A search for sparticles in zero lepton final states

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ABSTRACT

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Dedication

Chapter 1

Introduction

Particle physics is a remarkably successful field of scientific inquiry. The ability to precisely predict the properties of a exceedingly wide range of physical phenomena, such as the description of the cosmic microwave background [1, 2], the understanding of the anomalous magnetic dipole moment of the electron [3, 4], and the measurement of the number of weakly-interacting neutrino flavors [5] is truly amazing.

The theory that has allowed this range of predictions is the *Standard Model* of particle physics (SM). The Standard Model combines the electroweak theory of Glashow, Weinberg, and Salam [6–8] with the theory of the strong interactions, as first envisioned by Gell-Mann and Zweig [9, 10]. This quantum field theory (QFT) contains a tiny number of particles, whose interactions describe phenomena up to at least the TeV scale. These particles are manifestations of the fields of the Standard Model, after application of the Higgs Mechanism. The particle content of the SM consists only of the six quarks, the six leptons, the four gauge bosons, and the scalar Higgs boson.

Despite its impressive range of described phenomena, the Standard Model has some theoretical and experimental deficiencies. The SM contains 26 free parameters [1]. It would be more theoretically pleasing to understand these free parameters in terms of a more fundamental theory. The major theoretical concern of the Standard Model, as it pertains to this thesis, is the *hierachy problem*[11–15]. The light mass

[1] This is the Standard Model corrected to include neutrino masses. These parameters are the fermion masses (6 leptons, 6 quarks), CKM and PMNS mixing angles (8 angles, 2 CP-violating phases), W/Z/Higgs masses (3), the Higgs field expectation value, and the couplings of the strong, weak, and electromagnetic forces (3 α_{force}) .

of the Higgs boson (125 GeV) should be quadratically dependent on the scale of UV physics, due to the quantum corrections from high-energy physics processes. The most perplexing experimental issue is the existence of *dark matter*, as demonstrated by galactic rotation curves [16–22]. This data has shown that there exists additional matter which has not yet been seen interacting with the particles of the Standard Model. There is no particle in the SM which can act as a candidate for dark matter.

Both of these major issues, as well as numerous others, can be solved by the introduction of *supersymmetry* (SUSY) [15, 23–35]. In supersymmetric theories, each SM particles has a so-called *superpartner*, or sparticle partner, differing from given SM particle by 1/2 in spin. These theories solve the hierachy problem, since the quantum corrections induced from the superpartners exactly cancel those induced by the SM particles. In addition, these theories are usually constructed assuming $R-$parity, which can be thought of as the "charge" of supersymmetry, with SM particles having $R = 1$ and sparticles having $R = -1$. In collider experiments, since the incoming SM particles have total $R = 1$, the resulting sparticles are produced in pairs. This produces a rich phenomenology, which is characterized by significant hadronic activity and large missing transverse energy ($E_{\mathrm{T}}^{\mathrm{miss}}$), which provide significant discrimination against SM backgrounds [36].

Despite the power of searches for supersymmetry where $E_{\mathrm{T}}^{\mathrm{miss}}$ is a primary discriminating variable, there has been significant interest in the use of other variables to discriminate against SM backgrounds. These include searches employing variables such as αT, $M_{T,2}$, and the razor variables (M_R, R^2) [37–47]. In this thesis, we will present the first search for supersymmetry using the novel Recursive Jigsaw Reconstruction (RJR) technique. RJR can be considered the conceptual successor of the razor variables. We impose a particular final state "decay tree" on an events, which roughly corresponds to a simplified Feynmann diagram in decays containing weakly-interacting particles. We account for the missing degrees of freedom associated to the weakly-interacting particles by a series of simplifying assumptions, which allow us to calculate our variables of interest at each step in the decay tree. This allows an unprecedented understanding of the internal structure of the decay and the ability to construct additional variables to reject Standard Model backgrounds.

This thesis details a search for the superpartners of the gluon and quarks, the gluino and squarks, in final states with zero leptons, with 13.3 $\text{fb}^{-1}$of data using the ATLAS detector. We organzie the thesis as follows. The theoretical foundations of the Standard Model and supersymmetry are described in Chapters 2 and 3. The Large Hadron Collider and the ATLAS detector are presented in Chapters 4 and 5. Chapter 5 provides a detailed description of Recursive Jigsaw Reconstruction and a description of the variables used for the particular search presented in this thesis. Chapter 6 presents the details of the analysis, including details of the dataset, object reconstruction, and selections used. In Chapter 7, the final results are presented; since there is no evidence of a supersymmetric signal in the analysis, we present the final exclusion curves in simplified supersymmetric models.

Chapter 2

The Standard Model

2.1 Overview

A Standard Model is another name for a theory of the internal symmetry group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with its associated set of parameters. *The* Standard Model refers specifically to a Standard Model with the proper parameters to describe the universe. The SM is the culmination of years of work in both theoretical and experimental particle physics. In this thesis, we take the view that theorists construct a model with the field content and symmetries as inputs, and write down the most general Lagrangian consistent with those symmetries. Assuming this model is compatible with nature (in particular, the predictions of the model are consistent with previous experiments), experimentalists are responsible measuring the parameters of this model This will be applicable for this chapter and the following one.

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Additional theoretical background is in 9.6. The philosophy and notations are inspired by [48, 49].

2.2 Field Content

The Standard Model field content is

$$\begin{aligned}
\text{Fermions} &: Q_L(3,2)_{+1/3}, U_R(3,1)_{+4/3}, D_R(3,1)_{-2/3}, L_L(1,2)_{-1}, E_R(1,1)_{-2} \\
\text{Scalar (Higgs)} &: \phi(1,2)_{+1} \\
\text{Vector Fields} &: G^\mu(8,1)_0, W^\mu(1,3)_0, B^\mu(1,1)_0
\end{aligned} \tag{2.1}$$

where the $(A, B)_Y$ notation represents the irreducible representation under $SU(3)$ and $SU(2)$, with Y being the electroweak hypercharge. Each of these fermion fields has an additional index, representing the three generation of fermions.

We observed that Q_L, U_R, and D_R are triplets under $SU(3)_C$; these are the *quark* fields. The *color* group, $SU(3)_C$ is mediated by the *gluon* field $G^\mu(8,1)_0$, which has 8 degrees of freedom. The fermion fields $L_L(1,2)_{-1}$ and $E_R(1,1)_{-2}$ are singlets under $SU(3)_C$; we call them the *lepton* fields.

Next, we note the "left-handed" ("right-handed") fermion fields, denoted by L (R) subscript, The left-handed fields form doublets under $SU(2)_L$. These are mediated by the three degrees of freedom of the "W" fields $W^\mu(1,3)_0$. These fields only act on the left-handed particles of the Standard Model. This is the reflection of the "chirality" of the Standard Model; the left-handed and right-handed particles are treated differently by the electroweak forces. The right-handed fields, U_R, D_R, and E_R, are singlets under $SU(2)_L$.

The $U(1)_Y$ symmetry is associated to the $B^\mu(1,1)_0$ boson with one degree of freedom. The charge Y is known as the electroweak hypercharge.

To better understand the phenomenology of the Standard Model, let us investigate each of the *sectors* of the Standard Model separately.

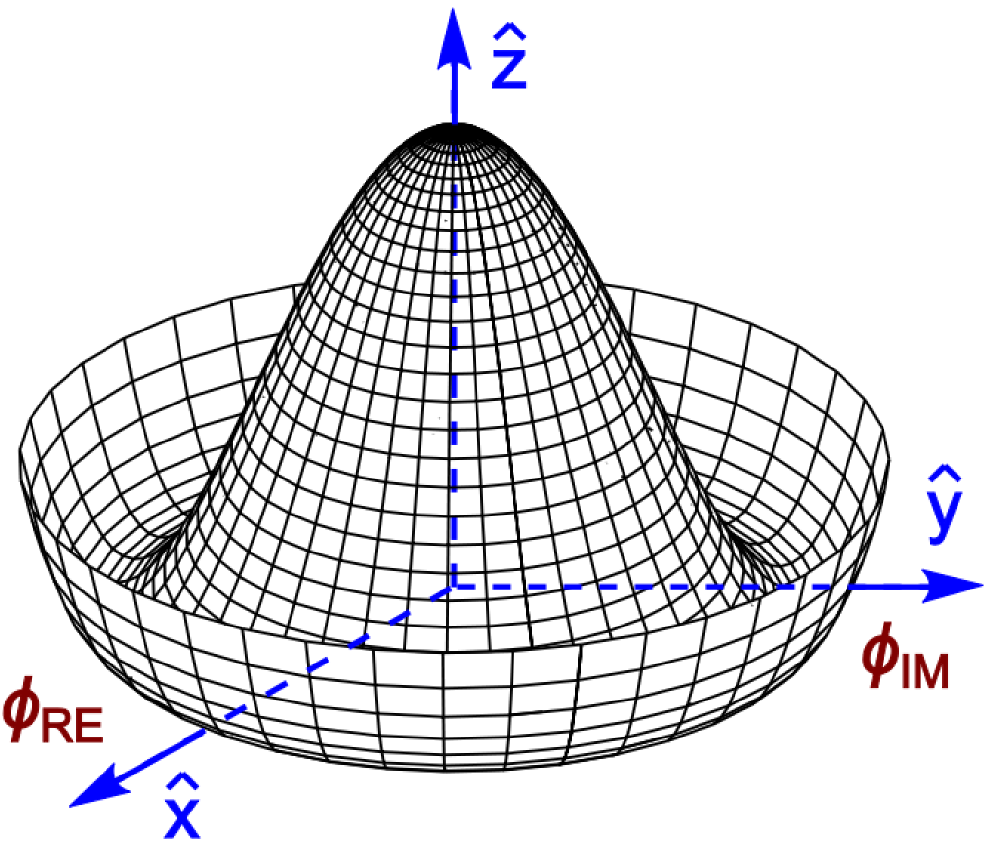
Electroweak sector

The electroweak sector refers to the $SU(2)_L \otimes U(1)_Y$ portion of the Standard Model gauge group. Following our philosophy of writing all gauge-invariant and renormalizable terms, the electroweak Lagrangian can be written as

$$\mathcal{L} = W_a^{\mu\nu} W_{\mu\nu}^a + B^{\mu\nu} B_{\mu\nu} + (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2. \tag{2.2}$$

where $W_a^{\mu\nu}$ are the three ($a = 1, 2, 3$) gauge bosons associated to the $SU(2)_L$ gauge group, $B^{\mu\nu}$ is the one gauge boson of the $U(1)_Y$ gauge group, and ϕ is the complex

Figure 2.1: Sombrero potential



Higgs multiplet. The covariant derivative D^μ is given by

$$D^\mu = \partial^\mu + \frac{ig}{2} W_a^\mu \sigma_a + \frac{ig'}{2} B^\mu \tag{2.3}$$

where $i\sigma_a$ are the Pauli matrices times the imaginary constant, which are the generators for $SU(2)_L$, and g and g' are the $SU(2)_L$ and $U(1)_Y$ coupling constants, respectively. The field strength tensors $W_a^{\mu\nu}$ and $B^{\mu\nu}$ are given by the commutator of the covariant derivative associated to each field

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W_a^{\mu\nu} &= \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\epsilon_{abc} W_a^\mu W_b^\nu, \qquad\qquad i = 1, 2, 3 \end{aligned} \tag{2.4}$$

The terms in the Lagrangian 2.2 proportional to μ^2 and λ make up the "Higgs potential" [50]. As normal (see Appendix 9.6), we restrict $\lambda > 0$ to guarantee our potential is bounded from below, and we also require $\mu^2 < 0$, which gives us the standard "sombrero" potential shown in 2.1.

This potential has infinitely many minima at $<\phi>=\sqrt{2m/\lambda}$; the ground state is *spontaneously* broken by the choice of ground state, which induces a vacuum expection value (VEV). Without loss of generality, we can choose the Higgs field ϕ to point in the real direction, and write the Higgs field ϕ in the following form :

$$\phi = \frac{1}{\sqrt{2}} \exp(\frac{i}{v}\sigma_a\theta_a) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{2.5}$$

We choose a gauge to rotate away the dependence on θ_a, such that we can write simply

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \tag{2.6}$$

Now, we can see how the masses of the vector bosons are generated from the application of the Higgs mechanism. We plug Eq.2.6 back into the electroweak Lagrangian, and only showing the relevant mass terms in the vacuum state where $h(x) = 0$ see that (dropping the Lorentz indices) :

$$\begin{aligned} \mathcal{L}_M &= \frac{1}{8} \left| \begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{g^2v^2}{8} \left[W_1^2 + W_2^2 + (\tfrac{g'}{g}B - W_3)^2 \right] \end{aligned} \tag{2.7}$$

Defining the *Weinberg* angle $\tan(\theta_W) = g'/g$ and the following *physical* fields :

$$\begin{aligned} W^\pm &= \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \\ Z^0 &= \cos\theta_W W_3 - \sin\theta_W B \\ A^0 &= \sin\theta_W W_3 + \cos\theta_W B \end{aligned} \tag{2.8}$$

we can write the piece of the Lagrangian associated to the vector boson masses as

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2v^2W^+W^- + \frac{1}{8}(g^2 + g'^2)v^2Z^0Z^0. \tag{2.9}$$

and we have the following values of the masses for the vector bosons :

$$\begin{aligned} m_W^2 &= \frac{1}{4}v^2g^2 \\ m_Z^2 &= \frac{1}{4}v^2(g^2 + g'^2) \\ m_A^2 &= 0 \end{aligned} \tag{2.10}$$

We thus see how the Higgs mechnanism gives rise to the masses of the $W^\pm$ and Z boson in the Standard Model; the mass of the photon is zero, as expected. The $SU(2)_L \otimes U(1)_Y$ symmetry of the initially massless $W_{1,2,3}$ and B fields is broken to the $U(1)_{EM}$. Of the four degrees of freedom in the complex Higgs doublet, three are "eaten" when we give mass to the $W^\pm$ and Z_0, while the other degree of freedom is the Higgs particle, as found in 2012 by the ATLAS and CMS collaborations [51, 52].

