Concordia University

Department of Computer Science and Software Engineering

SOEN 331 - S and U

Introduction to Formal Methods for Software Engineering

Assignment 1

Propositional and Predicate Logic, Structures, Binary Relations, Functions and Relational Calculus

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Contents

1 General information

Date posted: Thursday 28 January, 2021.

Date due: Thursday, February 11, 2021, by 23:59.

Weight: 25% of the overall grade.

2 Introduction

This is a team assignment. Each team should designate a leader who will submit the assignment electronically. There are **9** problems in this assignment, with a total of **100** points. You must prepare all your solutions in LaTeX and produce a single `pdf` file. Please make sure you include all names and id's of all contributing team members as the authors. Name the file after your team, e.g. `team1.pdf`.

3 Ground rules

This is an assessment exercise. You may not seek any assistance while expecting to receive credit. **You must work strictly within your team and seek no assistance for this project (from the instructor, the teaching assistants, fellow classmates and other teams or external help)**. Please note that you should **not** discuss the assignment during tutorials. Failure to do so will result in penalties or no credit.

All team members are expected to work relatively equally on each problem. The team leader has the responsibility to ensure that the team does not violate this rule. Failure to do so will result in penalties. In your submission, you must include only the names of those people who contributed to the assignment. Accommodating someone who did not contribute will result in penalties.

If there is any problem in the team (such as lack of contribution, etc.), the team leader must contact the instructor as soon as the problem appears.

4 Problems

4.1 Problem 1 (8 pts)

You are shown a set of four cards placed on a table, each of which has a **number** on one side and a **symbol** on the other side. The visible faces of the cards show the numbers **2** and **7**, and the symbols $\square$, and $\bigcirc$.

Which card(s) must you turn over in order to test the truth of the proposition that *"If a card has an odd number on one side, then it has the symbol* $\square$ *on the other side"*? Explain your reasoning by deciding for <u>each</u> card whether it should be turned over and why.

4.2 Problem 2 (8 pts)

Consider the predicate *asks(a, b)* that is interpreted as *"a has asked b out on a date."*

1. Translate the following into English: $\forall\, a\, \exists\, b\;\; asks(a, b)$ and $\exists\, y\, \forall\, x\;\; asks(a, b)$.

2. Can we claim that $\forall\, a\, \exists\, b\;\; asks(a, b) \rightarrow \exists\, y\, \forall\, x\;\; asks(a, b)$? Discuss in detail.

3. Can we claim that $\exists\, y\, \forall\, x\;\; asks(a, b) \rightarrow \forall\, a\, \exists\, b\;\; asks(a, b)$? Discuss in detail.

4.3 Problem 3 (12 pts)

Let $scientist(x)$ denote the statement "x is a scientist", and $honest(x)$ denote the statement "x is honest." Formalize the following sentences and indicate their corresponding formal type.

1. "No scientists are honest."
2. "All scientists are crooked."
3. "All scientists are honest."
4. "Some scientists are crooked."
5. "Some scientists are honest."
6. "No scientist is crooked."
7. "Some scientists are not crooked."
8. "Some scientists are not honest."

Identify pairs that are contradictories, contraries, subcontraries, and pairs that support subalteration (clearly indicating superaltern and subaltern).

4.4 Problem 4 (12 pts)

Consider list $\Lambda = \langle w, x, y, z \rangle$, deployed to implement a Queue Abstract Data Type.

1. Let the head of Λ correspond to the front position of the Queue. Implement operations `enqueue(el, Λ)` and `dequeue(Λ)` using list construction operations. In both cases we can refer to Λ' as the state of the list upon successful termination of one of its operations.

2. Let us now reverse the way we manipulate our data structure and let the head of Λ correspond to the rear of the Queue.

   (a) What would be the result of $cons(el, \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

   (b) What would be the result of $list(el, \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

   (c) What would be the result of $concat(list(el), \Lambda)$, and would it be a correct implementation for operation `enqueue(el, Λ)`?

4.5 Problem 5 (12 pts)

Let $A = \{0, 1, 2, 3, 4\}$ and relations R, S, T, and U on A defined as follows:

$$R = \{(0,0),(0,1),(0,3),(1,0),(1,1),(2,2),(3,0),(3,1),(3,3),(4,0),(4,1),(4,3),(4,4)\}$$

$$S = \{(0,1),(1,1),(2,3),(2,4),(3,0),(3,4),(4,0),(4,1),(4,4)\}$$

$$T = \{(0,3),(0,4),(2,1),(3,2),(4,2),(4,3)\}$$

$$U = \{(0,0),(0,1),(0,3),(1,0),(1,1),(1,3),(2,2),(3,0),(3,1),(3,3),(4,4)\}$$

Fill in the table below, using ✓, or ×.

