# Formale Semantik
# 08. Intensionalität

Roland Schäfer

Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena

Folien in Überarbeitung. Englische Teile (ab Woche 6) sind noch von 2007!

Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

# Inhalt

# Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

# Intensionality

# Some examples

- Stockhausen will write another opera.
- Had Arno Schmidt cut down on drinking, he would still be alive.
- Gustave Moreau believes that estheticism rules.

# Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (tense)
- ... and for just one state of affairs (modals, believe type verbs)

# What are intensions?

| **Type** | **Reference** | **Sense** |
|---|---|---|
| NP | individuals<br>*Venus* | individual concepts |
| VP | sets<br>*humming birds* | property concepts |
| S | 1 or 0<br>*I like cats.* | thoughts or **propositions** |

# Properties of intensions

- can't be just truth conditional
- encode knowledge about not just the actual but all possible and/or past/future states of affairs (PSOAs)
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values

# PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just thruth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

# A touch of parellel universes?

- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
  - A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
  - nothing of the above, but Maria was born 20 years earlier
  - nothing of the above, but A.S. rose from the dead in 2003, etc.

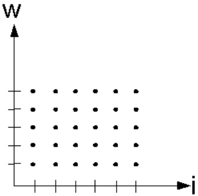
# A formal account of intensions

# Propositions and PSOAs

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

# Coordinates

- for each possible distinction in truth values of the whole of the propositional sentences: one possible world ($w \in W$)
- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$

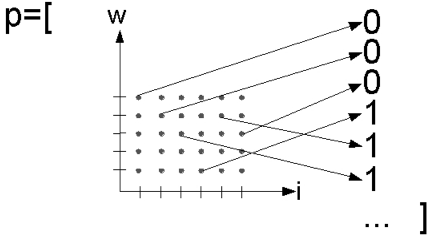


# The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

# Propositions as functions

- a propositional function *p*
- is a function from $W \times I$ to $\{0, 1\}$



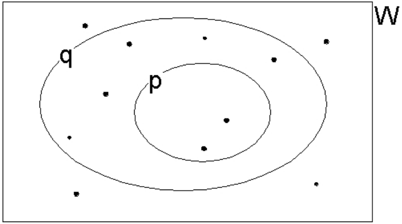
# Your evening prayer

- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

# Sets of worlds

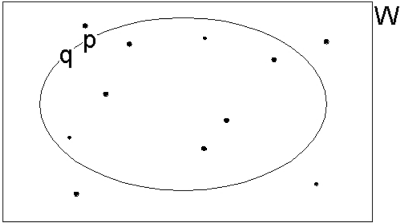
# Entailment

- defintion of intensions of sentences (propositions): characteristic functions
- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation: $p \subseteq q$:



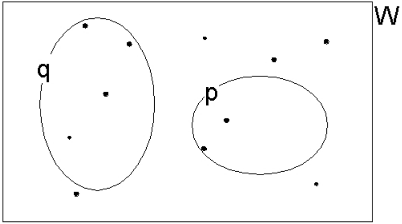
# Synonymy

- synonymy turns out as set equivalence:
- $p = q$



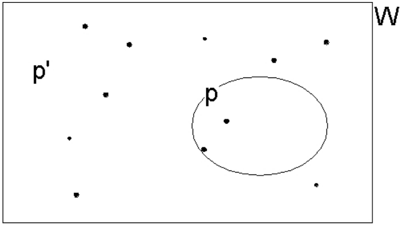
# Contradiction

- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



# Negation

- negation turns out as a complement:
- $p/W$

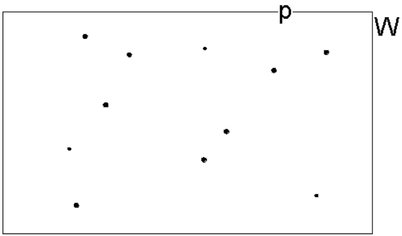


# Quantification over worlds

- new modal sentence/wff operators:
  - *necessarily p*: $\Box \mathbf{p}$
  - *possibly p*: $\Diamond \mathbf{p}$
- What does it mean for a proposition to be necessary/possible?

