

Formale Semantik

o8. Intensionalität

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stets aktuelle Fassungen: <https://github.com/rsling/VL-Deutsche-Syntax>

- 1 **Intensionality**
 - Problems with extensionality and non-dimensional models
 - Intensions
- 2 **A formal account of intensions**
 - Sets of PSOAs
 - Intensions as functions
 - Repeat after me...
- 3 **Sets of worlds**
 - Known relations
 - Modal operators
- 4 **Intensional Model Theory**
 - Ingredients of models
 - Evaluating individual constants
 - Set membership
 - Some peculiarities of \Box and \Diamond

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- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Intensionality

Some examples

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- Gustave Moreau **believes that** estheticism rules.

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- ... and for just one state of affairs (*modals, believe type verbs*)

What are intensions?

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts or propositions

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- not mental representations
- mediate between internal knowledge and truth-values

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- PSOAs are logically constrained
- observe the more than just truth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSA logic

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 - ▶ nothing of the above, but Maria was born 20 years earlier
 - ▶ nothing of the above, but A.S. rose from the dead in 2003, etc.

A formal account of intensions

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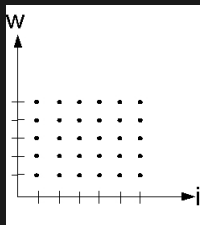
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- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

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- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$



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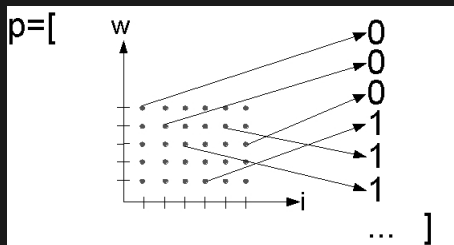
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- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

Propositions as functions

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- is a function from $W \times I$ to $\{0, 1\}$



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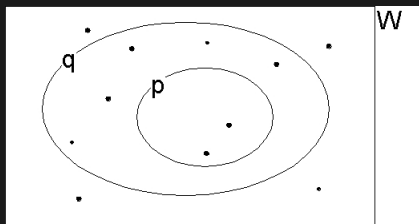
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- Hence, we call that function the intension of the sentence.

Sets of worlds

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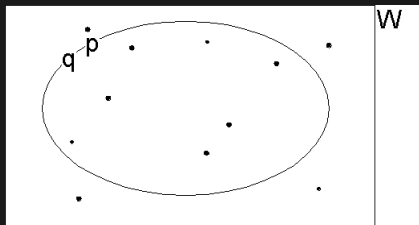
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- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation: $p \subseteq q$:



- synonymy turns out as set equivalence:

Synonymy

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- $p = q$

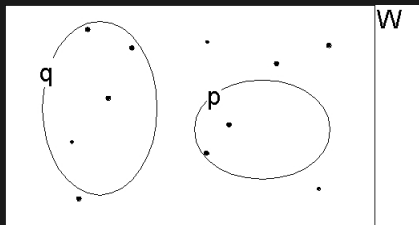


Contradiction

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Contradiction

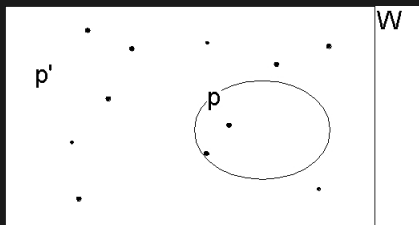
- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



- **negation** turns out as a **complement**:

Negation

- negation turns out as a complement:
- p/W



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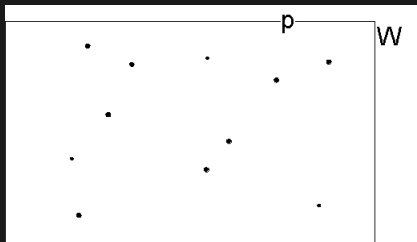
- new **modal** sentence/wff operators:
 - ▶ *necessarily* p : $\Box p$
 - ▶ *possibly* p : $\Diamond p$
- What does it mean for a proposition to be necessary/possible?

Necessity as universal quantification

- if $\Box p$ then $(\forall w) [p(w) = 1]$ (p as characteristic function)

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- such that $W = p$ (p as set):

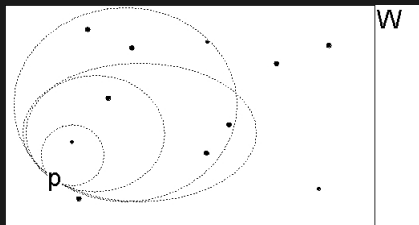


Possibility as existential quantification

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Intensional Model Theory

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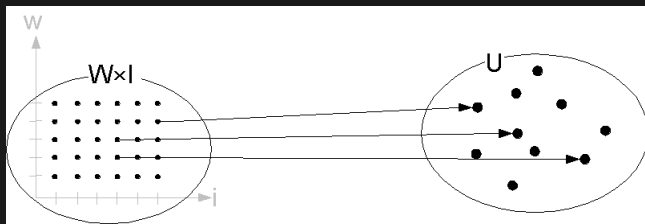
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- evaluate an expression α : $\llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)

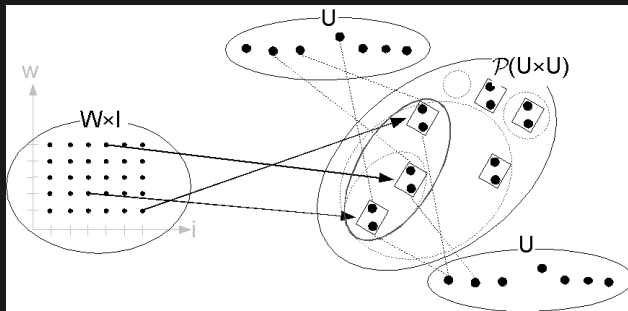
Intensional interpretation of individual constants

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)
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- for $\beta \in \text{Cons}_{\text{pred}_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ ($U^n = U_1 \times U_2 \times \dots \times U_n$)



The Chierchia approach: predicates/sentences

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- with: $\llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

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- nothing new here

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A similarity of \forall and \Box

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Some validities

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Kontakt

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