Time Series Models

Forecasting Method	Applicability
Moving average	No trend or seasonality
Simple exponential smoothing	No trend or seasonality
Holt's model	Trend but no seasonality
Winter's model	Trend and seasonality

1

Measures of Forecast Error

$$MSE_n = \frac{1}{n}\sum_{t=1}^{n} E_t^2$$

$$A_t = |E_t| \qquad MAD_n = \frac{1}{n}\sum_{t=1}^{n} A_t \qquad \sigma = 1.25 MAD$$

$$MAPE_n = \frac{\sum_{t=1}^{n} \left| \frac{E_t}{D_t} \right| 100}{n}$$

2

Why do Buffers Build? Why hold Inventory?

- Economies of scale
  - Fixed costs associated with batches
  - Quantity discounts
  - Trade Promotions

Cycle/Batch stock

- Uncertainty
  - Information Uncertainty
  - Supply/demand uncertainty

Safety stock

- Seasonal Variability

Seasonal stock

- Strategic
  - Flooding, availability

Strategic stock

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Cost of Inventory

- Physical holding cost
  (out-of-pocket)
- Financial holding cost
  (opportunity cost):
  - Cost of capital *r* (%/yr)
  - Annual opportunity cost
- Low responsiveness
  - to demand/market changes
  - to supply/quality changes

Holding cost

H = annual *unit* holding cost

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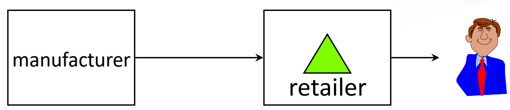
Radical Waves DB

The annual demand for the surfing boards is **800** which is met by contracting out the production to a factory in Hawaii. Radical Waves placed orders with the factory throughout the year for a total of **8 orders**, **each for 100 boards**. The boards are **shipped by air** in a dedicated container which could hold up to 1000 boards for **$2500**. **The cost per board is $250** and **the annual cost of capital is 20%.**

1. Evaluate the current policy.
2. What order size would you recommend for Radical Waves in current supply network?



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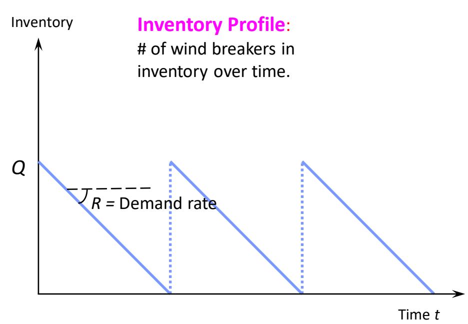
Economies of Scale: Inventory Build-Up Diagram

R: Annual demand *rate*,

Q: Number of jackets per replenishment order

- Number of orders per year = R/Q.
- Average number of boards in inventory = $Q/2$.



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Radical Waves: evaluation of current policy of ordering Q = 100 boards each time

1. What is average inventory I?
   - ❑ $I = Q/2 =$
   - ❑ Annual cost to hold one unit H =
   - ❑ Annual cost to hold I = *Holding cost* × *Inventory*

2. How often do we order?
   - ❑ Annual throughput R =
   - ❑ # of orders per year = *Throughput / Batch size*
   - ❑ Annual order cost = *Order cost* × *# of orders*

3. What is total cost?
   - ❑ TC = *Annual holding cost* + *Annual order cost* + *Annual purchasing cost* =

4. What happens if order size changes?

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Economies of Scale: Economic Order Quantity *EOQ*

R : Demand per year,

S : Setup or Order Cost ($/setup; $/order),

H : Marginal annual holding cost ($/per unit per year),

Q : Order quantity.

