# Formale Semantik o5. Pr–adikatenlogik

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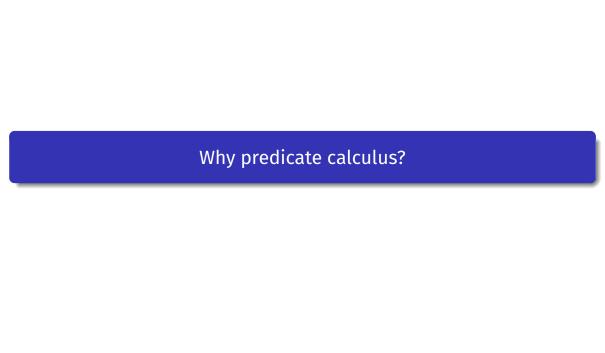
stets aktuelle Fassungen: https://github.com/rsling/VL-Deutsche-Syntax

## Inhalt

- Why predicate calculus?
- 2 The construction of PC
  - Atoms and syntax Semantics
  - More rules

- 3 Laws of PC
  - Negation and distributionMovement

  - Some in-class practice
  - Natural deduction in PC
    - Quantifier elimination
    - An example



# Weak compositionality in SL

- properties/relations vs. individuals
- Martin is an expert on inversion and Martin is a good climber.
- ...becomes  $E \wedge C$
- compositionality resticted to level of connected propositional atoms

## Some desirable deductions

- important generalizations about all and some individuals (which have property P)
- 'all P  $\rightarrow$  some P'
- 'Martin P  $\rightarrow$  some P'



### Atoms of PC

- individual variables:  $x, y, z, x_1, x_2 \dots$
- individual constants: a, b, c, ...
- variables and constants: terms
- predicate symbols (taking individual symbols or tuples of them):  $A, B, C, \ldots$
- quantifiers: existential  $\exists$  (or  $\lor$ ) and universal  $\forall$  (or  $\land$ )
- plus the connectives of SL

## Some syntax

- for an *n*-ary predicate P and terms  $t_1 ldots t_n$ ,  $P(t_1 ldots t_n)$  or  $Pt_1 ldots t_n$  is a wff.
- possible prefix, function (bracket) and infix notation:
   Pxy, P(x, y), xPy
- syntax for connectives from SL
- for any wff  $\phi$  and any variable x,  $(\exists x)\phi$  and  $(\forall x)\phi$  are wff's

## Semantic for individual constants

- denote individuals
- a model  $\mathcal{M}$  contains a set of individuals D
- the valuation function V (or F): from constants to individuals in D
- for some  $\mathcal{M}_1$ : D = {Martin, Kilroy, Scully}
- $V_{\mathcal{M}_1}(m) = Martin$
- $V_{\mathcal{M}_1}(k) = Kilroy$ ,  $V_{\mathcal{M}_1}(s) = Scully$

# Semantics for predicate symbols

- denote relations (sets of n-tuples)
- $\llbracket P \rrbracket^{\mathcal{M}_1} = \{ Martin, Kilroy \}$  or  $V_{\mathcal{M}_1}(P) = \{ Martin, Kilroy \}$
- $V_{\mathcal{M}_1}(Q) = \{\langle Martin, Kilroy \rangle, \langle Martin, Scully \rangle, \langle Kilroy, Kilroy \rangle, \langle Scully, Scully \rangle \}$
- s.t.  $\llbracket P(m) \rrbracket^{\mathcal{M}_1} = \llbracket P \rrbracket^{\mathcal{M}_1}(\llbracket m) \rrbracket^{\mathcal{M}_1}) = 1$  iff  $\llbracket m \rrbracket^{\mathcal{M}_1} \in \llbracket P \rrbracket^{\mathcal{M}_1}$

# Semantics for connectives and quantifiers

- connectives: 'apply to' formulas (semantically truth-valued), semantics as in SL
- $(\forall x)\phi$  = 1 iff  $\phi$  is true for every  $d \in D$  assigned to every occurrence of x in  $\phi$
- $(\exists x)\phi$  = 1 iff  $\phi$  is true for at least one  $d \in D$  assigned to every occurrence of x in  $\phi$
- algorithmic instruction to check wff's containing Q's
- check outside-in (unambiguous scoping)

