Formale Semantik

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stets aktuelle Fassungen: `https://github.com/rsling/VL-Deutsche-Syntax`

Inhalt

Inferenz und Bedeutung

Referentielle Semantik

Main textbooks

- Chierchia & McConnell-Ginet, *Meaning and Grammar*
- Partee, ter Meulen & Wall, *Mathematical Methods in Linguistics*
- Blackburn, Bos & Striegnitz, *Learn Prolog now!*
- Blackburn & Bos, *Computational Semantics for Natural Language*

Further reading

- Bucher, *Einführung in die angewandte Logik*
- Sag, Wasow & Bender, *Syntactic Theory*
- Dowty, *Tense, Time Adverbs, and Compositional Semantic Theory*
- Partee, *Noun Phrase Interpretation and Type-shifting Principles*
- Copestake, Flickinger & Sag *Minimal Recursion Semantics*

The three sessions

- Formal Semantics, 90 min. on Wednesday
- PROLOG, 30 min. on Wednesday
- Tutorial, 90 min. on Friday
- Summer course (implementation), 1 week

The first weeks: Preliminaries (subject to changes)

Session 1	Introduction to Referential Semantics (CM chap. 1 & 2)
Session 2	Set theory, ordering theory, statement logic (PMW chap. 1 - 6)
Session 3	Predicate calculi (PMW chap. 7 & 8)

The middle weeks: First steps (subject to changes)

Session 4	Quantification and model theory (CM chap. 3)
Session 5	Quantification in English (CM chap. 3)
Session 6	Intensionality (CM chap. 5)
Session 7	Tense, modals, complementizers (CM chap. 5)
Session 8	λ (CM chap. 7)

The final weeks: Advanced topics (subject to changes)

Session 9	Word meaning (CM chap. 8)
Session 10	Generalized quantifiers (CM chap. 7)
Session 11	Type shifting (Partee)
Session 12	Underspecified scope (Copestake *et al.*)
Session 13	Backup session
Session 14	Final test on 2004-07-13

What *meaning* could mean

- The meaning of an expression is the idea conveyed by it.
- ...is the mental image it creates.
- ...is what a speaker wants to achieve by uttering it.
- ...is the set of objects to which it refers (for example in the case of nouns).

What the study of meaning could be

- The study of the intellectual concepts perceivable in the world.
- ...of how the brain processes expressions, relates it to (fields of) cognitive concepts.
- ...of how a discourse of planful and intelligent agents (humans) is structured.
- ...of the correspondences between expressions and objects; and of how expressions are combined to be used productively.

What this class is about

- Which objects do words refer to?
- What makes sentences true?
- How is the informational value of sentences related to their logical structure?
- How can sentences be unambiguously interpreted?

What this class is not about

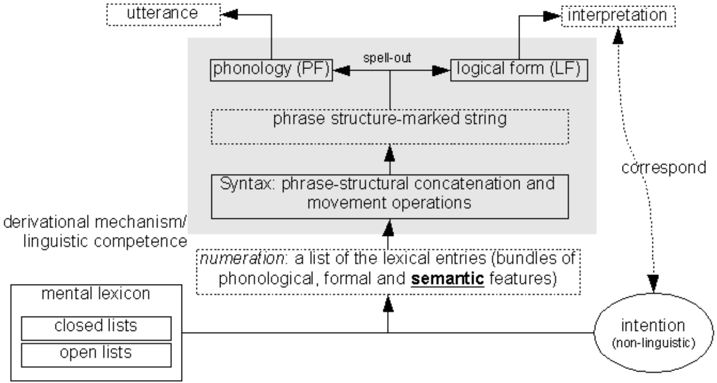
- what words mean,
- how the brain works with sentences,
- the structure of discourse (at least not much).

The theory of signs: a triangle

forms:

objects mental images

Semantics in the Chomskian T-model



LF is just the logical form

- No interpretation proper at LF.
- Movement transformations after the sentence has been uttered.
- At the LF level, sentences have a form compatible to their logic.
- Why? Syntax itself is often inadequate to express all alternatives of a sentence's logical representation.

Some properties of language

- aboutness
- referential nature
- informative
- objectiveness (of content)
- But which linguistic elements refer to what?

Names

an individual name $\longrightarrow$ one object in the world

Harald Schmidt

Common nouns

a common noun $\longrightarrow$ lots of objects

soldier

etc.

Adjectives

an adjective $\longrightarrow$ lots of different objects of different kinds

is human

a sentence $\longrightarrow$ a situation, a fact, ...

A humming bird is hovering over a red flower.

not at all
(object type mismatch)

Frege's Principle: Meaning is compositional

- *A humming bird* $\longrightarrow$ one of many individuals
- *is hovering* $\longrightarrow$ a property of that individual
- *over* $\longrightarrow$ a relation between individuals
- *a red* $\longrightarrow$ a property of another individual
- *flower* $\longrightarrow$ the other one of many individuals
- *is hovering over a red flower* $\longrightarrow$ a complex property.

Recursion: infinite use of finite means

- Frege's principle is indispensable!
- *Harald Schmidt is human.*
- *Harald Schmidt is human and tall.*
- *Harald Schmidt is human and tall and male.*
- *Harald Schmidt is human and tall and male and not blue.*
- *Harald Schmidt is human and tall and male and not blue and grumpy in the morning...*

Basic semantics judgements

- entailment
- presupposition
- ambiguity
- synonymy

Entailment: pure logic

- A: *This is electronic.*
- B: *This is a presentation.*
- C follows logically: *This is an electronic presentation.*
- $A, B \vdash C$
- $A \nvdash C$
- $B \nvdash C$

Entailment: pure logic, formally

- D: *Harald Schmidt is human.*
- E follows logically: *Something is human.*
- $D \vdash E$
- $D \wedge D$ follows logically: *Harald Schmidt is human and Harald Schmidt is human.*
- $D \vdash D \wedge D$

Tests: X entails Y if...

- When X is true, Y is true.
- A situation described by Y is also described by X.
- The information given by Y is fully contained in the information given by X.
- One cannot say X is true and Y is false.

Entailments?

- *Harald Schmidt is a talkmaster.* $\rightarrow$ *Harald Schmidt is human.*
- *Harald Schmidt is tall.* $\rightarrow$ *Someone is tall.*
- *Some humans are tall.* $\rightarrow$ *Harald Schmidt is tall.*
- *I have listened to Paul Kalkbrenner's new 12" on bpitchcontrol.* $\rightarrow$ *Paul Kalkbrenner has released a 12" on bpitchcontrol.*
- *After I had a Beck's, I installed RedHat on my PC.* $\rightarrow$ *I had a Beck's.*
- *After the bootloader had failed to boot RedHat on my PC, I had another Beck's.* $\rightarrow$ *RedHat has never booted on my PC.*
- *My flatmate likes Beck's.* $\rightarrow$ *My flatmate hates beer.*
- *Harald Schmidt cancelled his show.* $\rightarrow$ *Harald Schmidt's show was cancelled.*

Presuppostion: the background

- A: *Willy Brandt is the current chancelor of the FRG.*
- B: *If Willy Brandt is the current chancelor of the FRG, why doesn't he do something?*
- C: *Willy Brandt is not the current chancelor of the FRG.*
- A and B presuppose D: *Willy Brandt is alive.*, C doesn't.
- A, B, and C presuppose E: *There is a chancelor of the FRG.*
- Note: A $\vdash$ D, A $\vdash$ E
- But: B $\nvdash$ D, B $\nvdash$ E, C $\nvdash$ E

Presuppostion: two tests

- Presuppositions are triggered by all sorts of sentences (incl. negations, modals, conditionals, etc.).
- Presuppositions can be negated while the sentence which presupposes them remains true. Entailments cannot be negated while keeping the entailing sentence true.

Ambiguity in syntax

- *She saw the man with a telescope.*
- She [saw the man] with a telescope.
- She saw [the man with a telescope].

Ambiguity in semantics: scope

- *Everybody loves somebody.*
- Every person loves at least one other person.
  (Needn't be the same.)
- There is one person loved by everyone

Synonymy

- Lexical synonymy: *humming bird* $\overset{lex}{\equiv}$ *colibri*
- Compositionally (**equivalence**): *Mulder met his abducted sister after he broke into the secret army base.* $\equiv$ *Before meeting his abducted sister, Mulder broke into the secret army base.*
- A $\equiv$ B iff A $\vdash$ B and B $\vdash$ A

Noun-like expressions and complex NPs

- I saw a man.
- I saw the green wobbly thing crawling near.
- I saw it.

Problems with referential NPs

- *The dark subatomic particles in the universe have a total mass much larger than the visible subatomic particles.*
- *Problems with referential semantic theories don't concern Rumpletweezer.*
- and of course, vagueness (e.g., Sorites Paradox)

Problems with non-referential NPs

- *some guy*
- *not the faintest trace of blood*
- *any axiom of Zermelo-Fraenkel set theory*

Beyond pointin-at-and-naming

We need a logic to explain for effects like:

	my humming bird's favorite flower	is red
$\vdash$	some flower	is red

Some content-synonymous simple expressions

- a: *colibri*
- b: *humming bird*
- c: *a brunette lady*
- d: *a brown-haired dame*
- e: *the primates*
- f: *the apes and humans*
- a $\overset{lex}{\equiv}$ b, c $\overset{lex}{\equiv}$ d, e $\overset{lex}{\equiv}$ f

Some content-synonymous complex expressions

- A: *A colibri is hovering over a red flower.*
- B: *A humming bird is hovering over a red flower.*
- C: *Lauren Bacall was a brunette lady*
- D: *Lauren Bacall was a brown-haired dame*
- E: *Primates are intelligent.*
- F: *The apes and humans are inteligent.*
- A $\equiv$ B, C $\equiv$ D, E $\equiv$ F

Two axioms

- Ax1 Two expressions (e.g., NPs, sentences) that are synonymous have the same reference.
- Formally: $A \equiv B$ then $[\![A]\!] = [\![B]\!]$
- Note: $[\![A]\!]$ is applicable to simplex and complex expressions A; it just produces the reference of A.
- Ax2 If we replace expression B within expression A with the synonymous expression C, then A does not change its reference.
- Formally: If $[\![B]\!] = [\![C]\!]$ then $[\![[_A\ B]]\!] = [\![[_A\ C]]\!]$

One common property of sentences: the truth value

- A: *Lauren Bacall was a brunette lady.* (assumed to be true in the actual world)
- B: *My cat sleeps quietly.* (assumed to be true in the actual world)

First conclusion

- $[_{\mathrm{T}}A]$ = *The truth value of 'Lauren Bacall was a brunette lady' is 1.*
- $[_{\mathrm{T}}B]$ = *The truth value of 'My cat sleeps quietly' is 1.*
- Such that $\mathrm{A} \equiv [_{\mathrm{T}}A]$ and $\mathrm{B} \equiv [_{\mathrm{T}}B]$.
  (Check: Whenever A is true, $[_{\mathrm{T}}A]$ is true and v.v.)
- So, by Ax1 $[\![\mathrm{A}]\!] = [\![[_{\mathrm{T}}A]]\!]$
  and $[\![\mathrm{B}]\!] = [\![[_{\mathrm{T}}B]]\!]$

Second conclusion

- Check the denotations of the contained NPs:
  $[\![$*the truth value of A*$]\!] = [\![$*the truth value of B*$]\!] = 1$
- Such that by Ax2:
  $[\![[_T A]]\!] = [\![[_T B]]\!]$
- Why? Exchanging the referentially identical NPs 'the truth value of A' and 'the truth value of B' in the otherwise identical sentences '_ is 1' forces us to conclude by Ax2 that also the whole sentences must have the same reference. Our book (CM) is a bit vague on that point.

Final conclusion

$$[\![\text{A}]\!] = [\![[_T A]]\!] = [\![[_T B]]\!] = [\![\text{B}]\!] = 1$$

Sentences denote truth values.

Advantages of truth values

- indirect encoding of ‘richer’ semantics (One must know the truth conditions of a sentence and the state of affairs to decide about the truth of a sentence.)
- a minimal common semantic property of sentences
- easily computable in a formal system (binary)
- their logic provides a basis for ‘richer’ semantics (cf. second half of class)

Frege also thought, reference couldn't be all

Type	**Reference**	**Sense**
NP	individuals *Venus*	individual concepts
VP	sets *humming birds*	property concepts
S	1 or 0 *I like cats.*	thoughts

Some terminology

- *reference* = *extension* = what we're dealing with first
- *sense* = *intension* = what we will be dealing with later
- *proposition* = the intensions of sentences as informational content: The 'thought that S'.

Decomposing compositionality and composing truth

- How are sentences compositionally built up?
- What do their parts denote?
- How does the denotation of the parts contribute to the whole.
- T-sentences: S of L is true in v iff p.
- *S* a sentence, *L* a language, *v* a state of affairs, *p* a statement of the truth conditions.

