# Formale Semantik 05. Prädikatenlogik

#### Roland Schäfer

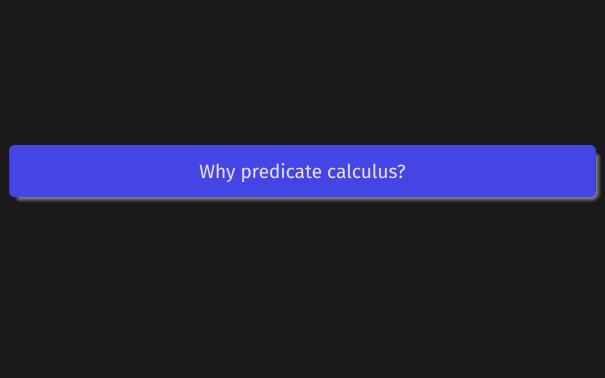
Institut für Germanistische Sprachwissenschaft Friedrich-Schiller-Universität Jena

stets aktuelle Fassungen: https://github.com/rsling/VL-Deutsche-Syntax

#### Inhalt

- 1 Why predicate calculus
- 2 The construction of PC
  - Atoms and syntax
  - Semantics
    More rules

- 2 Laws of PC
  - Negation and distribution
  - Movement
  - Some in-class practice
  - Natural deduction in PC
     Quantifier elimination
    - An example



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- ...becomes  $E \wedge C$
- compositionality resticted to level of connected propositional atoms

### Some desirable deductions

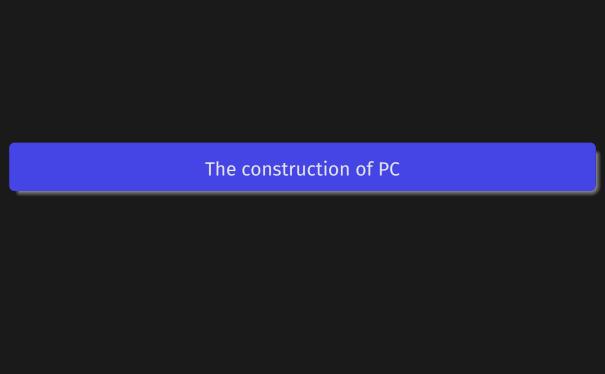
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- plus the connectives of SL

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- syntax for connectives from SL
- for any wff  $\phi$  and any variable x,  $(\exists x)\phi$  and  $(\forall x)\phi$  are wff's

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- s.t.  $\llbracket P(m) \rrbracket^{\mathcal{M}_1} = \llbracket P \rrbracket^{\mathcal{M}_1}(\llbracket m) \rrbracket^{\mathcal{M}_1}) = 1$  iff  $\llbracket m \rrbracket^{\mathcal{M}_1} \in \llbracket P \rrbracket^{\mathcal{M}_1}$

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- check outside-in (unambiguous scoping)

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  - ▶ if  $\exists \forall$  is true,  $\forall \exists$  follows

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- i.e.: Watch your variables!

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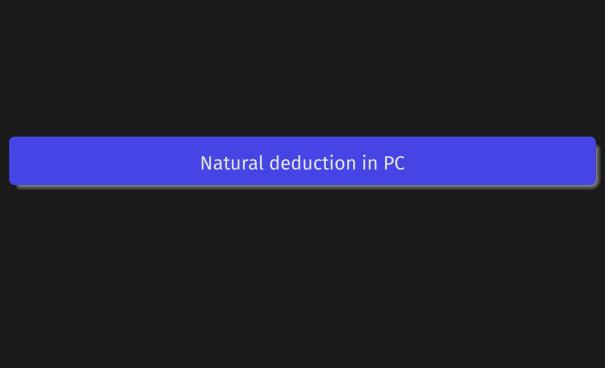
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- Some talkmasters are not musicians.

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- Some talkmasters are not musicians.
- Heiko Laux owns Kanzleramt records and does not like any Gigolo artist.
- Some <u>h</u>umans are neither <u>t</u>alkmasters nor do they <u>o</u>wn <u>K</u>anzleramt records.



•  $(\forall x)Px \rightarrow Pc$ 

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- $(\forall x)Px \rightarrow Pa$
- always applies

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- iff Pa was instantiated by  $-\forall$

•  $Pa \rightarrow (\exists x)Px$  for any individual constant a

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- for some  $(\exists x)Px$  and  $(\exists x)Qx$  the minimal individual might be different
- hence: When you apply EI, always use fresh constants!

• (1) Herr Keydana drives a Golf. (2) Anything that drives a golf is human or a complex program simulating an artificial neural net. (3) There are no programs s.a.a.n.n. which are complex enough to drive a Golf.

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- (∃x)Hx

### The proof

```
(1)
          Dk
(2)
          (\forall x)(Dx \rightarrow Hx \lor Px)
(3)
         \neg(\exists x)(Px \wedge Dx)
(4)
          (\forall x) \neg (Px \wedge Dx)
                                           3,QN
(5)
          (\forall x)(\neg Px \vee \neg Dx)
                                          4.DeM
(6)
         (\forall x)(Dx \rightarrow \neg Px)
                                          5,Comm,Impl
(7)
                                          6.−∀(1)
         Dk \rightarrow \neg Pk
(8)
         \neg Pk
                                           1.7.MP
(9)
                                           2,-∀(1)
          Dk \rightarrow Hk \lor Pk
(10)
          Hk \vee Pk
                                           1,9,MP
(11)
          Hk
                                           8,10,DS
                                           10,+∃
```

# Literatur I

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#### Autor

#### Kontakt

Prof. Dr. Roland Schäfer Institut für Germanistische Sprachwissenschaft Friedrich-Schiller-Universität Jena Fürstengraben 30 07743 Jena

https://rolandschaefer.net roland.schaefer@uni-jena.de

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