# Dependencies

• universal quantifiers can be swapped:

$$(\forall \mathbf{x})(\forall \mathbf{y})\phi \Leftrightarrow (\forall \mathbf{y})(\forall \mathbf{x})\phi$$

• same for existential quantifiers:

$$(\exists x)(\exists y)\phi \Leftrightarrow (\exists y)(\exists x)\phi$$

- whereas:  $(\exists x)(\forall y)\phi \Rightarrow (\forall y)(\exists x)\phi$
- example in  $\mathcal{M}_1$ :
  - $\qquad \qquad [ (\forall \mathbf{x}) \underline{(\exists \mathbf{y}) \mathbf{Q} \mathbf{x} \mathbf{y}} ]^{\mathcal{M}_1} = 1$
  - but:  $[(\exists y)(\forall x)Qxy]^{\mathcal{M}_1}=0$
  - direct consequence of algorithmic definition
  - if ∃∀ is true, ∀∃ follows

# Hints on quantifiers

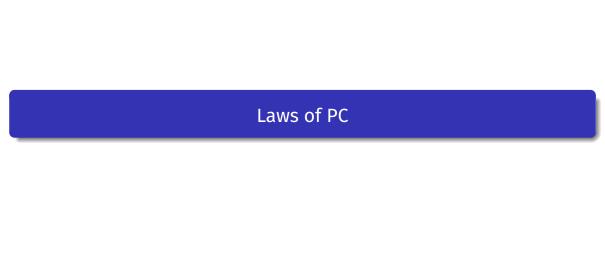
- domain of quantifiers: D (universe of discourse)
- $\forall x$  checks for truth of some predication for all individuals
- $\exists x (Px \land \neg Px)$  is a contradiction
- $\forall x (Wx \land \neg Wx)$  is a contradiciton,  $\forall x$  'checks' for an empty set by def.
- standard form of NL quantification:  $\forall x (Wx \rightarrow Bx)$  'All women are beautiful.'
- standard form of NL existential quantification:  $\exists x (Wx \land Bx)$  'Some woman is beautiful.'

# Functor/quantifier practice

- by def., functors take formulas, not terms:
  - ¬Wm 'Mary doesn't weep.'
  - ▶  $(\exists x)(Gx \land Wx)$  'Some girl weeps.'
  - ▶ \*W¬x
  - $\rightarrow$  \*( $\exists \neg x$ )(Gx)
- quantifiers take variables, not constants:
  - ▶  $(\forall x)(Ox \rightarrow Wx)$  'All ozelots are wildcats.'
  - ▶ \*(∀o)(Wo)
- negates the wff, not the q:
  - \* $(\neg \forall x)Px$  but  $\neg(\forall x)Px$

# Scope

- quantifiers bind variables
- free variables (constants) are unbound
- no double binding  $*(\forall x \exists x)Px$
- Q scope: only the first wff to its right:
  - ▶  $(\forall x)Px \lor Qx$
  - $(\forall x)(Px \lor Qx) = (\forall x)Px \lor (\forall x)Qx$
  - $\overline{(\exists x)Px} \to \underline{(\forall y)}(Qy \land Ry)$
  - ▶  $(\exists x)Px \land Qx$  (second x is a unbound)
- no double-naming



## Universal $\vee$ and $\wedge$

- $\exists$  and  $\forall$  'or' and 'and' over the universe of discourse (hence:  $\bigvee$  and  $\bigwedge$ )
- $(\forall x)$ Px  $\Leftrightarrow$  Px<sub>1</sub>  $\land$  Px<sub>2</sub>  $\land \dots \land$  Px<sub>n</sub> for all x<sub>n</sub> assigned to  $d_n \in D$
- $(\exists x)Px \Leftrightarrow Px_1 \lor Px_2 \lor \ldots \lor Px_n$  for all  $x_n$  assigned to  $d_n \in D$
- hence:  $\neg(\forall x) Px \Leftrightarrow \neg(Px_1 \land Px_2 \land \ldots \land Px_n)$
- with DeM:  $\overline{Px_1 \wedge Px_2 \wedge \ldots \wedge Px_n}$
- $\Leftrightarrow \overline{Px_1} \vee \overline{Px_2} \vee \ldots \vee \overline{Px_n}$
- $\Leftrightarrow (\exists x) \neg Px$

