Customer Analytics: Review & Practice session

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Customer Analytics

Reminders

- Peer evaluations for the PFG-bank presentations (Canvas)
- Intra-group peer evaluations: https://rsm-compute-01.ucsd.edu:4443/peer_eval/
- Exam will be in-person and run from 1pm – 4pm PT on Friday 3/22
- Come to room 1E107 at least 5 minutes before the start of the exam to find out if you will be in 1E106 or 1E107

Final exam: What you can expect

- Causality check-lists
- Customer Lifetime Value calculations (CLV)
- Manipulate data (e.g., transform variables, ‘bin’ a continuous variable)
- Exploratory Data Analysis (EDA)
- Linear and Logistic regression (incl prediction plots)
- Evaluate relative importance of explanatory variables (features) from an AD or ML model (permutation importance plots)
- Estimate interactions and generate plots from ML models to identify if an interaction exists
- Use training and test samples
- Generate predictions using Linear/Logistic regression, NN, Random Forests, XGBoost, etc.
- Create lift, gains, and profit charts and evaluate overfitting
- Determine profits and return on marketing expenditures
- Uplift modeling
- Estimate logistic regression on experimental data
- Bias-Variance tradeoff
- Tuning ML models using Cross-Validation
- Understand benefits and limitations of partial factorial design
- …

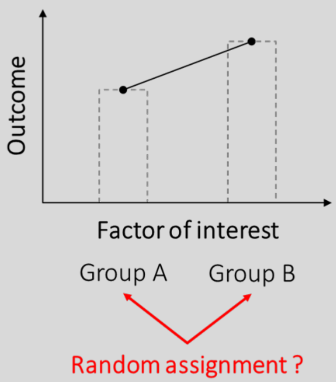
Topics and Tasks for Review session

- Task 1: Causality check list (2 in-class, 2 pre-work)
- Task 2: Linear regression
- Task 3: Logistic regression
- Task 4: (facebook.ipynb | interactions)
- Task 5: Customer Lifetime Value calculation (clv.xlsx and clv.ipynb)
- Task 6: (slow-auc.ipynb)
- Task 7: (bbb_sklearn.ipynb)
- Task 8: (bizware-review.ipynb | experimental design and logistic regression)
- Task 9: (impurity calculations)

The causality checklist

CHECK FOR PROBABILISTIC EQUIVALENCE

Were units randomly assigned to groups?



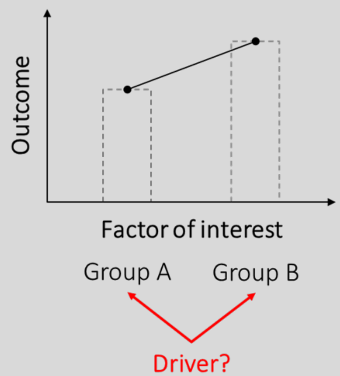
If no, then differences may not have a causal interpretation

If yes, then the analysis passes the causality checklist

IDENTIFY GROUP DRIVERS

Initial evaluation

What drivers influenced assignment of units to groups?



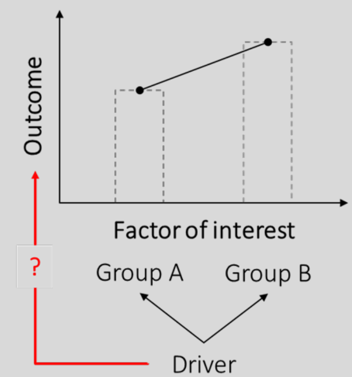
Digging deeper

- Did firm influence group assignment? If so, based on what drivers?
- Did units self select into groups? If so, based on what drivers?
- Are groups separated by time? If so, what outcome related drivers vary over time?

Note: Consider all possible drivers

CHECK FOR CONFOUNDS

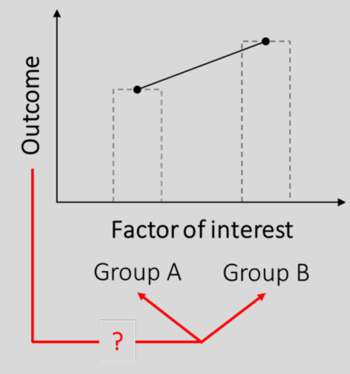
Could a driver have a direct effect on the outcome?



If yes, then the causality check fails because driver is a confound

CHECK FOR REVERSE CAUSALITY

Could group outcomes have a direct impact on the factor of interest?



If yes, then the causality check fails due to reverse causality

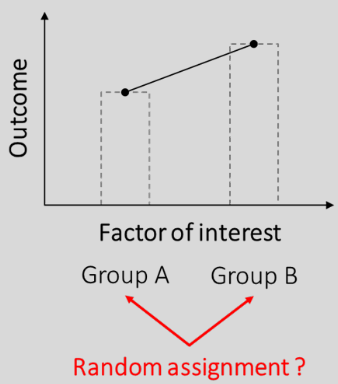
If no, then the analysis passes the causality checklist

If an analysis passes the causality checklist, we conclude that differences in the outcome variable across groups are **caused** by differences in the factor of interest

Review the Google Ads example

CHECK FOR PROBABILISTIC EQUIVALENCE

Were units randomly assigned to groups?

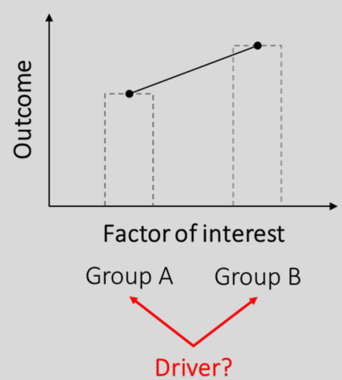


If no, then differences may not have a causal interpretation

If yes, then the analysis passes the causality checklist

IDENTIFY GROUP DRIVERS

What drivers influenced assignment of units to groups?

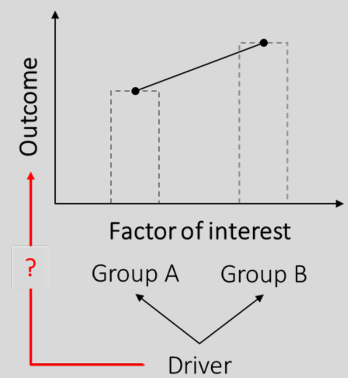


List drivers ...

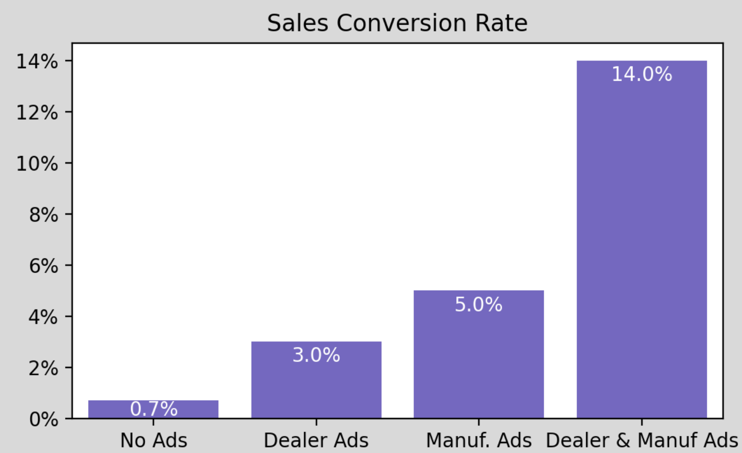
- Google search terms
- Interest in buying a car

CHECK FOR CONFOUNDS

Could a driver have a direct effect on the outcome?



The driver is a confound



Summary of insights from applying the causality checklist to the Google ads example

- **Causal claims**: (1) Google ads work, (2) Retailer and Manufacturer ads are complements
- **Factor of interest**: Car ad exposure
- **Groups**: Customers that saw (1) no ads, (2) retailer ads, (3) manufacturer ads, and (3) retailer and manufacturer ads
- **Outcome**: Sales conversion
- **Group assignment**: Not random
- **Drivers**: Firm influenced group assignment based on consumers' Google search terms reflecting interest in buying a car
- **Confound**: Yes. Interest in buying a car can have a direct effect on the likelihood of buying a car

TASK 1: Apply the checklist to evaluate causal statements

A company sells many snowmobiles in Canada but very few in Mexico. The company also advertises extensively in Canada but does not advertise at all in Mexico

Causal claim: Advertising works

Factor of interest:

Groups:

Outcome:

Group assignment:

Drivers of group assignment:

Confound:

Reverse causality:

TASK 1: Apply the checklist to evaluate causal statements

A company sells many snowmobiles in Canada but very few in Mexico. The company also advertises extensively in Canada but does not advertise at all in Mexico

Causal claim: Advertising works

Factor of interest:	Advertising
Groups:	Advertising (Canada) vs No-advertising (Mexico)
Outcome:	Sales
Group assignment:	Not Random
Drivers of group assignment:	Firm influenced group assignment, likely based on demand in Canada and Mexico (e.g., advertising budgets are often set as a % of sales)
Confound:	Demand = Sales
Reverse causality:	The **outcome** likely drives group assignment (i.e., Advertising > 0)

TASK 1: Apply the checklist to evaluate causal statements

Snowmobile sales are below expectations in January and a dealership in Toronto plans to run a promotion in February. During the promotional period an unexpected snow-storm hits the Toronto area. Sales of snowmobiles in February are 10% higher than expected

Causal claim: The promotion caused a 10% increase in sales

Factor of interest:

Groups:

Outcome:

Group assignment:

Drivers of group assignment:

Confound:

Reverse causality:

TASK 1: Apply the checklist to evaluate causal statements

Snowmobile sales are below expectations in January and a dealership in Toronto plans to run a promotion in February. During the promotional period an unexpected snow-storm hits the Toronto area. Sales of snowmobiles in February are 10% higher than expected

Causal claim: The promotion caused a 10% increase in sales

Factor of interest:	Promotion
Groups:	Promotion (February) vs No-promotion (January)
Outcome:	Sales
Group assignment:	Not random
Drivers of group assignment:	Groups are separated by time and weather is an outcome related driver
Confound:	Weather (snow-storms) can have a direct effect on sales
Reverse causality:	

TASK 1: Apply the checklist to evaluate causal statements (take home)

Doordash is a logistics software startup. Affiliated drivers deliver restaurant food to customers. A restaurant decides to put a link to Door Dash on their website, starting in January. The number of orders for take-out in January are 5% lower than in December

Causal claim: Adding the link to the Door Dash site caused a decrease in sales

Factor of interest:

Groups:

Outcome:

Group assignment:

Drivers of group assignment:

Confound:

Reverse causality?:

TASK 1: Apply the checklist to evaluate causal statements

Doordash is a logistics software startup. Affiliated drivers deliver restaurant food to customers. A restaurant decides to put a link to Door Dash on their website, starting in January. The number of orders for take-out in January are 5% lower than in December

Causal claim: Adding the link to the Door Dash site caused a decrease in sales

Factor of interest:	Link on Door Dash website
Groups:	No-link (December) vs Link (January)
Outcome:	Number of take-out orders
Group assignment:	Not random
Drivers of group assignment:	Groups are separated by time and demand for take-out may vary over time (e.g., holidays, new-years resolutions, …)
Confound:	Changing demand conditions can directly affect the number of take-out orders
Reverse causality?:	

TASK 1: Apply the checklist to evaluate causal statements (take home)

A manufacturer of kitchen knives has improved the quality of their product each year. The company also increased prices each year to cover the costs of these quality improvements. A regression of price on demand (i.e., demand = a + b x price) gives a coefficient for price very close to 0 that is not statistically significant

Causal claim: Customers are not sensitive to price changes so the manufacturer can continue to increase prices, even if quality is not improved

Factor of interest:

Groups:

Outcome:

Group assignment:

Drivers of group assignment:

Confound:

Reverse causality?:

TASK 1: Apply the checklist to evaluate causal statements

A manufacturer of kitchen knives has improved the quality of their product each year. The company also increased prices each year to cover the costs of these quality improvements. A regression of price on demand (i.e., demand = a + b x price) gives a coefficient for price very close to 0 that is not statistically significant

Causal claim: Customers are not sensitive to price changes so the manufacturer can continue to increase prices, even if quality is not improved

Factor of interest:	Price changes (Quality changes)
Groups:	Lower price (quality) vs Higher price (quality)
Outcome:	Demand
Group assignment:	Not Random
Drivers of group assignment:	Firm influences group assignment based on product quality. Group are also separated over time which connects to quality changes over time
Confound:	Product quality can have a direct effect on demand, independent of price