Quantum Chromodynamics

Quantum chromodynamics (or the theory of the *strong* force) characterizes the behavior of *colored* particles, collectively known as *partons*. The partons of the Standard Model are the (fermionic) quarks, and the (bosonic) gluons. The strong force is governed by $SU(3)_C$, an unbroken symmetry in the Standard Model, which implies the gluon remains massless. Defining the covariant derivative for QCD as

$$D^\mu = \partial^\mu + ig_sG_a^\mu L_a, a = 1, ..., 8 \tag{2.11}$$

where L_a are the generators of $SU(3)_C$, and g_s is the coupling constant of the strong force. The QCD Lagrangian then is given by

$$\mathcal{L}_{\text{QCD}} = i\bar{\psi}_f D_\mu \gamma^\mu \psi_f - \frac{1}{4}G_{a,\mu\nu}G_a^{\mu\nu} \tag{2.12}$$

where the summation over f is for quarks *families*, and $G_a^{\mu\nu}$ is the gluon field strength tensor, given by

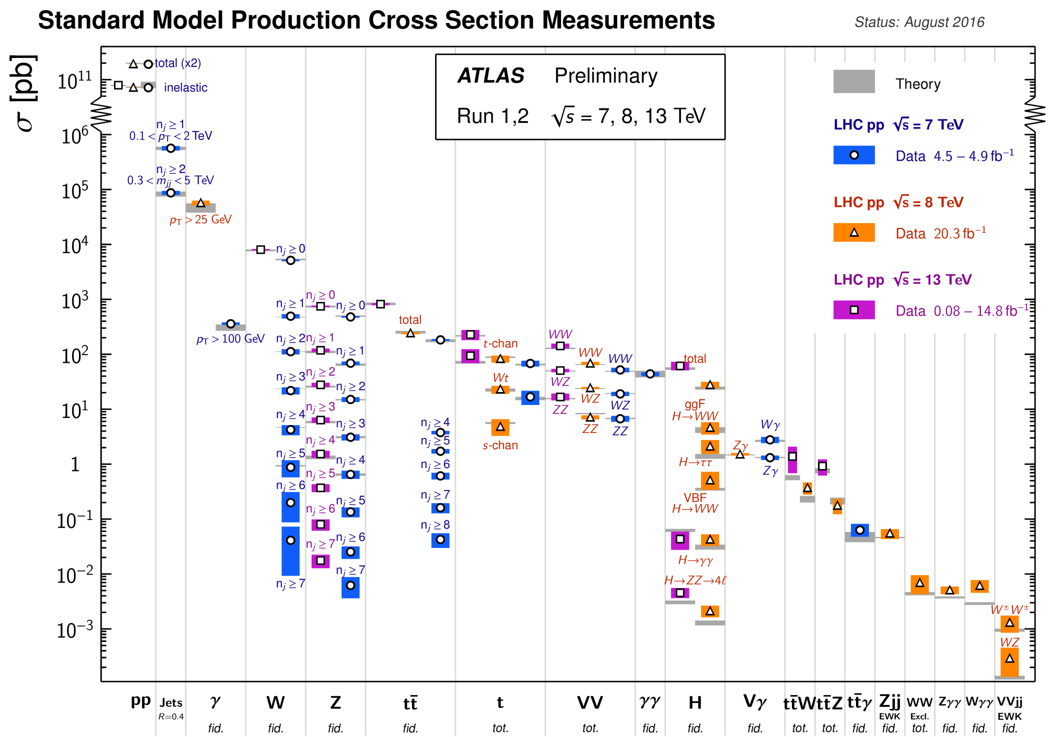
$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu, a, b, c = 1, ..., 8 \tag{2.13}$$

where f^{abc} are the structure constants of $SU(3)_C$, which are analogous to ϵ_{abc} for $SU(2)_L$. The kinetic term for the quarks is contained in the standard ∂_μ term, while the field strength term contains the interactions between the quarks and gluons, as well as the gluon self-interactions.

Written down in this simple form, the QCD Lagrangian does not seem much different from the QED Lagrangian, with the proper adjustments for the different group structures. The gluon is massless, like the photon, so one could näively expect an infinite range force, and it pays to understand why this is not the case. The reason for this fundamental difference is the gluon self-interactions arising in the field strength tensor term of the Lagrangian. This leads to the phenomena of *color confinement*, which describes how one only observes color-neutral particles alone in nature. In contrast to the electromagnetic force, particles which interact via the strong force experience a *greater* force as the distance between the particles increases. At long distances, the potential is given by $V(r) = -kr$. At some point, it is more energetically favorable to create additional partons out of the vacuum than continue pulling apart the existing partons, and the colored particles undergo *fragmentation*. This leads to *hadronization*. Bare quarks and gluons are actually observed as sprays of hadrons (primarly kaons and pions); these sprays are known as *jets*, which are what are observed by experiments.

It is important to recognize the importance of understanding these QCD interactions in high-energy hadron colliders such as the LHC. Since protons are hadrons, proton-proton collisions such as those produced by the LHC are primarily governed by the processes of QCD. In particular, by far the most frequent process observed in LHC experiments is dijet production from gluon-gluon interactions (see Fig.2.2). These

Figure 2.2: Cross-sections of various Standard Model processes



gluons that interact are part of the *sea* particles inside the proton; the simple $p = uud$ model does not apply. The main *valence* uud quarks are constantly interacting via gluons, which can themselves radiate gluons or split into quarks, and so on. A more useful understanding is given by the colloquially-known *bag* model [53, 54], where the proton is seen as a "bag" of (in principle) infinitely many partons, each with energy $E < \sqrt{s} = 6.5$ TeV. One then collides this (proton) bag with another, and views the products of this very complicated collision, where calculations include many loops in nonpertubative QCD calculations.

Fortunately, we are generally saved by the QCD factorization theorems [55]. This allows one to understand the hard (i.e. short distance or high energy) $2 \rightarrow 2$ parton process using the tools of perturbative QCD, while making series of approximations known as a *parton shower* model to understand the additional corrections from nonpertubative QCD. We will discuss the reconstruction of jets by experiments in Ch.5.

Fermions

We will now look more closely at the fermions in the Standard Model [56].

As noted earlier in Sec.2.2, the fermions of the Standard Model can be first distinguished between those that interact via the strong force (quarks) and those which do not (leptons).

There are six leptons in the Standard Model, which can be placed into three *generations*.

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \tag{2.14}$$

There is the electron (e), muon (μ), and tau (τ), each of which has an associated neutrino (ν_e, ν_μ, ν_τ). Each of the so-called charged ("electron-like") leptons has electromagnetic charge -1, while the neutrinos all have $q_{EM} = 0$.

Often in an experimental context, lepton is used to denote the stable electron and metastable muon, due to their striking experimental signatures. Taus are often treated separately, due to their much shorter lifetime of $\tau_\tau \sim 10^{-13}s$; these decay through hadrons or the other leptons, so often physics analyses at the LHC treat them as jets or leptons, as will be done in this thesis.

As the neutrinos are electrically neutral, nearly massless, and only interact via the weak force, it is quite difficult to observe them directly. Since LHC experiments rely overwhelmingly on electromagnetic interactions to observe particles, the presence of neutrinos is not observed directly. Neutrinos are instead observed by the conservation of four-momentum in the plane transverse to the proton-proton collisions, known as *missing transverse energy*.

There are six quarks in the Standard Model : up, down, charm, strange, top, and bottom. Quarks are similar organized into three generations :

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \tag{2.15}$$

where we speak of "up-like" quarks and "down-like" quarks.

Each up-like quark has charge $q_{up} = 2/3$, while the down-like quarks have $q_{down} = -1/3$. At the high energies of the LHC, one often makes the distinction between the light quarks (u, d, c, s), the bottom quark, and top quark. In general, due to the hadronization process described above, the light quarks, with masses $m_q <\sim 1.5 GeV$ are indistinguishable by LHC experiments. Their hadronic decay products generally have long lifetimes and they are reconstructed as jets.[1]. The bottom quark hadronizes primarily through the B-mesons, which generally travels a short distance before decaying to other hadrons. This allows one to distinguish decays via $b-$quarks form other jets; this procedure is known as *b-tagging* and will be discussed more in Ch.5. Due to its large mass, the top quark decays before it can hadronize; there are no bound states associated to the top quark. The top is of particular interest at the LHC; it has a striking signature through its most common decay mode $t \to Wb$. Decays via tops, especially $t\bar{t}$ are frequently an important signal decay mode, or an important background process.

Interactions in the Standard Model

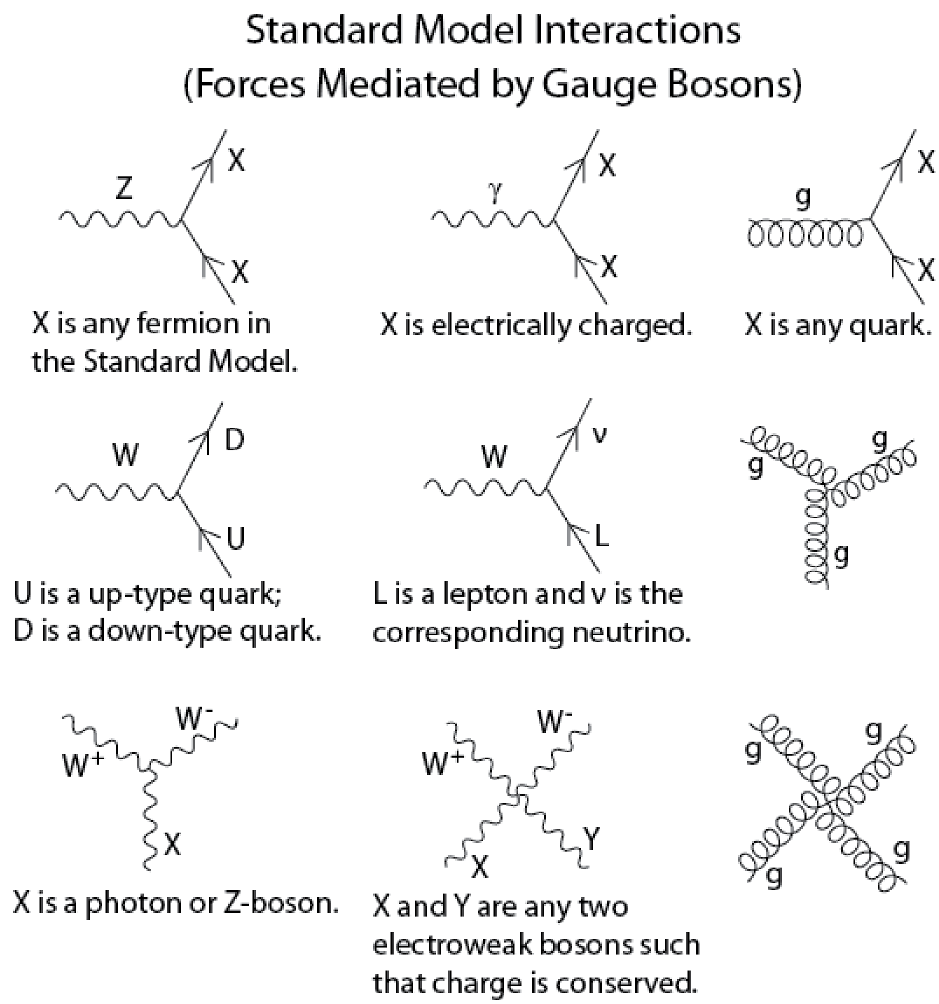
We briefly overview the entirety of the fundamental interactions of the Standard Model; these can also be found in 2.3.

The electromagnetic force, mediated by the photon, interacts with via a three-point coupling all charged particles in the Standard Model. The photon thus interacts with all the quarks, the charged leptons, and the charged $W^{\pm}$ bosons.

The weak force is mediated by three particles : the $W^{\pm}$ and the Z^0. The Z^0 can interacts with all fermions via a three-point coupling. A real Z_0 can thus decay to a fermion-antifermion pair of all SM fermions except the top quark, due to its large

[1]In some contexts, charm quarks are also treated as a separate category, although it is quite difficult to distinguish charm quarks from the other light quarks.

Figure 2.3: The interactions of the Standard Model



mass. The $W^{\pm}$ has two important three-point interactions with fermions. First, the $W^{\pm}$ can interact with an up-like quark and a down-like quark; an important example in LHC experiments is $t \to Wb$ The coupling constants for these interactions are encoded in the unitary matrix known as the Cabibbo–Kobayashi–Maskawa (CKM) matrix [57, 58], and are generally known as flavor-changing interactions. Secondly, the $W^{\pm}$ interacts with a charged lepton and its corresponding neutrino. In this case, the unitary matrix that corresponds to CKM matrix for quarks is the identity matrix, which forbids (fundamental) vertices such as $\mu \to We$. For leptons, instead this is a two-step process : $\mu \to \nu_m uW \to \nu_m u\bar{\nu}_e e$. Finally, there are the self-interactions of the weak gauge bosons. There is a three-point and four-point interaction; all combinations are allowed which conserve electric charge.

