

# Formale Semantik

## o8. Intensionalität

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Folien in Überarbeitung. Englische Teile (ab Woche 6) sind noch von 2007!  
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

- 1 **Intensionality**
  - Problems with extensionality and non-dimensional models
  - Intensions
- 2 **A formal account of intensions**
  - Sets of PSOAs
  - Intensions as functions
  - Repeat after me...
- 3 **Sets of worlds**
  - Known relations
  - Modal operators
- 4 **Intensional Model Theory**
  - Ingredients of models
  - Evaluating individual constants
  - Set membership
  - Some peculiarities of  $\Box$  and  $\Diamond$

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- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Intensionality

# Some examples

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- Gustave Moreau **believes that** estheticism rules.

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- ... and for just one state of affairs (*modals, believe type verbs*)

# What are intensions?

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts or <b>propositions</b>

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- not mental representations
- mediate between internal knowledge and truth-values

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- PSOAs are logically constrained
- observe the more than just truth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSA logic

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  - ▶ nothing of the above, but A.S. rose from the dead in 2003, etc.

A formal account of intensions

- assume a set of all PSOAs

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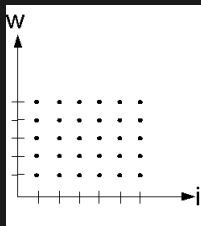
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- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

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- for each point in time: one possible temporal state of each world (instant  $i \in I$ )
- representation of temporarily ordered world-time coordinates  $\langle w, i \rangle \in W \times I$



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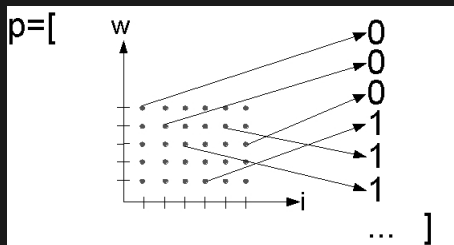
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- hence: every sentence is characterized by the set of worlds in which it is true
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- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

# Propositions as functions

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- a propositional function  $p$
- is a function from  $W \times I$  to  $\{0, 1\}$



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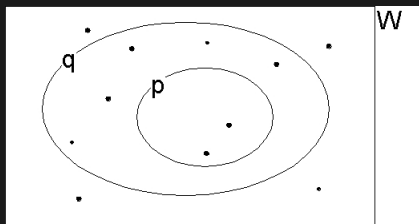
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- Hence, we call that function the intension of the sentence.

Sets of worlds

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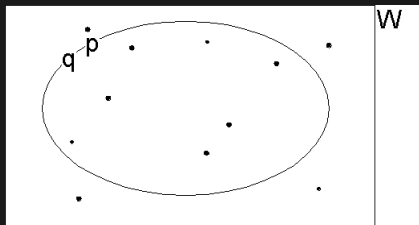
- definition of intensions of sentences (propositions): characteristic functions
- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation:  $p \subseteq q$ :



- synonymy turns out as set equivalence:

# Synonymy

- synonymy turns out as set equivalence:
- $p = q$



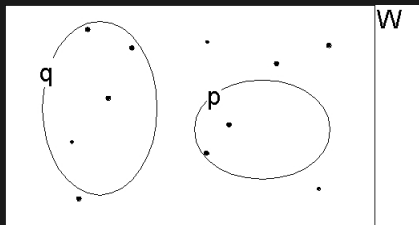
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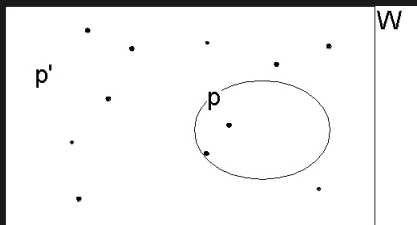
- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



- **negation** turns out as a **complement**:

# Negation

- negation turns out as a complement:
- $p/W$



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- What does it mean for a proposition to be necessary/possible?

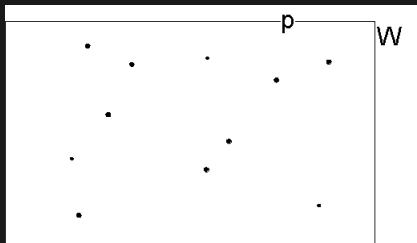
# Necessity as universal quantification

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- if  $\Box p$  then  $(\forall w) [p(w) = 1]$  ( $p$  as characteristic function)
- such that  $W = p$  ( $p$  as set):

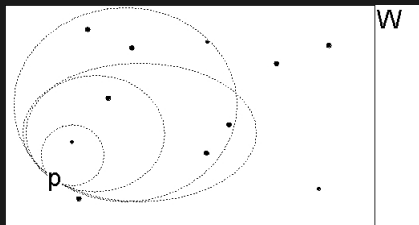


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## Intensional Model Theory

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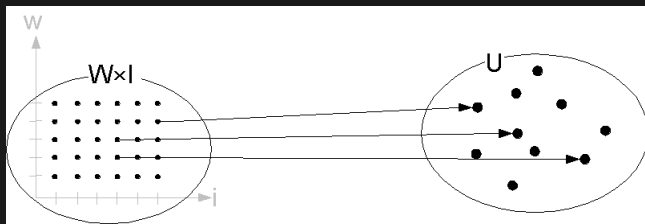
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  - ▶  $V$ , a valuation function for constants
- evaluate an expression  $\alpha$ :  $\llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)

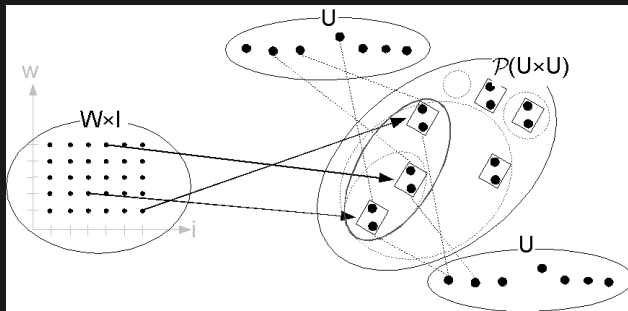
# Intensional interpretation of individual constants

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)
- for  $\beta \in \text{Cons}_{\text{ind}}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $U$



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- for  $\beta \in \text{Cons}_{\text{pred}_n}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $\wp U^n$  ( $U^n = U_1 \times U_2 \times \dots \times U_n$ )



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- simple sentences/predicates:  $\beta = \delta(t_1, t_2, \dots, t_n)$



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- with:  $\llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g} = V(t_1)(\langle w, i \rangle)$ , etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

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# Some validities

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## Kontakt

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