

# Formale Semantik

## 10. Montagues intensionale Logik

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Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

1 New types and up/down

2 The IL of PTQ

3 Examples

New types and up/down

# Beyond truth functionality

- $\llbracket \phi \rrbracket^{\mathcal{M},w,i,g}$  and  $\llbracket \mathbf{P} \rrbracket^{\mathcal{M},w,i,g}$  don't truth conditionally determine  $\llbracket \mathbf{P}\phi \rrbracket^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**,  $\diamond$ ,  $\square$  are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models

- for a name  $d$ :  $\llbracket d \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow b \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow b \\ \langle w_1, t_3 \rangle & \rightarrow b \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

- for an individual concept denoting expression  $m$ :

- $\llbracket m \rrbracket_{\mathcal{M},g}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow a \\ \langle w_2, t_1 \rangle & \rightarrow c \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow c \\ \langle w_1, t_3 \rangle & \rightarrow c \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

- for a one place predicate  $B$ :

- $\llbracket B \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow \{a, b, c\} \end{array} \right]$

- formula  $\phi$ :  $\llbracket \phi \rrbracket_{\mathcal{C}}^{\mathcal{M},g}$  is a function from indices to truth values

- $\llbracket B(m) \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow 1 \\ \langle w_2, t_1 \rangle & \rightarrow 1 \\ \langle w_1, t_2 \rangle & \rightarrow 0 \\ \langle w_2, t_2 \rangle & \rightarrow 0 \\ \langle w_1, t_3 \rangle & \rightarrow 1 \\ \langle w_2, t_3 \rangle & \rightarrow 1 \end{array} \right]$

- $\llbracket B(n) \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow 0 \\ \langle w_2, t_1 \rangle & \rightarrow 1 \\ \langle w_1, t_2 \rangle & \rightarrow 1 \\ \langle w_2, t_2 \rangle & \rightarrow 0 \\ \langle w_1, t_3 \rangle & \rightarrow 1 \\ \langle w_2, t_3 \rangle & \rightarrow 1 \end{array} \right]$



- again, the proposition  $\llbracket Bm \rrbracket_{\mathcal{C}}^{\mathcal{M},g}$  is a set of indices  $\langle w_i, t_j \rangle$
- from the extension at all indices, compute the intension
- $\llbracket \alpha \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = \llbracket \alpha \rrbracket^{\mathcal{M},w_i,t_j,g}$

- constant function on indices
- will play a great role, so remember!
- $\llbracket u \rrbracket_{\mathcal{M},g}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

# What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes **they denote intensions** (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

- Church/Montague: for an extension-denoting expression  $\alpha$ ,  $\hat{\alpha}$  denotes  $\alpha$ 's intension
- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}$
- $\alpha$  and  $\hat{\alpha}$  are just denoting expressions
- for an intension-denoting expression  $\alpha$ :  $\llbracket \check{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\langle w, t \rangle)$

# Down-up and up-down

- observe:  $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$
- but not always:  $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} \neq \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

- $k'$  intension:  $\llbracket k \rrbracket_{\mathcal{C}}^{\mathcal{M},g} =$ 

$$\left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & \left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_1, t_2 \rangle & \rightarrow & \left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{array} \right] \\ \langle w_2, t_1 \rangle & \rightarrow & \left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_2, t_2 \rangle & \rightarrow & \left[ \begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{array} \right] \end{array} \right]$$

- $k'$  extension (e.g., at  $\langle w_1, t_2 \rangle$ ):  $\llbracket k \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g}(\langle w_1, t_2 \rangle) =$
- $\llbracket k \rrbracket^{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix}$
- however:  $\llbracket \sim k \rrbracket^{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix}$
- since:  $\llbracket \sim k \rrbracket^{\mathcal{M}, w_1, t_1, g} = a$   
 $\llbracket \sim k \rrbracket^{\mathcal{M}, w_1, t_2, g} = b$   
 $\llbracket \sim k \rrbracket^{\mathcal{M}, w_2, t_1, g} = d$   
 $\llbracket \sim k \rrbracket^{\mathcal{M}, w_2, t_2, g} = b$

The IL of PTQ



# A typed higher order $\lambda$ language with $=$ and $\hat{\ } / \sim$

- $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbf{F}, \mathbf{P}, \Box, =$  (syncategorematically)
- $t, e \in Type$  ( $Con_{type}, Var_{type}$ )
- if  $a, b \in Type$ , then  $\langle a, b \rangle \in Type$
- if  $a \in Type$ , then  $\langle s, a \rangle \in Type$
- $s \notin Type$

# Meaningful expressions

- $ME_{type}$
- abstraction: if  $\alpha \in ME_a, \beta \in Var_b, \lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if  $\alpha \in ME_{\langle a,b \rangle}, \beta \in ME_a$  then  $\alpha(\beta) \in ME_b$
- if  $\alpha, \beta \in ME_a$  then  $\alpha = \beta \in ME_t$

# Interpretations of $\hat{\phantom{x}}$ and $\sim\phantom{x}$

- if  $\alpha \in ME_a$  then  $\hat{\alpha} \in ME_{s,a}$
- if  $\alpha \in ME_{\langle s,a \rangle}$  then  $\sim\alpha \in ME_a$

type	variables	constants
$e$	$x, y, z$	$a, b, c$
$\langle s, e \rangle$	$x, y, z$	—
$\langle e, t \rangle$	$X, Y$	$walk', A, B$
• $\langle \langle s, e \rangle, t \rangle$	$Q$	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	$P$	—
$\langle e, e \rangle$	$P$	$Sq$
$\langle e, \langle e, t \rangle \rangle$	$R$	$Gr, K$
$\langle e, \langle e, e \rangle \rangle$	—	$Plus$

