Formale Semantik

Roland Schäfer

Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena

Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: https://github.com/rsling/VL-Semantik

Inhalt

Inferenz und Bedeutung

Literatur | Einführungen

- Chierchia & McConnell-Ginet (2000) | GB-orientiert, nur die Kapitel von Chierchia
- Partee u. a. (1990) | wichtige Grundlagen (Algebra, Logik), viele Druckfehler

Literatur | weitere Empfehlungen

- Bucher (1998) | lesbare Logik-Einführung auf Deutsch
- Dowty u. a. (1981) | tolle Montague-Einführung von seinen Schülern
- Carpenter (1997) | prima Hardcore-Einführung mit Kategorialgrammatik
- Gutzmann (2019) | aktuelle Einführung auf Deutsch

Seminarverlauf

1. 9.10.2023 Diskussion: Wie schlussfolgern wir? Wie hängen unser Schlussfolgerungen mit Semantik zusammen?
2. 26.10.2023 Referentielle Semantik (Folien 2)
3. 02.11.2023 Mengen- und Funktionstheorie (Folien 3)
4. 09.11.2023 Aussagenlogik (Folien 4)
- 16.11.2023 Ausfall wegen Dienstreise
5. 23.11.2023 Prädikatenlogik (Folien 5)
6. 30.11.2023 Quantifikation und modelltheoretische Semantik (Folien 6)
7. 07.12.2023 Einfach getypte höherstufige λ-Sprachen (Folien 7)
8. 14.12.2023 Intensionalität (Folien 8)
9. 21.12.2023 Tempus und Modalität (Folien 9)
- 28.12.2023 Weihnachtsferien
- 04.01.2024 Weihnachtsferien
10. 11.01.2024 Montagues intensionale Logik (Folien 10)
11. 18.01.2024 *The Proper Treatment of Quantification in Ordinary English* (Montague 1973)
12. 25.01.2024 *Generalized Quantifiers and Natural Language* (Barwise & Cooper 1981)
13. 01.02.2024 *The Algebra of Events* (Bach 1986)
- 08.02.2024 Klausurenwoche/Einzelbesprechungen

Prüfungen

Einheitlicher Inhalt für alle Modul- und Examensprüfungen:

1. eine oder zwei inhaltlichen Fragen zu den Themen der *Sprachphilosophie*
   Die Liste der relevanten Texte wird rechtzeitig vor den Prüfungen eingeschränkt.
2. eine Logik-Aufgabe (natürliche Deduktion) – außer in mündlichen Prüfungen
3. eine Semantik-Aufgabe (kompositionale Modellierung eines Satzes)

Hausarbeiten nach Absprache.

Was folgt aus A?

Folgt B aus A? | Einfache Fälle

Folgt B aus A? | Schwierigere Fälle

Folgt B aus A? | Grammatik

Woher wissen Sie das?

Weltwissen vs. Logik

Inferenz | Induktion

Deduktion, Logik und Modelltheorie

Begriffe von „Bedeutung“

Die Bedeutung eines Ausdrucks ist …

- … die Idee, die er vermittelt
- … die mentale Repräsentation, die er erzeugt
- … was mit ihm bewirkt werden soll
- … die Menge der Dinge, auf die er verweist

Mögliche „Semantiken“

Semantik untersucht …

- … intellektuelle Konzepte, die überwiegend introspektiv erforschbar sind
- … die kognitive Verarbeitung und Repräsentation von Bedeutung
- … die Funktion von Ausdrücken in Kommunikationssituationen
- … Beziehungen zwischen Ausdrücken und Objekten und
  die Art der Kombination von Ausdrücken zur komplexeren Ausdrücken

Konkrete Fragen in diesem Seminar

Es dreht sich alles um die Beziehung von Sprache zur Welt!

- Auf welche Klassen von Objekten referieren auf welche Klassen von Ausdrücken?
- Wann sind Sätze wahr? (auch als Phänomen der Referenz!)
- Wie verhält sich die logische Struktur von Sätzen zu ihrem Informationsgehalt?
- Wie können Sätze eindeutig interpretiert werden,
  auch wenn sie mehrere Lesarten haben?

Keine Fragen in diesem Seminar

- Was ist die „Bedeutung“ von Wörtern und Sätzen jenseits ihrer Referenz?
- Wie verarbeitet das Gehirn Bedeutungen?
- Wie sind Diskurse strukturiert?

Programmatisches Schlussbild | Frage

Sind Sie nun kognitiver Linguist,
der sich für (probabilistische) mentale Kategorien interessiert,

oder glauben Sie daran,
dass Sprache unabhängig vom Menschen logische Eigenschaften hat?

Beides gleichzeitig geht ja nun wirklich nicht!

Kognition

- basierend auf Ähnlichkeiten von wahrgenommenen Objekten
- optimiert für schnelle Mustererkennung in allen Bereichen
- unscharfe Klassenbildung und Segmentierung der Ontologie
- parallele Verarbeitung (meistens mehrere Areale beteiligt)

Symbolische Systeme

- diskrete Symbole, wohldefinierte Semantik
- scharf getrennte Klassen von Symbolen
- eindeutige Referenz auf ontologische Objekte
- intrinsische (nicht emergente) logische Eigenschaften
  (Axiomatik, Schlussregeln usw.)
- sequentielle Verarbeitung/statische Deklaration
  (z. B. Python oder PROLOG; parallele Verarbeitung immer linearisierbar)

Klassisches kognitives Modell: Prototypentheorie (Rosch 1973)

Diskretes Symbol: Vogel … und demgegenüber …
Graduelles kognitives Konzept basierend auf Ähnlichkeiten / Prototypen:

 >  > 

Programmatisches Schlussbild | Antwort

Die ewige Schwachsinnsfrage: Sind Kiwis und Pinguine nun Vögel oder nicht?

Nur getoppt von: Erdbeeren sind gar keine Beeren, sondern Sammelnussfrüchte.

- Kognition | intrinsisch nicht diskret, sondern ähnlichkeitsbasiert und parallel
  - Netzwerkarchitektur
- Symbole = Phone, Morphe, Wörter, Phrasen, …| intrinsisch diskret und linear
  - akustisches Medium | Sagen Sie mal zwei Wörter gleichzeitig!
  - schriftliches Medium | Lesen Sie mal *Zettels Traum*!

→ Da wir nur akustisch oder über schriftliche Artefakte kommunizieren können, muss das Sprachsystem symbolisch sein.

→ Da es architekturbedingt nur nicht-symbolisch verarbeiten kann, muss das Gehirn symbolische Systeme so gut wie nötig und möglich emulieren.

Auch nicht-verschriftete Sprache muss medial bedingt logische Eigenschaften haben.
Kulturell bilden sich stärker symbolische Modi aus, vor allem durch Schrift.

Norm, Selbst- und Fremdkorrektur, Textplanung, intensionale Definitionen, Explizierung, ...

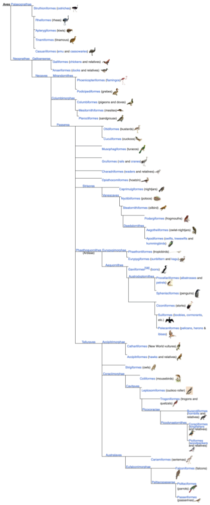
Warum wird das vor allem im Kontext von Schule, Fremdsprache und Bildungssprache diskutiert?

(= spontane Sprachproduktion)
weniger symbolische Eigenschaften
↓
↓
↓
↓
mehr symbolische Eigenschaften
(= reflektierte Sprachproduktion)

informelle Alltagssprache
formelle Alltagssprache
Bildungssprache
Wissenschaftssprache
Orthosprache
formales System

Und was ist denn nun mit Kiwis und Pinguinen?

Unser Verständnis der Welt führt zu genaueren und diskreten Kategorisierungen, wo dies nötig ist. Die Sprache folgt diesem Maß an Genauigkeit und Diskretheit!



Letzte Folie

- Viele Missverständnisse in der Linguistik basieren darauf,
  dass das eben Gesagte nicht dem allgemeinen Forschungsprogramm zugrundeliegt.
- Die Doppelnatur von Sprache führt dazu, dass sowohl rein formale Linguistik
  und sogenannte kognitive Linguistik scheinbar erfolgreich sind.
- Im Prinzip läuft aber die Linguistik aktuell weitgehend ins Leere.

- Modelltheoretische Semantik beschreibt einen essentiellen Teil von Sprache!
- Sie modelliert logische Eigenschaften und den Bezug zur realen objektiven Welt.

- Ganz am Rande zu generativer AI …
  - Erfolg | Sie modelliert völlig natürliche Grammatik.
  - … also alle Grammatiker bitte setzen!
  - Misserfolg | Sie weiß nichts über die Welt,
    es wirkt nur so wegen des immensen sprachlichen Inputs.
  - … eine Art fancy Papagei.

Referentielle Semantik

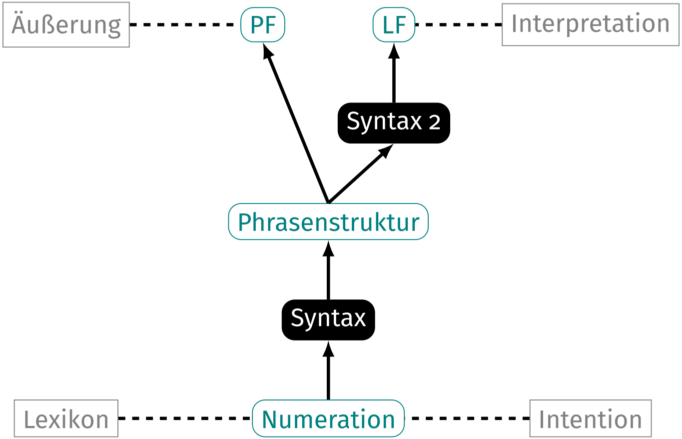
Ein neues semiotisches Dreieck

Im Sinn der letzten Woche interessiert uns nur die linke Seite.

Formen

Reale Objekte Mentale Konzepte

„Semantik“ im generativen T-Modell



Repräsentationsebenen

Im klassischen generativen Modell:

(In minimalistischen Modellen herrscht – Chomsky muss es mögen! – sowieso Anarchie.)

- keine echte Interpretation auf LF
- Bewegung nachdem der Satz geäußert wurde
- Herstellung einer logisch interpretierbaren Form auf LF
- Grund | Syntax kann nicht alle Interpretationen abbilden

Klassiker Quantorenskopus

Everybody loves somebody.

A Für alle Personen y gilt, dass es eine Person x gibt, für die gilt: y liebt x ($\forall y \exists x.L(y,x)$)

B Es gibt eine Person x, sodass für alle Personen y gilt: y liebt x ($\exists x \forall y.L(y,x)$)

Montagues direkte Interpretation

Sprache ist Logik ist Sprache …

A Entweder ist die Übersetzung in eine LF trivial und äquivalent zur PF/Syntax, oder sie fügt etwas hinzu, das der Sprache an sich fehlt.

B Sätze haben aber auch mit LF-Übersetzung nur die Bedeutungen, die sie sowieso haben (keine Hinzufügung).

→ Also ist die Übersetzung in LF trivial und äquivalent zur PF/Syntax.

→ Wir können Sätze direkt interpretieren (wie sie gesprochen/geschrieben werden).

- Montagues *lf* | direkte Übersetzung von sprachlichen in logische Ausdrücke

Interessante Eigenschaften von Sprache

- Aussagen über die/Teile der Welt
- Ausdrücke bezeichnen/referieren auf Dinge i. w. S.
- Informativität
- objektiv beurteilbar (z. B. Wahrheit von Sätzen)

- Aber welche sprachlichen Einheiten referieren auf was?

Referenz | Eigennamen

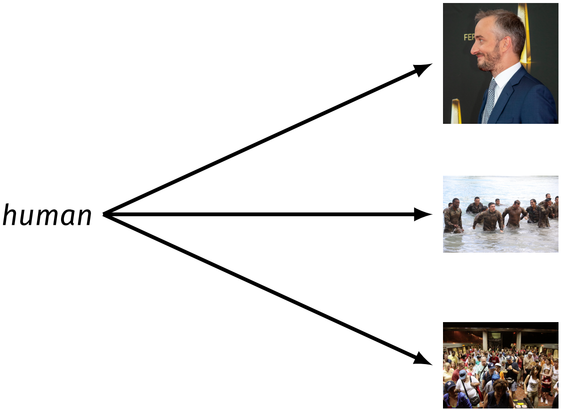
Ein Eigenname → genau ein Objekt in der Welt

Jan Böhmermann ⟶ 

Ein normales Nomen → eine Menge von Objekten in der Welt

soldier ⟶

Ein (intersektives) Adjektiv oder ein Verb → eine Menge von Objekten in der Welt



Ein Satz → in erster Näherung ein Sachverhalt

A humming bird is hovering over a red flower.

Nein! falsche Art von Objekt

(als Individuum)

Bedeutung ist kompositional!

- *humming bird* → die Menge der Kolibri-Objekte
- *a* → Existenzaussage für ein Element aus einer Menge
- *a humming bird* → Existenzaussage für ein Element x aus der Menge der Kolibri-Objekte
- *is hovering* → die Menge der schwebenden Objekten
- *a humming bird is hovering* → das existierende Kolibri-Objekt x ist auch ein Element der Menge der schwebenden Objekte
- *a red flower* → Existenzaussage für ein Element y aus der Schnittmenge der roten Objekte und der Blumen-Objekte
- *over* → die Relation zwischen Objekten (s. nächste Woche), die sich übereinander befinden
- *A Humming is hovering over a red flower.* →
  Es gibt ein Objekt x aus der Schnittmenge der Kolibri- und der schwebenden Objekte,
  und es gibt ein Objekt y aus der Schnittmenge der roten und der Blumen-Objekte,
  und x befindet sich über y.

Implikation (Entailment)

Mengen von Aussagesätzen implizieren andere Sätze.
Sätze (Implikationen) lassen sich aus anderen Sätzen (Axiome) beweisen.

A *Jan Böhmermann ist ein Mensch.*

B *Jan Böhmermann ist leutselig.*

C *Jan Böhmermann ist ein leutseliger Mensch.*

$A, B \vdash C$ | A und B implizieren C. (C ist beweisbar aus A und B.)

$A \nvdash C$ | A impliziert nicht C.

$B \nvdash C$ | B impliziert nicht C.

$A \vdash A \wedge A$ | *Jan Böhmermann ist ein Mensch* und *Jan Böhmermann ist ein Mensch.*

D *Irgendetwas ist ein Mensch.*

$A \vdash D$

Tests auf Implikation

Wenn diese Kriterien zutreffen, impliziert A B:

- Wenn A wahr ist, ist B auch immer wahr.
- Eine Situation, die von B beschrieben wird, wird auch von A beschrieben.
- Die Information in B ist vollständig in der Information in A enthalten.
- Man kann unter keinen Umständen sagen: *A ist wahr, aber B ist nicht wahr.*

Übung | Sind das Implikationen?

- Böhmermann ist Showmaster. ⊢ Böhmermann ist menschlich.
- Böhmermann ist nicht sehr groß. ⊢ Irgendjemand ist nicht sehr groß.
- Böhmermann ist nicht sehr groß. ⊢ Irgendjemand ist sehr groß.
- Manche Menschen sind leutselig. ⊢ Böhmermann ist leutselig.
- Ich habe das neue drip-133-Album gehört. ⊢ drip-133 hat ein neues Album veröffentlicht.
- Nachdem ich einen Sherry getrunken habe, habe ich den Kondensator getauscht.
  ⊢ Ich habe einen Sherry getrunken.
- Nachdem Linux nicht mehr startete, habe ich einen weiteren Sherry getrunken.
  ⊢ Linux ist noch nie gestartet.
- Mein ehemaliger Mitbewohner mag Becks.
  ⊢ Mein ehemaliger Mitbewohner könnte Sherry mögen.
- Böhmermann hat das heutige ZDF Magazin beendet.
  ⊢ Das heutige ZDF Magazin wurde beendet.