Task 2: Regression review (see linear-regression.ipynb)

```
Linear regression (OLS)
Data                 : diamonds
Response variable    : price
Explanatory variables: clarity, carat
Null hyp.: the effect of x on price is zero
Alt. hyp.: the effect of x on price is not zero

               coefficient  std.error  t.value p.value
Intercept        -6780.993    204.952  -33.086  < .001  ***
clarity[SI2]      2790.760    201.395   13.857  < .001  ***
clarity[SI1]      3608.531    200.508   17.997  < .001  ***
clarity[VS2]      4249.906    201.607   21.080  < .001  ***
clarity[VS1]      4461.956    204.592   21.809  < .001  ***
clarity[VVS2]     5109.476    210.207   24.307  < .001  ***
clarity[VVS1]     5027.669    214.251   23.466  < .001  ***
clarity[IF]       5265.170    233.658   22.534  < .001  ***
carat             8438.030     51.101  165.125  < .001  ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

R-squared: 0.904, Adjusted R-squared: 0.904
F-statistic: 3530.024 df(8, 2991), p.value < 0.001
Nr obs: 3,000
```

Click Ball Point Pens

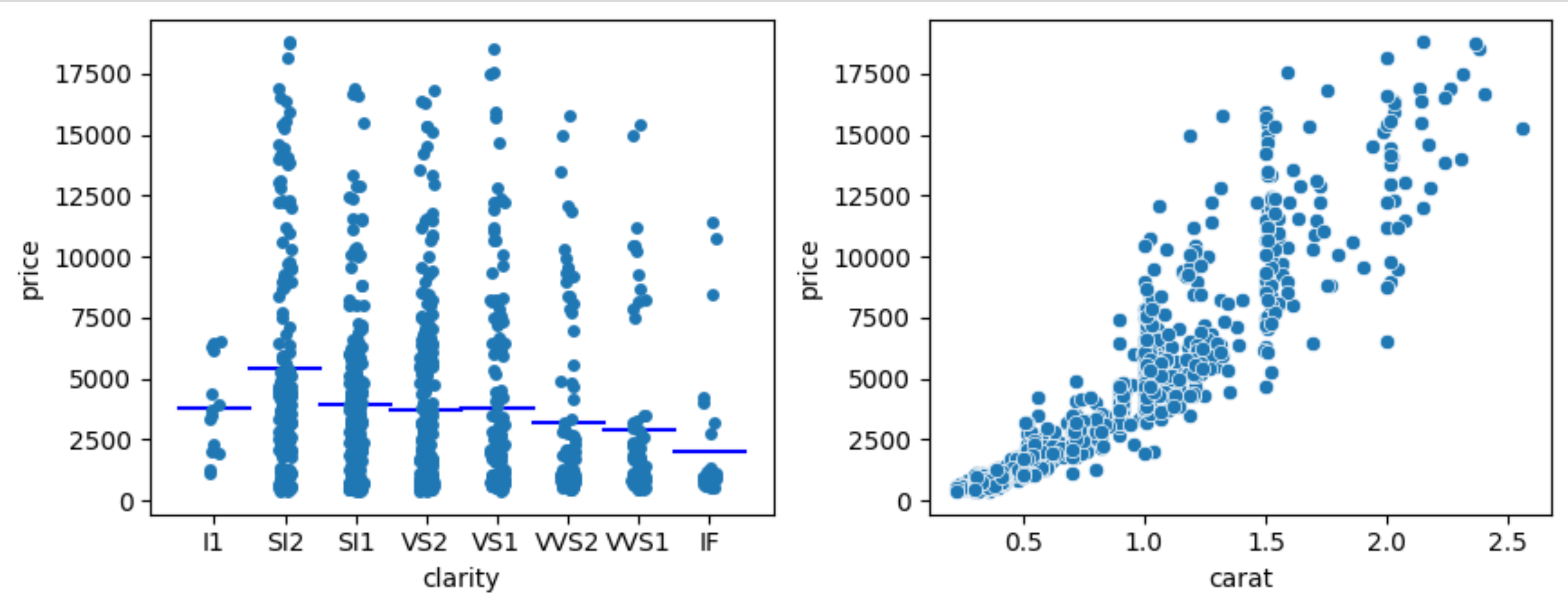
- **Company:** A national manufacturer of ball point pens.

- **Managerial problem:**
  - What is the value of an advertising spot?
  - How much should we pay sales reps?
  - Are the results the same when you include both advertising and sales reps in the model? If not, why not?

- **Data:** Sales data for 40 markets/territories along with measures of marketing effort

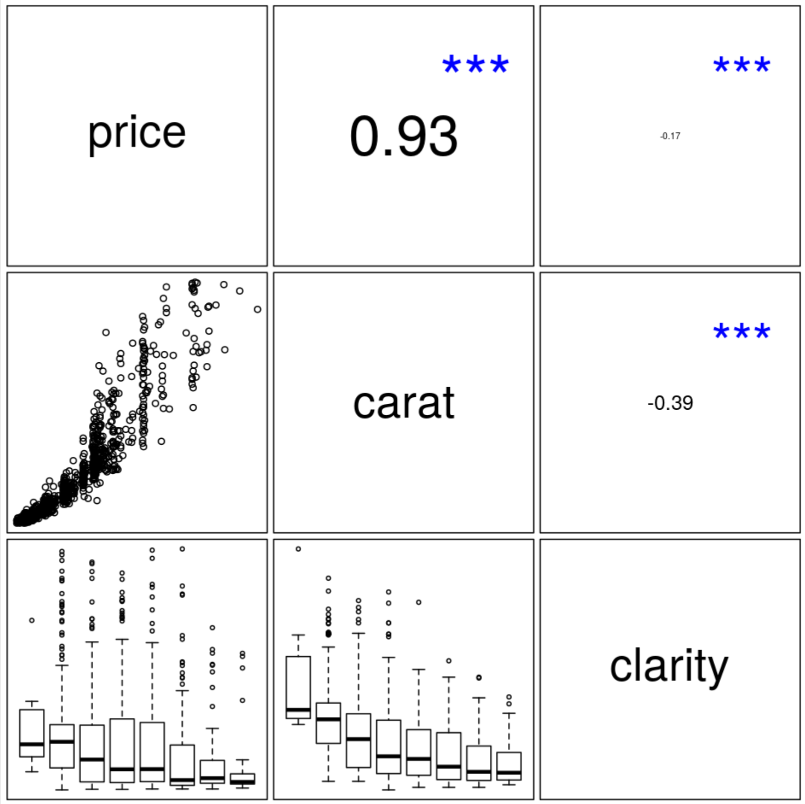
- Use **linear-regression.ipynb** and **data/click.pkl**

Omitted Variable Bias (OVB)



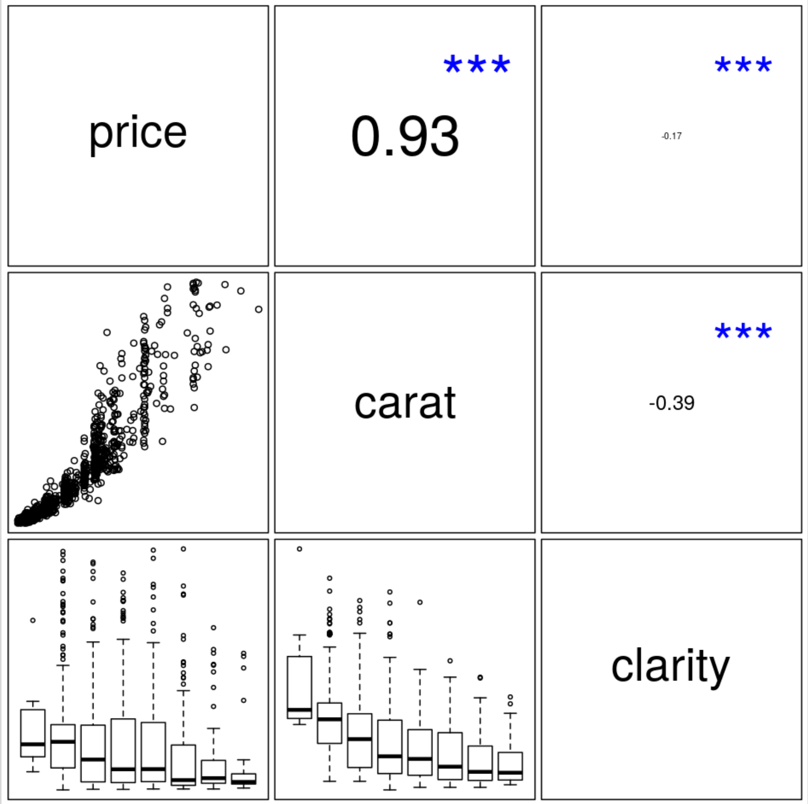
Omitted Variable Bias (OVB) and Multi-collinearity (MC)

```
              coefficient std.error t.value p.value
 (Intercept)     4194.775   616.530   6.804  < .001 ***
 clarity|SI2      905.414   639.415   1.416   0.157
 clarity|SI1     -196.198   633.401  -0.310   0.757
 clarity|VS2     -371.808   634.911  -0.586   0.558
 clarity|VS1     -405.594   643.823  -0.630   0.529
 clarity|VVS2    -856.955   658.518  -1.301   0.193
 clarity|VVS1   -1586.315   669.318  -2.370   0.018 *
 clarity|IF     -1783.078   730.540  -2.441   0.015 *

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

R-squared: 0.031,  Adjusted R-squared: 0.029
F-statistic: 13.759 df(7,2992), p.value < .001
Nr obs: 3,000
```



Omitted Variable Bias (OVB) and Multi-collinearity (MC)

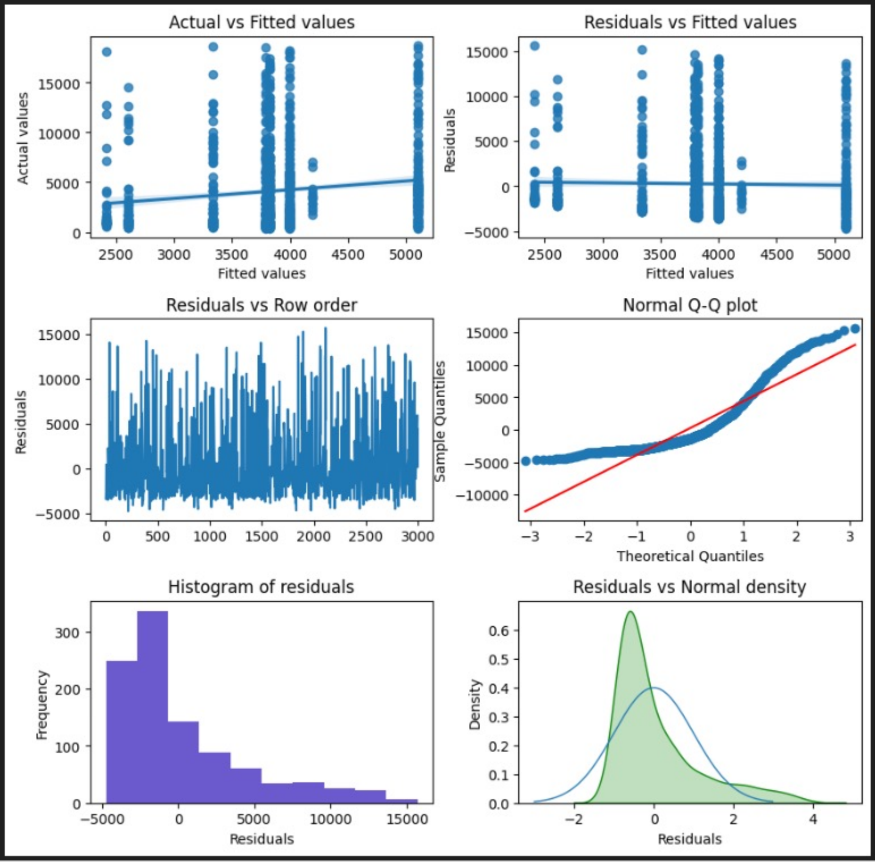
```
               coefficient std.error t.value p.value    
 (Intercept)     -6780.993   204.952 -33.086  < .001 ***
 carat            8438.030    51.101 165.125  < .001 ***
 clarity|SI2      2790.760   201.395  13.857  < .001 ***
 clarity|SI1      3608.531   200.508  17.997  < .001 ***
 clarity|VS2      4249.906   201.607  21.080  < .001 ***
 clarity|VS1      4461.956   204.592  21.809  < .001 ***
 clarity|VVS2     5109.476   210.207  24.307  < .001 ***
 clarity|VVS1     5027.669   214.251  23.466  < .001 ***
 clarity|IF       5265.170   233.658  22.534  < .001 ***

Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1

R-squared: 0.904,  Adjusted R-squared: 0.904
F-statistic: 3530.024 df(8,2991), p.value < .001
Nr obs: 3,000
```



