# Formale Semantik
# 10. Montagues intensionale Logik

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

# Inhalt

# New types and up/down

# Beyond truth functionality

- $[\![\phi]\!]^{\mathcal{M},w,i,g}$ and $[\![\mathbf{P}]\!]^{\mathcal{M},w,i,g}$ don't truth conditionally determine $[\![\mathbf{P}\phi]\!]^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**, $\Diamond$, $\Box$ are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models
- for a name $d$: $[\![d]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

- for an individual concept denoting expression *m*:
- $[\![m]\!]^{\mathcal{M},g}_{¢} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & c \\ \langle w_1, t_3 \rangle & \rightarrow & c \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

# $[\![\alpha]\!]^{\mathcal{M},g}_{\not c}$

- for a one place predicate $B$:
- $[\![B]\!]^{\mathcal{M},g}_{\not c} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{bmatrix}$

# Intensions of formulas

- formula $\phi$: $[\![\phi]\!]_{\not c}^{\mathcal{M},g}$ is a function from indices to truth values
- $[\![B(m)]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 1 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 0 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$
- $[\![B(n)]\!]_{\not c}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 0 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 1 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$

# Intensions of formulas

- again, the proposition $[\![Bm]\!]^{\mathcal{M},g}_{\not{c}}$ is a set of indices $(\langle w_i, t_j\rangle)$
- from the extension at all indices, compute the intension
- $[\![\alpha]\!]^{\mathcal{M},g}_{\not{c}}(\langle w_i, t_j\rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

# Intensions of variables

- constant function on indices
- will play a great role, so remember!
- $[\![u]\!]^{\mathcal{M},g}_{\not{c}}(\langle w_i, t_j\rangle) = g(u)$

# What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes they denote intensions (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

# Up and down

- Church/Montague: for an extension-denoting expression $\alpha$, $\hat{}\alpha$ denotes $\alpha$'s intension
- $[\![\hat{}Bm]\!]^{\mathcal{M},w,i,g} = [\![Bm]\!]^{\mathcal{M},g}_{\cancel{c}}$
- $\alpha$ and $\hat{}\alpha$ are just denoting expressions
- for an intension-denoting expression $\alpha$: $[\![\check{}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w, t\rangle)$

# Down-up and up-down

- observe: $[\![ {}^{\vee}{}^{\wedge}\alpha ]\!]^{\mathcal{M},w,i,g} = [\![ \alpha ]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t \rangle$
- but not always: $[\![ {}^{\wedge}{}^{\vee}\alpha ]\!]^{\mathcal{M},w,i,g} = [\![ \alpha ]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

# Non-equality

- *k*' intension: $[\![k]\!]^{\mathcal{M},g}_{c'} = \begin{bmatrix} \langle w_1,t_1\rangle & \rightarrow & \begin{bmatrix} \langle w_1,t_1\rangle & \rightarrow & a \\ \langle w_1,t_2\rangle & \rightarrow & a \\ \langle w_2,t_1\rangle & \rightarrow & a \\ \langle w_2,t_2\rangle & \rightarrow & a \end{bmatrix} \\ \langle w_1,t_2\rangle & \rightarrow & \begin{bmatrix} \langle w_1,t_1\rangle & \rightarrow & a \\ \langle w_1,t_2\rangle & \rightarrow & b \\ \langle w_2,t_1\rangle & \rightarrow & c \\ \langle w_2,t_2\rangle & \rightarrow & d \end{bmatrix} \\ \langle w_2,t_1\rangle & \rightarrow & \begin{bmatrix} \langle w_1,t_1\rangle & \rightarrow & c \\ \langle w_1,t_2\rangle & \rightarrow & b \\ \langle w_2,t_1\rangle & \rightarrow & d \\ \langle w_2,t_2\rangle & \rightarrow & a \end{bmatrix} \\ \langle w_2,t_2\rangle & \rightarrow & \begin{bmatrix} \langle w_1,t_1\rangle & \rightarrow & c \\ \langle w_1,t_2\rangle & \rightarrow & d \\ \langle w_2,t_1\rangle & \rightarrow & a \\ \langle w_2,t_2\rangle & \rightarrow & b \end{bmatrix} \end{bmatrix}$