Präsuppositionen sind schwächer als Implikationen.

A *Willy Brandt ist der gegenwärtige Kanzler Deutschlands.*

B *Wenn Willy Brandt der gegenwärtige Kanzler Deutschlands ist, trägt er eine große Verantwortung.*

C *Willy Brandt ist nicht der gegenwärtige Kanzler Deutschlands.*

D *Willy Brandt lebt.*

E *Es gibt einen Kanzler Deutschlands.*

- A und B präsupponieren D. = D ist eine Voraussetzung für eine erfolgreiche Interpretation von A und B.
- C präsupponiert nicht D.
- A, B und C präsupponieren E.

Tests auf Präsupposition

Die Unterschiede zur Implikation sind relevant.

- Nicht nur Aussagesätzen haben Präsuppositionen (Modale, Konditionale, ...)
- Negierte Sätze haben oft gleiche Präsuppositionen wie nicht-negierte.
- Präsuppositionen können negiert werden, und der Ausgangssatz bleibt wahr.
  (Geht nicht mit Implikationen.)

  F *Willy Brandt ist nicht Kanzler Deutschlands.*
  G *Es gibt einen Kanzler Deutschlands.*
  F präsupponiert G, bleibt aber wahr, wenn G falsch ist.

Synonymie

Synonyme Ausdrücke haben exakt die gleiche Referenz.

- lexikalische Synonymie | *humming bird* $\stackrel{lex}{\equiv}$ *colibri*
- kompositionale Synonymie

  Mulder traf seine entführte Schwester, nachdem er in die geheime Militärbasis eingebrochen war.

  $\equiv$ *Bevor er seine entführte Schwester traf, brach Mulder in die geheime Militärbasis ein.*

- $A \equiv B$ gdw $A \vdash B$ und $B \vdash A$ (gegenseitige Implikation)
- *gdw = genau dann wenn* | *iff = if and only if*

Natürliche Sprache und Implikation

Referentielle Semantik $\neq$ *einfaches Zeigen auf Objekte durch Sprache*.

Zusätzliche Logik für Fälle wie diesen (und viele andere):

	Die Lieblingsblume meines Kolibris	*ist rot.*
$\vdash$	*Eine Blume*	*ist rot.*

Sätze referieren aus Wahrheitswerte!

Um zu der gewünschten Logik zu kommen, zeigen wir jetzt,
dass Sätze auf Wahrheitswerte referieren.

Wahrheitswerte sind nur *wahr* und *falsch*.

Die Verben *denotieren* und *referieren auf* sind hier synonym.

Warten Sie bitte ein paar Wochen, wenn Sie diese Darstellung reduktionistisch finden.

Synonyme NPs

a *colibri*

b *humming bird*

$a \stackrel{lex}{\equiv} b$

c *a brunette lady*

d *a brown-haired dame*

$c \equiv d$

e *the primates*

f *the apes and humans*

$e \equiv f$

Synonymie von Konstituenten und Sätzen

Synonymie von Konstituenten im Satzkontext → Satzsynonymie

A *A colibri is hovering over a red flower.*

B *A humming bird is hovering over a red flower.*

$A \equiv B$ weil $a \equiv b$ und Satzkontext identisch

$[_{\mathrm{A}} a] \equiv [_{\mathrm{B}} b]$ wenn $a \equiv b$ und $[_{\mathrm{A}} \_] = [_{\mathrm{B}} \_]$

C *Lauren Bacall was a brunette lady.*

D *Lauren Bacall was a brown-haired dame.*

$C \equiv D$ weil $c \equiv d$ und Satzkontext identisch

E *Primates are intelligent.*

F *The apes and humans are intelligent.*

$E \equiv F$ weil $e \equiv f$ und Satzkontext identisch

Zwei Axiome

Referenz/Denotat eines Ausdrucks A als $[\![\mathrm{A}]\!]$
$[\![\cdot]\!]$ ist eine Funktion!

Erinnerung: Synonymität von Sätzen ist gegenseitige Implikation.

Ax1 Synonyme Ausdrücke (NPs, Verben, Sätze, ...) haben dieselbe Referenz.
Formal: $A \equiv B \leftrightarrow [\![\mathrm{A}]\!] = [\![\mathrm{B}]\!]$

Ax2 Wenn wir in Ausdruck C einen Ausdruck A durch
einen synonymen Ausdruck B ersetzen, behält C seine Referenz.
Formal: $[\![\mathrm{A}]\!] = [\![\mathrm{B}]\!] \rightarrow [\![[_{\mathrm{C}}\ \mathrm{A}]]\!] = [\![[_{\mathrm{C}}\ \mathrm{B}]]\!]$

Zwei wahre Sätze

Wahrheitswert von A und B | 1 bzw. *wahr* bzw. *true* oder *T*

A *Lauren Bacall was a brunette lady.*

B *My humming bird's favourite flower is red.*

Erste Schlussfolgerung

Einsetzen von A und B in Satzkontext T bzw. $[_T\_]$ (Aussage über Wahrheitswert)

T *The truth value of '_' is 1.*

$[_T A]$ *The truth value of 'Lauren Bacall was a brunette lady.' is 1.*

$[_T B]$ *The truth value of 'My humming bird's favourite flower is red.' is 1.*

folgt $A \equiv [_T A]$ und $B \equiv [_T B]$

mit Ax1 $[\![A]\!] = [\![[_T A]]\!]$ und $[\![B]\!] = [\![[_T B]]\!]$

Bitte bedenken: A und $[_T A]$ haben auch intuitiv „denselben Inhalt“.

Zweite Schlussforlgerung

In $[_{\mathsf{T}}A]$ und $[_{\mathsf{T}}B]$ sind A und B jeweils in einer NP eingebettet.

- $[\![\text{the truth value of A}]\!] = [\![\text{the truth value of B}]\!] = 1$
  mit Ax2 $[\![[_{\mathsf{T}}\text{A}]]\!] = [\![[_{\mathsf{T}}\text{B}]]\!]$
  damit $[\![\text{A}]\!] = [\![[_{\mathsf{T}}\text{A}]]\!] = [\![[_{\mathsf{T}}\text{B}]]\!] = [\![\text{B}]\!] = 1$
- Sätze referieren auf Wahrheitswerte.
  (Denn man kann das mit zwei beliebigen wahren Sätzen machen.)
- Achtung | Wahrheitswerte sind auch nur realweltliche Objekte.

Adäquatheit von Wahrheitswerten als Satzreferenten

Nicht so sinnlos, schwachsinnig, inhaltsleer, … wie oft vermutet

- Referentielle Semantik
  - Analyse der Referenten verschiedener Typen von Ausdrücken
  - Komposition von Sätzen
  - deduktive Logik für Sätze
  - Benennen der Wahrheitsbedingungen (→ Modelltheorie)

- minimale Gemeinsamkeit aller Sätze
- gut formal berechenbar (Binarität)
- reichhaltigere Semantik später (basierend auf Wahrheitswerten)

Grammatik- und Semantikfragmente

Konstruktive, schrittweise Annäherungen an sprachliche Modellierung

- Grammatikfragment | Ausschnitt einer Gesamtgrammatik
- erwünschte schrittweise Erweiterung von Fragmenten (vgl. HPSG)

- Konstruktion eines Semantik-Fragments
  - grammatische Kategorien und Referenzen von Wörtern
  - Grammatikmechanismen und zugehörige Bedeutungskonstruktion
  - Ergebnis | Semantik von Sätzen und Beitrag aller Konstituenten dazu

- T-Sätze
  - L eine Sprache, S ein Satz, v ein Sachverhalt, p eine Aussage über Wahrheitsbedingungen
  - S aus L ist wahr in v gdw p.

Das Lexikon

Die folgenden simplexen Ausdrücke sind Teil von F_1.
Kein anderer simplexer Ausdruck ist Teil von F_1.

1. N $\rightarrow$ *Herr Webelhuth, Frau Klenk, the Turm-Mensa*
2. V_i $\rightarrow$ *is relaxed, is creative, is stupid*
3. V_t $\rightarrow$ *prefers*
4. conj $\rightarrow$ *and, or*
5. neg $\rightarrow$ *it is not the case that*

Die Phrasenstrukturgrammatik von F_1

Folgende Kompositionsregeln sind Teil von F_1.
Keine andere Kompositionsregel ist Teil von F_1.

1. S $\rightarrow$ N VP
2. S $\rightarrow$ S conj S
3. S $\rightarrow$ neg S
4. VP $\rightarrow$ V_i
5. VP $\rightarrow$ V_t N

Referenz simplexer Ausdrücke

- $[\![\text{Herr Webelhuth}]\!]$ = Herr Webelhuth
- $[\![\text{Frau Klenk}]\!]$ = Frau Klenk
- $[\![\text{the Turm-Mensa}]\!]$ = the Turm-Mensa
- $[\![\text{is relaxed}]\!] = \{x{:}x \text{ is relaxed}\}$
- $[\![\text{is creative}]\!] = \{x{:}x \text{ is creative}\}$
- $[\![\text{is stupid}]\!] = \{x{:}x \text{ is stupid}\}$
- $[\![\text{prefers}]\!] = \{\langle x,y\rangle: x \text{ prefers } y\}$

Referenz von Funktionswörtern

Funktionswörter referieren auf Funktionen.

- $[\![\textit{neg}]\!] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$
- $[\![\textit{and}]\!] = \begin{bmatrix} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 0 \\ \langle 0,1 \rangle \rightarrow 0 \\ \langle 0,0 \rangle \rightarrow 0 \end{bmatrix}$
- $[\![\textit{or}]\!] = \begin{bmatrix} \langle 1,1 \rangle \rightarrow 1 \\ \langle 1,0 \rangle \rightarrow 1 \\ \langle 0,1 \rangle \rightarrow 1 \\ \langle 0,0 \rangle \rightarrow 0 \end{bmatrix}$

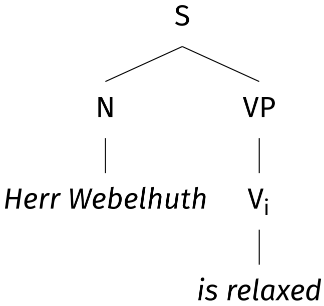
T-Sätze für F_1

- $[\![[_S \text{ N VP}]]\!]$ = 1 iff $[\![\text{N}]\!] \in [\![\text{VP}]\!]$, else 0
- $[\![[_S \text{ S1 conj S2}]]\!]$ = $[\![\text{conj}]\!](\langle[\![\text{S1}]\!],[\![\text{S2}]\!]\rangle)$
- $[\![[_S \text{ neg S}]]\!]$ = $[\![\text{neg}]\!]([\![\text{S}]\!])$
- $[\![[_{VP} \text{ V}_t \text{ N}]]\!]$ = $\{x: \langle x, [\![\text{N}]\!] \rangle \in [\![\text{V}_t]\!]\}$
- für einen nicht verzweigenden Knoten K und seine Tochter D: $[\![[_K D]]\!] = [\![D]\!]$
- Das geht alles eleganter. Bitte etwas Geduld!

Ist folgendes ein Satz aus F_1? *Herr Webelhuth is relaxed.*

- $[_N$ *Herr Webelhuth*$]$ mit Lexikonregel 1
- $[_{V_i}$ *is relaxed*$]$ mit Lexikonregel 2
- $[_{VP}$ $[_{V_i}$ *is relaxed*$]]$ mit Syntaxregel 4
- $[_S$ $[_N$ *Herr Webelhuth*$]$ $_{VP}$ $[_{V_i}$ *is relaxed*$]]$ mit Syntax 1

Syntax als Baum



v (Sachverhalt) | Herr Webelhuth (das ontologische Objekt) $\in$\{x: x is relaxed\}

- für N: $[\![$*Herr Webelhuth*$]\!]$ = Herr Webelhuth (das ontologische Objekt)
- für VP (und V_i): $[\![$*is relaxed*$]\!]$ = \{x: x is relaxed\} (enthält Herrn Webelhuth)
- für S: $[\![[_S \text{ N VP}]]\!] = 1$ iff $[\![\text{N}]\!] \in [\![\text{VP}]\!]$, else 0
- in v daher $[\![[_S$ *Herr Webelhuth is relaxed.*$]]\!]$=1

Semantik als Baum

$[\![\mathrm{S}]\!]$

$[\![\mathrm{N}]\!]$ $[\![\mathrm{VP}]\!]$

$[\![$*Herr Webelhuth*$]\!]$ $[\![\mathrm{V_i}]\!]$

$[\![$*is relaxed*$]\!]$

1 because Herr Webelhuth$\in$\{x:x is relaxed\}

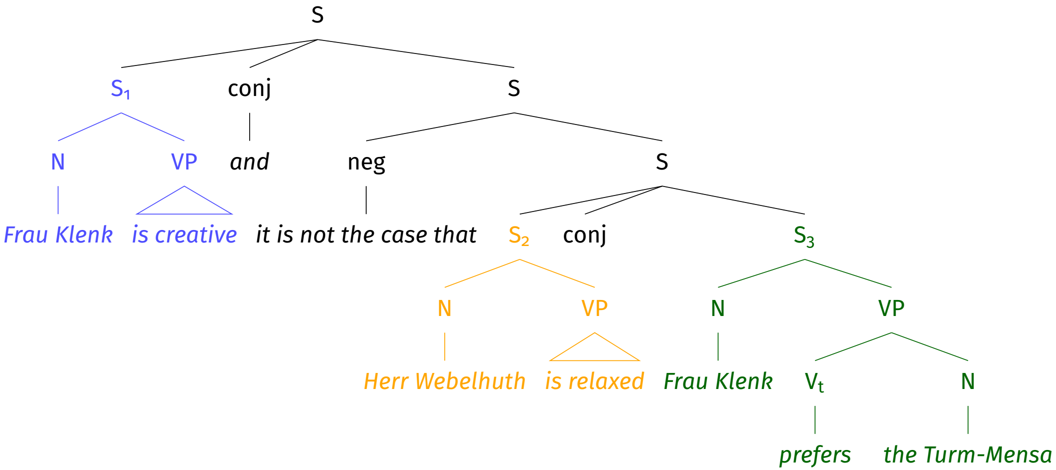
Herr Webelhuth {x:x is relaxed}

Herr Webelhuth {x:x is relaxed}

{x:x is relaxed}

Komplexere Phrasenstrukturen

[$_{S_1}$ *Frau Klenk is creative*] *and it is not the case that* [$_{S_2}$ *Herr Webelhuth is relaxed*] and [$_{S_3}$ *Frau Klenk prefers the Turm-Mensa*].



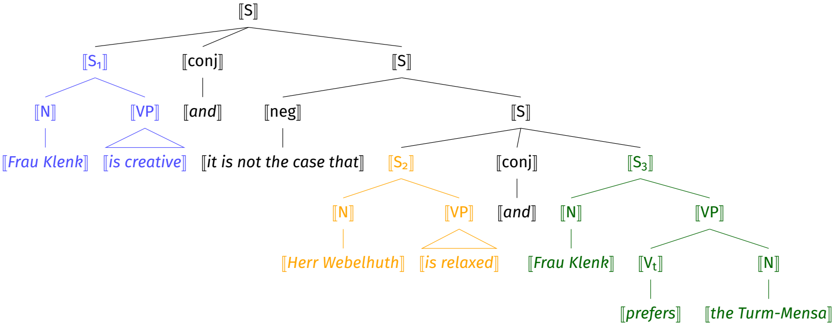
Interpretation

Die Situation/die Umstände v sind:

- Herr Webelhuth $\in \{x : x \text{ is relaxed}\}$
- Frau Klenk $\in \{x : x \text{ is creative}\}$
- $\langle$Frau Klenk, Turm-Mensa$\rangle \notin \{\langle x, y\rangle : x \text{ prefers } y\}$

Die Interpretation komplexerer Phrasenstrukturen ist einfach!

