Formale Semantik
10. Montagues intensionale Logik

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Stets aktuelle Fassungen: `https://github.com/rsling/VL-Semantik`

Inhalt

New types and up/down

Beyond truth functionality

- $[\![\phi]\!]^{\mathcal{M},w,i,g}$ and $[\![\mathbf{P}]\!]^{\mathcal{M},w,i,g}$ don't truth conditionally determine $[\![\mathbf{P}\phi]\!]^{\mathcal{M},w,i,g}$

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- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

$[\![\alpha]\!]^{\mathcal{M},g}_{\not{c}}$

- intension relative to models

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- for a name d: $[\![d]\!]^{\mathcal{M},g}_{\not{c}} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & b \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_2 \rangle & \rightarrow & b \\ \langle w_1, t_3 \rangle & \rightarrow & b \\ \langle w_2, t_3 \rangle & \rightarrow & b \end{bmatrix}$

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- $[\![\alpha]\!]^{\mathcal{M},g}_{\cancel{c}}(\langle w_i, t_j \rangle) = [\![\alpha]\!]^{\mathcal{M},w_i,t_j,g}$

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- $[\![u]\!]_{\cancel{c}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

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- alternatively: introduce rules which access an expression's extension/intension as appropriate

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- α and $\hat{}\alpha$ are just denoting expressions
- for an intension-denoting expression α: $[\![\check{}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},g}(\langle w,t\rangle)$

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- observe: $[\![{}^{\vee\wedge}\alpha]\!]^{\mathcal{M},w,i,g} = [\![\alpha]\!]^{\mathcal{M},w,i,g}$ for any $\langle w, t\rangle$

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- can easily be the case for intension-denoting expressions

Non-equality

- *k*' intension: $[\![k]\!]^{\mathcal{M},g}_{\mathcal{C}} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_1, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix} \\ \langle w_2, t_1 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{bmatrix} \\ \langle w_2, t_2 \rangle & \rightarrow & \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix} \end{bmatrix}$

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- however: $[\![\tilde{\ }k]\!]^{\mathcal{M},w_1,t_2,g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix}$
- since: $[\![\check{\ }k]\!]^{\mathcal{M},w_1,t_1,g} = a$
  $[\![\check{\ }k]\!]^{\mathcal{M},w_1,t_2,g} = b$
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The IL of PTQ

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- $s \notin Type$

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- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

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type	variables	constants
e	x, y, z	a, b, c
$\langle s, e\rangle$	x, y, z	–
$\langle e, t\rangle$	X, Y	$walk'$, A, B
$\langle\langle s, e\rangle, t\rangle$	Q	$rise'$, $change'$
$\langle s, \langle e, t\rangle\rangle$	P	–
$\langle e, e\rangle$	P	Sq
$\langle e, \langle e, t\rangle\rangle$	R	Gr, K
$\langle e, \langle e, e\rangle\rangle$	–	$Plus$

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- s.t. intension(index)=extention
- instead of: F(expression)(index)=extemsion

Some interpretations

- $[\![\lambda u \alpha]\!]^{\mathcal{M},w,i,g},\ u \in Var_b,\ \alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = [\![\alpha]\!]^{\mathcal{M},w,t,g'}$ with g' exactly like g except $g'(u) = x$

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- $[\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}$ is a function h from $W \times T$ to denotations of α's type s.t. at every $\langle w', t'\rangle \in W \times T$ $[\![\alpha]\!]^{\mathcal{M},w',t',g} = h(\langle w', t'\rangle) = [\![{}^\wedge\alpha]\!]^{\mathcal{M},w,i,g}(\langle w', t'\rangle)$

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  - $\langle s, \langle\langle s, t\rangle, t\rangle\rangle$ - properties of propositions
- from relations $\langle e, \langle e, t\rangle\rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t\rangle\rangle\rangle$

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- Expressions denote functions in the domain of indices.
- hence: $\langle s, a\rangle$ never applied to some typed argument (*s* is not a type!)
- useful thing: We never talk about indices!
- since often ${}^{\vee}\alpha(\beta)$ is needed for $\alpha \in ME_{\langle s,\langle e,t\rangle\rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

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- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{}\phi)$

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- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t\rangle$ not at current but at some past index qualify

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- extensions at all indices accessible via intension: those individuals bearing property $\langle e,t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not{c}}$ is a func. h s.t. for any property k, $h(\langle w,t\rangle)(k)$ is the set $k(\langle w,t'\rangle)$ for all $t' < t$.

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- 'former' as in 'a former member of this club'
- instead of $\langle\langle e, t\rangle, \langle e, t\rangle\rangle$
- intensionally: $\langle\langle s, \langle e, t\rangle\rangle, \langle e, t\rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t\rangle$ not at current but at some past index qualify
- formally: $[\![\mathbf{For}]\!]^{\mathcal{M},g}_{\not{c}}$ is a func. h s.t. for any property k, $h(\langle w, t\rangle)(k)$ is the set $k(\langle w, t'\rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w, t\rangle)(k)(x) = 1$ iff $k(\langle w, t'\rangle)(x) = 1$ for some $t' < t$.

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- λ conversion is restricted in IL!

Once again

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Literatur I

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Lizenz