Check residuals

```python
reg.plot("dashboard")
```

✓ 0.5s Python



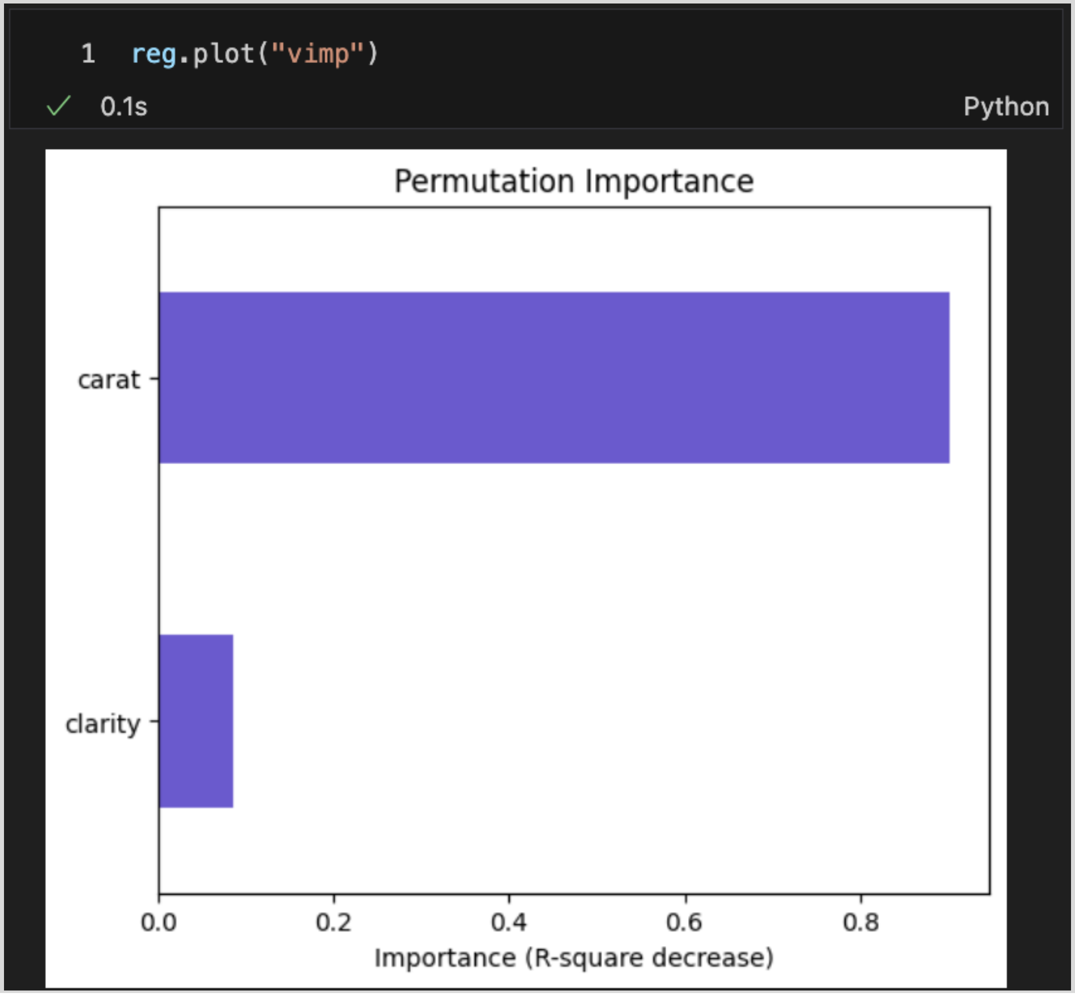
Model fit

```
1  reg.summary(main=False, fit=True)
```

✓ 0.0s Python

```
R-squared: 0.904, Adjusted R-squared: 0.904
F-statistic: 3530.024 df(8, 2991), p.value < 0.001
Nr obs: 3,000
```

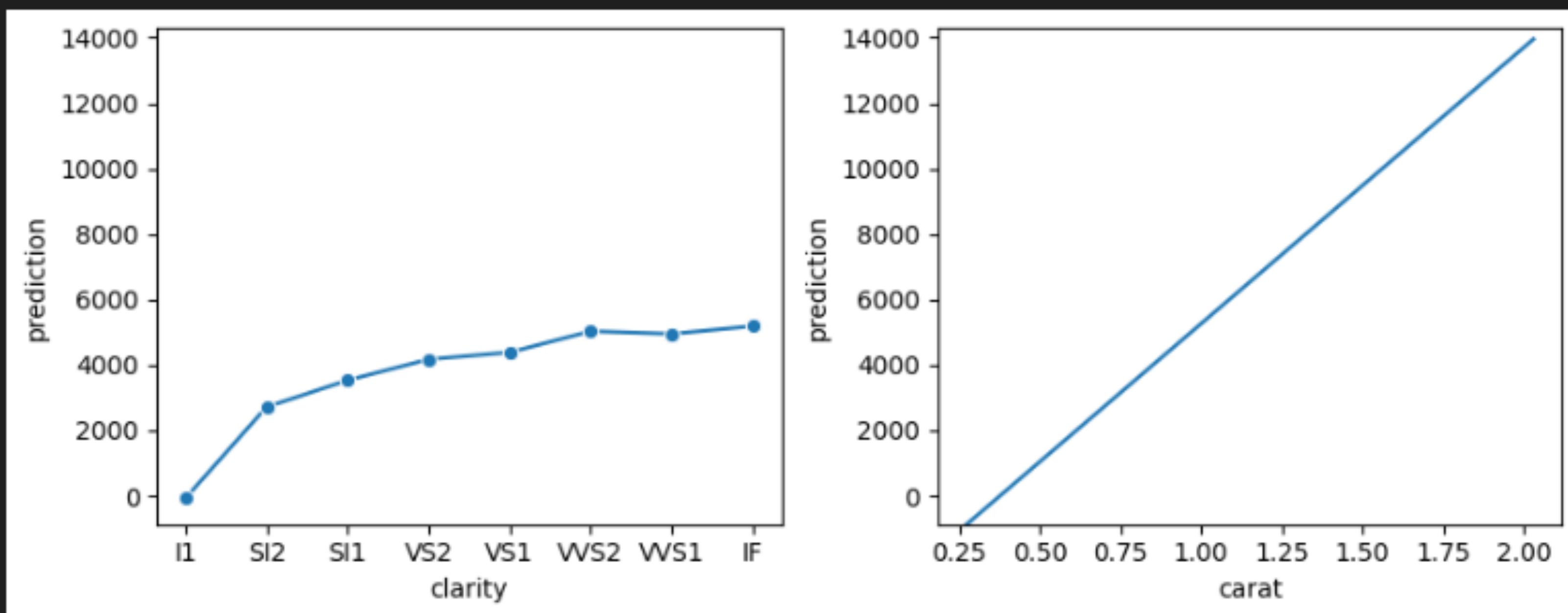
Variable importance



Variable effect

```
1   reg.plot("pred")
```

✓ 0.1s Python



Task 3: Logistic regression (see logistic-regression.ipynb)

```python
lr.coef.round(3)
```

✓ 0.0s Python

	index	OR	OR%	coefficient	std.error	z.value	p.value	
0	Intercept	0.048	-95.206	-3.038	0.063	-48.136	0.0	***
1	coupon	2.169	116.866	0.774	0.015	51.240	0.0	***
2	purch	1.095	9.539	0.091	0.005	17.879	0.0	***
3	last	0.933	-6.678	-0.069	0.002	-35.388	0.0	***

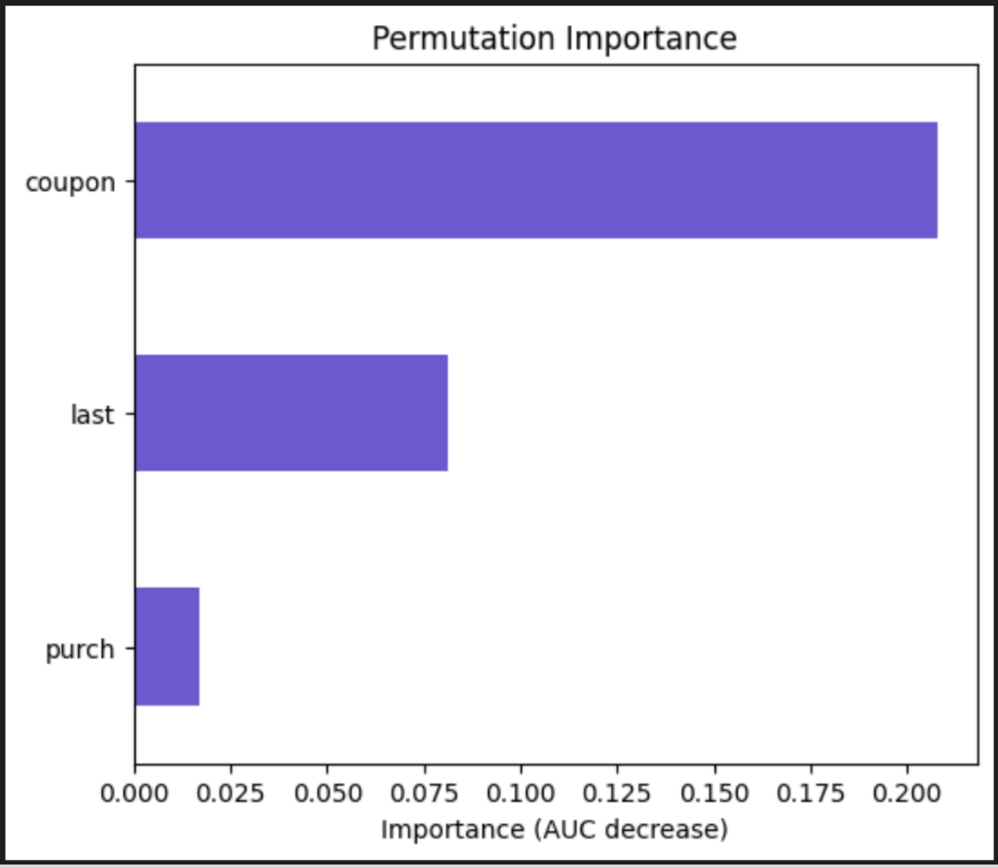
Model fit

```
Pseudo R-squared (McFadden): 0.208
Pseudo R-squared (McFadden adjusted): 0.208
Area under the RO Curve (AUC): 0.803
Log-likelihood: -9110.529, AIC: 18229.058, BIC: 18260.672
Chi-squared: 4796.899, df(3), p.value < 0.001
Nr obs: 20,000
```