# Non-equality

- $k'$ extension (e.g., at $\langle w_1, t_2\rangle$): $[\![k]\!]^{\mathcal{M},g}_{\not{c}}(\langle w_1, t_2\rangle) =$
- $[\![k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & c \\ \langle w_2, t_2\rangle & \rightarrow & d \end{bmatrix}$
- however: $[\![{}^{\frown}k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1\rangle & \rightarrow & a \\ \langle w_1, t_2\rangle & \rightarrow & b \\ \langle w_2, t_1\rangle & \rightarrow & d \\ \langle w_2, t_2\rangle & \rightarrow & b \end{bmatrix}$
- since: $[\![{}^{\smile}k]\!]^{\mathcal{M},w_1,t_1,g} = a$
  $[\![{}^{\smile}k]\!]^{\mathcal{M},w_1,t_2,g} = b$
  $[\![{}^{\smile}k]\!]^{\mathcal{M},w_2,t_1,g} = d$
  $[\![{}^{\smile}k]\!]^{\mathcal{M},w_2,t_2,g} = b$

# The IL of PTQ

# A typed higher order $\lambda$ language with = and $^\wedge$ / $^\vee$

- $\neg$, $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\mathbf{F}$, $\mathbf{P}$, $\Box$, $=$ (syncategorematically)
- $t, e \in Type$ ($Con_{type}$, $Var_{type}$)
- if $a, b \in Type$, then $\langle a, b\rangle \in Type$
- if $a \in Type$, then $\langle s, a\rangle \in Type$
- $s \notin Type$

# Meaningful expressions

- $ME_{type}$
- abstraction: if $\alpha \in ME_a$, $\beta \in Var_b$, $\lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if $\alpha \in ME_{\langle a,b \rangle}$, $\beta \in ME_a$ then $\alpha(\beta) \in ME_b$
- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

# Interpretations of ˆ and ˇ

- if $\alpha \in ME_a$ then $^\wedge\alpha \in ME_{s,a}$
- if $\alpha \in ME_{\langle s,a \rangle}$ then $^\vee\alpha \in ME_a$
-

| type | variables | constants |
|---|---|---|
| $e$ | $x, y, z$ | $a, b, c$ |
| $\langle s, e \rangle$ | $x, y, z$ | – |
| $\langle e, t \rangle$ | $X, Y$ | $walk', A, B$ |
| $\langle\langle s, e \rangle, t \rangle$ | $Q$ | $rise', change'$ |
| $\langle s, \langle e, t \rangle\rangle$ | $P$ | – |
| $\langle e, e \rangle$ | $P$ | Sq |
| $\langle e, \langle e, t \rangle\rangle$ | $R$ | $Gr, K$ |
| $\langle e, \langle e, e \rangle\rangle$ | – | $Plus$ |

# The model

- $\langle A, W, T, <, F \rangle$
- $D_{\langle a,b \rangle} = {D_b}^{D_a}$
- $D_{\langle s,a \rangle} = {D_a}^{W \times T}$
- ‘senses’ = **possible** denotations
- actual intensions chosen from the set of senses
- now: F(expression)=intenstion (itself a function)
- s.t. intension(index)=extention
- instead of: F(expression)(index)=extemsion

# Some interpretations

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g},\ u \in Var_b,\ \alpha \in ME_a$ is a function $h$ with domain $D_b$ s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with $g'$ exactly like $g$ except $g'(u) = x$
- $[\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}$ is a function $h$ from $W \times T$ to denotations of $\alpha$'s type s.t. at every $\langle w', t' \rangle \in W \times T$ $[\![\alpha]\!]^{\mathcal{M},w',t',g} = h(\langle w', t' \rangle) = [\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}(\langle w', t' \rangle)$