The strong force is mediated by the gluon, which as discussed above also carries the strong color charge. There is the fundamental three-point interaction, where a quark radiates a gluon. Additionally, there are the three-point and four-point gluon-only interactions.

2.3 Deficiencies of the Standard Model

At this point, it is quite easy to simply rest on our laurels. This relatively simple theory is capable of explaining a very wide range of phenomenom, which ultimately break down only to combinations of nine diagrams shown in Fig.2.3. Unfortunately, there are some unexplained problems with the Standard Model. We cannot go through all of the potential issues in this thesis, but we will motivate the primary issues which naturally lead one to *supersymmetry*, as we will see in Ch.3.

The Standard Model has many free paramaters; see Table 2.1 In general, we prefer models with less free parameters. A great example of this fact, and the primary experimental evidence for EWSB, is the relationship between the couplings of the weak force and the masses of the gauge bosons of the weak force :

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2\theta_W} \stackrel{?}{=} 1 \tag{2.16}$$

where ? indicates that this is a testable prediction of the Standard Model (in particular, that the gauge bosons gain mass through EWSB). This relationship has been measured within experimental and theoretical predictions. We would like to produce additional such relationships, which would exist if the Standard Model is a low-energy approximation of some other theory.

An additional issue is the lack of *gauge coupling unification*. The couplings of any quantum field theory "run" as a function of the distance scales (or inversely,

Table 2.1: Parameters of the Standard Model. For values dependent on the renormalization scheme, we use a combination of the on-shell normalization scheme [59–62] and modified minimal subtraction scheme with $m_{\bar{MS}}$ as indicated in the table[63]

.

m_e	Electron mass	511 keV
m_μ	Muon mass	105.7 MeV
m_τ	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV ($m_{\bar{MS}} = 2GeV$)
m_d	Down quark mass	4.4 MeV ($m_{\bar{MS}} = 2GeV$)
m_s	Strange quark mass	87 MeV ($m_{\bar{MS}} = 2GeV$)
m_c	Charm quark mass	1.32 GeV ($m_{\bar{MS}} = m_c$)
m_b	Bottom quark mass	4.24 GeV ($m_{\bar{MS}} = m_b$)
m_t	Top quark mass	172.7 GeV (on-shell renormalization)
θ_{12} CKM	12-mixing angle	13.1°
θ_{23} CKM	23-mixing angle	2.4°
θ_{13} CKM	13-mixing angle	0.2°
δ CKM	CP-violating Phase	0.995
g'	U(1) gauge coupling	0.357 ($m_{\bar{MS}} = m_Z$)
g	SU(2) gauge coupling	0.652 ($m_{\bar{MS}} = m_Z$)
g_s	SU(3) gauge coupling	1.221 ($m_{\bar{MS}} = m_Z$)
θQCD	QCD vacuum angle	∼0
VEV	Higgs vacuum expectation value	246 GeV
m_H	Higgs mass	125 GeV

energy scales) of the theory. The idea is closely related to the unification of the electromagnetic and weak forces at the so-called *electroweak scale* of $O(100 \text{ GeV})$. One would hope this behavior was repeated between the electroweak forces and the strong force at some suitable energy scale. The Standard Model does automatically not exhibit this behavior, as we can see in Fig.2.4.

The most significant problem with the Standard Model is the *hierarchy problem*. In its most straightforward incarnation, the Higgs scalar field is subject to quantum corrections through loop diagrams, as shown in Fig.2.5. For demonstration, we use the contributions from the top quark, since the top quark has the largest Higgs Yukawa coupling due to its large mass. In general, we should expect these corrections to quadratically dependent on the scale of the ultraviolet physics, Λ. Briefly assume there is no new physics before the Planck scale of gravity, $\Lambda_{\text{Planck}} = 10^{19}$ GeV. In this

Figure 2.4: The running of Standard Model gauge couplings. The Standard Model couplings do not unify at high energies, which indicates it cannot completely describe nature through the Planck scale.

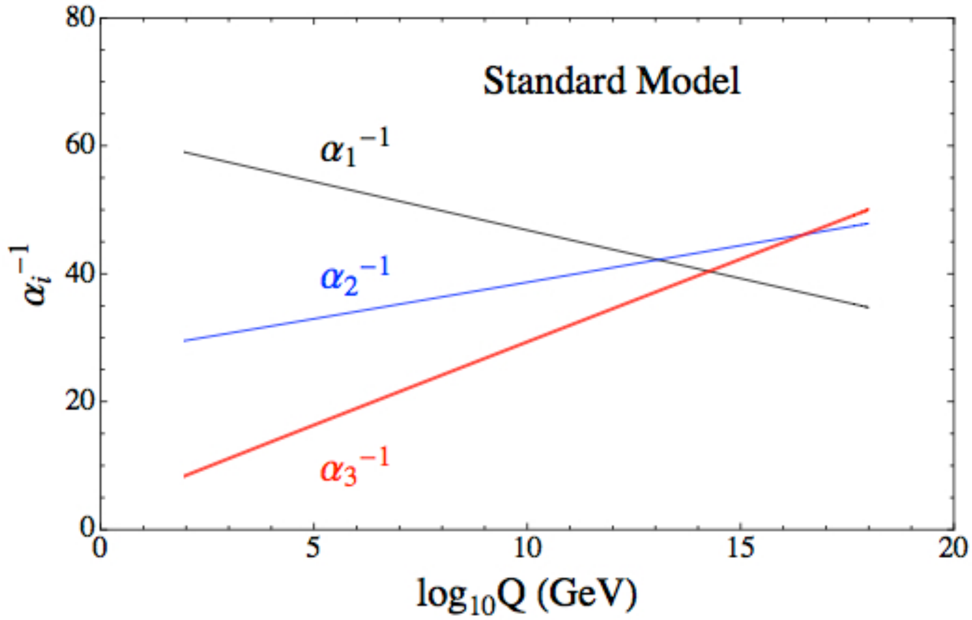
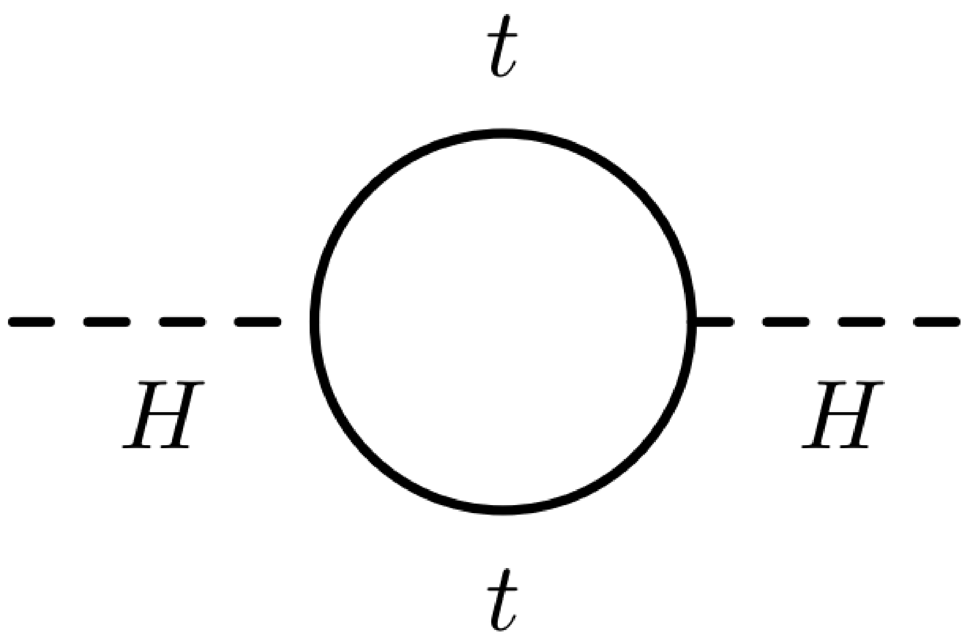


Figure 2.5: The dominant quantum loop correction to the Higgs mass in the Standard Model.



case, we expect the corrections to the Higgs mass like

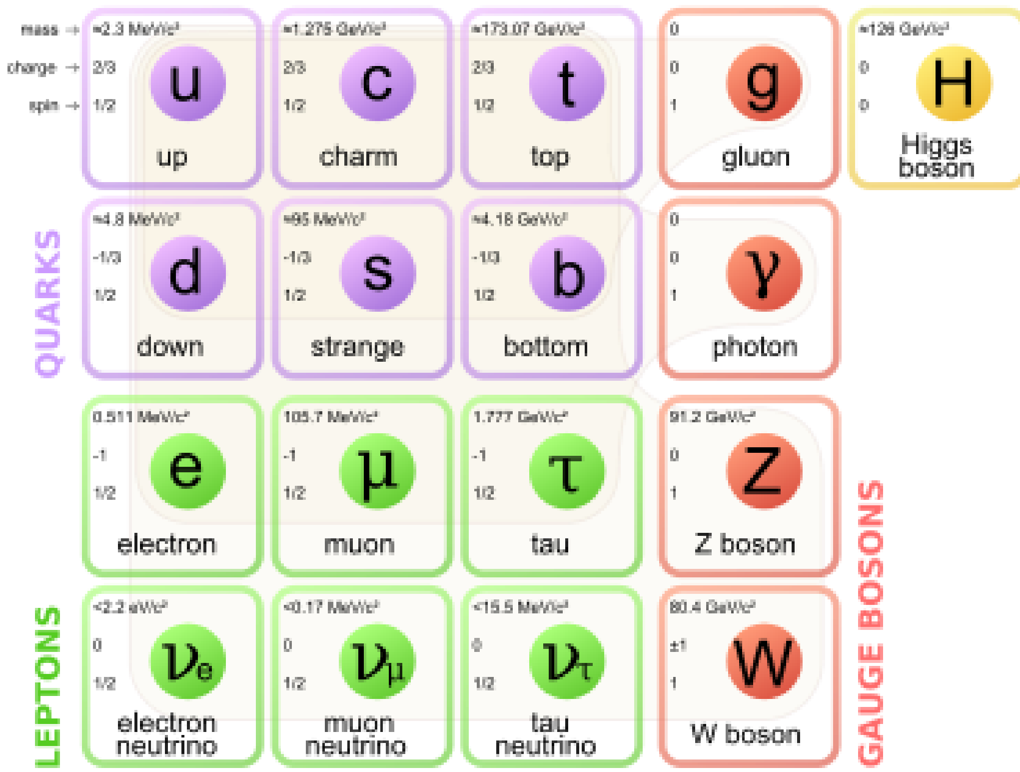
$$\delta m_H^2 \approx \left(\frac{m_t}{8\pi^2 {<}\phi{>}_{VEV}} \right)^2 \Lambda_{Planck}^2. \tag{2.17}$$

To achieve the miraculous cancellation required to get the observed Higgs mass of 125 GeV, one needs to then set the bare Higgs mass m_0, our input to the Standard Model Lagrangian, itself to a *precise* value $\sim 10^{19}$ GeV. This extraordinary level of parameter finetuning is quite undesirable, and within the framework of the Standard Model, there is little that can be done to alleviate this issue.

An additional concern, of a different nature, is the lack of a *dark matter* candidate in the Standard Model. Dark matter was discovered by observing galactic rotation curves, which showed that much of the matter that interacted gravitionally was invisible to our (electromagnetic) telescopes [16–22]. The postulation of the existence of dark matter, which interacts at least through gravity, allows one to understand these galatic rotation curves. Unfortunately, no particle in the Standard Model could possibly be the dark matter particle. The only candidate truly worth another look is the neutrino, but it has been shown that the neutrino content of the universe is simply too small to explain the galatic rotation curves [22, 64]. The experimental evidence from the galactic rotations curves thus show there *must* be additional physics beyond the Standard Model, which is yet to be understood.

In the next chapter, we will see how these problems can be alleviated by the theory of supersymmetry.

Figure 2.6: Particles of the Standard Model



Chapter 3

Supersymmetry

This chapter will introduce supersymmetry (SUSY) [15, 23–35]. We will begin by introducing the concept of a *superspace*, and discuss some general ingredients of supersymmetric theories. This will include a discussion of how the problems with the Standard Model described in Ch.2 are naturally fixed by these theories.

The next step is to discuss the particle content of the *Minimally Supersymmetric Standard Model* (MSSM). As its name implies, this theory contains the minimal additional particle content to make Standard Model supersymmetric. We then discuss the important phenomonological consequences of this theory, especially as it would be observed in experiments at the LHC.

3.1 Supersymmetric theories : from space to superspace

Coleman-Mandula "no-go" theorm

We begin the theoretical motivation for supersymmetry by citing the "no-go" theorem of Coleman and Mandula [65]. This theorem forbids *spin-charge unification*; it states that all quantum field theories which contain nontrivial interactions must be a direct product of the Poincarégroup of Lorentz symmetries, the internal product from of gauge symmetries, and the discrete symmetries of parity, charge conjugation, and time reversal. The assumptions which go into building the Coleman-Mandula theorem are quite restrictive, but there is one unique way out, which has become known as *supersymmetry* [26, 66]. In particular, we must introduce a *spinorial* group generator Q. Alternatively, and equivalently, this can be viewed as the addition of anti-commuting coordinates; space plus these new anti-commuting coordinates is then called *superspace* [67]. We will not investiage this view in detail, but it is also a quite intuitive and beautiful way to construct supersymmetry[15].

