

Formale Semantik

10. Montagues intensionale Logik

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Achtung: Folien in Überarbeitung. Englische Teile sind noch von 2007!
Stets aktuelle Fassungen: <https://github.com/rsling/VL-Semantik>

- 1 New types and up/down
 - Denoting intensions
 - Technical devices

- 2 The IL of PTQ

- Syntax
- Semantics
- Technical refinements

- 3 Examples

New types and up/down

- $\llbracket \phi \rrbracket^{\mathcal{M},w,i,g}$ and $\llbracket \mathbf{P} \rrbracket^{\mathcal{M},w,i,g}$ don't truth conditionally determine $\llbracket \mathbf{P}\phi \rrbracket^{\mathcal{M},w,i,g}$

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- again: individual concepts (variable function on indices) vs. names (constant)

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- for a name d : $\llbracket d \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow b \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow b \\ \langle w_1, t_3 \rangle & \rightarrow b \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

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- $\llbracket B(m) \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow 1 \\ \langle w_2, t_1 \rangle & \rightarrow 1 \\ \langle w_1, t_2 \rangle & \rightarrow 0 \\ \langle w_2, t_2 \rangle & \rightarrow 0 \\ \langle w_1, t_3 \rangle & \rightarrow 1 \\ \langle w_2, t_3 \rangle & \rightarrow 1 \end{array} \right]$

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- $\llbracket \alpha \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = \llbracket \alpha \rrbracket^{\mathcal{M},w_i,t_j,g}$

- constant function on indices

Intensions of variables

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- $\llbracket u \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

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- alternatively: introduce rules which access an expression's extension/intension as appropriate

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- α and $\hat{\alpha}$ are just denoting expressions
- for an intension-denoting expression α : $\llbracket \check{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\langle w, t \rangle)$

- observe: $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$ for any $\langle w, t \rangle$

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- but not always: $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$ for any $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

Non-equality

• k' intension: $\llbracket k \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g} =$

$$\left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_1, t_2 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{array} \right] \\ \langle w_2, t_1 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_2, t_2 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{array} \right] \end{array} \right]$$

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- since: $\llbracket \sim k \rrbracket_{\mathcal{M}, w_1, t_1, g} = a$
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The IL of PTQ

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type	variables	constants
e	x, y, z	a, b, c
$\langle s, e \rangle$	x, y, z	—
$\langle e, t \rangle$	X, Y	$walk', A, B$
• $\langle \langle s, e \rangle, t \rangle$	Q	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	P	—
$\langle e, e \rangle$	P	Sq
$\langle e, \langle e, t \rangle \rangle$	R	Gr, K
$\langle e, \langle e, e \rangle \rangle$	—	$Plus$

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- instead of: $F(\text{expression})(\text{index}) = \text{extension}$

Some interpretations

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- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}$ is a function h from $W \times T$ to denotations of α 's type s.t. at every $\langle w', t' \rangle \in W \times T$ $\llbracket \alpha \rrbracket^{\mathcal{M}, w', t', g} = h(\langle w', t' \rangle) = \llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}(\langle w', t' \rangle)$

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 - ▶ $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

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- useful thing: We never talk about indices!
- since often $\sim \alpha(\beta)$ is needed for $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

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- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{\phi})$

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- intensionally: $\langle\langle s, \langle e, t \rangle\rangle, \langle e, t \rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t \rangle$ not at current but at some past index qualify

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Kontakt

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