A phrase-structure grammar

- $\mathrm{S} \rightarrow \mathrm{N}\ \mathrm{VP}$
- $\mathrm{S} \rightarrow \mathrm{S}\ \mathrm{conj}\ \mathrm{S}$
- $\mathrm{S} \rightarrow \mathrm{neg}\ \mathrm{S}$
- $\mathrm{VP} \rightarrow \mathrm{V_i}$
- $\mathrm{VP} \rightarrow \mathrm{V_t}\ \mathrm{N}$

A lexicon

- N $\rightarrow$ *Herr Webelhuth, Frau Eckardt, the Turm-Mensa*
- V_i $\rightarrow$ *is relaxed, is creative, is stupid*
- V_t $\rightarrow$ *prefers*
- conj $\rightarrow$ *and, or*
- neg $\rightarrow$ *it is not the case that*

Simple denotiations

- $[\![$Herr Webelhuth$]\!]$ = Herr Webelhuth
- $[\![$Frau Eckardt$]\!]$ = Frau Eckardt
- $[\![$the Turm-Mensa$]\!]$ = the Turm-Mensa
- $[\![$is relaxed$]\!]$ = {x:x is relaxed}
- $[\![$is creative$]\!]$ = {x:x is creative}
- $[\![$is stupid$]\!]$ = {x:x is stupid}
- $[\![$prefers$]\!]$ = {$\langle$x,y$\rangle$: x prefers y}

Some words don't really 'denote', they act like functions

- $[\![\mathit{neg}]\!] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$
- $[\![\mathit{and}]\!] = \begin{bmatrix} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 0 \\ \langle 0,1 \rangle \rightarrow 0 \\ \langle 0,0 \rangle \rightarrow 0 \end{bmatrix}$
- $[\![\mathit{or}]\!] = \begin{bmatrix} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 1 \\ \langle 0,1 \rangle \rightarrow 1 \\ \langle 0,0 \rangle \rightarrow 0 \end{bmatrix}$

T-sentences: rule-to-rule

- $[\![[_S \text{ N VP}]]\!] = 1$ iff $[\![\text{N}]\!] \in [\![\text{VP}]\!]$, else 0
- $[\![[_S \text{ S1 conj S2}]]\!] = [\![\text{conj}]\!] (\langle [\![\text{S1}]\!], [\![\text{S2}]\!] \rangle)$
- $[\![[_S \text{ neg S}]]\!] = [\![\text{neg}]\!] ([\![\text{S}]\!])$
- $[\![[_{VP} \text{ V}_t \text{ N}]]\!] = \{x: \langle x, [\![\text{N}]\!] \rangle \in [\![\text{V}_t]\!]\}$
- semantics for non-branching nodes: pass-up

A starting point for our computation

Herr Webelhuth is relaxed.

- Circumstances (Model): Herr Webelhuth is an element of the set of relaxed individuals.
- (1) The syntax is well-formed by S $\rightarrow$ N VP
- (2) for N: $[\![$Herr Webelhuth$]\!]$ = Herr Webelhuth
- (3) for VP: $[\![$is relaxed$]\!]$ = {x: x is relaxed}
- (4) for S: $[\![[_S$ N VP$]]\!]$ = 1 iff $[\![$N$]\!] \in [\![$VP$]\!]$, else 0

A starting point for our computation

The tree:

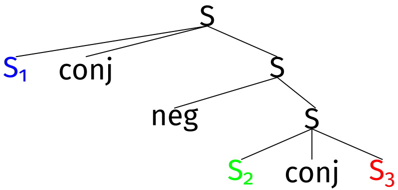
1 since $[\![\text{Herr Webelhuth}]\!] \in [\![\text{is relaxed}]\!]$

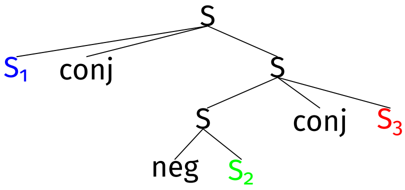
N VP

$[\![\text{Herr Webelhuth}]\!]$ $[\![\text{is relaxed}]\!]$

We compute syntactic representations, not flat sentences

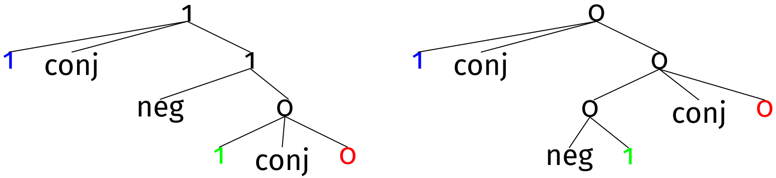
($_{S_1}$ Frau Eckardt is creative) and it is not the case that ($_{S_2}$ Herr Webehlhuth is relaxed) and ($_{S_3}$ Frau Eckardt prefers the Turm-Mensa).





A starting point for our computation

Circumstances: Herr Webelhuth is relaxed, Frau Eckardt is creative, and Frau Eckardt does not prefer the Turm-Mensa:



Mengen und Funktionen

What is a set?

- a freely defined unordered collection of discrete objects
  - numbers,
  - people,
  - pairs of shoes,
  - words, ...
- not necessarily for any purpose
- no object occurs more than once

Set definition and elements: $\in$

- $M_1 = \{a, b, c\}$
- N_1 = {‘my book’}
  vs. N_2 = {my book}
  vs. N_3 = {‘my’, ‘book’}
- ill-formed: N_4 = {‘my’, book}
- defined by a property of its members:
  M_2 = {x:x is one of the first three letters of the alphabet}
- alternatively:
  M_2 = {x||x is one of the first three letters of the alphabet}
- U: the universal set (contains every discrete object)

Equality: =

- Two sets with contain exactly the same members are *equal*.
- independent of definition:
  {a,b,c} =
  {x:x is one of the first three letters of the alphabet}
- {x:x is human} = {x:x is from the planet earth and x can speak}

Subsets: $\subseteq$

- A set N which holds no member which is not in M is a *subset* of M: N $\subseteq$ M
- $\{a\} \subseteq \{a, b, c\}$
- the inverse: the superset

Proper subsets: $\subset$

- A set N which holds no member which is not in M and which is not equal to M is a *proper subset* of M: N $\subset$ M
- So, actually: $\{a\} \subset \{a, b, c\}$ and $\{a, b, c\} \subseteq \{a, b, c\}$. Note that:
- $M \subseteq M$ but $M \not\subset M$
- $\{\{a\}\} \notin \{a, b, c\}$
- $\{\} \subset \{a, b, c\}$ (or any set), $\{\}$ is sometimes written $\emptyset$

Elements vs. subsets

- *All professors of English Linguistics are human.*
  Herr Webelhuth is a professor of English Linguistics.
- w = Herr Webelhuth
  E = the set of professors of English Linguistics
  H = the set of human beings
- $w \in E \;\&\; E \subset H \Rightarrow w \in H$

Elements vs. subsets

- But: *Professors of English Linguistics are numerous.*
- N = the set of sets with numerous members
- $w \in E \;\&\; E \in N \not\Rightarrow w \in P$
- Hence: *Herr Webelhuth is numerous.

Power sets: $\wp$

- For any set M: $\wp(M) = \{X \| X \subseteq M\}$
- for M=$\{a, b, c\}$:
  $\wp(M) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b, c\}, \{b, c\}\}$
- Why is the empty set in the power set of every set ...
- ...and why is the empty a set a proper subset of every set?

Union ∪ and intersection ∩

- For any sets M and N: $M \cup N = \{x||x \in M \textbf{ or } x \in N\}$
- if $M = \{a, b, c\}$ and $N = \{a, b, d\}$ then $M \cup N = \{a, b, c, d\}$
- For any sets M and N: $M \cap N = \{x||x \in M \textbf{ and } x \in N\}$
- if $M = \{a, b, c\}$ and $N = \{a, b\}$ then $M \cap N = \{a, b\}$
- as a general principle (Consitency): $M \subseteq N$ iff $M \cup N = N$ and $M \subseteq N$ iff $M \cap N = M$

Generalized union $\bigcup$ and intersection $\bigcap$

- $\bigcup M = \{x \| x \in Y$ **for some** $Y \in M\}$
- (a) if $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$ then $\bigcup M = \{a, b, c\}$
- (b) $M_1 = \{a\}$, $M_2 = \{a, b\}$, $M_3 = \{a, b, c\}$, $I = \{1, 2, 3\}$; $\bigcup_{i \in I} M = \{a, b, c\}$
- $\bigcap M = \{x \| x \in Y$ **for every** $Y \in M\}$
- (a) if $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$ then $\bigcap M = \{a\}$
- (b) $M_1 = \{a\}$, $M_2 = \{a, b\}$, $M_3 = \{a, b, c\}$, $I = \{1, 2, 3\}$; $\bigcap_{i \in I} M = \{a\}$

Difference - and complement $\setminus$ and $'$

- For any two sets M and N: $M - N = \{x \| x \in M \text{ and } x \notin N\}$
- $M = \{a, b, c\}$, $N = \{a\}$, $M - N = \{b, c\}$
- For any two sets M and N: $M \setminus N = \{x \| x \in N \text{ and } x \notin M\}$
- $O = \{a, b, c, k\}$ $M \setminus O = \{k\}$
- the universal complement: $M' = \{x \| x \in U \text{ and } x \notin M\}$
  (U the universal set)

Trivial equalities

- Idempotency: $M \cup M = M$, $M \cap M = M$
- Commutativity for $\cup$ and $\cap$: $M \cup N = N \cup M$ ...
- Associativiy for $\cup$ and $\cap$: $(M \cup N) \cup O = M \cup (N \cup O)$ ...
- Distributivity for $\cup$ and $\cap$: $M \cup (N \cap O) = (M \cup N) \cap (M \cup O)$ ...
- Identity: $M \cup \emptyset = X$, $M \cup U = U$ ...what about $\cap$

More interesting equalities

- Complement laws: $M \cup \emptyset = M$, $M'' = M$, $M \cap M' = \emptyset$, $X \cap U = U$
- DeMorgan: $(M \cup N)' = M' \cap X'$ ...

How to define an ordered pair

- ...without introducing ordered tuples as a new primitive
- take S=$\{\{a\},\{a,b\}\}$
- we write: $\langle a,b\rangle = \{\{a\},\{a,b\}\}$
- orderend n-tuples defined recursively
- $\langle a,b\rangle \neq \langle b,a\rangle$
- first and second coordinate of the tuple

Cartesian products

- sets of ordered pairs
- tupling each member of the first argument with each of the second
- $S_1 \times S_2 = \{\langle x, y\rangle \| x \in S_1 \text{ and } y \in S_2\}$
- for an arbitrary number of sets: $S_1 \times \dots \times S_n = \{\langle x_1, x_2, \dots, x_n\rangle \| x_i \in S_i\}$
- $\langle x_1, x_2, \dots, x_n\rangle$ abbreviated $\vec{x}$
- for $S \times S \times \cdots$: n-fold products
  $S^n = \{\vec{s} \| s_i \in S \text{ for } 1 \leq i \leq n\}$

Defintion of relations

- hold between (sets of) objects
- *x kicks y*, *x lives on the same floor as y*, ...
- formalization: Rab, aRb
- $a \in A$ and $b \in B$: $R \subseteq A \times B$,
  R is from A (**domain**) to B (**range**)
- R from A to A is in A

Complement, inverse

- complement $R' = \{\langle a, b \rangle \ \notin R\}$ for $R \subseteq A \times B$
  - R = the relation of teacherhood between a and b (the **arguments**)
  - R′ = all pairs $\langle b, a \rangle$ s.t. it is false that the first member is the teacher of the second member
- inverse: $R^{-1} = \{\langle b, a \rangle \| \langle a, b \rangle \in R\}$ for $R \subseteq A \times B$
  - R = the relation of teacherhood between a and b:
    Herr Webelhuth is the teacher of Herr Schäfer.
  - R^{-1} = all pairs $\langle b, a \rangle$ where *a* is the teacher of *b*:
    Herr Schäfer is the inverse-teacher of Herr Webelhuth.

Functions

- A function F from A to B is a relation s.t. for every $a \in A$ there is exactly on tuple $\langle a, b \rangle \in A \times B$ s.t. a is the first coordinate.
- partial function from A to B: for some $a \in A$ there is no tuple $\langle a, b \rangle \in A \times B$, F is not *defined for* some a

Injection, surjection, bijection

- B the range of F, F is **into** B
- F from A to B is **onto (a surjection)** B iff there is no $b_i \in B$ s.t. there is no $\langle a, b_i \rangle \in F$
- F from A to B is **one-to-one (an injection)** iff there are no two pairs s.t. $\langle a_i, b_j \rangle \in F$ and $\langle a_k, b_j \rangle \in F$
- one-to-one, onto, and total function: correspondence (bijection)

Composition

- One can take the range of a function and make it the domain of another function.
- A function $F_1 : A \rightarrow B$ and a function $F_2 : B \rightarrow C$ can be composed as $B(A(a))$, short $B \circ A$
- the compound function can be empty, it will be total if both A and B are bijections.

