case

October 20, 2024

[4]:
```python
import numpy as np
import pandas as pd
from scipy.stats import norm, f_oneway

import matplotlib.pyplot as plt
```

[5]:
```python
sales_dat = pd.read_csv('Sales.csv')
nyop_dat = pd.read_csv('NYOP.csv')
```

[6]:
```python
sales_dat.head()
```

[6]:

	Condition	NumberSold	Riders	MerchandiseRevenues
0	FR	77	12663	4592.41
1	FR	63	15561	6688.57
2	FR Charity	79	14796	6476.78
3	FR Charity	101	15796	5845.94
4	NYOP	1137	14077	4845.27

1 Flat Rate Pricing

$$H_0 : p_1 = p_2 H_a : p_1 \neq p_2$$

We are attempting to see difference in proportions between the charity and non charity cases. Under the null, we expect such proportions to be equal, whereas the alternative believes the proportions to be different.

[7]:
```python
flat_rates = sales_dat[(sales_dat['Condition'] == "FR") |␣
↪(sales_dat['Condition'] == "FR Charity")]
fr_rates = flat_rates.groupby('Condition')['NumberSold'].sum() / flat_rates.
↪groupby('Condition')['Riders'].sum()
fr_rates
```

[7]:
```
Condition
FR            0.004960
FR Charity    0.005884
dtype: float64
```

[8]:
```
n_s = flat_rates.groupby('Condition')['Riders'].sum()
n_s
```

[8]:
```
Condition
FR             28224
FR Charity     30592
Name: Riders, dtype: int64
```

[9]:
```
p1 = fr_rates.iloc[0]
p2 = fr_rates.iloc[1]
n1 = n_s.iloc[0]
n2 = n_s.iloc[1]
```

[10]:
```
pt1 = p1*(1-p1) / n1
pt2 = p2*(1-p2) / n2
denom = np.sqrt(pt1 + pt2)

test_stat = (p1-p2) / denom # Test statistic Z-score
test_stat
```

[10]:
```
-1.5264554280529021
```

[11]:
```
2 * (1 - norm.cdf(abs(test_stat))) # P value
```

[11]:
```
0.12689648269385967
```

There is a 12.689 chance that the FR and FR Charity sales values occur under a normal distribution. Hence, as it is over the 5 percent significance level, you will fail to reject the chance that there is no difference between the proportion of purchases. In other words, we see that the given proportions is likely enough to consider under a normal distribution.

2 NYOP Pricing

[12]:
```
NYOP_sales = sales_dat[(sales_dat['Condition'] == 'NYOP') |␣
↪(sales_dat['Condition'] == 'NYOP Charity')]
NYOP_sales
```

[12]:

	Condition	NumberSold	Riders	MerchandiseRevenues
4	NYOP	1137	14077	4845.27
5	NYOP	1233	14186	7038.63
6	NYOP Charity	539	12227	5690.59
7	NYOP Charity	628	13741	6003.44
8	NYOP Charity	626	18117	8557.47

[13]:
```
NYOP_rates = NYOP_sales.groupby('Condition')['NumberSold'].sum() / NYOP_sales.
↪groupby('Condition')['Riders'].sum()
```

```
NYOP_rates
```

[13]:
```
Condition
NYOP            0.083855
NYOP Charity    0.040671
dtype: float64
```

[14]:
```
n_s = NYOP_sales.groupby('Condition')['Riders'].sum()
n_s
```

[14]:
```
Condition
NYOP            28263
NYOP Charity    44085
Name: Riders, dtype: int64
```

[15]:
```
p1 = NYOP_rates.iloc[0]
p2 = NYOP_rates.iloc[1]
n1 = n_s.iloc[0]
n2 = n_s.iloc[1]
```

[16]:
```
pt1 = p1*(1-p1) / n1
pt2 = p2*(1-p2) / n2
denom = np.sqrt(pt1 + pt2)

test_stat = (p1-p2) / denom
test_stat # Z score test stat
```

[16]:
```
22.749707261972425
```

[17]:
```
2 * (1 - norm.cdf(abs(test_stat))) # P val
```

[17]:
```
0.0
```

There is a 0 percent chance that the NYOP purchase rates occur under a normal distribution. Therefore, you must reject the null hypothesis that there is no difference between the proportion of purchases of the NYOP and NYOP charity. Hence, we must conclude that for the NYOP case, that there is statistical difference between the charity and non charity case.