Supersymmetry transformations

A *supersymmetric* transformation Q transforms a bosonic state into a fermionic state, and vice versa :

$$Q\,|\text{Fermion}\rangle = |\text{Boson}\rangle \tag{3.1}$$

$$Q\,|\text{Boson}\rangle = |\text{Fermion}\rangle \tag{3.2}$$

To ensure this relation holds, Q must be an anticommuting spinor. Additionally, since spinors are inherently complex, $Q^\dagger$ must also be a generator of the supersymmetry transformation. Since Q and $Q^\dagger$ are spinor objects (with $s = 1/2$), we can see that supersymmetry must be a spacetime symmetry. The Haag-Lopuszanski-Sohnius extension [66] of the Coleman-Mandula theorem [65] is quite restrictive about the forms of such a symmetry. Here, we simply write the (anti-) commutation relations [15] :

$$Q_\alpha, Q^\dagger_{\dot\alpha} = -2\sigma_{\alpha\dot\alpha^\mu} P_\mu \tag{3.3}$$

$$Q_\alpha, Q_{\dot\beta} = Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta} = 0 \tag{3.4}$$

$$[P^\mu, Q_\alpha] = [P^\mu, Q^\dagger_{\dot\alpha}] = 0 \tag{3.5}$$

Supermultiplets

In a supersymmetric theory, we organize single-particle states into irreducible representations of the supersymmetric algebra which are known as *supermultiplets*. Each supermultiplet contains a fermion state $|\mathrm{F}\rangle$ and a boson state $|\mathrm{B}\rangle$; these two states are the known as *superpartners*. These are related by some combination of Q and $Q^\dagger$, up to a spacetime transformation. Q and $Q^\dagger$ commute with the mass-squared operator $-P^2$ and the operators corresponding to the gauge transformations [15]; in particular, the gauge interactions of the Standard Model. In an unbroken supersymmetric theory, this means the states $|\mathrm{F}\rangle$ and $|\mathrm{B}\rangle$ have exactly the same mass, electromagnetic charge, electroweak isospin, and color charges. One can also prove [15] that each supermultiplet contains the exact same number of bosonic (n_B) and fermion (n_F) degrees of freedom. We now explore the possible types of supermultiples one can find in a renormalizable supersymmetric theory.

Since each supermultiplet must contain a fermion state, the simplest type of supermultiplet contains a single Weyl fermion state ($n_F = 2$) which is paired with $n_B = 2$ scalar bosonic degrees of freedom. This is most conveniently constructed as single complex scalar field. We call this construction a *scalar supermultiplet* or *chiral supermultiplet*. The second name is indicative; only chiral supermultiplets can contain fermions whose right-handed and left-handed components transform differently under the gauge interactions (as of course happens in the Standard Model).

The second type of supermultiplet we construct is known as a *gauge* supermultiplet. We take a spin-1 gauge boson (which must be massless due to the gauge symmetry, so $n_B = 2$) and pair this with a single massless Weyl spinor[1]. The gauge bosons transform as the adjoint representation of the their respective gauge groups; their fermionic partners, which are known as gauginos, must also. In particular, the left-handed and right-handed components of the gaugino fermions have the same

[1]Choosing an $s = 3/2$ massless fermion leads to nonrenormalizable interactions.

gauge transformation properties.

Excluding gravity, this is the entire list of supermultiplets which can participate in renormalizable interactions in what is known as $N = 1$ supersymmetry. This means there is only one copy of the supersymmetry generators Q and $Q^\dagger$. This is essentially the only "easy" phenomenological choice, since it is the only choice in four dimensions which allows for the chiral fermions and parity violations built into the Standard Model, and we will not look further into $N > 1$ supersymmtry in this thesis.

The primary goal, after understanding the possible structures of the multiplets above, is to fit the Standard Model particles into a multiplet, and therefore make predictions about their supersymmetric partners. We explore this in the next section.

3.2 Minimally Supersymmetric Standard Model

To construct what is known as the MSSM [**susyPrimer** , 68–71], we need a few ingredients and assumptions. First, we match the Standard Model particles with their corresponding superpartners of the MSSM. We will also introduce the naming of the superpartners (also known as *sparticles*). We discuss a very common additional restraint imposed on the MSSM, known as $R-$parity. We also discuss the concept of soft supersymmetry breaking and how it manifests itself in the MSSM.

Chiral supermultiplets

The first thing we deduce is directly from Sec.**??**. The bosonic superpartners associated to the quarks and leptons *must* be spin 0, since the quarks and leptons must be arranged in a chiral supermultiplet. This is essentially the note above, since the chiral supermultiplet is the only one which can distinguish between the left-handed and right-handed components of the Standard Model particles. The superpartners of the quarks and leptons are known as *squarks* and *sleptons*, or *sfermions* in aggregate.

(for “scalar quarks”, “scalar leptons”, and “scalar fermion”[2]). The “s-” prefix can also be added to the individual quarks i.e. *selectron*, *sneutrino*, and *stop*. The notation is to add a $\sim$ over the corresponding Standard Model particle i.e. $\tilde{e}$, the selectron is the superpartner of the electron. The two-component Weyl spinors of the Standard Model must each have their own (complex scalar) partner i.e. e_L, e_R have two distinct partners : $\tilde{e_L}, \tilde{e_R}$. As noted above, the gauge interactions of any of the sfermions are identical to those of their Standard Model partners.

Due to the scalar nature of the Higgs, it must obviously lie in a chiral supermultiplet. To avoid gauge anomolies and ensure the correct Yukawa couplings to the quarks and leptons[15], we must add additional Higgs bosons to any supersymmetric theory. In the MSSM, we have two chiral supermultiplets. The SM (SUSY) parts of the multiplets are denoted $H_u(\tilde{H}_u)$ and $H_d(\tilde{H}_d)$. Writing out H_u and H_d explicitly:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \tag{3.6}$$

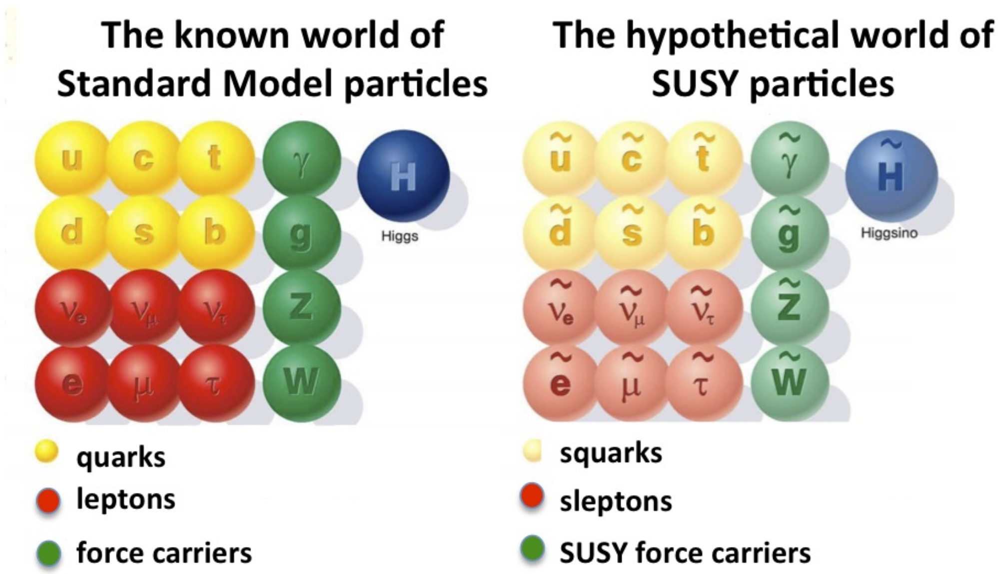
$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \tag{3.7}$$

$$\tag{3.8}$$

we see that H_u looks very similar to the SM Higgs with $Y = 1$, and H_d is symmetric to this with $+ \to -$, with $Y = -1$. The SM Higgs boson, h_0, is a linear superposition of the neutral components of these two doublets. The SUSY parts of the Higgs multiplets, $\tilde{H}_u$ and $\tilde{H}_d$, are each left-handed Weyl spinors. For generic spin-1/2 sparticles, we add the “-ino” suffix. We then call the partners of the two Higgs collectively the *Higgsinos*.

[2] The last one should probably have bigger scare quotes.

Figure 3.1: Particles of the MSSM



Gauge supermultiplets

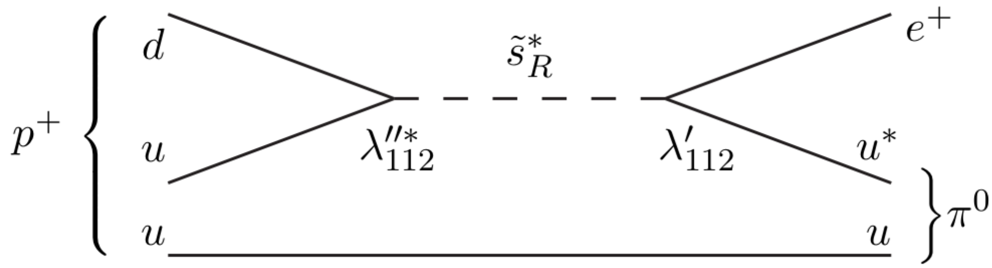
The superpartners of the gauge bosons must all be in gauge supermultiplets since they contain a spin-1 particle. Collectively, we refer to the superpartners of the gauge bosons as the gauginos.

The first gauge supermultiplet contains the gluon, and its superpartner, which is known as the *gluino*, denoted $\tilde{g}$. The gluon is of course the SM mediator of $SU(3)_C$; the gluino is also a colored particle, subject to $SU(3)_C$. From the SM before EWSB, we have the four gauge bosons of the electroweak symmetry group $SU(2)_L \otimes U(1)_Y$: $W^{1,2,3}$ and B^0. The superpartners of these particles are thus the *winos* $W^{\tilde{1},2,3}$ and *bino* $\tilde{B}^0$, where each is placed in another gauge supermultiplet with its corresponding SM particle. After EWSB, without breaking supersymmetry, we would also have the zino $\tilde{Z}^0$ and photino $\tilde{\gamma}$.

The entire particle content of the MSSM can be seen in Fig.3.1.

At this point, it's important to take a step back. Where are these particles? As stated above, supersymmetric theories require that the masses and all quantum

Figure 3.2: This Feynman diagram shows how proton decay is induced in the MSSM, if one does not impose $R-$parity.



numbers of the SM particle and its corresponding sparticle are the same. Of course, we have not observed a selectron, squark, or wino. The answer, as it often is, is that supersymmetry is *broken* by the vacuum state of nature [15].

$R-$parity

This section is a quick aside to the general story. $R-parity$ refers to an additional discrete symmetry which is often imposed on supersymmetric models. For a given particle state, we define

$$R = (-1)^{3(B-L)+2s} \tag{3.9}$$

where B, L is the baryon (lepton) number and s is the spin. The imposition of this symmetry forbids certain terms from the MSSM Lagrangian that would violate baryon and/or lepton number. This is required in order to prevent proton decay, as shown in Fig.3.2[3]. .

In supersymmetric models, this is a $\mathbb{Z}_2$ symmetry, where SM particles have $R = 1$ and sparticles have $R = -1$. We will take $R - parity$ as part of the definition of the MSSM. We will discuss later the *drastic* consequences of this symmetry on SUSY phenomenology

[3]Proton decay can actually be prevented by allowing only one of the four potential R-parity violating terms to survive.

Soft supersymmetry breaking

The fundamental idea of *soft* supersymmetry breaking[15, 34, 35, 72, 73] is that we would like to break supersymmetry without reintroducing the quadratic divergences we discussed at the end of Chapter 2. We write the Lagrangian in a form :

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \tag{3.10}$$

In this sense, the symmetry breaking is "soft", since we have separated out the completely symmetric terms from those soft terms which will not allow the quadratic divergences to the Higgs mass.

The explicitly allowed terms in the soft-breaking Lagrangian are [35].

- Mass terms for the scalar components of the chiral supermultipletss
- Mass terms for the Weyl spinor components of the gauge supermultipletss
- Trilinear couplings of scalar components of chiral supermultiplets

In particular, using the field content described above for the MSSM, the softly-broken portion of the MSSM Lagrangian can be writen

$$\begin{aligned}
\mathcal{L}_{\text{soft}} = & -\frac{1}{2}\left(M_3\tilde{g}\tilde{g} + M_2\tilde{W}\tilde{W} + M_1\tilde{B}\tilde{B} + c.c.\right) && (3.11)\\
& -\left(\tilde{u}a_u\tilde{Q}H_u - \tilde{d}a_d\tilde{Q}H_d - \tilde{e}a_e\tilde{L}H_d + c.c.\right) && (3.12)\\
& -\tilde{Q}^\dagger m_Q^2\tilde{Q} - \tilde{L}^\dagger m_L^2\tilde{L} - \tilde{u}m_u^2\tilde{u}^\dagger - \tilde{d}m_d^2\tilde{d}^\dagger - \tilde{e}m_e^2\tilde{e}^\dagger && (3.13)\\
& -m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_uH_d + cc). && (3.14)
\end{aligned}$$

where we have introduced the following notations :

1. M_3, M_2, M_1 are the gluino, wino, and bino masses.
2. a_u, a_d, a_e are complex 3×3 matrices in family space.
3. $m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2$ are hermitian 3×3 matrices in family space.

