Formale Semantik
10. Montagues intensionale Logik

Roland Schäfer

Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena

Folien in Überarbeitung. Englische Teile (ab Woche 8) sind noch von 2007!

Stets aktuelle Fassungen: https://github.com/rsling/VL-Semantik

Inhalt

New types and up/down

Beyond truth functionality

- $[\![\phi]\!]^{\mathcal{M},w,i,g}$ and $[\![\mathbf{P}]\!]^{\mathcal{M},w,i,g}$ don't truth conditionally determine $[\![\mathbf{P}\phi]\!]^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**, $\Diamond$, $\Box$ are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models
- for a name d: $[\![d]\!]^{\mathcal{M},g}_{\not{c}} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for an individual concept denoting expression *m*:
- $[\![m]\!]^{\mathcal{M},g}_{\not{c}} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & c \\ \langle w_1, t_3 \rangle & \rightarrow & c \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for a one place predicate B:
- $[\![B]\!]^{\mathcal{M},g}_{\not{c}} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{bmatrix}$

Intensions of formulas

- formula ϕ: $[\![\phi]\!]^{\mathcal{M},g}_{\not c}$ is a function from indices to truth values
- $[\![B(m)]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & 1 \\ \langle w_2, t_1\rangle & \rightarrow & 1 \\ \langle w_1, t_2\rangle & \rightarrow & 0 \\ \langle w_2, t_2\rangle & \rightarrow & 0 \\ \langle w_1, t_3\rangle & \rightarrow & 1 \\ \langle w_2, t_3\rangle & \rightarrow & 1 \end{bmatrix}$
- $[\![B(n)]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & 0 \\ \langle w_2, t_1\rangle & \rightarrow & 1 \\ \langle w_1, t_2\rangle & \rightarrow & 1 \\ \langle w_2, t_2\rangle & \rightarrow & 0 \\ \langle w_1, t_3\rangle & \rightarrow & 1 \\ \langle w_2, t_3\rangle & \rightarrow & 1 \end{bmatrix}$

Intensions of formulas

- again, the proposition $[\![Bm]\!]^{\mathcal{M},g}_{\cancel{c}}$ is a set of indices ($\langle w_i, t_j \rangle$)
- from the extension at all indices, compute the intension
- $[\![\alpha]\!]^{\mathcal{M},g}_{\cancel{c}}(\langle w_i, t_j \rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

Intensions of variables

- constant function on indices
- will play a great role, so remember!
- $[\![u]\!]^{\mathcal{M},g}_{\not{c}}(\langle w_i, t_j \rangle) = g(u)$

What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes they denote intensions (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

Up and down

- Church/Montague: for an extension-denoting expression α, $\hat{}\alpha$ denotes α's intension
- $[\![\hat{}Bm]\!]^{\mathcal{M},w,i,g} = [\![Bm]\!]^{\mathcal{M},g}_{\not{c}}$
- α and $\hat{}\alpha$ are just denoting expressions
- for an intension-denoting expression α: $[\![\check{}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w, t\rangle)$

Down-up and up-down

- observe: $[\![{}^{\vee}{}^{\wedge}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t\rangle$
- but not always: $[\![{}^{\wedge}{}^{\vee}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t\rangle$
- can easily be the case for intension-denoting expressions

Non-equality

- *k*' intension: $[\![k]\!]^{\mathcal{M},g}_{c} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_1, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix} \\ \langle w_2, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_2, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix} \end{bmatrix}$

Non-equality

- k' extension (e.g., at $\langle w_1, t_2 \rangle$): $[\![k]\!]^{\mathcal{M},g}_{\not c}(\langle w_1, t_2 \rangle) =$
- $[\![k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix}$
- however: $[\![{}^{\sim}k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix}$
- since: $[\![{}^{\smallsmile}k]\!]^{\mathcal{M},w_1,t_1,g} = a$
  $[\![{}^{\smallsmile}k]\!]^{\mathcal{M},w_1,t_2,g} = b$
  $[\![{}^{\smallsmile}k]\!]^{\mathcal{M},w_2,t_1,g} = d$
  $[\![{}^{\smallsmile}k]\!]^{\mathcal{M},w_2,t_2,g} = b$

The IL of PTQ

A typed higher order λ language with = and ^ / ˇ

- $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbf{F}, \mathbf{P}, \Box, =$ (syncategorematically)
- $t, e \in Type$ (Con_{type}, Var_{type})
- if $a, b \in Type$, then $\langle a, b \rangle \in Type$
- if $a \in Type$, then $\langle s, a \rangle \in Type$
- $s \notin Type$

Meaningful expressions

- ME_{type}
- abstraction: if $\alpha \in ME_a$, $\beta \in Var_b$, $\lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if $\alpha \in ME_{\langle a,b \rangle}$, $\beta \in ME_a$ then $\alpha(\beta) \in ME_b$
- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

Interpretations of ˆ and ˇ

- if $\alpha \in ME_a$ then $^\wedge\alpha \in ME_{s,a}$
- if $\alpha \in ME_{\langle s,a \rangle}$ then $^\vee\alpha \in ME_a$
-

type	variables	constants
e	x, y, z	a, b, c
$\langle s, e \rangle$	x, y, z	–
$\langle e, t \rangle$	X, Y	$walk', A, B$
$\langle\langle s, e \rangle, t \rangle$	Q	$rise', change'$
$\langle s, \langle e, t \rangle\rangle$	P	–
$\langle e, e \rangle$	P	Sq
$\langle e, \langle e, t \rangle\rangle$	R	Gr, K
$\langle e, \langle e, e \rangle\rangle$	–	$Plus$