Variable importance

```python
lr.plot("vimp")
```

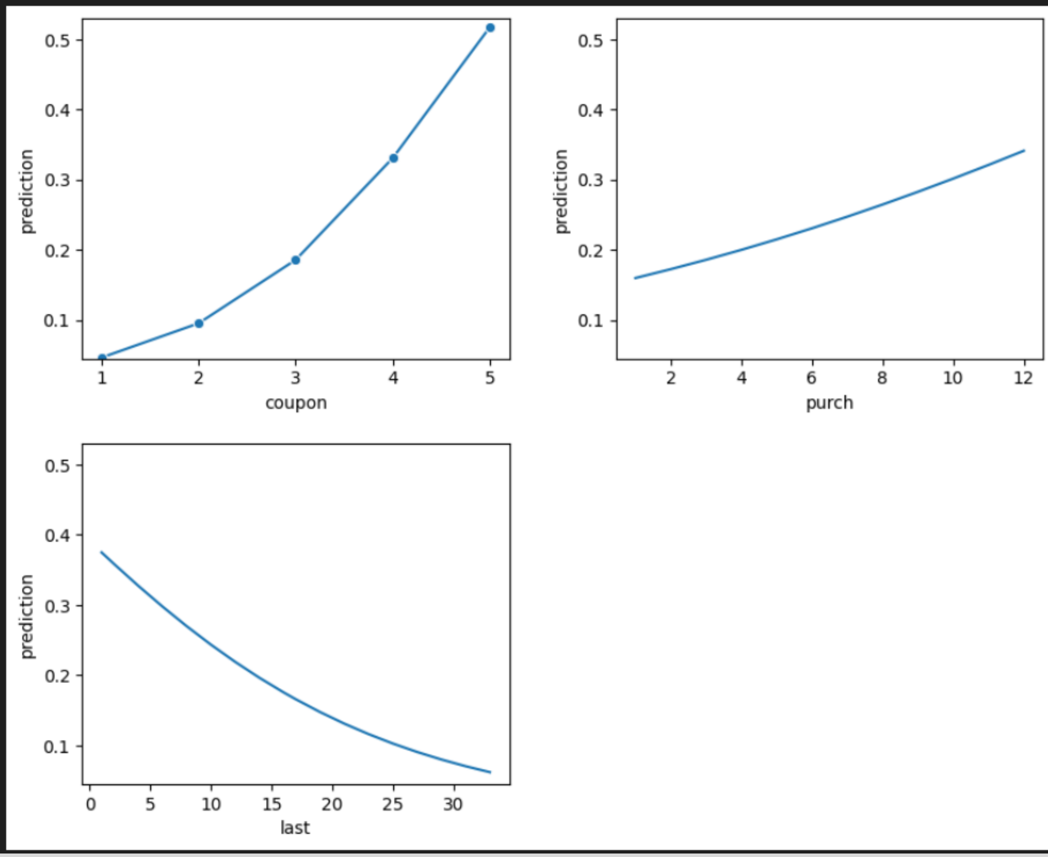
✓ 0.3s Python



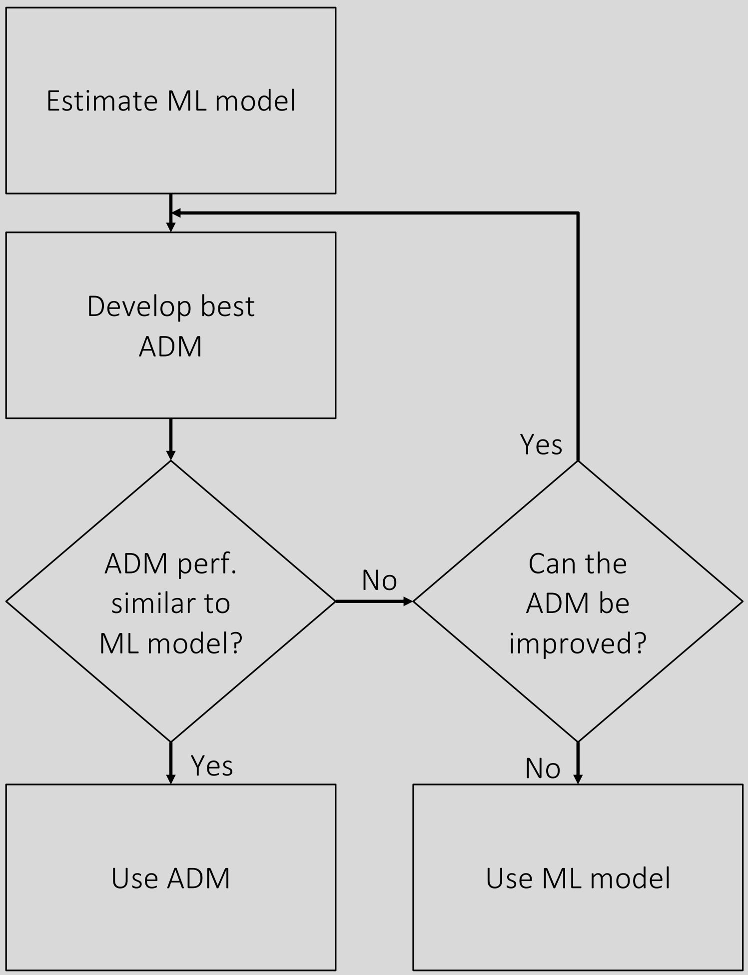
Variable importance

```python
lr.plot("pred")
```

✓ 0.1s Python



Machine Learning (ML) models can be used in combination with Analyst Driven Models (ADM)

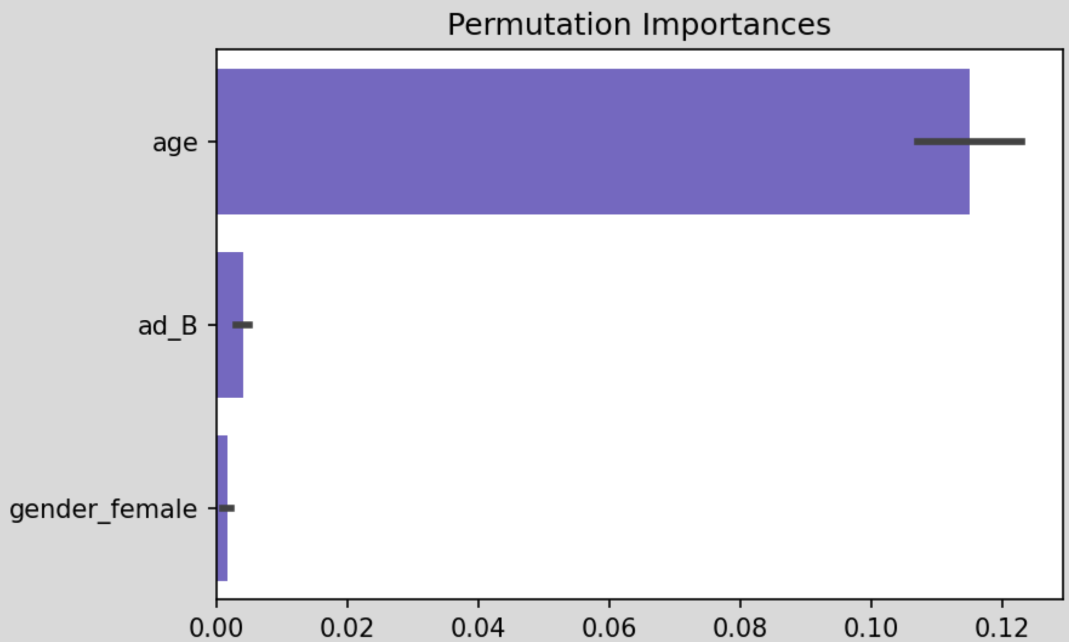


Core idea:

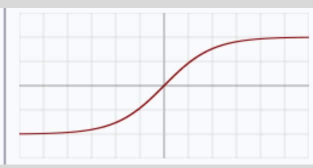
- Use ML model as performance benchmark
- Use ADM for interpretation

TASK 4: What predicts ad click-through? (see facebook.ipynb)

- Reproduce the plot on the right using a NN (1)
- How does the plot change as we add another node to the hidden layer, i.e., NN(2)? Why does it change?
- Develop a logistic regression model that achieves similar performance to the NN(2) model (use gainsplot)
- Use prediction plots to demonstrate key new effects are capture by the LR model



TanH



$$f(x) = \tanh(x) = \frac{(e^x - e^{-x})}{(e^x + e^{-x})}$$

TASK 5: Calculate CLV (use task-5-clv.xlsx or task-5-clv.ipynb)

				Years		
	Start of CLV Calc.	1	2	3	4	5
Revenues	$0	$400	$400			
Product/Service Costs	$0	$80	$80			
Marketing Costs	$0	$0	$0			
Customer Profit	$0	$320	$320			
Prob. of being active at end of period	100.00%	100.00%	59.00%	34.81%		
Profit expected on average	$0	$320.00	$188.80			
Present Value of Exp. Profits	$0	$320				

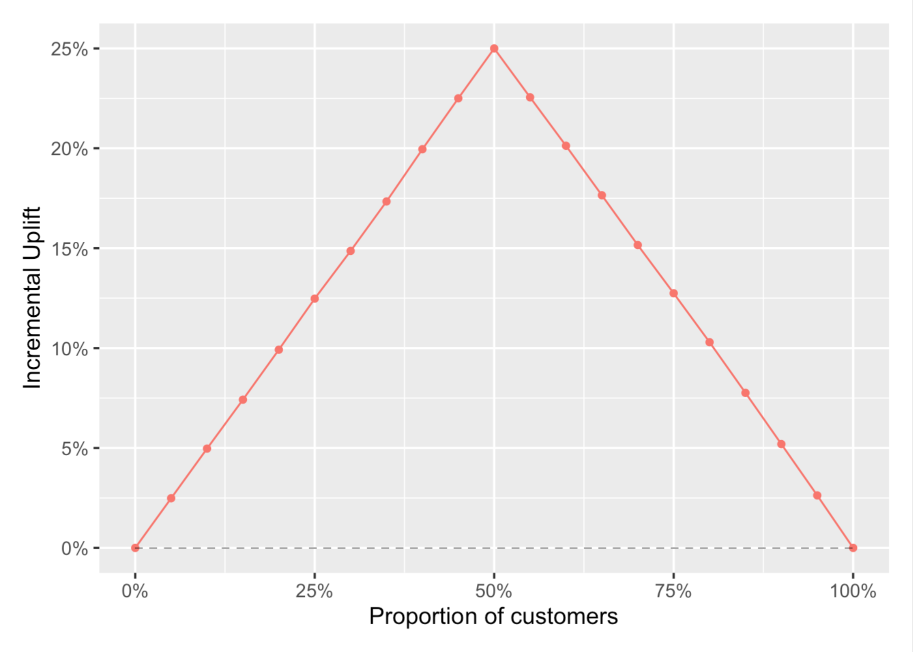
- Discount rate is 10% annually
- What is the churn rate? What about the retention rate?
- What assumption are we making about the timing of churn (Optimistic or Pessimistic)?
- What assumption are we making about the timing of payment (Optimistic or Pessimistic)?

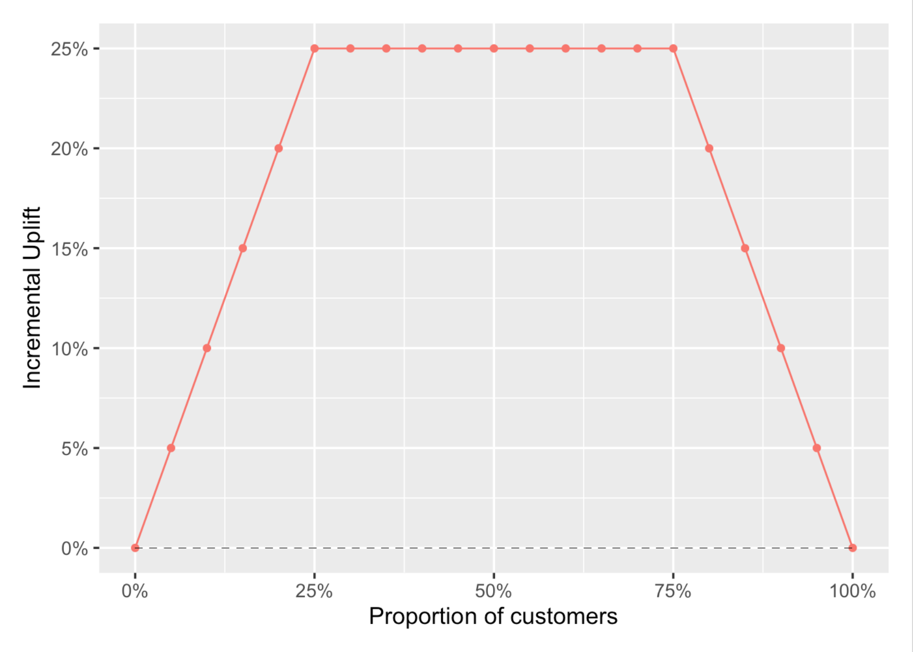
Calculate CLV - Solution

	Years					
	Start of CLV Calc.	1	2	3	4	5
Revenues	$0	$400	$400	$400	$400	$400
Product/Service Costs	$0	$80	$80	$80	$80	$80
Marketing Costs	$0	$0	$0	$0	$0	$0
Customer Profit	$0	$320	$320	$320	$320	$320
Prob. of being active at end of period	100.00%	100.00%	59.00%	34.81%	20.54%	12.12%
Profit expected on average	$0	$320.00	$188.80	$111.39	$65.72	$38.78
Present Value of Exp. Profits	$0	$320	$172	$92	$49	$26

- Discount rate is 10% annually
- What is the churn rate (41%)? What about the retention rate (59%)?
- What assumption are we making about the timing of churn (Optimistic)?
- What assumption are we making about the timing of payment (Optimistic)?