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
reflexive	for **every** $a \in A$: $\langle a, a \rangle \in R$	is as heavy as
		A: physical objects
irreflexive	for **every** $a \in A$: $\langle a, a \rangle \notin R$	is the father of
non-reflexive	for **some** $a \in A$: $\langle a, a \rangle \notin R$	has hurt

Symmetry

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
symmetric	for every $\langle a, b \rangle \in R$: $\langle b, a \rangle \in R$	has the same car as
asymmetric	for every $\langle a, b \rangle \in R$: $\langle b, a \rangle \notin R$	has a different car than
non-symmetric	for some $\langle a, b \rangle \in R$: $\langle b, a \rangle \notin R$	is the sister of
anti-symmetric	for every $\langle a, b \rangle \in R$: $a = b$	beats oneself
		not every human does

Transitivity

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
transitive	if $\langle a, b\rangle \in R$ and $\langle b, c\rangle \in R$ then $\langle a, c\rangle \in R$	is to the left of
intransitive	the above is never the case	is the father of
non-transitive	the above is sometimes not the case	likes

Connectedness

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
connected	for every $a, b \in A$, $a \neq b$: either $\langle a, b\rangle \in R$ or $\langle b, a\rangle \in R$	$>$ (A: the natural numbers)
non-connected	for some $a, b \in A$ the above is not the case	likes

Equivalence relations

- reflexive ($\langle a, a \rangle \in R$ for every a)
- symmetric ($\langle b, a \rangle \in R$ for every $\langle a, b \rangle$)
- transitive ($\langle a, b \rangle \in R$ & $\langle b, c \rangle \in R \rightarrow \langle a, c \rangle \in R$)
- *is as stupid as*
- partition the range into equivalence classes:
  $A = \{a, b, c, d\}$, for example $P_{A_1} = \{\{a, b\}, \{c\}, \{d\}\}$
- not $\{\{a\}, \{b, c\}\}$ or $\{\{a, b\}, \{b, c\}, \{d\}\}$

Defining ordering relations

An ordering relation R in A is ...

- transitive ($\langle a, b\rangle \in R$ & $\langle b, c\rangle \in R \rightarrow \langle a, c\rangle \in R$) ...plus ...
- irreflexive and asymmetric: **strict order**
- $A = \{a, b, c, d\}$, $R_1 = \{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$
- reflexive and anti-symmetric: **weak order**
- $A = \{a, b, c, d\}$, $R_1 = \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$

Orders: an example

- a strict order: *greater than* ($>$) in $\mathbb{N}$
- what is the corresponding weak order
- $\geq$

- **minimal**: x is not preceded
- **least**: x precedes every other lement
- **maximal**: x is not succeeded
- **greatest**: x succeeds every other element
- **well-ordering**: total order, every subset has a least element

The number of elements...

- $A = \{a, b, c\}$
- $B = \{a, b, c\}$
- obviously, $A = B$ (equal)
- there is an R from A to B s.t. $R = \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle\}$
- for every set C with the same number of elements (e.g., $C = \{1, 2, 3\}$): $R = \{\langle a, 1\rangle, \langle b, 2\rangle, \langle c, 3\rangle\}$
- such relations are one-to-one correspondences

Denumerable sets

- $\mathbb{N}$ is infinite
- for every A there is some R_{card}
  - a one-to-one correspondence
  - from A's members to the first *n* members of $\mathbb{N}$
  - s.t. *n* is the cardinality of A, $\|A\|$
- sets A,B with $\|A\| = \|B\|$ are equivalent
- $\|\mathbb{N}\| = \aleph^0$

A problem

- for some sets there is no such R_{card}
- no way of bringing their elements into an exhaustive linear order
- no problem with $\mathbb{Q}$:

$$
\begin{array}{ccccc}
 & \langle 0,1\rangle & \langle 0,2\rangle & \langle 0,3\rangle & \cdots \\
\langle 1,0\rangle & \langle 1,1\rangle & \langle 1,2\rangle & \langle 1,3\rangle & \cdots \\
\langle 2,0\rangle & \langle 2,1\rangle & \langle 2,2\rangle & \langle 2,3\rangle & \cdots \\
\vdots & \vdots & \vdots & \vdots &
\end{array}
$$

The non-denumerable real numbers

- now: $\mathbb{R}$
- some elements cannot be represented as an ordered pair of two elements of $\mathbb{N}$
- in $[0, 1]$, every real can be represented as $0.abcdefg\ldots$,
  $a, b, c, d, e, f, g, \ldots \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Trying to enumerate

- an enumeration of $[0,1]$ in $\mathbb{R}$?

$$
\begin{array}{llllllll}
x_1 & = & 0 & . & a_{11} & a_{12} & a_{13} & a_{14} & \ldots \\
x_2 & = & 0 & . & a_{21} & a_{22} & a_{23} & a_{24} & \ldots \\
x_3 & = & 0 & . & a_{31} & a_{32} & a_{33} & a_{34} & \ldots \\
\vdots & & \vdots & & & & & & \\
x_n & = & 0 & . & a_{n1} & a_{n2} & a_{n3} & a_{n4} & \ldots
\end{array}
$$

Failing to enumerate

- What about an x_m which differs from x_n at a_{nn}

$$
\begin{array}{cccccccccc}
x_1 & = & 0 & . & a_{11} & a_{12} & a_{13} & a_{14} & \dots \\
x_2 & = & 0 & . & a_{21} & a_{22} & a_{23} & a_{24} & \dots \\
x_3 & = & 0 & . & a_{31} & a_{32} & a_{33} & a_{34} & \dots \\
\vdots & & \vdots & & & & & & \\
x_n & = & 0 & . & a_{n1} & a_{n2} & a_{n3} & a_{nn} & \dots
\end{array}
$$

- It won't be in the array...
- $\mathbb{R}$ is non-denumerable
- If $\|A\| = \aleph^0$ then $\|\wp(A)\| = 2^{\aleph_0}$ (cf. Partee et al. 62f.)

Aussagenlogik

The book (PMW:87-246) deals with logic far more in-depth than we do. Only what is mentioned on the slides is relevant for the test. Reading the whole chapter from PMW will do you no harm, though.

Theories

- a collection of statements (propositions)
- axioms (statements accepted to be true)
- maybe based on observations (induction)
- statements that follow from the axioms (deduction)
- predictions beyond the axioms
- rechecking for usability: e.g., Russell's paradox

Proofs

- axioms: atomic truths of your theory
- theorem: a proposition you want to prove
- lemma: subsidiary propositions (used to prove the theorem)
- corollary: propositions proved while proving some axiom

A method of reasoning

- logic does not generate truths
- formalizing statements, predications etc.
- rules of deduction from axioms to theorems
- empirical (induction) and exact (deduction) science
- aiming at an adequate model of the world (e.g., heliocentric universe)

Why logic for semantics?

- truth-conditional
- compositional behavior of propositions and connectives
- a logic for entailments
- why, e.g.: *It is not the case that someone is happy.* $\rightarrow$ *Nobody is happy.*

Atomic formulas: statements

- statements/propositions = the atoms
- a propositional symbol p: a well-formed formula (wff)
- ex.: *Herr Keydana is a passionate cyclist.*: k
- $[\![k]\!]$=1 or 0 (depending on corresponding **model**)

Complex (molecular) formulas

- syntax: restricts the forms of wff's to make them interpretable
- define functors: functions in $\{0, 1\}$
- If p and q are wff's, then
  - $\neg p$
  - $p \vee q$
  - $p \wedge q$
  - $p \rightarrow q$
  - $p \leftrightarrow q$

  is also a wff (a molecular term).

Complex (molecular) formulas

- syntax: restricts forms of wff's to make them interpretable
- define functors: functions in $\{\langle 0,1\rangle, \langle 1,0\rangle, 0, 1\}$
- If p and q are wff's, then
  - $\neg p$ (negation)
  - $p \vee q$ (disjunction)
  - $p \wedge q$ (conjunction)
  - $p \rightarrow q$ (conditional)
  - $p \leftrightarrow q$ (biconditional)

  is also a wff.

Complex (molecular) formulas

- syntax: restricts forms of wff's to make them interpretable
- define functors: functions in $\{\langle 0,1\rangle, \langle 1,0\rangle, 0, 1\}$
- If *p* and *q* are wff's, then
  - $\neg p$ (negation - 'not')
  - $p \vee q$ (disjunction - 'or')
  - $p \wedge q$ (conjunction - 'and')
  - $p \rightarrow q$ (conditional - 'if')
  - $p \leftrightarrow q$ (biconditional - 'iff')

  is also a wff.

Functions and truth tables

- standard defintion:

$$[\![\neg]\!] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$$

- but most widely used: truth tables

$\neg$	p
0	1
1	0

Disjunction

p	$\vee$	q
1	1	1
1	1	0
0	1	1
0	0	0

- *Herr Keydana is a passionate cyclist* ***or*** *we all love logic.*
- $K \vee L$

Conjunction

p	$\wedge$	q
1	1	1
1	0	0
0	0	1
0	0	0

- *Herr Keydana is a passionate cyclist* ***and*** *we all love logic.*
- $K \wedge L$

Conditional

p	$\rightarrow$	q
1	1	1
1	0	0
0	1	1
0	1	0

- ***If*** *it rains,* ***then*** *the streets get wet.*
- $R \rightarrow S$

Any problems with that?

If it rains, the streets get wet.

- it is raining (1) , the streets are wet 1 : **1**
- it is raining (1) , the streets are dry 0 : **0**
- it is not raining (0) , the streets are wet 1 : **1**
- it is not raining (0) , the streets are dry 0 : **1**
- ex vero non sequitur falsum

Biconditional

p	$\leftrightarrow$	q
1	1	1
1	0	0
0	0	1
0	1	0

- ***If and only if** your score is above 50, **then** you pass the semantics exam.*
- $S \leftrightarrow P$

Scope of functors

- brackets are facultative
- or set non-default functor scope
- default scope

$$\text{scope} \; \Bigg\downarrow \; \begin{array}{c} \neg \\ \wedge \\ \vee \\ \rightarrow \\ \leftrightarrow \end{array} \; \Bigg\uparrow \; \text{binding strength}$$

An example

- $p \wedge \neg q \vee r \rightarrow \neg s$
- $p \wedge (\neg q) \vee r \rightarrow (\neg s)$
- $(p \wedge (\neg q)) \vee r \rightarrow (\neg s)$
- $((p \wedge (\neg q)) \vee r) \rightarrow (\neg s)$
- $(((p \wedge (\neg q)) \vee r) \rightarrow (\neg s))$

Large truth tables

- for n atoms in the term: 2^n lines
- alternating blocks of 1's and 0's under every atom
- $2^{(m-1)}$ times '1' followed by $2^{(m-1)}$ times '0' for the m-th atom from the right
- until 2^n lines are reached

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1			1		1			1
1			1		1			0
1			1		0			1
1			1		0			0
1			0		1			1
1			0		1			0
1			0		0			1
1			0		0			0
0			1		1			1
0			1		1			0
0			1		0			1
0			1		0			0
0			0		1			1
0			0		1			0
0			0		0			1
0			0		0			0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1		0	1		1		0	1
1		0	1		1		1	0
1		0	1		0		0	1
1		0	1		0		1	0
1		1	0		1		0	1
1		1	0		1		1	0
1		1	0		0		0	1
1		1	0		0		1	0
0		0	1		1		0	1
0		0	1		1		1	0
0		0	1		0		0	1
0		0	1		0		1	0
0		1	0		1		0	1
0		1	0		1		1	0
0		1	0		0		0	1
0		1	0		0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1		1		0	1
1	0	0	1		1		1	0
1	0	0	1		0		0	1
1	0	0	1		0		1	0
1	1	1	0		1		0	1
1	1	1	0		1		1	0
1	1	1	0		0		0	1
1	1	1	0		0		1	0
0	0	0	1		1		0	1
0	0	0	1		1		1	0
0	0	0	1		0		0	1
0	0	0	1		0		1	0
0	0	1	0		1		0	1
0	0	1	0		1		1	0
0	0	1	0		0		0	1
0	0	1	0		0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1		0	1
1	0	0	1	1	1		1	0
1	0	0	1	0	0		0	1
1	0	0	1	0	0		1	0
1	1	1	0	1	1		0	1
1	1	1	0	1	1		1	0
1	1	1	0	1	0		0	1
1	1	1	0	1	0		1	0
0	0	0	1	1	1		0	1
0	0	0	1	1	1		1	0
0	0	0	1	0	0		0	1
0	0	0	1	0	0		1	0
0	0	1	0	1	1		0	1
0	0	1	0	1	1		1	0
0	0	1	0	0	0		0	1
0	0	1	0	0	0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Assignments: a contingent example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Tautology

- take $p \vee \neg p$
- truth-table:

p	$\vee$	$\neg$	p
1	1	0	1
0	1	1	0

- true under every assignment, it is **valid**
- by *law of excluded middle*: for every P, $P \vee \neg P$ is true

Contradiction

- take $p \wedge \neg p$
- truth-table:

p	$\wedge$	$\neg$	p
1	0	0	1
0	0	1	0

- false under every assignment, called **contradictory**

Contingency

- take $p \wedge p$
- truth-table:

p	$\wedge$	p
1	1	1
0	0	0

- the truth value depends on the assignemt

What are laws?