- Herr Webelhuth $\in \{x : x \text{ is relaxed}\}$
- Frau Klenk $\in \{x : x \text{ is creative}\}$
- $\langle$Frau Klenk, Turm-Mensa$\rangle \notin \{\langle x, y\rangle : x \text{ prefers } y\}$



Das war aber nicht alles

Der zuletzt analysierte Satz ist strukturell ambig, und
und mit der strukturellen geht eine semantische Ambiguität einher.

Hausaufgabe: Analysieren Sie die Syntax und Semantik des Satzes
in der anderen Lesart nur mit den Mitteln von F_1.

Zusatzaufgabe

Entwickeln Sie ein ähnliches Fragment D_1 für das Deutsche mit Lexikon, Syntax und Semantik das die folgenden Sätze generiert. Lexikon und Konstituentenstruktur können Sie frei wählen.
Es hat einen guten Grund, dass wir Englisch als Objektsprache nehmen. Sie können für dieses Fragment des Deutschen Kasus entweder ignorieren, oder Sie probieren, Kasusunterschiede zu modellieren.

Lexikon von D_1:

- Herr Müller ist Aktivist.
- Frau Klann ist intelligent.
- Frau Klann begrüßt Herrn Müller.
- Frau Klann hustet.
- Wolken sind zahlreich.
  (Dieser Satz ist die Extra-Callenge. Bitte zuerst den Rest modellieren.)

Mengen und Funktionen

What is a set?

- a freely defined unordered collection of discrete objects
  - numbers,
  - people,
  - pairs of shoes,
  - words, ...
- not necessarily for any purpose
- no object occurs more than once

Set definition and elements: $\in$

- $\mathrm{M}_1 = \{a, b, c\}$
- $\mathrm{N}_1 = \{$‘my book’$\}$
  vs. $\mathrm{N}_2 = \{$my book$\}$
  vs. $\mathrm{N}_3 = \{$‘my’, ‘book’$\}$
- ill-formed: $\mathrm{N}_4 = \{$‘my’, book$\}$
- defined by a property of its members:
  $\mathrm{M}_2 = \{$x:x is one of the first three letters of the alphabet$\}$
- alternatively:
  $\mathrm{M}_2 = \{$x$\|$x is one of the first three letters of the alphabet$\}$
- U: the universal set (contains every discrete object)

Equality: =

- Two sets with contain exactly the same members are *equal*.
- independent of definition:
  {a,b,c} =
  {x:x is one of the first three letters of the alphabet}
- {x:x is human} = {x:x is from the planet earth and x can speak}

Subsets: $\subseteq$

- A set N which holds no member which is not in M is a *subset* of M: N $\subseteq$ M
- $\{a\} \subseteq \{a, b, c\}$
- the inverse: the superset

Proper subsets: $\subset$

- A set N which holds no member which is not in M and which is not equal to M is a *proper subset* of M: N $\subset$ M
- So, actually: $\{a\} \subset \{a, b, c\}$ and $\{a, b, c\} \subseteq \{a, b, c\}$. Note that:
- $M \subseteq M$ but $M \not\subset M$
- $\{\{a\}\} \notin \{a, b, c\}$
- $\{\} \subset \{a, b, c\}$ (or any set), $\{\}$ is sometimes written $\emptyset$

Elements vs. subsets

- *All professors of English Linguistics are human.*
  Herr Webelhuth is a professor of English Linguistics.
- w = Herr Webelhuth
  E = the set of professors of English Linguistics
  H = the set of human beings
- $w \in E \;\&\; E \subset H \Rightarrow w \in H$

Elements vs. subsets

- But: *Professors of English Linguistics are numerous.*
- N = the set of sets with numerous members
- $w \in E$ & $E \in N \not\Rightarrow w \in P$
- Hence: *Herr Webelhuth is numerous.

Power sets: $\wp$

- For any set M: $\wp(M) = \{X \| X \subseteq M\}$
- for M=$\{a, b, c\}$:
  $\wp(M) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, b, c\}, \{b, c\}\}$
- Why is the empty set in the power set of every set ...
- ...and why is the empty a set a proper subset of every set?

Union $\cup$ and intersection $\cap$

- For any sets M and N: $M \cup N = \{x \| x \in M$ **or** $x \in N\}$
- if $M = \{a, b, c\}$ and $N = \{a, b, d\}$ then $M \cup N = \{a, b, c, d\}$
- For any sets M and N: $M \cap N = \{x \| x \in M$ **and** $x \in N\}$
- if $M = \{a, b, c\}$ and $N = \{a, b\}$ then $M \cap N = \{a, b\}$
- as a general principle (Consitency): $M \subseteq N$ iff $M \cup N = N$ and $M \subseteq N$ iff $M \cap N = M$

Generalized union $\bigcup$ and intersection $\bigcap$

- $\bigcup M = \{x \| x \in Y$ **for some** $Y \in M\}$
- (a) if $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$ then $\bigcup M = \{a, b, c\}$
- (b) $M_1 = \{a\}$, $M_2 = \{a, b\}$, $M_3 = \{a, b, c\}$, $I = \{1, 2, 3\}$; $\bigcup_{i \in I} M = \{a, b, c\}$
- $\bigcap M = \{x \| x \in Y$ **for every** $Y \in M\}$
- (a) if $M = \{\{a\}, \{a, b\}, \{a, b, c\}\}$ then $\bigcap M = \{a\}$
- (b) $M_1 = \{a\}$, $M_2 = \{a, b\}$, $M_3 = \{a, b, c\}$, $I = \{1, 2, 3\}$; $\bigcap_{i \in I} M = \{a\}$

Difference - and complement $\setminus$ and $'$

- For any two sets M and N: $M - N = \{x \| x \in M \text{ and } x \notin N\}$
- $M = \{a, b, c\}$, $N = \{a\}$, $M - N = \{b, c\}$
- For any two sets M and N: $M \setminus N = \{x \| x \in N \text{ and } x \notin M\}$
- $O = \{a, b, c, k\}$ $M \setminus O = \{k\}$
- the universal complement: $M' = \{x \| x \in U \text{ and } x \notin M\}$
  (U the universal set)

Trivial equalities

- Idempotency: $M \cup M = M$, $M \cap M = M$
- Commutativity for $\cup$ and $\cap$: $M \cup N = N \cup M$ ...
- Associativiy for $\cup$ and $\cap$: $(M \cup N) \cup O = M \cup (N \cup O)$ ...
- Distributivity for $\cup$ and $\cap$: $M \cup (N \cap O) = (M \cup N) \cap (M \cup O)$ ...
- Identity: $M \cup \emptyset = X$, $M \cup U = U$ ...what about $\cap$

More interesting equalities

- Complement laws: $M \cup \emptyset = M$, $M'' = M$, $M \cap M' = \emptyset$, $X \cap U = U$
- DeMorgan: $(M \cup N)' = M' \cap X'$ ...

How to define an ordered pair

- ...without introducing ordered tuples as a new primitive
- take S=$\{\{a\},\{a,b\}\}$
- we write: $\langle a,b\rangle = \{\{a\},\{a,b\}\}$
- orderend n-tuples defined recursively
- $\langle a,b\rangle \neq \langle b,a\rangle$
- first and second coordinate of the tuple

Cartesian products

- sets of ordered pairs
- tupling each member of the first argument with each of the second
- $S_1 \times S_2 = \{\langle x, y\rangle \| x \in S_1 \text{ and } y \in S_2\}$
- for an arbitrary number of sets: $S_1 \times \cdots \times S_n = \{\langle x_1, x_2, \ldots, x_n\rangle \| x_i \in S_i\}$
- $\langle x_1, x_2, \ldots, x_n\rangle$ abbreviated $\vec{x}$
- for $S \times S \times \cdots$: n-fold products
  $S^n = \{\vec{s} \| s_i \in S \text{ for } 1 \leq i \leq n\}$

Defintion of relations

- hold between (sets of) objects
- *x kicks y, x lives on the same floor as y, ...*
- formalization: Rab, aRb
- $a \in A$ and $b \in B$: $R \subseteq A \times B$,
  R is from A (**domain**) to B (**range**)
- R from A to A is in A

Complement, inverse

- complement $R' = \{\langle a, b \rangle \ \notin R\}$ for $R \subseteq A \times B$
  - R = the relation of teacherhood between a and b (the **arguments**)
  - R′ = all pairs $\langle b, a \rangle$ s.t. it is false that the first member is the teacher of the second member
- inverse: $R^{-1} = \{\langle b, a \rangle \| \langle a, b \rangle \in R\}$ for $R \subseteq A \times B$
  - R = the relation of teacherhood between a and b:
    Herr Webelhuth is the teacher of Herr Schäfer.
  - R^{-1} = all pairs $\langle b, a \rangle$ where *a* is the teacher of *b*:
    Herr Schäfer is the inverse-teacher of Herr Webelhuth.

Functions

- A function F from A to B is a relation s.t. for every $a \in A$ there is exactly on tuple $\langle a, b \rangle \in A \times B$ s.t. a is the first coordinate.
- partial function from A to B: for some $a \in A$ there is no tuple $\langle a, b \rangle \in A \times B$, F is not *defined for* some a

Injection, surjection, bijection

- B the range of F, F is **into** B
- F from A to B is **onto (a surjection)** B iff there is no $b_i \in B$ s.t. there is no $\langle a, b_i \rangle \in F$
- F from A to B is **one-to-one (an injection)** iff there are no two pairs s.t. $\langle a_i, b_j \rangle \in F$ and $\langle a_k, b_j \rangle \in F$
- one-to-one, onto, and total function: correspondence (bijection)

Composition

- One can take the range of a function and make it the domain of another function.
- A function $F_1 : A \rightarrow B$ and a function $F_2 : B \rightarrow C$ can be composed as $B(A(a))$, short $B \circ A$
- the compound function can be empty, it will be total if both A and B are bijections.

Reflexivity

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
reflexive	for **every** $a \in A$: $\langle a, a \rangle \in R$	is as heavy as
		A: physical objects
irreflexive	for **every** $a \in A$: $\langle a, a \rangle \notin R$	is the father of
non-reflexive	for **some** $a \in A$: $\langle a, a \rangle \notin R$	has hurt

Symmetry

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
symmetric	for every $\langle a, b \rangle \in R$: $\langle b, a \rangle \in R$	has the same car as
asymmetric	for every $\langle a, b \rangle \in R$: $\langle b, a \rangle \notin R$	has a different car than
non-symmetric	for some $\langle a, b \rangle \in R$: $\langle b, a \rangle \notin R$	is the sister of
anti-symmetric	for every $\langle a, b \rangle \in R$: $a = b$	beats oneself not every human does

Transitivity

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
transitive	if $\langle a, b \rangle \in R$ and $\langle b, c \rangle \in R$ then $\langle a, c \rangle \in R$	is to the left of
intransitive	the above is never the case	is the father of
non-transitive	the above is sometimes not the case	likes

Connectedness

A relation R in $A = \{a, b, \ldots\}$ is...

	if	(ex.)
connected	for every $a, b \in A$, $a \neq b$: either $\langle a, b \rangle \in R$ or $\langle b, a \rangle \in R$	$>$ (A: the natural numbers)
non-connected	for some $a, b \in A$ the above is not the case	likes

Equivalence relations

- reflexive ($\langle a, a\rangle \in R$ for every a)
- symmetric ($\langle b, a\rangle \in R$ for every $\langle a, b\rangle$)
- transitive ($\langle a, b\rangle \in R$ & $\langle b, c\rangle \in R \rightarrow \langle a, c\rangle \in R$)
- *is as stupid as*
- partition the range into equivalence classes:
  $A = \{a, b, c, d\}$, for example $P_{A_1} = \{\{a, b\}, \{c\}, \{d\}\}$
- not $\{\{a\}, \{b, c\}\}$ or $\{\{a, b\}, \{b, c\}, \{d\}\}$

Defining ordering relations

An ordering relation R in A is ...

- transitive ($\langle a, b\rangle \in R$ & $\langle b, c\rangle \in R \rightarrow \langle a, c\rangle \in R$) ...plus ...
- irreflexive and asymmetric: **strict order**
- $A = \{a, b, c, d\}$, $R_1 = \{\langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$
- reflexive and anti-symmetric: **weak order**
- $A = \{a, b, c, d\}$, $R_1 = \{\langle a, a\rangle, \langle b, b\rangle, \langle c, c\rangle, \langle a, b\rangle, \langle b, c\rangle, \langle a, c\rangle\}$

Orders: an example

- a strict order: *greater than* ($>$) in $\mathbb{N}$
- what is the corresponding weak order
- $\geq$

- **minimal**: x is not preceded
- **least**: x precedes every other lement
- **maximal**: x is not succeeded
- **greatest**: x succeeds every other element
- **well-ordering**: total order, every subset has a least element

The number of elements...

- $A = \{a, b, c\}$
- $B = \{a, b, c\}$
- obviously, $A = B$ (equal)
- there is an R from A to B s.t. $R = \{\langle a, a \rangle, \langle b, b \rangle, \langle c, c \rangle\}$
- for every set C with the same number of elements (e.g., $C = \{1, 2, 3\}$): $R = \{\langle a, 1 \rangle, \langle b, 2 \rangle, \langle c, 3 \rangle\}$
- such relations are one-to-one correspondences

Denumerable sets

- $\mathbb{N}$ is infinite
- for every A there is some $\mathrm{R}_{\mathrm{card}}$
  - a one-to-one correspondence
  - from A's members to the first *n* members of $\mathbb{N}$
  - s.t. *n* is the cardinality of A, $\|A\|$
- sets A,B with $\|A\| = \|B\|$ are equivalent
- $\|\mathbb{N}\| = \aleph^0$

A problem

- for some sets there is no such R_{card}
- no way of bringing their elements into an exhaustive linear order
- no problem with $\mathbb{Q}$:

$$
\begin{array}{ccccc}
 & \langle 0,1 \rangle & \langle 0,2 \rangle & \langle 0,3 \rangle & \cdots \\
\langle 1,0 \rangle & \langle 1,1 \rangle & \langle 1,2 \rangle & \langle 1,3 \rangle & \cdots \\
\langle 2,0 \rangle & \langle 2,1 \rangle & \langle 2,2 \rangle & \langle 2,3 \rangle & \cdots \\
\vdots & \vdots & \vdots & \vdots &
\end{array}
$$

The non-denumerable real numbers

- now: $\mathbb{R}$
- some elements cannot be represented as an ordered pair of two elements of $\mathbb{N}$
- in $[0, 1]$, every real can be represented as $0.abcdefg\ldots$,
  $a, b, c, d, e, f, g, \ldots \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Trying to enumerate

- an enumeration of $[0, 1]$ in $\mathbb{R}$?

$$
\begin{array}{cccccccccc}
x_1 & = & 0 & . & a_{11} & a_{12} & a_{13} & a_{14} & \ldots \\
x_2 & = & 0 & . & a_{21} & a_{22} & a_{23} & a_{24} & \ldots \\
x_3 & = & 0 & . & a_{31} & a_{32} & a_{33} & a_{34} & \ldots \\
\vdots & & \vdots & & & & & & \\
x_n & = & 0 & . & a_{n1} & a_{n2} & a_{n3} & a_{n4} & \ldots
\end{array}
$$

Failing to enumerate

- What about an x_m which differs from x_n at a_{nn}

$$
\begin{array}{cccccccc}
x_1 & = & 0 & . & a_{11} & a_{12} & a_{13} & a_{14} & \dots \\
x_2 & = & 0 & . & a_{21} & a_{22} & a_{23} & a_{24} & \dots \\
x_3 & = & 0 & . & a_{31} & a_{32} & a_{33} & a_{34} & \dots \\
\vdots & & \vdots & & & & & & \\
x_n & = & 0 & . & a_{n1} & a_{n2} & a_{n3} & a_{nn} & \dots
\end{array}
$$

- It won't be in the array...
- $\mathbb{R}$ is non-denumerable
- If $\|A\| = \aleph^0$ then $\|\wp(A)\| = 2^{\aleph_0}$ (cf. Partee et al. 62f.)

