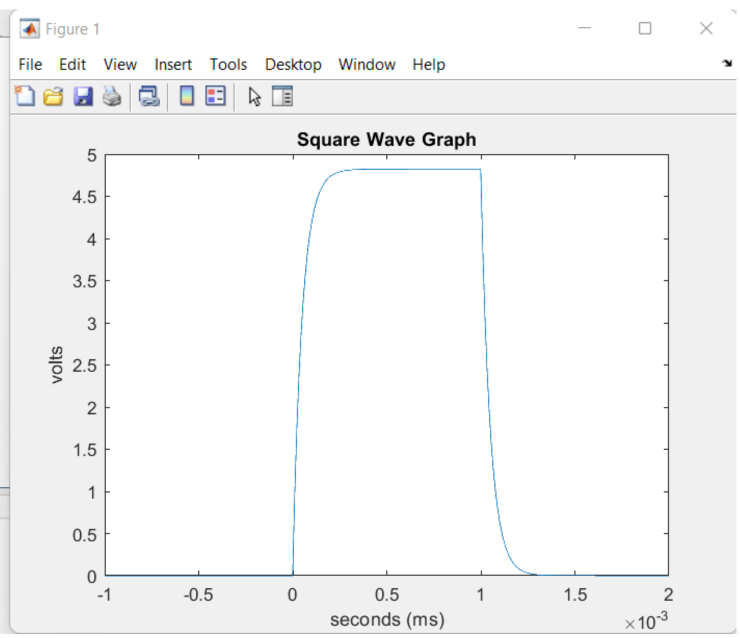
MATLAB Code:

ecen380_lab2.m | SDR_Rx.m | ecen_lab2_sym.m

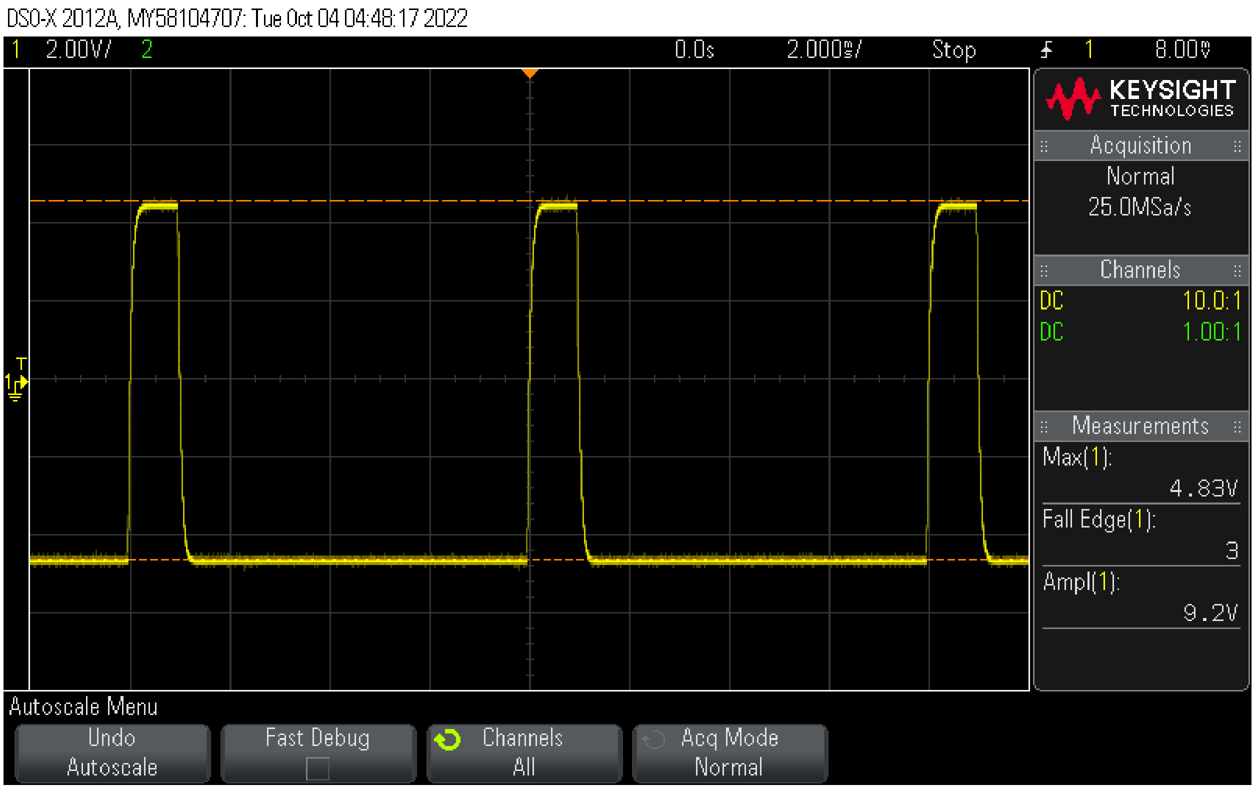
```
%%% MATLAB SCRIPT FOR LAB 2 %%%

C = 1.08e-6;
R_s = 50;
R_l = 599;
P = 1/(C*R_s);
t_step = 1e-6;
t = -1e-3:t_step:2.5e-3;

h = (1/(C*R_s))*exp(-t*(R_s+R_l)/(C*R_s*R_l));

t_0 = 0.001;
x = rectangularPulse(0, t_0, t);
y_c = 1e-14*conv(h,x);

t_c = -0.001:t_step:6e-3;
plot(t_c(1:3000), y_c(1:3000));
title("Square Wave Graph")
xlabel('seconds (ms)');
ylabel('volts');
```

Command Window

```
>> ecen380_lab2
>>
```



Oscilloscope Plot:



Conclusion:

Throughout this lab we were able to better understand how to calculate impulse response by hand. We learned that frequency response won't be infinite even though the amplitude looks like it might be when calculated in Matlab. This is due to the physical components, they are not able to support very high voltages. We also learned to calculate the convolution by hand and using matlab. Using Matlab to model the convolution was particularly insightful. We discovered that the conv() function in Matlab doesn't keep track of the time in seconds. To overcome this we had to create a separate array for time and keep track of it on our own. We also noticed some issues with the amplitude of our plot, however this will be better understood in later labs as we include and use sampling rates.