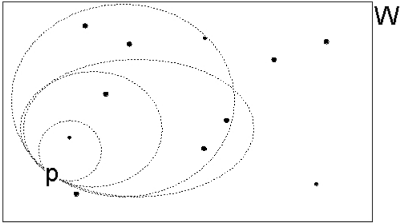
# Necessity as universal quantification

- if $\Box p$ then $(\forall w)\ [p(w) = 1]$ ($p$ as characteristic function)
- such that $W = p$ ($p$ as set):



# Possibility as existential quantification

- if $\Diamond p$ then $(\exists w)\,[p(w) = 1]$ (characteristic function)
- such that $p \neq \emptyset$ (set):



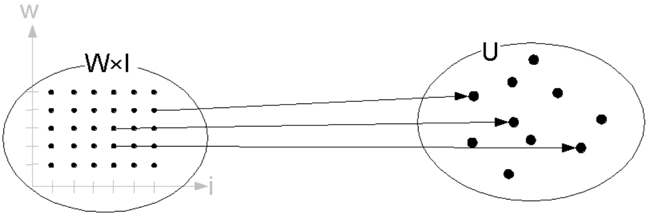
# Intensional Model Theory

# A larger tuple

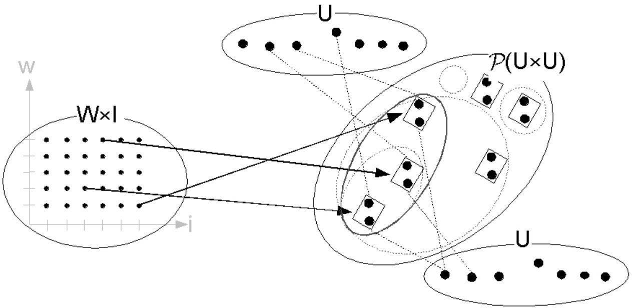
- $\mathcal{M} = \{W, I, <, U, V\}$
  - *W*, a set of worlds
  - *I*, a set of instants
  - $<$, an ordering relation in I
  - *U*, the set of individuals
  - *V*, a valuation function for constants
- evaluate an expression $\alpha$: $[\![\alpha]\!]^{\mathcal{M},w,i,g}$

# Intensional interpretation of individual constants

- *the President of the United States*, *the Pope*, *Bond* (in the sense of 'the actor currently playing Bond')
- *for* $\beta \in Cons_{ind}$, $V(\beta)$ is a function from $W \times I$ to $U$



- *walks* etc. denotes different sets (or CFs) at different $\langle w, i \rangle$ coordinates
- *for* $\beta \in Cons_{pred_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ $(U^n = U_1 \times U_2 \times \ldots \times U_n)$



# The Chierchia approach: predicates/sentences

- simple sentences/predicates: $\beta = \delta(t_1, t_2, \ldots, t_n)$
- $[\![\beta]\!]^{\mathcal{M},w,i,g} = 1$ iff
- $\langle [\![t_1]\!]^{\mathcal{M},w,i,g}, [\![t_2]\!]^{\mathcal{M},w,i,g}, \ldots, [\![t_n]\!]^{\mathcal{M},w,i,g} \rangle \in [\![\delta]\!]^{\mathcal{M},w,i,g}$
- with: $[\![t_1]\!]^{\mathcal{M},w,i,g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

# Quantification

- if $\psi = \forall x \phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $u \in U$
- ...$[\![\phi]\!]^{\mathcal{M},w,i,g[u/x]} = 1$
- nothing new here

# Modalities

- if $\psi = \Box x\phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $w' \in W$
- ...and all $i' \in I$
- ...$[\![\phi]\!]^{\mathcal{M},w',i',g} = 1$

# A similarity of $\forall$ and $\Box$

- as: $\forall x\,[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that: $\Box\,[\psi \rightarrow \phi] \rightarrow [\Box\psi \rightarrow \Box\phi]$
- but not vice-versa!

# Some validities

- $\exists x \Box P(x) \rightarrow \Box \exists x P(x)$
- $\exists x \Diamond P(x) \leftrightarrow \Diamond \exists x P(x)$
- $\forall x \Box P(x) \leftrightarrow \Box \forall x P(x)$ (Carnap-Barcan)
- $\forall x \Diamond P(x) \rightarrow \Diamond \forall x P(x)$

# Literatur I

# Autor

## Kontakt

Prof. Dr. Roland Schäfer
Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena
Fürstengraben 30
07743 Jena

https://rolandschaefer.net
roland.schaefer@uni-jena.de

# Lizenz