- notice: similarities of set theory and logic
- non-trivial exact nature of their equivalence
- laws state equivalences of (types of) wff
- truth-conservative rewriting of wff's
- any subformula which is a tautology (T) or contradiction (F):

  ignore by Identity Laws (Id.):
  - $(P \vee F) \Leftrightarrow P$, $(P \vee T) \Leftrightarrow T$
  - $(P \wedge F) \Leftrightarrow F$, $(P \wedge T) \Leftrightarrow P$

Equivalences: $\Leftrightarrow$

- $X \Leftrightarrow Y$: X has the same truth-conditions as Y
- derivability of laws and rules (convenient redundancies)
- Idempotency (Idemp.):
  - $(P \vee P) \Leftrightarrow P$
  - $(P \wedge P) \Leftrightarrow P$
  - *Peter walks and Peter walks.* $\Leftrightarrow$ *Peter walks.*

Simple laws

- Associative Laws for $\vee$ and $\wedge$ (Assoc.):
  - $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$
  - *((He walks or she talks) or we walk.)* $\Leftrightarrow$ *(He walks or (she talks or we walk.))*
- Commutative Laws for $\vee$ and $\wedge$ (Comm.):
  - $(P \vee Q) \Leftrightarrow (Q \vee P)$
  - *Peter walks or Sue snores.* $\Leftrightarrow$ *Sue snores or Peter walks.*
- Distributive Laws for $\vee\wedge$ and $\wedge\vee$ (Distr.):
  - $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
  - *(Sue snores) and (Peter walks or we talk).* $\Leftrightarrow$ *(Sue snores and Peter walks) or (Sue snores and we talk).*

Laws dealing with tautology and contradiction

- Complement Laws:
  - Tautology (T): $(P \lor \neg P) \Leftrightarrow \mathbf{T}$
  - Contradiction (F): $(P \land \neg P) \Leftrightarrow \mathbf{F}$
  - Double Negation (DN): $(\neg\neg P) \Leftrightarrow P$
  - *It is not the case that Sandy is not walking.*
    $\Leftrightarrow$ *Sandy is walking.*

Conditionals Laws

- **Implication** (Impl.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	P	$\vee$	Q
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

- **Contraposition** (Contr.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	Q	$\rightarrow$	$\neg$	P
1	1	1		0	1	1	0	1
1	0	0		1	0	0	0	1
0	1	1		0	1	1	1	0
0	1	0		1	0	1	1	0

DeMorgan (DeM)

- DeMorgan's Laws:
  - $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$
  - alternatively: $\overline{P \vee Q} \Leftrightarrow \overline{P} \wedge \overline{Q}$
  - $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
  - consequently: $\overline{\overline{P} \vee \overline{Q}} \Leftrightarrow \overline{\overline{P}} \wedge \overline{\overline{Q}} \Leftrightarrow P \wedge Q$

The Modus Ponens (MP)

- Definition:

P	$\rightarrow$	Q	premise 1
P			premise 2
		Q	conclusion

- or: $(P \rightarrow Q) \land (P) \rightarrow (Q)$
- *(1) If It rains, the streets get wet. (2) It is raining.*
  $\rightarrow$ *The streets are getting wet.*

MP: a truth table illustration

- Premises are always set to be true!
- the table:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- The conditional must be true.
- cancel the 'false' row

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- P must be true.
- cancel the 'false' rows, Q can only be true:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

The Modus Tollens (MT)

- Definition:

P	$\rightarrow$	Q
		$\neg Q$
$\neg P$		

- the table illustration:

P	$\rightarrow$	Q	
1	1	1	(by premise 2)
1	0	0	(by premise 1)
0	1	1	(by premise 2)
0	1	0	

The Syllogisms

- Hypothetical Syllogism (HS):
  - $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
  - *(1) If it rains, the streets get wet. (2) If the streets get wet, it smells nice.* $\rightarrow$ *If it rains, it smells nice.*
- Disjunctive Syllogism (DS):
  - $((P \vee Q) \wedge (\neg P)) \rightarrow (Q)$
  - *(1) Either Peter sleeps or Peter is awake. (2) Peter isn't awake.* $\rightarrow$ *Peter sleeps.*

Trivial rules

- Simplification (Simp.):
  - $(P \wedge Q) \rightarrow P$
  - *(1) It is raining and the sun is shining.* $\rightarrow$ *It is raining.*
- Conjunction (Conj.):
  - $(P) \wedge (Q) \rightarrow (P \wedge Q)$
  - *(1) It is raining. (2) The sun is shining.* $\rightarrow$ *It is raining and the sun is shining.*
- Addition (Add.):
  - $(P) \rightarrow (P \wedge Q)$
  - *(1) It is raining.* $\rightarrow$ *It is raining or the sun is shining.*
  - What if Q is instantiated as true or false by another premise?

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$
- The proof:

		$p \vee q$
1	$(p \vee q) \rightarrow \neg(r \wedge \neg s)$	
2	$r \wedge \neg s$	
	$p \vee q$	1,2,MT

Prädikatenlogik

Weak compositionality in SL

- properties/relations vs. individuals
- *Martin is an <u>e</u>xpert on inversion and Martin is a good <u>c</u>limber.*
- ...becomes $E \wedge C$
- compositionality resticted to level of connected propositional atoms

Some desirable deductions

- important generalizations about all and some individuals (which have property P)
- *‘all P* $\rightarrow$ *some P’*
- *‘Martin P* $\rightarrow$ *some P’*

Atoms of PC

- individual variables: $x, y, z, x_1, x_2 \ldots$
- individual constants: $a, b, c, \ldots$
- variables and constants: terms
- predicate symbols (taking individual symbols or tuples of them): $A, B, C, \ldots$
- quantifiers: existential $\exists$ (or $\bigvee$) and universal $\forall$ (or $\bigwedge$)
- plus the connectives of SL

Some syntax

- for an n-ary predicate P and terms $t_1 \ldots t_n$,
  $P(t_1 \ldots t_n)$ or $Pt_1 \ldots t_n$ is a wff.
- possible prefix, function (bracket) and infix notation:
  Pxy, $P(x, y)$, xPy
- syntax for connectives from SL
- for any wff ϕ and any variable x, $(\exists x)\phi$ and $(\forall x)\phi$ are wff's

Semantic for individual constants

- denote individuals
- a model $\mathcal{M}$ contains a set of individuals D
- the valuation function V (or F): from constants to individuals in D
- for some $\mathcal{M}_1$: $D = \{Martin, Kilroy, Scully\}$
- $V_{\mathcal{M}_1}(m) = Martin$
- $V_{\mathcal{M}_1}(k) = Kilroy$, $V_{\mathcal{M}_1}(s) = Scully$

Semantics for predicate symbols

- denote relations (sets of n-tuples)
- $[\![P]\!]^{\mathcal{M}_1} = \{Martin, Kilroy\}$ or $V_{\mathcal{M}_1}(P) = \{Martin, Kilroy\}$
- $V_{\mathcal{M}_1}(Q) = \{\langle Martin, Kilroy\rangle, \langle Martin, Scully\rangle, \langle Kilroy, Kilroy\rangle,$
  $\langle Scully, Scully\rangle\}$
- s.t. $[\![P(m)]\!]^{\mathcal{M}_1} = [\![P]\!]^{\mathcal{M}_1}([\![m)]\!]^{\mathcal{M}_1}) = 1$ iff $[\![m]\!]^{\mathcal{M}_1} \in [\![P]\!]^{\mathcal{M}_1}$

Semantics for connectives and quantifiers

- connectives: 'apply to' formulas (semantically truth-valued), semantics as in SL
- $(\forall x)\phi$ = 1 iff ϕ is true for every $d \in D$
  assigned to every occurence of x in ϕ
- $(\exists x)\phi$ = 1 iff ϕ is true for at least one $d \in D$
  assigned to every occurence of x in ϕ
- algorithmic instruction to check wff's containing Q's
- check outside-in (unambiguous scoping)

Dependencies

- universal quantifiers can be swapped:
  $(\forall x)(\forall y)\phi \Leftrightarrow (\forall y)(\forall x)\phi$
- same for existential quantifiers:
  $(\exists x)(\exists y)\phi \Leftrightarrow (\exists y)(\exists x)\phi$
- whereas: $(\exists x)(\forall y)\phi \Rightarrow (\forall y)(\exists x)\phi$
- example in $\mathcal{M}_1$:
  - $[\![\underline{(\forall x)\underline{(\exists y)Qxy}}]\!]^{\mathcal{M}_1} = 1$
  - but: $[\![\underline{(\exists y)\underline{(\forall x)Qxy}}]\!]^{\mathcal{M}_1} = 0$
  - direct consequence of algorithmic definition
  - if $\exists\forall$ is true, $\forall\exists$ follows

Hints on quantifiers

- domain of quantifiers: D (universe of discourse)
- $\forall x$ checks for truth of some predication for all individuals
- $\exists x(Px \wedge \neg Px)$ is a contradiction
- $\forall x(Wx \wedge \neg Wx)$ is a contradiciton,
  $\forall x$ 'checks' for an empty set by def.
- standard form of NL quantification:
  $\forall x(Wx \rightarrow Bx)$ 'All women are beautiful.'
- standard form of NL existential quantification:
  $\exists x(Wx \wedge Bx)$ 'Some woman is beautiful.'

Functor/quantifier practice

- by def., functors take formulas, not terms:
  - $\neg Wm$ 'Mary doesn't weep.'
  - $(\exists x)(Gx \wedge Wx)$ 'Some girl weeps.'
  - $^{*}W\neg x$
  - $^{*}(\exists\neg x)(Gx)$
- quantifiers take variables, not constants:
  - $(\forall x)(Ox \rightarrow Wx)$ 'All ozelots are wildcats.'
  - $^{*}(\forall o)(Wo)$
- $\neg$ negates the wff, not the q:
  $^{*}(\neg\forall x)Px$ but $\neg(\forall x)Px$

Scope

- quantifiers bind variables
- free variables (constants) are unbound
- no double binding $^*(\forall x \exists x)Px$
- Q scope: only the first wff to its right:
  - $\underline{(\forall x)Px} \vee Qx$
  - $\underline{(\forall x)(Px \vee Qx)} = \underline{(\forall x)Px} \vee \underline{(\forall x)Qx}$
  - $\underline{(\exists x)Px} \rightarrow \underline{(\forall y)(Qy \wedge Ry)}$
  - $\underline{(\exists x)Px} \wedge Qx$ (second x is a unbound)
- no double-naming

Universal $\vee$ and $\wedge$

- $\exists$ and $\forall$ 'or' and 'and' over the universe of discourse (hence: $\bigvee$ and $\bigwedge$)
- $(\forall x)Px \Leftrightarrow Px_1 \wedge Px_2 \wedge \ldots \wedge Px_n$ for all x_n assigned to $d_n \in D$
- $(\exists x)Px \Leftrightarrow Px_1 \vee Px_2 \vee \ldots \vee Px_n$ for all x_n assigned to $d_n \in D$
- hence: $\neg(\forall x)Px \Leftrightarrow \neg(Px_1 \wedge Px_2 \wedge \ldots \wedge Px_n)$
- with DeM: $\overline{Px_1 \wedge Px_2 \wedge \ldots \wedge Px_n}$
- $\Leftrightarrow \overline{Px_1} \vee \overline{Px_2} \vee \ldots \vee \overline{Px_n}$
- $\Leftrightarrow (\exists x)\neg Px$

Quantifier negation (QN)

- $\neg(\forall x)Px \Leftrightarrow (\exists x)\neg Px$
- $\neg(\exists x)Px \Leftrightarrow (\forall x)\neg Px$
- $\neg(\forall x)\neg Px \Leftrightarrow (\exists x)Px$
- $\neg(\exists x)\neg Px \Leftrightarrow (\forall x)Px$

The distribution laws

- the conjunction of universally quantified formulas:
  $\underline{(\forall x)(Px \wedge Qx)} \Leftrightarrow \underline{(\forall x)Px} \wedge \underline{(\forall x)Qx}$
- the disjunction of existentially quantified formulas:
  $\underline{(\exists x)(Px \vee Qx)} \Leftrightarrow \underline{(\exists x)Px} \vee \underline{(\exists x)Qx}$
- not v.v.: $(\forall x)Px \vee (\forall x)Qx \Rightarrow (\forall x)(Px \vee Qx)$
- why?

Quantifier movement (QM)

- desirable format: prefix + matrix
- Movement Laws for antecedents of conditionals:
  $(\exists x)Px \rightarrow \phi \Leftrightarrow (\forall x)(Px \rightarrow \phi)$
  $(\forall x)Px \rightarrow \phi \Leftrightarrow (\exists x)(Px \rightarrow \phi)$
- Movement Laws for Q's in disjunction, conjunction, and the consequent of conditionals: Just move them to the prefix!
- condition: x must not be free in ϕ.
- i.e.: Watch your variables!

