

# Formale Semantik

## o8. Intensionalität

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**Folien in Überarbeitung. Englische Teile (ab Woche 8) sind noch von 2007!**

Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

## 1 Intensionality

- Problems with extensionality and non-dimensional models
- Intensions

## 2 A formal account of intensions

- Sets of PSOAs
- Intensions as functions
- Repeat after me...

## 3 Sets of worlds

- Known relations
- Modal operators

## 4 Intensional Model Theory

- Ingredients of models
- Evaluating individual constants
- Set membership
- Some peculiarities of  $\Box$  and  $\Diamond$

# Targets for this week

- Understand that we have been exclusively dealing with extensions so far.
- Acknowledge that the approach fails in certain constructions.
- Learn how one can define an intensional calculus on top of the extensional one.
- See how that solves many problems with extensional logic for NL.

Intensionality

# Some examples

- Stockhausen **will** write another opera.
- **Had** Arno Schmidt cut down on drinking, he **would** still be alive.
- Gustave Moreau **believes that** estheticism rules.

# Simple extensions?

- syntactic types are no problem
- truth conditions impossible to define for static models (**tense**)
- ... and for just one state of affairs (**modals**, **believe type verbs**)

# What are intensions?

Type	Reference	Sense
NP	individuals <i>Venus</i>	individual concepts
VP	sets <i>humming birds</i>	property concepts
S	1 or 0 <i>I like cats.</i>	thoughts or <b>propositions</b>

- can't be just truth conditional
- encode knowledge about not just the actual but all possible and/or past/future states of affairs (PSOAs)
- therefore still involved in defining truth conditions
- not mental representations
- mediate between internal knowledge and truth-values



# PSOAs have their own logic

- PSOAs are logically constrained
- observe the more than just truth-valued failure of:
- *In 1985 Arno Schmidt will be planning to have finished 'Julia oder Die Gemälde' by August 1914.*
- incompatible to our knowledge of PSA logic

# A touch of parallel universes?

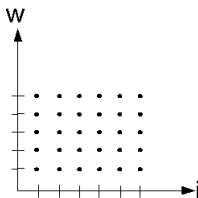
- *Maria could know Arno Schmidt in person.*
- is true not to facts but to an infinite number of optional SOAs s.t.:
  - ▶ A.S. is not a workaholic, does not drink 2 liters of coffee in the morning, does not drink a bottle of *Klarer* in the afternoon, consequently has never had any heart attacks
  - ▶ nothing of the above, but Maria was born 20 years earlier
  - ▶ nothing of the above, but A.S. rose from the dead in 2003, etc.

A formal account of intensions

- assume a set of all PSOAs
- PSOAs: determined by which propositions correspond to true sentences within the world they represent
- each proposition splits the set of PSOAs into two subsets:
  - ...the SOAs under which its corresponding sentence is true
  - ...the subset under which its corresponding sentence is false

# Coordinates

- for each possible distinction in truth values of the whole of the propositional sentences: **one possible world** ( $w \in W$ )
- for each point in time: **one possible temporal state of each world** (instant  $i \in I$ )
- representation of **temporally ordered world-time coordinates**  $\langle w, i \rangle \in W \times I$

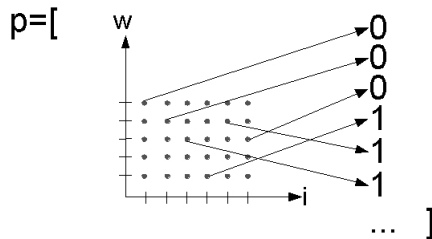


# The nature of propositions

- propositions = intensions of sentences (formulas)
- remember the condition: every possible truth-value configuration for the full set of possible sentences constitutes a member of the set of possible worlds
- hence: every sentence is characterized by the set of worlds in which it is true
- this characterization: its intension
- the proposition of a sentence/formula: the characteristic function of the set of world/world-time pairs in which it is true

# Propositions as functions

- a propositional function  $p$
- is a function from  $W \times I$  to  $\{0, 1\}$