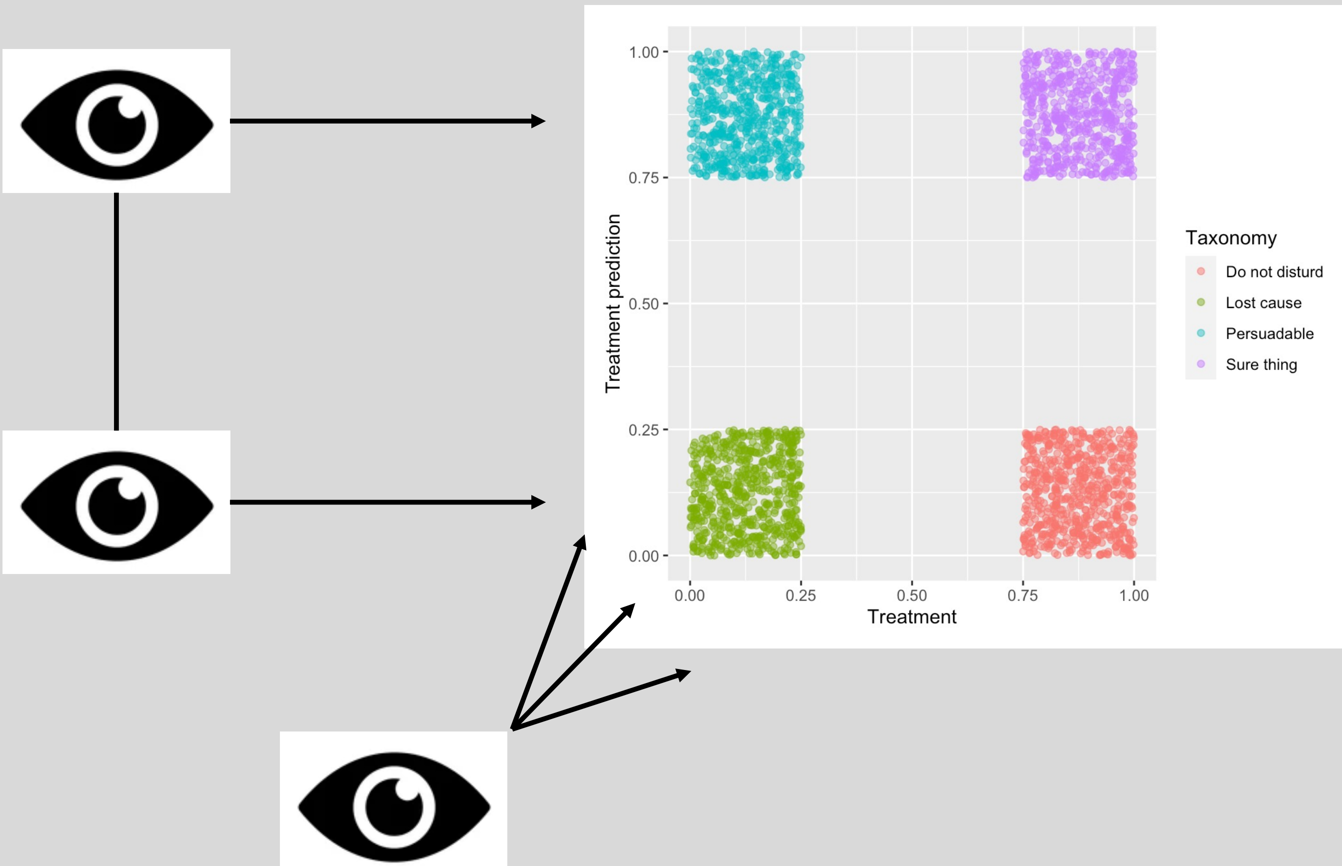
Incremental uplift plot based on the treatment predictions and uplift scores





Where are the optimal targeting points? Same or different?

Simulated data for the uplift taxonomy



What is the "viewpoint" for a propensity-to-buy model?

What is the "viewpoint" for an uplift model?

TASK 6: Evaluate model performance (review slow-auc.ipynb)

CONVERT A PROBABILITY TO A BINARY OUTCOME USING BREAKEVEN AS THE THRESHOLD

		Predicted	
		Pos.	Neg.
Actual	Pos.	TP	FN
	Neg.	FP	TN

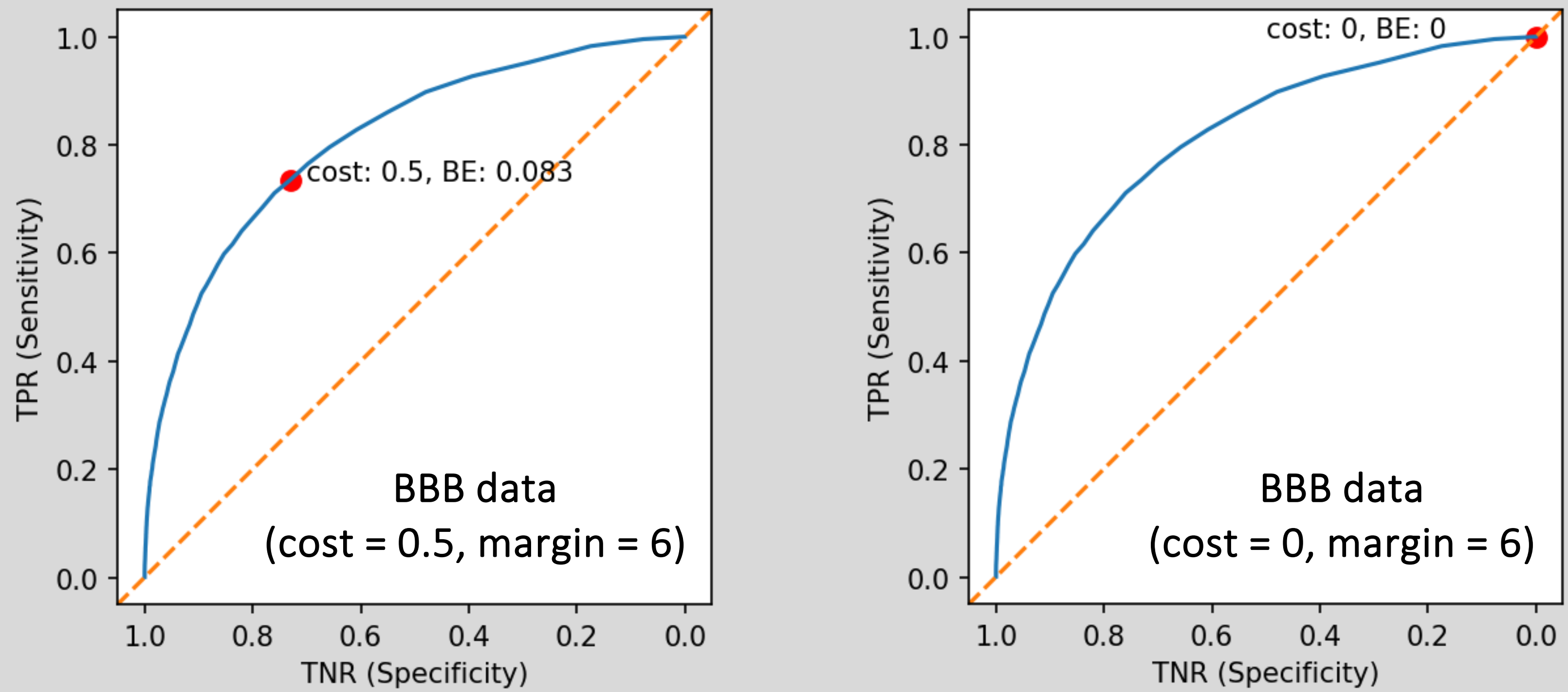
		Predicted	
		Pos.	Neg.
Actual	Pos.	655	176
	Neg.	10,871	16,176

- **TP**: True positive (predicted pos, actual pos)
- **TN**: True negative (predicted neg, actual neg)
- **FP**: False positive (predicted pos, actual neg)
- **FN**: False negative (predicted neg, actual pos)

Additional performance metrics used in practice

- **Accuracy**: Proportion of all outcomes that was correctly predicted as either positive or negative, i.e., (TP + TN) / (TP + TN + FP + FN)
- **Kappa**: Corrects the accuracy measure for the probability of generating a correct prediction purely by chance
- **True positive rate (TPR)**: Proportion of **actual positive outcomes** in the data that received a **positive prediction** (i.e., TP / (TP + FN)). Also known as **sensitivity** or **recall**
- **True negative rate (TNR)**: Proportion of **actual negative outcomes** in the data that received a **negative prediction** (i.e., TN / (TN + FP)). Also known as **specificity**
- **AUC:** Area Under the (ROC) Curve. The ROC curve plots the FPR against the TPR for all possible classification thresholds. AUC is the area under this curve. The maximum AUC value is 1 and the minimum value is 0.5

AUC is a measure of model performance at all possible thresholds



- **True positive rate (TPR)**: Proportion of **actual positive outcomes** in the data that received a **positive prediction** (i.e., TP / (TP + FN)). Also known as **sensitivity** or **recall**
- **True negative rate (TNR)**: Proportion of **actual negative outcomes** in the data that received a **negative prediction** (i.e., TN / (TN + FP)). Also known as **specificity**

Probabilistic interpretation of AUC

AUC is the probability that Pred(X) > Pred(Y) where X is a randomly selected buyer and Y is a randomly selected non-buyer

```
(
    np.random.choice(pred_did_buy, nr) >
    np.random.choice(pred_did_not_buy, nr)
).mean()
```

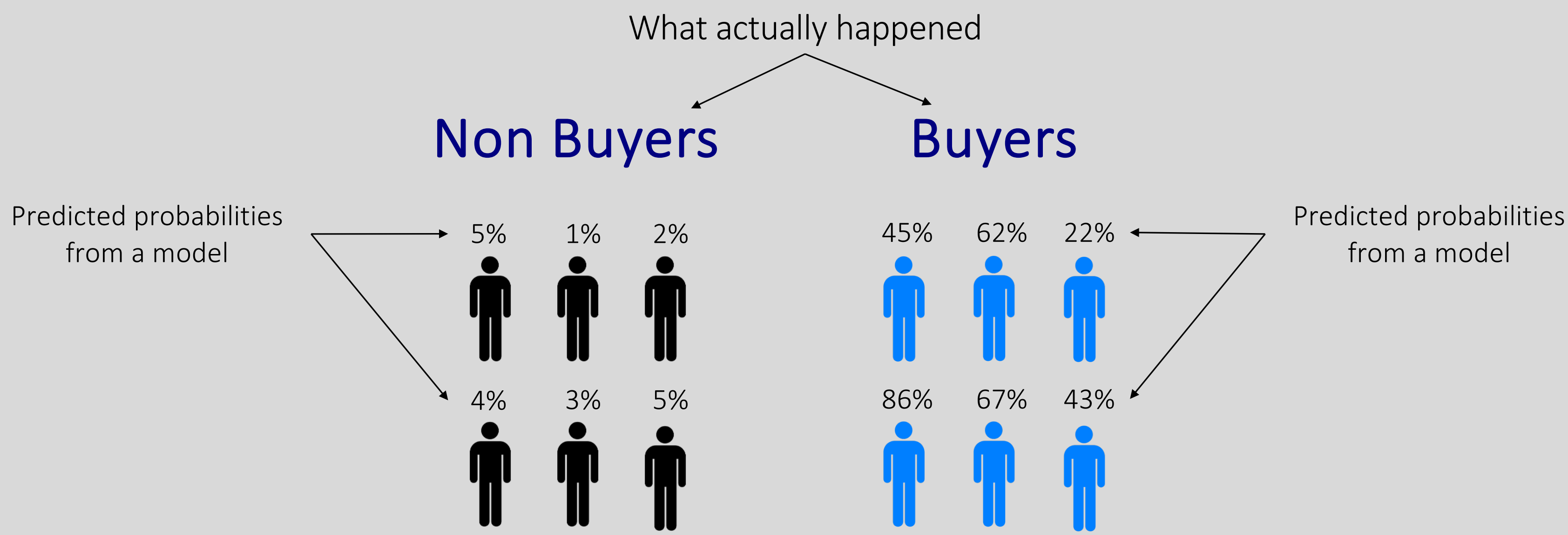
What does an AUC = 1 imply about “pred_did_buy” vs “pred_did_not_buy”?

What does an AUC = 0 imply about “pred_did_buy” vs “pred_did_not_buy”?

What does an AUC = 0.5 imply about “pred_did_buy” vs “pred_did_not_buy”?