4. $m^2_{H_u}, m^2_{H_u}, b$ are the SUSY-breaking contributions to the Higgs potential.

We have written matrix terms without any sort of additional notational decoration to indicate their matrix nature, and we now show why. The first term 1 are straightforward; these are just the straightforward mass terms for these fields. There are strong constraints on the off-diagonal terms for the matrices of 2 [74, 75]; for simplicity, we will assume that each $a_i, i = u, d, e$ is proportional to the Yukawa coupling matrix : $a_i = A_{i0}y_i$. The matrices in **??** can be similarly constrained by experiments [68, 75–82] Finally, we assume that the elements 4 contributing to the Higgs potential as well as all of the 1 terms must be real, which limits the possible CP-violating interactions to those of the Standard Model. We thus only consider flavor-blind, CP-conserving interactions within the MSSM.

The important mixing for mass and gauge interaction eigenstates in the MSSM occurs within electroweak sector, in a process akin to EWSB in the Standard Model. The neutral portions of the Higgsinos doublets and the neutral gauginos ($\tilde{H}^0_u, \tilde{H}^0_d, \tilde{B}^0, \tilde{W}^0$ of the gauge interaction basis mix to form what are known as the *neutralinos* of mass basis :

$$M_{\tilde{\chi}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \tag{3.15}$$

where $s(c)$ are the sine and cosine of angles related to EWSB, which introduced masses to the gauginos and higgsinos. Diagonalization of this matrix gives the four neutralino mass states, listed without loss of generality in order of increasing mass : $\tilde{\chi}^0_{1,2,3,4}$.

The neutralinos, especially the lightest neutralino $\tilde{chi}^0_1$, are important ingredients in SUSY phenomenology.

The same process can be done for the electrically charged gauginos with the charged portions of the Higgsino doublets along with the charged winos $(\tilde{H}_u^+, \tilde{H}_d^+, \tilde{W}^+, \tilde{W}^-$. This leads to the *charginos*, again in order of increasing mass : $\tilde{\chi}_{1,2}^{\pm}$.

3.3 Phenomenology

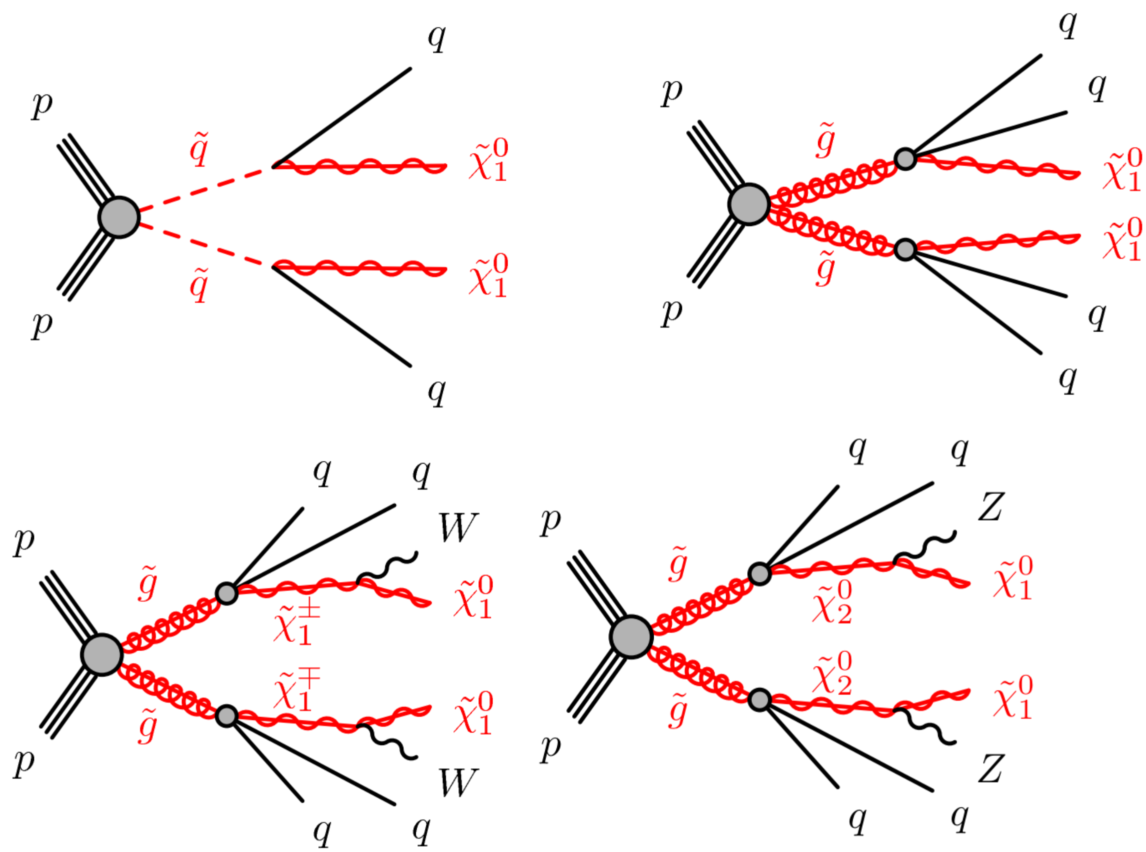
We are finally at the point where we can discuss the phenomenology of the MSSM, in particular as it manifests itself at the energy scales of the LHC.

As noted above in Sec.3.2, the assumption of $R-$parity has important consequences for MSSM phenomenology. The SM particles have $R = 1$, while the sparticles all have $R = -1$. Simply, this is the "charge" of supersymmetry. Since the particles of LHC collisions (pp) have total incoming $R = 1$, we must expect that all sparticles will be produced in *pairs*. An additional consequence of this symmetry is the fact that the lightest supersymmetric particle (LSP) is *stable*. Off each branch of the Feynmann diagram shown in Fig., we have $R = -1$, and this can only decay to another sparticle and a SM particle. Once we reach the lightest sparticle in the decay, it is absolutely stable. This leads to the common signature $E_{\mathrm{T}}^{\mathrm{miss}}$ for a generic SUSY signal.

For this thesis, we will be presenting an inclusive search for squarks and gluinos with zero leptons in the final state. This is a very interesting decay channel[4], due to the high cross-sections of $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{q}$ decays, as can be seen in Fig.**??** [83]. This is a direct consequence of the fact that these are the colored particles of the MSSM. Since the sparticles interact with the gauge groups of the SM in the same way as their SM partners, the colored sparticles, the squarks and gluinos, are produced and decay as governed by the color group $SU(3)_C$ with the strong coupling g_S. The digluino production is particularly copious, due to color factor corresponding to the color octet

[4] Prior to Run1, probably the most *most* interesting SUSY decay channel.

Figure 3.3: SUSY signals considered in this thesis



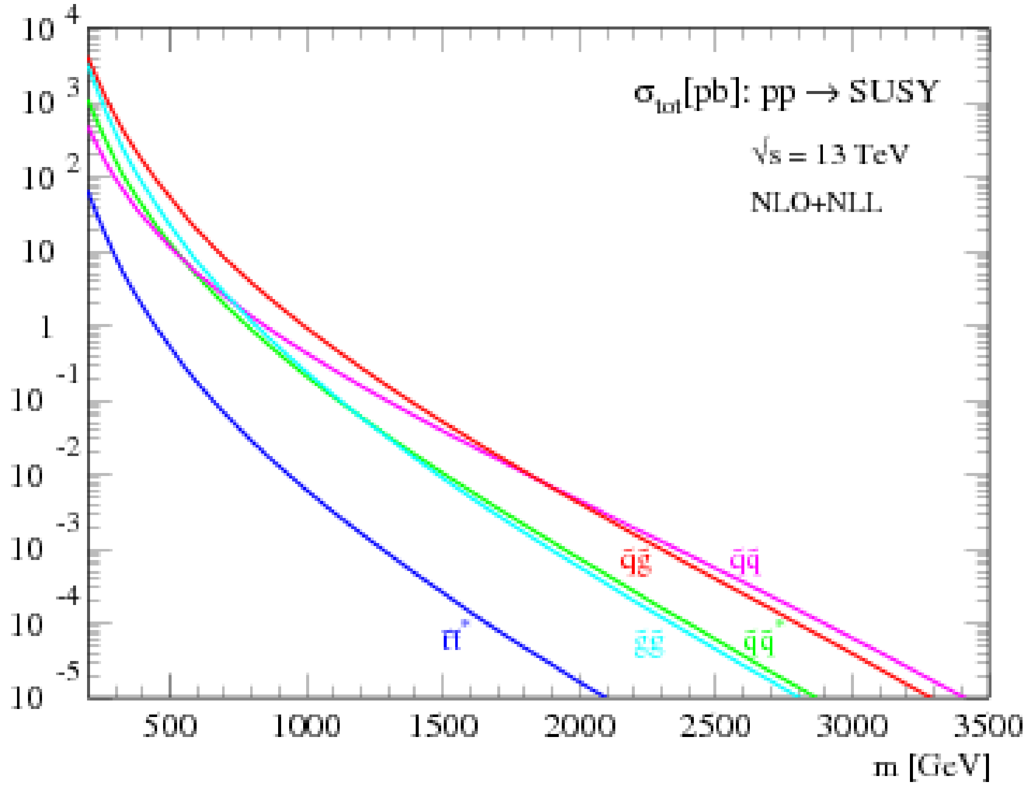
of $SU(3)C$.

In the case of disquark production, the most common decay mode of the squark in the MSSM is a decay directly to the LSP plus a single SM quark [15]. This means the basic search strategy of disquark production is two jets from the final state quarks, plus missing transverse energy for the LSPs. There are also cascade decays, the most common of which, and the only one considered in this thesis, is $\tilde{q} \rightarrow q\tilde{\chi}^{\pm} \rightarrow qW^{\pm}\chi^0$.

For digluino production, the most common decay is $\tilde{g} \rightarrow g\tilde{q}$, due to the large g_S coupling. The squark then decays as listed above. In this case, we generically search for four jets and missing transverse energy from the LSPs. We can also have the squark decay in association with a $W^{\pm}$ or Z^0; in this thesis, we are interested in those cases where this vector boson goes hadronically.

In the context of experimental searches for SUSY, we often consider *simplified*

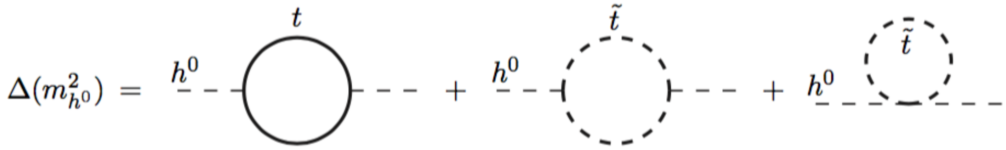
Figure 3.4: SUSY production cross-sections as a function of sparticle mass at $\sqrt{s} =$ 13 TeV.



models. These models make certain assumptions which allow easy comparisons of results by theorists and rival experimentalists. In the context of this thesis, the simplified models will make assumptions about the branching ratios described in the preceding paragraphs. In particular, we will often choose a model where the decay of interest occurs with 100% branching ratio. This is entirely for ease of interpretation by other physicists[5], but it is important to recognize that these are more a useful comparison tool, especially with limits, than a strict statement about the potential masses of sought-after beyond the Standard Model particle.

[5]In the author's opionion, this often leads to more confusion than comprehension. We will revisit the shortcomings of simplified models in the Conclusion to this thesis.

Figure 3.5: Loop diagrams correct the Higgs mass in the MSSM



3.4 How SUSY solves the problems with the SM

We now return to the issues with the Standard Model as described in Ch.2 to see how these issues are solved by supersymmetry.

Quadratic divergences to the Higgs mass

The quadratic divergences induced by the loop corrections to the Higgs mass, for example from the top Yukawa coupling, goes as

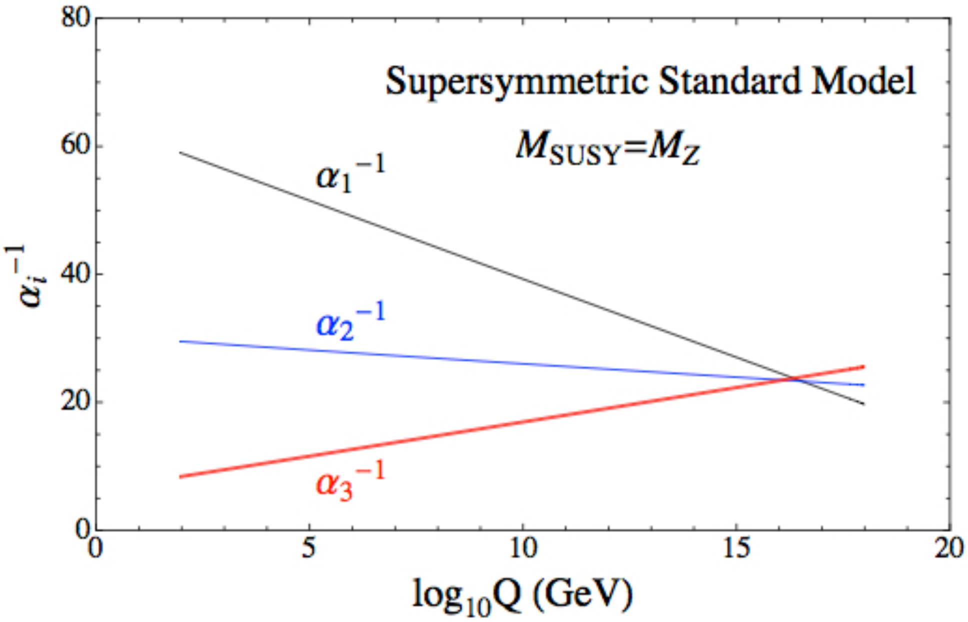
$$\delta m_H^2 \approx \left(\frac{m_t}{8\pi^2 <\phi>_{VEV}}\right)^2 \Lambda^2_{Planck}. \tag{3.16}$$

The miraculous thing about SUSY is each of these terms *automatically* comes with a term which exactly cancels this contribution[15]. The fermions and bosons have opposite signs in this loop diagram to all orders in pertubation theory, which completely solves the hierarchy problem. This is the most well-motivated reason for supersymmetry.