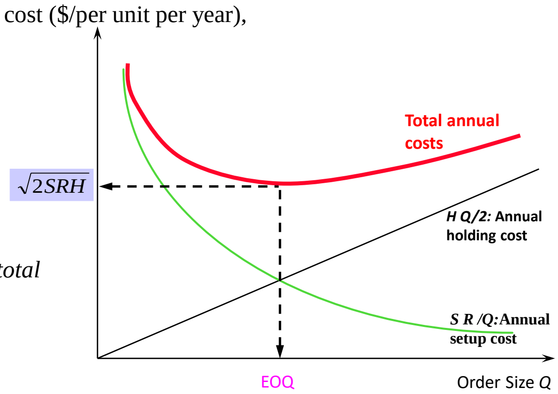
C : Cost per unit ($/unit),

r : Cost of capital (%/yr),

$H = r\,C.$

The order quantity that minimizes total

$$Q_{EOQ} = \sqrt{\frac{2SR}{H}}$$



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Optimal Economies of Scale:

For Radical Waves DB

R = 800 units/ year | C = \$ 250 / unit

r = 0.20/year | S = \$ 2,500 / order

Unit annual holding cost = H = 0.20/yr x \$250 = \$50/yr

Optimal order quantity = Q = sqrt(2 x 800 x 2500/50) = 283

Number of orders per year = R/Q = 2.8

Time between orders = Q/R = 0.35 yr = 4.2 months

Annual order cost = $(R/Q)S$ = \$7071/yr

Average inventory $I = Q/2$ = 141.5

Annual holding cost = $(Q/2)H$ =\$7071/yr

Average flow time $T = I/R$ = 0.18 yr = 2.1 months

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Optimal Economies of Scale:

Managerial Insights

$$Q_{EOQ} = \sqrt{\frac{2SR}{H}} \qquad C_{EOQ} = \sqrt{2SRH}$$

- How to grow: Amazon vs. Costco
- How cut inventories (economically smart)?

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Basic Batching: Lessons

- In deciding the optimal lot / batch size, **the trade off** is between **fixed order (setup) cost** and **holding cost**. It is changeover / setup cost and not time that is relevant.

- Aim for **smaller batch sizes** for products that have **high value** or become **obsolete quickly**.

- If demand increases by a factor of k, it is optimal to increase batch size by a factor of $\sqrt{k}$ and produce (order) $\sqrt{k}$ as often. *Flow time (and cycle inventory measured in days of demand) should decrease as demand increases without change in variety*.

- If lot size is to be reduced *without hurting total cost*, one has to reduce fixed order cost. To reduce lot size by a factor of k, order cost has to be reduced by a factor of k^2.

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Role of Leadtime *L*: *Radical Waves DB* cont.

- The lead time from when a *Radical Waves DB* places an order to when the order is received is **one month**. If demand is stable as before, when should the retailer place an order?

- I-Diagram:

- The two key decisions in inventory management are:
  - How much to order?
  - When to order?

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Radical Waves DB: Service levels & inventory management

- In reality, a *Radical Waves* DB's demand fluctuates from month to month. Monthly demand data for the past 10 years are uploaded in Canvas. The average monthly demand is about 67 boards (=800/12); the order lead time is a month; fixed order costs are $2,500/order and it costs $50 to hold one board in inventory during one year.
- Questions:
  1. If the Radical Waves uses the ordering policy discussed before, what will the probability of running out of stock in a given cycle, based on the past 10 years of data?
  2. They would like the stock-out probability to be smaller. How can they accomplish this?
  3. Specifically, how does it get the service level up to 95%?

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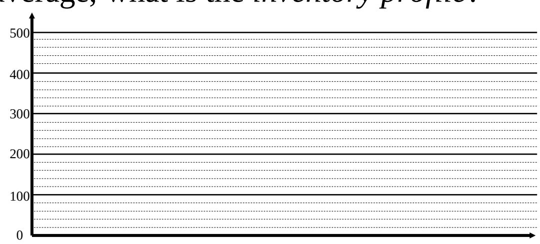
Example: say we increase ROP to 87 (and keep order size at Q = 283)