3 Section 2

3.1 Part A

[18]:
```
nyop_dat.head()
```

[18]:

	Condition	Number	Price
0	NYOP	1	1.00
1	NYOP	1	1.00

2	NYOP	1	0.01
3	NYOP	1	0.10
4	NYOP	1	0.01

[19]:
```
nyop_dat['UnitPrice'] = nyop_dat['Price'] / nyop_dat['Number']
nyop_dat.head()
```

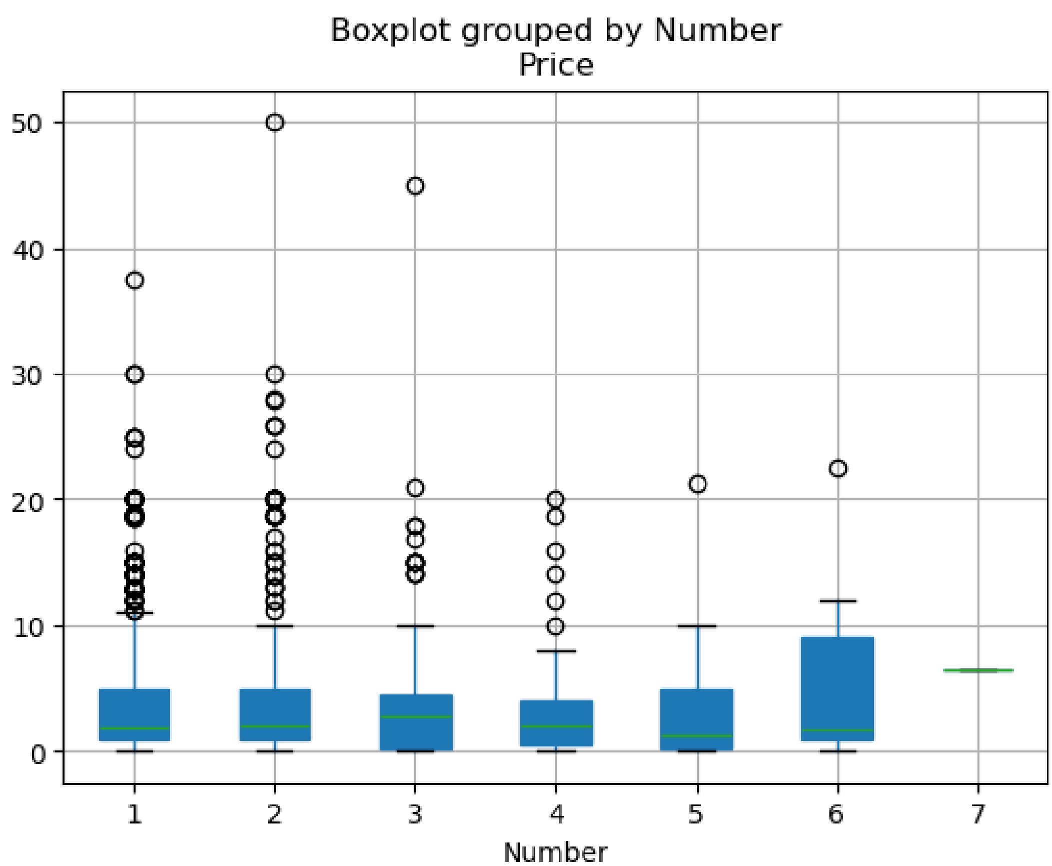
[19]:

	Condition	Number	Price	UnitPrice
0	NYOP	1	1.00	1.00
1	NYOP	1	1.00	1.00
2	NYOP	1	0.01	0.01
3	NYOP	1	0.10	0.10
4	NYOP	1	0.01	0.01