Let's formalize:

- *Paul Kalkbrenner is a musician and signed on bpitchcontrol.*
- *Herr S. installed RedHat and not every Linux distribution is easy to install.*
- *All talkmasters are human and Harald Schmidt is a talkmaster.*
- *Some talkmasters are not musicians.*
- *Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.*
- *Some humans are neither talkmasters nor do they own Kanzleramt records.*

Universal instantiation ($-\forall$) and generalization ($+\forall$)

- $(\forall x)Px \rightarrow Pa$
- always applies
- can use any variable/constant
- $Pa \rightarrow (\forall x)Px$
- iff *Pa* was instantiated by $-\forall$

Existential generalization ($+\exists$) and instantiation ($-\exists$)

- $Pa \rightarrow (\exists x)Px$ for any individual constant a
- always applies
- $(\exists x)Px \rightarrow Pa$ for some indiv. const.
- always applies (there is a minimal individual for $\exists x$)
- for some $(\exists x)Px$ and $(\exists x)Qx$ the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

One sample task

- (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.
- Formalize and prove: At least one human exists.
- (1) Dk
- (2) $(\forall x)(Dx \rightarrow Hx \vee Px)$
- (3) $\neg(\exists x)(Px \wedge Dx)$
- $(\exists x)Hx$

The proof

(1)	Dk	
(2)	$(\forall x)(Dx \rightarrow Hx \vee Px)$	
(3)	$\neg(\exists x)(Px \wedge Dx)$	
(4)	$(\forall x)\neg(Px \wedge Dx)$	3,QN
(5)	$(\forall x)(\neg Px \vee \neg Dx)$	4,DeM
(6)	$(\forall x)(Dx \rightarrow \neg Px)$	5,Comm,Impl
(7)	$Dk \rightarrow \neg Pk$	6,$-\forall$(1)
(8)	$\neg Pk$	1,7,MP
(9)	$Dk \rightarrow Hk \vee Pk$	2,$-\forall$(1)
(10)	$Hk \vee Pk$	1,9,MP
(11)	Hk	8,10,DS
$\therefore$	$(\exists x)Hx$	10,$+\exists$

Quantifikation und Modelltheorie

Back to semantics: F1

- before we turn to quantification in F1/F2 English:
- names refer to individuals
- itr. verbs refer to sets of individuals
- tr. verbs refer to sets of ordered pairs of individuals
- sentences refer to truth values

Reference of pronouns

- ***This** drives a Golf.*
- *this* = a pronominal NP
- denotes an individual
- but not rigidly
- fixed only within a specific context (SOA)

Pronouns and variables

- quantified expression: $(\forall x)Px$
- *for all assignments of 'this', 'this' has property P*
- Q evaluation in PC is algorithmic
- variables interpreted like definite pronominal NPs (within a fixed context)

Categories and lexicon

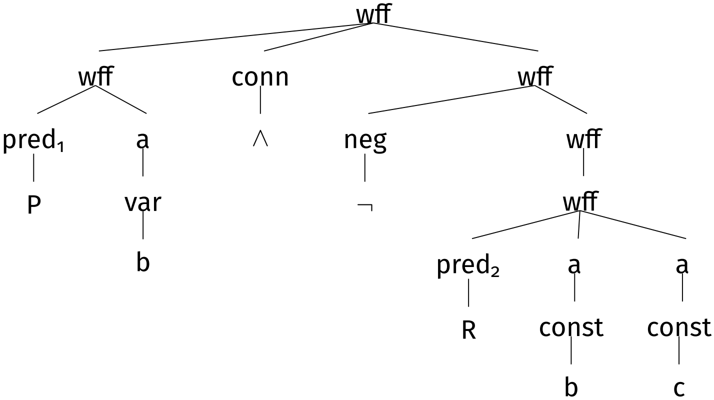
- $a \rightarrow$ const, var
- conn $\rightarrow \wedge, \vee, \rightarrow, \leftrightarrow$
- neg $\rightarrow \neg$
- Q $\rightarrow \exists, \forall$

Categories and lexicon

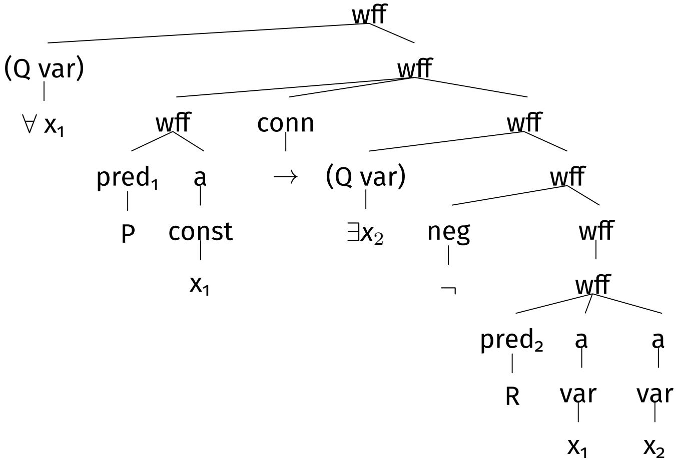
- $\text{pred}_1 \rightarrow$ P, Q
- $\text{pred}_2 \rightarrow$ R
- $\text{pred}_3 \rightarrow$ S
- const $\rightarrow$ b, c
- var $\rightarrow x_1, x_2, ..., x_n$

- wff $\rightarrow$ pred_n a_1 a_2 ...a_n
- wff $\rightarrow$ neg wff
- wff $\rightarrow$ wff con wff
- wff $\rightarrow$ (Q var) wff

A wff without Q



A wff with Q's


wff
(Q var)
wff
∀ x1
wff
conn
wff
pred1
a
→
(Q var)
wff
P
const
∃x2
neg
wff
x1
¬
wff
pred2
a
a
R
var
var
x1
x2

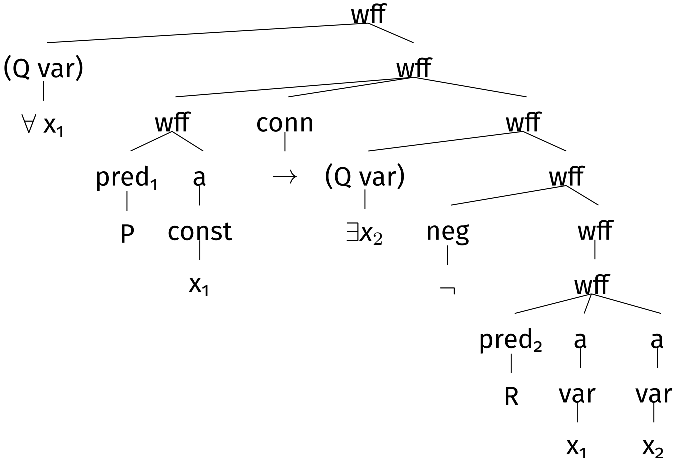
Definition of c-command

- Node A c-commands (constituent-commands) node B iff
  - A does not dominate B and
  - and the first branching node dominating A also dominates B.
- The definition in CM allows a node to dominate itself.

Configurational binding

- in configurational tree-structures:
- A variables is bound by the closest c-commanding coindexed quantifier.
- scope = binding domain

A wff with Q's


wff
(Q var)
∀ x1
wff
pred1
a
P
const
x1
conn
→
wff
(Q var)
∃x2
wff
neg
¬
wff
wff
pred2
a
a
R
var
var
x1
x2

Refinement of PC semantics

- remember T-sentences: **S of L is true in v iff p**.
- $\mathcal{M}$ is a model of the accessible universe of discourse
  - $\mathcal{M} = \langle U_n, V_n \rangle$
  - U_n = the set of accessible individuals (domain)
  - V_n = a valuation function which assigns
    - individuals to names
    - sets of n-tuples of indivuidals to $pred_n$
- **g** is function from variables to individuals in $\mathcal{M}$
- we evaluate: $[\![\alpha]\!]^{\mathcal{M}_n, g_n}$
- *the extension of α relative to $\mathcal{M}_n$ and g_n*

Fixed and context-bound denotation

- V_n valuates statically
- Q's require flexible valuation of pronominal matrices
- g_n is like V_n for constants, only flexible
- it can iterate through U_n
- initial assignment can be anything:

$$g_1 = \begin{bmatrix} x_1 \rightarrow \mathit{Herr\ Webelhuth} \\ x_2 \rightarrow \mathit{Frau\ Eckardt} \\ x_3 \rightarrow \mathit{Turm} - \mathit{Mensa} \end{bmatrix}$$

Iterating through U_n

- for each Q loop, one modification
- read $g_n\,[d/x_m]$ as
  '...relative to g_n where x_m is reassigned to d'
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[Eckardt/x_1]} = Frau\ Eckardt$
- $[\![x_2]\!]^{\mathcal{M}_1,g_1[[Eckardt/x_1]Mensa/x_2]} = Mensa$

Interpreting with g_n

- $[\![(\forall x_1)Px_1]\!]^{\mathcal{M}_1,g_1}$
- start with initial assignment: $[\![x_1]\!]^{\mathcal{M}_1,g_1} = \textit{Webelhuth}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- modify: $[\![x_1]\!]^{\mathcal{M}_1,g_1[\textit{Eckardt}/x_1]} = \textit{Eckardt}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- modify: $[\![x_1]\!]^{\mathcal{M}_1,g_1[\textit{Mensa}/x_1]} = \textit{Mensa}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- iff the answer was never 0, then $[\![(\forall x_1)Px_1]\!]^{\mathcal{M}_1,g_1} = 1$

Multiple Q's: subloops

- $[\![(\forall x_1)(\exists x_2)Px_1x_2]\!]^{\mathcal{M}_1,g_1}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1} = \mathit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1} = \mathit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\mathit{Webelhuth}/x_2]} = \mathit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\mathit{Mensa}/x_2]} = \mathit{Mensa}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[\mathit{Eckardt}/x_1]} = \mathit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\mathit{Eckardt}/x_1]} = \mathit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\mathit{Eckardt}/x_1]\mathit{Webelhuth}/x2]} = \mathit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\mathit{Eckardt}/x_1]\mathit{Mensa}/x2]} = \mathit{Mensa}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[\mathit{Mensa}/x_1]} = \mathit{Mensa}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\mathit{Mensa}/x_1]} = \mathit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\mathit{Mensa}/x_1]\mathit{Webelhuth}/x_2]} = \mathit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\mathit{Mensa}/x_1]\mathit{Mensa}/x_2]} = \mathit{Mensa}$

Natural weirdness

- quantifying expressions in NL beyond $\forall$ and $\exists$
- some seem to work differently:
- *All patients adore Dr. Rick Dagless M.D.*
  $(\forall x_1)Px_1 \rightarrow Ax_1d$ (ok)
- but: *Most patients adore Dr. Rick Dagless M.D.*
  $(MOST\ x_1)Px_1 \rightarrow Ax_1d$ (wrong interpretation)
- domain should be the set of patients, not individuals
- For NL: Assume that the checking domain for Q is the set denoted by CN.

Scope ambiguities

- c-command condition on binding/scope fails in NL
- no PNF's in NL
- Q and common noun (CN) usually in-situ (e.g., argument position)
- ambiguities independent of Q position
  - *Everybody loves somebody.* (*ELS*)
  - $(\forall x_1)(\exists x_2)Lx_1x_2$
  - $(\exists x_2)(\forall x_1)Lx_1x_2$
- Q ambiguity cannot be structural (e.g., $\exists$ will never c-command $\forall$)

Cases of overt movement and traces

- wh movement:
- *What$_i$ will Agent Cooper solve t_i?*
-
- passive movement:
- *(Laura Palmer)$_i$ was killed t_i.*
-
- raising verbs:
- *(Laura Palmer)$_i$ seems t_i to be dead.*
-

- construction of an independent representational level LF
- could use movement mechanism as used at surface level
- All quantifiers adjoin to the left periphery of S at LF.
- LF is constructed by syntactic rules!

Ambiguities at LF

- $[_{S''}$ *everybody*$_i$ $[_{S'}$ *somebody*$_j$ $[_S$ t_i *loves* t_j $]]]$
-
- $[_{S''}$ *somebody*$_j$ $[_{S'}$ *everybody*$_i$ $[_S$ t_i *loves* t_j $]]]$
-

$$[_{S}\, X\ NP\ Y\,] \;\Rightarrow\; [_{S'}\, NP_i\ [_{S}\, X\ t_i\ Y\,]]$$

- specify a PS as input and output
- QR rule also introduces coindexing of traces

Syntax

- copies all definitions from F1
- adds appropriate definitions of quantifying determiners etc.
  - $Det \rightarrow every, some$
  - $NP \rightarrow DetN_{common-count}$
- adds the QR rule
- assume introduction of reasonable syntactic types/rules without specifying
- assume admissible (reasonable, possible) models $\mathcal{M}$

$$[\![\,[\,[\textit{every}\ \beta]_i\ S]\,]\!]^{\mathcal{M},g} = 1 \textit{ iff for all } u \in U: \\ \textit{if } u \in [\![\beta]\!]^{\mathcal{M},g} \textit{ then } [\![S]\!]^{\mathcal{M},g[u/t_i]}$$

A sentence containing the trace t_i with an adjoined NP_i (which consists of *every* plus the common noun β) extend to 1 iff for each individual *u* in the universe *U* which is in the set referred to by the common noun β, *S* denotes 1 with *u* assigned to the pronominal trace t_i. *g* is modified iteratively to check that.

Semantics for QR output: *some, a*

$$[\![\,[\,[a\ \beta]_i\ S]\,]\!]^{\mathcal{M},g} = 1 \textit{ iff for some } u \in U: \\ u \in [\![\beta]\!]^{\mathcal{M},g} \textit{ and } [\![S]\!]^{\mathcal{M},g[u/t_i]}$$

(similar)

Einfach getypte höherstufige λ-Sprachen

Montague and the generative tradition

- Chierchia & McConnell-Ginet, Heim & Kratzer, etc.: GB-ish semantics
- both syntax and LF in phrase structures
- LF as a proper linguistic level of representation
- Montague: direct translation of NL into logic
- Monatgue's LF is just a notational system for NL semantics

Targets for this week

- Learn to tell the difference between the montagovian and generative approach.
- See the advantage of a general theory of typed languages.
- Understand how λ languages allow dramatically elegant formalizations.
- … while keeping in mind that these devices are extensions to our PC representation for NL semantics.

