

# Formale Semantik

## 10. Montagues intensionale Logik

Roland Schäfer

Institut für Germanistische Sprachwissenschaft  
Friedrich-Schiller-Universität Jena

Folien in Überarbeitung. Englische Teile (ab Woche 6) sind noch von 2007!  
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

- 1 New types and up/down
  - Denoting intensions
  - Technical devices

- 2 The IL of PTQ

- Syntax
- Semantics
- Technical refinements

- 3 Examples

New types and up/down

- $\llbracket \phi \rrbracket^{\mathcal{M},w,i,g}$  and  $\llbracket \mathbf{P} \rrbracket^{\mathcal{M},w,i,g}$  don't truth conditionally determine  $\llbracket \mathbf{P}\phi \rrbracket^{\mathcal{M},w,i,g}$

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- again: individual concepts (variable function on indices) vs. names (constant)

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- for a name  $d$ :  $\llbracket d \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow b \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow b \\ \langle w_1, t_3 \rangle & \rightarrow b \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

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- $\llbracket \alpha \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = \llbracket \alpha \rrbracket^{\mathcal{M},w_i,t_j,g}$

- constant function on indices

# Intensions of variables

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- $\alpha$  and  $\hat{\alpha}$  are just denoting expressions
- for an intension-denoting expression  $\alpha$ :  $\llbracket \check{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\langle w, t \rangle)$



- observe:  $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$

# Down-up and up-down

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- but not always:  $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$  for any  $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

# Non-equality

•  $k'$  intension:  $\llbracket k \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g} =$

$$\left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow \\ \langle w_1, t_2 \rangle & \rightarrow \\ \langle w_2, t_1 \rangle & \rightarrow \\ \langle w_2, t_2 \rangle & \rightarrow \end{array} \left[ \begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow a \\ \langle w_1, t_2 \rangle & \rightarrow a \\ \langle w_2, t_1 \rangle & \rightarrow a \\ \langle w_2, t_2 \rangle & \rightarrow a \\ \langle w_1, t_1 \rangle & \rightarrow a \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow c \\ \langle w_2, t_2 \rangle & \rightarrow d \\ \langle w_1, t_1 \rangle & \rightarrow c \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow d \\ \langle w_2, t_2 \rangle & \rightarrow a \\ \langle w_1, t_1 \rangle & \rightarrow c \\ \langle w_1, t_2 \rangle & \rightarrow d \\ \langle w_2, t_1 \rangle & \rightarrow a \\ \langle w_2, t_2 \rangle & \rightarrow b \end{array} \right] \right]$$

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- since:  $\llbracket \sim k \rrbracket_{\mathcal{M}, w_1, t_1, g} = a$   
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The IL of PTQ



# A typed higher order $\lambda$ language with $=$ and $\hat{\phantom{x}}/\sim$

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- $s \notin \text{Type}$

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- if  $\alpha, \beta \in ME_a$  then  $\alpha = \beta \in ME_t$

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type	variables	constants
$e$	$x, y, z$	$a, b, c$
$\langle s, e \rangle$	$x, y, z$	—
$\langle e, t \rangle$	$X, Y$	$walk', A, B$
• $\langle \langle s, e \rangle, t \rangle$	$Q$	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	$P$	—
$\langle e, e \rangle$	$P$	$Sq$
$\langle e, \langle e, t \rangle \rangle$	$R$	$Gr, K$
$\langle e, \langle e, e \rangle \rangle$	—	$Plus$

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- instead of:  $F(\text{expression})(\text{index}) = \text{extension}$

# Some interpretations

- $\llbracket \lambda u \alpha \rrbracket^{\mathcal{M}, w, i, g}$ ,  $u \in \text{Var}_b$ ,  $\alpha \in ME_a$  is a function  $h$  with domain  $D_b$  s.t.  $x \in D_b$ ,  
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- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}$  is a function  $h$  from  $W \times T$  to denotations of  $\alpha$ 's type s.t. at every  $\langle w', t' \rangle \in W \times T$   $\llbracket \alpha \rrbracket^{\mathcal{M}, w', t', g} = h(\langle w', t' \rangle) = \llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}(\langle w', t' \rangle)$

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  - ▶  $\langle \langle s, e \rangle, t \rangle$  - sets of individual concepts

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  - ▶  $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$  - properties of propositions
- from relations  $\langle e, \langle e, t \rangle \rangle$  to relations-in-intensions  $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$



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- since often  $\sim \alpha(\beta)$  is needed for  $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$  and  $\beta \in ME_e$ , abbr.  $\alpha\{\beta\}$

Examples

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- we could give  $\Box\phi$  as  $\mathbf{Nec}(\hat{\phi})$

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- $\lambda$  conversion is restricted in IL!

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## Kontakt

Prof. Dr. Roland Schäfer  
Institut für Germanistische Sprachwissenschaft  
Friedrich-Schiller-Universität Jena  
Fürstengraben 30  
07743 Jena

<https://rolandschaefer.net>  
[roland.schaefer@uni-jena.de](mailto:roland.schaefer@uni-jena.de)

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