

# Formale Semantik

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!  
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

# Inhalt

## 1 Inferenz und Bedeutung

- Organisation
- Schlussfolgern
- Grundfragen
- Programmatisches Schlussbild

## 2 Referentielle Semantik

- Linguistic theories
  - Semiotics
  - Generative Grammar
  - Levels of representation
- A referential framework
  - The simple case
  - Complex cases
- Some fundamental semantic notions
  - Entailment
  - Presupposition
  - Ambiguity, Synonymy, Vagueness, ...
- From reference to sense
  - Referential and non-referential NPs
  - A 'reference' for complex terms?
  - Sentences refer to 0 and 1
  - Sense and reference
- We're talking in fragments: F1
  - A syntax
  - The semantics: individuals, sets, functions, T-sentences
  - Bottom-up evaluation

## 3 Mengen und Funktionen

- Sets and Functions
  - The naive concept
  - Elements, subsets, power sets
  - Union, intersection, etc.
- Functions and Relations
  - Ordered pairs/sets, n-tuples, Cartesian products
  - Relations

## 4 Trees

- C-command
- Model theory
  - Models and valuations
  - Assignment functions
  - Modified assignment functions
- Problems with natural language
  - Restricted quantification
  - Variable binding and scope
  - Pre-spellout movement
  - LF movement
- Quantification in English: F2
  - Movement rules
  - Fragment F2

## 7 Einfach getypete höherstufige $\lambda$ -Sprachen

- Preliminaries
  - Different but related semantics
  - Sets and characteristic functions
  - Functional application
- Simply typed languages
  - New names for old categories
  - The syntax of types
  - Higher orders
  - Summed up semantics for a higher-order language
- Lambda languages
  - From set constructor to the functional  $\lambda$  abstractor
  - General syntax/semantics for  $\lambda$  languages
  - A glimpse at quantification in Montague's system

## 8 Intensionalität

- Intensionality
  - Problems with extensionality and non-dimensional models
  - Intensions
- A formal account of intensions

## Inferenz und Bedeutung

- Chierchia & McConnell-Ginet (2000) | GB-orientiert, nur die Kapitel von Chierchia
- Partee u.a. (1990) | wichtige Grundlagen (Algebra, Logik), viele Druckfehler

- [Bucher \(1998\)](#) | lesbare Logik-Einführung auf Deutsch
- [Dowty u. a. \(1981\)](#) | tolle Montague-Einführung von seinen Schülern
- [Carpenter \(1997\)](#) | prima Hardcore-Einführung mit Kategorialgrammatik
- [Gutzmann \(2019\)](#) | aktuelle Einführung auf Deutsch

# Seminarverlauf

- 1 9.10.2023 Diskussion: Wie schlussfolgern wir? Wie hängen unser Schlussfolgerungen mit Semantik zusammen?
- 2 26.10.2023 Referentielle Semantik (Folien 2)
- 3 02.11.2023 Mengen- und Funktionstheorie (Folien 3)
- 4 09.11.2023 Aussagenlogik (Folien 4)
- 16.11.2023 Ausfall wegen Dienstreise
- 5 23.11.2023 Prädikatenlogik (Folien 5)
- 6 30.11.2023 Quantifikation und modelltheoretische Semantik (Folien 6)
- 7 07.12.2023 Einfach getypte höherstufige  $\lambda$ -Sprachen (Folien 7)
- 8 14.12.2023 Intensionalität (Folien 8)
- 9 21.12.2023 Tempus und Modalität (Folien 9)
- 28.12.2023 Weihnachtsferien
- 04.01.2024 Weihnachtsferien
- 10 11.01.2024 Montagues intensionale Logik (Folien 10)
- 11 18.01.2024 *The Proper Treatment of Quantification in Ordinary English* (Montague 1973)
- 12 25.01.2024 *Generalized Quantifiers and Natural Language* (Barwise & Cooper 1981)
- 13 01.02.2024 *The Algebra of Events* (Bach 1986)
- 08.02.2024 Klausurenwoche/Einzelbesprechungen

Einheitlicher Inhalt für alle Modul- und Examensprüfungen:

- 1** eine oder zwei inhaltlichen Fragen zu den Themen der *Sprachphilosophie*

Die Liste der relevanten Texte wird rechtzeitig vor den Prüfungen eingeschränkt.

- 2** eine Logik-Aufgabe (natürliche Deduktion) – außer in mündlichen Prüfungen
- 3** eine Semantik-Aufgabe (kompositionale Modellierung eines Satzes)

Hausarbeiten nach Absprache.

# Was folgt aus A?

# Folgt B aus A? | Einfache Fälle

# Folgt B aus A? | Schwierigere Fälle

# Folgt B aus A? | Grammatik

# Woher wissen Sie das?

# Weltwissen vs. Logik









Die Bedeutung eines Ausdrucks ist ...

- ... die Idee, die er vermittelt
- ... die mentale Repräsentation, die er erzeugt
- ... was mit ihm bewirkt werden soll
- ... die Menge der Dinge, auf die er verweist

Semantik untersucht ...

- ... intellektuelle Konzepte, die überwiegend introspektiv erforschbar sind
- ... die kognitive Verarbeitung und Repräsentation von Bedeutung
- ... die Funktion von Ausdrücken in Kommunikationssituationen
- ... Beziehungen zwischen Ausdrücken und Objekten und  
die Art der Kombination von Ausdrücken zur komplexeren Ausdrücken

Es dreht sich alles um die Beziehung von Sprache zur Welt!

- Auf welche Klassen von Objekten referieren auf welche Klassen von Ausdrücken?
- Wann sind Sätze wahr? (auch als Phänomen der Referenz!)
- Wie verhält sich die logische Struktur von Sätzen zu ihrem Informationsgehalt?
- Wie können Sätze eindeutig interpretiert werden,  
auch wenn sie mehrere Lesarten haben?

- Was ist die „Bedeutung“ von Wörtern und Sätzen jenseits ihrer Referenz?
- Wie verarbeitet das Gehirn Bedeutungen?
- Wie sind Diskurse strukturiert?

Sind Sie nun kognitiver Linguist,  
der sich für (probabilistische) **mentale Kategorien** interessiert,  
oder glauben Sie daran,  
dass Sprache **unabhängig vom Menschen logische Eigenschaften** hat?

Beides gleichzeitig geht ja nun wirklich nicht!

## Kognition

- basierend auf Ähnlichkeiten von wahrgenommenen Objekten
- optimiert für schnelle Mustererkennung **in allen Bereichen**
- unscharfe Klassenbildung und Segmentierung der Ontologie
- parallele Verarbeitung (meistens mehrere Areale beteiligt)

## Symbolische Systeme

- diskrete Symbole, wohldefinierte Semantik
- scharf getrennte Klassen von Symbolen
- eindeutige Referenz auf ontologische Objekte
- intrinsische (nicht emergente) logische Eigenschaften  
(Axiomatik, Schlussregeln usw.)
- sequentielle Verarbeitung/statische Deklaration  
(z. B. Python oder PROLOG; parallele Verarbeitung immer linearisierbar)

Klassisches kognitives Modell: **Prototypentheorie** (Rosch 1973)

Diskretes Symbol: **Vogel** ... und demgegenüber ...

Graduelles kognitives Konzept basierend auf Ähnlichkeiten/Prototypen:



&gt;



&gt;



Die ewige Schwachsinnfrage: Sind Kiwis und Pinguine nun **Vögel** oder nicht?

Nur getoppt von: Erdbeeren sind gar keine Beeren, sondern Sammelnussfrüchte.

- Kognition | **intrinsisch nicht diskret**, sondern ähnlichkeitsbasiert und **parallel**
  - ▶ Netzwerkarchitektur
- Symbole = Phone, Morphe, Wörter, Phrasen, ... | **intrinsisch** diskret und **linear**
  - ▶ **akustisches** Medium | Sagen Sie mal zwei Wörter gleichzeitig!
  - ▶ **schriftliches** Medium | Lesen Sie mal Zettels Traum!
- Da wir nur akustisch oder über schriftliche Artefakte kommunizieren können, **muss das Sprachsystem symbolisch sein**.
- Da es architekturbedingt nur nicht-symbolisch verarbeiten kann, **muss das Gehirn symbolische Systeme so gut wie nötig und möglich emulieren**.

Auch nicht-verschriftete Sprache muss medial bedingt logische Eigenschaften haben.  
Kulturell bilden sich stärker symbolische Modi aus, vor allem durch Schrift.

Norm, Selbst- und Fremdkorrektur, Textplanung, intensionale Definitionen, Explizierung, ...

Warum wird das vor allem im Kontext von Schule, Fremdsprache und Bildungssprache diskutiert?

(= spontane Sprachproduktion)

weniger symbolische Eigenschaften



mehr symbolische Eigenschaften

(= reflektierte Sprachproduktion)

informelle Alltagssprache

formelle Alltagssprache

Bildungssprache

Wissenschaftssprache

Orthosprache

formales System

# Und was ist denn nun mit Kiwis und Pinguinen?

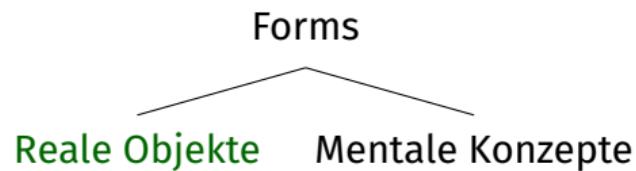
Unser Verständnis der Welt führt zu genaueren und diskreten Kategorisierungen, wo dies nötig ist. Die Sprache folgt diesem Maß an Genauigkeit und Diskretheit!



- Viele Missverständnisse in der Linguistik basieren darauf, dass das eben Gesagte nicht dem allgemeinen Forschungsprogramm zugrundeliegt.
- Die Doppelnatur von Sprache führt dazu, dass sowohl rein formale Linguistik und sogenannte kognitive Linguistik scheinbar erfolgreich sind.
- Im Prinzip läuft aber die Linguistik aktuell weitgehend ins Leere.
- Modelltheoretische Semantik beschreibt einen essentiellen Teil von Sprache!
- Sie modelliert logische Eigenschaften und den Bezug zur realen objektiven Welt.
- Ganz am Rande zu generativer AI ...
  - ▶ Erfolg | Sie modelliert völlig natürliche Grammatik.
  - ▶ ... also alle Grammatiker bitte setzen!
  - ▶ Misserfolg | Sie weiß nichts über die Welt, es wirkt nur so wegen des immensen sprachlichen Inputs.
  - ▶ ... eine Art fancy Papagei.

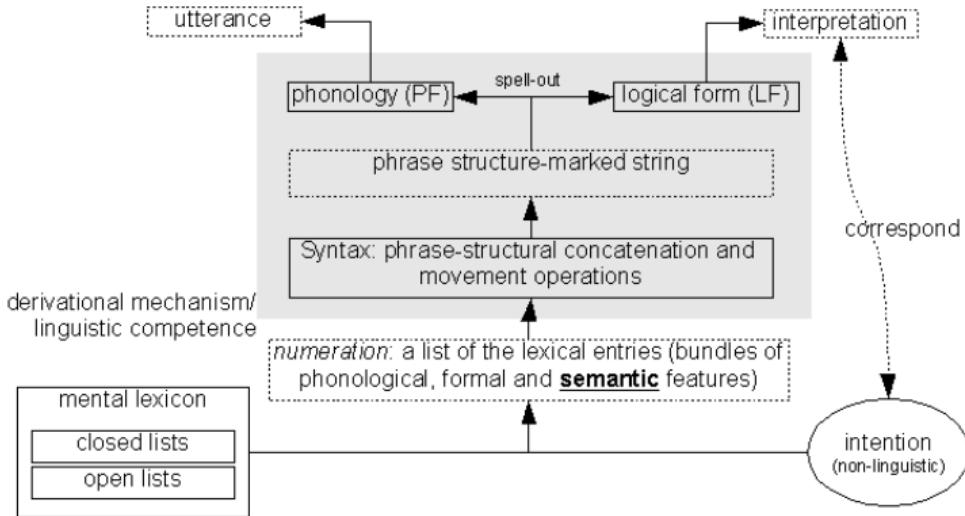
## Referentielle Semantik

# Ein neues semiotisches Dreieck



# The theory of signs: a triangle

# Semantics in the Chomskian T-model



# LF is just the logical form

- No interpretation proper at LF.
- Movement transformations after the sentence has been uttered.
- At the LF level, sentences have a form compatible to their logic.
- Why? Syntax itself is often inadequate to express all alternatives of a sentence's logical representation.

# Some properties of language

- aboutness
- referential nature
- informative
- objectiveness (of content)
- But which linguistic elements refer to what?

an individual name → one object in the world

*Harald Schmidt*



# Common nouns

a common noun → lots of objects

*soldier*



etc.

# Adjectives

an adjective → lots of different objects of different kinds

*is human*



# Sentences

a sentence



a situation. a fact, ...



*A humming bird  
is hovering over  
a red flower.*

not at all  
(object type mismatch)



# Frege's Principle: Meaning is compositional

- *A humming bird* → one of many individuals
- *is hovering* → a property of that individual
- *over* → a relation between individuals
- *a red* → a property of another individual
- *flower* → the other one of many individuals
- *is hovering over a red flower* → a complex property.

# Recursion: infinite use of finite means

- Frege's principle is indispensable!
- *Harald Schmidt is human.*
- *Harald Schmidt is human and tall.*
- *Harald Schmidt is human and tall and male.*
- *Harald Schmidt is human and tall and male and not blue.*
- *Harald Schmidt is human and tall and male and not blue and grumpy in the morning...*

# Basic semantics judgements

- entailment
- presupposition
- ambiguity
- synonymy

- A: *This is electronic.*
- B: *This is a presentation.*
- C follows logically: *This is an electronic presentation.*
- $A, B \vdash C$
- $A \not\vdash C$
- $B \not\vdash C$

# Entailment: pure logic, formally

- $D$ : *Harald Schmidt is human.*
- $E$  follows logically: *Something is human.*
- $D \vdash E$
- $D \wedge D$  follows logically: *Harald Schmidt is human and Harald Schmidt is human.*
- $D \vdash D \wedge D$

## Tests: X entails Y if...

- When X is true, Y is true.
- A situation described by Y is also described by X.
- The information given by Y is fully contained in the information given by X.
- One cannot say X is true and Y is false.

# Entailments?

- *Harald Schmidt is a talkmaster.* → *Harald Schmidt is human.*
- *Harald Schmidt is tall.* → *Someone is tall.*
- *Some humans are tall.* → *Harald Schmidt is tall.*
- *I have listened to Paul Kalkbrenner's new 12" on bpitchcontrol.* → *Paul Kalkbrenner has released a 12" on bpitchcontrol.*
- *After I had a Beck's, I installed RedHat on my PC.* → *I had a Beck's.*
- *After the bootloader had failed to boot RedHat on my PC, I had another Beck's.* → *RedHat has never booted on my PC.*
- *My flatmate likes Beck's.* → *My flatmate hates beer.*
- *Harald Schmidt cancelled his show.* → *Harald Schmidt's show was cancelled.*

# Presupposition: the background

- A: *Willy Brandt is the current chancelor of the FRG.*
- B: *If Willy Brandt is the current chancelor of the FRG, why doesn't he do something?*
- C: *Willy Brandt is not the current chancelor of the FRG.*
- A and B presuppose D: *Willy Brandt is alive.*, C doesn't.
- A, B, and C presuppose E: *There is a chancelor of the FRG.*
- Note:  $A \vdash D$ ,  $A \vdash E$
- But:  $B \not\vdash D$ ,  $B \not\vdash E$ ,  $C \not\vdash E$

## Presupposition: two tests

- Presuppositions are triggered by all sorts of sentences (incl. negations, modals, conditionals, etc.).
- Presuppositions can be negated while the sentence which presupposes them remains true. Entailments cannot be negated while keeping the entailing sentence true.

# Ambiguity in syntax

- *She saw the man with a telescope.*
- She [saw the man] with a telescope.
- She saw [the man with a telescope].

