

Formale Semantik

o8. Intensionalität

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

1 Intensionality

- Problems with extensionality and non-dimensional models
- Intensions

2 A formal account of intensions

- Sets of PSOAs
- Intensions as functions
- Repeat after me...

3 Sets of worlds

- Known relations
- Modal operators

4 Intensional Model Theory

- Ingredients of models
- Evaluating individual constants
- Set membership
- Some peculiarities of \Box and \Diamond

Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Intensionality

Some examples

- Stockhausen **will** write another opera.
- **Had** Arno Schmidt cut down on drinking, he **would** still be alive.
- Gustave Moreau **believes that** estheticism rules.

Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (**tense**)
- ... and for just one state of affairs (**modals**, **believe type verbs**)

What are intensions?

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts or propositions

- can't be just truth conditional
- encode knowledge about not just the actual but all possible and/or past/future states of affairs (PSOAs)
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values

PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just truth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSA logic

A touch of parallel universes?

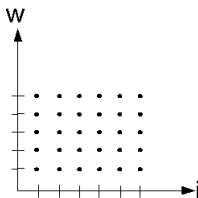
- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
 - ▶ A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
 - ▶ nothing of the above, but Maria was born 20 years earlier
 - ▶ nothing of the above, but A.S. rose from the dead in 2003, etc.

A formal account of intensions

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
 - ...the SOAs under which its corresponding sentence is true
 - ...the subset under which its corresponding sentence is false

Coordinates

- for each possible distinction in truth values of the whole of the propositional sentences: **one possible world** ($w \in W$)
- for each point in time: **one possible temporal state of each world** (instant $i \in I$)
- representation of **temporally ordered world-time coordinates** $\langle w, i \rangle \in W \times I$

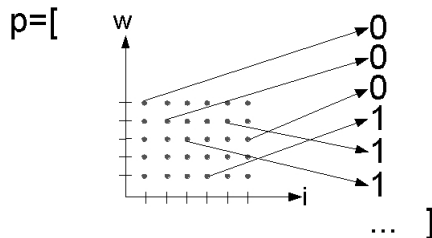


The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

Propositions as functions

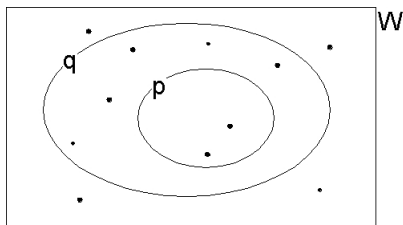
- a propositional function p
- is a function from $W \times I$ to $\{0, 1\}$



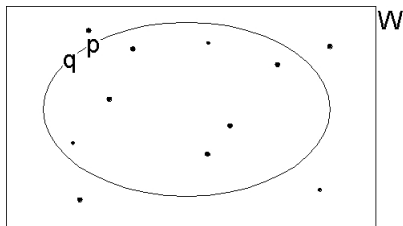
- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

Sets of worlds

- definition of intensions of sentences (propositions): characteristic functions
- **equivalently: propositions are sets of possible worlds**
- **entailment** turns out as a **subset-relation**: $p \subseteq q$:

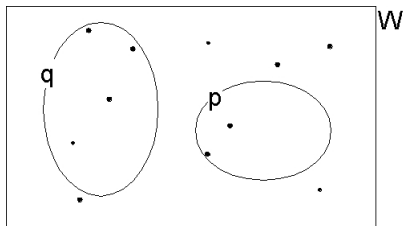


- synonymy turns out as set equivalence:
- $p = q$



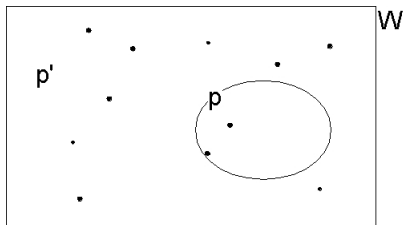
Contradiction

- **contradiction** turns out as an **empty intersection**:
- $p \cap q = \emptyset$



Negation

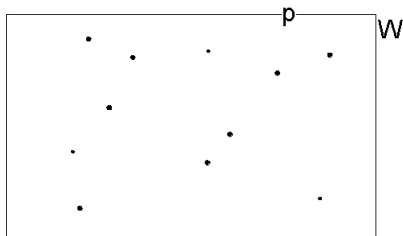
- **negation** turns out as a **complement**:
- p/W



- new **modal** sentence/wff operators:
 - ▶ *necessarily* p : $\Box p$
 - ▶ *possibly* p : $\Diamond p$
- What does it mean for a proposition to be necessary/possible?

