

Formale Semantik

07. Einfach getypte höherstufige L-Sprachen

Roland Schäfer

Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena

Folien in Überarbeitung. Englische Teile (ab Woche 6) sind noch von 2007!

Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

1 Preliminaries

- Different but related semantics
- Sets and characteristic functions
- Functional application

2 Simply typed languages

- New names for old categories

- The syntax of types
- Higher orders
- Summed up semantics for a higher-order language

3 Lambda languages

- From set constructor to the functional λ abstractor
- General syntax/semantics for λ languages
- A glimpse at quantification in Montague's system

Preliminaries

Montague and the generative tradition

- Chierchia & McConnell-Ginet, Heim & Kratzer, etc.: GB-ish semantics
- both syntax and LF in phrase structures
- LF as a proper linguistic level of representation
- Montague: direct translation of NL into logic
- Montague's LF is just a notational system for NL semantics

Targets for this week

- Learn to tell the difference between the montagovian and generative approach.
- See the advantage of a general theory of typed languages.
- Understand how λ languages allow dramatically elegant formalizations.
- ... while keeping in mind that these devices are extensions to our PC representation for NL semantics.

- denotations in set/function-theoretic terms
- a **characteristic function (CF)** \mathcal{S} of a set S :
 $\mathcal{S}(a) = 1$ *iff* $a \in S$, *else* 0
- a CF ‘checks’ individuals into a set
- denotations can be stated as sets or their CF

- interpretation for $[_S \text{ NP VP}]$:
 $\llbracket [_S \text{ NP VP}] \rrbracket^{\mathcal{M},g} = 1$ iff $\llbracket \text{NP} \rrbracket^{\mathcal{M},g} \in \llbracket \text{VP} \rrbracket^{\mathcal{M},g}$
- Montague generally used CF's in definitions
- evaluating $[_S [_{\text{NP}} \text{ Mary}] [_{\text{VP}} \text{ sleeps}]]$ as a matter of **functional application (FA)**:
 - ▶ $\llbracket \text{Mary} \rrbracket^{\mathcal{M},g} = \text{Mary in } \mathcal{M}$
 - ▶ $\llbracket \text{sleeps} \rrbracket^{\mathcal{M},g}$ be the CF of the set of sleepers in \mathcal{M}
 - ▶ $\llbracket S \rrbracket^{\mathcal{M},g} = \llbracket \text{sleeps} \rrbracket^{\mathcal{M},g}(\llbracket \text{Mary} \rrbracket^{\mathcal{M},g})$
 - ▶ ideally: generalize to all nodes

The superscript notation

- all functions from S_1 to S_2
- $S_2^{S_1}$
- for $T = \{0, 1\}$
 - ▶ T^D : all pred_1
 - ▶ $T^{D \times D}$: all pred_2
-

Simply typed languages

- base for Dowty et al.: L_1 , a first-order predicate language as we know it
- semantic renaming of types:
 - ▶ terms: $\langle e \rangle$ (entity-denoting)
 - ▶ formulas: $\langle t \rangle$ (truth-valued)
 - ▶ pred_1 : $\langle e, t \rangle$
 - ▶ pred_2 : $\langle e, \langle e, t \rangle \rangle$

Possible denotations of types

- D_α possible denotation (a set) of expressions of type α
- $D_{\langle e \rangle} = U$ (Dowty et al.'s A)
- $D_{\langle t \rangle} = \{0, 1\}$
- recursively: $D_{\langle \alpha, \beta \rangle} = D_{\langle \beta \rangle}^{D_{\langle \alpha \rangle}}$
- e.g., $D_{\langle e, t \rangle} = D_{\langle t \rangle}^{D_{\langle e \rangle}}$
- $D_{\langle e, \langle e, t \rangle \rangle} = (D_{\langle t \rangle}^{D_{\langle e \rangle}})^{D_{\langle e \rangle}}$
- just a systematic way of naming types, model-theoretic interpretations still by V, g

- in our PS syntax: S as start symbol
- in the typed system: sentences should be of type $\langle t \rangle$
- **complex types: functions from $\langle e \rangle$ to $\langle t \rangle$**
or generally **from any (complex) type to any (complex) type**

- **saturation** of complex types by FA:
 - ▶ γ is of type $\langle e, \langle e, t \rangle \rangle$, δ of $\langle e, t \rangle$, α and β of $\langle e \rangle$
 - ▶ then $\gamma(\alpha)$ is of type $\langle e, t \rangle$
 - ▶ and $\delta(\beta)$ is of type $\langle t \rangle$
- for any pred₂ P and its arguments a_1, a_2 , $P(a_2)(a_1)$ is a wff
- connectives are of types $\langle t, t \rangle$ (\neg), $\langle t, \langle t, t \rangle \rangle$ (\wedge , etc.)