# Some examples

- $\alpha = \beta$ at $\langle w, t \rangle$ might be true, but $\hat{}\alpha = \hat{}\beta$ need not be 1 at that same index
- on types:
  - $e$ - individuals
  - $\langle s, e \rangle$ - individual concepts ('present Queen of England')
  - $\langle s, \langle e, t \rangle \rangle$ - properties of inidviduals
  - $\langle e, t \rangle$ - sets of individuals
  - $\langle \langle s, e \rangle, t \rangle$- sets of individual concepts

# Some examples

- on properties:
  - $\langle s, \langle a, t \rangle \rangle$ - properties of denotations of $a$-type expressions
  - $\langle s, \langle e, t \rangle \rangle$ - properties of individuals
  - $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

# On indices

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence: $\langle s, a \rangle$ never applied to some typed argument ($s$ is not a type!)
- useful thing: We never talk about indices!
- since often ${}^{\vee}\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s,\langle e,t\rangle\rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

# Examples

# Nec

- former problem with **Nec** as $\langle t, t \rangle$: non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle\langle s,t\rangle,t\rangle}$ - $\{0,1\}^{(\{0,1\}^{W \times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{\ }\phi)$

# For

- ‘former’ as in ‘a former member of this club’
- instead of $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$
- intensionally: $\langle\langle s,\langle e,t\rangle\rangle,\langle e,t\rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e,t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not{c}}$ is a func. $h$ s.t. for any property $k$, $h(\langle w,t\rangle)(k)$ is the set $k(\langle w,t'\rangle)$ for all $t' < t$.
- So, for any individual $x$ $h(\langle w,t\rangle)(k)(x) = 1$ iff $k(\langle w,t'\rangle)(x) = 1$ for some $t' < t$.

# Bel

- relations between individuals and propositions
- $\langle\langle s,t\rangle,\langle e,t\rangle\rangle$
- $\mathbf{Bel}({}^\wedge(B(m))(j))$ *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $[\![B(m)]\!]^{M,w_2,t_1,g} = 1$ since $[\![m]\!]^{M,w_2,t_1,g} = [\![n]\!]^{M,w_2,t_1,g}$
- however: $[\![{}^\wedge(B(m))]\!]^{M,w_2,t_1,g} \neq [\![{}^\wedge(B(n))]\!]^{M,w_2,t_1,g}$

# de dicto

- $\mathbf{Bel}(\,\hat{}\,(B(m))(j))$ 'John believes that Miss America is bald.'
- $\mathbf{Bel}(\,\hat{}\,(B(n))(j))$ 'John believes that Norma is bald.'
- needn't be equal: John can take worlds other than $\langle w_2, t_1\rangle$ into account where $[\![n]\!] \neq [\![m]\!]$
- $\alpha = \beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$ is true iff $\alpha$ is not in the scope of $\hat{}\,, \mathbf{F}, \mathbf{P}, \Box$ (oblique contexts)
- however: $\hat{}\alpha = \hat{}\beta \rightarrow \left[\phi \leftrightarrow \phi^{[\alpha/\beta]}\right]$

# de re

- like so: $\lambda x\,[\mathbf{Bel}(\hat{}\,[B(x)])(j)]\,(m)$
- the above is true at an index $\langle w, t\rangle$ iff $[\![\mathbf{Bel}(\hat{}\,[B(x)])(j)]\!]^{w,t} = 1$
  if $[\![m]\!]^{w,t} = x$, i.e. if John is in a believe-rel with $\hat{}(B(x))$
  s.t. $g(x) = m$ (by semantics of $\lambda$)
- Why is $\hat{}(B(x))$ not equal to $\hat{}(B(m))$?
- constant *m*: non-rigid designator relativized to indices
- variable *x*: a rigid designator by def. of *g* (for the relevant checking case with $g(x) = MissAmerica$
- the above: a belief about 'whoever *m* is'
- $\lambda$ conversion is restricted in IL!

# Once again

- *John believes that a republican will win.*
- $\exists x\,[Rx \wedge \mathbf{Bel}(j, {}^\wedge\,[\mathbf{F}W(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x\,[R(x) \wedge W(x)])$

# Literatur I

# Autor

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# Lizenz