Gauge coupling unification

An additional motivation for supersymmetry is seen by the gauge coupling unification high scales. In the Standard Model, as we saw the gauge couplings fail to unify at high energies. In the MSSM and many other forms of supersymmetry, the gauge couplings unify at high energy, as can be seen in Fig.**??**. This provides additional aesthetic motivation for supersymmetric theories.

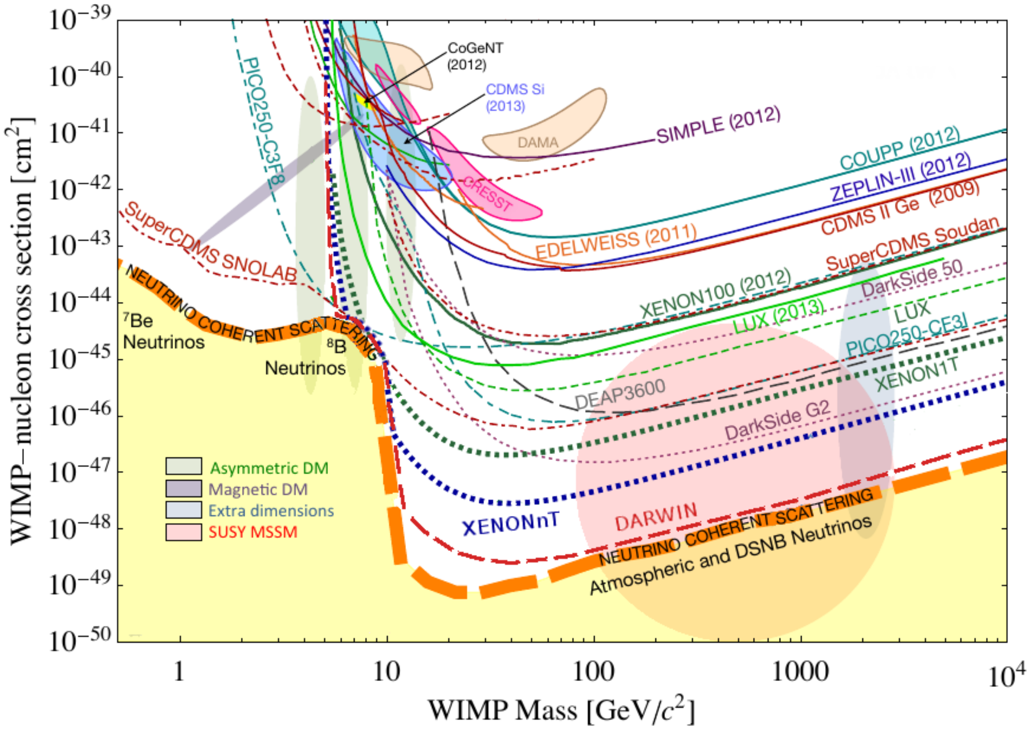
Figure 3.6: The running of Standard Model gauge couplings; compare to Fig.2.4. The MSSM gauge couplings nearly intersect at high energies.



Dark matter

As we discussed previously, the lack of any dark matter candidate in the Standard Model naturally leads to beyond the Standard Model theories. In the Standard Model, there is a natural dark matter candidate in the lightest supersymmetric particle[15] The LSP would in dark matter experiments be called a *weakly-interacting massive particle* (WIMP), which is a type of cold dark matter [22, 84]. These WIMPS would only interact through the weak force and gravity, which is exactly as a model like the MSSM predicts for the neutralino. In Fig.3.7, we can see the current WIMP exclusions for a given mass. The range of allowed masses which have not been excluded for LSPs and WIMPs have significant overlap. This provides additional motivation outside of the context of theoretical details.

Figure 3.7: WIMP exclusions from direct dark matter detection experiments.



3.5 Conclusions

Supersymmetry is the most well-motivated theory for physics beyond the Standard Model. It provides a solution to the hierarchy problem, leads to gauge coupling unification, and provides a dark matter candidate consistent with galatic rotation curves. As noted in this chapter, due to the LSPs in the final state, most SUSY searches require a significant amount of missing transverse energy in combination with jets of high transverse momentum. However, there is some opportunity to do better than this, especially in final states where one has two weakly-interacting LSPs on opposite sides of some potentially complicated decay tree. We will see how this is done in Ch.??.

Chapter 4

The Large Hadron Collider

The Large Hadron Collider (LHC) produces high-energy protons which are collided at the center of multiple large experiments at CERN on the outskirts of Geneva, Switzerland [85]. The LHC produces the highest energy collisions in the world, with design center-of-mass energy of $\sqrt{s} = 14$ TeV, which allows the experiments to investigate physics far beyond the reach of previous colliders. This chapter will summarize the basics of accelerator physics, especially with regards to discovering physics beyond the Standard Model. We will describe the CERN accelerator complex and the LHC.

4.1 Basics of Accelerator Physics

This section follows closely the presentation of [86].

Simple particle accelerators simply rely on the acceleration of charged particles in a static electric field. Given a field of strength E, charge q, and mass m, this is simply

$$a = \frac{qE}{m}. \tag{4.1}$$

For a given particle with a given mass and charge, this is limited by the static electric field which can be produced, which in turn is limited by electrical breakdown at high voltages.

There are two complementary solutions to this issue. First, we use the *radio frequency acceleration* technique. We call the devices used for this *RF cavities*. The cavities produce a time-varied electric field, which oscillate such that the charged particles passing through it are accelerated towards the design energy of the RF cavity. This oscillation also induces the particles into *bunches*, since particles which are slightly off in energy from that induced by the RF cavity are accelerated towards the design energy.

Second, one bends the particles in a magnetic field, which allows them to pass through the same RF cavity over and over. This second process is often limited by *synchrotron radiation*, which describes the radiation produced when a charged particle is accelerated. The power radiated is

$$P \sim \frac{1}{r^2}\left(E/m\right)^4 \tag{4.2}$$

where r is the radius of curvature and E, m is the energy (mass) of the charged particle. Given an energy which can be produced by a given set of RF cavities (which is *not* limited by the mass of the particle), one then has two options to increase the actual collision energy : increase the radius of curvature or use a heavier particle. Practically speaking, the easiest options for particles in a collider are protons and electrons, since they are (obviously) copious in nature and do not decay[1]. Given the dependence on mass, we can see why protons are used to reach the highest energies. The tradeoff for this is that protons are not point particles, and we thus we don't know the exact incoming four-vectors of the protons, as discussed in Ch.2.

The particle *beam* referes to the bunches all together An important property of a beam of a particular energy E, moving in uniform magnetic field B, containing particles of momentum p is the *beam rigidity* :

$$R \equiv rB = p/c. \tag{4.3}$$

The linear relation between r and p, or alternatively B and p have important consequences for LHC physics. For hadron colliders, this is the limiting factor on

[1]Muon colliders are a really cool option at high energies, since the relativistic γ factor gives them a relatively long lifetime in the lab frame.

going to higher energy scales; one needs a proportionally larger magnetic field to keep the beam accelerating in a circle.

Besides the rigidity of the beam, the most important quantities to characterize a beam are known as the (normalized) *emittance* ϵ_N and the *betatron function* β. These quantities determine the transverse size σ of a relativistic beam $v \overset{\sim}{<} c$ beam : $\sigma^2 = \beta^* \epsilon_N / \gamma_{\mathrm{rel}}$, where β^* is the value of the betatron function at the collision point and γ_{rel} is the Lorentz factor.

These quantities determine the *instaneous luminosity* L of a collider, which combined with the cross-section σ of a particular physics process, give the rate of this physics process :

$$R = L\sigma. \tag{4.4}$$

The instaneous luminosity L is given by :

$$L = \frac{f_{\mathrm{rev}} N_b^2 F}{4\pi\sigma^2} = \frac{f_{\mathrm{rev}} n N_b^2 \gamma_{\mathrm{rel}} F}{4\pi\beta^* \epsilon_N}. \tag{4.5}$$

Here we have introduced the frequency of revolutions f_{rev}, the number of bunches n, the number of protons per bunch N_b^2, and a geometric factor F related to the crossing angle of the beams.

The *integrated luminosity* $\int L$ gives the total number of a particular physics process P, with cross-section σ_{P}.

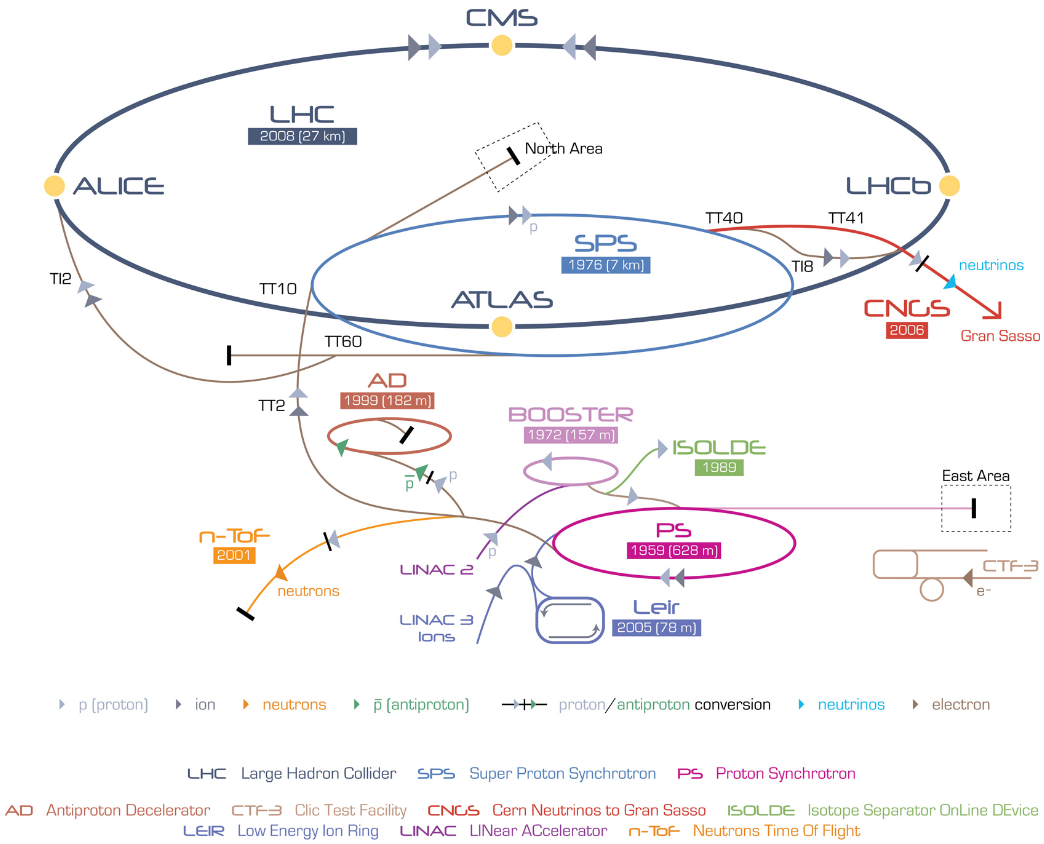
$$N_{\mathrm{P}} = \sigma_{\mathrm{P}} \int L. \tag{4.6}$$

Due to this simple relation, one can also quantify the “amount of data delivered” by a collider simply by $\int L$.

4.2 Accelerator Complex

The Large Hadron Collider is the last accelerator in a chain of accelerators which together form the CERN accelerator complex, which can be seen in 4.1. The protons

Figure 4.1: The CERN accelerator complex.



begin their journey to annilation in a hydrogen source, where they are subsequently ionized. The first acceleration occurs in the Linac 2, a linear accelerator composed of RF cavities. The protons leave the Linac 2 at an energy of 50 MeV and enter the Proton Synchrotron Booster (PSB). The PSB contains four superimposed rings, which accelerate the protons to 1.4 GeV. The protons are then injected into the Proton Synchrotron (PS). This synchrotron increases the energy up to 25 GeV. After leaving the PS, the protons enter the Super Proton Synchrotron (SPS). This is the last step before entering the LHC ring, and the protons are accelerated to 450 GeV. From the SPS, the protons are injected into the beam pipes of the LHC. The process to fill the LHC rings with proton bunches from start to finish typically takes about four minutes.

