Formale Semantik 05. Prädikatenlogik

Roland Schäfer

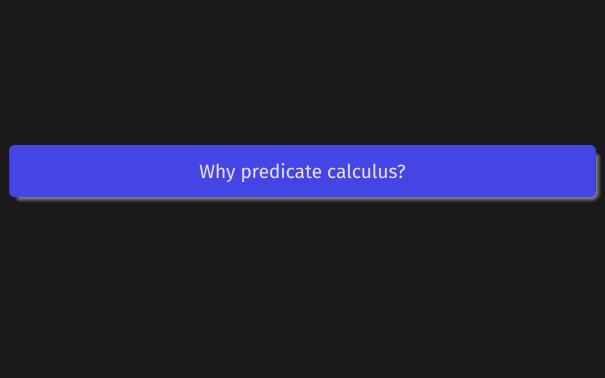
Institut für Germanistische Sprachwissenschaft Friedrich-Schiller-Universität Jena

Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007! Stets aktuelle Fassungen: https://github.com/rsling/VL-Semantik

Inhalt

- 1 Why predicate calculus
- 2 The construction of PC
 - Atoms and syntax
 - Semantics
 More rules

- 2 Laws of PC
 - Negation and distribution
 - Movement
 - Some in-class practice
 - Natural deduction in PC
 Quantifier elimination
 - An example



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- ...becomes $E \wedge C$
- compositionality resticted to level of connected propositional atoms

Some desirable deductions

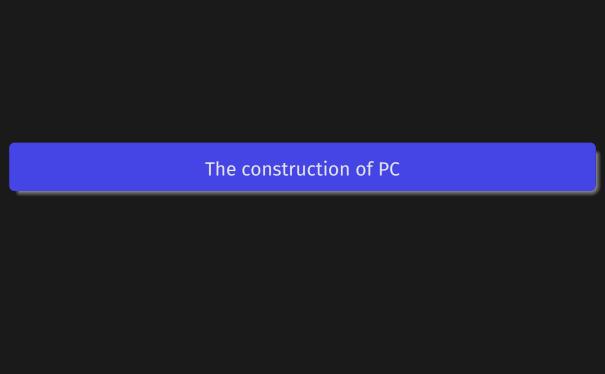
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- 'Martin P \rightarrow some P'



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- plus the connectives of SL

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- syntax for connectives from SL
- for any wff ϕ and any variable x, $(\exists x)\phi$ and $(\forall x)\phi$ are wff's

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- s.t. $\llbracket P(m) \rrbracket^{\mathcal{M}_1} = \llbracket P \rrbracket^{\mathcal{M}_1}(\llbracket m) \rrbracket^{\mathcal{M}_1}) = 1$ iff $\llbracket m \rrbracket^{\mathcal{M}_1} \in \llbracket P \rrbracket^{\mathcal{M}_1}$

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- check outside-in (unambiguous scoping)

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• same for existential quantifiers: $(\exists x)(\exists y)\phi \Leftrightarrow (\exists y)(\exists x)\phi$

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- example in \mathcal{M}_1 :
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 - ▶ if $\exists \forall$ is true, $\forall \exists$ follows

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- ¬ negates the wff, not the q:
 - * $(\neg \forall x)$ Px but $\neg(\forall x)$ Px

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- hence: $\neg(\forall x)Px \Leftrightarrow \neg(Px_1 \land Px_2 \land \ldots \land Px_n)$

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- i.e.: Watch your variables!

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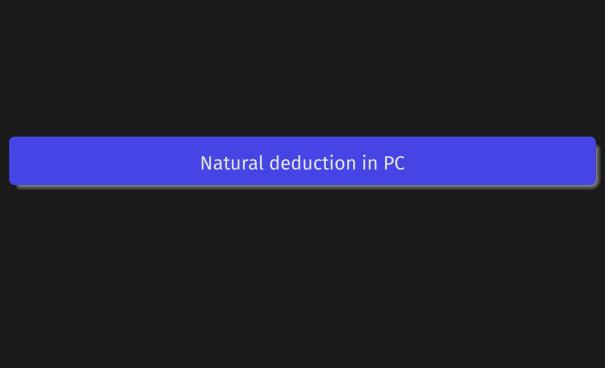
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- Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.
- Some <u>h</u>umans are neither <u>t</u>alkmasters nor do they <u>o</u>wn <u>K</u>anzleramt records.



• $(\forall x)Px \rightarrow Pc$

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- hence: When you apply EI, always use fresh constants!

• (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.

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- (∃x)Hx

The proof

```
(1)
          Dk
(2)
          (\forall x)(Dx \rightarrow Hx \lor Px)
(3)
         \neg(\exists x)(Px \wedge Dx)
(4)
          (\forall x) \neg (Px \wedge Dx)
                                           3,QN
(5)
          (\forall x)(\neg Px \vee \neg Dx)
                                          4.DeM
(6)
         (\forall x)(Dx \rightarrow \neg Px)
                                          5,Comm,Impl
(7)
                                          6.−∀(1)
         Dk \rightarrow \neg Pk
(8)
         \neg Pk
                                           1.7.MP
(9)
                                           2,-∀(1)
          Dk \rightarrow Hk \lor Pk
(10)
          Hk \vee Pk
                                           1,9,MP
(11)
          Hk
                                           8,10,DS
                                           10,+∃
```

Literatur I

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