3.2 Part B

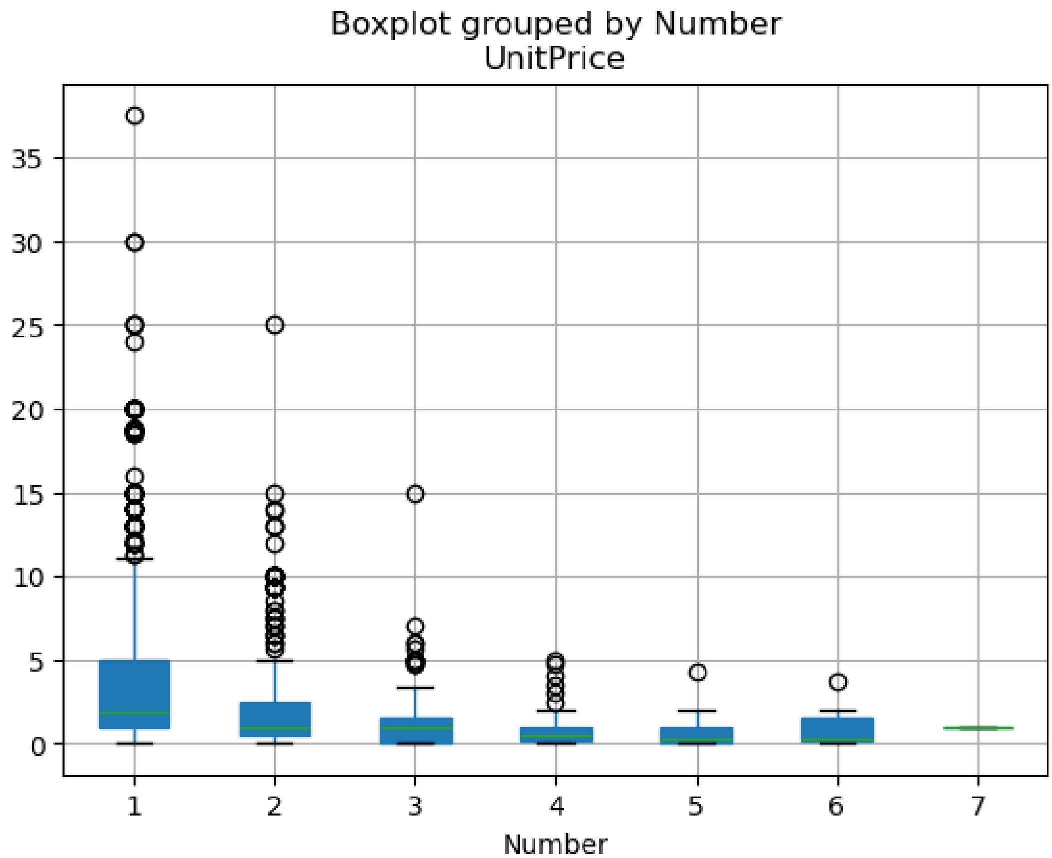
[20]:
```
nyop_dat['Number'] = pd.Categorical(nyop_dat['Number'])
```

[21]:
```
nyop_dat.boxplot(column='Price',by='Number', patch_artist = True)
plt.show()
```



Under the the various number of photos bought against the Price, we see that there is no strong relationship between the two variables. Although under 50 percent of the time, we see that the price paid for photos is between 0 and 10 dollars, there are a significant spread of out outliers that consider at each price level except for the 7 photo case.

[22]:
```
nyop_dat.boxplot(column='UnitPrice',by='Number', patch_artist = True)
plt.show()
```



Unlike the price relationship, we see in this case there is a clear negative relationship between the number and unit price for the photos paid under the NYOP case. Overall, as the number of photos increases, we see that the price paid per photo decreases. This seems to hold under the center spread between the 25th and 75th percentile and the outlier values.

3.3 Part C

[23]:
```
avg_unit_price = nyop_dat.groupby('Condition')['UnitPrice'].mean()
avg_unit_price
```

[23]:
```
Condition
NYOP            1.040439
NYOP Charity    5.680480
Name: UnitPrice, dtype: float64
```

There seems to be a substantial difference between the unit price of the charity and non charity cases. To do this, we can compare the difference in means under the two conditions with a two sided test.

3.4 Part D

$$H_0 : UnitPrice_{\text{NYOP}} = UnitPrice_{\text{NYOP Charity}} H_a : UnitPrice_{\text{NYOP}} \neq UnitPrice_{\text{NYOP Charity}}$$

Under the null we expect the unit price between the charity and non charity case to be the same; or in other words, there is no significance difference in the average unit price for both conditions.