4.3 Large Hadron Collider

The Large Hadron Collider is the final step in the CERN accelerator complex, and produces the collisions analyzed in this thesis. From the point of view of experimentalists on the general-purpose ATLAS and CMS experiments, the main goal of the LHC is to deliver collisions at the highest possible energy, with the highest possible instaneous luminosity. The LHC was installed in the existing 27 km tunnel used by the Large Electron Positron (LEP) collider [87]. This allowed the existing accelerator complex at CERN, described in the previous section, to be used as the injection system to prepare the protons up to 450 GeV. Many aspects of the LHC design were decided by this very fact, and specified the options allowed to increase the energy or luminosity. In particular, the radius of the tunnel was already specified; from Eq.4.3, this implies the momentum (or energy) of the beam is entirely determined by the magnetic field. Given the 27 km circumference of the LEP tunnel, one can calculate the required magnetic field to reach the 7 TeV per proton design energy of the LHC :

$$r = C/2\pi = 4.3 \text{ km} \tag{4.7}$$

$$\rightarrow B = \frac{p}{rc} = 5 \text{ T} \tag{4.8}$$

In fact, the LHC consists of 8 528 m straight portions consisting of RF cavities, used to accelerate the particles, and 8 circular portions which bend the protons around the LHC ring. These circular portions actually have a slightly smaller radius of curvature $r = 2804$ m, and we require $B = 8.33$ T. To produce this large field, we need to use superconducting magnets, as discussed in the next section.

Magnets

There are many magnets used by the LHC machine, but the most important are the 1232 dipole magnets; a schematic is shown in Fig.4.2 and a photograph is shown in

Figure 4.2: Schematic of an LHC dipole magnet.

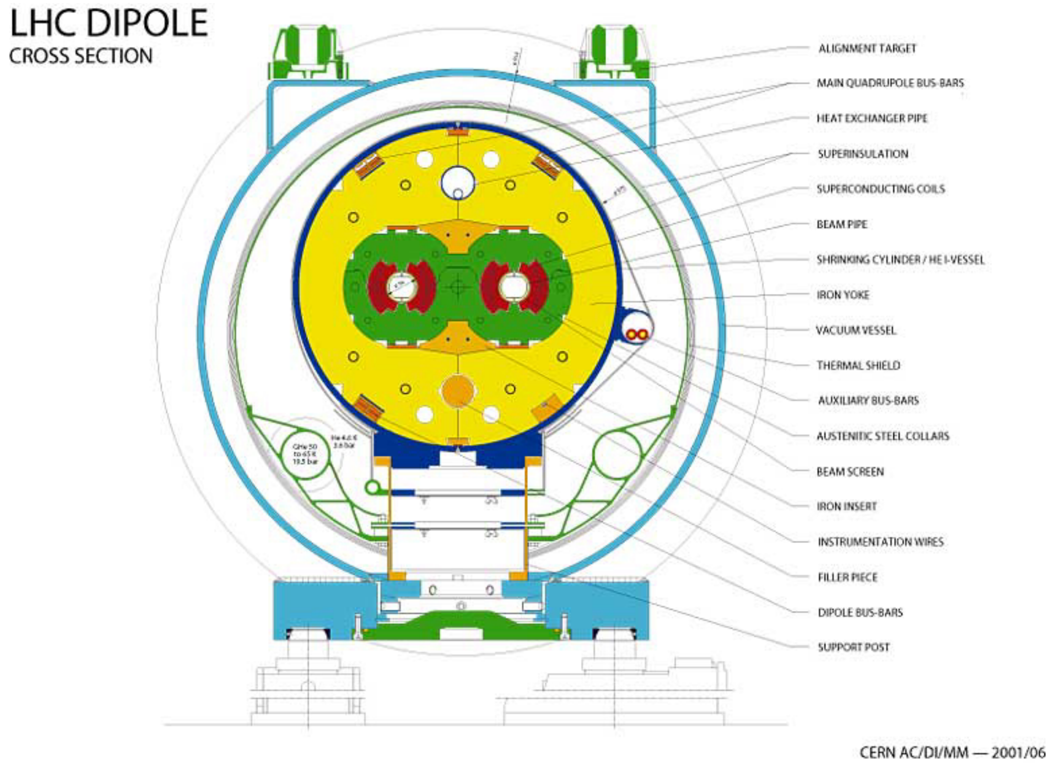


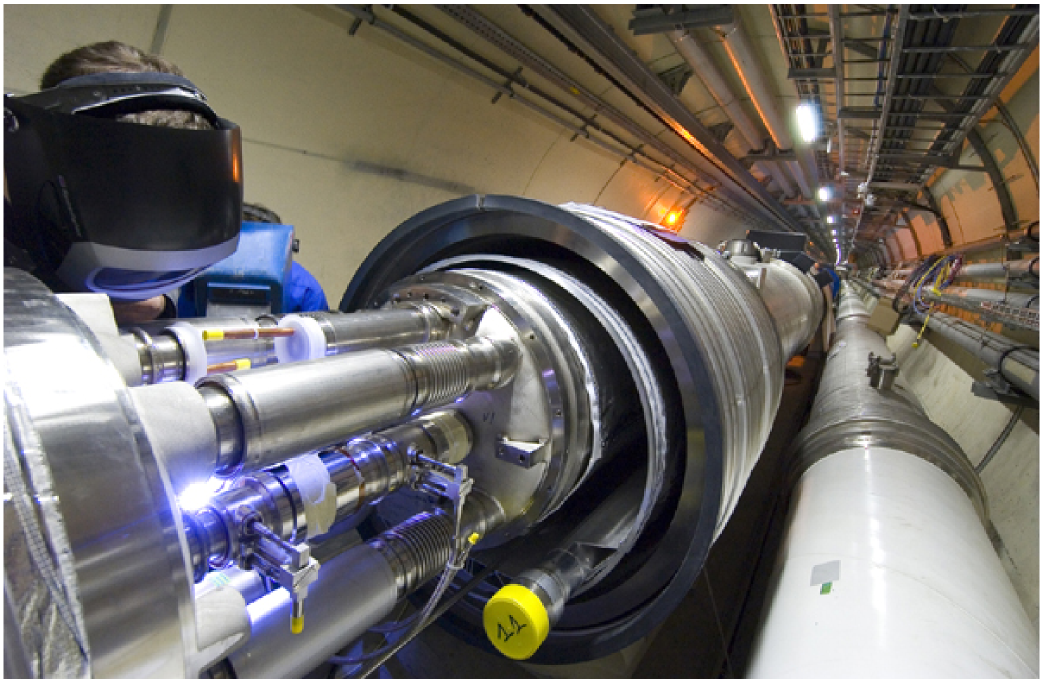
Fig.4.3.

The magnets are made of Niobium and Titanium. The maximum field strength is 10 T when cooled to 1.9 Kelvin. The magnets are cooled by superfluid helium, which is supplied by a large cryogenic system. Due to heating between the eight helium refrigerators and the beampipe, the helium is cooled in the refrigerators to 1.8 K.

A failure in the cooling system can cause what is known as a *quench*. If the temperature goes above the critical superconducting temperature, the metal loses its superconducting properties, which leads to a large resistance in the metal. This leads to rapid temperature increases, and can cause extensive damages if not controlled.

The dipole magnets are 16.5 meters long with a diameter of 0.57 meters. There are two individual beam pipes inside each magnet, which allows the dipoles to house the beams travelling in both directions around the LHC ring. They curve slightly, at an angle of 5.1 mrad, which carefully matches the curvature of the ring. The

Figure 4.3: Photograph of a technician connecting an LHC dipole magnet.



beampipes inside of the magnets are held in high vacuum, to avoid stray particles interacting with the beam.

4.4 Dataset Delivered by the LHC

In this thesis, we analyze the data delivered by the LHC to ATLAS in the 2015 and 2016 datasets. The beam parameters relevant to this dataset are available in Table 4.1.

The peak instaneous luminosity delivered in 2015 (2016) was $L = 5.2\,(11)\,\mathrm{cm}^{-2}\mathrm{s}^{-1} \times 10^{33}$. One can note that the instaneous luminosity delivered in the 2016 dataset exceeds the design luminosity of the LHC. The total integrated luminosity delivered was $13.3\,\mathrm{fb}^{-1}$. In Figure 4.4, we display the integrated luminosity as a function of day for 2015 and 2016.

Figure 4.4: Integrated Luminosity delivered by the LHC and collected by ATLAS in the 2015 and 2016 datasets.

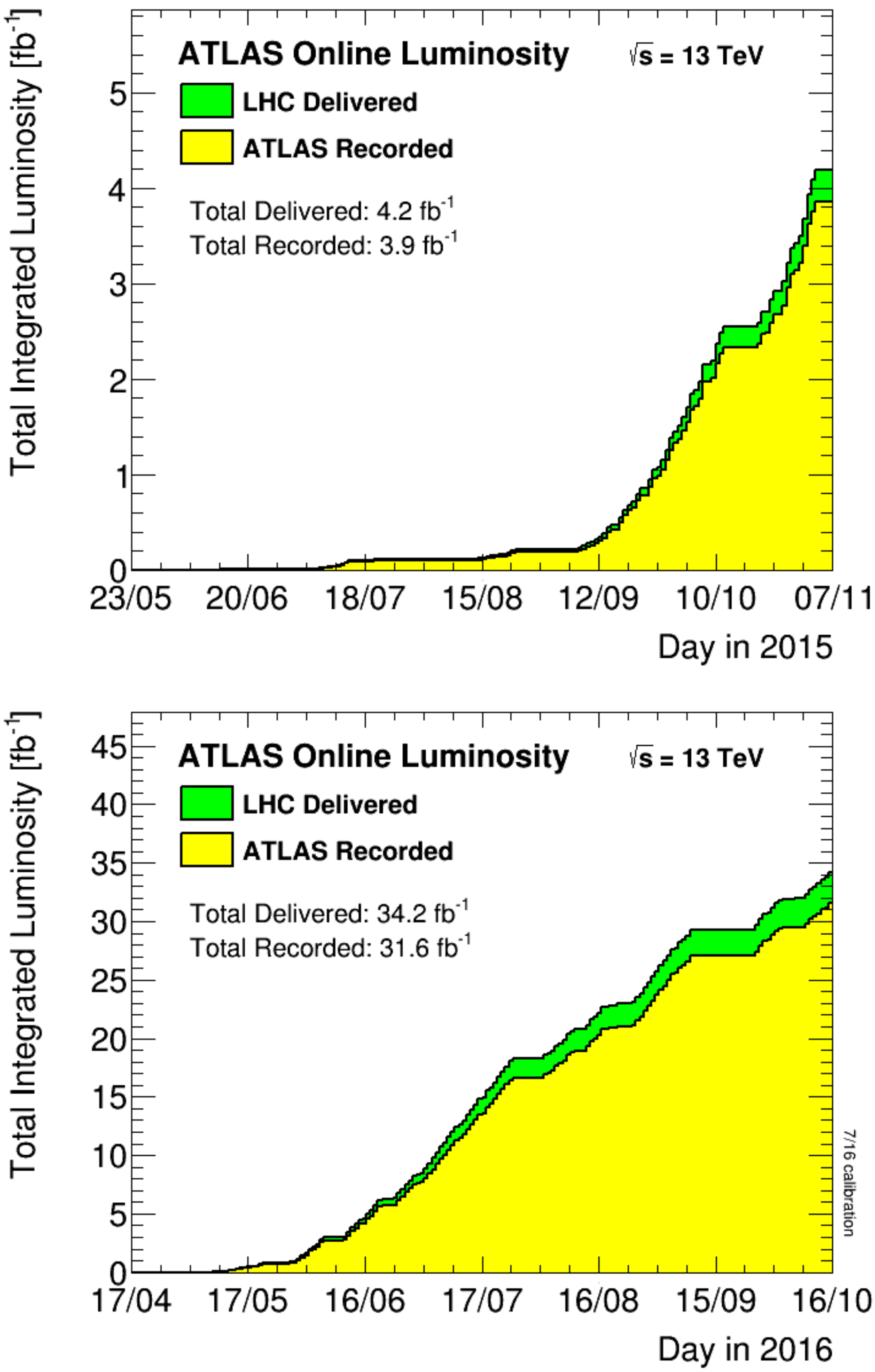
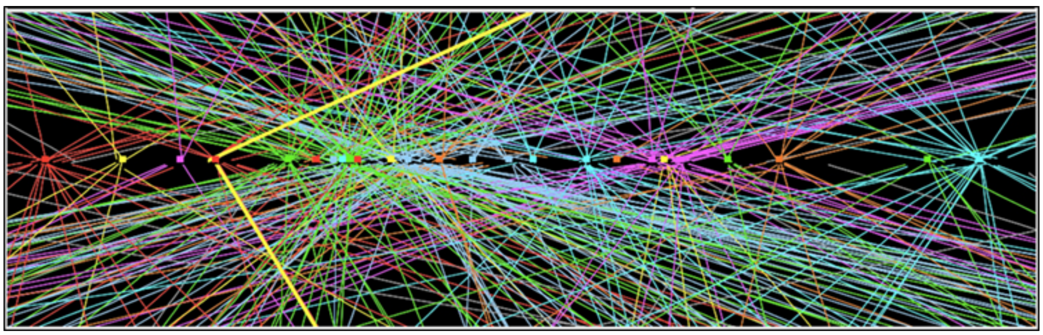


Table 4.1: Beam parameters of the Large Hadron Collider.

Parameter	Injection	Extraction
Energy (GeV)	450	7000
Rigidity (T-m)	3.8	23353
Bunch spacing (ns)	25	25
Design Luminosity ($cm^{-2}s^{-1} \times 10^{3}4$)	-	1.0
Bunches per proton beam	2808	2808
Protons per bunch	1.15 e11	1.15 e11
Beam lifetime (hr)	-	10
Normalized Emittance ϵ_N (mm μrad)	3.3	3.75
Betatron function at collision point β^* (cm)	-	55

Figure 4.5: Simulated event with many pileup vertices.



