Formale Semantik
04. Aussagenlogik

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stets aktuelle Fassungen: `https://github.com/rsling/VL-Deutsche-Syntax`

Inhalt

1. What logic is about
   - On reasoning
   - Where we need logic
2. Statement calculus
   - Formalization: Recursive Syntax
   - Interpretation
   - Laws of the PropC
   - Rules of Inference
   - Proof

The book (PMW:87-246) deals with logic far more in-depth than we do. Only what is mentioned on the slides is relevant for the test. Reading the whole chapter from PMW will do you no harm, though.

What logic is about

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- rechecking for usability: e.g., Russell's paradox

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- theorem: a proposition you want to prove
- lemma: subsidiary propositions (used to prove the theorem)
- corollary: propositions proved while proving some axiom

A method of reasoning

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- logic does not generate truths
- formalizing statements, predications etc.
- rules of deduction from axioms to theorems
- empirical (induction) and exact (deduction) science
- aiming at an adequate model of the world (e.g., heliocentric universe)

Why logic for semantics?

- truth-conditional

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- compositional behavior of propositions and connectives

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Why logic for semantics?

- truth-conditional
- compositional behavior of propositions and connectives
- a logic for entailments
- why, e.g.: *It is not the case that someone is happy.* $\rightarrow$ *Nobody is happy.*

Statement calculus

Atomic formulas: statements

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- statements/propositions = the atoms
- a propositional symbol *p*: a well-formed formula (wff)
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- $[\![k]\!]$=1 or 0 (depending on corresponding **model**)

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  - $p \vee q$ (disjunction)
  - $p \wedge q$ (conjunction)
  - $p \rightarrow q$ (conditional)
  - $p \leftrightarrow q$ (biconditional)

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  - $\neg p$ (negation - 'not')
  - $p \vee q$ (disjunction - 'or')
  - $p \wedge q$ (conjunction - 'and')
  - $p \rightarrow q$ (conditional - 'if')
  - $p \leftrightarrow q$ (biconditional - 'iff')

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Functions and truth tables

- standard defintion:

$$[\![\neg]\!] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$$

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- but most widely used: truth tables

$\neg$	p
0	1
1	0

Disjunction

p	$\vee$	q
1	1	1
1	1	0
0	1	1
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- $K \vee L$

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p	$\rightarrow$	q
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1	0	0
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p	$\rightarrow$	q
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- ***If*** *it rains,* ***then*** *the streets get wet.*

Conditional

p	$\rightarrow$	q
1	1	1
1	0	0
0	1	1
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- ***If*** *it rains,* ***then*** *the streets get wet.*
- $R \rightarrow S$

Any problems with that?

If it rains, the streets get wet.

- it is raining (1) , the streets are wet 1 : **1**

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- it is not raining (0) , the streets are wet 1 : **1**

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- ex vero non sequitur falsum

Biconditional

p	$\leftrightarrow$	q
1	1	1
1	0	0
0	0	1
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p	$\leftrightarrow$	q
1	1	1
1	0	0
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- ***If and only if*** *your score is above 50,* ***then*** *you pass the semantics exam.*

Biconditional

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- $S \leftrightarrow P$

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- or set non-default functor scope
- default scope

$$\text{scope} \left\downarrow \begin{array}{c} \neg \\ \wedge \\ \vee \\ \rightarrow \\ \leftrightarrow \end{array} \right\uparrow \text{binding strength}$$

An example

- $p \wedge \neg q \vee r \rightarrow \neg s$

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- $p \wedge (\neg q) \vee r \rightarrow (\neg s)$

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- $p \wedge (\neg q) \vee r \rightarrow (\neg s)$
- $(p \wedge (\neg q)) \vee r \rightarrow (\neg s)$
- $((p \wedge (\neg q)) \vee r) \rightarrow (\neg s)$

An example

- $p \land \neg q \lor r \to \neg s$
- $p \land (\neg q) \lor r \to (\neg s)$
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- $(((p \land (\neg q)) \lor r) \to (\neg s))$

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- for *n* atoms in the term: 2^n lines
- alternating blocks of 1's and 0's under every atom
- $2^{(m-1)}$ times '1' followed by $2^{(m-1)}$ times '0' for the *m*-th atom from the right
- until 2^n lines are reached

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1			1		1			1
1			1		1			0
1			1		0			1
1			1		0			0
1			0		1			1
1			0		1			0
1			0		0			1
1			0		0			0
0			1		1			1
0			1		1			0
0			1		0			1
0			1		0			0
0			0		1			1
0			0		1			0
0			0		0			1
0			0		0			0

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p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1		0	1		1		0	1
1		0	1		1		1	0
1		0	1		0		0	1
1		0	1		0		1	0
1		1	0		1		0	1
1		1	0		1		1	0
1		1	0		0		0	1
1		1	0		0		1	0
0		0	1		1		0	1
0		0	1		1		1	0
0		0	1		0		0	1
0		0	1		0		1	0
0		1	0		1		0	1
0		1	0		1		1	0
0		1	0		0		0	1
0		1	0		0		1	0

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1	0	0	1		0		0	1
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1	1	1	0		1		0	1
1	1	1	0		1		1	0
1	1	1	0		0		0	1
1	1	1	0		0		1	0
0	0	0	1		1		0	1
0	0	0	1		1		1	0
0	0	0	1		0		0	1
0	0	0	1		0		1	0
0	0	1	0		1		0	1
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1	1	1	0	1	0		0	1
1	1	1	0	1	0		1	0
0	0	0	1	1	1		0	1
0	0	0	1	1	1		1	0
0	0	0	1	0	0		0	1
0	0	0	1	0	0		1	0
0	0	1	0	1	1		0	1
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1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
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0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Assignments: a contingent example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
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- true under every assignment, it is **valid**