[24]:
```python
import pyrsm as rsm
```

3.5 Part E

[25]:
```python
cm = rsm.basics.compare_means({'NYOP': nyop_dat}, var1='Condition',␣
↪var2='UnitPrice', alt_hyp='two-sided')
cm.summary()
```

```
Pairwise mean comparisons (t-test)
Data      : NYOP
Variables : Condition, UnitPrice
Samples   : independent
Confidence: 0.95
Adjustment: None
   Condition  mean    n  n_missing    sd    se    me
        NYOP  1.04 1641          0 1.305 0.032 0.063
NYOP Charity  5.68 1457          0 4.670 0.122 0.240
          Null hyp.                         Alt. hyp.  diff p.value
NYOP = NYOP Charity NYOP not equal to NYOP Charity -4.64  < .001 ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- Type 1: Under a significance level level of 5 percent, there is a 5 percent chance of claiming that the average unit price under the two conditions are difference when there is none.
- Type 2: If we fail to reject the null hypothesis to state that there is no difference when there actually was.

Regardless, under both circumstances, we see that because the p value is smaller than 0.001, there is an extremely small likelihood that a Type 1 or even a Type 2 is committed under this difference of means t-test.

3.6 Part F

[26]:
```
one_pic = nyop_dat[nyop_dat['Number'] == 1]
six_pic = nyop_dat[nyop_dat['Number'] == 6]
```

[27]:
```
cm2 = rsm.basics.compare_means({'one_pic': one_pic}, var1='Condition',␣
↪var2='UnitPrice', alt_hyp='two-sided')
cm2.summary()
```

```
Pairwise mean comparisons (t-test)
Data      : one_pic
Variables : Condition, UnitPrice
Samples   : independent
Confidence: 0.95
Adjustment: None
   Condition  mean    n  n_missing    sd    se    me
        NYOP 1.177 1162          0 1.432 0.042 0.082
NYOP Charity 5.941 1203          0 4.830 0.139 0.273
          Null hyp.                       Alt. hyp.   diff p.value
NYOP = NYOP Charity NYOP not equal to NYOP Charity -4.765  < .001 ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Under this condition, we see that the p value for the 1 picture condition is extremely small. Hence, we reject the null hypothesis that there is a difference between the mean unit price between the charity and non charity case.

[28]:
```
cm3 = rsm.basics.compare_means({'six_pic': six_pic}, var1='Condition',␣
↪var2='UnitPrice', alt_hyp='two-sided')
cm3.summary()
```

```
Pairwise mean comparisons (t-test)
Data      : six_pic
Variables : Condition, UnitPrice
Samples   : independent
Confidence: 0.95
Adjustment: None
   Condition  mean  n  n_missing    sd    se    me
        NYOP 0.495  6          0 0.615 0.251 0.646
NYOP Charity 1.970  3          0 1.795 1.036 4.459
          Null hyp.                       Alt. hyp.   diff p.value
NYOP = NYOP Charity NYOP not equal to NYOP Charity -1.475   0.288

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Under this condition, we see that the p value for the 6 picture condition is at 0.288; this is the large compared to the 0.05 significance level. Hence, we fail to reject the null hypothesis that there is a difference between the mean unit price between the charity and non charity case.

3.7 Part G

$$H_0 : \mu_1 = \mu_2 H_a : \mu_1 \neq \mu_2$$

Under the 6 picture case, the null states that the means between the charity and non charity case is the same. The alternative hypothesis case states that the means between the two cases are not the same.

[29]:
```
six_pic.head()
```

[29]:
```
     Condition  Number  Price  UnitPrice
296       NYOP       6   0.06       0.01
356       NYOP       6   6.00       1.00
382       NYOP       6   0.90       0.15
623       NYOP       6   0.06       0.01
1445      NYOP       6   9.00       1.50
```

[30]:
```
six_nyop = six_pic[six_pic['Condition'] == 'NYOP']['UnitPrice']
six_char = six_pic[six_pic['Condition'] == 'NYOP Charity']['UnitPrice']
```

[31]:
```
from scipy.stats import t
```

[32]:
```
xbar_1 = six_nyop.mean()
xbar_2 = six_char.mean()
s1 = six_nyop.std(ddof=1)
s2 = six_char.std(ddof=1)
n1 = len(six_nyop)
n2 = len(six_char)
```

[33]:
```
SE = np.sqrt((s1**2 / n1) + (s2**2 / n2))
```

[34]:
```
t_val = (xbar_1 - xbar_2) / SE
t_val
```

[34]:
```
-1.3830917803702296
```

[35]:
```
numer = (s1**2 / n1 + s2**2 / n2)**2
denom = (s1**2 / n1)**2 / (n1 -1) + (s2**2 / n2)**2 / (n2-1)
dof = numer/denom # Use welch's t test, so variances are not equal
```

[36]:
```
pval = (1 - t.cdf(abs(t_val), df=dof)) * 2
pval
```

[36]:
```
0.2884114566627223
```

According to this difference of means test, we see that the test coincides as part f. Overall, we fail to reject the null hypothesis as the p value is much larger than the 0.05 significance level.

4 Economics

5 Part A + B

5.1 FR Case

[37]:
```
sales_dat
```

[37]:

	Condition	NumberSold	Riders	MerchandiseRevenues
0	FR	77	12663	4592.41
1	FR	63	15561	6688.57
2	FR Charity	79	14796	6476.78
3	FR Charity	101	15796	5845.94
4	NYOP	1137	14077	4845.27
5	NYOP	1233	14186	7038.63
6	NYOP Charity	539	12227	5690.59
7	NYOP Charity	628	13741	6003.44
8	NYOP Charity	626	18117	8557.47

[38]:
```
fr_data = sales_dat[(sales_dat['Condition'] == 'FR') | (sales_dat['Condition']␣
↪== 'FR Charity')].copy()
fr_data['cogs'] = fr_data['NumberSold'] * 1.2
fr_data['revenues'] = fr_data['NumberSold'] * 12.95
fr_data.loc[fr_data['Condition'] == 'FR Charity', 'revenues'] = fr_data.
↪loc[fr_data['Condition'] == 'FR Charity', 'revenues'] * 0.5
fr_data['profit'] = fr_data['revenues'] - fr_data['cogs']
```

[39]:
```
fr_avg_profit = fr_data.groupby('Condition')['profit'].mean()
fr_avg_profit
```

[39]:
```
Condition
FR            822.50
FR Charity    474.75
Name: profit, dtype: float64
```

5.2 NYOP Case

[40]:
```
nyop_dat
```

[40]:

	Condition	Number	Price	UnitPrice
0	NYOP	1	1.00	1.00
1	NYOP	1	1.00	1.00
2	NYOP	1	0.01	0.01
3	NYOP	1	0.10	0.10
4	NYOP	1	0.01	0.01
…	…	…	…	…
3093	NYOP Charity	1	9.38	9.38