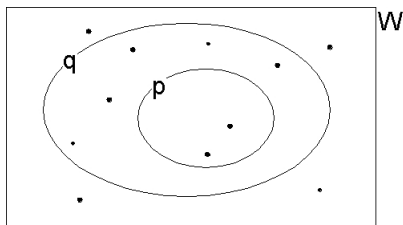


- If we know the state of affairs, we know for every sentence whether it is true!
- If we know which sentences are true, we know the state of affairs!
- It is quite difficult to state what other kind of knowledge (or information) should exist. So for now we assume there isn't any.
- Since we agree that sentences denote truth values, and that the truth of a sentence depends on the state of affairs (=world), the function from all possible worlds to truth values characterizes sentences under all thinkable conditions.
- Hence, we call that function the intension of the sentence.

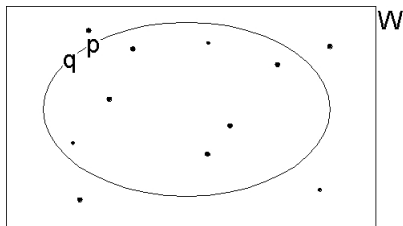


Sets of worlds

- definition of intensions of sentences (propositions): characteristic functions
- **equivalently: propositions are sets of possible worlds**
- **entailment** turns out as a **subset-relation**:  $p \subseteq q$ :

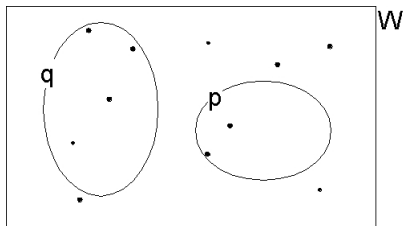


- synonymy turns out as set equivalence:
- $p = q$



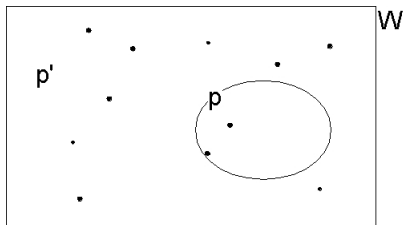
# Contradiction

- **contradiction** turns out as an **empty intersection**:
- $p \cap q = \emptyset$



# Negation

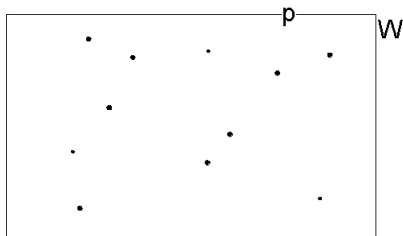
- negation turns out as a complement:
- $p/W$



- new **modal** sentence/wff operators:
  - ▶ *necessarily*  $p$ :  $\Box p$
  - ▶ *possibly*  $p$ :  $\Diamond p$
- What does it mean for a proposition to be necessary/possible?

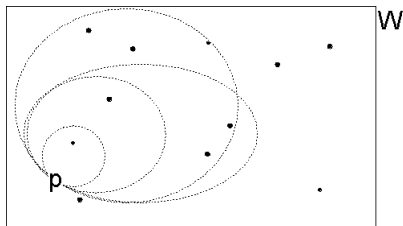
# Necessity as universal quantification

- if  $\Box p$  then  $(\forall w) [p(w) = 1]$  ( $p$  as characteristic function)
- such that  $W = p$  ( $p$  as set):



# Possibility as existential quantification

- if  $\Diamond p$  then  $(\exists w) [p(w) = 1]$  (characteristic function)
- such that  $p \neq \emptyset$  (set):





