Formale Semantik
08. Intensionalität

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

Inhalt

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- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Intensionality

Some examples

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- Gustave Moreau believes that estheticism rules.

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- ... and for just one state of affairs (modals, believe type verbs)

What are intensions?

Type	**Reference**	**Sense**
NP	individuals *Venus*	individual concepts
VP	sets *humming birds*	property concepts
S	1 or 0 *I like cats.*	thoughts or **propositions**

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- mediate between internal knowledge and truth-values

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- observe the more than just thruth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

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  - nothing of the above, but A.S. rose from the dead in 2003, etc.

A formal account of intensions

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- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

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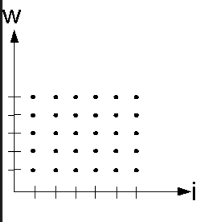
- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$



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- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

Propositions as functions

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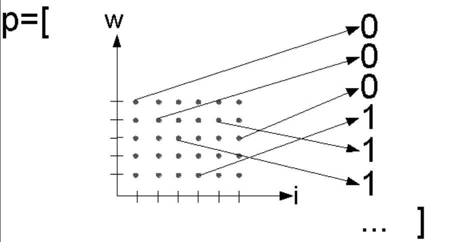
- is a function from $W \times I$ to $\{0,1\}$



Your evening prayer

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- Hence, we call that function the intension of the sentence.

Sets of worlds

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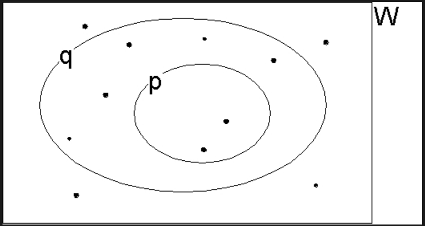
- entailment turns out as a subset-relation: $p \subseteq q$:



Synonymy

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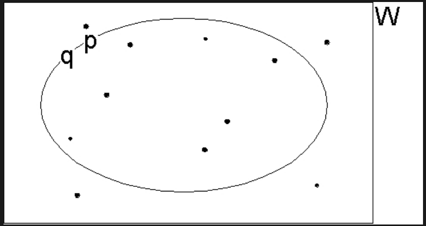
- $p = q$



Contradiction

- contradiction turns out as an empty intersection:

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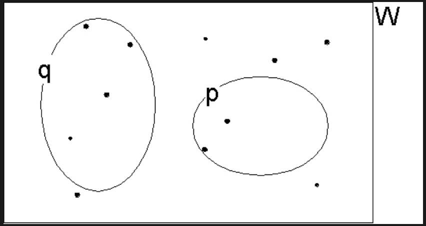
- $p \cap q = \emptyset$



Negation

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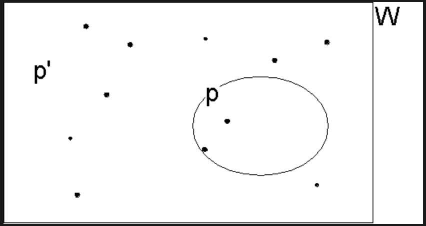
- p/W



Quantification over worlds

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- What does it mean for a proposition to be necessary/possible?

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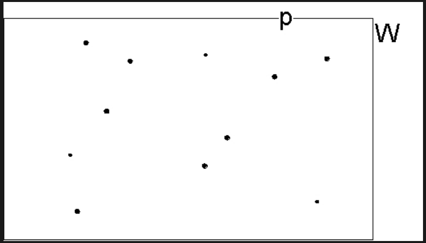
- such that $W = p$ (p as set):



Possibility as existential quantification

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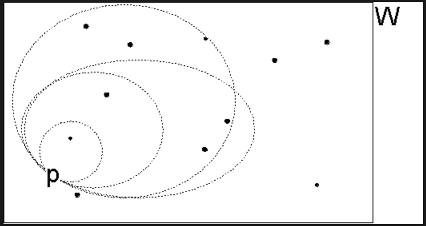
- such that $p \neq \emptyset$ (set):



Intensional Model Theory

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  - V, a valuation function for constants
- evaluate an expression α: $[\![\alpha]\!]^{\mathcal{M},w,i,g}$

Intensional interpretation of individual constants

- *the President of the United States*, *the Pope*, *Bond* (in the sense of ‘the actor currently playing Bond’)

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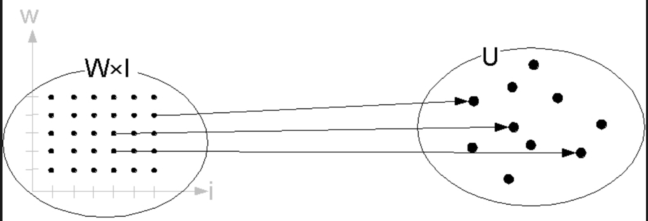

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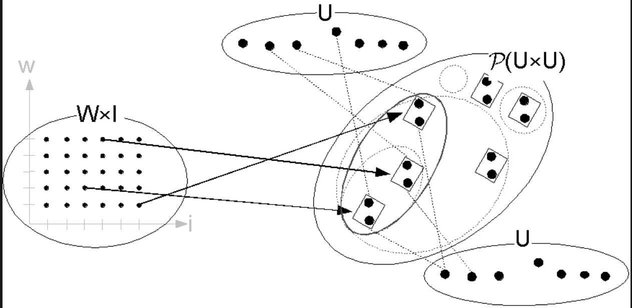
- *for* $\beta \in Cons_{pred_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ $(U^n = U_1 \times U_2 \times \ldots \times U_n)$



The Chierchia approach: predicates/sentences

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- with: $[\![t_1]\!]^{\mathcal{M},w,i,g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

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Literatur I

Autor

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Lizenz