

Formale Semantik

10. Montagues intensionale Logik

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stets aktuelle Fassungen: <https://github.com/rsling/VL-Deutsche-Syntax>

1 New types and up/down

- Denoting intensions
- Technical devices

2 The IL of PTQ

- Syntax
- Semantics
- Technical refinements

3 Examples

New types and up/down

Beyond truth functionality

- $\llbracket \phi \rrbracket^{\mathcal{M},w,i,g}$ and $\llbracket \mathbf{P} \rrbracket^{\mathcal{M},w,i,g}$ don't truth conditionally determine $\llbracket \mathbf{P}\phi \rrbracket^{\mathcal{M},w,i,g}$
- *Iceland was once covered with a glacier.*
- **F**, **B**, \diamond , \square are not fully truth functional
- Leibnitz Law of identity of individuals for logics containing '=' failing in opaque contexts
- 'former', 'alleged', etc. are not intersective adjectives like 'red'
- Frege: sometimes expressions **denote a sense**
- again: individual concepts (variable function on indices) vs. names (constant)

- intension relative to models

- for a name d : $\llbracket d \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow b \\ \langle w_2, t_1 \rangle & \rightarrow b \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow b \\ \langle w_1, t_3 \rangle & \rightarrow b \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

- for an individual concept denoting expression m :

- $\llbracket m \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow a \\ \langle w_2, t_1 \rangle & \rightarrow c \\ \langle w_1, t_2 \rangle & \rightarrow b \\ \langle w_2, t_2 \rangle & \rightarrow c \\ \langle w_1, t_3 \rangle & \rightarrow c \\ \langle w_2, t_3 \rangle & \rightarrow b \end{array} \right]$

- for a one place predicate B :

- $\llbracket B \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow \{a, b\} \\ \langle w_2, t_1 \rangle & \rightarrow \{b, c\} \\ \langle w_1, t_2 \rangle & \rightarrow \{a, c\} \\ \langle w_2, t_2 \rangle & \rightarrow \{a\} \\ \langle w_1, t_3 \rangle & \rightarrow \{b, c\} \\ \langle w_2, t_3 \rangle & \rightarrow \{a, b, c\} \end{array} \right]$

- formula ϕ : $\llbracket \phi \rrbracket_{\mathcal{C}}^{\mathcal{M},g}$ is a function from indices to truth values

- $\llbracket B(m) \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow 1 \\ \langle w_2, t_1 \rangle & \rightarrow 1 \\ \langle w_1, t_2 \rangle & \rightarrow 0 \\ \langle w_2, t_2 \rangle & \rightarrow 0 \\ \langle w_1, t_3 \rangle & \rightarrow 1 \\ \langle w_2, t_3 \rangle & \rightarrow 1 \end{array} \right]$

- $\llbracket B(n) \rrbracket_{\mathcal{C}}^{\mathcal{M},g} = \left[\begin{array}{ll} \langle w_1, t_1 \rangle & \rightarrow 0 \\ \langle w_2, t_1 \rangle & \rightarrow 1 \\ \langle w_1, t_2 \rangle & \rightarrow 1 \\ \langle w_2, t_2 \rangle & \rightarrow 0 \\ \langle w_1, t_3 \rangle & \rightarrow 1 \\ \langle w_2, t_3 \rangle & \rightarrow 1 \end{array} \right]$

- again, the proposition $\llbracket Bm \rrbracket_{\mathcal{C}}^{\mathcal{M},g}$ is a set of indices $\langle w_i, t_j \rangle$
- from the extension at all indices, compute the intension
- $\llbracket \alpha \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = \llbracket \alpha \rrbracket^{\mathcal{M},w_i,t_j,g}$

- constant function on indices
- will play a great role, so remember!
- $\llbracket u \rrbracket_{\mathcal{C}}^{\mathcal{M},g}(\langle w_i, t_j \rangle) = g(u)$

What expressions denote

- sometimes expressions denote individuals, sets of individuals, truth values...
- and sometimes **they denote intensions** (functions)
- alternatively: introduce rules which access an expression's extension/intension as appropriate

- Church/Montague: for an extension-denoting expression α , $\hat{\alpha}$ denotes α 's intension
- $\llbracket \hat{Bm} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket Bm \rrbracket_{\hat{c}}^{\mathcal{M}, g}$
- α and $\hat{\alpha}$ are just denoting expressions
- for an intension-denoting expression α : $\llbracket \check{\alpha} \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, g}(\langle w, t \rangle)$

- observe: $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$ for any $\langle w, t \rangle$
- but not always: $\llbracket \sim \alpha \rrbracket^{\mathcal{M}, w, i, g} = \llbracket \alpha \rrbracket^{\mathcal{M}, w, i, g}$ for any $\langle w, t \rangle$
- can easily be the case for intension-denoting expressions