Aussagenlogik

The book (PMW:87-246) deals with logic far more in-depth than we do. Only what is mentioned on the slides is relevant for the test. Reading the whole chapter from PMW will do you no harm, though.

Theories

- a collection of statements (propositions)
- axioms (statements accepted to be true)
- maybe based on observations (induction)
- statements that follow from the axioms (deduction)
- predictions beyond the axioms
- rechecking for usability: e.g., Russell’s paradox

Proofs

- axioms: atomic truths of your theory
- theorem: a proposition you want to prove
- lemma: subsidiary propositions (used to prove the theorem)
- corollary: propositions proved while proving some axiom

A method of reasoning

- logic does not generate truths
- formalizing statements, predications etc.
- rules of deduction from axioms to theorems
- empirical (induction) and exact (deduction) science
- aiming at an adequate model of the world (e.g., heliocentric universe)

Why logic for semantics?

- truth-conditional
- compositional behavior of propositions and connectives
- a logic for entailments
- why, e.g.: *It is not the case that someone is happy.* $\rightarrow$ *Nobody is happy.*

Atomic formulas: statements

- statements/propositions = the atoms
- a propositional symbol *p*: a well-formed formula (wff)
- ex.: *Herr Keydana is a passionate cyclist.*: *k*
- $[\![k]\!]$=1 or 0 (depending on corresponding **model**)

Complex (molecular) formulas

- syntax: restricts the forms of wff's to make them interpretable
- define functors: functions in $\{0, 1\}$
- If p and q are wff's, then
  - $\neg p$
  - $p \vee q$
  - $p \wedge q$
  - $p \rightarrow q$
  - $p \leftrightarrow q$

  is also a wff (a molecular term).

Complex (molecular) formulas

- syntax: restricts forms of wff's to make them interpretable
- define functors: functions in $\{\langle 0,1\rangle, \langle 1,0\rangle, 0, 1\}$
- If *p* and *q* are wff's, then
  - $\neg p$ (negation)
  - $p \vee q$ (disjunction)
  - $p \wedge q$ (conjunction)
  - $p \rightarrow q$ (conditional)
  - $p \leftrightarrow q$ (biconditional)

  is also a wff.

Complex (molecular) formulas

- syntax: restricts forms of wff's to make them interpretable
- define functors: functions in $\{\langle 0,1\rangle, \langle 1,0\rangle, 0, 1\}$
- If *p* and *q* are wff's, then
  - $\neg p$ (negation - 'not')
  - $p \vee q$ (disjunction - 'or')
  - $p \wedge q$ (conjunction - 'and')
  - $p \rightarrow q$ (conditional - 'if')
  - $p \leftrightarrow q$ (biconditional - 'iff')

  is also a wff.

Functions and truth tables

- standard defintion:

$$[\![\neg]\!] = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$$

- but most widely used: truth tables

$\neg$	p
0	1
1	0

Disjunction

p	$\vee$	q
1	1	1
1	1	0
0	1	1
0	0	0

- *Herr Keydana is a passionate cyclist **or** we all love logic.*
- $K \vee L$

Conjunction

p	$\wedge$	q
1	1	1
1	0	0
0	0	1
0	0	0

- *Herr Keydana is a passionate cyclist **and** we all love logic.*
- $K \wedge L$

Conditional

p	$\rightarrow$	q
1	1	1
1	0	0
0	1	1
0	1	0

- ***If*** *it rains,* ***then*** *the streets get wet.*
- $R \rightarrow S$

Any problems with that?

If it rains, the streets get wet.

- it is raining (1) , the streets are wet 1 : **1**
- it is raining (1) , the streets are dry 0 : **0**
- it is not raining (0) , the streets are wet 1 : **1**
- it is not raining (0) , the streets are dry 0 : **1**
- ex vero non sequitur falsum

Biconditional

p	$\leftrightarrow$	q
1	1	1
1	0	0
0	0	1
0	1	0

- ***If and only if** your <u>s</u>core is above 50, **then** you <u>p</u>ass the semantics exam.*
- $S \leftrightarrow P$

Scope of functors

- brackets are facultative
- or set non-default functor scope
- default scope

$$\text{scope} \left\downarrow \begin{array}{c} \neg \\ \wedge \\ \vee \\ \rightarrow \\ \leftrightarrow \end{array} \right\uparrow \text{binding strength}$$

An example

- $p \land \neg q \lor r \to \neg s$
- $p \land (\neg q) \lor r \to (\neg s)$
- $(p \land (\neg q)) \lor r \to (\neg s)$
- $((p \land (\neg q)) \lor r) \to (\neg s)$
- $(((p \land (\neg q)) \lor r) \to (\neg s))$

Large truth tables

- for *n* atoms in the term: 2^n lines
- alternating blocks of 1's and 0's under every atom
- $2^{(m-1)}$ times '1' followed by $2^{(m-1)}$ times '0' for the *m*-th atom from the right
- until 2^n lines are reached

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1			1		1			1
1			1		1			0
1			1		0			1
1			1		0			0
1			0		1			1
1			0		1			0
1			0		0			1
1			0		0			0
0			1		1			1
0			1		1			0
0			1		0			1
0			1		0			0
0			0		1			1
0			0		1			0
0			0		0			1
0			0		0			0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1		0	1		1		0	1
1		0	1		1		1	0
1		0	1		0		0	1
1		0	1		0		1	0
1		1	0		1		0	1
1		1	0		1		1	0
1		1	0		0		0	1
1		1	0		0		1	0
0		0	1		1		0	1
0		0	1		1		1	0
0		0	1		0		0	1
0		0	1		0		1	0
0		1	0		1		0	1
0		1	0		1		1	0
0		1	0		0		0	1
0		1	0		0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1		1		0	1
1	0	0	1		1		1	0
1	0	0	1		0		0	1
1	0	0	1		0		1	0
1	1	1	0		1		0	1
1	1	1	0		1		1	0
1	1	1	0		0		0	1
1	1	1	0		0		1	0
0	0	0	1		1		0	1
0	0	0	1		1		1	0
0	0	0	1		0		0	1
0	0	0	1		0		1	0
0	0	1	0		1		0	1
0	0	1	0		1		1	0
0	0	1	0		0		0	1
0	0	1	0		0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1		0	1
1	0	0	1	1	1		1	0
1	0	0	1	0	0		0	1
1	0	0	1	0	0		1	0
1	1	1	0	1	1		0	1
1	1	1	0	1	1		1	0
1	1	1	0	1	0		0	1
1	1	1	0	1	0		1	0
0	0	0	1	1	1		0	1
0	0	0	1	1	1		1	0
0	0	0	1	0	0		0	1
0	0	0	1	0	0		1	0
0	0	1	0	1	1		0	1
0	0	1	0	1	1		1	0
0	0	1	0	0	0		0	1
0	0	1	0	0	0		1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

An example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Assignments: a contingent example

p	$\wedge$	$\neg$	q	$\vee$	r	$\rightarrow$	$\neg$	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Tautology

- take $p \vee \neg p$
- truth-table:

p	$\vee$	$\neg$	p
1	1	0	1
0	1	1	0

- true under every assignment, it is **valid**
- by *law of excluded middle*: for every P, $P \vee \neg P$ is true

Contradiction

- take $p \wedge \neg p$
- truth-table:

p	$\wedge$	$\neg$	p
1	0	0	1
0	0	1	0

- false under every assignment, called **contradictory**

Contingency

- take $p \wedge p$
- truth-table:

p	$\wedge$	p
1	1	1
0	0	0

- the truth value depends on the assignemt

What are laws?

- notice: similarities of set theory and logic
- non-trivial exact nature of their equivalence
- laws state equivalences of (types of) wff
- truth-conservative rewriting of wff's
- any subformula which is a tautology (T) or contradiction (F):

  ignore by Identity Laws (Id.):
  - $(P \vee F) \Leftrightarrow P$, $(P \vee T) \Leftrightarrow T$
  - $(P \wedge F) \Leftrightarrow F$, $(P \wedge T) \Leftrightarrow P$

- X $\Leftrightarrow$ Y: X has the same truth-conditions as Y
- derivability of laws and rules (convenient redundancies)
- Idempotency (Idemp.):
  - $(P \vee P) \Leftrightarrow P$
  - $(P \wedge P) \Leftrightarrow P$
  - *Peter walks and Peter walks.* $\Leftrightarrow$ *Peter walks.*

Simple laws

- Associative Laws for $\vee$ and $\wedge$ (Assoc.):
  - $((P \vee Q) \vee R) \Leftrightarrow (P \vee (Q \vee R))$
  - *((He walks or she talks) or we walk.)* $\Leftrightarrow$ *(He walks or (she talks or we walk.))*
- Commutative Laws for $\vee$ and $\wedge$ (Comm.):
  - $(P \vee Q) \Leftrightarrow (Q \vee P)$
  - *Peter walks or Sue snores.* $\Leftrightarrow$ *Sue snores or Peter walks.*
- Distributive Laws for $\vee\wedge$ and $\wedge\vee$ (Distr.):
  - $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
  - *(Sue snores) and (Peter walks or we talk).* $\Leftrightarrow$ *(Sue snores and Peter walks) or (Sue snores and we talk).*

Laws dealing with tautology and contradiction

- Complement Laws:
  - Tautology (T): $(P \vee \neg P) \Leftrightarrow \mathbf{T}$
  - Contradiction (F): $(P \wedge \neg P) \Leftrightarrow \mathbf{F}$
  - Double Negation (DN): $(\neg\neg P) \Leftrightarrow P$
  - *It is not the case that Sandy is not walking.*
    $\Leftrightarrow$ *Sandy is walking.*

Conditionals Laws

- **Implication** (Impl.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	P	$\vee$	Q
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

- **Contraposition** (Contr.):

P	$\rightarrow$	Q	$\Leftrightarrow$	$\neg$	Q	$\rightarrow$	$\neg$	P
1	1	1		0	1	1	0	1
1	0	0		1	0	0	0	1
0	1	1		0	1	1	1	0
0	1	0		1	0	1	1	0

DeMorgan (DeM)

- DeMorgan's Laws:
  - $\neg(P \vee Q) \Leftrightarrow (\neg P \wedge \neg Q)$
  - alternatively: $\overline{P \vee Q} \Leftrightarrow \overline{P} \wedge \overline{Q}$
  - $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
  - consequently: $\overline{\overline{P} \vee \overline{Q}} \Leftrightarrow \overline{\overline{P}} \wedge \overline{\overline{Q}} \Leftrightarrow P \wedge Q$

The Modus Ponens (MP)

- Definition:

P	$\rightarrow$	Q	premise 1
P			premise 2
		Q	conclusion

- or: $(P \rightarrow Q) \wedge (P) \rightarrow (Q)$
- *(1) If It rains, the streets get wet. (2) It is raining.*
  $\rightarrow$ *The streets are getting wet.*

MP: a truth table illustration

- Premises are always set to be true!
- the table:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- The conditional must be true.
- cancel the ‘false’ row

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- P must be true.
- cancel the ‘false’ rows, Q can only be true:

P	$\rightarrow$	Q
1	1	1
1	0	0
0	1	1
0	1	0

The Modus Tollens (MT)

- Definition:

P	$\rightarrow$	Q
		$\neg Q$
$\neg P$		

- the table illustration:

P	$\rightarrow$	Q	
1	1	1	(by premise 2)
1	0	0	(by premise 1)
0	1	1	(by premise 2)
0	1	0	

The Syllogisms

- Hypothetical Syllogism (HS):
  - $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
  - *(1) If it rains, the streets get wet. (2) If the streets get wet, it smells nice. $\rightarrow$ If it rains, it smells nice.*
- Disjunctive Syllogism (DS):
  - $((P \vee Q) \wedge (\neg P)) \rightarrow (Q)$
  - *(1) Either Peter sleeps or Peter is awake. (2) Peter isn't awake. $\rightarrow$ Peter sleeps.*

Trivial rules

- Simplification (Simp.):
  - $(P \wedge Q) \rightarrow P$
  - *(1) It is raining and the sun is shining.* $\rightarrow$ *It is raining.*
- Conjunction (Conj.):
  - $(P) \wedge (Q) \rightarrow (P \wedge Q)$
  - *(1) It is raining. (2) The sun is shining.* $\rightarrow$ *It is raining and the sun is shining.*
- Addition (Add.):
  - $(P) \rightarrow (P \wedge Q)$
  - *(1) It is raining.* $\rightarrow$ *It is raining or the sun is shining.*
  - What if Q is instantiated as true or false by another premise?

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$
- The proof:

		$p \vee q$
1	$(p \vee q) \rightarrow \neg(r \wedge \neg s)$	
2	$r \wedge \neg s$	
	$p \vee q$	1,2,MT

Prädikatenlogik

Weak compositionality in SL

- properties/relations vs. individuals
- *Martin is an expert on inversion and Martin is a good climber.*
- ...becomes $E \wedge C$
- compositionality resticted to level of connected propositional atoms

Some desirable deductions

- important generalizations about all and some individuals (which have property P)
- *‘all P $\rightarrow$ some P’*
- *‘Martin P $\rightarrow$ some P’*

Atoms of PC

- individual variables: $x, y, z, x_1, x_2 \ldots$
- individual constants: $a, b, c, \ldots$
- variables and constants: terms
- predicate symbols (taking individual symbols or tuples of them): $A, B, C, \ldots$
- quantifiers: existential $\exists$ (or $\bigvee$) and universal $\forall$ (or $\bigwedge$)
- plus the connectives of SL

Some syntax

- for an n-ary predicate P and terms $t_1 \dots t_n$, $P(t_1 \dots t_n)$ or $Pt_1 \dots t_n$ is a wff.
- possible prefix, function (bracket) and infix notation: Pxy, $P(x, y)$, xPy
- syntax for connectives from SL
- for any wff ϕ and any variable x, $(\exists x)\phi$ and $(\forall x)\phi$ are wff's

Semantic for individual constants

- denote individuals
- a model $\mathcal{M}$ contains a set of individuals D
- the valuation function V (or F): from constants to individuals in D
- for some $\mathcal{M}_1$: $D = \{Martin, Kilroy, Scully\}$
- $V_{\mathcal{M}_1}(m) = Martin$
- $V_{\mathcal{M}_1}(k) = Kilroy$, $V_{\mathcal{M}_1}(s) = Scully$

Semantics for predicate symbols

- denote relations (sets of n-tuples)
- $[\![P]\!]^{\mathcal{M}_1} = \{Martin, Kilroy\}$ or $V_{\mathcal{M}_1}(P) = \{Martin, Kilroy\}$
- $V_{\mathcal{M}_1}(Q) = \{\langle Martin, Kilroy\rangle, \langle Martin, Scully\rangle, \langle Kilroy, Kilroy\rangle, \langle Scully, Scully\rangle\}$
- s.t. $[\![P(m)]\!]^{\mathcal{M}_1} = [\![P]\!]^{\mathcal{M}_1}([\![m)]\!]^{\mathcal{M}_1}) = 1$ iff $[\![m]\!]^{\mathcal{M}_1} \in [\![P]\!]^{\mathcal{M}_1}$

- connectives: ‘apply to’ formulas (semantically truth-valued), semantics as in SL
- $(\forall x)\phi$ = 1 iff ϕ is true for every $d \in D$
  assigned to every occurence of x in ϕ
- $(\exists x)\phi$ = 1 iff ϕ is true for at least one $d \in D$
  assigned to every occurence of x in ϕ
- algorithmic instruction to check wff’s containing Q’s
- check outside-in (unambiguous scoping)

Dependencies

- universal quantifiers can be swapped:
  $(\forall x)(\forall y)\phi \Leftrightarrow (\forall y)(\forall x)\phi$
- same for existential quantifiers:
  $(\exists x)(\exists y)\phi \Leftrightarrow (\exists y)(\exists x)\phi$
- whereas: $(\exists x)(\forall y)\phi \Rightarrow (\forall y)(\exists x)\phi$
- example in $\mathcal{M}_1$:
  - $[\![\underline{(\forall x)\underline{(\exists y)Qxy}}]\!]^{\mathcal{M}_1} = 1$
  - but: $[\![\underline{(\exists y)\underline{(\forall x)Qxy}}]\!]^{\mathcal{M}_1} = 0$
  - direct consequence of algorithmic definition
  - if $\exists\forall$ is true, $\forall\exists$ follows

Hints on quantifiers

- domain of quantifiers: D (universe of discourse)
- $\forall x$ checks for truth of some predication for all individuals
- $\exists x(Px \wedge \neg Px)$ is a contradiction
- $\forall x(Wx \wedge \neg Wx)$ is a contradiciton,
  $\forall x$ 'checks' for an empty set by def.
- standard form of NL quantification:
  $\forall x(Wx \rightarrow Bx)$ 'All women are beautiful.'
- standard form of NL existential quantification:
  $\exists x(Wx \wedge Bx)$ 'Some woman is beautiful.'