Denotations

- denotations in set/function-theoretic terms
- a characteristic function (CF) $\mathcal{S}$ of a set S:
  $\mathcal{S}(a) = 1$ *iff* $a \in S$, *else* 0
- a CF ‘checks’ individuals into a set
- denotations can be stated as sets or their CF

Generalizing combinatory semantic operations

- interpretation for [$_S$ NP VP]:
  $[\![\,[_S\ NP\ VP]\,]\!]^{\mathcal{M},g} = 1$ *iff* $[\![NP]\!]^{\mathcal{M},g} \in [\![VP]\!]^{\mathcal{M},g}$
- Montague generally used CF's in definitions
- evaluating [$_S$ [$_{NP}$ *Mary*] [$_{VP}$ *sleeps*]] as a matter of functional application (FA):
  - $[\![Mary]\!]^{\mathcal{M},g} =$ *Mary in* $\mathcal{M}$
  - $[\![sleeps]\!]^{\mathcal{M},g}$ *be the CF of the set of sleepers in* $\mathcal{M}$
  - $[\![S]\!]^{\mathcal{M},g} = [\![sleeps]\!]^{\mathcal{M},g}([\![Mary]\!]^{\mathcal{M},g})$
  - ideally: generalize to all nodes

The superscript notation

- all functions from S_1 to S_2
- $S_2^{S_1}$
- for $T = \{0, 1\}$
  - T^D: all $pred_1$
  - $T^{D \times D}$: all $pred_2$
-

Some new names

- base for Dowty et al.: L_1, a first-order predicate language as we know it
- semantic renaming of types:
  - terms: $\langle e \rangle$ (entity-denoting)
  - formulas: $\langle t \rangle$ (truth-valued)
  - pred_1: $\langle e, t \rangle$
  - pred_2: $\langle e, \langle e, t \rangle \rangle$

Possible denotations of types

- D_α possible denotation (a set) of expressions of type α
- $D_{\langle e \rangle} = U$ (Dowty et al.'s A)
- $D_{\langle t \rangle} = \{0, 1\}$
- recursively: $D_{\langle \alpha, \beta \rangle} = D_{\langle \beta \rangle}^{D_{\langle \alpha \rangle}}$
- e.g., $D_{\langle e,t \rangle} = D_{\langle t \rangle}^{D_{\langle e \rangle}}$
- $D_{\langle e, \langle e,t \rangle \rangle} = (D_{\langle t \rangle}^{D_{\langle e \rangle}})^{D_{\langle e \rangle}}$
- just a systematic way of naming types, model-theoretic interpretations still by V, g

Defining types

- in our PS syntax: S as start symbol
- in the typed system: sentences should be of type $\langle t \rangle$
- complex types: functions from $\langle e \rangle$ to $\langle t \rangle$
  or generally from any (complex) type to any (comlex) type

Complex types as functions

- saturation of complex types by FA:
  - γ *is of type* $\langle e, \langle e, t \rangle \rangle$, δ *of* $\langle e, t \rangle$, α *and* β *of* $\langle e \rangle$
  - then $\gamma(\alpha)$ *is of type* $\langle e, t \rangle$
  - and $\delta(\beta)$ *is of type* $\langle t \rangle$
- for any pred_2 P and its arguments a_1, a_2, $P(a_2)(a_1)$ is a wff
- connectives are of types $\langle t, t \rangle$ ($\neg$), $\langle t, \langle t, t \rangle \rangle$ ($\wedge$, etc.)

General semantics of typed languages

- generalized CF/FA approach
- $\langle e \rangle$-types (terms):
  $[\![a_n]\!]^{\mathcal{M},g} = V(a_n)$
  $[\![x_n]\!]^{\mathcal{M},g} = g(x_n)$
- the rest: functional application
  $[\![\delta(\alpha)]\!]^{\mathcal{M},g} = [\![\delta]\!]^{\mathcal{M},g}([\![\alpha]\!]^{\mathcal{M},g})$

Refinement

- *Type* is the set of types
- recursively defined complex types $\langle a, b\rangle$: infinite
- type label $\langle\alpha\rangle$
- vs. set of meaningful expressions of that type: $ME_{\langle\alpha\rangle}$

Higher order

- first order languages: variables over individuals ($\langle e \rangle$-types)
- n-order: variables over higher types ($\langle e, t \rangle$-types etc.)
- $P_{\langle e,t \rangle}$ or $Q_{\langle e,\langle e,t \rangle\rangle}$: constants of higher types
- so: $v_{1_{\langle e,t \rangle}}\ [v_1(m)]$
- if $V(m) = Mary$, v_1 is the set of all of Mary's properties

Typing variables

- we write:
  - $v_{n_{\langle\alpha\rangle}}$ for the n-th variable of type $\langle\alpha\rangle$
  - Dowty et al.: $v_{n,\langle\alpha\rangle}$
- alternatively abbreviated by old symbols x_1, a, P, etc.

Constants, variables, functions

- non-logical constant α: $[\![\alpha]\!]^{\mathcal{M},g} = V(\alpha)$
- variable α: $[\![\alpha]\!]^{\mathcal{M},g} = V(\alpha)$
- $\alpha \in \langle a, b \rangle,\ \beta \in a$, then $[\![\alpha(\beta)]\!]^{\mathcal{M},g} = [\![\alpha]\!]^{\mathcal{M},g}([\![\beta]\!]^{\mathcal{M},g})$

Logical constants and quantifiers

- logical constants interpreted as functions in {0,1} as usual
- if $v_{1_{\langle\alpha\rangle}}$ is a variable and $\phi \in ME_t$
  then $[\![(\forall v_1)\phi]\!]^{\mathcal{M},g} = 1$*iff*
  for all $a \in D_\alpha$ $[\![\phi]\!]^{\mathcal{M},g[a/v_1]} = 1$

An example

- quantified variable of type $\langle e, t \rangle$: $v_{0_{\langle e,t \rangle}}$
- $\forall v_{0_{\langle e,t \rangle}} \left[v_{0_{\langle e,t \rangle}}(j) \rightarrow v_{0_{\langle e,t \rangle}}(d) \right]$
- for $j, d \in ME_{\langle e \rangle}$
- one property of every individual: being alone in its union set
- hence, $j = d$
- else in $\forall v_{0_{\langle e,t \rangle}}$, $\forall$ wouldn't hold

Defining *non*

- productive adjectival prefix: *non-adjacent*, *non-local*, etc.
- inverting the characteristic function of the adjective
- result denotes complement of the original adjective in $D_{\langle e \rangle}$
- *adjective*: $\langle e, t \rangle$, *non*: $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$
- a function h s.t. for every $k \in D_{\langle e,t \rangle}$ and every $d \in D_{\langle e \rangle}$
  $(h(k))(d) = 1$ iff $k(d) = 0$ and
  $(h(k))(d) = 0$ iff $k(d) = 1$

Argument deletion

- understood objects in: *I eat.* - *Vanity kills.* - etc.
- *eat* is in $ME_{\langle e,\langle e,t\rangle\rangle}$
- assume a silent logical constant: R_O in $ME_{\langle\langle e,\langle e,t\rangle\rangle,\langle e,t\rangle\rangle}$
- a function h s.t. for all $k \in D_{\langle e,\langle e,t\rangle\rangle}$ and all $d \in D_{\langle e\rangle}$
  $h(k)(d) = 1$ iff there is some $d' \in D_{\langle e\rangle}$ s.t. k(d′)(d)=1
- passives as similar subject deletion

All there is to λ

- a new variable binder
- allows abstraction over wff's of arbitrary complexity
- similar to $\{x \| \phi\}$ (read as 'the set of all x s.t. ϕ')
- we get $\lambda x\,[\phi]$
- on Montague's typewriter: $\hat{x}\,[\phi]$
- does not create a set but a function which can be taken as the CF of a set

λ abstraction

- for every wff ϕ, any $x \in Var$, and any $a \in Con$
- λ abstraction: $\phi \rightarrow \lambda x \left[\phi^{[a/x]}\right](a)$
- read $\phi^{(a/x)}$ as '*phi* with every a replaced by x'
- x can be of any type

Two informal examples

- $\lambda x_{\langle e \rangle}\ [L(x)]$ is the characteristic function of the set of those individuals $d \in D_{\langle e \rangle}$ which have property L
- $\lambda x_{\langle e,t \rangle}\ [x(l)]$ is the characteristic function of the set of those properties $k \in D_{\langle e,t \rangle}$ that the individual l has

λ conversion

- $\lambda x\,[L(x)]$ is the abstract of $L(a)$ (with some individual a)
- hence, it holds: $\lambda x\,[L(x)]\,(a) \Leftrightarrow L(a)$
- for every wff ϕ, any $x \in Var$, and any $a \in Con$
- λ conversion: $\lambda x\,[\phi]\,(a) \rightarrow \phi^{[x/a]}$

- $\lambda x\,[\phi]\,(a) \leftrightarrow \phi^{[x/a]}$
- not just syntactically, since truth conditions are equivalent
- $\lambda x\,[\phi]\,(a) \Leftrightarrow \phi^{[x/a]}$
- notice: $\lambda x_{\langle\alpha\rangle}\,[\phi]$ is in $ME_{\langle\alpha,t\rangle}$
- while ϕ (as a wff) is in $ME_{\langle t\rangle}$

The full rules

- Dowty et al., 102f. (*Syn C.10* and *Sem 10*)
- If $\alpha \in ME_\alpha$ and $u \in Var_b$, then $\lambda u\,[\alpha] \in ME_{\langle b,a \rangle}$.
- If $\alpha \in ME_a$ and $u \in Var_b$ then $[\![\lambda u\,[\alpha]]\!]^{\mathcal{M},g}$ is that function h from D_b into D_a s.t. for all objects k in D_b, $h(k)$ is equal to $[\![\alpha]\!]^{\mathcal{M},g[k/u]}$.

The *non* example revised (Dowty et al., 104)

- $\forall x \forall v_{0\langle e,t \rangle} \left[(\mathbf{non}(v_{0_{\langle e,t \rangle}}))(x) \leftrightarrow \neg(v_{0_{\langle e,t \rangle}}(x)) \right]$
- $\forall v_{0_{\langle e,t \rangle}} \left[\lambda x \left[(\mathbf{non}(v_{0_{\langle e,t \rangle}}))(x) \right] = \lambda x \left[\neg(v_{0_{\langle e,t \rangle}}(x)) \right] \right]$
- $\forall v_{0_{\langle e,t \rangle}} \left[\mathbf{non}(v_{0_{\langle e,t \rangle}}) = \lambda x \left[\neg(v_{0_{\langle e,t \rangle}}(x)) \right] \right]$
  (since $\lambda x\, [\mathbf{non}(v)(x)]$ is unnecessarily abstract/η reduction)
- $\lambda v_{0_{\langle e,t \rangle}} \left[\mathbf{non}(v_{0_{\langle e,t \rangle}}) \right] = \lambda v_{0_{\langle e,t \rangle}} \left[\lambda x \left[\neg(v_{0_{\langle e,t \rangle}}(x)) \right] \right]$
- and since that is about all assignments for $\lambda v_{0_{\langle e,t \rangle}}$:
  $\mathbf{non} = \lambda v_{0_{\langle e,t \rangle}} \left[\lambda x \left[\neg v_{0_{\langle e,t \rangle}}(x) \right] \right]$

Mary is non-adjacent.

(translate 'adjacent' as $c_{0_{\langle e,t \rangle}}$, 'Mary' as $c_{0_{\langle e \rangle}}$, ignore the copula)

The behavior of quantified NPs

- syntactically like referential NPs
- semantically like PC quantifiers
- *Every student walks.*: $\forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \rightarrow c_{1_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \right]$
- *Some student walks.*: $\forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \wedge c_{1_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \right]$
- making referential NPs and QNPs the same type?

A higher type

- $\lambda v_{0_{\langle e,t \rangle}} \forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \rightarrow v_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \right]$
- a second order function
- characterizes the set of all predicates true of every student
- equally: $\lambda v_{0_{\langle e,t \rangle}} \exists v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \wedge v_{0_{\langle e,t \rangle}} \left(v_{0_{\langle e \rangle}} \right) \right]$

Combining with some predicate

Intensionalität

Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Some examples

- Stockhausen will write another opera.
- Had Arno Schmidt cut down on drinking, he would still be alive.
- Gustave Moreau believes that estheticism rules.

Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (tense)
- … and for just one state of affairs (modals, believe type verbs)

What are intensions?

Type	Reference	Sense
NP	individuals *Venus*	individual concepts
VP	sets *humming birds*	property concepts
S	1 or 0 *I like cats.*	thoughts or **propositions**

Properties of intensions

- can't be just truth conditional
- encode knowledge about not just the actual but all possible and/or past/future states of affairs (PSOAs)
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values

PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just thruth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

A touch of parellel universes?

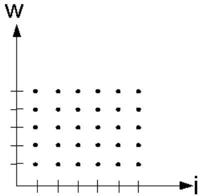
- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
  - A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
  - nothing of the above, but Maria was born 20 years earlier
  - nothing of the above, but A.S. rose from the dead in 2003, etc.