The model

- $\langle A, W, T, <, F\rangle$
- $D_{\langle a,b\rangle} = {D_b}^{D_a}$
- $D_{\langle s,a\rangle} = {D_a}^{W\times T}$
- 'senses' = **possible** denotations
- actual intensions chosen from the set of senses
- now: F(expression)=intenstion (itself a function)
- s.t. intension(index)=extention
- instead of: F(expression)(index)=extemsion

Some interpretations

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g},\ u \in Var_b,\ \alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with g' exactly like g except $g'(u) = x$
- $[\![\hat{}\alpha]\!]^{\mathcal{M},w,i,g}$ is a function h from $W \times T$ to denotations of α's type s.t. at every $\langle w', t' \rangle \in W \times T$ $[\![\alpha]\!]^{\mathcal{M},w',t',g} = h(\langle w', t' \rangle) = [\![\hat{}\alpha]\!]^{\mathcal{M},w,i,g}(\langle w', t' \rangle)$

Some examples

- $\alpha = \beta$ at $\langle w, t \rangle$ might be true, but $\hat{}\alpha = \hat{}\beta$ need not be 1 at that same index
- on types:
  - e - individuals
  - $\langle s, e \rangle$ - individual concepts ('present Queen of England')
  - $\langle s, \langle e, t \rangle \rangle$ - properties of inidviduals
  - $\langle e, t \rangle$ - sets of individuals
  - $\langle \langle s, e \rangle, t \rangle$- sets of individual concepts

Some examples

- on properties:
  - $\langle s, \langle a, t\rangle\rangle$ - properties of denotations of a-type expressions
  - $\langle s, \langle e, t\rangle\rangle$ - properties of individuals
  - $\langle s, \langle\langle s, t\rangle, t\rangle\rangle$ - properties of propositions
- from relations $\langle e, \langle e, t\rangle\rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t\rangle\rangle\rangle$

On indices

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence: $\langle s, a\rangle$ never applied to some typed argument (s is not a type!)
- useful thing: We never talk about indices!
- since often ${}^{\vee}\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s,\langle e,t\rangle\rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

Examples

Nec

- former problem with **Nec** as $\langle t, t \rangle$: non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle\langle s,t\rangle,t\rangle}$ - $\{0,1\}^{(\{0,1\}^{W \times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{}\phi)$

For

- 'former' as in 'a former member of this club'
- instead of $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$
- intensionally: $\langle\langle s, \langle e, t\rangle\rangle, \langle e, t\rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not{c}}$ is a func. h s.t. for any property k, $h(\langle w, t\rangle)(k)$ is the set $k(\langle w, t'\rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w, t\rangle)(k)(x) = 1$ iff $k(\langle w, t'\rangle)(x) = 1$ for some $t' < t$.

Bel

- relations between individuals and propositions
- $\langle\langle s, t\rangle, \langle e, t\rangle\rangle$
- $\mathbf{Bel}(\hat{}(B(m))(j))$ *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $[\![B(m)]\!]^{M,w_2,t_1,g} = 1$ since $[\![m]\!]^{M,w_2,t_1,g} = [\![n]\!]^{M,w_2,t_1,g}$
- however: $[\![\hat{}(B(m))]\!]^{M,w_2,t_1,g} \neq [\![\hat{}(B(n))]\!]^{M,w_2,t_1,g}$

de dicto

- $\mathbf{Bel}(\hat{}(B(m))(j))$ ‘John believes that Miss America is bald.’
- $\mathbf{Bel}(\hat{}(B(n))(j))$ ‘John believes that Norma is bald.’
- needn’t be equal: John can take worlds other than $\langle w_2, t_1 \rangle$ into account where $[\![n]\!] \neq [\![m]\!]$
- $\alpha = \beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$ is true iff α is not in the scope of $\hat{}, \mathbf{F}, \mathbf{P}, \Box$ (oblique contexts)
- however: $\hat{}\alpha = \hat{}\beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$

de re

- like so: $\lambda x\,[\mathbf{Bel}(\hat{}\,[B(x)])(j)]\,(m)$
- the above is true at an index $\langle w, t\rangle$ iff $[\![\mathbf{Bel}(\hat{}\,[B(x)])(j)]\!]^{w,t} = 1$ if $[\![m]\!]^{w,t} = x$, i.e. if John is in a believe-rel with $\hat{}(B(x))$ s.t. $g(x) = m$ (by semantics of λ)
- Why is $\hat{}(B(x))$ not equal to $\hat{}(B(m))$?
- constant m: non-rigid designator relativized to indices
- variable x: a rigid designator by def. of g (for the relevant checking case with $g(x) = \mathit{MissAmerica}$
- the above: a belief about 'whoever m is'
- λ conversion is restricted in IL!

Once again

- *John believes that a republican will win.*
- $\exists x\,[Rx \wedge \mathbf{Bel}(j, {}^\wedge\,[\mathbf{F}W(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x\,[R(x) \wedge W(x)])$

Literatur I

Autor

Kontakt

Prof. Dr. Roland Schäfer
Institut für Germanistische Sprachwissenschaft
Friedrich-Schiller-Universität Jena
Fürstengraben 30
07743 Jena

https://rolandschaefer.net
roland.schaefer@uni-jena.de

Lizenz

Creative Commons BY-SA-3.0-DE

Dieses Werk ist unter einer Creative Commons Lizenz vom Typ *Namensnennung - Weitergabe unter gleichen Bedingungen 3.0 Deutschland* zugänglich. Um eine Kopie dieser Lizenz einzusehen, konsultieren Sie `http://creativecommons.org/licenses/by-sa/3.0/de/` oder wenden Sie sich brieflich an Creative Commons, Postfach 1866, Mountain View, California, 94042, USA.