```
3094   NYOP Charity      1   1.00       1.00
3095   NYOP Charity      1   0.93       0.93
3096   NYOP Charity      1   9.38       9.38
3097   NYOP Charity      1   1.87       1.87

[3098 rows x 4 columns]
```

[41]:
```python
nyop_dat_rev = nyop_dat.copy()
nyop_dat_rev['Number'] = (nyop_dat_rev['Number']).astype(int)
nyop_dat_rev['cogs'] = nyop_dat_rev['Number'] * 1.2
nyop_dat_rev['revenues'] = nyop_dat_rev['UnitPrice'] * nyop_dat_rev['Number']
nyop_dat_rev.loc[nyop_dat_rev['Condition'] == 'NYOP Charity', 'revenues'] = nyop_dat_rev.loc[nyop_dat_rev['Condition'] == 'NYOP Charity', 'revenues'] * 0.5
nyop_dat_rev['profit'] = nyop_dat_rev['revenues'] - nyop_dat_rev['cogs']
nyop_dat_rev
```

[41]:
```
         Condition  Number  Price  UnitPrice  cogs  revenues  profit
0             NYOP       1   1.00       1.00   1.2     1.000  -0.200
1             NYOP       1   1.00       1.00   1.2     1.000  -0.200
2             NYOP       1   0.01       0.01   1.2     0.010  -1.190
3             NYOP       1   0.10       0.10   1.2     0.100  -1.100
4             NYOP       1   0.01       0.01   1.2     0.010  -1.190
…              …     …     …         …   …       …     …
3093  NYOP Charity       1   9.38       9.38   1.2     4.690   3.490
3094  NYOP Charity       1   1.00       1.00   1.2     0.500  -0.700
3095  NYOP Charity       1   0.93       0.93   1.2     0.465  -0.735
3096  NYOP Charity       1   9.38       9.38   1.2     4.690   3.490
3097  NYOP Charity       1   1.87       1.87   1.2     0.935  -0.265