# Ambiguity in semantics: scope

- *Everybody loves somebody.*
- Every person loves at least one other person.  
(Needn't be the same.)
- There is one person loved by everyone

# Synonymy

- Lexical synonymy: *humming bird*  $\overset{\text{lex}}{\equiv}$  *colibri*
- Compositionally (**equivalence**): *Mulder met his abducted sister after he broke into the secret army base.*  $\equiv$  *Before meeting his abducted sister, Mulder broke into the secret army base.*
- A  $\equiv$  B iff A  $\vdash$  B and B  $\vdash$  A

# Noun-like expressions and complex NPs

- I saw a man.
- I saw the green wobbly thing crawling near.
- I saw it.

- *The dark subatomic particles in the universe have a total mass much larger than the visible subatomic particles.*
- *Problems with referential semantic theories don't concern Rumpletweezer.*
- and of course, vagueness (e.g., Sorites Paradox)

# Problems with non-referential NPs

- *some guy*
- *not the faintest trace of blood*
- *any axiom of Zermelo-Fraenkel set theory*

# Beyond pointin-at-and-naming

We need a logic to explain for effects like:

- my humming bird's favorite flower    is red
- └ some flower                         is red

# Some content-synonymous simple expressions

- a: *colibri*
- b: *humming bird*
- c: *a brunette lady*
- d: *a brown-haired dame*
- e: *the primates*
- f: *the apes and humans*
- $a \stackrel{\text{lex}}{\equiv} b, c \stackrel{\text{lex}}{\equiv} d, e \stackrel{\text{lex}}{\equiv} f$

# Some content-synonymous complex expressions

- A: *A colibri is hovering over a red flower.*
- B: *A humming bird is hovering over a red flower.*
- C: Lauren Bacall was *a brunette lady*
- D: Lauren Bacall was *a brown-haired dame*
- E: *Primates* are intelligent.
- F: *The apes and humans* are intelligent.
- A ≡ B, C ≡ D, E ≡ F

## Two axioms

- **Ax1** Two expressions (e.g., NPs, sentences) that are synonymous have the same reference.
- Formally:  $A \equiv B$  then  $\llbracket A \rrbracket = \llbracket B \rrbracket$
- Note:  $\llbracket A \rrbracket$  is applicable to simplex and complex expressions A; it just produces the reference of A.
- **Ax2** If we replace expression B within expression A with the synonymous expression C, then A does not change its reference.
- Formally: If  $\llbracket B \rrbracket = \llbracket C \rrbracket$  then  $\llbracket \llbracket_A B \rrbracket \rrbracket = \llbracket \llbracket_A C \rrbracket \rrbracket$

## One common property of sentences: the truth value

- A: *Lauren Bacall was a brunette lady.* (assumed to be true in the actual world)
- B: *My cat sleeps quietly.* (assumed to be true in the actual world)

# First conclusion

- $[_{\mathbb{T}} A] = \text{The truth value of 'Lauren Bacall was a brunette lady' is } 1.$
- $[_{\mathbb{T}} B] = \text{The truth value of 'My cat sleeps quietly' is } 1.$
- Such that  $A \equiv [_{\mathbb{T}} A]$  and  $B \equiv [_{\mathbb{T}} B]$ .  
(Check: Whenever A is true,  $[_{\mathbb{T}} A]$  is true and v.v.)
- So, by Ax1  $\llbracket A \rrbracket = \llbracket [_{\mathbb{T}} A] \rrbracket$   
and  $\llbracket B \rrbracket = \llbracket [_{\mathbb{T}} B] \rrbracket$

## Second conclusion

- Check the denotations of the contained NPs:  
 $\llbracket \text{the truth value of } A \rrbracket = \llbracket \text{the truth value of } B \rrbracket = 1$
- Such that by Ax2:  
 $\llbracket [T A] \rrbracket = \llbracket [T B] \rrbracket$
- Why? Exchanging the referentially identical NPs ‘the truth value of A’ and ‘the truth value of B’ in the otherwise identical sentences ‘\_ is 1’ forces us to conclude by Ax2 that also the whole sentences must have the same reference. Our book (CM) is a bit vague on that point.

$$\llbracket A \rrbracket = \llbracket [\tau A] \rrbracket = \llbracket [\tau B] \rrbracket = \llbracket B \rrbracket = 1$$

Sentences denote truth values.

# Advantages of truth values

- indirect encoding of ‘richer’ semantics (One must know the truth conditions of a sentence and the state of affairs to decide about the truth of a sentence.)
- a minimal common semantic property of sentences
- easily computable in a formal system (binary)
- their logic provides a basis for ‘richer’ semantics (cf. second half of class)

# Frege also thought, reference couldn't be all

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts

# Some terminology

- *reference* = *extension* = what we're dealing with first
- *sense* = *intension* = what we will be dealing with later
- *proposition* = the intensions of sentences as informational content: The ‘thought that S’.

# Decomposing compositionality and composing truth

- How are sentences compositionally built up?
- What do their parts denote?
- How does the denotation of the parts contribute to the whole.
- T-sentences: **S of L is true in v iff p.**
- *S* a sentence, *L* a language, *v* a state of affairs, *p* a statement of the truth conditions.

# A phrase-structure grammar

- $S \rightarrow N \text{ VP}$
- $S \rightarrow S \text{ conj } S$
- $S \rightarrow \text{neg } S$
- $\text{VP} \rightarrow V_i$
- $\text{VP} \rightarrow V_t \text{ N}$

- N → *Herr Webelhuth, Frau Eckardt, the Turm-Mensa*
- V<sub>i</sub> → *is relaxed, is creative, is stupid*
- V<sub>t</sub> → *prefers*
- conj → *and, or*
- neg → *it is not the case that*

# Simple denotiations

- $\llbracket \text{Herr Webelhuth} \rrbracket = \text{Herr Webelhuth}$
- $\llbracket \text{Frau Eckardt} \rrbracket = \text{Frau Eckardt}$
- $\llbracket \text{the Turm-Mensa} \rrbracket = \text{the Turm-Mensa}$
- $\llbracket \text{is relaxed} \rrbracket = \{x : x \text{ is relaxed}\}$
- $\llbracket \text{is creative} \rrbracket = \{x : x \text{ is creative}\}$
- $\llbracket \text{is stupid} \rrbracket = \{x : x \text{ is stupid}\}$
- $\llbracket \text{prefers} \rrbracket = \{\langle x, y \rangle : x \text{ prefers } y\}$

# Some words don't really 'denote', they act like functions

- $\llbracket \text{neg} \rrbracket = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$
- $\llbracket \text{and} \rrbracket = \begin{bmatrix} \langle 1, 1 \rangle \rightarrow 1 \\ \langle 1, 0 \rangle \rightarrow 0 \\ \langle 0, 1 \rangle \rightarrow 0 \\ \langle 0, 0 \rangle \rightarrow 0 \end{bmatrix}$
- $\llbracket \text{or} \rrbracket = \begin{bmatrix} \langle 1, 1 \rangle \rightarrow 1 \\ \langle 1, 0 \rangle \rightarrow 1 \\ \langle 0, 1 \rangle \rightarrow 1 \\ \langle 0, 0 \rangle \rightarrow 0 \end{bmatrix}$

## T-sentences: rule-to-rule

- $\llbracket [S \ N \ VP] \rrbracket = 1$  iff  $\llbracket N \rrbracket \in \llbracket VP \rrbracket$ , else 0
- $\llbracket [S \ S_1 \ \text{conj} \ S_2] \rrbracket = \llbracket \text{conj} \rrbracket (\langle \llbracket S_1 \rrbracket, \llbracket S_2 \rrbracket \rangle)$
- $\llbracket [S \ \text{neg} \ S] \rrbracket = \llbracket \text{neg} \rrbracket (\llbracket S \rrbracket)$
- $\llbracket [VP \ V_t \ N] \rrbracket = \{x: \langle x, \llbracket N \rrbracket \rangle \in \llbracket V_t \rrbracket\}$
- semantics for non-branching nodes: pass-up

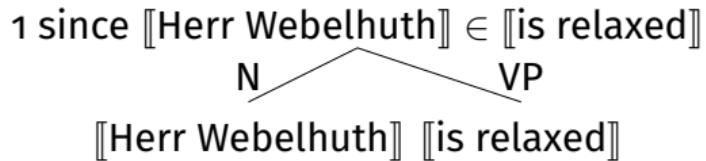
# A starting point for our computation

*Herr Webelhuth is relaxed.*

- Circumstances (Model): Herr Webelhuth is an element of the set of relaxed individuals.
- (1) The syntax is well-formed by  $S \rightarrow N VP$
- (2) for  $N$ :  $\llbracket \text{Herr Webelhuth} \rrbracket = \text{Herr Webelhuth}$
- (3) for  $VP$ :  $\llbracket \text{is relaxed} \rrbracket = \{x: x \text{ is relaxed}\}$
- (4) for  $S$ :  $\llbracket [S N VP] \rrbracket = 1 \text{ iff } \llbracket N \rrbracket \in \llbracket VP \rrbracket, \text{ else } 0$

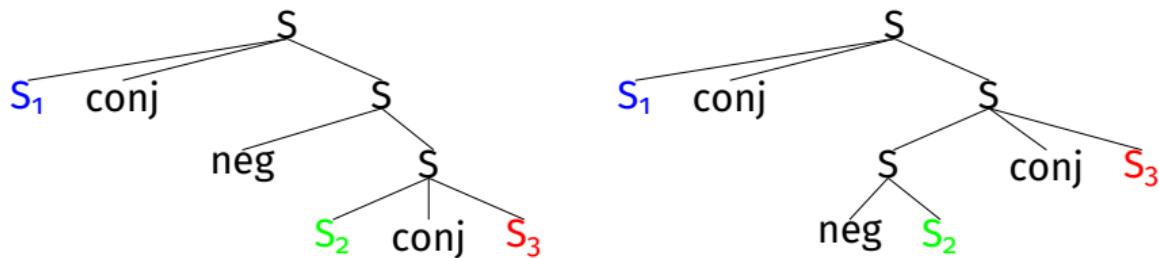
# A starting point for our computation

The tree:



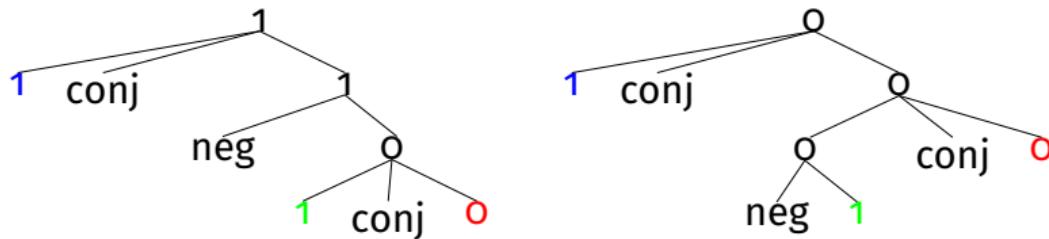
# We compute syntactic representations, not flat sentences

$(S_1 \text{ Frau Eckardt is creative})$  and it is not the case that  $(S_2 \text{ Herr Webehlhuth is relaxed})$  and  $(S_3 \text{ Frau Eckardt prefers the Turm-Mensa})$ .



# A starting point for our computation

Circumstances: Herr Webelhuth is relaxed, Frau Eckardt is creative, and Frau Eckardt does not prefer the Turm-Mensa:



## Mengen und Funktionen

# What is a set?

- a freely defined unordered collection of discrete objects
  - ▶ numbers,
  - ▶ people,
  - ▶ pairs of shoes,
  - ▶ words, ...
- not necessarily for any purpose
- no object occurs more than once

- $M_1 = \{a, b, c\}$
- $N_1 = \{\text{'my book'}$   
vs.  $N_2 = \{\text{my book}\}$   
vs.  $N_3 = \{\text{'my', 'book'}$
- ill-formed:  $N_4 = \{\text{'my', book}\}$
- defined by a property of its members:  
 $M_2 = \{x : x \text{ is one of the first three letters of the alphabet}\}$
- alternatively:  
 $M_2 = \{x \mid x \text{ is one of the first three letters of the alphabet}\}$
- **U**: the universal set (contains every discrete object)

# Equality: =

- Two sets with contain exactly the same members are *equal*.
- independent of definition:  
 $\{a,b,c\} = \{x : x \text{ is one of the first three letters of the alphabet}\}$
- $\{x : x \text{ is human}\} = \{x : x \text{ is from the planet earth and } x \text{ can speak}\}$

## Subsets: $\subseteq$

- A set N which holds no member which is not in M is a *subset* of M:  $N \subseteq M$
- $\{a\} \subseteq \{a, b, c\}$
- the inverse: the **superset**

## Proper subsets: $\subset$

- A set  $N$  which holds no member which is not in  $M$  and which is not equal to  $M$  is a *proper subset* of  $M$ :  $N \subset M$
- So, actually:  $\{a\} \subset \{a, b, c\}$  and  $\{a, b, c\} \subseteq \{a, b, c\}$ . Note that:
  - $M \subseteq M$  but  $M \not\subset M$
  - $\{\{a\}\} \not\subseteq \{a, b, c\}$
  - $\{\} \subset \{a, b, c\}$  (or any set),  $\{\}$  is sometimes written  $\emptyset$

- All professors of English Linguistics are human.  
*Herr Webelhuth is a professor of English Linguistics.*
- $w = \text{Herr Webelhuth}$   
 $E = \text{the set of professors of English Linguistics}$   
 $H = \text{the set of human beings}$
- $w \in E \ \& \ E \subset H \Rightarrow w \in H$

- But: *Professors of English Linguistics are numerous.*
- $N$  = the set of sets with numerous members
- $w \in E \& E \in N \not\Rightarrow w \in P$
- Hence: \*Herr Webelhuth is numerous.

## Power sets: $\wp$

- For any set  $M$ :  $\wp(M) = \{X \mid X \subseteq M\}$
- for  $M = \{a, b, c\}$ :  
$$\wp(M) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$$
- Why is the empty set in the power set of every set ...
- ...and why is the empty set a proper subset of every set?

# Union $\cup$ and intersection $\cap$

- For any sets  $M$  and  $N$ :  $M \cup N = \{x \mid x \in M \text{ or } x \in N\}$
- if  $M = \{a, b, c\}$  and  $N = \{a, b, d\}$  then  $M \cup N = \{a, b, c, d\}$
- For any sets  $M$  and  $N$ :  $M \cap N = \{x \mid x \in M \text{ and } x \in N\}$
- if  $M = \{a, b, c\}$  and  $N = \{a, b\}$  then  $M \cap N = \{a, b\}$
- as a general principle (Consistency):  $M \subseteq N$  iff  $M \cup N = N$  and  $M \subseteq N$  iff  $M \cap N = M$

# Generalized union $\bigcup$ and intersection $\bigcap$

- $\bigcup M = \{x \mid x \in Y \text{ for some } Y \in M\}$
- (a) if  $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$  then  $\bigcup M = \{a, b, c\}$
- (b)  $M_1 = \{a\}, M_2 = \{a, b\}, M_3 = \{a, b, c\}, I = \{1, 2, 3\}; \bigcup_{i \in I} M_i = \{a, b, c\}$
- $\bigcap M = \{x \mid x \in Y \text{ for every } Y \in M\}$
- (a) if  $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$  then  $\bigcap M = \{a\}$
- (b)  $M_1 = \{a\}, M_2 = \{a, b\}, M_3 = \{a, b, c\}, I = \{1, 2, 3\}; \bigcap_{i \in I} M_i = \{a\}$

# Difference - and complement \ and '

- For any two sets M and N:  $M - N = \{x \mid x \in M \text{ and } x \notin N\}$
- $M = \{a, b, c\}, N = \{a\}, M - N = \{b, c\}$
- For any two sets M and N:  $M \setminus N = \{x \mid x \in N \text{ and } x \notin M\}$
- $O = \{a, b, c, k\} M \setminus O = \{k\}$
- the universal complement:  $M' = \{x \mid x \in U \text{ and } x \notin M\}$   
(U the universal set)

# Trivial equalities

- Idempotency:  $M \cup M = M, M \cap M = M$
- Commutativity for  $\cup$  and  $\cap$ :  $M \cup N = N \cup M \dots$
- Associativity for  $\cup$  and  $\cap$ :  $(M \cup N) \cup O = M \cup (N \cup O) \dots$
- Distributivity for  $\cup$  and  $\cap$ :  $M \cup (N \cap O) = (M \cup N) \cap (M \cup O) \dots$
- Identity:  $M \cup \emptyset = X, M \cup U = U \dots$  what about  $\cap$

# More interesting equalities

- Complement laws:  $M \cup \emptyset = M$ ,  $M'' = M$ ,  $M \cap M' = \emptyset$ ,  $X \cap U = U$
- DeMorgan:  $(M \cup N)' = M' \cap N'$  ...