		R	S	T	U
Reflexive					
Irreflexive					
Symmetric					
Asymmetric					
Antisymmetric					
Transitive					
Equivalence					
Partial order					

4.6 Problem 6 (8 pts)

Consider the relation *"is a subtype of"* over the set {*rectangle*, *quadrilateral*, *square*, *parellelogram*, *rhombus*}.

1. Is this an *equivalence relation*?

2. Is this relation a *partial order*? If so, create a *Hasse diagram*, and identify *minimal* and *maximal* elements.

4.7 Problem 7 (8 pts)

Consider the set $A = \{w, x, y, z\}$, and the relations

$$S = \{(w, x), (w, y), (x, w), (x, x), (z, x)\}$$

$$T = \{(w, w), (w, y), (x, w), (x, x), (x, z), (y, w), (y, y), (y, z)\}$$

Find the following compositions:

1. $S \circ T$
2. $T \circ S$
3. $T^{-1} \circ S^{-1}$

NOTE: Some authors (e.g. Rosen) adopt a different ordering of operands than the one we use in our lecture notes. Please follow the ordering (and the definition) of the lecture notes.

4.8 Problem 8 (12 pts)

Consider sets $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{a, b, c, d, e, f\}$.

1. Determine the type of the correspondence in each of the following cases, or indicate if the correspondence is not a function.

   (a) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 3 \mapsto a\}$

   (b) $\{1 \mapsto a, 2 \mapsto d, 3 \mapsto a, 4 \mapsto f, 5 \mapsto d, 6 \mapsto c\}$

   (c) $\{1 \mapsto c, 2 \mapsto b, 3 \mapsto d, 4 \mapsto e, 5 \mapsto e, 6 \mapsto f\}$

   (d) $\{1 \mapsto b, 2 \mapsto c, 3 \mapsto e, 4 \mapsto d, 5 \mapsto f, 6 \mapsto a\}$

   Fill in the table below, using ✓, or $\times$.

	Injective	Surjective	Bijective	Neither injective nor surjective	Not a function
(a)					
(b)					
(c)					
(d)					

2. Is it possible to construct a function $f : A \to B$ which is surjective and not injective? Discuss.

4.9 Problem 9 (20 pts)

Consider the following relation:

$$laptops : Model \leftrightarrow Brand$$

where

$$
\begin{array}{l}
laptops = \\
\quad \{ \\
\quad\quad legion5 \mapsto lenovo, \\
\quad\quad macbookair \mapsto apple, \\
\quad\quad xps15 \mapsto dell, \\
\quad\quad spectre \mapsto hp, \\
\quad\quad xps13 \mapsto dell, \\
\quad\quad swift3 \mapsto acer, \\
\quad\quad macbookpro \mapsto apple, \\
\quad\quad dragonfly \mapsto hp, \\
\quad\quad envyx360 \mapsto hp \\
\quad \}
\end{array}
$$

1. What is the domain and the range of the relation?

2. What is the result of the expression

   $$\{xps15, xps13, swift3, envyx360\} \lhd laptops$$

   What is the meaning of operator $\lhd$ and where would you deploy such operator in the context of a database management system?

3. What is the result of the expression

   $$laptops \rhd \{lenovo, hp\}$$

   What is the meaning of operator $\rhd$ and where would you deploy such operator in the context of a database management system?

4. What is the result of the expression

$$\{legion5, xps15, xps13, dragonfly\} \lhd laptops$$

   What is the meaning of operator $\lhd$ and where would you deploy such operator in the context of a database management system?

5. What is the result of the expression

$$laptops \rhd \{apple, dell, hp\}$$

   What is the meaning of operator $\rhd$ and where would you deploy such operator in the context of a database management system?

6. Consider the following expression

$$laptop \oplus \{ideapad \mapsto lenovo\}$$

   (a) What is the result of the expression?

   (b) What is the meaning of operator $\oplus$ and where would you deploy such operator in the context of a database management system?

   (c) Does the result of the expression have a permanent effect on the database (relation)? If not, describe in detail how would you ensure a permanent effect.

5 What to submit

Please submit your `pdf` file at the Electronic Assignment Submission portal

(`https://fis.encs.concordia.ca/eas`)

under **Theory Assignment 1**.

END OF ASSIGNMENT.