1. On average, what is the stock level when the replenishment arrives?
2. On average, what is the *inventory profile*?



3. What is the probability that we run out of stock?
4. How do we get that stock-out probability down to 5%?

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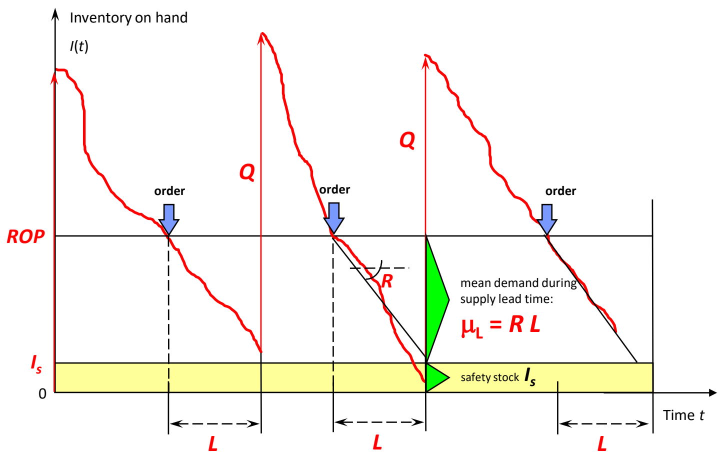
Safety Stocks



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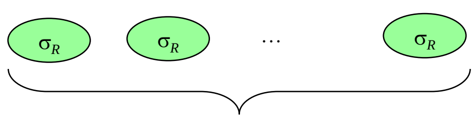
Safety stock: How find s of lead time demand?
A Fundamental Statistics Result: The Portfolio Effect



Sum of N *independent* random variables, each with identical standard deviation σ_R, has standard deviation =

Applications:

Demand over the leadtime L has standard deviation = $s_R \times \sqrt{L}$

Pooled demand over N regions or products has standard deviation =

$$s_R \times \sqrt{N}$$

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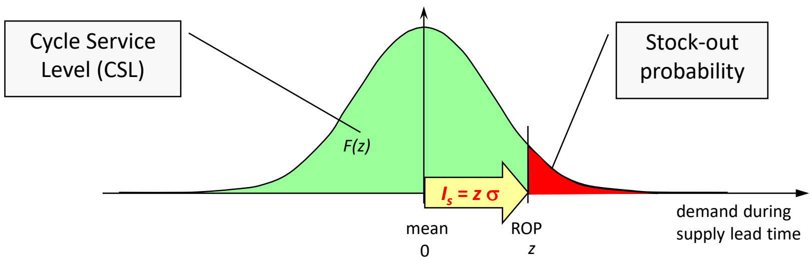
Safety Stocks & Service Levels: The relationship



- Raise ROP until we reach appropriate SL
- To do numbers, we need:
  - Mean μ and stdev s of demand during lead time
  - Either Excel or tables with z-value such that CSL = $F(z)$

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1. How to find service level (given ROP)?
2. How to find re-order point (given SL)?

L	Lead time		
D	Demand per unit time	Mean: R Std. Dev.: σ_R	
D_L	Demand during lead time	Mean: μ_L Std. Dev.: σ_L	μ_L = RL $\sigma_L = \sigma_R \sqrt{L}$

1. Given ROP, find SL $= P(\text{no stock out})$

   $= P(\text{demand during lead time} \leq ROP)$

   $= F(z^* = (ROP - m_L)/s_L)$

2. Given SL, find ROP $= m_L + \boldsymbol{I_s}$

   $= m_L + z^* s_L$ [use table to get z^*]

Safety stock $\boldsymbol{I_s} = z^* s_L$; Reorder point $\boldsymbol{ROP} = m_L + \boldsymbol{I_s}$

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Learning Objectives: *Batching & Economies of Scale*

- Increasing batch size Q of order (or production) increases average inventories (and thus flow times).
  - Average inventory for a batch size of Q is $Q/2$.
- The optimal batch size minimizes supply chain costs by trading off setup cost and holding cost and is given by the EOQ formula.
- To reduce batch size, one must *reduce setup cost (time).*
- **Economies of scale** are manifested by **the square-root relationship** between Q_{EOQ} and (R, S):
  - If demand increases by a factor of 4, it is optimal to increase batch size by a factor of 2 and produce (order) twice as often.
  - To reduce batch size by a factor of 2, setup cost has to be reduced by a factor of 4.

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Learning Objectives safety stocks

$$I_s = z^* \sigma_R \sqrt{L}$$

- Safety stock is a hedge against uncertainty
- Which factors drive s*afety stock* ?
  - level of service z
    - » Impact of increased service level on required safety stock
  - demand variability or forecast error s_R,
  - delivery lead time L for the same level of service,
  - delivery lead time variability for the same level of service.
- Applications:
  - How measure safety stock in practice?
  - What is the value of a better information system?