# How to define an ordered pair

- ...without introducing ordered tuples as a new primitive
- take  $S = \{\{a\}, \{a, b\}\}$
- we write:  $\langle a, b \rangle = \{\{a\}, \{a, b\}\}$
- ordered n-tuples defined recursively
- $\langle a, b \rangle \neq \langle b, a \rangle$
- first and second coordinate of the tuple

# Cartesian products

- sets of ordered pairs
- tupling each member of the first argument with each of the second
- $S_1 \times S_2 = \{\langle x, y \rangle \mid x \in S_1 \text{ and } y \in S_2\}$
- for an arbitrary number of sets:  $S_1 \times \cdots \times S_n = \{\langle x_1, x_2, \dots, x_n \rangle \mid x_i \in S_i\}$
- $\langle x_1, x_2, \dots, x_n \rangle$  abbreviated  $\vec{x}$
- for  $S \times S \times \cdots$ : n-fold products  
 $S^n = \{\vec{s} \mid s_i \in S \text{ for } 1 \leq i \leq n\}$

# Definition of relations

- hold between (sets of) objects
- $x$  kicks  $y$ ,  $x$  lives on the same floor as  $y$ , ...
- formalization:  $Rab$ ,  $aRb$
- $a \in A$  and  $b \in B$ :  $R \subseteq A \times B$ ,  
 $R$  is from  $A$  (**domain**) to  $B$  (**range**)
- $R$  from  $A$  to  $A$  is **in A**

- complement  $R' = \{\langle a, b \rangle \notin R\}$  for  $R \subseteq A \times B$ 
  - ▶  $R$  = the relation of teacherhood between  $a$  and  $b$  (the **arguments**)
  - ▶  $R'$  = all pairs  $\langle b, a \rangle$  s.t. it is false that the first member is the teacher of the second member
- inverse:  $R^{-1} = \{\langle b, a \rangle \mid \langle a, b \rangle \in R\}$  for  $R \subseteq A \times B$ 
  - ▶  $R$  = the relation of teacherhood between  $a$  and  $b$ :  
*Herr Webelhuth is the teacher of Herr Schäfer.*
  - ▶  $R^{-1}$  = all pairs  $\langle b, a \rangle$  where  $a$  is the teacher of  $b$ :  
*Herr Schäfer is the inverse-teacher of Herr Webelhuth.*

- A function  $F$  from  $A$  to  $B$  is a relation s.t. for every  $a \in A$  there is exactly one tuple  $\langle a, b \rangle \in A \times B$  s.t.  $a$  is the first coordinate.
- partial function from  $A$  to  $B$ : for some  $a \in A$  there is no tuple  $\langle a, b \rangle \in A \times B$ ,  $F$  is not defined for some  $a$

# Injection, surjection, bijection

- B the range of F, F is **into** B
- F from A to B is **onto (a surjection)** B iff there is no  $b_i \in B$  s.t. there is no  $\langle a, b_i \rangle \in F$
- F from A to B is **one-to-one (an injection)** iff there are no two pairs s.t.  $\langle a_i, b_j \rangle \in F$  and  $\langle a_k, b_j \rangle \in F$
- one-to-one, onto, and total function: correspondence (bijection)

# Composition

- One can take the range of a function and make it the domain of another function.
- A function  $F_1 : A \rightarrow B$  and a function  $F_2 : B \rightarrow C$  can be composed as  $B(A(a))$ , short  $B \circ A$
- the compound function can be empty, it will be total if both A and B are bijections.

# Reflexivity

A relation  $R$  in  $A = \{a, b, \dots\}$  is...

	if	(ex.)
reflexive	for <b>every</b> $a \in A$ : $\langle a, a \rangle \in R$	is as heavy as A: physical objects
irreflexive	for <b>every</b> $a \in A$ : $\langle a, a \rangle \notin R$	is the father of
non-reflexive	for <b>some</b> $a \in A$ : $\langle a, a \rangle \notin R$	has hurt

# Symmetry

A relation  $R$  in  $A = \{a, b, \dots\}$  is...

	if	(ex.)
symmetric	for every $\langle a, b \rangle \in R$ : $\langle b, a \rangle \in R$	has the same car as
asymmetric	for every $\langle a, b \rangle \in R$ : $\langle b, a \rangle \notin R$	has a different car than
non-symmetric	for some $\langle a, b \rangle \in R$ : $\langle b, a \rangle \notin R$	is the sister of
anti-symmetric	for every $\langle a, b \rangle \in R$ : $a = b$	beats oneself not every human does

# Transitivity

A relation  $R$  in  $A = \{a, b, \dots\}$  is...

	if	(ex.)
transitive	if $\langle a, b \rangle \in R$ and $\langle b, c \rangle \in R$ then $\langle a, c \rangle \in R$	is to the left of
intransitive	the above is never the case	is the father of
non-transitive	the above is sometimes not the case	likes

# Connectedness

A relation  $R$  in  $A = \{a, b, \dots\}$  is...

	if	(ex.)
connected	for every $a, b \in A, a \neq b$ : either $\langle a, b \rangle \in R$ or $\langle b, a \rangle \in R$	$>$ (A: the natural numbers)
non-connected	for some $a, b \in A$ the above is not the case	likes

- **reflexive** ( $\langle a, a \rangle \in R$  for every  $a$ )
- **symmetric** ( $\langle b, a \rangle \in R$  for every  $\langle a, b \rangle$ )
- **transitive** ( $\langle a, b \rangle \in R \& \langle b, c \rangle \in R \rightarrow \langle a, c \rangle \in R$ )
- *is as stupid as*
- partition the range into equivalence classes:  
 $A = \{a, b, c, d\}$ , for example  $P_{A_1} = \{\{a, b\}, \{c\}, \{d\}\}$
- **not**  $\{\{a\}, \{b, c\}\}$  or  $\{\{a, b\}, \{b, c\}, \{d\}\}$

# Defining ordering relations

An ordering relation R in A is ...

- transitive ( $\langle a, b \rangle \in R \ \& \ \langle b, c \rangle \in R \rightarrow \langle a, c \rangle \in R$ ) ...plus ...
- irreflexive and asymmetric: **strict order**
- $A = \{a, b, c, d\}$ ,  $R_1 = \{\langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\}$
- reflexive and anti-symmetric: **weak order**
- $A = \{a, b, c, d\}$ ,  $R_1 = \{\langle a, a \rangle, \langle b, b \rangle, \langle c, c \rangle, \langle a, b \rangle, \langle b, c \rangle, \langle a, c \rangle\}$

# Orders: an example

- a strict order: *greater than* ( $>$ ) in  $\mathbb{N}$
- what is the corresponding weak order
- $\geq$

- **minimal:**  $x$  is not preceded
- **least:**  $x$  precedes every other element
- **maximal:**  $x$  is not succeeded
- **greatest:**  $x$  succeeds every other element
- **well-ordering:** total order, every subset has a least element

# The number of elements...

- $A = \{a, b, c\}$
- $B = \{a, b, c\}$
- obviously,  $A = B$  (equal)
- there is an  $R$  from A to B s.t.  $R = \{\langle a, a \rangle, \langle b, b \rangle, \langle c, c \rangle\}$
- for every set C with the same number of elements  
(e.g.,  $C = \{1, 2, 3\}$ ):  $R = \{\langle a, 1 \rangle, \langle b, 2 \rangle, \langle c, 3 \rangle\}$
- such relations are one-to-one correspondences

- $\mathbb{N}$  is infinite
- for every  $A$  there is some  $R_{\text{card}}$ 
  - ▶ a one-to-one correspondence
  - ▶ from  $A$ 's members to the first  $n$  members of  $\mathbb{N}$
  - ▶ s.t.  $n$  is the **cardinality of  $A$ ,  $\|A\|$**
- sets  $A, B$  with  $\|A\| = \|B\|$  are **equivalent**
- $\|\mathbb{N}\| = \aleph^0$

# A problem

- for some sets there is no such  $R_{\text{card}}$
- no way of bringing their elements into an exhaustive linear order
- no problem with  $\mathbb{Q}$ :  $\langle 0, 1 \rangle$      $\langle 0, 2 \rangle$      $\langle 0, 3 \rangle$     ...

$\langle 1, 0 \rangle$      $\langle 1, 1 \rangle$      $\langle 1, 2 \rangle$      $\langle 1, 3 \rangle$     ...

$\langle 2, 0 \rangle$      $\langle 2, 1 \rangle$      $\langle 2, 2 \rangle$      $\langle 2, 3 \rangle$     ...

⋮    ⋮    ⋮    ⋮

# The non-denumerable real numbers

- now:  $\mathbb{R}$
- some elements cannot be represented as an ordered pair of two elements of  $\mathbb{N}$
- in  $[0, 1]$ , every real can be represented as  $0.\underline{abcdefg\dots}$ ,  
 $a, b, c, d, e, f, g, \dots \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

# Trying to enumerate

- an enumeration of  $[0, 1]$  in  $\mathbb{R}$ ?

$$\begin{array}{ccccccccc} x_1 & = & 0 & . & a_{11} & a_{12} & a_{13} & a_{14} & \dots \\ x_2 & = & 0 & . & a_{21} & a_{22} & a_{23} & a_{24} & \dots \\ x_3 & = & 0 & . & a_{31} & a_{32} & a_{33} & a_{34} & \dots \\ \vdots & & \vdots & & & & & & \\ x_n & = & 0 & . & a_{n1} & a_{n2} & a_{n3} & a_{n4} & \dots \end{array}$$

# Failing to enumerate

- What about an  $x_m$  which differs from  $x_n$  at  $a_{nn}$

$x_1$	=	0	.	$a_{11}$	$a_{12}$	$a_{13}$	$a_{14}$	...
$x_2$	=	0	.	$a_{21}$	$a_{22}$	$a_{23}$	$a_{24}$	...
$x_3$	=	0	.	$a_{31}$	$a_{32}$	$a_{33}$	$a_{34}$	...
$\vdots$		$\vdots$						
$x_n$	=	0	.	$a_{n1}$	$a_{n2}$	$a_{n3}$	$a_{nn}$	...

- It won't be in the array...
- $\mathbb{R}$  is non-denumerable
- If  $\|A\| = \aleph_0$  then  $\|\wp(A)\| = 2^{\aleph_0}$  (cf. Partee et al. 62f.)

# Aussagenlogik

The book (PMW:87-246) deals with logic far more in-depth than we do. Only what is mentioned on the slides is relevant for the test. Reading the whole chapter from PMW will do you no harm, though.

- a collection of statements (propositions)
- axioms (statements accepted to be true)
- maybe based on observations (induction)
- statements that follow from the axioms (deduction)
- predictions beyond the axioms
- rechecking for usability: e.g., Russell's paradox

- **axioms**: atomic truths of your theory
- **theorem**: a proposition you want to prove
- **lemma**: subsidiary propositions (used to prove the theorem)
- **corollary**: propositions proved while proving some axiom

# A method of reasoning

- logic does not generate truths
- formalizing statements, predications etc.
- rules of deduction from axioms to theorems
- empirical (induction) and exact (deduction) science
- aiming at an adequate model of the world (e.g., heliocentric universe)

# Why logic for semantics?

- truth-conditional
- compositional behavior of propositions and connectives
- a logic for entailments
- why, e.g.: *It is not the case that someone is happy.* → *Nobody is happy.*

# Atomic formulas: statements

- statements/propositions = the atoms
- a propositional symbol  $p$ : a well-formed formula ([wff](#))
- ex.: *Herr Keydana is a passionate cyclist.*  $\therefore k$
- $\llbracket k \rrbracket = 1$  or 0 (depending on corresponding **model**)

# Complex (molecular) formulas

- **syntax:** restricts the forms of wff's to make them interpretable
- define functors: functions in  $\{0, 1\}$
- If  $p$  and  $q$  are wff's, then
  - ▶  $\neg p$
  - ▶  $p \vee q$
  - ▶  $p \wedge q$
  - ▶  $p \rightarrow q$
  - ▶  $p \leftrightarrow q$

is also a wff (a **molecular term**).

# Complex (molecular) formulas

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- define functors: functions in  $\{\langle 0, 1 \rangle, \langle 1, 0 \rangle, 0, 1\}$
- If  $p$  and  $q$  are wff's, then
  - ▶  $\neg p$  (negation)
  - ▶  $p \vee q$  (disjunction)
  - ▶  $p \wedge q$  (conjunction)
  - ▶  $p \rightarrow q$  (conditional)
  - ▶  $p \leftrightarrow q$  (biconditional)

is also a wff.

# Complex (molecular) formulas

- **syntax:** restricts forms of wff's to make them interpretable
- define functors: functions in  $\{\langle 0, 1 \rangle, \langle 1, 0 \rangle, 0, 1\}$
- If  $p$  and  $q$  are wff's, then
  - ▶  $\neg p$  (negation - 'not')
  - ▶  $p \vee q$  (disjunction - 'or')
  - ▶  $p \wedge q$  (conjunction - 'and')
  - ▶  $p \rightarrow q$  (conditional - 'if')
  - ▶  $p \leftrightarrow q$  (biconditional - 'iff')

is also a wff.

- standard definition:

$$[\neg] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$$

- but most widely used: [truth tables](#)

$\neg$	$p$
0	1
1	0

# Disjunction

$p$	$\vee$	$q$
1	1	1
1	1	0
0	1	1
0	0	0

- Herr Keydana is a passionate cyclist **or** we all love logic.
- $K \vee L$

# Conjunction

$p$	$\wedge$	$q$
1	1	1
1	0	0
0	0	1
0	0	0

- Herr Keydana is a passionate cyclist **and** we all love logic.
- $K \wedge L$

# Conditional

$p$	$\rightarrow$	$q$
1	1	1
1	0	0
0	1	1
0	1	0

- **If it rains, then the streets get wet.**
- $R \rightarrow S$

**If it rains, the streets get wet.**

- it is raining (1) , the streets are wet 1 : 1
- it is raining (1) , the streets are dry 0 : 0
- it is not raining (0) , the streets are wet 1 : 1
- it is not raining (0) , the streets are dry 0 : 1
- ex vero non sequitur falsum

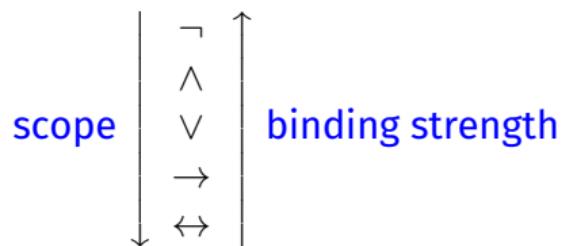
# Biconditional

$p$	$\leftrightarrow$	$q$
1	1	1
1	0	0
0	0	1
0	1	0

- **If and only if** your score is above 50, **then** you pass the semantics exam.
- $S \leftrightarrow P$

# Scope of functors

- brackets are facultative
- or set non-default functor scope
- default scope



# An example

- $p \wedge \neg q \vee r \rightarrow \neg s$
- $p \wedge (\neg q) \vee r \rightarrow (\neg s)$
- $(p \wedge (\neg q)) \vee r \rightarrow (\neg s)$
- $((p \wedge (\neg q)) \vee r) \rightarrow (\neg s)$
- $((p \wedge (\neg q)) \vee r) \rightarrow (\neg s)$

# Large truth tables

- for  $n$  atoms in the term:  $2^n$  lines
- alternating blocks of 1's and 0's under every atom
- $2^{(m-1)}$  times '1' followed by  $2^{(m-1)}$  times '0' for the  $m$ -th atom from the right
- until  $2^n$  lines are reached