- $\langle A, W, T, <, F \rangle$
- $D_{\langle a, b \rangle} = D_b^{D_a}$
- $D_{\langle s, a \rangle} = D_a^{W \times T}$
- 'senses' = **possible** denotations
- actual intensions chosen from the set of senses
- now:  $F(\text{expression}) = \text{intension}$  (itself a function)
- s.t.  $\text{intension}(\text{index}) = \text{extension}$
- instead of:  $F(\text{expression})(\text{index}) = \text{extension}$

# Some interpretations

- $\llbracket \lambda u \alpha \rrbracket^{\mathcal{M}, w, i, g}$ ,  $u \in \text{Var}_b$ ,  $\alpha \in ME_a$  is a function  $h$  with domain  $D_b$  s.t.  $x \in D_b$ ,  
 $h(x) = \llbracket \alpha \rrbracket^{\mathcal{M}, w, t, g'}$  with  $g'$  exactly like  $g$  except  $g'(u) = x$
- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}$  is a function  $h$  from  $W \times T$  to denotations of  $\alpha$ 's type s.t. at every  
 $\langle w', t' \rangle \in W \times T$   $\llbracket \alpha \rrbracket^{\mathcal{M}, w', t', g} = h(\langle w', t' \rangle) = \llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}(\langle w', t' \rangle)$

# Some examples

- $\alpha = \beta$  at  $\langle w, t \rangle$  might be true, but  $\hat{\alpha} = \hat{\beta}$  need not be 1 at that same index
- on types:
  - ▶  $e$  - individuals
  - ▶  $\langle s, e \rangle$  - individual concepts ('present Queen of England')
  - ▶  $\langle s, \langle e, t \rangle \rangle$  - properties of individuals
  - ▶  $\langle e, t \rangle$  - sets of individuals
  - ▶  $\langle \langle s, e \rangle, t \rangle$  - sets of individual concepts

- on properties:
  - ▶  $\langle s, \langle a, t \rangle \rangle$  - properties of denotations of  $a$ -type expressions
  - ▶  $\langle s, \langle e, t \rangle \rangle$  - properties of individuals
  - ▶  $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$  - properties of propositions
- from relations  $\langle e, \langle e, t \rangle \rangle$  to relations-in-intensions  $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence:  $\langle s, a \rangle$  never applied to some typed argument ( $s$  is not a type!)
- useful thing: We never talk about indices!
- since often  $\sim \alpha(\beta)$  is needed for  $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$  and  $\beta \in ME_e$ , abbr.  $\alpha\{\beta\}$



## Examples

- former problem with **Nec** as  $\langle t, t \rangle$ : non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle \langle s, t \rangle, t \rangle} - \{0, 1\}^{(\{0, 1\}^{W \times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give  $\Box\phi$  as  $\mathbf{Nec}(\hat{\phi})$

- ‘former’ as in ‘a former member of this club’
- instead of  $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$
- intensionally:  $\langle\langle s, \langle e, t \rangle \rangle, \langle e, t \rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property  $\langle e, t \rangle$  not at current but at some past index qualify
- formally:  $\llbracket \text{For} \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g}$  is a func.  $h$  s.t. for any property  $k$ ,  $h(\langle w, t \rangle)(k)$  is the set  $k(\langle w, t' \rangle)$  for all  $t' < t$ .
- So, for any individual  $x$   $h(\langle w, t \rangle)(k)(x) = 1$  iff  $k(\langle w, t' \rangle)(x) = 1$  for some  $t' < t$ .

- relations between individuals and propositions
- $\langle \langle s, t \rangle, \langle e, t \rangle \rangle$
- $\text{Bel}(\wedge(B(m))(j))$  *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $\llbracket B(m) \rrbracket^{M, w_2, t_1, g} = 1$  since  $\llbracket m \rrbracket^{M, w_2, t_1, g} = \llbracket n \rrbracket^{M, w_2, t_1, g}$
- however:  $\llbracket \wedge(B(m)) \rrbracket^{M, w_2, t_1, g} \neq \llbracket \wedge(B(n)) \rrbracket^{M, w_2, t_1, g}$

- $\text{Bel}(\hat{(B(m))}(j))$  'John believes that Miss America is bald.'
- $\text{Bel}(\hat{(B(n))}(j))$  'John believes that Norma is bald.'
- needn't be equal: John can take worlds other than  $\langle w_2, t_1 \rangle$  into account where  $\llbracket n \rrbracket \neq \llbracket m \rrbracket$
- $\alpha = \beta \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$  is true iff  $\alpha$  is not in the scope of  $\hat{, \mathbf{F}, \mathbf{P}, \square}$  (oblique contexts)
- however:  $\hat{\alpha} = \hat{\beta} \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$

- like so:  $\lambda x [\mathbf{Bel}(\hat{[B(x)]})(j)](m)$
- the above is true at an index  $\langle w, t \rangle$  iff  $\llbracket \mathbf{Bel}(\hat{[B(x)]})(j) \rrbracket^{w,t} = 1$   
if  $\llbracket m \rrbracket^{w,t} = x$ , i.e. if John is in a believe-rel with  $\hat{[B(x)]}$   
s.t.  $g(x) = m$  (by semantics of  $\lambda$ )
- Why is  $\hat{[B(x)]}$  not equal to  $\hat{[B(m)]}$ ?
- constant  $m$ : non-rigid designator relativized to indices
- variable  $x$ : a rigid designator by def. of  $g$  (for the relevant checking case with  $g(x) = \text{MissAmerica}$ )
- the above: a belief about 'whoever  $m$  is'
- $\lambda$  conversion is restricted in IL!

- *John believes that a republican will win.*
- $\exists x [Rx \wedge \mathbf{Bel}(j, \wedge [\mathbf{FW}(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x [R(x) \wedge W(x)])$





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