

Formale Semantik

04. Aussagenlogik

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

- 1 What logic is about
 - On reasoning
 - Where we need logic
- 2 Statement calculus

- Formalization: Recursive Syntax
- Interpretation
- Laws of the PropC
- Rules of Inference
- Proof

What logic is about

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- rechecking for usability: e.g., Russell's paradox

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- **lemma**: subsidiary propositions (used to prove the theorem)
- **corollary**: propositions proved while proving some axiom

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- logic does not generate truths
- formalizing statements, predications etc.
- rules of deduction from axioms to theorems
- empirical (induction) and exact (deduction) science
- aiming at an adequate model of the world (e.g., heliocentric universe)

Why logic for semantics?

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Why logic for semantics?

- truth-conditional
- compositional behavior of propositions and connectives
- a logic for entailments
- why, e.g.: *It is not the case that someone is happy.* \rightarrow *Nobody is happy.*

Statement calculus

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- $\llbracket k \rrbracket = 1$ or 0 (depending on corresponding **model**)

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 - ▶ $\neg p$ (negation - 'not')
 - ▶ $p \vee q$ (disjunction - 'or')
 - ▶ $p \wedge q$ (conjunction - 'and')
 - ▶ $p \rightarrow q$ (conditional - 'if')
 - ▶ $p \leftrightarrow q$ (biconditional - 'iff')

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- standard definition:

$$\llbracket \neg \rrbracket = \begin{bmatrix} 1 \rightarrow 0 \\ 0 \rightarrow 1 \end{bmatrix}$$

Functions and truth tables

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- but most widely used: **truth tables**

\neg	p
0	1
1	0

Disjunction

p	\vee	q
1	1	1
1	1	0
0	1	1
0	0	0

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- $K \vee L$

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Conditional

p	\rightarrow	q
1	1	1
1	0	0
0	1	1
0	1	0

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- $R \rightarrow S$

Any problems with that?

If it rains, the streets get wet.

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- ex vero non sequitur falsum

Biconditional

p	\leftrightarrow	q
1	1	1
1	0	0
0	0	1
0	1	0

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- ***If and only if*** your score is above 50, ***then*** you pass the semantics exam.

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- $S \leftrightarrow P$

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- or set non-default functor scope
- default scope



An example

- $p \wedge \neg q \vee r \rightarrow \neg s$

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- $p \wedge (\neg q) \vee r \rightarrow (\neg s)$

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Large truth tables

- for n atoms in the term: 2^n lines

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- for n atoms in the term: 2^n lines
- alternating blocks of 1's and 0's under every atom
- $2^{(m-1)}$ times '1' followed by $2^{(m-1)}$ times '0' for the m -th atom from the right
- until 2^n lines are reached

An example

p	\wedge	\neg	q	\vee	r	\rightarrow	\neg	s
1			1		1			1
1			1		1			0
1			1		0			1
1			1		0			0
1			0		1			1
1			0		1			0
1			0		0			1
1			0		0			0
0			1		1			1
0			1		1			0
0			1		0			1
0			1		0			0
0			0		1			1
0			0		1			0
0			0		0			1
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p	\wedge	\neg	q	\vee	r	\rightarrow	\neg	s
1		0	1		1		0	1
1		0	1		1		1	0
1		0	1		0		0	1
1		0	1		0		1	0
1		1	0		1		0	1
1		1	0		1		1	0
1		1	0		0		0	1
1		1	0		0		1	0
0		0	1		1		0	1
0		0	1		1		1	0
0		0	1		0		0	1
0		0	1		0		1	0
0		1	0		1		0	1
0		1	0		1		1	0
0		1	0		0		0	1
0		1	0		0		1	0

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p	\wedge	\neg	q	\vee	r	\rightarrow	\neg	s
1	0	0	1		1		0	1
1	0	0	1		1		1	0
1	0	0	1		0		0	1
1	0	0	1		0		1	0
1	1	1	0		1		0	1
1	1	1	0		1		1	0
1	1	1	0		0		0	1
1	1	1	0		0		1	0
0	0	0	1		1		0	1
0	0	0	1		1		1	0
0	0	0	1		0		0	1
0	0	0	1		0		1	0
0	0	1	0		1		0	1
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0	0	1	0		0		0	1
0	0	1	0		0		1	0

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1	0	0	1	1	1		1	0
1	0	0	1	0	0		0	1
1	0	0	1	0	0		1	0
1	1	1	0	1	1		0	1
1	1	1	0	1	1		1	0
1	1	1	0	1	0		0	1
1	1	1	0	1	0		1	0
0	0	0	1	1	1		0	1
0	0	0	1	1	1		1	0
0	0	0	1	0	0		0	1
0	0	0	1	0	0		1	0
0	0	1	0	1	1		0	1
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1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
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1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
0	0	0	1	1	1	0	0	1
0	0	0	1	1	1	1	1	0
0	0	0	1	0	0	1	0	1
0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
0	0	1	0	1	1	1	1	0
0	0	1	0	0	0	1	0	1
0	0	1	0	0	0	1	1	0

Assignments: a contingent example

p	\wedge	\neg	q	\vee	r	\rightarrow	\neg	s
1	0	0	1	1	1	0	0	1
1	0	0	1	1	1	1	1	0
1	0	0	1	0	0	1	0	1
1	0	0	1	0	0	1	1	0
1	1	1	0	1	1	0	0	1
1	1	1	0	1	1	1	1	0
1	1	1	0	1	0	0	0	1
1	1	1	0	1	0	1	1	0
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0	0	0	1	1	1	1	1	0
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0	0	0	1	0	0	1	1	0
0	0	1	0	1	1	0	0	1
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- by *law of excluded middle*: for every P , $P \vee \neg P$ is true