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1			1		1			1
1			1		1			0
1			1		0			1
1			1		0			0
1			0		1			1
1			0		1			0
1			0		0			1
1			0		0			0
0			1		1			1
0			1		1			0
0			1		0			1
0			1		0			0
0			0		1			1
0			0		1			0
0			0		0			1
0			0		0			0

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1		0	1		1		0	1
1		0	1		1		1	0
1		0	1		0		0	1
1		0	1		0		1	0
1		1	0		1		0	1
1		1	0		1		1	0
1		1	0		0		0	1
1		1	0		0		1	0
0		0	1		1		0	1
0		0	1		1		1	0
0		0	1		0		0	1
0		0	1		0		1	0
0		1	0		1		0	1
0		1	0		1		1	0
0		1	0		0		0	1
0		1	0		0		1	0

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1	0	0	1		1		0	1
1	0	0	1		1		1	0
1	0	0	1		0		0	1
1	0	0	1		0		1	0
1	1	1	0		1		0	1
1	1	1	0		1		1	0
1	1	1	0		0		0	1
1	1	1	0		0		1	0
0	0	0	1		1		0	1
0	0	0	1		1		1	0
0	0	0	1		0		0	1
0	0	0	1		0		1	0
0	0	1	0		1		0	1
0	0	1	0		1		1	0
0	0	1	0		0		0	1
0	0	1	0		0		1	0

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1	0	0	1	1	1		0	1
1	0	0	1	1	1		1	0
1	0	0	1	0	0		0	1
1	0	0	1	0	0		1	0
1	1	0	1	1	1		0	1
1	1	0	1	1	1		1	0
1	1	0	1	0	0		0	1
1	1	0	1	0	0		1	0
0	0	1	1	1	1		0	1
0	0	1	1	1	1		1	0
0	0	1	0	0	0		0	1
0	0	1	0	0	0		1	0
0	0	1	0	1	1		0	1
0	0	1	0	1	1		1	0
0	0	1	0	0	0		0	1
0	0	1	0	0	0		1	0

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

# An example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

# Assignments: a contingent example

$p$	$\wedge$	$\neg$	$q$	$\vee$	$r$	$\rightarrow$	$\neg$	$s$
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

# Tautology

- take  $p \vee \neg p$

- truth-table:

$p$	$\vee$	$\neg$	$p$
1	1	0	1
0	1	1	0

- true under every assignment, it is **valid**
- by *law of excluded middle*: for every  $P$ ,  $P \vee \neg P$  is true

# Contradiction

- take  $p \wedge \neg p$

$p$	$\wedge$	$\neg$	$p$
1	0	0	1
0	0	1	0

- truth-table:  $\begin{array}{|c|c|c|c|} \hline p & \wedge & \neg & p \\ \hline 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ \hline \end{array}$
- false under every assignment, called **contradictory**

- take  $p \wedge p$

- truth-table:

$p$	$\wedge$	$p$
1	1	1
0	0	0

- the truth value depends on the assignment

# What are laws?

- notice: similarities of set theory and logic
- non-trivial exact nature of their equivalence
- laws state equivalences of (types of) wff
- truth-conservative rewriting of wff's
- any subformula which is a tautology ( $T$ ) or contradiction ( $F$ ):

ignore by Identity Laws (Id.):

- ▶  $(P \vee F) \Leftrightarrow P, (P \vee T) \Leftrightarrow T$
- ▶  $(P \wedge F) \Leftrightarrow F, (P \wedge T) \Leftrightarrow P$

- $X \Leftrightarrow Y$ : X has the same truth-conditions as Y
- derivability of laws and rules (convenient redundancies)
- **Idempotency (Idemp.):**
  - ▶  $(P \vee P) \Leftrightarrow P$
  - ▶  $(P \wedge P) \Leftrightarrow P$
  - ▶ *Peter walks and Peter walks.*  $\Leftrightarrow$  *Peter walks.*

- **Associative Laws for  $\vee$  and  $\wedge$  (Assoc.):**
  - ▶  $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$
  - ▶  $((\text{He walks or she talks}) \text{ or we walk.}) \Leftrightarrow (\text{He walks or (she talks or we walk.)})$
- **Commutative Laws for  $\vee$  and  $\wedge$  (Comm.):**
  - ▶  $(P \vee Q) \Leftrightarrow (Q \vee P)$
  - ▶  $\text{Peter walks or Sue snores.} \Leftrightarrow \text{Sue snores or Peter walks.}$
- **Distributive Laws for  $\vee\wedge$  and  $\wedge\vee$  (Distr.):**
  - ▶  $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
  - ▶  $(\text{Sue snores}) \text{ and } (\text{Peter walks or we talk}).$   
 $\Leftrightarrow (\text{Sue snores and Peter walks}) \text{ or } (\text{Sue snores and we talk}).$

- Complement Laws:

- ▶ Tautology (T):  $(P \vee \neg P) \Leftrightarrow \mathbf{T}$
- ▶ Contradiction (F):  $(P \wedge \neg P) \Leftrightarrow \mathbf{F}$
- ▶ Double Negation (DN):  $(\neg\neg P) \Leftrightarrow P$
- ▶ *It is not the case that Sandy is not walking.*  
 $\Leftrightarrow$  *Sandy is walking.*

# Conditionals Laws

- **Implication (Impl.):**

$P$	$\rightarrow$	$Q$	$\Leftrightarrow$	$\neg$	$P$	$\vee$	$Q$
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

- **Contraposition (Contr.):**

$P$	$\rightarrow$	$Q$	$\Leftrightarrow$	$\neg$	$Q$	$\rightarrow$	$\neg$	$P$
1	1	1		0	1	1	0	1
1	0	0		1	0	0	0	1
0	1	1		0	1	1	1	0
0	1	0		1	0	1	1	0

- DeMorgan's Laws:

- ▶  $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$
- ▶ alternatively:  $\overline{P \vee Q} \Leftrightarrow \overline{P} \wedge \overline{Q}$
- ▶  $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
- ▶ consequently:  $\overline{\overline{P} \vee \overline{Q}} \Leftrightarrow \overline{\overline{P}} \wedge \overline{\overline{Q}} \Leftrightarrow P \wedge Q$

# The Modus Ponens (MP)

- Definition:

$P$	$\rightarrow$	$Q$	premise 1
$P$			premise 2
		$Q$	conclusion

- or:  $(P \rightarrow Q) \wedge (P) \rightarrow (Q)$
- (1) *If it rains, the streets get wet.* (2) *It is raining.*  
→ *The streets are getting wet.*

# MP: a truth table illustration

- Premises are always set to be true!
- the table:

$P$	$\rightarrow$	$Q$
1	1	1
1	0	0
0	1	1
0	1	0

## MP: a truth table illustration

- The conditional must be true.
- cancel the ‘false’ row

$P$	$\rightarrow$	$Q$
1	1	1
1	0	0
0	1	1
0	1	0

## MP: a truth table illustration

- $P$  must be true.
- cancel the ‘false’ rows, Q can only be true:

$P$	$\rightarrow$	$Q$
1	1	1
1	0	0
0	1	1
0	1	0

# The Modus Tollens (MT)

- Definition:

$P$	$\rightarrow$	$Q$
		$\neg Q$
		$\neg P$

- the table illustration:

$P$	$\rightarrow$	$Q$	
1	1	1	(by premise 2)
1	0	0	(by premise 1)
0	1	1	(by premise 2)
0	1	0	

# The Syllogisms

- Hypothetical Syllogism (HS):

- ▶  $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
- ▶ (1) *If it rains, the streets get wet.* (2) *If the streets get wet, it smells nice.*  $\rightarrow$  *If it rains, it smells nice.*

- Disjunctive Syllogism (DS):

- ▶  $((P \vee Q) \wedge (\neg P)) \rightarrow (Q)$
- ▶ (1) *Either Peter sleeps or Peter is awake.* (2) *Peter isn't awake.*  
 $\rightarrow$  *Peter sleeps.*

- Simplification (Simp.):
  - ▶  $(P \wedge Q) \rightarrow P$
  - ▶ (1) *It is raining and the sun is shining.*  $\rightarrow$  *It is raining.*
- Conjunction (Conj.):
  - ▶  $(P) \wedge (Q) \rightarrow (P \wedge Q)$
  - ▶ (1) *It is raining.* (2) *The sun is shining.*  $\rightarrow$  *It is raining and the sun is shining.*
- Addition (Add.):
  - ▶  $(P) \rightarrow (P \wedge Q)$
  - ▶ (1) *It is raining.*  $\rightarrow$  *It is raining or the sun is shining.*
  - ▶ What if Q is instantiated as true or false by another premise?

# A sample proof

- Prove  $p \vee q$  from  $(p \vee q) \rightarrow \neg(r \wedge \neg s)$  and  $r \wedge \neg s$
- The proof:

$$\frac{\begin{array}{l} 1 \quad (p \vee q) \rightarrow \neg(r \wedge \neg s) \\ 2 \quad r \wedge \neg s \end{array}}{p \vee q} \quad 1,2,\text{MT}$$

# Prädikatenlogik

# Weak compositionality in SL

- properties/relations vs. individuals
- *Martin is an expert on inversion and Martin is a good climber.*
- ...becomes  $E \wedge C$
- compositionality restricted to level of connected propositional atoms

# Some desirable deductions

- important generalizations about all and some individuals (which have property P)
- '*all P → some P*'
- '*Martin P → some P*'

- individual **variables**:  $x, y, z, x_1, x_2 \dots$
- individual **constants**:  $a, b, c, \dots$
- variables and constants: **terms**
- **predicate symbols** (taking individual symbols or tuples of them):  $A, B, C, \dots$
- **quantifiers**: existential  $\exists$  (or  $\vee$ ) and universal  $\forall$  (or  $\wedge$ )
- plus the connectives of SL

# Some syntax

- for an  $n$ -ary predicate  $P$  and terms  $t_1 \dots t_n$ ,  
 $P(t_1 \dots t_n)$  or  $Pt_1 \dots t_n$  is a wff.
- possible prefix, function (bracket) and infix notation:  
 $Pxy$ ,  $P(x, y)$ ,  $xPy$
- syntax for connectives from SL
- for any wff  $\phi$  and any variable  $x$ ,  $(\exists x)\phi$  and  $(\forall x)\phi$  are wff's

# Semantic for individual constants

- denote individuals
- a model  $\mathcal{M}$  contains a set of individuals  $D$
- the valuation function  $V$  (or  $F$ ): from constants to individuals in  $D$
- for some  $\mathcal{M}_1$ :  $D = \{Martin, Kilroy, Scully\}$
- $V_{\mathcal{M}_1}(m) = Martin$
- $V_{\mathcal{M}_1}(k) = Kilroy, V_{\mathcal{M}_1}(s) = Scully$

# Semantics for predicate symbols

- denote relations (sets of n-tuples)
- $\llbracket P \rrbracket^{\mathcal{M}_1} = \{Martin, Kilroy\}$  or  $V_{\mathcal{M}_1}(P) = \{Martin, Kilroy\}$
- $V_{\mathcal{M}_1}(Q) = \{\langle Martin, Kilroy \rangle, \langle Martin, Scully \rangle, \langle Kilroy, Kilroy \rangle, \langle Scully, Scully \rangle\}$
- s.t.  $\llbracket P(m) \rrbracket^{\mathcal{M}_1} = \llbracket P \rrbracket^{\mathcal{M}_1}(\llbracket m \rrbracket^{\mathcal{M}_1}) = 1$  iff  $\llbracket m \rrbracket^{\mathcal{M}_1} \in \llbracket P \rrbracket^{\mathcal{M}_1}$

# Semantics for connectives and quantifiers

- **connectives:** ‘apply to’ formulas (semantically truth-valued), semantics as in SL
- $(\forall x)\phi = 1$  iff  $\phi$  is true for every  $d \in D$   
assigned to every occurrence of  $x$  in  $\phi$
- $(\exists x)\phi = 1$  iff  $\phi$  is true for at least one  $d \in D$   
assigned to every occurrence of  $x$  in  $\phi$
- algorithmic instruction to check wff’s containing Q’s
- check outside-in (unambiguous scoping)

- universal quantifiers can be swapped:

$$(\forall x)(\forall y)\phi \Leftrightarrow (\forall y)(\forall x)\phi$$

- same for existential quantifiers:

$$(\exists x)(\exists y)\phi \Leftrightarrow (\exists y)(\exists x)\phi$$

- whereas:  $(\exists x)(\forall y)\phi \Rightarrow (\forall y)(\exists x)\phi$

- example in  $\mathcal{M}_1$ :

- ▶  $\llbracket (\forall x)(\exists y)Qxy \rrbracket^{\mathcal{M}_1} = 1$

- ▶ but:  $\llbracket (\exists y)(\forall x)Qxy \rrbracket^{\mathcal{M}_1} = 0$

- ▶ direct consequence of algorithmic definition

- ▶ if  $\exists\forall$  is true,  $\forall\exists$  follows

# Hints on quantifiers

- domain of quantifiers:  $D$  (universe of discourse)
- $\forall x$  checks for truth of some predication for all individuals
- $\exists x(Px \wedge \neg Px)$  is a contradiction
- $\forall x(Wx \wedge \neg Wx)$  is a contradiction,  
 $\forall x$  ‘checks’ for an empty set by def.
- standard form of NL quantification:  
 $\forall x(Wx \rightarrow Bx)$  ‘All women are beautiful.’
- standard form of NL existential quantification:  
 $\exists x(Wx \wedge Bx)$  ‘Some woman is beautiful.’

- by def., functors take formulas, not terms:
  - ▶  $\neg Wm$  'Mary doesn't weep.'
  - ▶  $(\exists x)(Gx \wedge Wx)$  'Some girl weeps.'
  - ▶  $^*W\neg x$
  - ▶  $^*(\exists \neg x)(Gx)$
- quantifiers take variables, not constants:
  - ▶  $(\forall x)(Ox \rightarrow Wx)$  'All ozelots are wildcats.'
  - ▶  $^*(\forall o)(Wo)$
- $\neg$  negates the wff, not the q:  
 $^*(\neg \forall x)Px$  but  $\neg(\forall x)Px$

- quantifiers bind variables
- free variables (constants) are unbound
- no double binding  $\ast(\forall x \exists x)Px$
- Q scope: only the first wff to its right:
  - ▶  $(\forall x)Px \vee Qx$
  - ▶  $(\forall x)(Px \vee Qx)$  =  $(\forall x)Px \vee$   $(\forall x)Qx$
  - ▶  $(\exists x)Px \rightarrow (\forall y)(Qy \wedge Ry)$
  - ▶  $(\exists x)Px \wedge Qx$  (second x is a unbound)
- no double-naming

## Universal $\vee$ and $\wedge$

- $\exists$  and  $\forall$  ‘or’ and ‘and’ over the universe of discourse (hence:  $\vee$  and  $\wedge$ )
- $(\forall x)Px \Leftrightarrow Px_1 \wedge Px_2 \wedge \dots \wedge Px_n$  for all  $x_n$  assigned to  $d_n \in D$
- $(\exists x)Px \Leftrightarrow Px_1 \vee Px_2 \vee \dots \vee Px_n$  for all  $x_n$  assigned to  $d_n \in D$
- hence:  $\neg(\forall x)Px \Leftrightarrow \neg(Px_1 \wedge Px_2 \wedge \dots \wedge Px_n)$
- with DeM:  $\overline{Px_1 \wedge Px_2 \wedge \dots \wedge Px_n}$
- $\Leftrightarrow \overline{Px_1} \vee \overline{Px_2} \vee \dots \vee \overline{Px_n}$
- $\Leftrightarrow (\exists x)\neg Px$

# Quantifier negation (QN)

- $\neg(\forall x)Px \Leftrightarrow (\exists x)\neg Px$
- $\neg(\exists x)Px \Leftrightarrow (\forall x)\neg Px$
- $\neg(\forall x)\neg Px \Leftrightarrow (\exists x)Px$
- $\neg(\exists x)\neg Px \Leftrightarrow (\forall x)Px$

- the conjunction of universally quantified formulas:  
$$\underline{(\forall x)(Px \wedge Qx)} \Leftrightarrow \underline{(\forall x)Px} \wedge \underline{(\forall x)Qx}$$
- the disjunction of existentially quantified formulas:  
$$\underline{(\exists x)(Px \vee Qx)} \Leftrightarrow \underline{(\exists x)Px} \vee \underline{(\exists x)Qx}$$
- not v.v.:  $(\forall x)Px \vee (\forall x)Qx \Rightarrow (\forall x)(Px \vee Qx)$
- why?