[3098 rows x 7 columns]
```

[42]:
```python
nyop_daily = (nyop_dat_rev.loc[nyop_dat_rev['Condition'] == 'NYOP', :].groupby('Condition').sum() / 2)['profit']
nyop_char_daily = (nyop_dat_rev.loc[nyop_dat_rev['Condition'] == 'NYOP Charity', :].groupby('Condition').sum() / 3)['profit']
```

[43]:
```python
nyop_daily_prof = pd.concat([nyop_daily, nyop_char_daily])
nyop_daily_prof
```

[43]:
```
Condition
NYOP            -334.100000
NYOP Charity     885.518333
Name: profit, dtype: float64
```

5.2.1 All daily Profit

[44]:
```
all_daily = pd.concat([nyop_daily_prof, fr_avg_profit])
all_daily.sort_values(ascending=False)
```

[44]:
```
Condition
NYOP Charity    885.518333
FR              822.500000
FR Charity      474.750000
NYOP           -334.100000
Name: profit, dtype: float64
```

[45]:
```
all_daily.idxmax() # The highest profit strategy
```

[45]:
```
'NYOP Charity'
```

6 Part C

[46]:
```
charity_dat = fr_data[fr_data['Condition'].str.contains('Charity')]
charity_dat
```

[46]:
```
     Condition  NumberSold  Riders  MerchandiseRevenues   cogs  revenues  \
2  FR Charity          79   14796              6476.78   94.8   511.525
3  FR Charity         101   15796              5845.94  121.2   653.975

    profit
2  416.725
3  532.775
```

[47]:
```
chairty_rev_fr = ((charity_dat['NumberSold'] * 12.95 -␣
  ↪charity_dat['NumberSold'] * 1.2)).sum()
chairty_rev_fr
```

[47]:
```
2115.0
```

[48]:
```
nyop_dat_charity = nyop_dat_rev[nyop_dat_rev['Condition'].str.
  ↪contains('Charity')].copy()
nyop_dat_charity.loc[:,'total'] = nyop_dat_charity.loc[:,'revenues'] * 2
charity_rev_nyop = nyop_dat_charity['total'].sum() - (nyop_dat_charity['cogs'].
  ↪sum())
charity_rev_nyop
```

[48]:
```
7464.710000000002
```

[49]:
```
charity_profit = pd.DataFrame({'profit': [charity_rev_nyop, chairty_rev_fr],␣
  ↪'Condition': ['NYOP Charity', 'FR Charity']})
```

```
charity_profit
```

[49]:

	profit	Condition
0	7464.71	NYOP Charity
1	2115.00	FR Charity

Considering the total profits between both the park and charity, we see that the NYOP strategies outperform the FR method.

[50]:
```
charity_profit.sort_values('profit').loc[0, 'Condition'] #The highest strategy␣
↪for making largest societal profit.
```

[50]: 'NYOP Charity'

7 Part D

[51]:
```
sales_dat
```

[51]:

	Condition	NumberSold	Riders	MerchandiseRevenues
0	FR	77	12663	4592.41
1	FR	63	15561	6688.57
2	FR Charity	79	14796	6476.78
3	FR Charity	101	15796	5845.94
4	NYOP	1137	14077	4845.27
5	NYOP	1233	14186	7038.63
6	NYOP Charity	539	12227	5690.59
7	NYOP Charity	628	13741	6003.44
8	NYOP Charity	626	18117	8557.47

[52]:
```
all_daily_df = pd.DataFrame(all_daily).reset_index()
all_daily_df = all_daily_df[all_daily_df['Condition'].str.contains("Charity")␣
↪== False]
all_daily_df
```

[52]:

	Condition	profit
0	NYOP	-334.1
2	FR	822.5

Get all the profit metrics into daily profit values to convert them to yearly terms

[53]:
```
charity_profit.loc[charity_profit['Condition'] == 'NYOP Charity', 'profit'] =␣
↪charity_profit.loc[charity_profit['Condition'] == 'NYOP Charity', 'profit'] /
↪ 3
charity_profit.loc[charity_profit['Condition'] == 'FR Charity', 'profit'] =␣
↪charity_profit.loc[charity_profit['Condition'] == 'FR Charity', 'profit'] / 2
```

[54]:
```
charity_profit = pd.concat([all_daily_df, charity_profit])
charity_profit
```

[54]:
```
      Condition       profit
0          NYOP  -334.100000
2            FR   822.500000
0  NYOP Charity  2488.236667
1    FR Charity  1057.500000
```

[55]:
```
charity_profit['Yearly_Profit'] = charity_profit['profit'] * 365
charity_profit = charity_profit.sort_values('Yearly_Profit') # The difference␣
 ↪between the strategy profits
charity_profit
```

[55]:
```
      Condition       profit  Yearly_Profit
0          NYOP  -334.100000 -121946.500000
2            FR   822.500000  300212.500000
1    FR Charity  1057.500000  385987.500000
0  NYOP Charity  2488.236667  908206.383333
```

[56]:
```
charity_profit.iloc[-1, 2] - charity_profit.iloc[0, 2]
```

[56]:
```
1030152.8833333335
```

There is a $1030152 difference between the the NYOP Charity and NYOP conditions that are the most and least profitable pricing strategies.

8 Q5

[57]:
```
grouped_sales = sales_dat.groupby('Condition').sum().reset_index()
grouped_sales
```

[57]:
```
      Condition  NumberSold  Riders  MerchandiseRevenues
0            FR         140   28224             11280.98
1    FR Charity         180   30592             12322.72
2          NYOP        2370   28263             11883.90
3  NYOP Charity        1793   44085             20251.50
```

[58]:
```
grouped_sales['MerchSpendingPerRider'] = grouped_sales['MerchandiseRevenues'] /␣
 ↪grouped_sales['Riders']
grouped_sales['PhotoBuyerMerch'] = grouped_sales['MerchandiseRevenues'] /␣
 ↪grouped_sales['NumberSold']
grouped_sales
```

[58]:
```
      Condition  NumberSold  Riders  MerchandiseRevenues  \
0            FR         140   28224             11280.98
```