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Replenishment Policies

1. **Continuous review**
   – Inventory is continuously tracked
   – Order for a lot size *Q* is placed when the inventory declines to the reorder point (*ROP*)
2. **Periodic review**
   – Inventory status is checked at regular periodic intervals
   – Order is placed to raise the inventory level to a specified threshold

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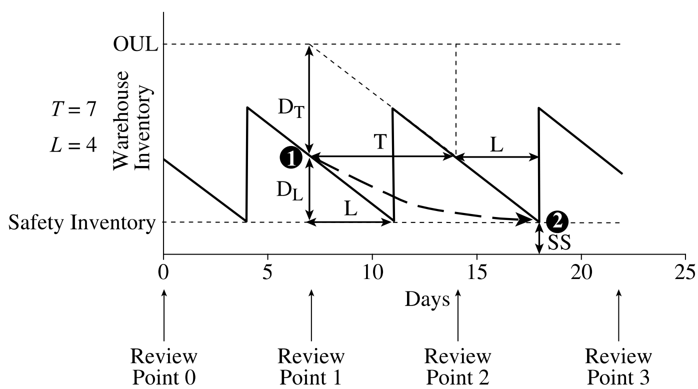
Periodic Review Policy



Periodic Review Policy with Replenishment Lead time L = 4 days, and Review Interval T = 7 days

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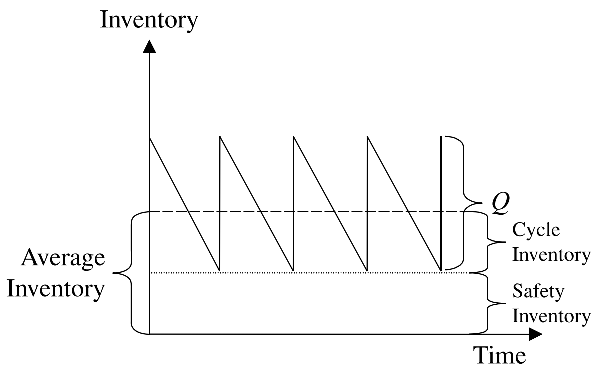
Cycle Inventory & Safety Inventory



Average Inventory Profile

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Radical Waves DB: Periodic Review Policy

- In reality, a *Radical Waves* DB's demand fluctuates from month to month. In fact, monthly demand at a store had a standard deviation of about 15 boards → assume roughly normally distributed. Recall that average monthly demand was about 67 boards (=800/12); the order lead time is a month; fixed order costs are $2,500/order and it costs $50 to hold one board in inventory during one year. **Instead of continuously reviewing the inventory level, *Radical Waves* reviews inventory every two months.**

- Questions:
  1. When reviewed, the current inventory level was 100 boards, how many boards they have to order to have 95% service level?

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Measuring Product Availability

1. **Product fill rate (fr)**
   – Fraction of product demand satisfied from product in inventory
2. **Order fill rate**
   – Fraction of orders filled from available inventory
3. **Cycle service level (CSL)**
   – Fraction of replenishment cycles that end with all customer demand being met
   – **Replenishment cycle** – the interval between two successive replenishment deliveries

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Evaluating Fill Rate

- **Expected shortage per replenishment cycle** (*ESC*) is the average units of demand that are not satisfied from inventory in stock per replenishment cycle
- Product fill rate

$$fr = 1 - \frac{ESC}{Q} = \frac{(Q - ESC)}{Q}$$

- If you know the Fill Rate: $ESC = (1 - fr)Q$

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Example: Periodic Review Policy

Periodic Review Policy with Replenishment Lead time *L* = 4 days, and Review Interval *T* = 7 days
Daily demand is given in Canvas for the past 1000 days.
Target cycle service level is 90%.

Calculate the following:

OUL:
Average Order Quantity:
Average Cycle Stock:
Average Safety Stock:
Average Inventory
Average Flow Time:
Annual Inventory Turn-over":

Average Fill rate:

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