Formale Semantik
10. Montagues intensionale Logik

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stets aktuelle Fassungen: `https://github.com/rsling/VL-Deutsche-Syntax`

Inhalt

New types and up/down

Beyond truth functionality

- $\llbracket \phi \rrbracket^{\mathcal{M},w,i,g}$ and $\llbracket \mathbf{P} \rrbracket^{\mathcal{M},w,i,g}$ don't truth conditionally determine $\llbracket \mathbf{P}\phi \rrbracket^{\mathcal{M},w,i,g}$

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- again: individual concepts (variable function on indices) vs. names (constant)

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- for a name d: $[\![d]\!]_{\mathscr{C}}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

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- $$[\![m]\!]^{\mathcal{M},g}_{\mathcal{C}'} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & c \\ \langle w_1, t_3 \rangle & \rightarrow & c \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$$

- for a one place predicate *B*:

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- $[\![B]\!]_{\mathcal{G}'}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow & \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow & \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow & \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow & \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow & \{a, b, c\} \end{bmatrix}$

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- $[\![B(m)]\!]^{\mathcal{M},g}_{\mathcal{C}'} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 1 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 0 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$

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- $[\![B(n)]\!]^{\mathcal{M},g}_{\mathcal{C}'} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & 0 \\ \langle w_2, t_1 \rangle & \rightarrow & 1 \\ \langle w_1, t_2 \rangle & \rightarrow & 1 \\ \langle w_2, t_2 \rangle & \rightarrow & 0 \\ \langle w_1, t_3 \rangle & \rightarrow & 1 \\ \langle w_2, t_3 \rangle & \rightarrow & 1 \end{bmatrix}$

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- $[\![\alpha]\!]_{g'}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

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- $[\![u]\!]_{\mathcal{G}'}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

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- alternatively: introduce rules which access an expression's extension/intension as appropriate

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Up and down

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- $[\![\,\hat{}Bm\,]\!]^{\mathcal{M},w,i,g} = [\![Bm]\!]^{\mathcal{M},g}_{\mathscr{C}}$
- α and $\hat{\alpha}$ are just denoting expressions
- for an intension-denoting expression α: $[\![\,\check{}\alpha\,]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w, t\rangle)$

- observe: $[\![\check{}\hat{}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t \rangle$

Down-up and up-down

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- can easily be the case for intension-denoting expressions

- k' intension: $[\![k]\!]_{c'}^{\mathcal{M},g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_1, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix} \\ \langle w_2, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_2, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix} \end{bmatrix}$

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- however: $[\![{}^\sim k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \to & a \\ \langle w_1, t_2 \rangle & \to & b \\ \langle w_2, t_1 \rangle & \to & d \\ \langle w_2, t_2 \rangle & \to & b \end{bmatrix}$

- since: $[\![{}^\sim k]\!]^{\mathcal{M},w_1,t_1,g} = a$
  $[\![{}^\sim k]\!]^{\mathcal{M},w_1,t_2,g} = b$
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The IL of PTQ

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A typed higher order λ language with = and ˆ/ ˘

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- $s \notin$ *Type*

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- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

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type	variables	constants
e	x, y, z	a, b, c
$\langle s, e \rangle$	x, y, z	–
$\langle e, t \rangle$	X, Y	$walk', A, B$
$\langle \langle s, e \rangle, t \rangle$	Q	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	P	–
$\langle e, e \rangle$	P	Sq
$\langle e, \langle e, t \rangle \rangle$	R	Gr, K
$\langle e, \langle e, e \rangle \rangle$	–	$Plus$

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- instead of: F(expression)(index)=extemsion

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g}$, $u \in Var_b$, $\alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with g' exactly like g except $g'(u) = x$

Some interpretations

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- $\llbracket {}^{\wedge}\alpha \rrbracket^{\mathcal{M},w,i,g}$ is a function h from $W \times T$ to denotations of α's type s.t. at every $\langle w', t' \rangle \in W \times T$ $\llbracket \alpha \rrbracket^{\mathcal{M},w',t',g} = h(\langle w', t' \rangle) = \llbracket {}^{\wedge}\alpha \rrbracket^{\mathcal{M},w,i,g}(\langle w', t' \rangle)$

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  - $\langle \langle s, e \rangle, t \rangle$ - sets of individual concepts

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  - $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

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- since often $\breve{}\,\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

Examples

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- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{}\,\phi)$

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- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]_{g'}^{\mathcal{M}, g}$ is a func. h s.t. for any property k, $h(\langle w, t\rangle)(k)$ is the set $k(\langle w, t'\rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w, t\rangle)(k)(x) = 1$ iff $k(\langle w, t'\rangle)(x) = 1$ for some $t' < t$.

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- λ conversion is restricted in IL!

Once again

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- $\mathbf{Bel}(j, \mathbf{F}\exists x\,[R(x) \wedge W(x)])$

Literatur I

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