```
1    FR Charity         180   30592            12322.72
2          NYOP        2370   28263            11883.90
3  NYOP Charity        1793   44085            20251.50

   MerchSpendingPerRider  PhotoBuyerMerch
0               0.399695        80.578429
1               0.402809        68.459556
2               0.420476         5.014304
3               0.459374        11.294757
```

[59]:
```python
sales_dat['MerchSpendingPerRider'] = sales_dat['MerchandiseRevenues'] /␣
 ↪sales_dat['Riders']
sales_dat['PhotoBuyerMerch'] = sales_dat['MerchandiseRevenues'] /␣
 ↪sales_dat['NumberSold']
sales_dat
```

[59]:
```
      Condition  NumberSold  Riders  MerchandiseRevenues  \
0            FR          77   12663              4592.41
1            FR          63   15561              6688.57
2    FR Charity          79   14796              6476.78
3    FR Charity         101   15796              5845.94
4          NYOP        1137   14077              4845.27
5          NYOP        1233   14186              7038.63
6  NYOP Charity         539   12227              5690.59
7  NYOP Charity         628   13741              6003.44
8  NYOP Charity         626   18117              8557.47

   MerchSpendingPerRider  PhotoBuyerMerch
0               0.362664        59.641688
1               0.429829       106.167778
2               0.437739        81.984557
3               0.370090        57.880594
4               0.344198         4.261451
5               0.496167         5.708540
6               0.465412        10.557681
7               0.436900         9.559618
8               0.472345        13.670080
```

[60]:
```python
conditions = [group['MerchandiseRevenues'].values for name, group in sales_dat.
 ↪groupby('Condition')]
f_statistic, p_value = f_oneway(*conditions)


print(f"P-value: {p_value:.4f}")
```

```
P-value: 0.8343
```

$$H_0 : \text{revenue}_{cond_1} = \text{revenue}_{cond_2} = ... \, H_a : \text{revenue}_{cond_1} \neq \text{revenue}_{cond_2} \neq ...$$

H0: Merchandise revenues change with different conditions

H1: Merchandise revenues do not change with different conditions

P-value is 0.8343, which is greater than 0.05, so we fail to reject the null hypothesis. Merchandise revenues do not change with different conditions. So merchandise sales should not be a concern.

The general concern for merchandise sales is related to the crowding out effect of photo purchases against merchandise purchases. Overall, we see that the merchandise spending per rider does not change across the condition they are facing without considering the fact that each person did or did not buy any photos. If anything we see that there is no identifiable relationship between the merchandise sales and photo purchasing. Under examination of the aggregated data, we see that merchandise revenue per rider is the highest under the NYOP + charity condition. Overall, we can see that incorporating SSR into the park's pricing strategy may create perception of positive corporate social responsibility which minimizes the firm's profit driven motivations.