Formale Semantik
08. Intensionalität

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Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

Inhalt

Kernfragen dieser Woche

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Definition des intensionalen Kalküls auf Basis des extensionalen.

Wozu Intensionalität?

Intensionalität | Beispiele

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  - und ohne Modellierung von Möglichkeit und Notwendigkeit (Modalverben, modale Adverbiale, *glauben*-Verben)

Was sind Intensionen?

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Bedeutung (Extension) und Sinn (Intension)

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Bedeutung (Extension) und Sinn (Intension)

Synt. Typ	Bedeutung	Sinn
NP	Individuum *Venus*	Individuenkonzept
VP	Menge *Kolibri*	Eigenschaftskonzept
S	{0,1} *Ich mag Kolibris.*	Gedanke/Proposition

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- mediate between internal knowledge and truth-values

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- PSOAs are logically constrained
- observe the more than just thruth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSOA logic

A touch of parellel universes?

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  - nothing of the above, but A.S. rose from the dead in 2003, etc.

Formale Modellierung von Intensionen

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- ...the SOAs under which its corresponding sentence is true
- ...the subset under which its corresponding sentence is false

Coordinates

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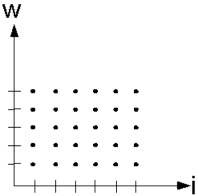
- for each point in time: one possible temporal state of each world (instant $i \in I$)
- representation of temporarily ordered world-time coordinates $\langle w, i \rangle \in W \times I$



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- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

Propositions as functions

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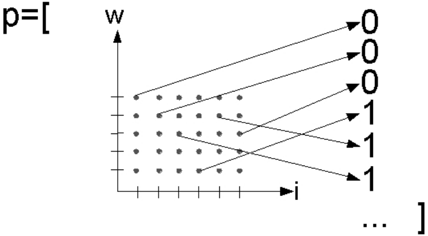
- is a function from $W \times I$ to $\{0, 1\}$



Your evening prayer

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- Hence, we call that function the intension of the sentence.

Mengen von Welten

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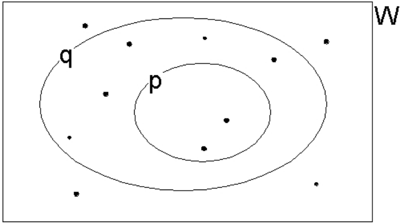
- entailment turns out as a subset-relation: $p \subseteq q$:



Synonymy

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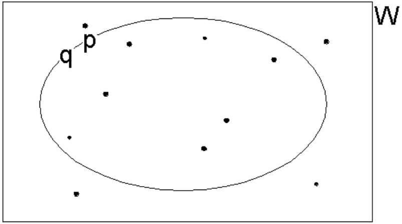
- $p = q$



Contradiction

- contradiction turns out as an empty intersection:

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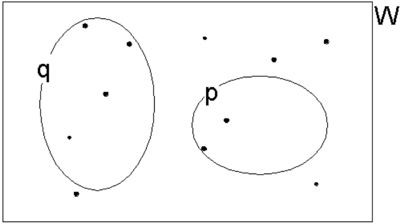
- $p \cap q = \emptyset$



Negation

- negation turns out as a complement:

Negation

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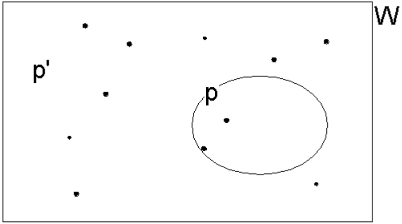
- p/W



Quantification over worlds

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- What does it mean for a proposition to be necessary/possible?

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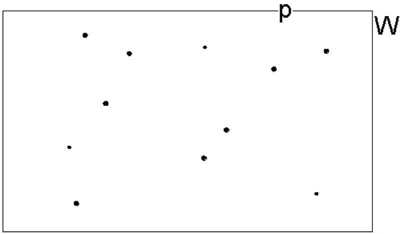
- such that $W = p$ (p as set):



Possibility as existential quantification

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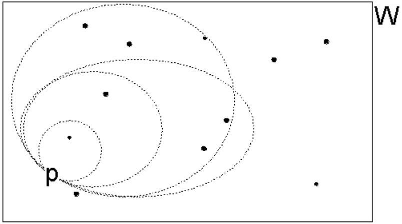
- such that $p \neq \emptyset$ (set):



Intensionale Modelltheory

A larger tuple

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  - *V*, a valuation function for constants
- evaluate an expression α: $[\![\alpha]\!]^{\mathcal{M},w,i,g}$

Intensional interpretation of individual constants

- *the President of the United States*, *the Pope*, *Bond* (in the sense of ‘the actor currently playing Bond’)

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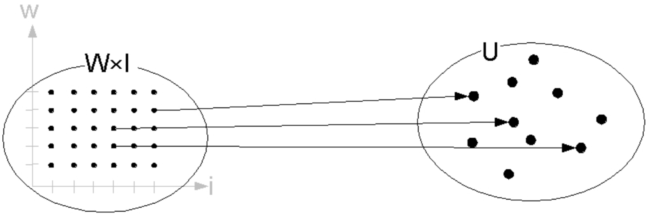

… and pred_ns

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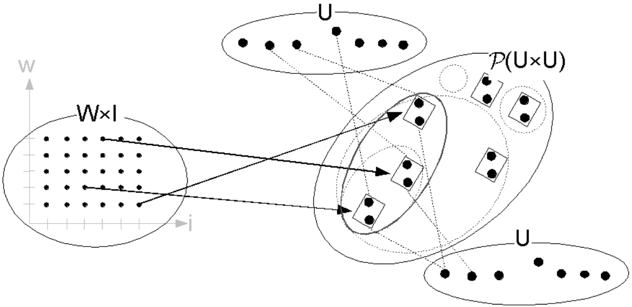
… and pred$_n$s

- *walks* etc. denotes different sets (or CFs) at different $\langle w, i\rangle$ coordinates
- *for* $\beta \in Cons_{pred_n}$, $V(\beta)$ is a function from $W \times I$ to $\wp U^n$ $(U^n = U_1 \times U_2 \times \ldots \times U_n)$



The Chierchia approach: predicates/sentences

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- with: $[\![t_1]\!]^{\mathcal{M},w,i,g} = V(t_1)(\langle w, i \rangle)$, etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

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Literatur I

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Lizenz