# Quantifier movement (QM)

- desirable format: prefix + matrix
- Movement Laws for antecedents of conditionals:  
 $(\exists x)Px \rightarrow \phi \Leftrightarrow (\forall x)(Px \rightarrow \phi)$   
 $(\forall x)Px \rightarrow \phi \Leftrightarrow (\exists x)(Px \rightarrow \phi)$
- Movement Laws for Q's in disjunction, conjunction, and the consequent of conditionals: Just move them to the prefix!
- condition:  $x$  must not be free in  $\phi$ .
- i.e.: Watch your variables!

## Let's formalize:

- Paul Kalkbrenner is a musician and signed on bpitchcontrol.
- Herr S. installed RedHat and not every Linux distribution is easy to install.
- All talkmasters are human and Harald Schmidt is a talkmaster.
- Some talkmasters are not musicians.
- Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.
- Some humans are neither talkmasters nor do they own Kanzleramt records.

# Universal instantiation ( $\neg\forall$ ) and generalization ( $+\forall$ )

- $(\forall x)Px \rightarrow Pa$
- always applies
- can use any variable/constant
- $Pa \rightarrow (\forall x)Px$
- iff  $Pa$  was instantiated by  $\neg\forall$

## Existential generalization ( $+ \exists$ ) and instantiation ( $- \exists$ )

- $Pa \rightarrow (\exists x)Px$  for any individual constant a
- always applies
- $(\exists x)Px \rightarrow Pa$  for some indiv. const.
- always applies (there is a minimal individual for  $\exists x$ )
- for some  $(\exists x)Px$  and  $(\exists x)Qx$  the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

# One sample task

- (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.
- Formalize and prove: At least one human exists.
- (1)  $Dk$
- (2)  $(\forall x)(Dx \rightarrow Hx \vee Px)$
- (3)  $\neg(\exists x)(Px \wedge Dx)$
- $(\exists x)Hx$

# The proof

(1)	$Dk$	
(2)	$(\forall x)(Dx \rightarrow Hx \vee Px)$	
(3)	$\neg(\exists x)(Px \wedge Dx)$	
(4)	$(\forall x)\neg(Px \wedge Dx)$	3, QN
(5)	$(\forall x)(\neg Px \vee \neg Dx)$	4, DeM
(6)	$(\forall x)(Dx \rightarrow \neg Px)$	5, Comm, Impl
(7)	$Dk \rightarrow \neg Pk$	6, $\neg\forall(1)$
(8)	$\neg Pk$	1, 7, MP
(9)	$Dk \rightarrow Hk \vee Pk$	2, $\neg\forall(1)$
(10)	$Hk \vee Pk$	1, 9, MP
(11)	$Hk$	8, 10, DS
∴	$(\exists x)Hx$	10, + $\exists$

## Quantifikation und Modelltheorie

- before we turn to quantification in F1/F2 English:
- names refer to individuals
- itr. verbs refer to sets of individuals
- tr. verbs refer to sets of ordered pairs of individuals
- sentences refer to truth values

# Reference of pronouns

- ***This*** drives a *Golf*.
- *this* = a pronominal NP
- denotes an individual
- but not rigidly
- fixed only within a specific context (SOA)

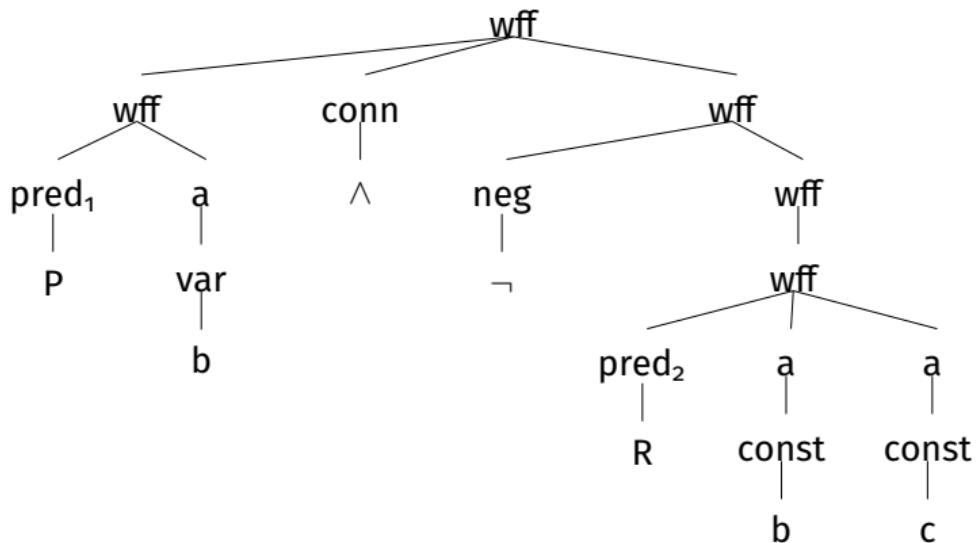
- quantified expression:  $(\forall x)Px$
- *for all assignments of 'this', 'this' has property P*
- Q evaluation in PC is algorithmic
- variables interpreted like definite pronominal NPs (within a fixed context)

- $a \rightarrow \text{const, var}$
- $\text{conn} \rightarrow \wedge, \vee, \rightarrow, \leftrightarrow$
- $\text{neg} \rightarrow \neg$
- $\mathbf{Q} \rightarrow \exists, \forall$

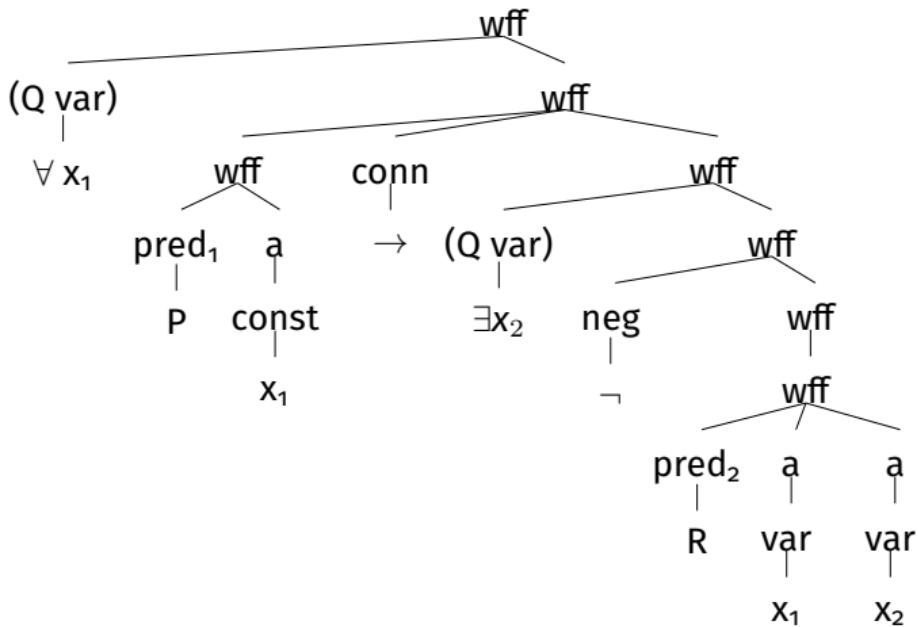
- $\text{pred}_1 \rightarrow P, Q$
- $\text{pred}_2 \rightarrow R$
- $\text{pred}_3 \rightarrow S$
- $\text{const} \rightarrow b, c$
- $\text{var} \rightarrow x_1, x_2, \dots, x_n$

- $wff \rightarrow \text{pred}_n a_1 a_2 \dots a_n$
- $wff \rightarrow \text{neg } wff$
- $wff \rightarrow wff \text{ con } wff$
- $wff \rightarrow (\text{Q var}) wff$

# A wff without Q



# A wff with Q's



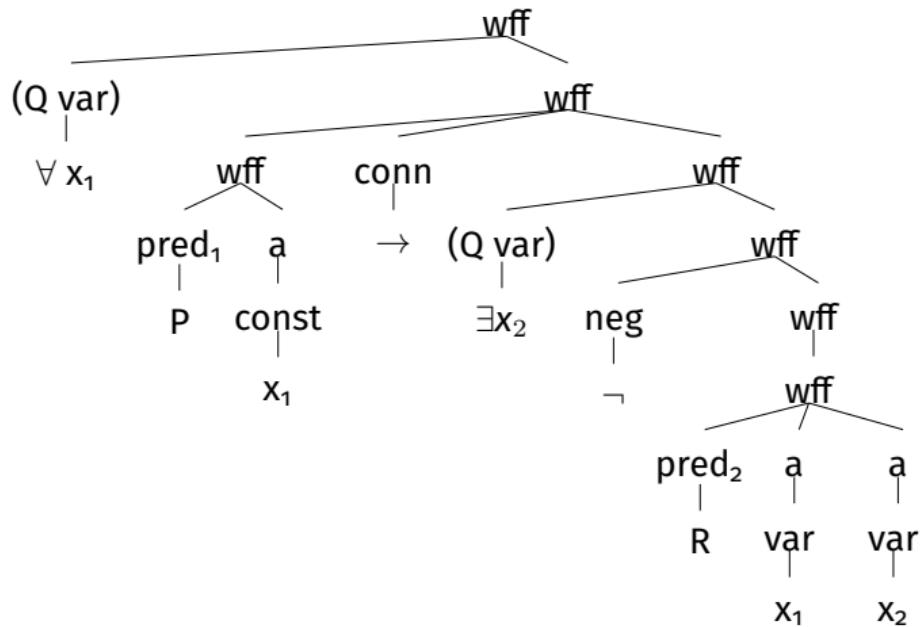
## Definition of c-command

- Node A **c-commands** (constituent-commands) node B iff
  - ▶ A does not dominate B and
  - ▶ and the first branching node dominating A also dominates B.
- The definition in CM allows a node to dominate itself.

# Configurational binding

- in configurational tree-structures:
- A variables is bound by the closest c-commanding coindexed quantifier.
- scope = binding domain

# A wff with Q's



# Refinement of PC semantics

- remember T-sentences: **S of L is true in v iff p.**
- $\mathcal{M}$  is a model of the accessible universe of discourse
  - ▶  $\mathcal{M} = \langle U_n, V_n \rangle$
  - ▶  $U_n$  = the set of accessible individuals (**domain**)
  - ▶  $V_n$  = a **valuation function** which assigns
    - ★ individuals to names
    - ★ sets of n-tuples of individuals to  $\text{pred}_n$
- $g$  is function from variables to individuals in  $\mathcal{M}$
- we evaluate:  $\llbracket \alpha \rrbracket^{\mathcal{M}_n, g_n}$
- *the extension of  $\alpha$  relative to  $\mathcal{M}_n$  and  $g_n$*

- $V_n$  evaluates statically
- Q's require flexible valuation of pronominal matrices
- $g_n$  is like  $V_n$  for constants, only flexible
- it can iterate through  $U_n$
- initial assignment can be anything:

$$g_1 = \begin{bmatrix} x_1 \rightarrow \textit{Herr Webelhuth} \\ x_2 \rightarrow \textit{Frau Eckardt} \\ x_3 \rightarrow \textit{Turm – Mensa} \end{bmatrix}$$

# Iterating through $U_n$

- for each Q loop, one modification
- read  $g_n [d/x_m]$  as  
‘...relative to  $g_n$  where  $x_m$  is reassigned to  $d$ ’
- $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1[Eckardt/x_1]} = \textit{Frau Eckardt}$
- $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1[[Eckardt/x_1]\textit{Mensa}/x_2]} = \textit{Mensa}$

# Interpreting with $g_n$

- $\llbracket (\forall x_1) Px_1 \rrbracket^{\mathcal{M}_1, g_1}$
- start with initial assignment:  $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1} = \text{Webelhuth}$   
check:  $\llbracket Px_1 \rrbracket^{\mathcal{M}_1, g_1}$
- modify:  $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1[\text{Eckardt}/x_1]} = \text{Eckardt}$   
check:  $\llbracket Px_1 \rrbracket^{\mathcal{M}_1, g_1}$
- modify:  $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1[\text{Mensa}/x_1]} = \text{Mensa}$   
check:  $\llbracket Px_1 \rrbracket^{\mathcal{M}_1, g_1}$
- iff the answer was never 0, then  $\llbracket (\forall x_1) Px_1 \rrbracket^{\mathcal{M}_1, g_1} = 1$

# Multiple Q's: subloops

- $\llbracket (\forall x_1)(\exists x_2)Px_1x_2 \rrbracket^{\mathcal{M}_1, g_1}$
- $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1} = \text{Webelhuth}$ 
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1} = \text{Eckardt}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [\text{Webelhuth}/x_2]} = \text{Webelhuth}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [\text{Mensa}/x_2]} = \text{Mensa}$
- $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1 [\text{Eckardt}/x_1]} = \text{Eckardt}$ 
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [\text{Eckardt}/x_1]} = \text{Eckardt}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [[\text{Eckardt}/x_1]\text{Webelhuth}/x_2]} = \text{Webelhuth}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [[\text{Eckardt}/x_1]\text{Mensa}/x_2]} = \text{Mensa}$
- $\llbracket x_1 \rrbracket^{\mathcal{M}_1, g_1 [\text{Mensa}/x_1]} = \text{Mensa}$ 
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [\text{Mensa}/x_1]} = \text{Eckardt}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [[\text{Mensa}/x_1]\text{Webelhuth}/x_2]} = \text{Webelhuth}$
  - ▶  $\llbracket x_2 \rrbracket^{\mathcal{M}_1, g_1 [[\text{Mensa}/x_1]\text{Mensa}/x_2]} = \text{Mensa}$

- quantifying expressions in NL beyond  $\forall$  and  $\exists$
- some seem to work differently:
- *All patients adore Dr. Rick Dagless M.D.*  
 $(\forall x_1)Px_1 \rightarrow Ax_1d$  (ok)
- but: *Most patients adore Dr. Rick Dagless M.D.*  
 $(\text{MOST } x_1)Px_1 \rightarrow Ax_1d$  (wrong interpretation)
- domain should be the set of patients, not individuals
- For NL: Assume that the checking domain for Q is the set denoted by CN.

# Scope ambiguities

- c-command condition on binding/scope fails in NL
- no PNF's in NL
- Q and common noun (CN) usually **in-situ** (e.g., argument position)
- **ambiguities independent of Q position**
  - ▶ *Everybody loves somebody.* (ELS)
  - ▶  $(\forall x_1)(\exists x_2)Lx_1x_2$
  - ▶  $(\exists x_2)(\forall x_1)Lx_1x_2$
- **Q ambiguity cannot be structural** (e.g.,  $\exists$  will never c-command  $\forall$ )

# Cases of overt movement and traces

- **wh** movement:
  - *What<sub>i</sub> will Agent Cooper solve t<sub>j</sub>?*
  -
- **passive** movement:
  - *(Laura Palmer)<sub>i</sub> was killed t<sub>j</sub>.*
  -
- **raising** verbs:
  - *(Laura Palmer)<sub>i</sub> seems t<sub>j</sub> to be dead.*
  -

# Levels of representation

- construction of an independent representational level LF
- could use movement mechanism as used at surface level
- All quantifiers adjoin to the left periphery of S at LF.
- LF is constructed by syntactic rules!