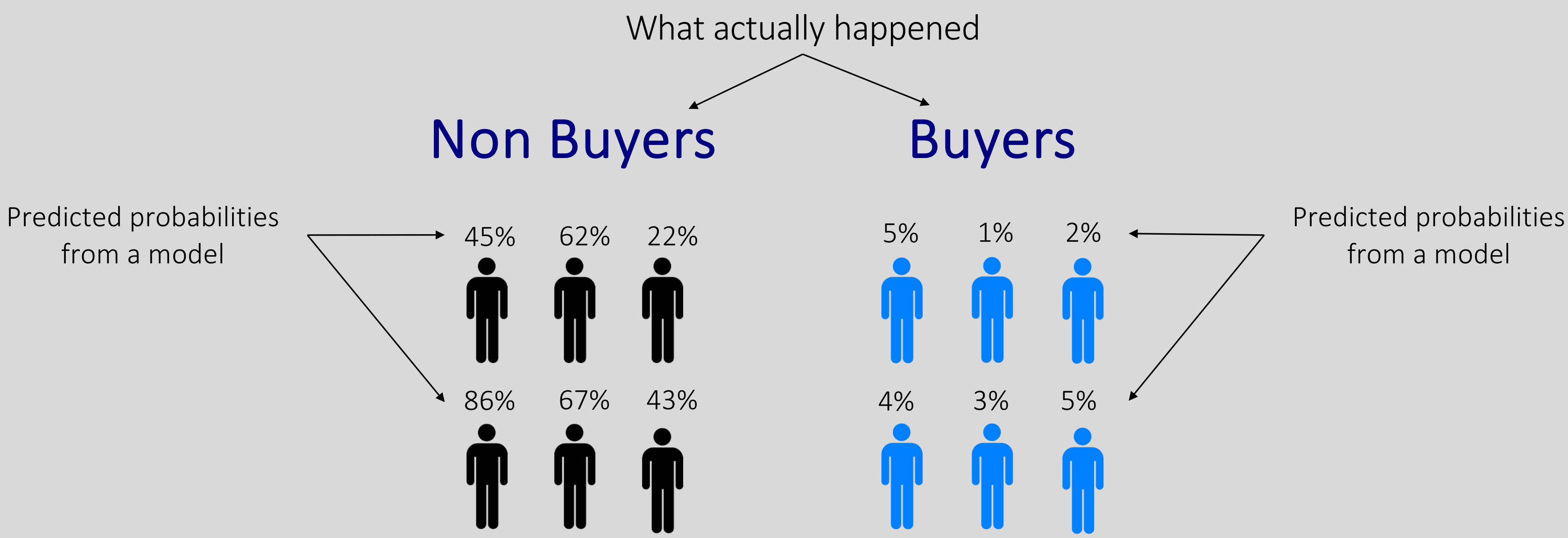
https://www.alexejgossmann.com/auc/

Probabilistic interpretation of AUC



What does an AUC = 1 imply about “pred_did_buy” vs “pred_did_not_buy”?

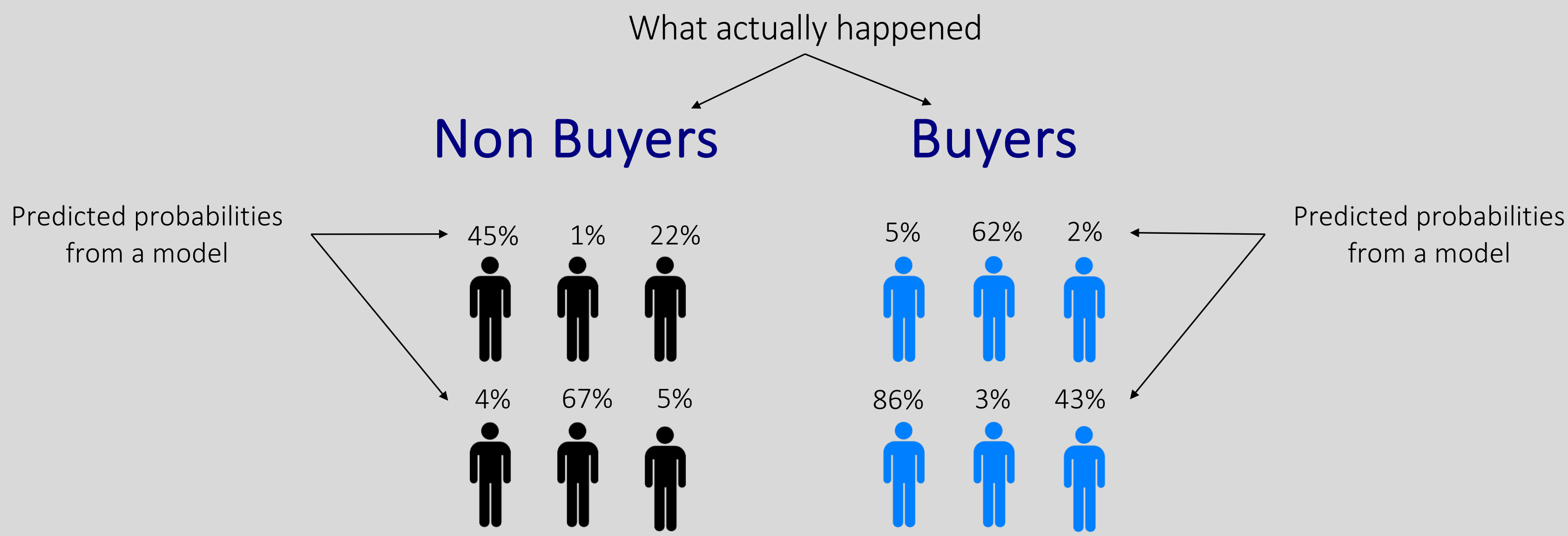
Probabilistic interpretation of AUC



What does an AUC = 0 imply about “pred_did_buy” vs “pred_did_not_buy”?

Is this a useful model?

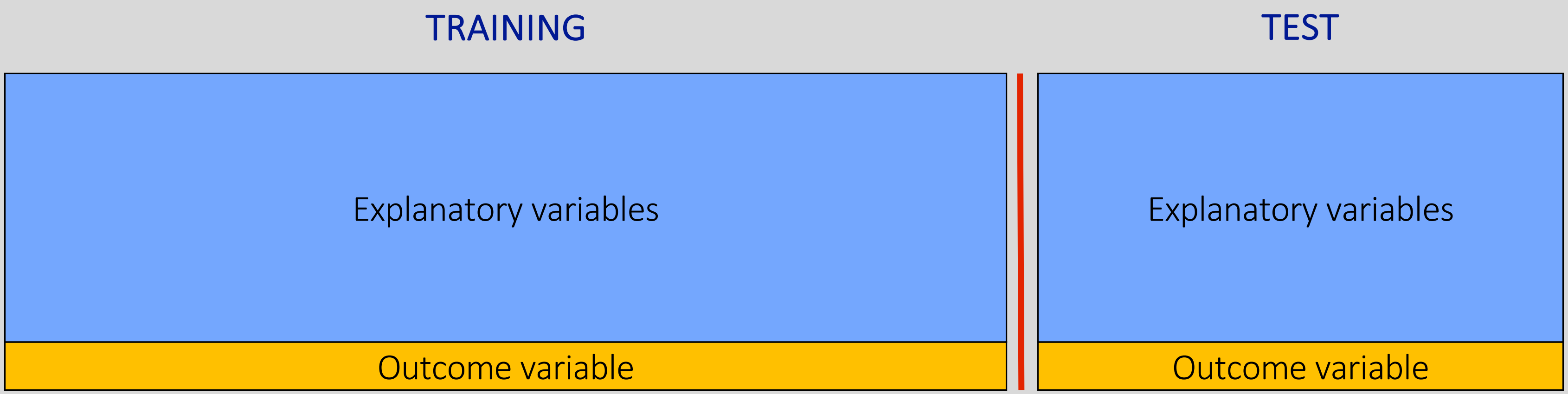
Probabilistic interpretation of AUC



What does an AUC = 0.5 imply about “pred_did_buy” vs “pred_did_not_buy”?

Is this a useful model?

TASK 7: How to “tune” hyper parameters to avoid overfitting?

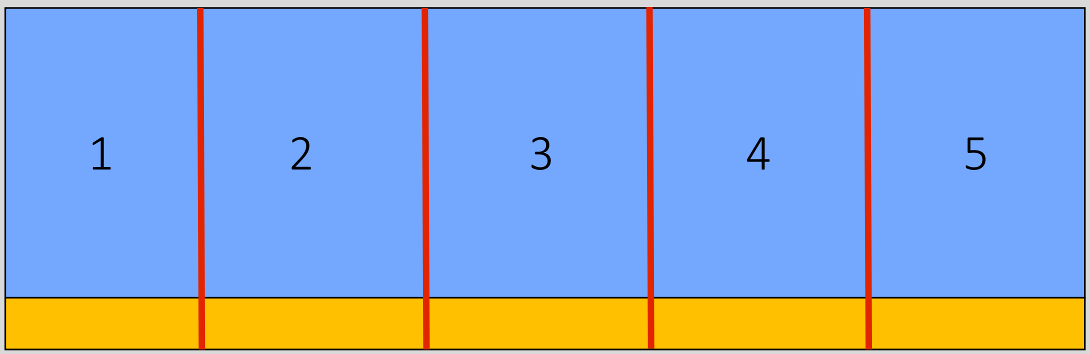


SIZE 1 2 3 4 5

DECAY 0 0.1 0.2 0.3 0.4 0.5

	Size				
Decay	1	2	3	4	5
0	(1 , 0)	(2 , 0)	(3 , 0)	(4 , 0)	(5 , 0)
0.1	(1, 0.1)	(2, 0.1)	(3, 0.1)	(4, 0.1)	(5, 0.1)
0.2	(1, 0.2)	(2, 0.2)	(3, 0.2)	(4, 0.2)	(5, 0.2)
0.3	(1, 0.3)	(2, 0.3)	(3, 0.3)	(4, 0.3)	(5, 0.3)
0.4	(1, 0.4)	(2, 0.4)	(3, 0.4)	(4, 0.4)	(5, 0.4)
0.5	(1, 0.5)	(2, 0.5)	(3, 0.5)	(4, 0.5)	(5, 0.5)

K-fold cross validation to “tune” hyper parameters



TRAIN	VALIDATE
1-4	5
2-5	1
3-1	2
4-2	3
5-3	4

HYPER PARAMETER GRID

	Size				
Decay	1	2	3	4	5
0	(1 , 0)	(2 , 0)	(3 , 0)	(4 , 0)	(5 , 0)
0.1	(1, 0.1)	(2, 0.1)	(3, 0.1)	(4, 0.1)	(5, 0.1)
0.2	(1, 0.2)	(2, 0.2)	(3, 0.2)	(4, 0.2)	(5, 0.2)
0.3	(1, 0.3)	(2, 0.3)	(3, 0.3)	(4, 0.3)	(5, 0.3)
0.4	(1, 0.4)	(2, 0.4)	(3, 0.4)	(4, 0.4)	(5, 0.4)
0.5	(1, 0.5)	(2, 0.5)	(3, 0.5)	(4, 0.5)	(5, 0.5)

The model associated with each cell in the “grid” is evaluated 5 times in a training-validation pair. The average performance metric for each grid cell is then used to determine the best hyper parameters to use.