- k' intension: $\llbracket k \rrbracket_{\mathcal{M},g}^{\mathcal{M},g} =$

$$\left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & a \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_1, t_2 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{array} \right] \\ \langle w_2, t_1 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & a \end{array} \right] \\ \langle w_2, t_2 \rangle & \rightarrow & \left[\begin{array}{lcl} \langle w_1, t_1 \rangle & \rightarrow & c \\ \langle w_1, t_2 \rangle & \rightarrow & d \\ \langle w_2, t_1 \rangle & \rightarrow & a \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{array} \right] \end{array} \right]$$

- k' extension (e.g., at $\langle w_1, t_2 \rangle$): $\llbracket k \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g}(\langle w_1, t_2 \rangle) =$

- $\llbracket k \rrbracket_{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & c \\ \langle w_2, t_2 \rangle & \rightarrow & d \end{bmatrix}$

- however: $\llbracket \sim k \rrbracket_{\mathcal{M}, w_1, t_2, g} = \begin{bmatrix} \langle w_1, t_1 \rangle & \rightarrow & a \\ \langle w_1, t_2 \rangle & \rightarrow & b \\ \langle w_2, t_1 \rangle & \rightarrow & d \\ \langle w_2, t_2 \rangle & \rightarrow & b \end{bmatrix}$

- since: $\llbracket \sim k \rrbracket_{\mathcal{M}, w_1, t_1, g} = a$
 $\llbracket \sim k \rrbracket_{\mathcal{M}, w_1, t_2, g} = b$
 $\llbracket \sim k \rrbracket_{\mathcal{M}, w_2, t_1, g} = d$
 $\llbracket \sim k \rrbracket_{\mathcal{M}, w_2, t_2, g} = b$

The IL of PTQ

A typed higher order λ language with $=$ and \wedge / \sim

- $\neg, \wedge, \vee, \rightarrow, \leftrightarrow, \mathbf{F}, \mathbf{P}, \Box, =$ (syncategorematically)
- $t, e \in \text{Type}$ ($\text{Con}_{\text{type}}, \text{Var}_{\text{type}}$)
- if $a, b \in \text{Type}$, then $\langle a, b \rangle \in \text{Type}$
- if $a \in \text{Type}$, then $\langle s, a \rangle \in \text{Type}$
- $s \notin \text{Type}$

- ME_{type}
- abstraction: if $\alpha \in ME_a, \beta \in Var_b, \lambda\beta\alpha \in ME_{\langle b,a \rangle}$
- FA: if $\alpha \in ME_{\langle a,b \rangle}, \beta \in ME_a$ then $\alpha(\beta) \in ME_b$
- if $\alpha, \beta \in ME_a$ then $\alpha = \beta \in ME_t$

Interpretations of $\hat{}$ and \sim

- if $\alpha \in ME_a$ then $\hat{\alpha} \in ME_{s,a}$
- if $\alpha \in ME_{\langle s,a \rangle}$ then $\sim\alpha \in ME_a$

type	variables	constants
e	x, y, z	a, b, c
$\langle s, e \rangle$	x, y, z	—
$\langle e, t \rangle$	X, Y	$walk', A, B$
• $\langle \langle s, e \rangle, t \rangle$	Q	$rise', change'$
$\langle s, \langle e, t \rangle \rangle$	P	—
$\langle e, e \rangle$	P	Sq
$\langle e, \langle e, t \rangle \rangle$	R	Gr, K
$\langle e, \langle e, e \rangle \rangle$	—	$Plus$

- $\langle A, W, T, <, F \rangle$
- $D_{\langle a,b \rangle} = D_b^{D_a}$
- $D_{\langle s,a \rangle} = D_a^{W \times T}$
- 'senses' = **possible** denotations
- actual intensions chosen from the set of senses
- now: $F(\text{expression}) = \text{intension (itself a function)}$
- s.t. $\text{intension}(\text{index}) = \text{extension}$
- instead of: $F(\text{expression})(\text{index}) = \text{extension}$

Some interpretations

- $\llbracket \lambda u \alpha \rrbracket^{\mathcal{M}, w, i, g}$, $u \in \text{Var}_b$, $\alpha \in ME_a$ is a function h with domain D_b s.t. $x \in D_b$, $h(x) = \llbracket \alpha \rrbracket^{\mathcal{M}, w, t, g'}$ with g' exactly like g except $g'(u) = x$
- $\llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}$ is a function h from $W \times T$ to denotations of α 's type s.t. at every $\langle w', t' \rangle \in W \times T$ $\llbracket \alpha \rrbracket^{\mathcal{M}, w', t', g} = h(\langle w', t' \rangle) = \llbracket \hat{\alpha} \rrbracket^{\mathcal{M}, w, i, g}(\langle w', t' \rangle)$