# Ambiguities at LF

- $[_{S''} \text{everybody}_i [_{S'} \text{somebody}_j [_S t_i \text{ loves } t_j ]]]$
- 
- $[_{S''} \text{somebody}_j [_{S'} \text{everybody}_i [_S t_i \text{ loves } t_j ]]]$
-

# The Q raising rule

$$[s X NP Y] \Rightarrow [s' NP_i [s X t_i Y]]$$

- specify a PS as input and output
- QR rule also introduces coindexing of traces

- copies all definitions from F1
- adds appropriate definitions of quantifying determiners etc.
  - ▶  $\text{Det} \rightarrow \text{every}, \text{some}$
  - ▶  $\text{NP} \rightarrow \text{DetN}_{\text{common-count}}$
- adds the QR rule
- assume introduction of reasonable syntactic types/rules without specifying
- assume admissible (reasonable, possible) models  $\mathcal{M}$

## Semantics for QR output: *every*

$$\llbracket [ [ \text{every } \beta ] ; S ] \rrbracket^{\mathcal{M}, g} = 1 \text{ iff for all } u \in U : \\ \text{if } u \in \llbracket \beta \rrbracket^{\mathcal{M}, g} \text{ then } \llbracket S \rrbracket^{\mathcal{M}, g[u/t_i]}$$

A sentence containing the trace  $t_i$  with an adjoined  $NP_i$  (which consists of *every* plus the common noun  $\beta$ ) extends to 1 iff for each individual  $u$  in the universe  $U$  which is in the set referred to by the common noun  $\beta$ ,  $S$  denotes 1 with  $u$  assigned to the pronominal trace  $t_i$ .  $g$  is modified iteratively to check that.

## Semantics for QR output: *some*, *a*

$\llbracket [ [a \ \beta]_i \ S] \rrbracket^{\mathcal{M}, g} = 1$  iff for some  $u \in U$ :  
 $u \in \llbracket \beta \rrbracket^{\mathcal{M}, g}$  and  $\llbracket S \rrbracket^{\mathcal{M}, g[u/t_i]}$

(similar)

Einfach getypte höherstufige  $\lambda$ -Sprachen

- Chierchia & McConnell-Ginet, Heim & Kratzer, etc.: GB-ish semantics
- both syntax and LF in phrase structures
- LF as a proper linguistic level of representation
- Montague: direct translation of NL into logic
- Monatgue's LF is just a notational system for NL semantics

# Targets for this week

- Learn to tell the difference between the montagovian and generative approach.
- See the advantage of a general theory of typed languages.
- Understand how  $\lambda$  languages allow dramatically elegant formalizations.
- ... while keeping in mind that these devices are extensions to our PC representation for NL semantics.

- denotations in set/function-theoretic terms
- a characteristic function (CF)  $\mathcal{S}$  of a set  $S$ :  
 $\mathcal{S}(a) = 1 \text{ iff } a \in S, \text{ else } 0$
- a CF ‘checks’ individuals into a set
- denotations can be stated as sets or their CF

# Generalizing combinatory semantic operations

- interpretation for  $[_S NP VP]$ :  
 $\llbracket [s NP VP] \rrbracket^{\mathcal{M}, g} = 1 \text{ iff } \llbracket NP \rrbracket^{\mathcal{M}, g} \in \llbracket VP \rrbracket^{\mathcal{M}, g}$
- Montague generally used CF's in definitions
- evaluating  $[_S [_{NP} Mary] [_{VP} sleeps] ]$  as a matter of functional application (FA):
  - ▶  $\llbracket Mary \rrbracket^{\mathcal{M}, g} = \text{Mary in } \mathcal{M}$
  - ▶  $\llbracket sleeps \rrbracket^{\mathcal{M}, g}$  be the CF of the set of sleepers in  $\mathcal{M}$
  - ▶  $\llbracket S \rrbracket^{\mathcal{M}, g} = \llbracket sleeps \rrbracket^{\mathcal{M}, g}(\llbracket Mary \rrbracket^{\mathcal{M}, g})$
  - ▶ ideally: generalize to all nodes

# The superscript notation

- all functions from  $S_1$  to  $S_2$
- $S_2^{S_1}$
- for  $T = \{0, 1\}$ 
  - ▶  $T^D$ : all  $\text{pred}_1$
  - ▶  $T^{D \times D}$ : all  $\text{pred}_2$
-

# Some new names

- base for Dowty et al.:  $L_1$ , a first-order predicate language as we know it
- semantic renaming of types:
  - ▶ terms:  $\langle e \rangle$  (entity-denoting)
  - ▶ formulas:  $\langle t \rangle$  (truth-valued)
  - ▶  $\text{pred}_1$ :  $\langle e, t \rangle$
  - ▶  $\text{pred}_2$ :  $\langle e, \langle e, t \rangle \rangle$

# Possible denotations of types

- $D_\alpha$  possible denotation (a set) of expressions of type  $\alpha$
- $D_{\langle e \rangle} = U$  (Dowty et al.'s  $A$ )
- $D_{\langle t \rangle} = \{0, 1\}$
- recursively:  $D_{\langle \alpha, \beta \rangle} = D_{\langle \beta \rangle}^{D_{\langle \alpha \rangle}}$
- e.g.,  $D_{\langle e, t \rangle} = D_{\langle t \rangle}^{D_{\langle e \rangle}}$
- $D_{\langle e, \langle e, t \rangle \rangle} = (D_{\langle t \rangle}^{D_{\langle e \rangle}})^{D_{\langle e \rangle}}$
- just a systematic way of naming types, model-theoretic interpretations still by  $V, g$

# Defining types

- in our PS syntax: S as start symbol
- in the typed system: sentences should be of type  $\langle t \rangle$
- complex types: functions from  $\langle e \rangle$  to  $\langle t \rangle$   
or generally from any (complex) type to any (complex) type

# Complex types as functions

- saturation of complex types by FA:
  - ▶  $\gamma$  is of type  $\langle e, \langle e, t \rangle \rangle$ ,  $\delta$  of  $\langle e, t \rangle$ ,  $\alpha$  and  $\beta$  of  $\langle e \rangle$
  - ▶ then  $\gamma(\alpha)$  is of type  $\langle e, t \rangle$
  - ▶ and  $\delta(\beta)$  is of type  $\langle t \rangle$
- for any pred<sub>2</sub>  $P$  and its arguments  $a_1, a_2$ ,  $P(a_2)(a_1)$  is a wff
- connectives are of types  $\langle t, t \rangle$  ( $\neg$ ),  $\langle t, \langle t, t \rangle \rangle$  ( $\wedge$ , etc.)

# General semantics of typed languages

- generalized CF/FA approach
- $\langle e \rangle$ -types (terms):  
 $\llbracket a_n \rrbracket^{\mathcal{M}, g} = V(a_n)$   
 $\llbracket x_n \rrbracket^{\mathcal{M}, g} = g(x_n)$
- the rest: functional application  
 $\llbracket \delta(\alpha) \rrbracket^{\mathcal{M}, g} = \llbracket \delta \rrbracket^{\mathcal{M}, g}(\llbracket \alpha \rrbracket^{\mathcal{M}, g})$

- Type is the set of types
- recursively defined complex types  $\langle a, b \rangle$ : infinite
- type label  $\langle \alpha \rangle$
- vs. set of meaningful expressions of that type:  $ME_{\langle \alpha \rangle}$

- first order languages: variables over individuals ( $\langle e \rangle$ -types)
- n-order: **variables over higher types** ( $\langle e, t \rangle$ -types etc.)
- $P_{\langle e, t \rangle}$  or  $Q_{\langle e, \langle e, t \rangle \rangle}$ : constants of higher types
- so:  $v_{1_{\langle e, t \rangle}} [v_1(m)]$
- if  $V(m) = Mary$ ,  $v_1$  is the set of all of Mary's properties

# Typing variables

- we write:
  - ▶  $v_{n,\langle\alpha\rangle}$  for the n-th variable of type  $\langle\alpha\rangle$
  - ▶ Dowty et al.:  $v_{n,\langle\alpha\rangle}$
- alternatively abbreviated by old symbols  $x_1, a, P$ , etc.

# Constants, variables, functions

- non-logical constant  $\alpha$ :  $\llbracket \alpha \rrbracket^{\mathcal{M}, g} = V(\alpha)$
- variable  $\alpha$ :  $\llbracket \alpha \rrbracket^{\mathcal{M}, g} = V(\alpha)$
- $\alpha \in \langle a, b \rangle$ ,  $\beta \in a$ , then  $\llbracket \alpha(\beta) \rrbracket^{\mathcal{M}, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\llbracket \beta \rrbracket^{\mathcal{M}, g})$

# Logical constants and quantifiers

- logical constants interpreted as functions in {0,1} as usual
- if  $v_{1_{(\alpha)}}$  is a variable and  $\phi \in ME_t$   
then  $\llbracket (\forall v_1)\phi \rrbracket^{\mathcal{M},g} = 1$  iff  
for all  $a \in D_\alpha$   $\llbracket \phi \rrbracket^{\mathcal{M},g[a/v_1]} = 1$

# An example

- quantified variable of type  $\langle e, t \rangle$ :  $v_{0_{\langle e, t \rangle}}$
- $\forall v_{0_{\langle e, t \rangle}} [v_{0_{\langle e, t \rangle}}(j) \rightarrow v_{0_{\langle e, t \rangle}}(d)]$
- for  $j, d \in ME_{\langle e \rangle}$
- one property of every individual: being alone in its union set
- hence,  $j = d$
- else in  $\forall v_{0_{\langle e, t \rangle}}, \forall$  wouldn't hold

# Defining *non*

- productive adjetival prefix: *non-adjacent*, *non-local*, etc.
- inverting the characteristic function of the adjective
- result denotes complement of the original adjective in  $D_{\langle e \rangle}$
- *adjective*:  $\langle e, t \rangle$ , *non*:  $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$
- a function  $h$  s.t. for every  $k \in D_{\langle e, t \rangle}$  and every  $d \in D_{\langle e \rangle}$   
 $(h(k))(d) = 1$  iff  $k(d) = 0$  and  
 $(h(k))(d) = 0$  iff  $k(d) = 1$

# Argument deletion

- understood objects in: *I eat.* - *Vanity kills.* - etc.
- *eat* is in  $ME_{\langle e, \langle e, t \rangle \rangle}$
- assume a silent logical constant:  $R_O$  in  $ME_{\langle \langle e, \langle e, t \rangle \rangle, \langle e, t \rangle \rangle}$
- a function  $h$  s.t. for all  $k \in D_{\langle e, \langle e, t \rangle \rangle}$  and all  $d \in D_{\langle e \rangle}$   
 $h(k)(d) = 1$  iff there is some  $d' \in D_{\langle e \rangle}$  s.t.  $k(d')(d) = 1$
- passives as similar subject deletion

# All there is to $\lambda$

- a new variable binder
- allows abstraction over wff's of arbitrary complexity
- similar to  $\{x \mid \phi\}$  (read as 'the set of all  $x$  s.t.  $\phi$ ')
- we get  $\lambda x [\phi]$
- on Montague's typewriter:  $\hat{x} [\phi]$
- does not create a set but a function which can be taken as the CF of a set

# $\lambda$ abstraction

- for every wff  $\phi$ , any  $x \in Var$ , and any  $a \in Con$
- $\lambda$  abstraction:  $\phi \rightarrow \lambda x [\phi^{[a/x]}] (a)$
- read  $\phi^{(a/x)}$  as ‘*phi* with every  $a$  replaced by  $x$ ’
- $x$  can be of any type

## Two informal examples

- $\lambda x_{\langle e \rangle} [L(x)]$  is the characteristic function of the set of those individuals  $d \in D_{\langle e \rangle}$  which have property  $L$
- $\lambda x_{\langle e,t \rangle} [x(l)]$  is the characteristic function of the set of those properties  $k \in D_{\langle e,t \rangle}$  that the individual  $l$  has

# $\lambda$ conversion

- $\lambda x [L(x)]$  is the abstract of  $L(a)$  (with some individual  $a$ )
- hence, it holds:  $\lambda x [L(x)](a) \Leftrightarrow L(a)$
- for every wff  $\phi$ , any  $x \in Var$ , and any  $a \in Con$
- $\lambda$  conversion:  $\lambda x [\phi](a) \rightarrow \phi^{[x/a]}$

- $\lambda x [\phi] (a) \leftrightarrow \phi^{[x/a]}$
- not just syntactically, since truth conditions are equivalent
- $\lambda x [\phi] (a) \Leftrightarrow \phi^{[x/a]}$
- notice:  $\lambda x_{\langle \alpha \rangle} [\phi]$  is in  $ME_{\langle \alpha, t \rangle}$
- while  $\phi$  (as a wff) is in  $ME_{\langle t \rangle}$

- Dowty et al., 102f. (*Syn C.10* and *Sem 10*)
- If  $\alpha \in ME_\alpha$  and  $u \in Var_b$ , then  $\lambda u [\alpha] \in ME_{\langle b,a \rangle}$ .
- If  $\alpha \in ME_a$  and  $u \in Var_b$  then  $\llbracket \lambda u [\alpha] \rrbracket^{\mathcal{M},g}$  is that function  $h$  from  $D_b$  into  $D_a$  s.t. for all objects  $k$  in  $D_b$ ,  $h(k)$  is equal to  $\llbracket \alpha \rrbracket^{\mathcal{M},g[k/u]}$ .

## The *non* example revised (Dowty et al., 104)

- $\forall x \forall v_{0\langle e, t \rangle} \left[ (\mathbf{non}(v_{0\langle e, t \rangle}))(x) \leftrightarrow \neg(v_{0\langle e, t \rangle}(x)) \right]$
- $\forall v_{0\langle e, t \rangle} \left[ \lambda x \left[ (\mathbf{non}(v_{0\langle e, t \rangle}))(x) \right] = \lambda x \left[ \neg(v_{0\langle e, t \rangle}(x)) \right] \right]$
- $\forall v_{0\langle e, t \rangle} \left[ \mathbf{non}(v_{0\langle e, t \rangle}) = \lambda x \left[ \neg(v_{0\langle e, t \rangle}(x)) \right] \right]$   
(since  $\lambda x [\mathbf{non}(v)(x)]$  is unnecessarily abstract/ $\eta$  reduction)
- $\lambda v_{0\langle e, t \rangle} \left[ \mathbf{non}(v_{0\langle e, t \rangle}) = \lambda v_{0\langle e, t \rangle} \left[ \lambda x \left[ \neg(v_{0\langle e, t \rangle}(x)) \right] \right] \right]$
- and since that is about all assignments for  $\lambda v_{0\langle e, t \rangle}$ :  
 $\mathbf{non} = \lambda v_{0\langle e, t \rangle} \left[ \lambda x \left[ \neg(v_{0\langle e, t \rangle}(x)) \right] \right]$

*Mary is non-adjacent.*

(translate 'adjacent' as  $c_{0\langle e,t \rangle}$ , 'Mary' as  $c_{0\langle e \rangle}$ , ignore the copula)

# The behavior of quantified NPs

- syntactically like referential NPs
- semantically like PC quantifiers
- *Every student walks.*:  $\forall v_{0\langle e \rangle} [c_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \rightarrow c_{1\langle e,t \rangle}(v_{0\langle e \rangle})]$
- *Some student walks.*:  $\exists v_{0\langle e \rangle} [c_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \wedge c_{1\langle e,t \rangle}(v_{0\langle e \rangle})]$
- making referential NPs and QNPs **the same type?**

# A higher type

- $\lambda v_{0\langle e,t\rangle} \forall v_{0\langle e\rangle} [c_{0\langle e,t\rangle}(v_{0\langle e\rangle}) \rightarrow v_{0\langle e,t\rangle}(v_{0\langle e\rangle})]$
- a second order function
- characterizes the set of all predicates true of every student
- equally:  $\lambda v_{0\langle e,t\rangle} \exists v_{0\langle e\rangle} [c_{0\langle e,t\rangle}(v_{0\langle e\rangle}) \wedge v_{0\langle e,t\rangle}(v_{0\langle e\rangle})]$

# Combining with some predicate

# Intensionalität

# Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

## Some examples

- Stockhausen **will** write another opera.
- **Had** Arno Schmidt cut down on drinking, he **would** still be alive.
- Gustave Moreau **believes that** estheticism rules.

# Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (**tense**)
- ... and for just one state of affairs (**modals, believe type verbs**)

# What are intensions?

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts or <b>propositions</b>

- can't be just truth conditional
- encode knowledge about not just the actual but all **possible** and/or past/future **states of affairs (PSOAs)**
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values

## PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just truth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

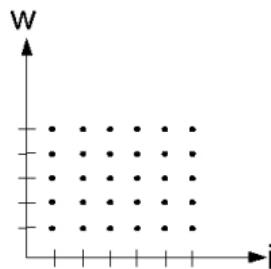
# A touch of parallel universes?

- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
  - ▶ A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
  - ▶ nothing of the above, but Maria was born 20 years earlier
  - ▶ nothing of the above, but A.S. rose from the dead in 2003, etc.

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
  - ...the SOAs under which its corresponding sentence is true
  - ...the subset under which its corresponding sentence is false

# Coordinates

- for each possible distinction in truth values of the whole of the propositional sentences: one possible world ( $w \in W$ )
- for each point in time: one possible temporal state of each world (instant  $i \in I$ )
- representation of temporarily ordered world-time coordinates  $\langle w, i \rangle \in W \times I$

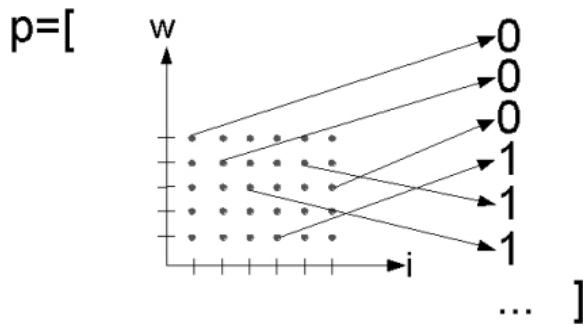


# The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- **the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true**

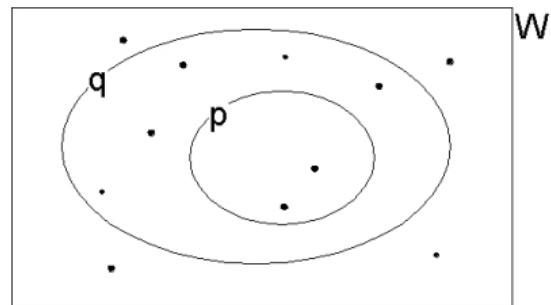
# Propositions as functions

- a propositional function  $p$
- is a function from  $W \times I$  to  $\{0, 1\}$



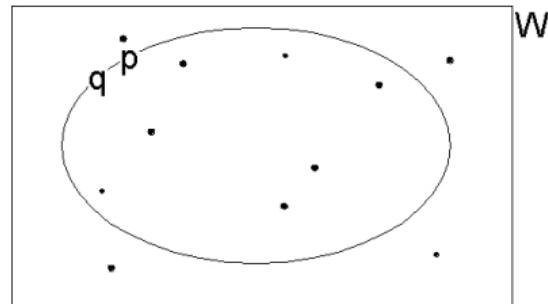
- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

- definition of intensions of sentences (propositions): characteristic functions
- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation:  $p \subseteq q$ :



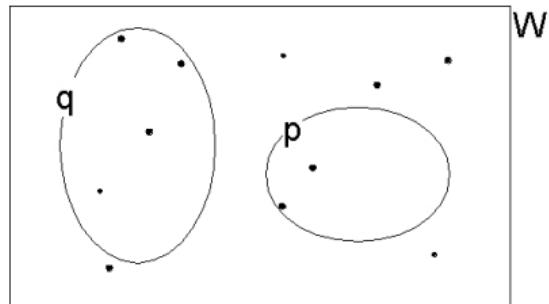
# Synonymy

- **synonymy** turns out as **set equivalence**:
- $p = q$



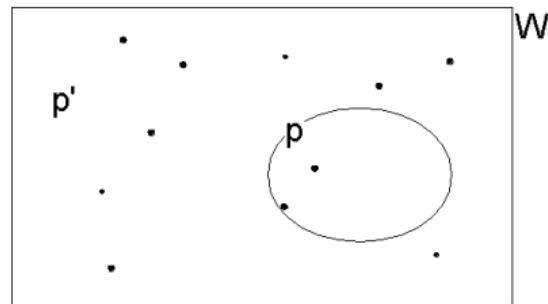
# Contradiction

- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



# Negation

- negation turns out as a complement:
- $p/W$

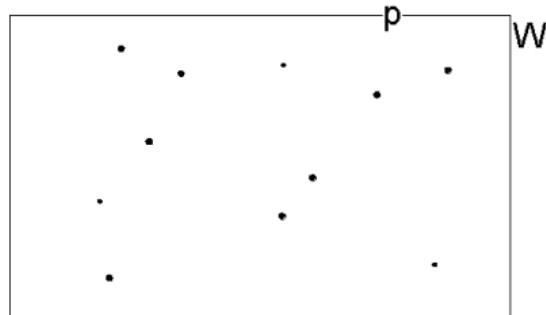


# Quantification over worlds

- new **modal** sentence/wff operators:
  - ▶ *necessarily p*:  $\Box p$
  - ▶ *possibly p*:  $\Diamond p$
- What does it mean for a proposition to be necessary/possible?

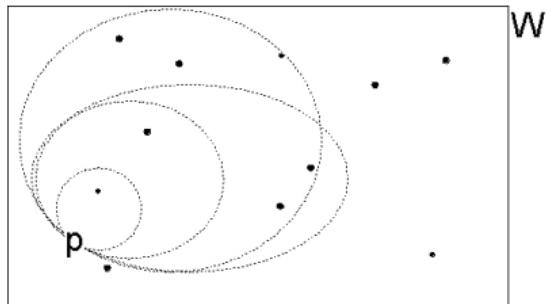
# Necessity as universal quantification

- if  $\Box p$  then  $(\forall w) [p(w) = 1]$  ( $p$  as characteristic function)
- such that  $W = p$  ( $p$  as set):



# Possibility as existential quantification

- if  $\Diamond p$  then  $(\exists w) [p(w) = 1]$  (characteristic function)
- such that  $p \neq \emptyset$  (set):

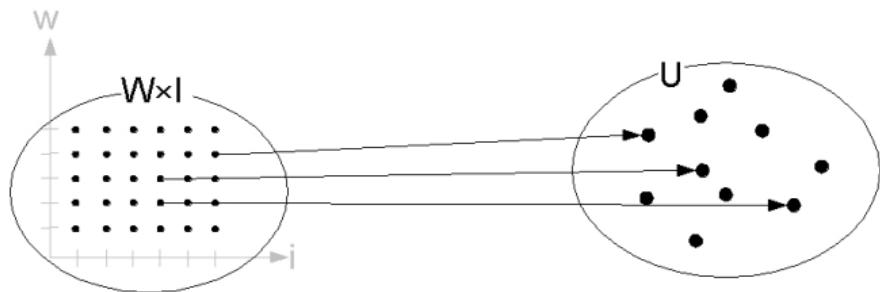


# A larger tuple

- $\mathcal{M} = \{W, I, <, U, V\}$ 
  - ▶  $W$ , a set of worlds
  - ▶  $I$ , a set of instants
  - ▶  $<$ , an ordering relation in  $I$
  - ▶  $U$ , the set of individuals
  - ▶  $V$ , a valuation function for constants
- evaluate an expression  $\alpha$ :  $\llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$

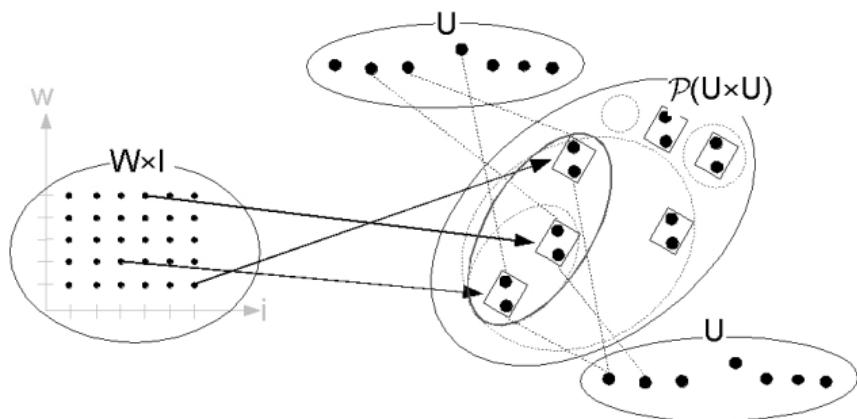
# Intensional interpretation of individual constants

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)
- for  $\beta \in \text{Cons}_{\text{ind}}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $U$



## ... and $\text{pred}_n$ s

- $\text{walks}$  etc. denotes different sets (or CFs) at different  $\langle w, i \rangle$  coordinates
- for  $\beta \in \text{Cons}_{\text{pred}_n}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $\wp U^n$  ( $U^n = U_1 \times U_2 \times \dots \times U_n$ )



# The Chierchia approach: predicates/sentences

- simple sentences/predicates:  $\beta = \delta(t_1, t_2, \dots, t_n)$
- $[\![\beta]\!]^{\mathcal{M}, w, i, g} = 1$  iff
- $\langle [\![t_1]\!]^{\mathcal{M}, w, i, g}, [\![t_2]\!]^{\mathcal{M}, w, i, g}, \dots, [\![t_n]\!]^{\mathcal{M}, w, i, g} \rangle \in [\![\delta]\!]^{\mathcal{M}, w, i, g}$
- with:  $[\![t_1]\!]^{\mathcal{M}, w, i, g} = V(t_1)(\langle w, i \rangle)$ , etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

- if  $\psi = \forall x\phi$  then
- ...  $\llbracket\psi\rrbracket^{\mathcal{M}, w, i, g} = 1$  iff for all  $u \in U$
- ...  $\llbracket\phi\rrbracket^{\mathcal{M}, w, i, g[u/x]} = 1$
- nothing new here

- if  $\psi = \Box x\phi$  then
- ... $\llbracket\psi\rrbracket^{\mathcal{M}, w, i, g} = 1$  iff for all  $w' \in W$
- ...and all  $i' \in I$
- ... $\llbracket\phi\rrbracket^{\mathcal{M}, w', i', g} = 1$

# A similarity of $\forall$ and $\square$

- as:  $\forall x [P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that:  $\square [\psi \rightarrow \phi] \rightarrow [\square \psi \rightarrow \square \phi]$
- but not vice-versa!

# Some validities

- $\exists x \square P(x) \rightarrow \square \exists x P(x)$
- $\exists x \diamond P(x) \leftrightarrow \diamond \exists x P(x)$
- $\forall x \square P(x) \leftrightarrow \square \forall x P(x)$  (Carnap-Barcan)
- $\forall x \diamond P(x) \rightarrow \diamond \forall x P(x)$

## Tempus und Modalität

# Targets for this week

- Understand how simple tense logic can be represented by operators shifting  $i$  indices.
- See why tense operators are sentence operators.
- See why a multi-dimensional theory of tenses and a better handling of tense embedding are required.
- See how we restrict (different types of) propositional backgrounds.
- Understand how opaque contexts affect meaning (incl. *believe* type verbs).
- Get a first idea of why we need the *up* operator  $\wedge$ .

- **present:** no operator ( $\phi$  ‘it is the case that  $\phi$ ’)
- **past:**  $P(\phi$  ‘it was the case that  $\phi$ ’)
- **future:**  $F(\phi$  ‘it will be the case that  $\phi$ ’)
- it will always be the case... ( $G = \neg F \neg \phi$ )
- it was always the case... ( $H = \neg P \neg \phi$ )

# Evaluation

- $\text{PD}(a)$  ‘Arno Schmidt (has?) died.’
- relative to the current  $\langle w, i \rangle$ :  $\llbracket \text{PD}(a) \rrbracket^{\mathcal{M}, w, i, g}$
- ...is true iff there is some  $i'$ ,  $\langle i', i \rangle \in <$  and
- $\llbracket \text{PD}(a) \rrbracket^{\mathcal{M}, w, i', g} = 1$

# Like it or not...

- tense operators (TOp) are sentence (wff) Op's
- raise it to sentence-scopal position
- TP/IP position is motivated by copular/auxiliary elements
- *He is stupid.* vs. *Kare-wa bakarashi-i.*
- *He was stupid.* vs. *Kare-wa bakarashi-katta.*
- *What<sub>i</sub>; did you expect t<sub>j</sub>?* vs. *Nani-o yokishi-ta-ka.*

# New ps rules

- $T' \rightarrow TVP$  (adds tense to VP)
- $TP \rightarrow NP T'$
- $TP \rightarrow TP \ conj \ TP$
- $TP \rightarrow neg \ TP$
- $[_{TP} \ NP \ T \ VP] \Rightarrow [_{TP} \ T \ NP \ VP]$  (T raising)

# Quantification over instants

- $\llbracket \mathbf{PTP} \rrbracket^{\mathcal{M}, w, i, g} = 1$
- iff among all  $\langle i_n, i \rangle \in <$
- there is **at least one** s.t.  $\llbracket \mathbf{TP} \rrbracket^{\mathcal{M}, w, i', g} = 1$

- $U$ : domain of quantification
- $V(\beta)$ : non-relativized function for all  $\beta$  which are not a proper name
- $V(\beta)(\langle w, i \rangle)$ :  $V$  evaluates  $\beta$  to a function from world-time pairs to the denotata of the predicate (sets of individuals, tuples of them, etc.)

- NL tenses beyond TOp's:
- *Arno Schmidt had already read Poe when he started writing 'Zettels Traum'.*
- *Gosh, I forgot to feed the cat.*
- shifts of evaluation time

	past (R<S)	present (R,S)	future (S<R)
anterior(E<R)	E<R<S <i>er war gegangen</i>	E<R,S <i>er ist gegangen</i>	S<E<R S,E<R E<S<R <i>er wird gegangen sein</i>
simple(E,R)	E,R<S <i>er ging</i>	E,R,S <i>er geht</i>	S<E,R <i>er wird gehen</i>
posterior(R<E)	R<E<S R<S,E R<S,E R<S<E <i>*er würde gehen</i>	R,S<E <i>er wird gehen</i>	S<R<E <i>*er wird gehen werden</i>

# Embedded tenses and adverbials

- *A man was born who will be king.*
- **P**(a man is born **F**(who be king)) ?
- *Yesterday, Maria woke up happy.*
- **Y(P(Maria wake up happy))** ?

# Types of modal expressions

- tense forms: *I eat up to 100 nachos a minute.*
- mood: *Resonderet alias minus sapienter.*
- modal auxiliaries: *Herr Webelhuth can look like Michael Moore.*
- adverbs: *Maybe Herr Keydana will show up.*
- affixes: *Frau Eckardt is recognizable.*

# The logical form of modal operators

- like tense: **sentence operators**
- modal Aux in English is tense-insensitive (evidence for *Infl*)
- $\Box$  and  $\Diamond$  in intensional predicate calculi (IPC): exploit the full set of possible worlds
- in NL: evaluation of modal expressions against restricted **conversational backgrounds**

# The background

- different sets of possible worlds under consideration for different types of modal expressions
- different types of modality: different sets of admitted possible worlds
- we call the conversationally relevant background **the set of  $\langle w, i \rangle$  pairs relevant to the interpretation of the sentence**

- Agent Cooper **cannot** solve the mystery.
- translated into root modal IPC:  $\neg\Diamond S(c, m)$
- wrong interpretation: Under no possible circumstances can Cooper solve the mystery.
- usually, some **obvious facts constitute the background**:
  - ▶ he could, but some relevant information is missing
  - ▶ he could, but is sick
  - ▶ he could, but ...

- *Leo Johnson must be the murderer of Laura Palmer.*
- in accordance with the **known facts** (e.g., in episode 7 of *Twin Peaks*):
  - ▶ Leo Johnson is a violent person.
  - ▶ Leo smuggles cocaine, Laura was addicted to it.
  - ▶ Leo is connected to Jacques Renault who is the bartender of *One Eyed Jack's* where Laura worked as a prostitute.
  - ▶ ...
- which constitute the epistemic background, the sentence is true
- known facts narrow down the root background

- *Agent Cooper must not solve the mystery.*
- assume:
  - ▶ there is some U.S. law which allows a local sheriff to ask the FBI to keep out of local murder investigations
  - ▶ Sheriff Truman has asked the FBI headquarters to keep out of the Palmer investigation
  - ▶ as a special agent, Cooper is required to obey Bureau policy
- Deontic backgrounds are narrowed down by **normative rules** and **moral ideals**.
- statable in propositional form (ten commandments, law, ...)