Propositions and PSOAs

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

Coordinates

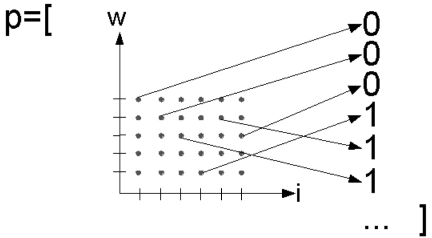
- for each possible distinction in truth values of the whole of the propositional sentences: one possible world ($w \in W$)
- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$



The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

- a propositional function *p*
- is a function from $W \times I$ to $\{0, 1\}$

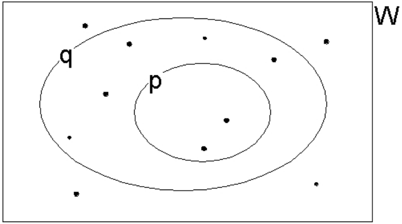


Your evening prayer

- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

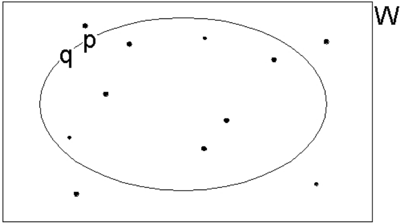
Entailment

- defintion of intensions of sentences (propositions): characteristic functions
- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation: $p \subseteq q$:



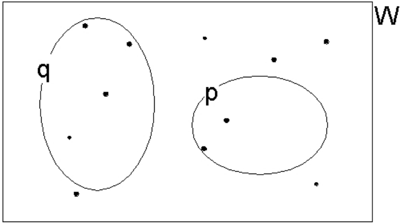
Synonymy

- synonymy turns out as set equivalence:
- $p = q$



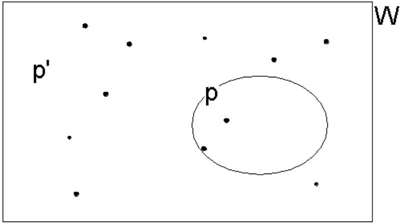
Contradiction

- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



Negation

- negation turns out as a complement:
- p/W

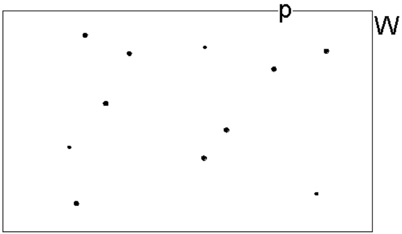


Quantification over worlds

- new modal sentence/wff operators:
  - *necessarily p*: $\Box \mathbf{p}$
  - *possibly p*: $\Diamond \mathbf{p}$
- What does it mean for a proposition to be necessary/possible?

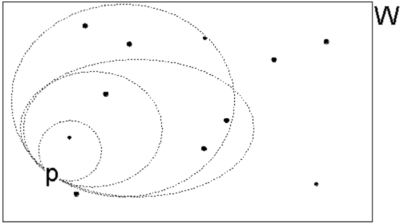
Necessity as universal quantification

- if $\Box p$ then $(\forall w)\ [p(w) = 1]$ (p as characteristic function)
- such that $W = p$ (p as set):



Possibility as existential quantification

- if $\Diamond p$ then $(\exists w)\,[p(w) = 1]$ (characteristic function)
- such that $p \neq \emptyset$ (set):

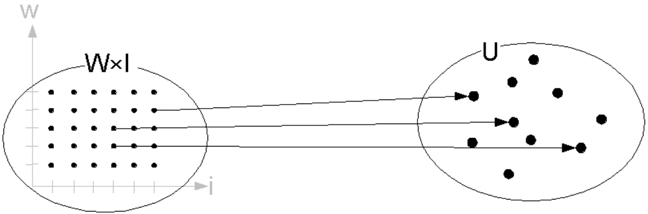


A larger tuple

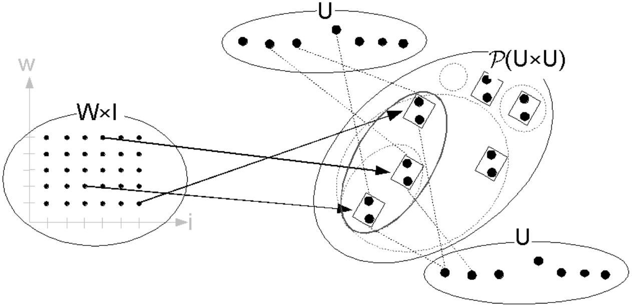
- $\mathcal{M} = \{W, I, <, U, V\}$
  - *W*, a set of worlds
  - *I*, a set of instants
  - $<$, an ordering relation in I
  - *U*, the set of individuals
  - *V*, a valuation function for constants
- evaluate an expression α: $[\![\alpha]\!]^{\mathcal{M},w,i,g}$

Intensional interpretation of individual constants

- *the President of the United States*, *the Pope*, *Bond* (in the sense of 'the actor currently playing Bond')
- *for* $\beta \in Cons_{ind}$, $V(\beta)$ is a function from $W \times I$ to U



- *walks* etc. denotes different sets (or CFs) at different $\langle w, i\rangle$ coordinates
- *for* $\beta \in Cons_{pred_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ $(U^n = U_1 \times U_2 \times \ldots \times U_n)$



The Chierchia approach: predicates/sentences

- simple sentences/predicates: $\beta = \delta(t_1, t_2, \ldots, t_n)$
- $[\![\beta]\!]^{\mathcal{M},w,i,g} = 1$ iff
- $\langle [\![t_1]\!]^{\mathcal{M},w,i,g}, [\![t_2]\!]^{\mathcal{M},w,i,g}, \ldots, [\![t_n]\!]^{\mathcal{M},w,i,g} \rangle \in [\![\delta]\!]^{\mathcal{M},w,i,g}$
- with: $[\![t_1]\!]^{\mathcal{M},w,i,g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

Quantification

- if $\psi = \forall x \phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $u \in U$
- ...$[\![\phi]\!]^{\mathcal{M},w,i,g[u/x]} = 1$
- nothing new here

Modalities

- if $\psi = \Box x \phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $w' \in W$
- ...and all $i' \in I$
- ...$[\![\phi]\!]^{\mathcal{M},w',i',g} = 1$

A similarity of $\forall$ and $\Box$

- as: $\forall x\,[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that: $\Box\,[\psi \rightarrow \phi] \rightarrow [\Box\psi \rightarrow \Box\phi]$
- but not vice-versa!

Some validities

- $\exists x \Box P(x) \rightarrow \Box \exists x P(x)$
- $\exists x \Diamond P(x) \leftrightarrow \Diamond \exists x P(x)$
- $\forall x \Box P(x) \leftrightarrow \Box \forall x P(x)$ (Carnap-Barcan)
- $\forall x \Diamond P(x) \rightarrow \Diamond \forall x P(x)$

Tempus und Modalität

Targets for this week

- Understand how simple tense logic can be represented by operators shifting *i* indices.
- See why tense operators are sentence operators.
- See why a multi-dimensional theory of tenses and a better handling of tense embedding are required.
- See how we restrict (different types of) propositional backgrounds.
- Understand how opaque contexts affect meaning (incl. *believe* type verbs).
- Get a first idea of why we need the *up* operator $^\wedge$.

Will, was... and always

- present: no operator (ϕ 'it is the case that ϕ')
- past: $\mathbf{P}$ ($\mathbf{P}\phi$ 'it was the case that ϕ')
- future: $\mathbf{F}$ ($\mathbf{F}\phi$ 'it will be the case that ϕ')
- it will always be the case... ($\mathbf{G} = \neg\mathbf{F}\neg\phi$)
- it was always the case... ($\mathbf{H} = \neg\mathbf{P}\neg\phi$)

Evaluation

- $\mathbf{P}D(a)$ 'Arno Schmidt (has?) died.'
- relative to the current $\langle w, i \rangle$: $[\![\mathbf{P}D(a)]\!]^{\mathcal{M},w,i,g}$
- ...is true iff there is some i', $\langle i', i \rangle \in <$ and
- $[\![\mathbf{P}D(a)]\!]^{\mathcal{M},w,i',g} = 1$

Like it or not...

- tense operators (TOp) are sentence (wff) Op's
- raise it to sentence-scopal position
- TP/IP position is motivated by copular/auxiliary elements
- *He **is** stupid.* vs. *Kare-wa bakarashi-**i**.*
- *He **was** stupid.* vs. *Kare-wa bakarashi-**katta**.*
- *What$_i$ **did** you expect t$_i$?* vs. *Nani-o yokishi-**ta**-ka.*

New ps rules

- $T' \rightarrow T VP$ (adds tense to VP)
- $TP \rightarrow NP\ T'$
- $TP \rightarrow TP\ conj\ TP$
- $TP \rightarrow neg\ TP$
- $[_{TP}\ NP\ T\ VP] \Rightarrow [_{TP}\ T\ NP\ VP]$ (T raising)

Quantification over instants

- $[\![\mathbf{P}\mathit{TP}]\!]^{\mathcal{M},w,i,g} = 1$
- iff among all $\langle i_n, i \rangle \in <$
- there is at least one s.t. $[\![\mathit{TP}]\!]^{\mathcal{M},w,i',g} = 1$

Valuations as in Chierchia's M_3

- *U*: domain of quantification
- $V(\beta)$: non-relativized function for all β which are not a proper name
- $V(\beta)(\langle w, i \rangle)$: V valuates β to a function from world-time pairs to the denotata of the predicate (sets of individuals, tuples of them, etc.)

- NL tenses beyond TOp's:
- *Arno Schmidt had already read Poe when he started writing 'Zettels Traum'.*
- *Gosh, I forgot to feed the cat.*
- shifts of evaluation time

	past (R<S)	present (R,S)	future (S<R)
anterior(E<R)	E<R<S _er war gegangen_	E<R,S _er ist gegangen_	S<E<R S,E<R E<S<R _er wird gegangen sein_
simple(E,R)	E,R<S _er ging_	E,R,S _er geht_	S<E,R _er wird gehen_
posterior(R<E)	R<E<S R<S,E R<S,E R<S<E *_er würde gehen_	R,S<E _er wird gehen_	S<R<E *_er wird gehen werden_

Embedded tenses and adverbials

- *A man was born who will be king.*
- **P**(a man is born **F**(who be king)) ?
- *Yesterday, Maria woke up happy.*
- **Y**(**P**(Maria wake up happy)) ?

Types of modal expressions

- tense forms: *I eat up to 100 nachos a minute.*
- mood: *Responderet alius minus sapienter.*
- modal auxiliaries: *Herr Webelhuth can look like Michael Moore.*
- adverbs: *Maybe Herr Keydana will show up.*
- affixes: *Frau Eckardt is recognizable.*

The logical form of modal operators

- like tense: sentence operators
- modal *Aux* in English is tense-insensitive (evidence for *Infl*)
- $\Box$ and $\Diamond$ in intensional predicate calculi (IPC): exploit the full set of possible worlds
- in NL: evaluation of modal expressions against restricted conversational backgrounds

The background

- different sets of possible worlds under consideration for different types of modal expressions
- different types of modality: different sets of admitted possible worlds
- we call the conversationally relevant background the set of $\langle w, i \rangle$ pairs relevant to the interpretation of the sentence

Root/Logical modality

- *Agent Cooper cannot solve the mystery.*
- translated into root modal IPC: $\neg\Diamond S(c, m)$
- wrong interpretation: Under no possible circumstances can Cooper solve the mystery.
- usually, some obvious facts constitute the background:
  - he could, but some relevant information is missing
  - he could, but is sick
  - he could, but ...

Epistemic modality

- *Leo Johnson must be the murderer of Laura Palmer.*
- in accordance with the known facts (e.g., in episode 7 of *Twin Peaks*):
  - Leo Johnson is a violent person.
  - Leo smuggles cocaine, Laura was addicted to it.
  - Leo is connected to Jacques Renault who is the bartender of *One Eyed Jack's* where Laura worked as a prostitute.
  - ...
- which constitute the epistemic background, the sentence is true
- known facts narrow down the root background

Deontic modality

- *Agent Cooper must not solve the mystery.*
- assume:
  - there is some U.S. law which allows a local sheriff to ask the FBI to keep out of local murder investigations
  - Sheriff Truman has asked the FBI headquarters to keep out of the Palmer investigation
  - as a special agent, Cooper is required to obey Bureau policy
- Deontic backgrounds are narrowed down by normative rules and moral ideals.
- statable in propositional form (ten commandments, law, ...)

Sets of propositions

- specify the kind of background against which you evaluate under the given situation
- we need:
  a function from $\langle w, i \rangle$ to the relevant background set of $\langle w_n, i_m \rangle$
- reuse g:
  $g(\langle w, i \rangle) = \{p_1, p_2, \ldots, p_n\} = \{\langle w, i \rangle_1, \langle w, i \rangle_2, \ldots, \langle w, i \rangle_n\}$
- such that all possible worlds are: $\bigcap g(\langle w, i \rangle)$

CP structures: *that*

- *that* is a complementizer, it turns a sentence into an argument.
- ps rule: $CP \rightarrow C\ IP$
- *[$_{IP}$ Racine believes [$_{CP}$ that [$_{IP}$ theatre rules]]]*
- CP (fully fledged sentence) receives theta role by *believe* under government.