Functor/quantifier practice

- by def., functors take formulas, not terms:
  - $\neg Wm$ 'Mary doesn't weep.'
  - $(\exists x)(Gx \wedge Wx)$ 'Some girl weeps.'
  - $^{*}W\neg x$
  - $^{*}(\exists\neg x)(Gx)$
- quantifiers take variables, not constants:
  - $(\forall x)(Ox \rightarrow Wx)$ 'All ozelots are wildcats.'
  - $^{*}(\forall o)(Wo)$
- $\neg$ negates the wff, not the q:
  $^{*}(\neg\forall x)Px$ but $\neg(\forall x)Px$

Scope

- quantifiers bind variables
- free variables (constants) are unbound
- no double binding $^*(\forall x \exists x)Px$
- Q scope: only the first wff to its right:
  - $\underline{(\forall x)Px} \vee Qx$
  - $\underline{(\forall x)(Px \vee Qx)} = \underline{(\forall x)Px} \vee \underline{(\forall x)Qx}$
  - $\underline{(\exists x)Px} \rightarrow \underline{(\forall y)(Qy \wedge Ry)}$
  - $\underline{(\exists x)Px} \wedge Qx$ (second x is a unbound)
- no double-naming

- $\exists$ and $\forall$ 'or' and 'and' over the universe of discourse (hence: $\bigvee$ and $\bigwedge$)
- $(\forall x)Px \Leftrightarrow Px_1 \land Px_2 \land \ldots \land Px_n$ for all x_n assigned to $d_n \in D$
- $(\exists x)Px \Leftrightarrow Px_1 \lor Px_2 \lor \ldots \lor Px_n$ for all x_n assigned to $d_n \in D$
- hence: $\neg(\forall x)Px \Leftrightarrow \neg(Px_1 \land Px_2 \land \ldots \land Px_n)$
- with DeM: $\overline{Px_1 \land Px_2 \land \ldots \land Px_n}$
- $\Leftrightarrow \overline{Px_1} \lor \overline{Px_2} \lor \ldots \lor \overline{Px_n}$
- $\Leftrightarrow (\exists x)\neg Px$

Quantifier negation (QN)

- $\neg(\forall x)Px \Leftrightarrow (\exists x)\neg Px$
- $\neg(\exists x)Px \Leftrightarrow (\forall x)\neg Px$
- $\neg(\forall x)\neg Px \Leftrightarrow (\exists x)Px$
- $\neg(\exists x)\neg Px \Leftrightarrow (\forall x)Px$

The distribution laws

- the conjunction of universally quantified formulas:
  $\underline{(\forall x)(Px \wedge Qx)} \Leftrightarrow \underline{(\forall x)Px} \wedge \underline{(\forall x)Qx}$
- the disjunction of existentially quantified formulas:
  $\underline{(\exists x)(Px \vee Qx)} \Leftrightarrow \underline{(\exists x)Px} \vee \underline{(\exists x)Qx}$
- not v.v.: $(\forall x)Px \vee (\forall x)Qx \Rightarrow (\forall x)(Px \vee Qx)$
- why?

Quantifier movement (QM)

- desirable format: prefix + matrix
- Movement Laws for antecedents of conditionals:
  $(\exists x)Px \rightarrow \phi \Leftrightarrow (\forall x)(Px \rightarrow \phi)$
  $(\forall x)Px \rightarrow \phi \Leftrightarrow (\exists x)(Px \rightarrow \phi)$
- Movement Laws for Q's in disjunction, conjunction, and the consequent of conditionals: Just move them to the prefix!
- condition: x must not be free in ϕ.
- i.e.: Watch your variables!

Let’s formalize:

- *Paul Kalkbrenner is a musician and signed on bpitchcontrol.*
- *Herr S. installed RedHat and not every Linux distribution is easy to install.*
- *All talkmasters are human and Harald Schmidt is a talkmaster.*
- *Some talkmasters are not musicians.*
- *Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.*
- *Some humans are neither talkmasters nor do they own Kanzleramt records.*

Universal instantiation ($-\forall$) and generalization ($+\forall$)

- $(\forall x)Px \rightarrow Pa$
- always applies
- can use any variable/constant
- $Pa \rightarrow (\forall x)Px$
- iff *Pa* was instantiated by $-\forall$

Existential generalization ($+\exists$) and instantiation ($-\exists$)

- $Pa \rightarrow (\exists x)Px$ for any individual constant a
- always applies
- $(\exists x)Px \rightarrow Pa$ for some indiv. const.
- always applies (there is a minimal individual for $\exists x$)
- for some $(\exists x)Px$ and $(\exists x)Qx$ the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

One sample task

- (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.
- Formalize and prove: At least one human exists.
- (1) Dk
- (2) $(\forall x)(Dx \rightarrow Hx \vee Px)$
- (3) $\neg(\exists x)(Px \wedge Dx)$
- $(\exists x)Hx$

(1)	Dk	
(2)	$(\forall x)(Dx \rightarrow Hx \vee Px)$	
(3)	$\neg(\exists x)(Px \wedge Dx)$	
(4)	$(\forall x)\neg(Px \wedge Dx)$	3,QN
(5)	$(\forall x)(\neg Px \vee \neg Dx)$	4,DeM
(6)	$(\forall x)(Dx \rightarrow \neg Px)$	5,Comm,Impl
(7)	$Dk \rightarrow \neg Pk$	6,$-\forall$(1)
(8)	$\neg Pk$	1,7,MP
(9)	$Dk \rightarrow Hk \vee Pk$	2,$-\forall$(1)
(10)	$Hk \vee Pk$	1,9,MP
(11)	Hk	8,10,DS
$\therefore$	$(\exists x)Hx$	10,$+\exists$

Quantifikation und Modelltheorie

Back to semantics: F1

- before we turn to quantification in F1/F2 English:
- names refer to individuals
- itr. verbs refer to sets of individuals
- tr. verbs refer to sets of ordered pairs of individuals
- sentences refer to truth values

Reference of pronouns

- ***This** drives a Golf.*
- *this* = a pronominal NP
- denotes an individual
- but not rigidly
- fixed only within a specific context (SOA)

Pronouns and variables

- quantified expression: $(\forall x)Px$
- *for all assignments of 'this', 'this' has property P*
- Q evaluation in PC is algorithmic
- variables interpreted like definite pronominal NPs (within a fixed context)

Categories and lexicon

- $a \rightarrow$ const, var
- conn $\rightarrow \wedge, \vee, \rightarrow, \leftrightarrow$
- neg $\rightarrow \neg$
- Q $\rightarrow \exists, \forall$

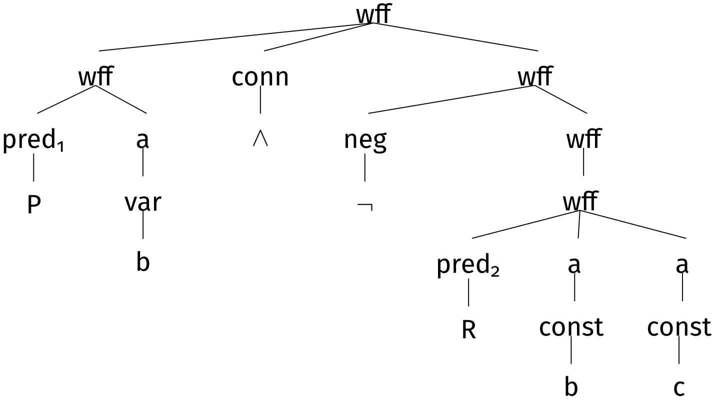
Categories and lexicon

- $\text{pred}_1 \rightarrow \text{P, Q}$
- $\text{pred}_2 \rightarrow \text{R}$
- $\text{pred}_3 \rightarrow \text{S}$
- $\text{const} \rightarrow \text{b, c}$
- $\text{var} \rightarrow x_1, x_2, ..., x_n$

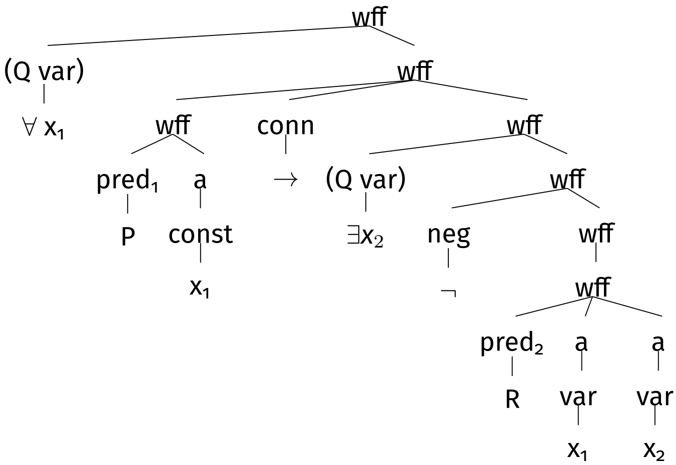
Phrase structure

- wff $\rightarrow$ pred$_n$ a$_1$ a$_2$ ...a$_n$
- wff $\rightarrow$ neg wff
- wff $\rightarrow$ wff con wff
- wff $\rightarrow$ (Q var) wff

A wff without Q



A wff with Q's


wff
(Q var)
wff
∀ x1
wff
conn
wff
pred1
a
→
(Q var)
wff
P
const
∃x2
neg
wff
x1
¬
wff
pred2
a
a
R
var
var
x1
x2

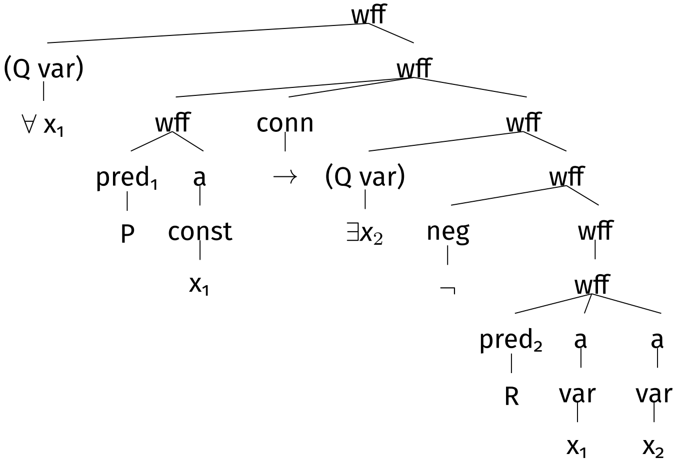
Definition of c-command

- Node A c-commands (constituent-commands) node B iff
  - A does not dominate B and
  - and the first branching node dominating A also dominates B.
- The definition in CM allows a node to dominate itself.

Configurational binding

- in configurational tree-structures:
- A variables is bound by the closest c-commanding coindexed quantifier.
- scope = binding domain

A wff with Q's


wff
(Q var)
wff
∀ x1
wff
conn
wff
pred1
a
→
(Q var)
wff
P
const
∃x2
neg
wff
x1
¬
wff
pred2
a
a
R
var
var
x1
x2

Refinement of PC semantics

- remember T-sentences: **S of L is true in v iff p**.
- $\mathcal{M}$ is a model of the accessible universe of discourse
  - $\mathcal{M} = \langle U_n, V_n \rangle$
  - U_n = the set of accessible individuals (domain)
  - V_n = a valuation function which assigns
    - individuals to names
    - sets of n-tuples of indivuiduals to $pred_n$
- **g** is function from variables to individuals in $\mathcal{M}$
- we evaluate: $[\![\alpha]\!]^{\mathcal{M}_n, g_n}$
- *the extension of α relative to $\mathcal{M}_n$ and g_n*

Fixed and context-bound denotation

- $\mathrm{V_n}$ valuates statically
- Q's require flexible valuation of pronominal matrices
- g_n is like $\mathrm{V_n}$ for constants, only flexible
- it can iterate through $\mathrm{U_n}$
- initial assignment can be anything:

$$g_1 = \begin{bmatrix} x_1 \rightarrow \textit{Herr Webelhuth} \\ x_2 \rightarrow \textit{Frau Eckardt} \\ x_3 \rightarrow \textit{Turm} - \textit{Mensa} \end{bmatrix}$$

Iterating through U_n

- for each Q loop, one modification
- read $g_n\,[d/x_m]$ as
  '...relative to g_n where x_m is reassigned to d'
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[Eckardt/x_1]} = \textit{Frau Eckardt}$
- $[\![x_2]\!]^{\mathcal{M}_1,g_1[[Eckardt/x_1]Mensa/x_2]} = \textit{Mensa}$

Interpreting with g_n

- $[\![(\forall x_1)Px_1]\!]^{\mathcal{M}_1,g_1}$
- start with initial assignment: $[\![x_1]\!]^{\mathcal{M}_1,g_1} = \mathit{Webelhuth}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- modify: $[\![x_1]\!]^{\mathcal{M}_1,g_1[\mathit{Eckardt}/x_1]} = \mathit{Eckardt}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- modify: $[\![x_1]\!]^{\mathcal{M}_1,g_1[\mathit{Mensa}/x_1]} = \mathit{Mensa}$
  check: $[\![Px_1]\!]^{\mathcal{M}_1,g_1}$
- iff the answer was never 0, then $[\![(\forall x_1)Px_1]\!]^{\mathcal{M}_1,g_1} = 1$

Multiple Q's: subloops

- $[\![(\forall x_1)(\exists x_2)Px_1x_2]\!]^{\mathcal{M}_1,g_1}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1} = \textit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1} = \textit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\textit{Webelhuth}/x_2]} = \textit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\textit{Mensa}/x_2]} = \textit{Mensa}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[\textit{Eckardt}/x_1]} = \textit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\textit{Eckardt}/x_1]} = \textit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\textit{Eckardt}/x_1]\textit{Webelhuth}/x2]} = \textit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\textit{Eckardt}/x_1]\textit{Mensa}/x2]} = \textit{Mensa}$
- $[\![x_1]\!]^{\mathcal{M}_1,g_1[\textit{Mensa}/x_1]} = \textit{Mensa}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[\textit{Mensa}/x_1]} = \textit{Eckardt}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\textit{Mensa}/x_1]\textit{Webelhuth}/x_2]} = \textit{Webelhuth}$
  - $[\![x_2]\!]^{\mathcal{M}_1,g_1[[\textit{Mensa}/x_1]\textit{Mensa}/x_2]} = \textit{Mensa}$

- quantifying expressions in NL beyond $\forall$ and $\exists$
- some seem to work differently:
- *All patients adore Dr. Rick Dagless M.D.*
  $(\forall x_1)Px_1 \rightarrow Ax_1d$ (ok)
- but: *Most patients adore Dr. Rick Dagless M.D.*
  $(MOST\ x_1)Px_1 \rightarrow Ax_1d$ (wrong interpretation)
- domain should be the set of patients, not individuals
- For NL: Assume that the checking domain for Q is the set denoted by CN.