# Sets of propositions

- specify the kind of background against which you evaluate under the given situation
- we need:  
a function from  $\langle w, i \rangle$  to the relevant background set of  $\langle w_n, i_m \rangle$
- reuse  $g$ :  
 $g(\langle w, i \rangle) = \{p_1, p_2, \dots, p_n\} = \{\langle w, i \rangle_1, \langle w, i \rangle_2, \dots, \langle w, i \rangle_n\}$
- such that all possible worlds are:  $\bigcap g(\langle w, i \rangle)$

- *that* is a complementizer, it turns a sentence into an argument.
- ps rule:  $CP \rightarrow CIP$
- $[_{IP} Racine believes [_{CP} that [_{IP} theatre rules]]]$
- CP (fully fledged sentence) receives theta role by *believe* under government.

- gerunds:  
 $[_{IP} Stockhausen has plans [_{IP} to write another 29 hour opera]]$
- incomplete embedded IP, no subject
- internal theta role of *has plans*: to IP
- external theta role of *write*: to ?
- PRO, controlled by the subject of *has plans*:  
 $[_{IP} Stockhausen has plans [_{IP} PRO to write another 29 hour opera]]$

# Propositional attitudes

- verbs like *believe*: **propositional attitude verbs**
- content of the believe: a piece of information held to be true by the believer, hence a proposition, a  $\langle w_n, i_m \rangle$
- signalling one element in the background assumed by the believer
- belief:  $\langle w, i \rangle$  is an element of the proposition of CP

## Translating that as $\wedge$

- value of propositional attitude (PA) verbs: functions  $[\langle w, i \rangle \rightarrow \langle u_n, p \rangle]$  with  $u_n \in U$ ,  $p$  a proposition (set of  $\langle w_n, i_m \rangle$ ) and compatible to  $u_n$ 's background
- $up(\wedge\chi)$ : an operator which gives the intension of an expression  $\chi$
- the full logic of  $\wedge$  and  $\neg$  as designed by Montague next week
- $\wedge$  rids us of the problem that the belief content looks truth-conditional (a sentence) but doesn't contribute to the embedding sentence's truth-value. PA verbs take intensions as arguments.

- Quine's story: Ralph knows...
- Bernard J.Ortcutt, the nice guy on the beach.
- He sees a strange guy with a hat in the dark alley - a spy?
- Ortcutt just likes to behave funny on the way to his pub...
- and actually is sinister guy in the alley!
- Only Ralph doesn't know.

# Is Ralph insane?

- What's the truth value of...
- *Ralph believes that the guy from the beach is a spy.*
- true: since Orcutt and the guy in the hat are one individual
- false: since Ralph doesn't know that and in a way 'doesn't believe it'

- the Russelian interpretation for *the* like  $\exists$  with a uniqueness condition (as a GQ):  
$$\lambda Q \lambda P [\exists x [Q(x) \wedge P(x)] \wedge \forall y [Q(y) \leftrightarrow y = x]]$$
- in a raising framework: ambiguity between *THE* and *believe*
- [<sub>IP</sub> the guy from the beach; [<sub>IP</sub> Ralph believes [<sub>CP</sub> that  $x_i$  is a spy]]]
- makes the sentence true: the *de re* reading
- Ralph believes [<sub>CP</sub> that [<sub>IP</sub> the guy from the beach; [<sub>IP</sub>  $x_i$  is a spy]]]
- makes the sentence false: the *de dicto* reading

- *Yuri Gagarin might now have been the first man in space.*
- some Mickey Mouse LFs:
- $\Diamond \text{THE(first-man-in-space)}(\text{not-be-Gagarin})$
- at some  $\langle w_n, i_m \rangle$  the first individual in space is not Y.G.
- $\text{THE(first-man-in-space)}(\Diamond[\text{not-be-Gagarin}])$
- at  $\langle w, i \rangle$  the first individual in space (definitely Y.G.) is not Y.G. in an accessible world
- Names are rigid designators across world-time-pairs, definite descriptions aren't.

- CP has its own subject, to-IPs don't (PRO)
- PRO must be interpreted, in our examples by coindexation with the matrix subject
- infinitive embedding verbs: functions from world-time pairs to sets of individuals which have a certain property, the intension of a predicate  $\hat{P}$
- *John tries to sing.*
- $\text{try}(j, \hat{\text{swim}})$

Montagues intentionale Logik

- $\llbracket \phi \rrbracket^{\mathcal{M}, w, i, g}$  and  $\llbracket P \rrbracket^{\mathcal{M}, w, i, g}$  don't truth conditionally determine  $\llbracket P\phi \rrbracket^{\mathcal{M}, w, i, g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**,  $\diamond$ ,  $\square$  are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models

- for a name  $d$ :  $\llbracket d \rrbracket_{\mathcal{C}}^{\mathcal{M}, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for an individual concept denoting expression  $m$ :

$$\bullet \llbracket m \rrbracket_{\mathcal{C}}^{\mathcal{M}, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & c \\ \langle w_1, t_3 \rangle & \rightarrow & c \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$$

- for a one place predicate  $B$ :

$$\bullet \quad \llbracket B \rrbracket_{\mathcal{C}}^{\mathcal{M}, g} = \left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{array} \right]$$

# Intensions of formulas

- formula  $\phi$ :  $\llbracket \phi \rrbracket_{\mathcal{C}}^{\mathcal{M}, g}$  is a function from indices to truth values

$$\bullet \quad \llbracket B(m) \rrbracket_{\mathcal{C}}^{\mathcal{M}, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 1 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 0 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$$

$$\bullet \quad \llbracket B(n) \rrbracket_{\mathcal{C}}^{\mathcal{M}, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 0 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 1 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$$

- again, the proposition  $\llbracket Bm \rrbracket_{\mathcal{Q}}^{\mathcal{M}, g}$  is a set of indices  $(\langle w_i, t_j \rangle)$
- from the extension at all indices, compute the intension
- $\llbracket \alpha \rrbracket_{\mathcal{Q}}^{\mathcal{M}, g}(\langle w_i, t_j \rangle) = \llbracket \alpha \rrbracket^{\mathcal{M}, w_i, t_j, g}$

# Intensions of variables

- constant function on indices
- will play a great role, so remember!
- $\llbracket u \rrbracket_{\varsigma'}^{\mathcal{M}, g}(\langle w_i, t_j \rangle) = g(u)$

# What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes **they denote intensions** (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

- Church/Montague: for an extension-denoting expression  $\alpha$ ,  $\hat{\alpha}$  denotes  $\alpha$ 's intension
- $\llbracket \hat{Bm} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket Bm \rrbracket_{\hat{\alpha}}^{\mathcal{M}, g}$
- $\alpha$  and  $\hat{\alpha}$  are just denoting expressions
- for an intension-denoting expression  $\alpha$ :  $\llbracket \check{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\langle w, t \rangle)$

- observe:  $\llbracket \neg \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$
- but not always:  $\llbracket \neg \alpha \rrbracket^{\mathcal{M}, w, i, g} \neq \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

# Non-equality

- $k'$  intension:  $\llbracket k \rrbracket_{\mathcal{C}'}^{\mathcal{M}, g} =$

$$\begin{array}{ccl} \langle w_1, t_1 \rangle & \rightarrow & \left[ \begin{array}{ccc} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \\ \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \\ \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \\ \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{array} \right] \\ \langle w_1, t_2 \rangle & \rightarrow & \\ \langle w_2, t_1 \rangle & \rightarrow & \\ \langle w_2, t_2 \rangle & \rightarrow & \end{array}$$

# Non-equality

- $k'$  extension (e.g., at  $\langle w_1, t_2 \rangle$ ):  $\llbracket k \rrbracket_{\mathcal{M}}^{\mathcal{M}, g}(\langle w_1, t_2 \rangle) =$

- $\llbracket k \rrbracket^{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix}$

- however:  $\llbracket \neg k \rrbracket^{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix}$

- since:  $\llbracket \neg k \rrbracket^{\mathcal{M}, w_1, t_1, g} = a$

$$\llbracket \neg k \rrbracket^{\mathcal{M}, w_1, t_2, g} = b$$

$$\llbracket \neg k \rrbracket^{\mathcal{M}, w_2, t_1, g} = d$$

$$\llbracket \neg k \rrbracket^{\mathcal{M}, w_2, t_2, g} = b$$

# A typed higher order $\lambda$ language with $=$ and $\wedge / \vee$

- $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbf{F}, \mathbf{P}, \square, =$  (syncategorematically)
- $t, e \in Type$  ( $Con_{type}, Var_{type}$ )
- if  $a, b \in Type$ , then  $\langle a, b \rangle \in Type$
- if  $a \in Type$ , then  $\langle s, a \rangle \in Type$
- $s \notin Type$

# Meaningful expressions

- $ME_{type}$
- abstraction: if  $\alpha \in ME_a$ ,  $\beta \in Var_b$ ,  $\lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if  $\alpha \in ME_{\langle a,b \rangle}$ ,  $\beta \in ME_a$  then  $\alpha(\beta) \in ME_b$
- if  $\alpha, \beta \in ME_a$  then  $\alpha = \beta \in ME_t$

# Interpretations of $\wedge$ and $\neg$

- if  $\alpha \in ME_a$  then  $\wedge\alpha \in ME_{s,a}$
- if  $\alpha \in ME_{\langle s,a \rangle}$  then  $\neg\alpha \in ME_a$

type	variables	constants
$e$	$x, y, z$	$a, b, c$
$\langle s, e \rangle$	$x, y, z$	—
$\langle e, t \rangle$	$X, Y$	$walk', A, B$
• $\langle \langle s, e \rangle, t \rangle$	$Q$	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	$P$	—
$\langle e, e \rangle$	$P$	$Sq$
$\langle e, \langle e, t \rangle \rangle$	$R$	$Gr, K$
$\langle e, \langle e, e \rangle \rangle$	—	$Plus$

# The model

- $\langle A, W, T, <, F \rangle$
- $D_{\langle a,b \rangle} = D_b^{D_a}$
- $D_{\langle s,a \rangle} = D_a^{W \times T}$
- ‘senses’ = **possible** denotations
- actual intensions chosen from the set of senses
- now:  $F(\text{expression}) = \text{intension}$  (itself a function)
- s.t.  $\text{intension(index)} = \text{extention}$
- instead of:  $F(\text{expression})(\text{index}) = \text{extension}$

# Some interpretations

- $\llbracket \lambda u \alpha \rrbracket^{\mathcal{M}, w, i, g}$ ,  $u \in \text{Var}_b$ ,  $\alpha \in ME_a$  is a function  $h$  with domain  $D_b$  s.t.  $x \in D_b$ ,  
 $h(x) = \llbracket \alpha \rrbracket^{\mathcal{M}, w, t, g'}$  with  $g'$  exactly like  $g$  except  $g'(u) = x$
- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}$  is a function  $h$  from  $W \times T$  to denotations of  $\alpha$ 's type s.t. at every  
 $\langle w', t' \rangle \in W \times T$   $\llbracket \alpha \rrbracket^{\mathcal{M}, w', t', g} = h(\langle w', t' \rangle) = \llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}(\langle w', t' \rangle)$

## Some examples

- $\alpha = \beta$  at  $\langle w, t \rangle$  might be true, but  ${}^\wedge\alpha = {}^\wedge\beta$  need not be 1 at that same index
- on types:
  - ▶  $e$  - individuals
  - ▶  $\langle s, e \rangle$  - individual concepts ('present Queen of England')
  - ▶  $\langle s, \langle e, t \rangle \rangle$  - properties of individuals
  - ▶  $\langle e, t \rangle$  - sets of individuals
  - ▶  $\langle \langle s, e \rangle, t \rangle$  - sets of individual concepts

# Some examples

- on properties:
  - ▶  $\langle s, \langle a, t \rangle \rangle$  - properties of denotations of  $a$ -type expressions
  - ▶  $\langle s, \langle e, t \rangle \rangle$  - properties of individuals
  - ▶  $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$  - properties of propositions
- from relations  $\langle e, \langle e, t \rangle \rangle$  to relations-in-intensions  $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence:  $\langle s, a \rangle$  never applied to some typed argument ( $s$  is not a type!)
- useful thing: We never talk about indices!
- since often  $\mathbf{\tilde{\alpha}}(\beta)$  is needed for  $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$  and  $\beta \in ME_e$ , abbr.  $\alpha\{\beta\}$

- former problem with **Nec** as  $\langle t, t \rangle$ : non-compositional extensional interpretation
- $\text{Nec} \in ME_{\langle \langle s, t \rangle, t \rangle} - \{0, 1\}^{(\{0,1\}^{W \times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give  $\Box\phi$  as  $\text{Nec}(\wedge\phi)$

- ‘former’ as in ‘a former member of this club’
- instead of  $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$
- intensionally:  $\langle\langle s, \langle e, t \rangle \rangle, \langle e, t \rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property  $\langle e, t \rangle$  not at current but at some past index qualify
- formally:  $\llbracket \text{For} \rrbracket_{\zeta'}^{\mathcal{M}, g}$  is a func.  $h$  s.t. for any property  $k$ ,  $h(\langle w, t \rangle)(k)$  is the set  $k(\langle w, t' \rangle)$  for all  $t' < t$ .
- So, for any individual  $x$   $h(\langle w, t \rangle)(k)(x) = 1$  iff  $k(\langle w, t' \rangle)(x) = 1$  for some  $t' < t$ .

- relations between individuals and propositions
- $\langle \langle s, t \rangle, \langle e, t \rangle \rangle$
- $\text{Bel}(\wedge(B(m))(j))$  John believes that Miss America is bald.
- take the model from page 134 (Dowty et al.):
- $\llbracket B(m) \rrbracket^{M, w_2, t_1, g} = 1$  since  $\llbracket m \rrbracket^{M, w_2, t_1, g} = \llbracket n \rrbracket^{M, w_2, t_1, g}$
- however:  $\llbracket \wedge(B(m)) \rrbracket^{M, w_2, t_1, g} \neq \llbracket \wedge(B(n)) \rrbracket^{M, w_2, t_1, g}$

- $\text{Bel}(\hat{\wedge}(B(m))(j))$  ‘John believes that Miss America is bald.’
- $\text{Bel}(\hat{\wedge}(B(n))(j))$  ‘John believes that Norma is bald.’
- needn’t be equal: John can take worlds other than  $\langle w_2, t_1 \rangle$  into account where  $\llbracket n \rrbracket \neq \llbracket m \rrbracket$
- $\alpha = \beta \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$  is true iff  $\alpha$  is not in the scope of  $\hat{\wedge}, \mathbf{F}, \mathbf{P}, \Box$  (oblique contexts)
- however:  $\hat{\wedge}\alpha = \hat{\wedge}\beta \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$

- like so:  $\lambda x [\text{Bel}(\wedge [B(x)])(j)](m)$
- the above is true at an index  $\langle w, t \rangle$  iff  $[\![\text{Bel}(\wedge [B(x)])(j)]\!]^{w,t} = 1$   
if  $[\![m]\!]^{w,t} = x$ , i.e. if John is in a belief-rel with  $\wedge(B(x))$   
s.t.  $g(x) = m$  (by semantics of  $\lambda$ )
- Why is  $\wedge(B(x))$  not equal to  $\wedge(B(m))$ ?
- constant  $m$ : non-rigid designator relativized to indices
- variable  $x$ : a rigid designator by def. of  $g$  (for the relevant checking case with  $g(x) = \text{MissAmerica}$ )
- the above: a belief about ‘whoever  $m$  is’
- $\lambda$  conversion is restricted in IL!

# Once again

- *John believes that a republican will win.*
- $\exists x [Rx \wedge \text{Bel}(j, \wedge [\text{FW}(x)])]$
- $\text{Bel}(j, \mathbf{F} \exists x [R(x) \wedge W(x)])$

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