Tautology

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- truth-table:

p	$\lor$	$\neg$	p
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- true under every assignment, it is **valid**
- by *law of excluded middle*: for every P, $P \lor \neg P$ is true

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- false under every assignment, called **contradictory**

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- Idempotency (Idemp.):
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  - *Peter walks and Peter walks.* $\Leftrightarrow$ *Peter walks.*

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  - *Peter walks or Sue snores.* $\Leftrightarrow$ *Sue snores or Peter walks.*

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- Distributive Laws for $\vee\wedge$ and $\wedge\vee$ (Distr.):

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  - $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$
  - *((He walks or she talks) or we walk.)* $\Leftrightarrow$ *(He walks or (she talks or we walk.))*
- Commutative Laws for $\vee$ and $\wedge$ (Comm.):
  - $(P \vee Q) \Leftrightarrow (Q \vee P)$
  - *Peter walks or Sue snores.* $\Leftrightarrow$ *Sue snores or Peter walks.*
- Distributive Laws for $\vee\wedge$ and $\wedge\vee$ (Distr.):
  - $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$

Simple laws

- Associative Laws for $\vee$ and $\wedge$ (Assoc.):
  - $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$
  - *((He walks or she talks) or we walk.)* $\Leftrightarrow$ *(He walks or (she talks or we walk.))*
- Commutative Laws for $\vee$ and $\wedge$ (Comm.):
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  - *Peter walks or Sue snores.* $\Leftrightarrow$ *Sue snores or Peter walks.*
- Distributive Laws for $\vee\wedge$ and $\wedge\vee$ (Distr.):
  - $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
  - *(Sue snores) and (Peter walks or we talk).* $\Leftrightarrow$ *(Sue snores and Peter walks) or (Sue snores and we talk).*

Laws dealing with tautology and contradiction

- Complement Laws:

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Laws dealing with tautology and contradiction

- Complement Laws:
  - Tautology (T): $(P \vee \neg P) \Leftrightarrow \mathbf{T}$
  - Contradiction (F): $(P \wedge \neg P) \Leftrightarrow \mathbf{F}$
  - Double Negation (DN): $(\neg\neg P) \Leftrightarrow P$
  - *It is not the case that Sandy is not walking.*
    $\Leftrightarrow$ *Sandy is walking.*

Conditionals Laws

- **Implication** (Impl.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	P	$\vee$	Q
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

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- **Contraposition** (Contr.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	Q	$\rightarrow$	$\neg$	P
1	1	1		0	1	1	0	1
1	0	0		1	0	0	0	1
0	1	1		0	1	1	1	0
0	1	0		1	0	1	1	0

DeMorgan (DeM)

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  - $\neg(P \lor Q) \Leftrightarrow (\neg P \land \neg Q)$
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  - $\neg(P \land Q) \Leftrightarrow (\neg P \lor \neg Q)$

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- DeMorgan's Laws:
  - $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$
  - alternatively: $\overline{P \vee Q} \Leftrightarrow \overline{P} \wedge \overline{Q}$
  - $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
  - consequently: $\overline{\overline{P} \vee \overline{Q}} \Leftrightarrow \overline{\overline{P}} \wedge \overline{\overline{Q}} \Leftrightarrow P \wedge Q$

The Modus Ponens (MP)

- Definition:

P	$\rightarrow$	Q	premise 1
P			premise 2
		Q	conclusion

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The Modus Ponens (MP)

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- or: $(P \rightarrow Q) \wedge (P) \rightarrow (Q)$
- *(1) If It rains, the streets get wet. (2) It is raining.*
  $\rightarrow$ *The streets are getting wet.*

MP: a truth table illustration

- Premises are always set to be true!

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- the table:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

- The conditional must be true.
- cancel the ‘false’ row

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- P must be true.
- cancel the 'false' rows, Q can only be true:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

The Modus Tollens (MT)

- Definition:

$$\boxed{\begin{array}{lll} P & \rightarrow & Q \\ & & \neg Q \\ \hline \neg P & & \end{array}}$$

The Modus Tollens (MT)

- Definition:

P	$\rightarrow$	Q
		$\neg Q$
$\neg P$		

- the table illustration:

P	$\rightarrow$	Q	
1	1	1	(by premise 2)
1	0	0	(by premise 1)
0	1	1	(by premise 2)
0	1	0	

The Syllogisms

- Hypothetical Syllogism (HS):
  - $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
  - *(1) If it rains, the streets get wet. (2) If the streets get wet, it smells nice. $\rightarrow$ If it rains, it smells nice.*

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- Disjunctive Syllogism (DS):
  - $((P \vee Q) \wedge (\neg P)) \rightarrow (Q)$
  - *(1) Either Peter sleeps or Peter is awake. (2) Peter isn't awake. $\rightarrow$ Peter sleeps.*

Trivial rules

- Simplification (Simp.):
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Trivial rules

- Simplification (Simp.):
  - $(P \land Q) \rightarrow P$
  - *(1) It is raining and the sun is shining.* $\rightarrow$ *It is raining.*
- Conjunction (Conj.):
  - $(P) \land (Q) \rightarrow (P \land Q)$
  - *(1) It is raining. (2) The sun is shining.* $\rightarrow$ *It is raining and the sun is shining.*
- Addition (Add.):
  - $(P) \rightarrow (P \land Q)$
  - *(1) It is raining.* $\rightarrow$ *It is raining or the sun is shining.*
  - What if Q is instantiated as true or false by another premise?

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$
- The proof:

		$p \vee q$
1	$(p \vee q) \rightarrow \neg(r \wedge \neg s)$	
2	$r \wedge \neg s$	
	$p \vee q$	1,2,MT

Literatur I

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