Scope ambiguities

- c-command condition on binding/scope fails in NL
- no PNF's in NL
- Q and common noun (CN) usually in-situ (e.g., argument position)
- ambiguities independent of Q position
  - *Everybody loves somebody.* (*ELS*)
  - $(\forall x_1)(\exists x_2)Lx_1x_2$
  - $(\exists x_2)(\forall x_1)Lx_1x_2$
- Q ambiguity cannot be structural (e.g., $\exists$ will never c-command $\forall$)

Cases of overt movement and traces

- wh movement:
- *What$_i$ will Agent Cooper solve t$_i$?*
-
- passive movement:
- *(Laura Palmer)$_i$ was killed t$_i$.*
-
- raising verbs:
- *(Laura Palmer)$_i$ seems t$_i$ to be dead.*
-

Levels of representation

- construction of an independent representational level LF
- could use movement mechanism as used at surface level
- All quantifiers adjoin to the left periphery of S at LF.
- LF is constructed by syntactic rules!

Ambiguities at LF

- $[_{S''}$ *everybody*$_i$ $[_{S'}$ *somebody*$_j$ $[_S$ t_i *loves* t_j]]]
-
- $[_{S''}$ *somebody*$_j$ $[_{S'}$ *everybody*$_i$ $[_S$ t_i *loves* t_j]]]
-

The Q raising rule

$$[_S\, X\ NP\ Y\,] \;\Rightarrow\; [_{S'}\, NP_i\ [_S\, X\ t_i\ Y\,]]$$

- specify a PS as input and output
- QR rule also introduces coindexing of traces

Syntax

- copies all definitions from F1
- adds appropriate definitions of quantifying determiners etc.
  - $Det \rightarrow every, some$
  - $NP \rightarrow DetN_{common-count}$
- adds the QR rule
- assume introduction of reasonable syntactic types/rules without specifying
- assume admissible (reasonable, possible) models $\mathcal{M}$

Semantics for QR output: *every*

$$\llbracket [\,[\textit{every}\ \beta]_i\ S]\rrbracket^{\mathcal{M},g} = 1 \textit{ iff for all } u \in U: \\ \textit{if } u \in \llbracket\beta\rrbracket^{\mathcal{M},g} \textit{ then } \llbracket S\rrbracket^{\mathcal{M},g[u/t_i]}$$

A sentence containing the trace t_i with an adjoined NP_i (which consists of *every* plus the common noun β) extend to 1 iff for each individual *u* in the universe *U* which is in the set referred to by the common noun β, *S* denotes 1 with *u* assigned to the pronominal trace t_i. *g* is modified iteratively to check that.

Semantics for QR output: *some, a*

$$[\![\,[\,[a\ \beta]_i\ S]\,]\!]^{\mathcal{M},g} = 1 \textit{ iff for some } u \in U: \\ u \in [\![\beta]\!]^{\mathcal{M},g} \textit{ and } [\![S]\!]^{\mathcal{M},g[u/t_i]}$$

(similar)

Einfach getypte höherstufige λ-Sprachen

Montague and the generative tradition

- Chierchia & McConnell-Ginet, Heim & Kratzer, etc.: GB-ish semantics
- both syntax and LF in phrase structures
- LF as a proper linguistic level of representation
- Montague: direct translation of NL into logic
- Monatgue's LF is just a notational system for NL semantics

Targets for this week

- Learn to tell the difference between the montagovian and generative approach.
- See the advantage of a general theory of typed languages.
- Understand how λ languages allow dramatically elegant formalizations.
- … while keeping in mind that these devices are extensions to our PC representation for NL semantics.

Denotations

- denotations in set/function-theoretic terms
- a characteristic function (CF) $\mathcal{S}$ of a set S:
  $\mathcal{S}(a) = 1$ *iff* $a \in S,$ *else* 0
- a CF ‘checks’ individuals into a set
- denotations can be stated as sets or their CF

Generalizing combinatory semantic operations

- interpretation for [$_S$ NP VP]:
  $[\![[_S\ NP\ VP]]\!]^{\mathcal{M},g} = 1$ *iff* $[\![NP]\!]^{\mathcal{M},g} \in [\![VP]\!]^{\mathcal{M},g}$
- Montague generally used CF's in definitions
- evaluating [$_S$ [$_{NP}$ *Mary*] [$_{VP}$ *sleeps*]] as a matter of functional application (FA):
  - $[\![\textit{Mary}]\!]^{\mathcal{M},g} =$ *Mary in* $\mathcal{M}$
  - $[\![\textit{sleeps}]\!]^{\mathcal{M},g}$ *be the CF of the set of sleepers in* $\mathcal{M}$
  - $[\![S]\!]^{\mathcal{M},g} = [\![\textit{sleeps}]\!]^{\mathcal{M},g}([\![\textit{Mary}]\!]^{\mathcal{M},g})$
  - ideally: generalize to all nodes

The superscript notation

- all functions from S_1 to S_2
- $S_2^{S_1}$
- for $T = \{0, 1\}$
  - T^D: all $pred_1$
  - $T^{D \times D}$: all $pred_2$
-

Some new names

- base for Dowty et al.: L_1, a first-order predicate language as we know it
- semantic renaming of types:
  - terms: $\langle e \rangle$ (entity-denoting)
  - formulas: $\langle t \rangle$ (truth-valued)
  - pred_1: $\langle e, t \rangle$
  - pred_2: $\langle e, \langle e, t \rangle \rangle$

Possible denotations of types

- D_α possible denotation (a set) of expressions of type α
- $D_{\langle e\rangle} = U$ (Dowty et al.'s A)
- $D_{\langle t\rangle} = \{0, 1\}$
- recursively: $D_{\langle\alpha,\beta\rangle} = D_{\langle\beta\rangle}^{D_{\langle\alpha\rangle}}$
- e.g., $D_{\langle e,t\rangle} = D_{\langle t\rangle}^{D_{\langle e\rangle}}$
- $D_{\langle e,\langle e,t\rangle\rangle} = (D_{\langle t\rangle}^{D_{\langle e\rangle}})^{D_{\langle e\rangle}}$
- just a systematic way of naming types, model-theoretic interpretations still by V, g

Defining types

- in our PS syntax: S as start symbol
- in the typed system: sentences should be of type $\langle t \rangle$
- complex types: functions from $\langle e \rangle$ to $\langle t \rangle$
  or generally from any (complex) type to any (comlex) type

Complex types as functions

- saturation of complex types by FA:
  - γ *is of type* $\langle e, \langle e, t\rangle\rangle$, δ *of* $\langle e, t\rangle$, α *and* β *of* $\langle e\rangle$
  - then $\gamma(\alpha)$ *is of type* $\langle e, t\rangle$
  - and $\delta(\beta)$ *is of type* $\langle t\rangle$
- for any pred_2 P and its arguments a_1, a_2, $P(a_2)(a_1)$ is a wff
- connectives are of types $\langle t, t\rangle$ ($\neg$), $\langle t, \langle t, t\rangle\rangle$ ($\wedge$, etc.)

General semantics of typed languages

- generalized CF/FA approach
- $\langle e \rangle$-types (terms):
  $[\![a_n]\!]^{\mathcal{M},g} = V(a_n)$
  $[\![x_n]\!]^{\mathcal{M},g} = g(x_n)$
- the rest: functional application
  $[\![\delta(\alpha)]\!]^{\mathcal{M},g} = [\![\delta]\!]^{\mathcal{M},g}([\![\alpha]\!]^{\mathcal{M},g})$

Refinement

- *Type* is the set of types
- recursively defined complex types $\langle a, b \rangle$: infinite
- type label $\langle \alpha \rangle$
- vs. set of meaningful expressions of that type: $ME_{\langle \alpha \rangle}$

Higher order

- first order languages: variables over individuals ($\langle e \rangle$-types)
- n-order: variables over higher types ($\langle e, t \rangle$-types etc.)
- $P_{\langle e,t \rangle}$ or $Q_{\langle e,\langle e,t \rangle\rangle}$: constants of higher types
- so: $v_{1_{\langle e,t \rangle}} [v_1(m)]$
- if $V(m) = Mary$, v_1 is the set of all of Mary's properties

Typing variables

- we write:
  - $v_{n_{\langle\alpha\rangle}}$ for the n-th variable of type $\langle\alpha\rangle$
  - Dowty et al.: $v_{n,\langle\alpha\rangle}$
- alternatively abbreviated by old symbols x_1, a, P, etc.

Constants, variables, functions

- non-logical constant α: $[\![\alpha]\!]^{\mathcal{M},g} = V(\alpha)$
- variable α: $[\![\alpha]\!]^{\mathcal{M},g} = V(\alpha)$
- $\alpha \in \langle a, b \rangle,\ \beta \in a$, then $[\![\alpha(\beta)]\!]^{\mathcal{M},g} = [\![\alpha]\!]^{\mathcal{M},g}([\![\beta]\!]^{\mathcal{M},g})$

- logical constants interpreted as functions in {0,1} as usual
- if $v_{1_{\langle\alpha\rangle}}$ is a variable and $\phi \in ME_t$
  then $[\![(\forall v_1)\phi]\!]^{\mathcal{M},g} = 1$ *iff*
  for all $a \in D_\alpha$ $[\![\phi]\!]^{\mathcal{M},g[a/v_1]} = 1$

An example

- quantified variable of type $\langle e, t\rangle$: $v_{0_{\langle e,t\rangle}}$
- $\forall v_{0_{\langle e,t\rangle}} \left[v_{0_{\langle e,t\rangle}}(j) \rightarrow v_{0_{\langle e,t\rangle}}(d) \right]$
- for $j, d \in ME_{\langle e\rangle}$
- one property of every individual: being alone in its union set
- hence, $j = d$
- else in $\forall v_{0_{\langle e,t\rangle}}$, $\forall$ wouldn't hold

Defining *non*

- productive adjectival prefix: *non-adjacent*, *non-local*, etc.
- inverting the characteristic function of the adjective
- result denotes complement of the original adjective in $D_{\langle e \rangle}$
- *adjective*: $\langle e, t \rangle$, *non*: $\langle \langle e, t \rangle, \langle e, t \rangle \rangle$
- a function h s.t. for every $k \in D_{\langle e,t \rangle}$ and every $d \in D_{\langle e \rangle}$
  $(h(k))(d) = 1$ iff $k(d) = 0$ and
  $(h(k))(d) = 0$ iff $k(d) = 1$

Argument deletion

- understood objects in: *I eat.* - *Vanity kills.* - etc.
- *eat* is in $ME_{\langle e,\langle e,t\rangle\rangle}$
- assume a silent logical constant: R_O in $ME_{\langle\langle e,\langle e,t\rangle\rangle,\langle e,t\rangle\rangle}$
- a function h s.t. for all $k \in D_{\langle e,\langle e,t\rangle\rangle}$ and all $d \in D_{\langle e\rangle}$ $h(k)(d) = 1$ iff there is some $d' \in D_{\langle e\rangle}$ s.t. k(d′)(d)=1
- passives as similar subject deletion

All there is to λ

- a new variable binder
- allows abstraction over wff's of arbitrary complexity
- similar to $\{x \| \phi\}$ (read as 'the set of all x s.t. ϕ')
- we get $\lambda x\,[\phi]$
- on Montague's typewriter: $\hat{x}\,[\phi]$
- does not create a set but a function which can be taken as the CF of a set

- for every wff ϕ, any $x \in Var$, and any $a \in Con$
- λ abstraction: $\phi \to \lambda x \left[\phi^{[a/x]}\right] (a)$
- read $\phi^{(a/x)}$ as '*phi* with every *a* replaced by *x*'
- *x* can be of any type

Two informal examples

- $\lambda x_{\langle e\rangle}\,[L(x)]$ is the characteristic function of the set of those individuals $d \in D_{\langle e\rangle}$ which have property L
- $\lambda x_{\langle e,t\rangle}\,[x(l)]$ is the characteristic function of the set of those properties $k \in D_{\langle e,t\rangle}$ that the individual l has

λ conversion

- $\lambda x\,[L(x)]$ is the abstract of $L(a)$ (with some individual a)
- hence, it holds: $\lambda x\,[L(x)]\,(a) \Leftrightarrow L(a)$
- for every wff ϕ, any $x \in Var$, and any $a \in Con$
- λ conversion: $\lambda x\,[\phi]\,(a) \rightarrow \phi^{[x/a]}$

λ in and out

- $\lambda x\,[\phi]\,(a) \leftrightarrow \phi^{[x/a]}$
- not just syntactically, since truth conditions are equivalent
- $\lambda x\,[\phi]\,(a) \Leftrightarrow \phi^{[x/a]}$
- notice: $\lambda x_{\langle\alpha\rangle}\,[\phi]$ is in $ME_{\langle\alpha,t\rangle}$
- while ϕ (as a wff) is in $ME_{\langle t\rangle}$

The full rules

- Dowty et al., 102f. (*Syn C.10* and *Sem 10*)
- If $\alpha \in ME_\alpha$ and $u \in Var_b$, then $\lambda u\,[\alpha] \in ME_{\langle b,a \rangle}$.
- If $\alpha \in ME_a$ and $u \in Var_b$ then $[\![\lambda u\,[\alpha]]\!]^{\mathcal{M},g}$ is that function h from D_b into D_a s.t. for all objects k in D_b, $h(k)$ is equal to $[\![\alpha]\!]^{\mathcal{M},g[k/u]}$.

- $\forall x \forall v_{0\langle e,t\rangle} \left[(\mathbf{non}(v_{0_{\langle e,t\rangle}}))(x) \leftrightarrow \neg(v_{0_{\langle e,t\rangle}}(x)) \right]$
- $\forall v_{0_{\langle e,t\rangle}} \left[\lambda x \left[(\mathbf{non}(v_{0_{\langle e,t\rangle}}))(x) \right] = \lambda x \left[\neg(v_{0_{\langle e,t\rangle}}(x)) \right] \right]$
- $\forall v_{0_{\langle e,t\rangle}} \left[\mathbf{non}(v_{0_{\langle e,t\rangle}}) = \lambda x \left[\neg(v_{0_{\langle e,t\rangle}}(x)) \right] \right]$
  (since $\lambda x\, [\mathbf{non}(v)(x)]$ is unnecessarily abstract/η reduction)
- $\lambda v_{0_{\langle e,t\rangle}} \left[\mathbf{non}(v_{0_{\langle e,t\rangle}}) = \lambda v_{0_{\langle e,t\rangle}} \left[\lambda x \left[\neg(v_{0_{\langle e,t\rangle}}(x)) \right] \right] \right]$
- and since that is about all assignments for $\lambda v_{0_{\langle e,t\rangle}}$:
  $\mathbf{non} = \lambda v_{0_{\langle e,t\rangle}} \left[\lambda x \left[\neg v_{0_{\langle e,t\rangle}}(x) \right] \right]$

Mary is non-adjacent.

(translate 'adjacent' as $c_{0_{\langle e,t \rangle}}$, 'Mary' as $c_{0_{\langle e \rangle}}$, ignore the copula)

The behavior of quantified NPs

- syntactically like referential NPs
- semantically like PC quantifiers
- *Every student walks.*: $\forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \rightarrow c_{1_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \right]$
- *Some student walks.*: $\forall v_{0_{\langle e \rangle}} \left[c_{0_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \wedge c_{1_{\langle e,t \rangle}}(v_{0_{\langle e \rangle}}) \right]$
- making referential NPs and QNPs the same type?

