Formale Semantik 05. Pr–adikatenlogik

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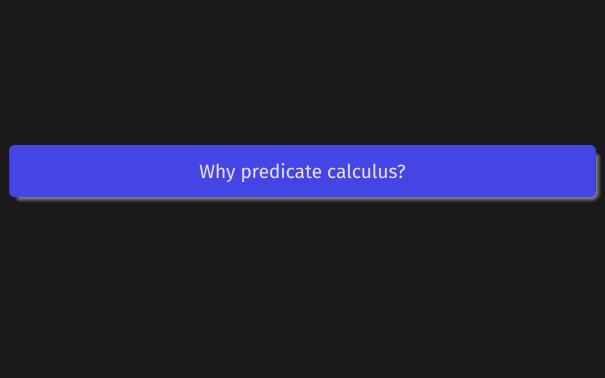
stets aktuelle Fassungen: https://github.com/rsling/VL-Deutsche-Syntax

Inhalt

- The construction of PC
 - Atoms and syntax Semantics
 - More rules

- Laws of PC
 - Negation and distributionMovement

 - Some in-class practice
 - Natural deduction in PC Quantifier elimination
 - An example



• properties/relations vs. individuals

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- …becomes E ∧ C
- compositionality resticted to level of connected propositional atoms

Some desirable deductions

• important generalizations about all and some individuals (which have property P)

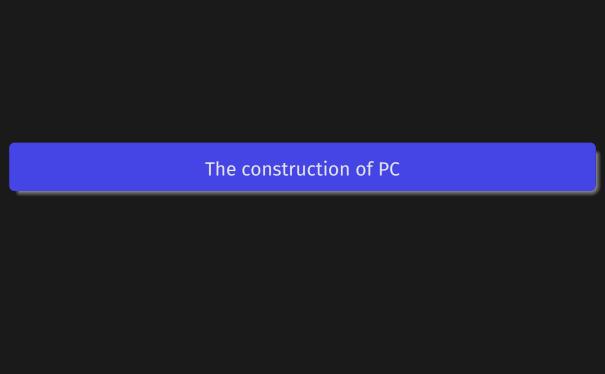
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- quantifiers: existential \exists (or \lor) and universal \forall (or \land)
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- for any wff ϕ and any variable x, $(\exists x)\phi$ and $(\forall x)\phi$ are wff's

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- s.t. $\llbracket P(m) \rrbracket^{\mathcal{M}_1} = \llbracket P \rrbracket^{\mathcal{M}_1}(\llbracket m) \rrbracket^{\mathcal{M}_1}) = 1$ iff $\llbracket m \rrbracket^{\mathcal{M}_1} \in \llbracket P \rrbracket^{\mathcal{M}_1}$

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 - if ∃∀ is true, ∀∃ follows

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- ¬ negates the wff, not the q:
 - $*(\neg \forall x)Px$ but $\neg(\forall x)Px$

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- i.e.: Watch your variables!

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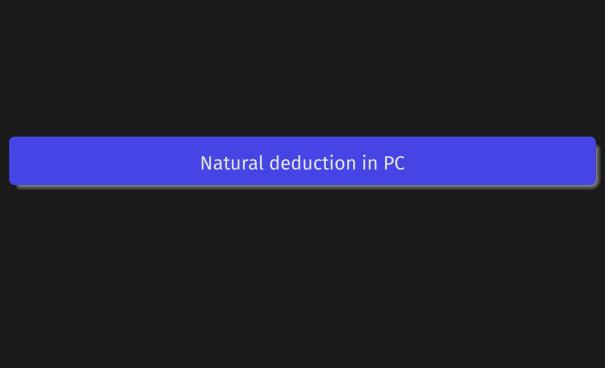
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- Some <u>h</u>umans are neither <u>t</u>alkmasters nor do they <u>o</u>wn <u>K</u>anzleramt records.



 $(\forall x) Px \rightarrow Pc$

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- always applies

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- iff Pa was instantiated by $-\forall$

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- hence: When you apply EI, always use fresh constants!

• (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.

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- (∃x)Hx

The proof

```
(1)
          Dk
(2)
          (\forall x)(Dx \rightarrow Hx \lor Px)
(3)
         \neg(\exists x)(Px \wedge Dx)
(4)
          (\forall x) \neg (Px \wedge Dx)
                                           3,QN
(5)
          (\forall x)(\neg Px \vee \neg Dx)
                                          4.DeM
(6)
         (\forall x)(Dx \rightarrow \neg Px)
                                          5,Comm,Impl
(7)
                                          6,−∀(1)
          Dk \rightarrow \neg Pk
(8)
         \neg Pk
                                           1.7.MP
(9)
         Dk \rightarrow Hk \vee Pk
                                           2,-∀(1)
(10)
          Hk \vee Pk
                                           1,9,MP
(11)
          Hk
                                           8,10,DS
                                           10,+∃
```

Literatur I

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Autor

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