- generalized CF/FA approach
- $\langle e \rangle$ -types (terms):
 - $\llbracket a_n \rrbracket^{\mathcal{M},g} = V(a_n)$
 - $\llbracket x_n \rrbracket^{\mathcal{M},g} = g(x_n)$
- the rest: functional application
 - $\llbracket \delta(\alpha) \rrbracket^{\mathcal{M},g} = \llbracket \delta \rrbracket^{\mathcal{M},g}(\llbracket \alpha \rrbracket^{\mathcal{M},g})$

- *Type* is the set of types
- recursively defined complex types $\langle a, b \rangle$: infinite
- type label $\langle \alpha \rangle$
- vs. set of meaningful expressions of that type: $ME_{\langle \alpha \rangle}$

- first order languages: variables over individuals ($\langle e \rangle$ -types)
- n-order: **variables over higher types** ($\langle e, t \rangle$ -types etc.)
- $P_{\langle e, t \rangle}$ or $Q_{\langle e, \langle e, t \rangle \rangle}$: constants of higher types
- so: $v_{1_{\langle e, t \rangle}} [v_1(m)]$
- if $V(m) = \text{Mary}$, v_1 is the set of all of Mary's properties

- we write:
 - ▶ $v_{n\langle\alpha\rangle}$ for the n-th variable of type $\langle\alpha\rangle$
 - ▶ Dowty et al.: $v_{n,\langle\alpha\rangle}$
- alternatively abbreviated by old symbols x_1 , a , P , etc.

- non-logical constant α : $\llbracket \alpha \rrbracket^{\mathcal{M},g} = V(\alpha)$
- variable α : $\llbracket \alpha \rrbracket^{\mathcal{M},g} = V(\alpha)$
- $\alpha \in \langle a, b \rangle$, $\beta \in a$, then $\llbracket \alpha(\beta) \rrbracket^{\mathcal{M},g} = \llbracket \alpha \rrbracket^{\mathcal{M},g}(\llbracket \beta \rrbracket^{\mathcal{M},g})$

- logical constants interpreted as functions in $\{0,1\}$ as usual
- if $v_{1\langle\alpha\rangle}$ is a variable and $\phi \in ME_t$
then $\llbracket (\forall v_1)\phi \rrbracket^{\mathcal{M},g} = 1$ iff
for all $a \in D_\alpha$ $\llbracket \phi \rrbracket^{\mathcal{M},g[a/v_1]} = 1$

An example

- quantified variable of type $\langle e, t \rangle$: $v_{0_{\langle e, t \rangle}}$
- $\forall v_{0_{\langle e, t \rangle}} \left[v_{0_{\langle e, t \rangle}}(j) \rightarrow v_{0_{\langle e, t \rangle}}(d) \right]$
- for $j, d \in ME_{\langle e \rangle}$
- one property of every individual: being alone in its union set
- hence, $j = d$
- else in $\forall v_{0_{\langle e, t \rangle}}$, \forall wouldn't hold

- productive adjectival prefix: *non-adjacent*, *non-local*, etc.
- inverting the characteristic function of the adjective
- result denotes complement of the original adjective in $D_{\langle e \rangle}$
- *adjective*: $\langle e, t \rangle$, *non*: $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$
- a function h s.t. for every $k \in D_{\langle e, t \rangle}$ and every $d \in D_{\langle e \rangle}$
 $(h(k))(d) = 1$ iff $k(d) = 0$ and
 $(h(k))(d) = 0$ iff $k(d) = 1$

- **understood objects** in: *I eat.* - *Vanity kills.* - etc.
- *eat* is in $ME_{\langle e, \langle e, t \rangle \rangle}$
- assume a **silent logical constant**: R_O in $ME_{\langle \langle e, \langle e, t \rangle \rangle, \langle e, t \rangle \rangle}$
- a function h s.t. for all $k \in D_{\langle e, \langle e, t \rangle \rangle}$ and all $d \in D_{\langle e \rangle}$
 $h(k)(d) = 1$ iff there is some $d' \in D_{\langle e \rangle}$ s.t. $k(d')(d) = 1$
- passives as similar subject deletion

Lambda languages

All there is to λ

- a new variable binder
- allows abstraction over wff's of arbitrary complexity
- similar to $\{x \mid \phi\}$ (read as 'the set of all x s.t. ϕ ')
 - we get $\lambda x [\phi]$
- on Montague's typewriter: $\hat{x} [\phi]$
- does not create a set but a function which can be taken as the CF of a set

- for every wff ϕ , any $x \in Var$, and any $a \in Con$
- λ abstraction: $\phi \rightarrow \lambda x [\phi^{[a/x]}] (a)$
- read $\phi^{(a/x)}$ as ‘ ϕ with every a replaced by x ’
- x can be of any type

