Controller

We use impedance controller to determine forces in xyz directions w.r.t. world frame. In terms of the motor torque, we can multiply the Jacobian transpose with xyz forces, then add the friction compensation of the motor to get desired torques the motor applied.

$$\begin{bmatrix}\tau_{M1}\\ \tau_{M2}\\ \tau_{M3}\end{bmatrix} = \begin{bmatrix} J^T * \begin{bmatrix}F_X\\ F_Y\\ F_Z\end{bmatrix}\end{bmatrix} + \begin{bmatrix}\tau_{M1FricComp}\\ \tau_{M2FricComp}\\ \tau_{M3FricComp}\end{bmatrix} \tag{1}$$

To get a better performance in our mission, our controller need to have the agility of varying impedance in arbitrary direction w.r.t. world frame. We can use the rotation matrix multiplication to get the desired position and velocity in N frame, where N frame are some rotated world frame. Given the desired path and known obstacles position w.r.t. our robot, we know the N frame in different task, s.t. we can calculate the rotation matrix. We can use then different value of Kp and Kd to control the impedance of our robot in xyz direction w.r.t. N frame. The original expression of torque can be expressed as

$$\begin{bmatrix} J^T * \begin{bmatrix}F_X\\ F_Y\\ F_Z\end{bmatrix}\end{bmatrix} = \begin{bmatrix} J^T * R_N^W * \begin{bmatrix} Kp * R_W^N * \begin{bmatrix}(x_W^d - x_W)\\ (y_W^d - y_W)\\ (z_W^d - z_W)\end{bmatrix} + Kd * R_W^N * \begin{bmatrix}(\dot{x}_W^d - \dot{x}_W)\\ (\dot{y}_W^d - \dot{y}_W)\\ (\dot{z}_W^d - \dot{z}_W)\end{bmatrix}\end{bmatrix}\end{bmatrix} \tag{2}$$

Kp and Kd are variables in different task to let our robot to have soft contact(small impedance) with obstacles, where

$$Kp = \begin{bmatrix} Kp_{X_N} & 0 & 0\\ 0 & Kp_{Y_N} & 0\\ 0 & 0 & Kp_{Z_N}\end{bmatrix} \tag{3}$$

$$Kd = \begin{bmatrix} Kd_{X_N} & 0 & 0\\ 0 & Kd_{Y_N} & 0\\ 0 & 0 & Kd_{Z_N}\end{bmatrix} \tag{4}$$