Weak *Infl* and *PRO*

- gerunds:
  [$_{IP}$ Stockhausen has plans [$_{IP}$ to write another 29 hour opera]]
- incomplete embedded IP, no subject
- internal theta role of *has plans*: to IP
- external theta role of *write*: to ?
- PRO, controlled by the subject of *has plans*:
  [$_{IP}$ Stockhausen has plans [$_{IP}$ PRO to write another 29 hour opera]]

Propositional attitudes

- verbs like *believe*: propositional attitude verbs
- content of the believe: a pice of information held to be true by the believer, hence a proposition, a $\langle w_n, i_m \rangle$
- signalling one element in the background assumed by the believer
- belief: $\langle w, i \rangle$ is an element of the proposition of CP

- value of propositional attitude (PA) verbs: functions $[\langle w,i\rangle \rightarrow \langle u_n,p\rangle]$ with $u_n \in U$, p a proposition (set of $\langle w_n,i_m\rangle$) and compatible to u$_n$'s background
- *up* ($^\wedge\chi$): an operator which gives the intension of an expression χ
- the full logic of $^\wedge$ and $^\vee$ as designed by Montague next week
- $^\wedge$rids us of the problem that the belief content looks truth-conditional (a sentence) but doesn't contribute to the embedding sentence's truth-value. PA verbs take intensions as arguments.

Meet B.J. Ortcutt

- Quine's story: Ralph knows...
- Bernard J.Ortcutt, the nice guy on the beach.
- He sees a strange guy with a hat in the dark alley - a spy?
- Ortcutt just likes to behave funny on the way to his pub...
- and actually is sinister guy in the alley!
- Only Ralph doesn't know.

Is Ralph insane?

- What's the truth value of...
- *Ralph believes that the guy from the beach is a spy.*
- true: since Ortcutt and the guy in the hat are one individual
- false: since Ralph doesn't know that and in a way 'doesn't believe it'

de dicto and *de re*

- the Russelian interpretation for *the* like $\exists$ with a uniqueness condition (as a GQ): $\lambda Q \lambda P\, [\exists x\, [Q(x) \wedge P(x)] \wedge \forall y\, [Q(y) \leftrightarrow y = x]]$
- in a raising framework: ambiguity between *THE* and *believe*
- $[_{IP}$ the guy from the beach$_i$ $[_{IP}$ Ralph believes $[_{CP}$ that x_i is a spy]]]
- makes the sentence true: the *de re* reading
- Ralph believes $[_{CP}$ that $[_{IP}$ the guy from the beach$_i$ $[_{IP}$ x_i is a spy]]]
- makes the sentence false: the *de dicto* reading

Rigid designators

- *Yuri Gagarin might now have been the first man in space.*
- some Mickey Mouse LFs:
- $\Diamond$ THE(first-man-in-space)(not-be-Gagarin)
- at some $\langle w_n, i_m \rangle$ the first individual in space is not Y.G.
- THE(first-man-in-space)($\Diamond$[not-be-Gagarin])
- at $\langle w, i \rangle$ the first individual in space (definitely Y.G.) is not Y.G. in an accessible world
- Names are rigid designators across world-time-pairs, definite descriptions aren't.

Chierchia's formalization

- CP has its own subject, *to*-IPs don't (PRO)
- PRO must be interpreted, in our examples by coindexation with the matrix subject
- infinitive embedding verbs: functions from world-time pairs to sets of individuals which have a certain property, the intension of a predicate $^\wedge P$
- *John tries to sing.*
- $try(j, {}^\wedge swim)$

Montagues intentionale Logik

Beyond truth functionality

- $[\![\phi]\!]^{\mathcal{M},w,i,g}$ and $[\![\mathbf{P}]\!]^{\mathcal{M},w,i,g}$ don't truth conditionally determine $[\![\mathbf{P}\phi]\!]^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**, $\Diamond$, $\Box$ are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models
- for a name d: $[\![d]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for an individual concept denoting expression *m*:
- $[\![m]\!]^{\mathcal{M},g}_{\not{c}} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_2, t_1\rangle & \rightarrow & c \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_2\rangle & \rightarrow & c \\ \langle w_1, t_3\rangle & \rightarrow & c \\ \langle w_2, t_3\rangle & \rightarrow & b \end{bmatrix}$

- for a one place predicate *B*:
- $[\![B]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{bmatrix}$

- formula ϕ: $[\![\phi]\!]_{\not c}^{\mathcal{M},g}$ is a function from indices to truth values
- $[\![B(m)]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 1 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 0 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$
- $[\![B(n)]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 0 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 1 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$

Intensions of formulas

- again, the proposition $[\![Bm]\!]^{\mathcal{M},g}_{\not{c}}$ is a set of indices ($\langle w_i, t_j \rangle$)
- from the extension at all indices, compute the intension
- $[\![\alpha]\!]^{\mathcal{M},g}_{\not{c}}(\langle w_i, t_j \rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

Intensions of variables

- constant function on indices
- will play a great role, so remember!
- $[\![u]\!]_{\not{c}}^{\mathcal{M},g}(\langle w_i, t_j\rangle) = g(u)$

What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes they denote intensions (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

Up and down

- Church/Montague: for an extension-denoting expression α, ${}^\wedge\alpha$ denotes α's intension
- $[\![{}^\wedge Bm]\!]^{\mathcal{M},w,i,g} = [\![Bm]\!]^{\mathcal{M},g}_{\cancel{c}}$
- α and ${}^\wedge\alpha$ are just denoting expressions
- for an intension-denoting expression α: $[\![{}^\vee\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w, t\rangle)$

Down-up and up-down

- observe: $\llbracket \check{}\hat{}\alpha \rrbracket^{\mathcal{M},w,i,g} = \llbracket \alpha \rrbracket^{\mathcal{M},w,i,g}$ for any $\langle w,t \rangle$
- but not always: $\llbracket \hat{}\check{}\alpha \rrbracket^{\mathcal{M},w,i,g} = \llbracket \alpha \rrbracket^{\mathcal{M},w,i,g}$ for any $\langle w,t \rangle$
- can easily be the case for intension-denoting expressions

Non-equality

- *k*' intension: $[\![k]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_1, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix} \\ \langle w_2, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_2, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix} \end{bmatrix}$

Non-equality

- k' extension (e.g., at $\langle w_1, t_2\rangle$): $[\![k]\!]_{\cent}^{\mathcal{M},g}(\langle w_1, t_2\rangle) =$
- $[\![k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & c \\ \langle w_2, t_2\rangle & \rightarrow & d \end{bmatrix}$
- however: $[\![\tilde{}k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & d \\ \langle w_2, t_2\rangle & \rightarrow & b \end{bmatrix}$
- since: $[\![\check{}k]\!]^{\mathcal{M},w_1,t_1,g} = a$
  $[\![\check{}k]\!]^{\mathcal{M},w_1,t_2,g} = b$
  $[\![\check{}k]\!]^{\mathcal{M},w_2,t_1,g} = d$
  $[\![\check{}k]\!]^{\mathcal{M},w_2,t_2,g} = b$

A typed higher order λ language with = and ˆ / ˇ

- $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\mathbf{F}$, $\mathbf{P}$, $\Box$, $=$ (syncategorematically)
- $t, e \in Type$ (Con_{type}, Var_{type})
- if $a, b \in Type$, then $\langle a, b \rangle \in Type$
- if $a \in Type$, then $\langle s, a \rangle \in Type$
- $s \notin Type$

Meaningful expressions

- ME_{type}
- abstraction: if $\alpha \in ME_a$, $\beta \in Var_b$, $\lambda\beta\alpha \in ME_{\langle b,a\rangle}$
- FA: if $\alpha \in ME_{\langle a,b\rangle}$, $\beta \in ME_a$ then $\alpha(\beta) \in ME_b$
- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

- if $\alpha \in ME_a$ then $\hat{}\alpha \in ME_{s,a}$
- if $\alpha \in ME_{\langle s,a\rangle}$ then $\check{}\alpha \in ME_a$
-
type	variables	constants
e	x, y, z	a, b, c
$\langle s, e\rangle$	x, y, z	—
$\langle e, t\rangle$	X, Y	$walk', A, B$
$\langle\langle s, e\rangle, t\rangle$	Q	$rise', change'$
$\langle s, \langle e, t\rangle\rangle$	P	—
$\langle e, e\rangle$	P	Sq
$\langle e, \langle e, t\rangle\rangle$	R	Gr, K
$\langle e, \langle e, e\rangle\rangle$	—	$Plus$

The model

- $\langle A, W, T, <, F\rangle$
- $D_{\langle a,b\rangle} = {D_b}^{D_a}$
- $D_{\langle s,a\rangle} = {D_a}^{W\times T}$
- 'senses' = **possible** denotations
- actual intensions chosen from the set of senses
- now: F(expression)=intenstion (itself a function)
- s.t. intension(index)=extention
- instead of: F(expression)(index)=extemsion

Some interpretations

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g},\ u \in Var_b,\ \alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with g' exactly like g except $g'(u) = x$
- $[\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}$ is a function h from $W \times T$ to denotations of α's type s.t. at every $\langle w', t' \rangle \in W \times T$ $[\![\alpha]\!]^{\mathcal{M},w',t',g} = h(\langle w', t' \rangle) = [\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}(\langle w', t' \rangle)$

Some examples

- $\alpha = \beta$ at $\langle w, t \rangle$ might be true, but $\hat{}\alpha = \hat{}\beta$ need not be 1 at that same index
- on types:
  - e - individuals
  - $\langle s, e \rangle$ - individual concepts ('present Queen of England')
  - $\langle s, \langle e, t \rangle \rangle$ - properties of inidviduals
  - $\langle e, t \rangle$ - sets of individuals
  - $\langle \langle s, e \rangle, t \rangle$- sets of individual concepts

Some examples

- on properties:
  - $\langle s, \langle a, t \rangle \rangle$ - properties of denotations of a-type expressions
  - $\langle s, \langle e, t \rangle \rangle$ - properties of individuals
  - $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

On indices

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence: $\langle s, a\rangle$ never applied to some typed argument (s is not a type!)
- useful thing: We never talk about indices!
- since often ${}^{\vee}\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s,\langle e,t\rangle\rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

Nec

- former problem with **Nec** as $\langle t, t\rangle$: non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle\langle s,t\rangle,t\rangle}$ - $\{0, 1\}^{(\{0,1\}^{W\times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{}\,\phi)$

For

- 'former' as in 'a former member of this club'
- instead of $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
- intensionally: $\langle\langle s,\langle e,t\rangle\rangle,\langle e,t\rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e,t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not c}$ is a func. h s.t. for any property k, $h(\langle w,t\rangle)(k)$ is the set $k(\langle w,t'\rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w,t\rangle)(k)(x) = 1$ iff $k(\langle w,t'\rangle)(x) = 1$ for some $t' < t$.

Bel

- relations between individuals and propositions
- $\langle\langle s, t\rangle, \langle e, t\rangle\rangle$
- $\mathbf{Bel}(\hat{\ }(B(m))(j))$ *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $[\![B(m)]\!]^{M,w_2,t_1,g} = 1$ since $[\![m]\!]^{M,w_2,t_1,g} = [\![n]\!]^{M,w_2,t_1,g}$
- however: $[\![\hat{\ }(B(m))]\!]^{M,w_2,t_1,g} \neq [\![\hat{\ }(B(n))]\!]^{M,w_2,t_1,g}$

de dicto

- $\mathbf{Bel}(\hat{}(B(m))(j))$ ‘John believes that Miss America is bald.’
- $\mathbf{Bel}(\hat{}(B(n))(j))$ ‘John believes that Norma is bald.’
- needn’t be equal: John can take worlds other than $\langle w_2, t_1 \rangle$ into account where $[\![n]\!] \neq [\![m]\!]$
- $\alpha = \beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$ is true iff α is not in the scope of $\hat{}, \mathbf{F}, \mathbf{P}, \Box$ (oblique contexts)
- however: $\hat{}\alpha = \hat{}\beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$

de re

- like so: $\lambda x\, [\mathbf{Bel}(\hat{}\ [B(x)])(j)]\ (m)$
- the above is true at an index $\langle w, t\rangle$ iff $[\![\mathbf{Bel}(\hat{}\ [B(x)])(j)]\!]^{w,t} = 1$
  if $[\![m]\!]^{w,t} = x$, i.e. if John is in a believe-rel with $\hat{}(B(x))$
  s.t. $g(x) = m$ (by semantics of λ)
- Why is $\hat{}(B(x))$ not equal to $\hat{}(B(m))$?
- constant *m*: non-rigid designator relativized to indices
- variable *x*: a rigid designator by def. of *g* (for the relevant checking case with $g(x) = MissAmerica$
- the above: a belief about 'whoever *m* is'
- λ conversion is restricted in IL!

Once again

- *John believes that a republican will win.*
- $\exists x\,[Rx \wedge \mathbf{Bel}(j, \hat{}\,[\mathbf{F}W(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x\,[R(x) \wedge W(x)])$

Literatur I

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Lizenz