Two informal examples

- $\lambda x_{\langle e \rangle} [L(x)]$ is the characteristic function of the set of those individuals $d \in D_{\langle e \rangle}$ which have property L
- $\lambda x_{\langle e, t \rangle} [x(l)]$ is the characteristic function of the set of those properties $k \in D_{\langle e, t \rangle}$ that the individual l has

- $\lambda x [L(x)]$ is the abstract of $L(a)$ (with some individual a)
- hence, it holds: $\lambda x [L(x)] (a) \Leftrightarrow L(a)$
- for every wff ϕ , any $x \in Var$, and any $a \in Con$
- λ conversion: $\lambda x [\phi] (a) \rightarrow \phi^{[x/a]}$

- $\lambda x [\phi] (a) \leftrightarrow \phi^{[x/a]}$
- not just syntactically, since truth conditions are equivalent
- $\lambda x [\phi] (a) \Leftrightarrow \phi^{[x/a]}$
- notice: $\lambda x_{\langle \alpha \rangle} [\phi]$ is in $ME_{\langle \alpha, t \rangle}$
- while ϕ (as a wff) is in $ME_{\langle t \rangle}$

- Dowty et al., 102f. (*Syn C.10* and *Sem 10*)
- If $\alpha \in ME_\alpha$ and $u \in Var_b$, then $\lambda u [\alpha] \in ME_{\langle b, \alpha \rangle}$.
- If $\alpha \in ME_a$ and $u \in Var_b$ then $\llbracket \lambda u [\alpha] \rrbracket^{\mathcal{M}, g}$ is that function h from D_b into D_a s.t. for all objects k in D_b , $h(k)$ is equal to $\llbracket \alpha \rrbracket^{\mathcal{M}, g[k/u]}$.

The *non* example revised (Dowty et al., 104)

- $\forall x \forall v_{0\langle e,t \rangle} \left[(\mathbf{non}(v_{0\langle e,t \rangle}))(x) \leftrightarrow \neg(v_{0\langle e,t \rangle}(x)) \right]$
- $\forall v_{0\langle e,t \rangle} \left[\lambda x \left[(\mathbf{non}(v_{0\langle e,t \rangle}))(x) \right] = \lambda x \left[\neg(v_{0\langle e,t \rangle}(x)) \right] \right]$
- $\forall v_{0\langle e,t \rangle} \left[\mathbf{non}(v_{0\langle e,t \rangle}) = \lambda x \left[\neg(v_{0\langle e,t \rangle}(x)) \right] \right]$
(since $\lambda x [\mathbf{non}(v)(x)]$ is unnecessarily abstract/ η reduction)
- $\lambda v_{0\langle e,t \rangle} \left[\mathbf{non}(v_{0\langle e,t \rangle}) = \lambda v_{0\langle e,t \rangle} \left[\lambda x \left[\neg(v_{0\langle e,t \rangle}(x)) \right] \right] \right]$
- and since that is about all assignments for $\lambda v_{0\langle e,t \rangle}$:
 $\mathbf{non} = \lambda v_{0\langle e,t \rangle} \left[\lambda x \left[\neg v_{0\langle e,t \rangle}(x) \right] \right]$

Mary is non-adjacent.

(translate 'adjacent' as $c_{0\langle e,t \rangle}$, 'Mary' as $c_{0\langle e \rangle}$, ignore the copula)

The behavior of quantified NPs

- syntactically like referential NPs
- semantically like PC quantifiers
- *Every student walks.*: $\forall v_{0\langle e \rangle} \left[c_{0\langle e, t \rangle}(v_{0\langle e \rangle}) \rightarrow c_{1\langle e, t \rangle}(v_{0\langle e \rangle}) \right]$
- *Some student walks.*: $\forall v_{0\langle e \rangle} \left[c_{0\langle e, t \rangle}(v_{0\langle e \rangle}) \wedge c_{1\langle e, t \rangle}(v_{0\langle e \rangle}) \right]$
- making referential NPs and QNPs **the same type?**

A higher type

- $\lambda v_{0\langle e,t \rangle} \forall v_{0\langle e \rangle} \left[c_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \rightarrow v_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \right]$
- a second order function
- characterizes the set of all predicates true of every student
- equally: $\lambda v_{0\langle e,t \rangle} \exists v_{0\langle e \rangle} \left[c_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \wedge v_{0\langle e,t \rangle}(v_{0\langle e \rangle}) \right]$

Combining with some predicate

Kontakt

Prof. Dr. Roland Schäfer
Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena
Fürstengraben 30
07743 Jena

<https://rolandschaefer.net>
roland.schaefer@uni-jena.de

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