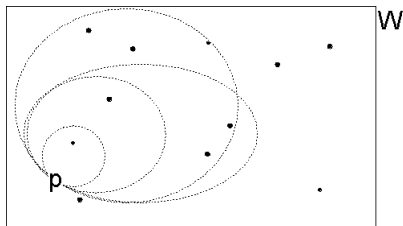
Necessity as universal quantification

- if $\Box p$ then $(\forall w) [p(w) = 1]$ (p as characteristic function)
- such that $W = p$ (p as set):



Possibility as existential quantification

- if $\Diamond p$ then $(\exists w) [p(w) = 1]$ (characteristic function)
- such that $p \neq \emptyset$ (set):



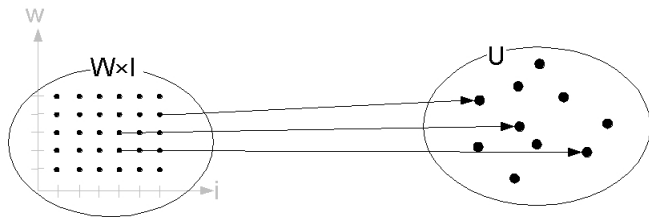
Intensional Model Theory

A larger tuple

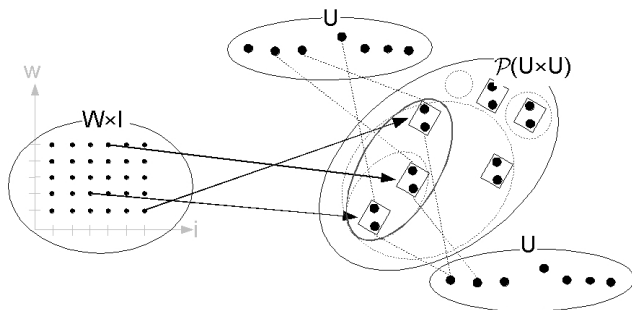
- $\mathcal{M} = \{W, I, <, U, V\}$
 - ▶ W , a set of worlds
 - ▶ I , a set of instants
 - ▶ $<$, an ordering relation in I
 - ▶ U , the set of individuals
 - ▶ V , a valuation function for constants
- evaluate an expression α : $\llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$

Intensional interpretation of individual constants

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)
- for $\beta \in \text{Cons}_{\text{ind}}$, $V(\beta)$ is a function from $W \times I$ to U



- *walks* etc. denotes different sets (or CFs) at different $\langle w, i \rangle$ coordinates
- for $\beta \in \text{Cons}_{\text{pred}_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ ($U^n = U_1 \times U_2 \times \dots \times U_n$)



The Chierchia approach: predicates/sentences

- simple sentences/predicates: $\beta = \delta(t_1, t_2, \dots, t_n)$
- $\llbracket \beta \rrbracket^{\mathcal{M}, w, i, g} = 1$ iff
- $\langle \llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g}, \llbracket t_2 \rrbracket^{\mathcal{M}, w, i, g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w, i, g} \rangle \in \llbracket \delta \rrbracket^{\mathcal{M}, w, i, g}$
- with: $\llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

- if $\psi = \forall x\phi$ then
- $\dots \llbracket \psi \rrbracket^{\mathcal{M}, w, i, g} = 1$ iff for all $u \in U$
- $\dots \llbracket \phi \rrbracket^{\mathcal{M}, w, i, g[u/x]} = 1$
- nothing new here

- if $\psi = \Box x \phi$ then
- $\dots \llbracket \psi \rrbracket^{\mathcal{M}, w, i, g} = 1$ iff for all $w' \in W$
- ...and all $i' \in I$
- $\dots \llbracket \phi \rrbracket^{\mathcal{M}, w', i', g} = 1$

A similarity of \forall and \Box

- as: $\forall x [P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that: $\Box [\psi \rightarrow \phi] \rightarrow [\Box \psi \rightarrow \Box \phi]$
- **but not vice-versa!**

Some validities

- $\exists x \Box P(x) \rightarrow \Box \exists x P(x)$
- $\exists x \Diamond P(x) \leftrightarrow \Diamond \exists x P(x)$
- $\forall x \Box P(x) \leftrightarrow \Box \forall x P(x)$ (Carnap-Barcan)
- $\forall x \Diamond P(x) \rightarrow \Diamond \forall x P(x)$

Kontakt

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