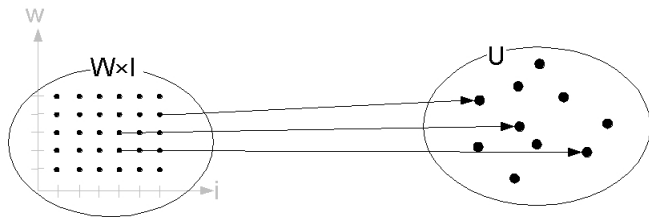
## Intensional Model Theory

# A larger tuple

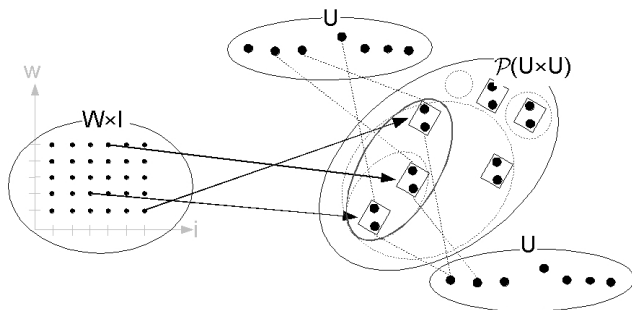
- $\mathcal{M} = \{W, I, <, U, V\}$ 
  - ▶  $W$ , a set of worlds
  - ▶  $I$ , a set of instants
  - ▶  $<$ , an ordering relation in  $I$
  - ▶  $U$ , the set of individuals
  - ▶  $V$ , a valuation function for constants
- evaluate an expression  $\alpha$ :  $\llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$

# Intensional interpretation of individual constants

- *the President of the United States, the Pope, Bond* (in the sense of ‘the actor currently playing Bond’)
- for  $\beta \in \text{Cons}_{\text{ind}}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $U$



- *walks* etc. denotes different sets (or CFs) at different  $\langle w, i \rangle$  coordinates
- for  $\beta \in \text{Cons}_{\text{pred}_n}$ ,  $V(\beta)$  is a function from  $W \times I$  to  $\wp U^n$  ( $U^n = U_1 \times U_2 \times \dots \times U_n$ )



# The Chierchia approach: predicates/sentences

- simple sentences/predicates:  $\beta = \delta(t_1, t_2, \dots, t_n)$
- $\llbracket \beta \rrbracket^{\mathcal{M}, w, i, g} = 1$  iff
- $\langle \llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g}, \llbracket t_2 \rrbracket^{\mathcal{M}, w, i, g}, \dots, \llbracket t_n \rrbracket^{\mathcal{M}, w, i, g} \rangle \in \llbracket \delta \rrbracket^{\mathcal{M}, w, i, g}$
- with:  $\llbracket t_1 \rrbracket^{\mathcal{M}, w, i, g} = V(t_1)(\langle w, i \rangle)$ , etc.
- In an intensional type-theoretic language, we could define new functional types and try to use FA where possible.

- if  $\psi = \forall x\phi$  then
- ... $\llbracket \psi \rrbracket^{\mathcal{M}, w, i, g} = 1$  iff for all  $u \in U$
- ... $\llbracket \phi \rrbracket^{\mathcal{M}, w, i, g[u/x]} = 1$
- nothing new here

- if  $\psi = \Box x \phi$  then
- ... $\llbracket \psi \rrbracket^{\mathcal{M}, w, i, g} = 1$  iff for all  $w' \in W$
- ...and all  $i' \in I$
- ... $\llbracket \phi \rrbracket^{\mathcal{M}, w', i', g} = 1$

## A similarity of $\forall$ and $\Box$

- as:  $\forall x [P(x) \rightarrow Q(x)] \rightarrow [\forall x P(x) \rightarrow \forall x Q(x)]$
- and not vice-versa
- it holds that:  $\Box [\psi \rightarrow \phi] \rightarrow [\Box \psi \rightarrow \Box \phi]$
- **but not vice-versa!**



# Some validities

- $\exists x \Box P(x) \rightarrow \Box \exists x P(x)$
- $\exists x \Diamond P(x) \leftrightarrow \Diamond \exists x P(x)$
- $\forall x \Box P(x) \leftrightarrow \Box \forall x P(x)$  (Carnap-Barcan)
- $\forall x \Diamond P(x) \rightarrow \Diamond \forall x P(x)$



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