TASK 7: K-fold cross validation to "tune" hyper parameters for NN (classification) – see bbb_sklearn.ipynb

```
nr_hnodes = range(1, 5)
hls = list(zip(nr_hnodes)) + list(zip(nr_hnodes, nr_hnodes))
hls
```

```
[(1,), (2,), (3,), (4,), (1, 1), (2, 2), (3, 3), (4, 4)]
```

```
param_grid = {"hidden_layer_sizes": hls, "alpha": [0.001, 0.01, 0.05]}
scoring = {"AUC": "roc_auc"}
```

```
clf_cv = GridSearchCV(
    clf, param_grid, scoring=scoring, cv=5, n_jobs=4, refit="AUC", verbose=5
).fit(Xs[training == 1], y[training == 1])
```

```
Fitting 5 folds for each of 24 candidates, totalling 120 fits
```

TASK 8: Experimental design and partial factorials (see bizware-review.ipynb)

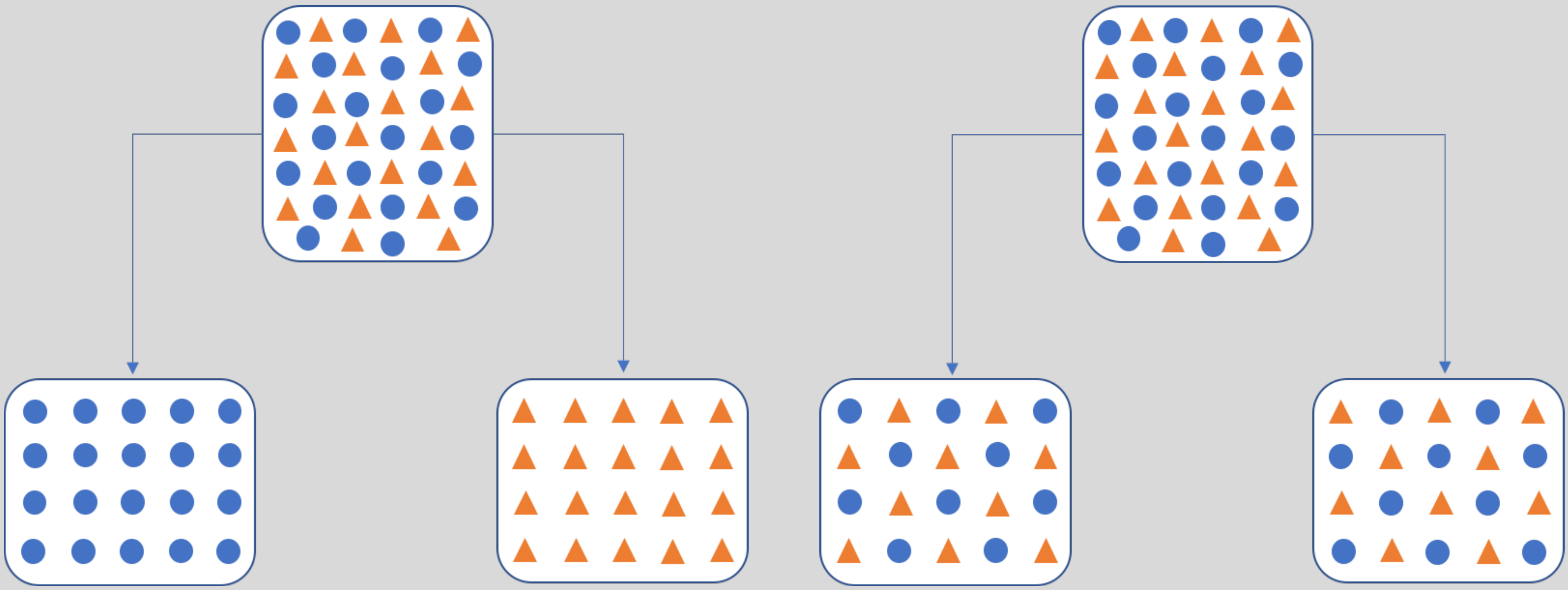
price	message	promotion	response
USD150	speed	trial	0.14
USD150	power	gift	0.40
USD160	power	trial	0.09
USD160	speed	gift	0.13
USD170	power	trial	0.06
USD170	speed	gift	0.10
USD180	speed	trial	0.01
USD180	power	gift	0.07

source: Boost your Marketing ROI with Experimental Design (HBR)
Authors: Eric Almquist and Gordon Wyner

Assume the sample size for each cell was 2,000

- Generate a partial factorial design using information about factors and levels shown in the response table (**use the radiant browser interface or functions directly from radiant.design**)
- Did you get the same design? Why (not)?
- Estimate a logistic regression based on the response table shown and predict response for all profiles
- Use **data/bizware.xls**
- What are the 2 top offers?
- What are the 2 worst offers?

Task 9: Decision trees -- Best possible split vs worst possible split



Classification trees split the data by filtering on a variable

buyer	a	b
yes	1	1
no	0	1
yes	1	1
no	0	1
yes	1	0
no	0	0
yes	1	0
no	0	0

50% buyer, 50% non-buyers

```
1  tf[tf.a == 1]
```

✓ 0.1s

	buyer	a	b
0	yes	1	1
2	yes	1	1
4	yes	1	0
6	yes	1	0

100% buyers

```
1  tf[tf.a == 0]
```

✓ 0.4s

	buyer	a	b
1	no	0	1
3	no	0	1
5	no	0	0
7	no	0	0

100% non-buyers

Classification trees split the data by filtering on a variable

buyer	a	b
yes	1	1
no	0	1
yes	1	1
no	0	1
yes	1	0
no	0	0
yes	1	0
no	0	0

50% buyer, 50% non-buyers

```
1  tf[tf.b == 1]
```

✓ 0.3s

	buyer	a	b
0	yes	1	1
1	no	0	1
2	yes	1	1
3	no	0	1

50% buyer, 50% non-buyers

```
1  tf[tf.b == 0]
```

✓ 0.2s

	buyer	a	b
4	yes	1	0
5	no	0	0
6	yes	1	0
7	no	0	0

50% buyer, 50% non-buyers

Regression trees also split the data by filtering on a variable

sales	a	b
20	1	1
10	0	1
20	1	1
10	0	1
20	1	0
10	0	0
20	1	0
10	0	0

SSE = 200

```
1  tf[tf.a == 1]
```

✓ 0.2s

	sales	a	b
0	20	1	1
2	20	1	1
4	20	1	0
6	20	1	0

SSE = 0

```
1  tf[tf.a == 0]
```

✓ 0.2s

	sales	a	b
1	10	0	1
3	10	0	1
5	10	0	0
7	10	0	0

SSE = 0

Regression trees also split the data by filtering on a variable

sales	a	b
20	1	1
10	0	1
20	1	1
10	0	1
20	1	0
10	0	0
20	1	0
10	0	0

SSE = 200

```
1  tf[tf.b == 1]
```

✓ 0.2s

	sales	a	b
0	20	1	1
1	10	0	1
2	20	1	1
3	10	0	1

SSE = 100

```
1  tf[tf.b == 0]
```

✓ 0.2s

	sales	a	b
4	20	1	0
5	10	0	0
6	20	1	0
7	10	0	0

SSE = 100

TASK 10: Calculate "node impurity" when there are two classes (CART)

$$I(A) = p_1 \times (1 - p_1) + p_2 \times (1 - p_2)$$

$$\Delta I = N(A)I(A) - N(A_L)I(A_L) - N(A_R)I(A_R)$$

$I(A)$ is the level of *impurity* in the node we want to split

$N(A)$ is the number of observations in the node we want to split

$I(A_L)$ and $I(A_R)$ represent the level of *impurity* in the node's children after the split

$N(A_L)$ and $N(A_L)$ are the number of observations in the node's children after the split

Creating a decision tree starts at the root (node)

Female variable:

- Cross tab "female" and "response"
- Calculate the reduction in impurity from the split

```
Pivot table
Data        : cart_demo50
Categorical : response female

 female yes  no Total
    yes 130  87   217
     no  71 114   185
  Total 201 201   402
```

female == “yes” vs female == “no”

9.26

- Root: 402 x (201/402 x (1-201/402) + 201/402 x (1 - 201/402))

Now evaluate all possible splits of the root node using age

Age variable:

- Cross tab "age" and "response"
- Calculate the reduction in impurity for each split

```
Pivot table
Data          : cart_demo50
Categorical : response age

   age  yes  no  Total
     1   36  71    107
     2   80  72    152
     3   85  58    143
 Total  201 201    402
```

- age == 1 vs age == 2 | age == 3 — 7.80
- age == 2 vs age == 1 | age == 3 — 0.34
- age == 3 vs age == 1 | age == 2 — 3.96

-
-
-

Finally, evaluate all possible splits of the root node using income

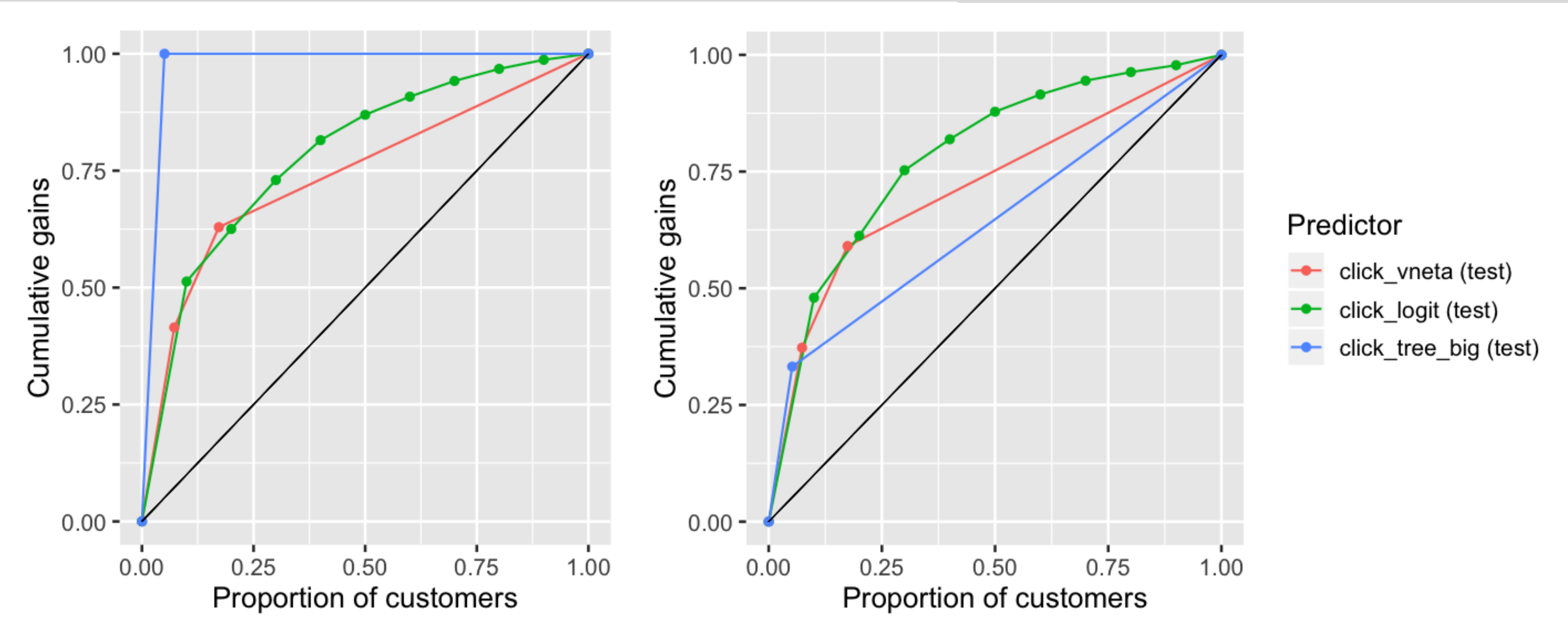
Income variable:

- Cross tab "income" and "response"
- Calculate reduction in impurity for each split

```
Pivot table
Data          : cart_demo50
Categorical : response income

 income yes  no Total
      1  42  39    81
      2  74 107   181
      3  85  55   140
  Total 201 201   402
```

- income == 1 vs income == 2 | income == 3 — 0.01
- income == 2 vs income == 1 | income == 3 — 5.47
- income == 3 vs income == 1 | income == 2 — 4.93

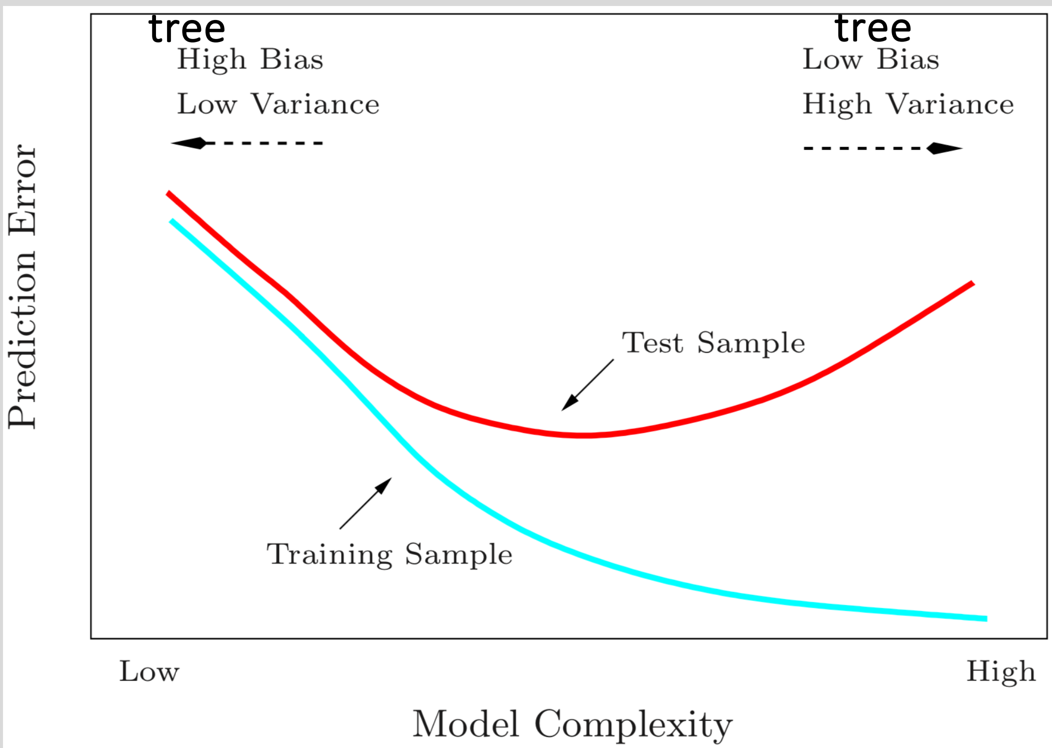
An un-pruned decision tree over-fits the training data massively!



