# Formale Semantik 05. Prädikatenlogik

#### Roland Schäfer

Institut für Germanistische Sprachwissenschaft Friedrich-Schiller-Universität Jena

Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007! Stets aktuelle Fassungen: https://github.com/rsling/VL-Semantik

### Inhalt

- Warum Prädikatenlogik?
- 2 Syntax und Semantik

- 3 Äquivalenzer
- 4 Schlussregeln



### Kaum Kompositionalität in der Aussagenlogik

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  - $\triangleright$  wird zu  $e \land c$

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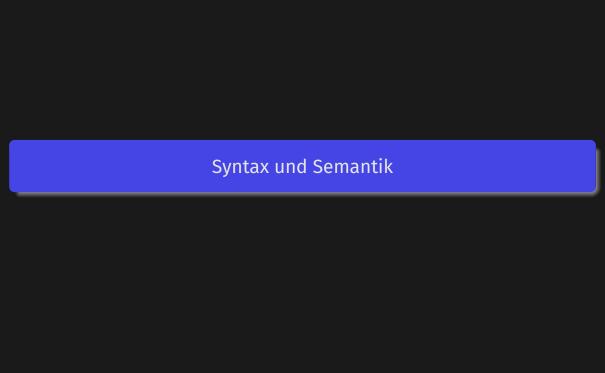
### Deduktion mit Quantifikation

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- plus alle Funktoren der Aussagenlogik

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Wffs aus Prädikaten, Termen und Quantoren

•  $P(t_1, \dots, t_n)$  ist eine Wff wenn  $P \in P_n$  und  $t_1, \dots, t_n \in T$ 

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- Nichts anderes ist eine Wff in PL.
   Hinweis | Eigentlich ist Wff auch als Menge aller Wffs definiert.

### Semantik | Modelle und Individuen

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Modell | Die (Beschreibung der) Welt, relativ zu derer Wffs ausgewertet werden

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Bedeutung von Prädikaten | Relationen (Mengen von Tupeln)

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  - ▶ Martin (m) schläft ( $R_1$ ):  $[R_1(m)]^{\mathcal{M}_1} = 1$  weil  $[m]^{\mathcal{M}_1} = M$  artin und Martin  $\in [R_1]^{\mathcal{M}_1}$
  - ▶ Martin (m) jagt (R<sub>3</sub>) Kilroy (k):  $[R_3(m,k)]^{\mathcal{M}_1} = 0$  weil  $[m]^{\mathcal{M}_1} = Martin$  und  $[k]^{\mathcal{M}_1} = Kilroy$  und  $\langle Martin, Kilroy \rangle \notin [R_3]^{\mathcal{M}_1}$

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  - ▶ if  $\exists \forall$  is true,  $\forall \exists$  follows

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- standard form of NL quantification:  $\forall x (Wx \rightarrow Bx)$  'All women are beautiful.'

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- ∀x(Wx ∧ ¬Wx) is a contradiciton,
   ∀x 'checks' for an empty set by def.
- standard form of NL quantification:  $\forall x (Wx \rightarrow Bx)$  'All women are beautiful.'
- standard form of NL existential quantification:  $\exists x(Wx \land Bx)$  'Some woman is beautiful.'

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- ¬ negates the wff, not the q:
  - \* $(\neg \forall x)$ Px but  $\neg(\forall x)$ Px

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#### Universal ∨ and ∧

•  $\exists$  and  $\forall$  'or' and 'and' over the universe of discourse (hence:  $\bigvee$  and  $\bigwedge$ )

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#### Universal $\vee$ and $\wedge$

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- $(\exists x)Px \Leftrightarrow Px_1 \lor Px_2 \lor ... \lor Px_n$  for all  $x_n$  assigned to  $d_n \in D$
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- $\Leftrightarrow \overline{Px_1} \vee \overline{Px_2} \vee \ldots \vee \overline{Px_n}$
- $\Leftrightarrow$   $(\exists x) \neg Px$

•  $\neg(\forall x)Px \Leftrightarrow (\exists x)\neg Px$ 

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- why?

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- i.e.: Watch your variables!

• Paul Kalkbrenner is a musician and signed on bpitchcontrol.

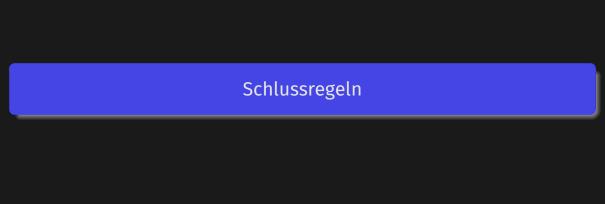
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(∀x)Px → Pa

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- for some  $(\exists x)Px$  and  $(\exists x)Qx$  the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

• (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.

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- (∃x)Hx

## The proof

```
(1)
          Dk
(2)
          (\forall x)(Dx \rightarrow Hx \lor Px)
(3)
         \neg(\exists x)(Px \wedge Dx)
(4)
          (\forall x) \neg (Px \wedge Dx)
                                           3,QN
(5)
          (\forall x)(\neg Px \vee \neg Dx)
                                          4.DeM
(6)
         (\forall x)(Dx \rightarrow \neg Px)
                                          5,Comm,Impl
(7)
                                          6.−∀(1)
         Dk \rightarrow \neg Pk
(8)
         \neg Pk
                                           1.7.MP
(9)
                                           2,-∀(1)
          Dk \rightarrow Hk \lor Pk
(10)
          Hk \vee Pk
                                           1,9,MP
(11)
          Hk
                                           8,10,DS
                                           10,+∃
```

# Literatur I

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#### Autor

#### Kontakt

Prof. Dr. Roland Schäfer Institut für Germanistische Sprachwissenschaft Friedrich-Schiller-Universität Jena Fürstengraben 30 07743 Jena

https://rolandschaefer.net roland.schaefer@uni-jena.de

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