Formale Semantik
07. Einfach getypte höherstufige L-Sprachen

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Folien in Überarbeitung. Englische Teile (ab Woche 7) sind noch von 2007!
Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

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1. Preliminaries
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3. Lambda languages
   - A glimpse at quantification in Montague's system

Kernfragen in dieser Woche

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Wie unterscheidet sich Montagues System von GB-Semantik?

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Welche Rolle spielen Typen?

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Welche Rolle spielen Typen?

Was sind λ-Sprachen?
Und woher kennen Sie den λ-Operator eigentlich schon?

Preliminaries

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- LF as a proper linguistic level of representation
- Montague: direct translation of NL into logic
- Monatgue's LF is just a notational system for NL semantics

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- See the advantage of a general theory of typed languages.
- Understand how λ languages allow dramatically elegant formalizations.
- … while keeping in mind that these devices are extensions to our PC representation for NL semantics.

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- denotations can be stated as sets or their CF

Generalizing combinatory semantic operations

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  - ideally: generalize to all nodes

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- $D_{\langle e, \langle e,t \rangle \rangle} = (D_{\langle t \rangle}^{D_{\langle e \rangle}})^{D_{\langle e \rangle}}$
- just a systematic way of naming types, model-theoretic interpretations still by V, g

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- complex types: functions from $\langle e \rangle$ to $\langle t \rangle$
  or generally from any (complex) type to any (comlex) type

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- for any pred$_2$ P and its arguments a_1, a_2, $P(a_2)(a_1)$ is a wff
- connectives are of types $\langle t, t\rangle$ ($\neg$), $\langle t, \langle t, t\rangle\rangle$ ($\wedge$, etc.)

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  $[\![x_n]\!]^{\mathcal{M},g} = g(x_n)$
- the rest: functional application
  $[\![\delta(\alpha)]\!]^{\mathcal{M},g} = [\![\delta]\!]^{\mathcal{M},g}([\![\alpha]\!]^{\mathcal{M},g})$

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- vs. set of meaningful expressions of that type: $ME_{\langle\alpha\rangle}$

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- if $V(m) = Mary$, v_1 is the set of all of Mary's properties

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- alternatively abbreviated by old symbols x_1, a, P, etc.

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- $\alpha \in \langle a, b \rangle,\ \beta \in a$, then $[\![\alpha(\beta)]\!]^{\mathcal{M},g} = [\![\alpha]\!]^{\mathcal{M},g}([\![\beta]\!]^{\mathcal{M},g})$

Logical constants and quantifiers

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- if $v_{1_{\langle\alpha\rangle}}$ is a variable and $\phi \in ME_t$
  then $[\![(\forall v_1)\phi]\!]^{\mathcal{M},g} = 1 \textit{iff}$
  for all $a \in D_\alpha$ $[\![\phi]\!]^{\mathcal{M},g[a/v_1]} = 1$

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- else in $\forall v_{0_{\langle e,t\rangle}}$, $\forall$ wouldn't hold

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  $(h(k))(d) = 1$ iff $k(d) = 0$ and
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- passives as similar subject deletion

Lambda languages

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- we get $\lambda x\,[\phi]$
- on Montague's typewriter: $\hat{x}\,[\phi]$
- does not create a set but a function which can be taken as the CF of a set

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- x can be of any type

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- $\lambda x_{\langle e,t \rangle}\,[x(l)]$ is the characteristic function of the set of those properties $k \in D_{\langle e,t \rangle}$ that the individual l has

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- for every wff ϕ, any $x \in Var$, and any $a \in Con$
- λ conversion: $\lambda x\,[\phi]\,(a) \rightarrow \phi^{[x/a]}$

λ in and out

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The *non* example revised (Dowty et al., 104)

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- and since that is about all assignments for $\lambda v_{0_{\langle e,t\rangle}}$:
  $\mathbf{non} = \lambda v_{0_{\langle e,t\rangle}} \left[\lambda x \left[\neg v_{0_{\langle e,t\rangle}}(x) \right] \right]$

Mary is non-adjacent.

(translate ‘adjacent’ as $c_{0_{\langle e,t\rangle}}$, ‘Mary’ as $c_{0_{\langle e\rangle}}$, ignore the copula)

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- making referential NPs and QNPs the same type?

A higher type

- $\lambda v_{0_{\langle e,t \rangle}} \forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \rightarrow v_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \right]$

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- $\lambda \boldsymbol{v}0_{\langle e,t\rangle} \forall \boldsymbol{v}0_{\langle e\rangle} \left[\boldsymbol{c}0_{\langle e,t\rangle}(\boldsymbol{v}0_{\langle e\rangle}) \rightarrow \boldsymbol{v}0_{\langle e,t\rangle}(\boldsymbol{v}0_{\langle e\rangle}) \right]$
- a second order function
- characterizes the set of all predicates true of every student

A higher type

- $\lambda V_{0_{\langle e,t\rangle}} \forall V_{0_{\langle e\rangle}} \left[C_{0_{\langle e,t\rangle}} \left(V_{0_{\langle e\rangle}} \right) \rightarrow V_{0_{\langle e,t\rangle}} \left(V_{0_{\langle e\rangle}} \right) \right]$
- a second order function
- characterizes the set of all predicates true of every student
- equally: $\lambda V_{0_{\langle e,t\rangle}} \exists V_{0_{\langle e\rangle}} \left[C_{0_{\langle e,t\rangle}} \left(V_{0_{\langle e\rangle}} \right) \wedge V_{0_{\langle e,t\rangle}} \left(V_{0_{\langle e\rangle}} \right) \right]$

Combining with some predicate

Literatur I

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