“Ensembles” of trees address key weakness of single decision trees

Small decision tree

Large decision tree



Underfitting

Overfitting

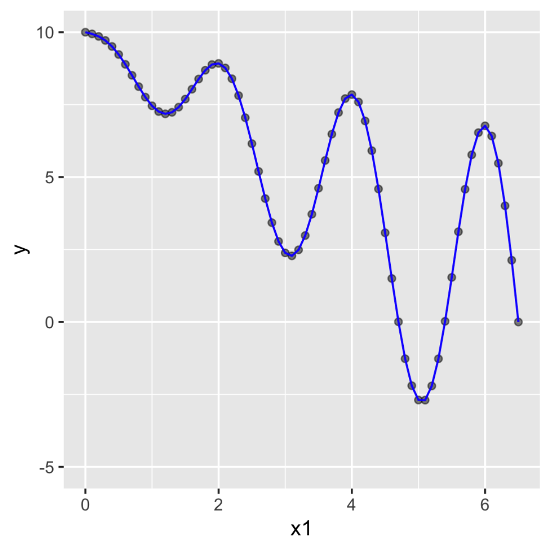
- Random Forests combine many large (overfit) decision tree to reduce variance
- Boosted Decision tree combine may small (underfit) decision trees to reduce bias
- Graph source: **The Elements of Statistical Learning**

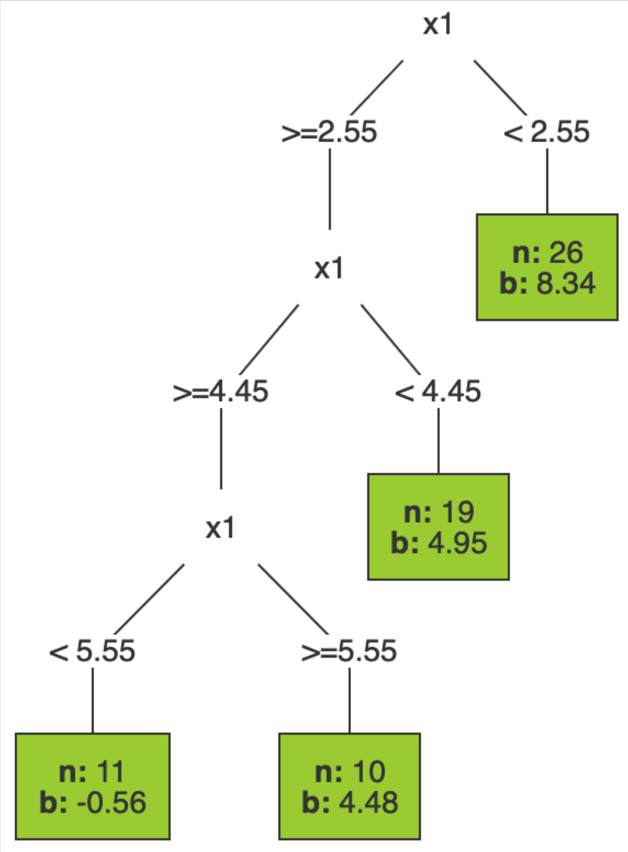
How does a random forests work?

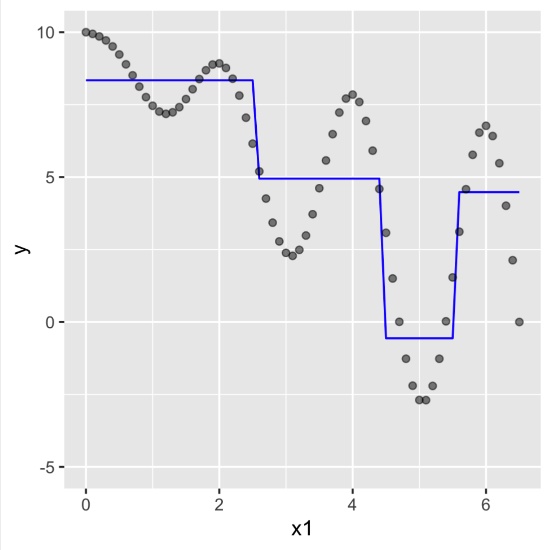
RANDOM FOREST IDEA

- Algorithm adds randomness to address overfitting for decision trees (Breiman and Cutler)
- Key idea is to create many decision trees, each of based on a
  - randomly chosen subsample of the data
  - randomly chosen subset of the explanatory variables at each node
- Can be used with different decision tree algorithms (e.g., CART)
- Very accurate predictor that can handle large numbers of explanatory variables [**WHY?**]

How do Boosted Decision Trees work?

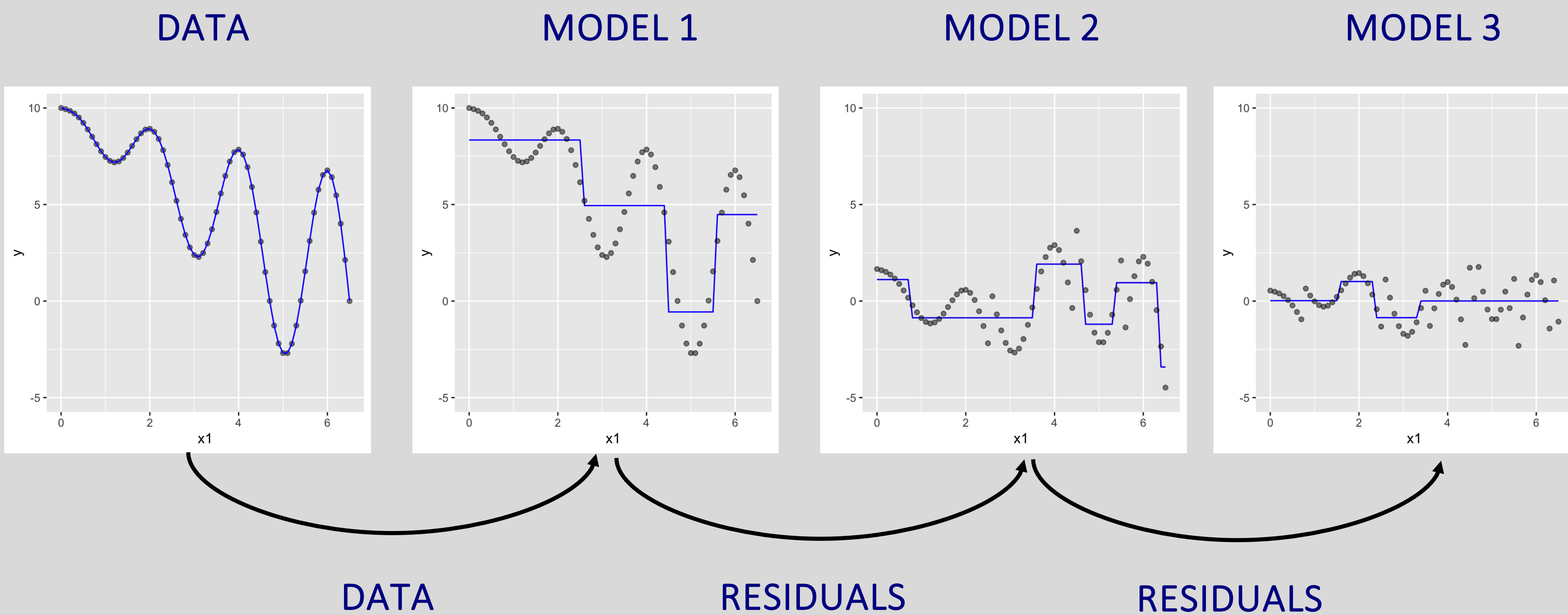




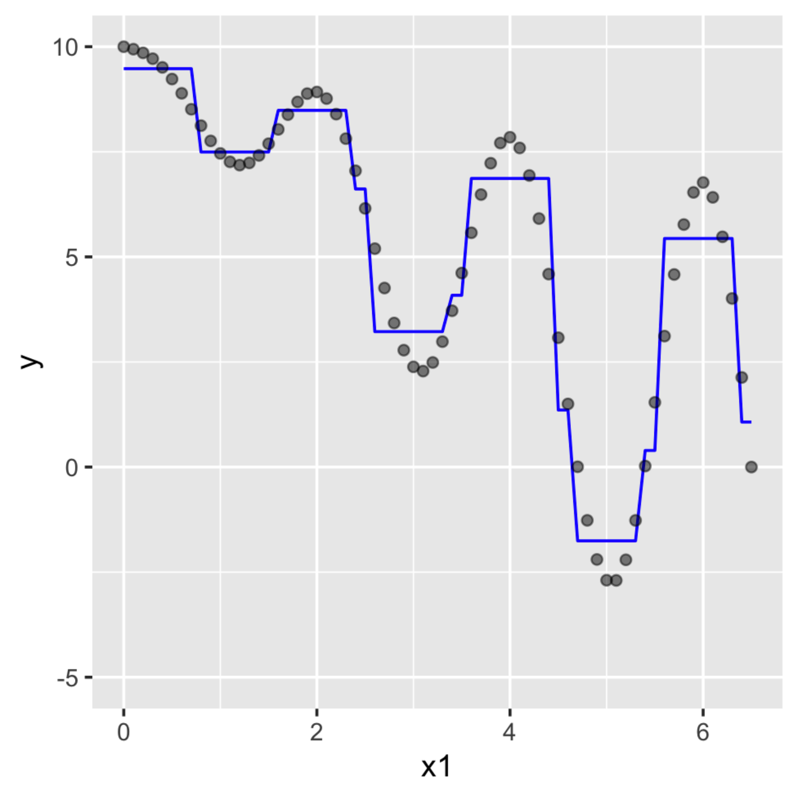


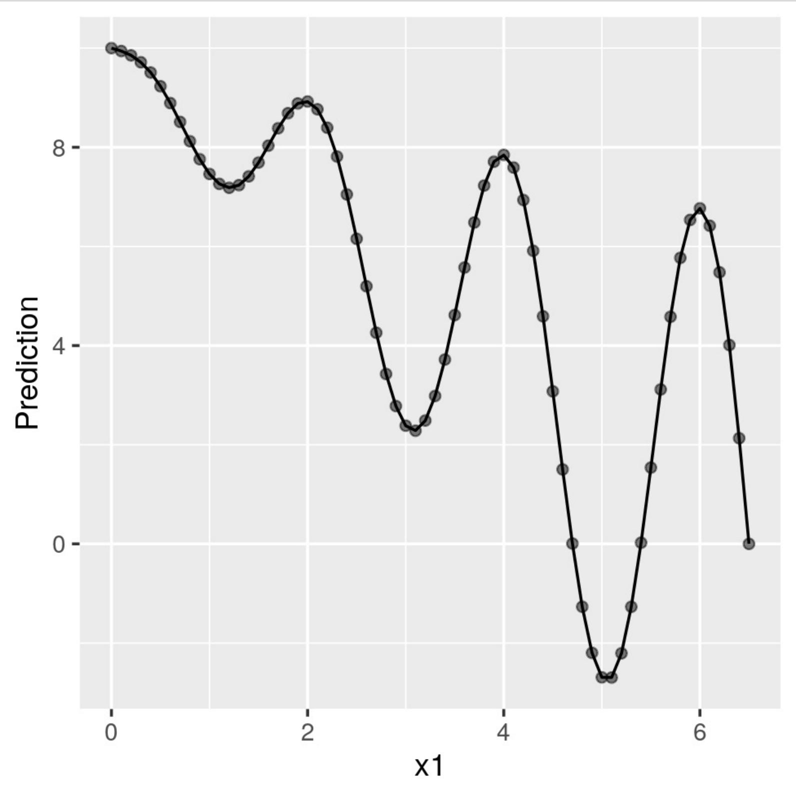
Task 10: Review code for a simplified boosted regression tree (see task-10-boosting_regression.ipynb)

Boosted Decision Trees combine “weak learners” applied to residuals from previous model(s)



To generate a final prediction, we can sum the 3 tree predictions





- Note: The above prediction uses a “learning rate” of 1. In practice, we would use a much smaller number (e.g., 0.01) and build (many) more trees