Some examples

- $\alpha = \beta$ at $\langle w, t \rangle$ might be true, but $\hat{\alpha} = \hat{\beta}$ need not be 1 at that same index
- on types:
 - ▶ e - individuals
 - ▶ $\langle s, e \rangle$ - individual concepts ('present Queen of England')
 - ▶ $\langle s, \langle e, t \rangle \rangle$ - properties of individuals
 - ▶ $\langle e, t \rangle$ - sets of individuals
 - ▶ $\langle \langle s, e \rangle, t \rangle$ - sets of individual concepts

- on properties:
 - ▶ $\langle s, \langle a, t \rangle \rangle$ - properties of denotations of a -type expressions
 - ▶ $\langle s, \langle e, t \rangle \rangle$ - properties of individuals
 - ▶ $\langle s, \langle \langle s, t \rangle, t \rangle \rangle$ - properties of propositions
- from relations $\langle e, \langle e, t \rangle \rangle$ to relations-in-intensions $\langle s, \langle e, \langle e, t \rangle \rangle \rangle$

- In IL indices are never denoted by expressions!
- Expressions denote functions in the domain of indices.
- hence: $\langle s, a \rangle$ never applied to some typed argument (s is not a type!)
- useful thing: We never talk about indices!
- since often $\sim \alpha(\beta)$ is needed for $\alpha \in ME_{\langle s, \langle e, t \rangle \rangle}$ and $\beta \in ME_e$, abbr. $\alpha\{\beta\}$

Examples

- former problem with **Nec** as $\langle t, t \rangle$: non-compositional extensional interpretation
- $\mathbf{Nec} \in ME_{\langle \langle s, t \rangle, t \rangle} - \{0, 1\}^{(\{0, 1\}^{W \times T})}$
- from (from indices to truth values = propositions) to truth values
- we could give $\Box\phi$ as $\mathbf{Nec}(\hat{\phi})$

- ‘former’ as in ‘a former member of this club’
- instead of $\langle\langle e, t \rangle, \langle e, t \rangle\rangle$
- intensionally: $\langle\langle s, \langle e, t \rangle \rangle, \langle e, t \rangle\rangle$
- extensions at all indices accessible via intension: those individuals bearing property $\langle e, t \rangle$ not at current but at some past index qualify
- formally: $\llbracket \text{For} \rrbracket_{\mathcal{M}, g}^{\mathcal{M}, g}$ is a func. h s.t. for any property k , $h(\langle w, t \rangle)(k)$ is the set $k(\langle w, t' \rangle)$ for all $t' < t$.
- So, for any individual x $h(\langle w, t \rangle)(k)(x) = 1$ iff $k(\langle w, t' \rangle)(x) = 1$ for some $t' < t$.

- relations between individuals and propositions
- $\langle \langle s, t \rangle, \langle e, t \rangle \rangle$
- $\text{Bel}(\wedge(B(m))(j))$ *John believes that Miss America is bald.*
- take the model from page 134 (Dowty et al.):
- $\llbracket B(m) \rrbracket^{M, w_2, t_1, g} = 1$ since $\llbracket m \rrbracket^{M, w_2, t_1, g} = \llbracket n \rrbracket^{M, w_2, t_1, g}$
- however: $\llbracket \wedge(B(m)) \rrbracket^{M, w_2, t_1, g} \neq \llbracket \wedge(B(n)) \rrbracket^{M, w_2, t_1, g}$

- $\text{Bel}(\hat{}(B(m))(j))$ 'John believes that Miss America is bald.'
- $\text{Bel}(\hat{}(B(n))(j))$ 'John believes that Norma is bald.'
- needn't be equal: John can take worlds other than $\langle w_2, t_1 \rangle$ into account where $\llbracket n \rrbracket \neq \llbracket m \rrbracket$
- $\alpha = \beta \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$ is true iff α is not in the scope of $\hat{}, \mathbf{F}, \mathbf{P}, \square$ (oblique contexts)
- however: $\hat{}\alpha = \hat{}\beta \rightarrow [\phi \leftrightarrow \phi^{[\alpha/\beta]}]$

- like so: $\lambda x [\mathbf{Bel}(\hat{B}(x))(j)](m)$
- the above is true at an index $\langle w, t \rangle$ iff $\llbracket \mathbf{Bel}(\hat{B}(x))(j) \rrbracket^{w,t} = 1$
if $\llbracket m \rrbracket^{w,t} = x$, i.e. if John is in a believe-rel with $\hat{B}(x)$
s.t. $g(x) = m$ (by semantics of λ)
- Why is $\hat{B}(x)$ not equal to $\hat{B}(m)$?
- constant m : non-rigid designator relativized to indices
- variable x : a rigid designator by def. of g (for the relevant checking case with $g(x) = \text{MissAmerica}$)
- the above: a belief about 'whoever m is'
- λ conversion is restricted in IL!

- *John believes that a republican will win.*
- $\exists x [Rx \wedge \mathbf{Bel}(j, \wedge [\mathbf{FW}(x)])]$
- $\mathbf{Bel}(j, \mathbf{F}\exists x [R(x) \wedge W(x)])$

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