# Quantifier negation (QN)

- $\neg(\forall x)Px \Leftrightarrow (\exists x)\neg Px$
- $\neg(\exists x)Px \Leftrightarrow (\forall x)\neg Px$
- $\neg(\forall x)\neg Px \Leftrightarrow (\exists x)Px$
- $\bullet \ \neg(\exists x)\neg Px \Leftrightarrow (\forall x)Px$

## The distribution laws

• the conjunction of universally quantified formulas:

$$(\forall x)(Px \land Qx) \Leftrightarrow (\forall x)Px \land (\forall x)Qx$$

• the disjunction of existentially quantified formulas:

$$(\exists x)(Px \lor Qx) \Leftrightarrow (\exists x)Px \lor (\exists x)Qx$$

- not v.v.:  $(\forall x)Px \lor (\forall x)Qx \Rightarrow (\forall x)(Px \lor Qx)$
- why?

## Quantifier movement (QM)

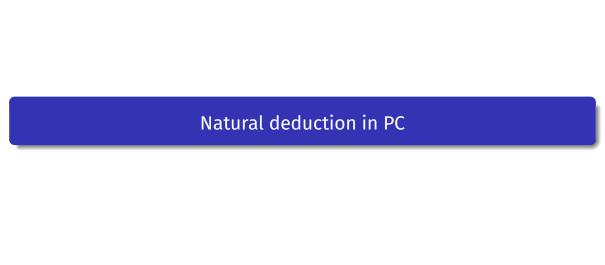
- desirable format: prefix + matrix
- Movement Laws for antecedents of conditionals:

$$(\exists x) Px \to \phi \Leftrightarrow (\forall x) (Px \to \phi) (\forall x) Px \to \phi \Leftrightarrow (\exists x) (Px \to \phi)$$

- Movement Laws for Q's in disjunction, conjunction, and the consequent of conditionals: Just move them to the prefix!
- condition: x must not be free in  $\phi$ .
- i.e.: Watch your variables!

### Let's formalize:

- Paul Kalkbrenner is a musician and signed on bpitchcontrol.
- Herr <u>S</u>. installed <u>RedHat</u> and not every <u>Linux</u> distribution is <u>e</u>asy to install.
- All talkmasters are human and Harald Schmidt is a talkmaster.
- Some talkmasters are not musicians.
- Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.
- Some <u>h</u>umans are neither <u>t</u>alkmasters nor do they <u>o</u>wn <u>K</u>anzleramt records.



# Universal instantiation ( $-\forall$ ) and generalization ( $+\forall$ )

- $(\forall x)Px \rightarrow Pa$
- always applies
- can use any variable/constant
- $Pa \rightarrow (\forall x)Px$
- iff Pa was instantiated by  $-\forall$

# Existential generalization $(+\exists)$ and instantiation $(-\exists)$

- $Pa \rightarrow (\exists x)Px$  for any individual constant a
- always applies
- $(\exists x)Px \rightarrow Pa$  for some indiv. const.
- always applies (there is a minimal individual for  $\exists x$ )
- for some  $(\exists x)Px$  and  $(\exists x)Qx$  the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

# One sample task

- (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.
- Formalize and prove: At least one human exists.
- (1) Dk
- (2)  $(\forall x)(Dx \rightarrow Hx \lor Px)$
- (3)  $\neg(\exists x)(Px \wedge Dx)$
- (∃x)Hx

# The proof

(1)	Dk	
(2)	$(\forall x)(Dx \rightarrow Hx \lor Px)$	
(3)	$\neg(\exists x)(Px \wedge Dx)$	
(4)	$(\forall x)\neg(Px\wedge Dx)$	3,QN
(5)	$(\forall x)(\neg Px \vee \neg Dx)$	4,DeM
(6)	$(\forall x)(Dx \rightarrow \neg Px)$	5,Comm,Impl
(7)	D k  o  eg P k	6,−∀(1)
(8)	$\neg Pk$	1,7,MP
(9)	Dk  o Hk ee Pk	2,−∀(1)
(10)	$Hk \lor Pk$	1,9,MP
(11)	Hk	8,10,DS
<i>:</i> .	$(\exists x)Hx$	10,+∃

# Literatur I

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