A higher type

- $\lambda v_{0_{\langle e,t\rangle}} \forall v_{0_{\langle e\rangle}} \left[c_{0_{\langle e,t\rangle}}(v_{0_{\langle e\rangle}}) \rightarrow v_{0_{\langle e,t\rangle}}(v_{0_{\langle e\rangle}}) \right]$
- a second order function
- characterizes the set of all predicates true of every student
- equally: $\lambda v_{0_{\langle e,t\rangle}} \exists v_{0_{\langle e\rangle}} \left[c_{0_{\langle e,t\rangle}}(v_{0_{\langle e\rangle}}) \wedge v_{0_{\langle e,t\rangle}}(v_{0_{\langle e\rangle}}) \right]$

Combining with some predicate

Intensionalität

Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Some examples

- Stockhausen will write another opera.
- Had Arno Schmidt cut down on drinking, he would still be alive.
- Gustave Moreau believes that estheticism rules.

Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (tense)
- … and for just one state of affairs (modals, believe type verbs)

What are intensions?

Type	Reference	Sense
NP	individuals *Venus*	individual concepts
VP	sets *humming birds*	property concepts
S	1 or 0 *I like cats.*	thoughts or **propositions**

Properties of intensions

- can't be just truth conditional
- encode knowledge about not just the actual but all possible and/or past/future states of affairs (PSOAs)
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values

PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just thruth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

A touch of parellel universes?

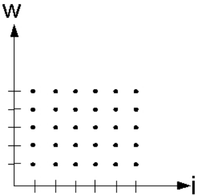
- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
  - A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
  - nothing of the above, but Maria was born 20 years earlier
  - nothing of the above, but A.S. rose from the dead in 2003, etc.

Propositions and PSOAs

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

Coordinates

- for each possible distinction in truth values of the whole of the propositional sentences: one possible world ($w \in W$)
- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$

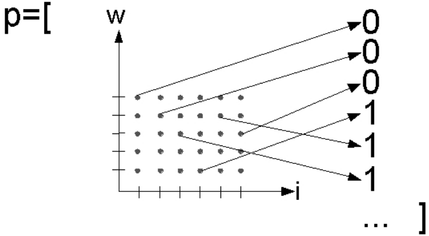


The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

Propositions as functions

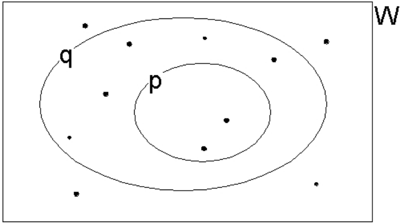
- a propositional function *p*
- is a function from $W \times I$ to $\{0, 1\}$



Your evening prayer

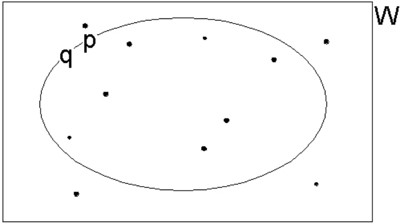
- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

- defintion of intensions of sentences (propositions): characteristic functions
- equivalently: propositions are sets of possible worlds
- entailment turns out as a subset-relation: $p \subseteq q$:



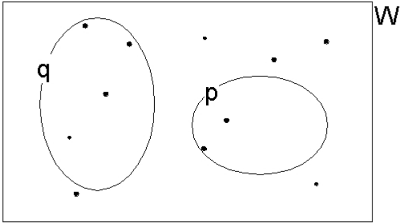
Synonymy

- synonymy turns out as set equivalence:
- $p = q$



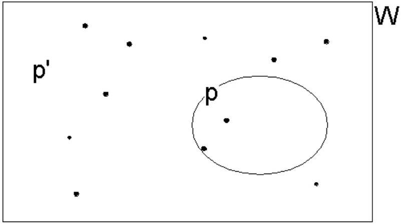
Contradiction

- contradiction turns out as an empty intersection:
- $p \cap q = \emptyset$



Negation

- negation turns out as a complement:
- p/W

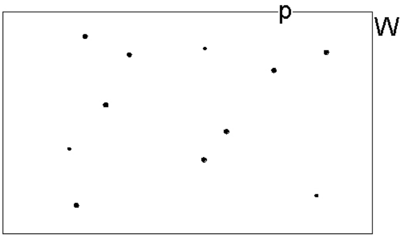


Quantification over worlds

- new modal sentence/wff operators:
  - *necessarily p*: $\Box \mathbf{p}$
  - *possibly p*: $\Diamond \mathbf{p}$
- What does it mean for a proposition to be necessary/possible?

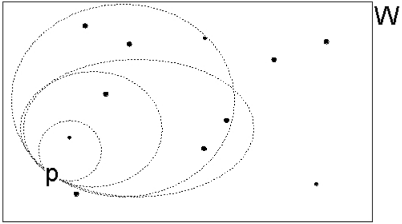
Necessity as universal quantification

- if $\Box p$ then $(\forall w)\ [p(w) = 1]$ (p as characteristic function)
- such that $W = p$ (p as set):



Possibility as existential quantification

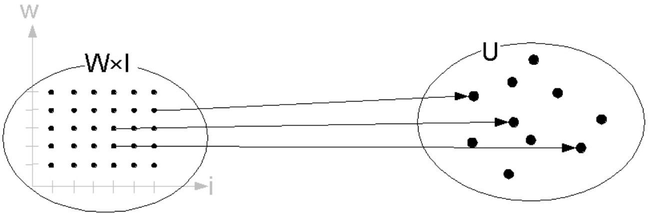
- if $\Diamond p$ then $(\exists w)\,[p(w) = 1]$ (characteristic function)
- such that $p \neq \emptyset$ (set):



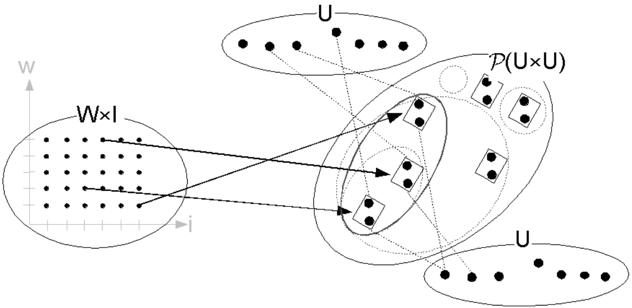
A larger tuple

- $\mathcal{M} = \{W, I, <, U, V\}$
  - W, a set of worlds
  - I, a set of instants
  - $<$, an ordering relation in I
  - U, the set of individuals
  - V, a valuation function for constants
- evaluate an expression α: $[\![\alpha]\!]^{\mathcal{M},w,i,g}$

- *the President of the United States*, *the Pope*, *Bond* (in the sense of ‘the actor currently playing Bond’)
- *for* $\beta \in Cons_{ind}$, $V(\beta)$ is a function from $W \times I$ to U



- *walks* etc. denotes different sets (or CFs) at different $\langle w, i \rangle$ coordinates
- *for* $\beta \in Cons_{pred_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ $(U^n = U_1 \times U_2 \times \ldots \times U_n)$



- simple sentences/predicates: $\beta = \delta(t_1, t_2, \ldots, t_n)$
- $[\![\beta]\!]^{\mathcal{M},w,i,g} = 1$ iff
- $\langle [\![t_1]\!]^{\mathcal{M},w,i,g}, [\![t_2]\!]^{\mathcal{M},w,i,g}, \ldots, [\![t_n]\!]^{\mathcal{M},w,i,g} \rangle \in [\![\delta]\!]^{\mathcal{M},w,i,g}$
- with: $[\![t_1]\!]^{\mathcal{M},w,i,g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

Quantification

- if $\psi = \forall x \phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $u \in U$
- ...$[\![\phi]\!]^{\mathcal{M},w,i,g[u/x]} = 1$
- nothing new here

Modalities

- if $\psi = \Box x \phi$ then
- ...$[\![\psi]\!]^{\mathcal{M},w,i,g} = 1$ iff for all $w' \in W$
- ...and all $i' \in I$
- ...$[\![\phi]\!]^{\mathcal{M},w',i',g} = 1$

A similarity of $\forall$ and $\Box$

- as: $\forall x\,[P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that: $\Box\,[\psi \rightarrow \phi] \rightarrow [\Box\psi \rightarrow \Box\phi]$
- but not vice-versa!

Some validities

- $\exists x \Box P(x) \rightarrow \Box \exists x P(x)$
- $\exists x \Diamond P(x) \leftrightarrow \Diamond \exists x P(x)$
- $\forall x \Box P(x) \leftrightarrow \Box \forall x P(x)$ (Carnap-Barcan)
- $\forall x \Diamond P(x) \rightarrow \Diamond \forall x P(x)$

Tempus und Modalität

Targets for this week

- Understand how simple tense logic can be represented by operators shifting *i* indices.
- See why tense operators are sentence operators.
- See why a multi-dimensional theory of tenses and a better handling of tense embedding are required.
- See how we restrict (different types of) propositional backgrounds.
- Understand how opaque contexts affect meaning (incl. *believe* type verbs).
- Get a first idea of why we need the *up* operator $\hat{}$.

Will, was... and always

- present: no operator (ϕ 'it is the case that ϕ')
- past: $\mathbf{P}$ ($\mathbf{P}\phi$ 'it was the case that ϕ')
- future: $\mathbf{F}$ ($\mathbf{F}\phi$ 'it will be the case that ϕ')
- it will always be the case... ($\mathbf{G} = \neg\mathbf{F}\neg\phi$)
- it was always the case... ($\mathbf{H} = \neg\mathbf{P}\neg\phi$)

Evaluation

- $\mathbf{P}D(a)$ 'Arno Schmidt (has?) died.'
- relative to the current $\langle w, i \rangle$: $[\![\mathbf{P}D(a)]\!]^{\mathcal{M},w,i,g}$
- ...is true iff there is some i', $\langle i', i \rangle \in <$ and
- $[\![\mathbf{P}D(a)]\!]^{\mathcal{M},w,i',g} = 1$

Like it or not...

- tense operators (TOp) are sentence (wff) Op's
- raise it to sentence-scopal position
- TP/IP position is motivated by copular/auxiliary elements
- *He **is** stupid.* vs. *Kare-wa bakarashi-**i**.*
- *He **was** stupid.* vs. *Kare-wa bakarashi-**katta**.*
- *What$_i$ **did** you expect t$_i$?* vs. *Nani-o yokishi-**ta**-ka.*

New ps rules

- $T' \rightarrow T VP$ (adds tense to VP)
- $TP \rightarrow NP\ T'$
- $TP \rightarrow TP\ conj\ TP$
- $TP \rightarrow neg\ TP$
- $[_{TP}\ NP\ T\ VP] \Rightarrow [_{TP}\ T\ NP\ VP]$ (T raising)

Quantification over instants

- $[\![\mathbf{P}\mathit{TP}]\!]^{\mathcal{M},w,i,g} = 1$
- iff among all $\langle i_n, i \rangle \in <$
- there is at least one s.t. $[\![\mathit{TP}]\!]^{\mathcal{M},w,i',g} = 1$

Valuations as in Chierchia's M_3

- *U*: domain of quantification
- $V(\beta)$: non-relativized function for all β which are not a proper name
- $V(\beta)(\langle w, i\rangle)$: V valuates β to a function from world-time pairs to the denotata of the predicate (sets of individuals, tuples of them, etc.)

Natural tenses

- NL tenses beyond TOp's:
- *Arno Schmidt had already read Poe when he started writing 'Zettels Traum'.*
- *Gosh, I forgot to feed the cat.*
- shifts of evaluation time

Reichenbach

	past (R<S)	present (R,S)	future (S<R)
anterior(E<R)	E<R<S *er war gegangen*	E<R,S *er ist gegangen*	S<E<R S,E<R E<S<R *er wird gegangen sein*
simple(E,R)	E,R<S *er ging*	E,R,S *er geht*	S<E,R *er wird gehen*
posterior(R<E)	R<E<S R<S,E R<S,E R<S<E **er würde gehen*	R,S<E *er wird gehen*	S<R<E **er wird gehen werden*

Embedded tenses and adverbials

- *A man was born who will be king.*
- **P**(a man is born **F**(who be king)) ?
- *Yesterday, Maria woke up happy.*
- **Y**(**P**(Maria wake up happy)) ?

Types of modal expressions

- tense forms: *I eat up to 100 nachos a minute.*
- mood: *Responderet alius minus sapienter.*
- modal auxiliaries: *Herr Webelhuth can look like Michael Moore.*
- adverbs: *Maybe Herr Keydana will show up.*
- affixes: *Frau Eckardt is recognizable.*

The logical form of modal operators

- like tense: sentence operators
- modal *Aux* in English is tense-insensitive (evidence for *Infl*)
- $\Box$ and $\Diamond$ in intensional predicate calculi (IPC): exploit the full set of possible worlds
- in NL: evaluation of modal expressions against restricted conversational backgrounds

The background

- different sets of possible worlds under consideration for different types of modal expressions
- different types of modality: different sets of admitted possible worlds
- we call the conversationally relevant background the set of $\langle w, i \rangle$ pairs relevant to the interpretation of the sentence

Root/Logical modality

- *Agent Cooper cannot solve the mystery.*
- translated into root modal IPC: $\neg\Diamond S(c, m)$
- wrong interpretation: Under no possible circumstances can Cooper solve the mystery.
- usually, some obvious facts constitute the background:
  - he could, but some relevant information is missing
  - he could, but is sick
  - he could, but ...

Epistemic modality

- *Leo Johnson must be the murderer of Laura Palmer.*
- in accordance with the known facts (e.g., in episode 7 of *Twin Peaks*):
  - Leo Johnson is a violent person.
  - Leo smuggles cocaine, Laura was addicted to it.
  - Leo is connected to Jacques Renault who is the bartender of *One Eyed Jack's* where Laura worked as a prostitute.
  - ...
- which constitute the epistemic background, the sentence is true
- known facts narrow down the root background

Deontic modality

- *Agent Cooper must not solve the mystery.*
- assume:
  - there is some U.S. law which allows a local sheriff to ask the FBI to keep out of local murder investigations
  - Sheriff Truman has asked the FBI headquarters to keep out of the Palmer investigation
  - as a special agent, Cooper is required to obey Bureau policy
- Deontic backgrounds are narrowed down by normative rules and moral ideals.
- statable in propositional form (ten commandments, law, ...)

Sets of propositions

- specify the kind of background against which you evaluate under the given situation
- we need:
  a function from $\langle w, i\rangle$ to the relevant background set of $\langle w_n, i_m\rangle$
- reuse g:
  $g(\langle w, i\rangle) = \{p_1, p_2, \ldots, p_n\} = \{\langle w, i\rangle_1, \langle w, i\rangle_2, \ldots, \langle w, i\rangle_n\}$
- such that all possible worlds are: $\bigcap g(\langle w, i\rangle)$

CP structures: *that*

- *that* is a complementizer, it turns a sentence into an argument.
- ps rule: *CP* $\rightarrow$ *C IP*
- *[$_{IP}$ Racine believes [$_{CP}$ that [$_{IP}$ theatre rules]]]*
- CP (fully fledged sentence) receives theta role by *believe* under government.