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- false under every assignment, called **contradictory**

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- the truth value depends on the assignment

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 $(\text{He walks or } (\text{she talks or we walk.}))$

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 - ▶ $((\text{He walks or she talks}) \text{ or we walk.}) \Leftrightarrow (\text{He walks or (she talks or we walk.)})$
- **Commutative Laws for \vee and \wedge (Comm.):**
 - ▶ $(P \vee Q) \Leftrightarrow (Q \vee P)$
 - ▶ $\text{Peter walks or Sue snores.} \Leftrightarrow \text{Sue snores or Peter walks.}$
- **Distributive Laws for $\vee \wedge$ and $\wedge \vee$ (Distr.):**
 - ▶ $(P \vee (Q \wedge R)) \Leftrightarrow ((P \vee Q) \wedge (P \vee R))$
 - ▶ $(\text{Sue snores}) \text{ and } (\text{Peter walks or we talk.}) \Leftrightarrow (\text{Sue snores and Peter walks}) \text{ or } (\text{Sue snores and we talk.})$

Laws dealing with tautology and contradiction

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Laws dealing with tautology and contradiction

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- ▶ Tautology (T): $(P \vee \neg P) \Leftrightarrow \mathbf{T}$
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- ▶ Double Negation (DN): $(\neg\neg P) \Leftrightarrow P$
- ▶ *It is not the case that Sandy is not walking.*
 \Leftrightarrow *Sandy is walking.*

- **Implication** (Impl.):

P	\rightarrow	Q	\Leftrightarrow	\neg	P	\vee	Q
1	1	1		0	1	1	1
1	0	0		0	1	0	0
0	1	1		1	0	1	1
0	1	0		1	0	1	0

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0	1	1		1	0	1	1
0	1	0		1	0	1	0

- **Contraposition (Contr.):**

P	\rightarrow	Q	\Leftrightarrow	\neg	Q	\rightarrow	\neg	P
1	1	1		0	1	1	0	1
1	0	0		1	0	0	0	1
0	1	1		0	1	1	1	0
0	1	0		1	0	1	1	0

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- DeMorgan's Laws:

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- ▶ alternatively: $\overline{P \vee Q} \Leftrightarrow \overline{P} \wedge \overline{Q}$
- ▶ $\neg(P \wedge Q) \Leftrightarrow (\neg P \vee \neg Q)$
- ▶ consequently: $\overline{\overline{P} \vee \overline{Q}} \Leftrightarrow \overline{\overline{P}} \wedge \overline{\overline{Q}} \Leftrightarrow P \wedge Q$

The Modus Ponens (MP)

- Definition:

$P \rightarrow Q$	premise 1
P	premise 2
Q	conclusion

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- (1) *If It rains, the streets get wet.* (2) *It is raining.*
 \rightarrow *The streets are getting wet.*

MP: a truth table illustration

- Premises are always set to be true!

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- the table:

P	\rightarrow	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- The conditional must be true.
- cancel the 'false' row

P	\rightarrow	Q
1	1	1
1	0	0
0	1	1
0	1	0

MP: a truth table illustration

- P must be true.
- cancel the 'false' rows, Q can only be true:

P	\rightarrow	Q
1	1	1
1	0	0
0	1	1
0	1	0

The Modus Tollens (MT)

- Definition:

P	\rightarrow	Q
		$\neg Q$
$\neg P$		

The Modus Tollens (MT)

- Definition:

P	\rightarrow	Q
		$\neg Q$
$\neg P$		

- the table illustration:

P	\rightarrow	Q	
1	1	1	(by premise 2)
1	0	0	(by premise 1)
0	1	1	(by premise 2)
0	1	0	

- Hypothetical Syllogism (HS):
 - ▶ $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
 - ▶ (1) *If it rains, the streets get wet.* (2) *If the streets get wet, it smells nice.* \rightarrow *If it rains, it smells nice.*

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- ▶ $((P \rightarrow Q) \wedge (Q \rightarrow R)) \rightarrow (P \rightarrow R)$
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- Disjunctive Syllogism (DS):

- ▶ $((P \vee Q) \wedge (\neg P)) \rightarrow (Q)$
- ▶ (1) *Either Peter sleeps or Peter is awake.* (2) *Peter isn't awake.* \rightarrow *Peter sleeps.*

- Simplification (Simp.):

- ▶ $(P \wedge Q) \rightarrow P$
- ▶ (1) *It is raining and the sun is shining.* \rightarrow *It is raining.*

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- **Addition (Add.):**
 - ▶ $(P) \rightarrow (P \vee Q)$
 - ▶ (1) *It is raining.* \rightarrow *It is raining or the sun is shining.*
 - ▶ What if Q is instantiated as true or false by another premise?

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$

A sample proof

- Prove $p \vee q$ from $(p \vee q) \rightarrow \neg(r \wedge \neg s)$ and $r \wedge \neg s$
- The proof:

		$p \vee q$
1	$(p \vee q) \rightarrow \neg(r \wedge \neg s)$	
2	$r \wedge \neg s$	
<hr/>		
	$p \vee q$	1,2,MT

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