Weak *Infl* and *PRO*

- gerunds:
  [$_{IP}$ Stockhausen has plans [$_{IP}$ to write another 29 hour opera]]
- incomplete embedded IP, no subject
- internal theta role of *has plans*: to IP
- external theta role of *write*: to ?
- PRO, controlled by the subject of *has plans*:
  [$_{IP}$ Stockhausen has plans [$_{IP}$ PRO to write another 29 hour opera]]

Propositional attitudes

- verbs like *believe*: propositional attitude verbs
- content of the believe: a pice of information held to be true by the believer, hence a proposition, a $\langle w_n, i_m \rangle$
- signalling one element in the background assumed by the believer
- belief: $\langle w, i \rangle$ is an element of the proposition of CP

Translating *that* as ^

- value of propositional attitude (PA) verbs: functions $[\langle w, i\rangle \rightarrow \langle u_n, p\rangle]$ with $u_n \in U$, p a proposition (set of $\langle w_n, i_m\rangle$) and compatible to u$_n$'s background
- *up* ($^\wedge\chi$): an operator which gives the intension of an expression χ
- the full logic of $^\wedge$ and $^\vee$ as designed by Montague next week
- $^\wedge$rids us of the problem that the belief content looks truth-conditional (a sentence) but doesn't contribute to the embedding sentence's truth-value. PA verbs take intensions as arguments.

Meet B.J. Ortcutt

- Quine's story: Ralph knows...
- Bernard J.Ortcutt, the nice guy on the beach.
- He sees a strange guy with a hat in the dark alley - a spy?
- Ortcutt just likes to behave funny on the way to his pub...
- and actually is sinister guy in the alley!
- Only Ralph doesn't know.

Is Ralph insane?

- What's the truth value of...
- *Ralph believes that the guy from the beach is a spy.*
- true: since Ortcutt and the guy in the hat are one individual
- false: since Ralph doesn't know that and in a way 'doesn't believe it'

- the Russelian interpretation for *the* like $\exists$ with a uniqueness condition (as a GQ): $\lambda Q \lambda P\, [\exists x\, [Q(x) \wedge P(x)] \wedge \forall y\, [Q(y) \leftrightarrow y = x]]$
- in a raising framework: ambiguity between *THE* and *believe*
- $[_{IP}$ the guy from the beach$_i$ $[_{IP}$ Ralph believes $[_{CP}$ that x_i is a spy$]]]$
- makes the sentence true: the *de re* reading
- Ralph believes $[_{CP}$ that $[_{IP}$ the guy from the beach$_i$ $[_{IP}$ x_i is a spy$]]]$
- makes the sentence false: the *de dicto* reading

Rigid designators

- *Yuri Gagarin might now have been the first man in space.*
- some Mickey Mouse LFs:
- $\Diamond$ THE(first-man-in-space)(not-be-Gagarin)
- at some $\langle w_n, i_m \rangle$ the first individual in space is not Y.G.
- THE(first-man-in-space)($\Diamond$[not-be-Gagarin])
- at $\langle w, i \rangle$ the first individual in space (definitely Y.G.) is not Y.G. in an accessible world
- Names are rigid designators across world-time-pairs, definite descriptions aren't.

Chierchia's formalization

- CP has its own subject, *to*-IPs don't (PRO)
- PRO must be interpreted, in our examples by coindexation with the matrix subject
- infinitive embedding verbs: functions from world-time pairs to sets of individuals which have a certain property, the intension of a predicate $^\wedge P$
- *John tries to sing.*
- $try(j, {}^\wedge swim)$

Montagues intentionale Logik

Beyond truth functionality

- $[\![\phi]\!]^{\mathcal{M},w,i,g}$ and $[\![\mathbf{P}]\!]^{\mathcal{M},w,i,g}$ don't truth conditionally determine $[\![\mathbf{P}\phi]\!]^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**, $\Diamond$, $\Box$ are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models
- for a name d: $[\![d]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & b \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_2\rangle & \rightarrow & b \\ \langle w_1, t_3\rangle & \rightarrow & b \\ \langle w_2, t_3\rangle & \rightarrow & b \end{bmatrix}$

- for an individual concept denoting expression *m*:
- $[\![m]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & c \\ \langle w_1, t_3 \rangle & \rightarrow & c \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for a one place predicate *B*:
- $[\![B]\!]^{\mathcal{M},g}_{¢} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{bmatrix}$

Intensions of formulas

- formula ϕ: $[\![\phi]\!]^{\mathcal{M},g}_{\not c}$ is a function from indices to truth values
- $[\![B(m)]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & 1 \\ \langle w_2, t_1\rangle & \rightarrow & 1 \\ \langle w_1, t_2\rangle & \rightarrow & 0 \\ \langle w_2, t_2\rangle & \rightarrow & 0 \\ \langle w_1, t_3\rangle & \rightarrow & 1 \\ \langle w_2, t_3\rangle & \rightarrow & 1 \end{bmatrix}$
- $[\![B(n)]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & 0 \\ \langle w_2, t_1\rangle & \rightarrow & 1 \\ \langle w_1, t_2\rangle & \rightarrow & 1 \\ \langle w_2, t_2\rangle & \rightarrow & 0 \\ \langle w_1, t_3\rangle & \rightarrow & 1 \\ \langle w_2, t_3\rangle & \rightarrow & 1 \end{bmatrix}$

Intensions of formulas

- again, the proposition $[\![Bm]\!]_{\not{c}}^{\mathcal{M},g}$ is a set of indices ($\langle w_i, t_j \rangle$)
- from the extension at all indices, compute the intension
- $[\![\alpha]\!]_{\not{c}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

Intensions of variables

- constant function on indices
- will play a great role, so remember!
- $[\![u]\!]_{\not{c}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes they denote intensions (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

Up and down

- Church/Montague: for an extension-denoting expression α, $\hat{}\alpha$ denotes α's intension
- $[\![\hat{}Bm]\!]^{\mathcal{M},w,i,g} = [\![Bm]\!]^{\mathcal{M},g}_{\cancel{c}}$
- α and $\hat{}\alpha$ are just denoting expressions
- for an intension-denoting expression α: $[\![\check{}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w, t\rangle)$

Down-up and up-down

- observe: $\llbracket {}^{\vee}{}^{\wedge}\alpha \rrbracket^{\mathcal{M},w,i,g} = \llbracket \alpha \rrbracket^{\mathcal{M},w,i,g}$ for any $\langle w, t \rangle$
- but not always: $\llbracket {}^{\wedge}{}^{\vee}\alpha \rrbracket^{\mathcal{M},w,i,g} = \llbracket \alpha \rrbracket^{\mathcal{M},w,i,g}$ for any $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

Non-equality

- *k*' intension: $[\![k]\!]^{\mathcal{M},g}_{\not{c}} = \left[\begin{array}{lll} \langle w_1,t_1\rangle & \rightarrow & \left[\begin{array}{lll} \langle w_1,t_1\rangle & \rightarrow & a \\ \langle w_1,t_2\rangle & \rightarrow & a \\ \langle w_2,t_1\rangle & \rightarrow & a \\ \langle w_2,t_2\rangle & \rightarrow & a \end{array}\right] \\ \langle w_1,t_2\rangle & \rightarrow & \left[\begin{array}{lll} \langle w_1,t_1\rangle & \rightarrow & a \\ \langle w_1,t_2\rangle & \rightarrow & b \\ \langle w_2,t_1\rangle & \rightarrow & c \\ \langle w_2,t_2\rangle & \rightarrow & d \end{array}\right] \\ \langle w_2,t_1\rangle & \rightarrow & \left[\begin{array}{lll} \langle w_1,t_1\rangle & \rightarrow & c \\ \langle w_1,t_2\rangle & \rightarrow & b \\ \langle w_2,t_1\rangle & \rightarrow & d \\ \langle w_2,t_2\rangle & \rightarrow & a \end{array}\right] \\ \langle w_2,t_2\rangle & \rightarrow & \left[\begin{array}{lll} \langle w_1,t_1\rangle & \rightarrow & c \\ \langle w_1,t_2\rangle & \rightarrow & d \\ \langle w_2,t_1\rangle & \rightarrow & a \\ \langle w_2,t_2\rangle & \rightarrow & b \end{array}\right] \end{array}\right]$

Non-equality

- *k*' extension (e.g., at $\langle w_1, t_2\rangle$): $\llbracket k \rrbracket_{\not{g}}^{\mathcal{M},g}(\langle w_1, t_2\rangle) =$
- $\llbracket k \rrbracket^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & c \\ \langle w_2, t_2\rangle & \rightarrow & d \end{bmatrix}$
- however: $\llbracket {}^{\sim}k \rrbracket^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & d \\ \langle w_2, t_2\rangle & \rightarrow & b \end{bmatrix}$
- since: $\llbracket {}^{\smile}k \rrbracket^{\mathcal{M},w_1,t_1,g} = a$
  $\llbracket {}^{\smile}k \rrbracket^{\mathcal{M},w_1,t_2,g} = b$
  $\llbracket {}^{\smile}k \rrbracket^{\mathcal{M},w_2,t_1,g} = d$
  $\llbracket {}^{\smile}k \rrbracket^{\mathcal{M},w_2,t_2,g} = b$

A typed higher order λ language with = and ^ / ˇ

- $\neg,\ \wedge,\ \vee,\ \rightarrow,\ \leftrightarrow,\ \mathbf{F},\ \mathbf{P},\ \Box,\ =$ (syncategorematically)
- $t, e \in Type$ (Con_{type}, Var_{type})
- if $a, b \in Type$, then $\langle a, b \rangle \in Type$
- if $a \in Type$, then $\langle s, a \rangle \in Type$
- $s \notin Type$

Meaningful expressions

- ME_{type}
- abstraction: if $\alpha \in ME_a$, $\beta \in Var_b$, $\lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if $\alpha \in ME_{\langle a,b \rangle}$, $\beta \in ME_a$ then $\alpha(\beta) \in ME_b$
- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

Interpretations of ^ and ˇ

- if $\alpha \in ME_a$ then $\hat{}\alpha \in ME_{s,a}$
- if $\alpha \in ME_{\langle s,a \rangle}$ then $\check{}\alpha \in ME_a$
-

type	variables	constants
e	x, y, z	a, b, c
$\langle s, e \rangle$	x, y, z	—
$\langle e, t \rangle$	X, Y	$walk', A, B$
$\langle\langle s, e \rangle, t \rangle$	Q	$rise', change'$
$\langle s, \langle e, t \rangle\rangle$	P	—
$\langle e, e \rangle$	P	Sq
$\langle e, \langle e, t \rangle\rangle$	R	Gr, K
$\langle e, \langle e, e \rangle\rangle$	—	$Plus$

The model

- $\langle A, W, T, <, F \rangle$
- $D_{\langle a,b \rangle} = {D_b}^{D_a}$
- $D_{\langle s,a \rangle} = {D_a}^{W \times T}$
- ‘senses’ = **possible** denotations
- actual intensions chosen from the set of senses
- now: F(expression)=intenstion (itself a function)
- s.t. intension(index)=extention
- instead of: F(expression)(index)=extemsion

Some interpretations

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g},\ u \in Var_b,\ \alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with g' exactly like g except $g'(u) = x$
- $[\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}$ is a function h from $W \times T$ to denotations of α's type s.t. at every $\langle w', t' \rangle \in W \times T$ $[\![\alpha]\!]^{\mathcal{M},w',t',g} = h(\langle w', t' \rangle) = [\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}(\langle w', t' \rangle)$

Some examples

- $\alpha = \beta$ at $\langle w, t \rangle$ might be true, but $\hat{}\alpha = \hat{}\beta$ need not be 1 at that same index
- on types:
  - e - individuals
  - $\langle s, e \rangle$ - individual concepts ('present Queen of England')
  - $\langle s, \langle e, t \rangle \rangle$ - properties of inidviduals
  - $\langle e, t \rangle$ - sets of individuals
  - $\langle \langle s, e \rangle, t \rangle$- sets of individual concepts

Some examples

- on properties:
  - $\langle s, \langle a, t \rangle \rangle$ - properties of denotations of a-type expressions
  - $\langle s, \langle e, t \rangle \rangle$ - properties of individuals
  - $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

On indices

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence: $\langle s, a \rangle$ never applied to some typed argument (s is not a type!)
- useful thing: We never talk about indices!
- since often ${}^{\vee}\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s,\langle e,t\rangle\rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

Nec

- former problem with **Nec** as $\langle t, t\rangle$: non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle\langle s,t\rangle,t\rangle}$ - $\{0,1\}^{(\{0,1\}^{W\times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give $\Box\phi$ as $\mathbf{Nec}({}^\wedge\phi)$

For

- 'former' as in 'a former member of this club'
- instead of $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
- intensionally: $\langle\langle s,\langle e,t\rangle\rangle,\langle e,t\rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e,t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not{c}}$ is a func. h s.t. for any property k, $h(\langle w,t\rangle)(k)$ is the set $k(\langle w,t'\rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w,t\rangle)(k)(x) = 1$ iff $k(\langle w,t'\rangle)(x) = 1$ for some $t' < t$.

Bel

- relations between individuals and propositions
- $\langle\langle s,t\rangle,\langle e,t\rangle\rangle$
- $\mathbf{Bel}(\hat{}(B(m))(j))$ *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $[\![B(m)]\!]^{M,w_2,t_1,g} = 1$ since $[\![m]\!]^{M,w_2,t_1,g} = [\![n]\!]^{M,w_2,t_1,g}$
- however: $[\![\hat{}(B(m))]\!]^{M,w_2,t_1,g} \neq [\![\hat{}(B(n))]\!]^{M,w_2,t_1,g}$

de dicto

- $\mathbf{Bel}(\hat{}(B(m))(j))$ ‘John believes that Miss America is bald.’
- $\mathbf{Bel}(\hat{}(B(n))(j))$ ‘John believes that Norma is bald.’
- needn’t be equal: John can take worlds other than $\langle w_2, t_1 \rangle$ into account where $[\![n]\!] \neq [\![m]\!]$
- $\alpha = \beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$ is true iff α is not in the scope of $\hat{}, \mathbf{F}, \mathbf{P}, \Box$ (oblique contexts)
- however: $\hat{}\alpha = \hat{}\beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$

- like so: $\lambda x\,[\mathbf{Bel}(\hat{}\,[B(x)])(j)]\,(m)$
- the above is true at an index $\langle w,t\rangle$ iff $[\![\mathbf{Bel}(\hat{}\,[B(x)])(j)]\!]^{w,t}=1$ if $[\![m]\!]^{w,t}=x$, i.e. if John is in a believe-rel with $\hat{}(B(x))$ s.t. $g(x)=m$ (by semantics of λ)
- Why is $\hat{}(B(x))$ not equal to $\hat{}(B(m))$?
- constant *m*: non-rigid designator relativized to indices
- variable *x*: a rigid designator by def. of *g* (for the relevant checking case with $g(x)=\mathit{MissAmerica}$
- the above: a belief about 'whoever *m* is'
- λ conversion is restricted in IL!

Once again

- *John believes that a republican will win.*
- $\exists x\,[Rx \wedge \mathbf{Bel}(j, \hat{}\,[\mathbf{F}W(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x\,[R(x) \wedge W(x)])$

Literatur I

Bucher, Theodor. 1998. *Einführung in die angewandte Logik*. 2. Aufl. Bd. 2231 (Sammlung Göschen). Berlin: de Gruyter.

Carpenter, Bob. 1997. *Type-logical semantics*. Cambridge, MA: MIT Press.

Chierchia, Gennaro & Sally McConnell-Ginet. 2000. *Meaning and grammar: An introduction to semantics*. 2. Aufl. Cambridge, MA: MIT Press.

Dowty, David R., Robert E. Wall & Stanley Peters. 1981. *Introduction to Montague semantics*. Dordrecht: Kluwer.

Gutzmann, Daniel. 2019. *Semantik: Eine Einführung*. Stuttgart: Metzler.

Partee, Barbara, Alice ter Meulen & Robert E. Wall. 1990. *Mathematical methods in linguistics*. Dordrecht: Kluwer.

Rosch, Eleanor. 1973. Natural categories. *Cognitive Psychology* 4(3), 328–350.

Autor

Kontakt

Prof. Dr. Roland Schäfer
Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena
Fürstengraben 30
07743 Jena

https://rolandschaefer.net
roland.schaefer@uni-jena.de

Lizenz