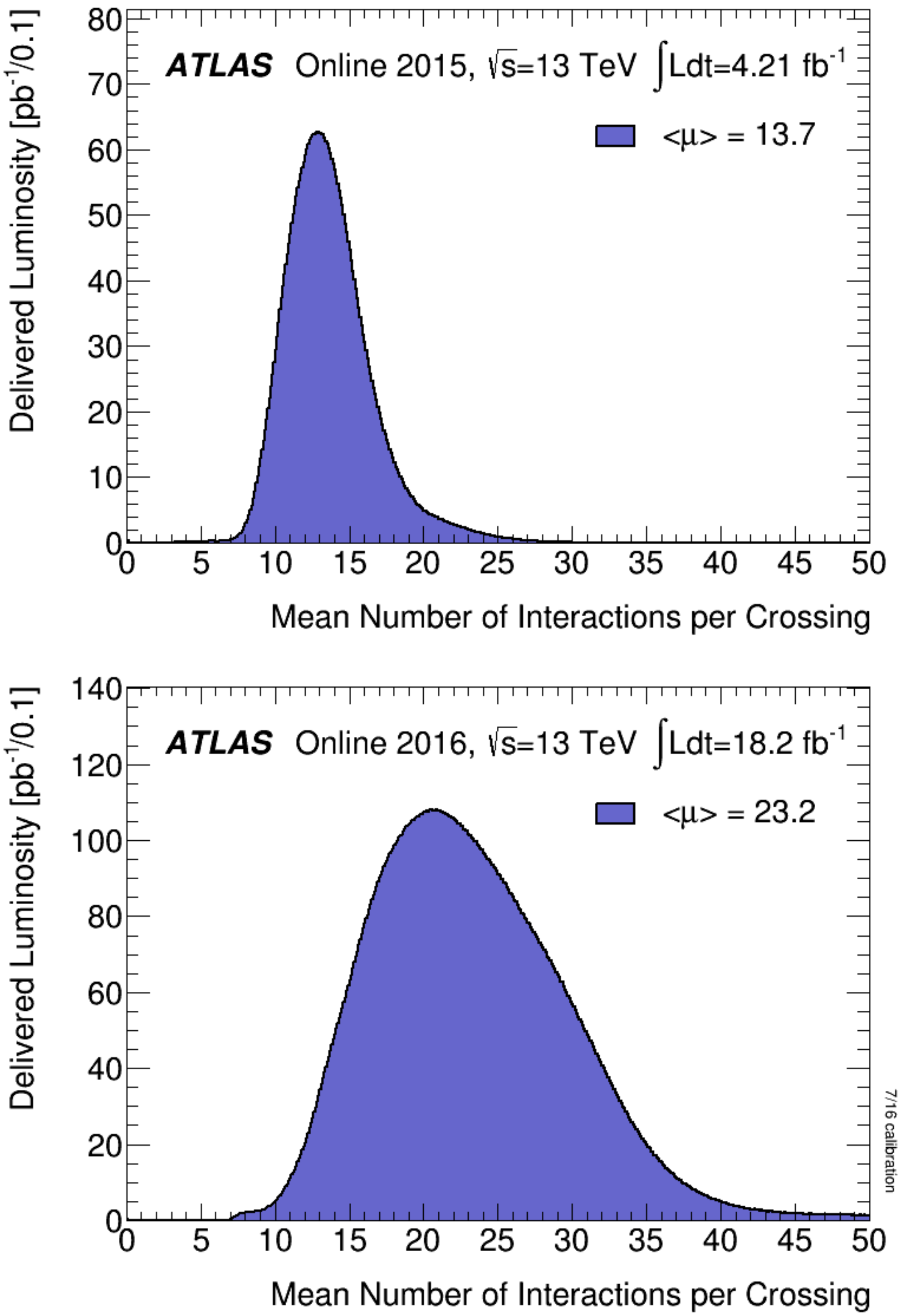
Pileup

Pileup is the term for the additional proton-proton interactions which occur during each bunch crossing of the LHC. At the beginning of the LHC physics program, there had not been a collider which averaged more than a single interaction per bunch crossing. In the LHC, each bunch crossing (or *event*) generally contains multiple proton-proton interactions. An simulated event with many *vertices* can be seen in Fig.4.5 The so-called *primary vertex* (or *hard scatter vertex*) refers to the vertex which has the highest Σp_T^2; this summation occurs over the *tracks* in the detector, which we will describe later. We then distinguish between *in-time* pileup and *out-of-time* pileup. In-time pileup refers to the additional proton-proton interactions which occur in the event. Out-of-time pileup refers to effects related to proton-proton interactions previous bunch crossings.

We quantify in-time pileup by the number of "primary"[2] vertices in a particular event. To quantify the out-of-time pileup, we use the average number of interactions per bunch crossing $< \mu >$ over some human-scale time. In Figure 4.6, we show the distribution of μ for the dataset used in this thesis.

[2] *The* primary vertex is as defined above, but we unfortunately use the same name here.

Figure 4.6: Mean number of interactions per bunch crossing in the 2015 and 2016 datasets.



Chapter 5

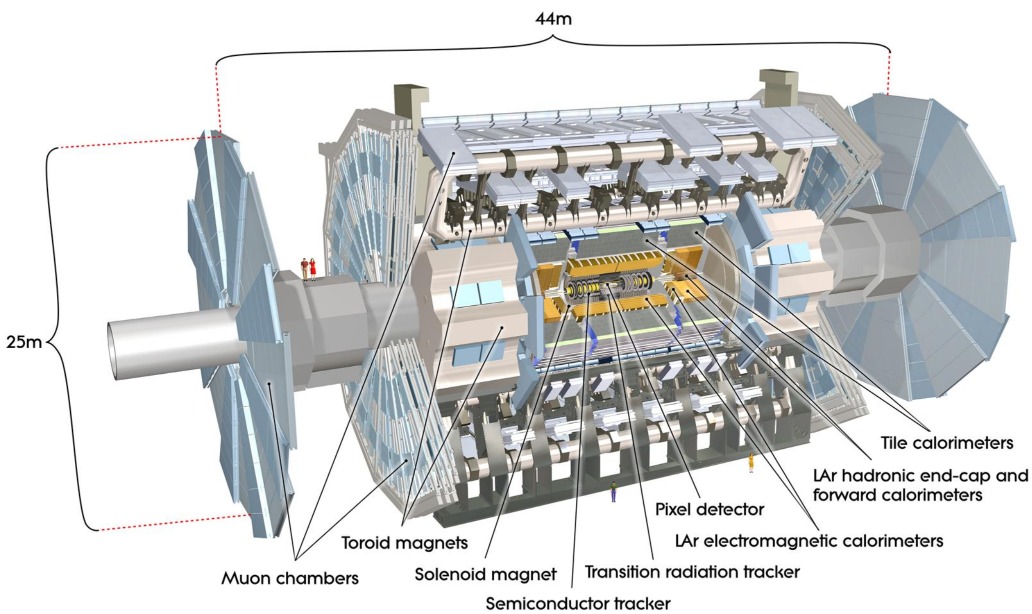
The ATLAS detector

The dataset analyzed in this thesis was taken by the ATLAS detector [88], which is located at the "Point 1" cavern of the LHC beampipe, just across the street from the main CERN campus. The much-maligned acronym stands for *A Toriodal LHC ApparatuS*. ATLAS is a massive cylindrical detector, with a radius of 12.5 m and a length of 44 m, with nearly hermitic coverage around the collision point. It consists of multiple subdetectors; each plays a role in ATLAS's ultimate purpose of measuring the energy, momentum, and type of the particles produced in collisions delivered by the LHC. These subdetectors are immersed in a hybrid solenoid-toroid magnet system whichs forces charged particles to curve, which allows for precise measurements of their momenta. These magnetic fields are maximized in the central solenoid magnet, which contains a magnetic field of 2 T. A schematic of the detector can be seen in 5.1.

The *inner detector* (ID) lies closest to the collision point, and contains three separate subdetectors. It provides pseudorapidity[1]coverage of $|\eta| < 2.5$ for charged particles to interact with the tracking material. The tracks reconstructed from the inner detector hits are used to reconstruct the primary vertices, as noted in Ch.**??**,

[1] ATLAS uses a right-handed Cartesian coordinate system; the origin is defined by the nominal beam interaction point. The positive-z direction is defined by the incoming beam travelling counterclockwise around the LHC. The positive-x direction points towards the center of the LHC ring from the origin, and the positive-y direction points upwards towards the sky. For particles of transverse (in the $x-y$ plane) momentum $p_T = \sqrt{p_x^2 + p_y^2}$ and energy E, it is generally most convenient fully describe this particle's kinematics as measured by the detector in the (p_T, ϕ, η, E) basis. The angle $\phi = \arctan(p_y/p_x)$ is the standard azimuthal angle, and $\eta = \ln \tan(\theta/2)$ is known as the pseudorapidity, and defined based on the standard polar angle $\theta = \arccos(p_z/p_T)$. For locations of i.e. detector elements, both (r, ϕ, η) and (z, ϕ, η) can be useful.

Figure 5.1: The ATLAS detector

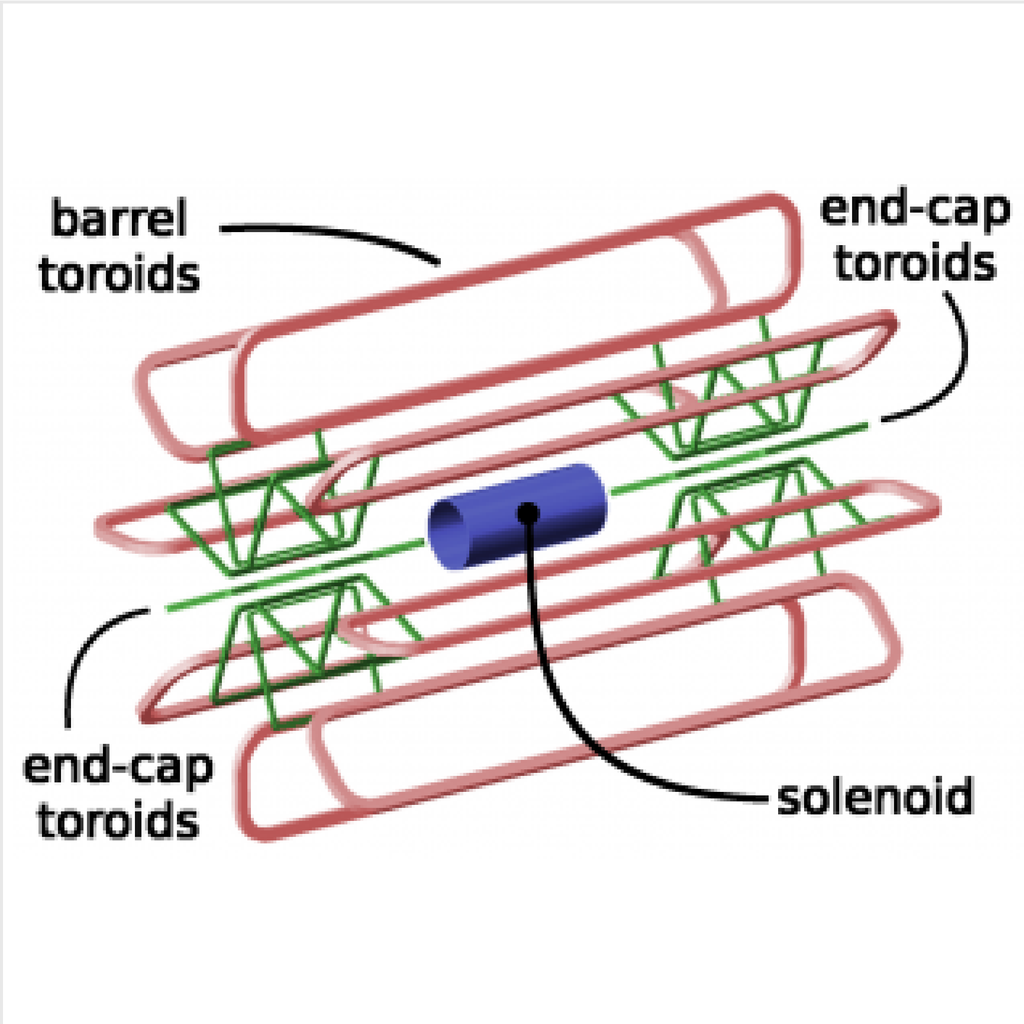


and to determine the momemta of charged particles. The ATLAS *calorimeter* consists of two subdetectors, known as the *electromagnetic* and *hadronic* calorimeters. These detectors stop particles in their detector material, and measure the energy deposition inside, which measures the energy of the particles deposited. The calorimeters provide coverage out to pseudorapidity of $|\eta| < 4.9$. The muon spectrometer is aptly named; it is specifically used for muons, which are the only particles which generally reach the outer portions of the detector. In this region, we have the large tracking systems of the muon spectrometer, which provide precise measurements of muon momenta. The muon spectrometer has pseudorapidity coverage of $|\eta| < 2.7$.

5.1 Magnets

ATLAS contains multiple magnetic systems; primarily, we are concerned with the solenoid, used by the inner detector, and the toroids located outside of the ATLAS calorimeter. A schematic is shown in Fig.5.2. These magnetic fields are used to bend

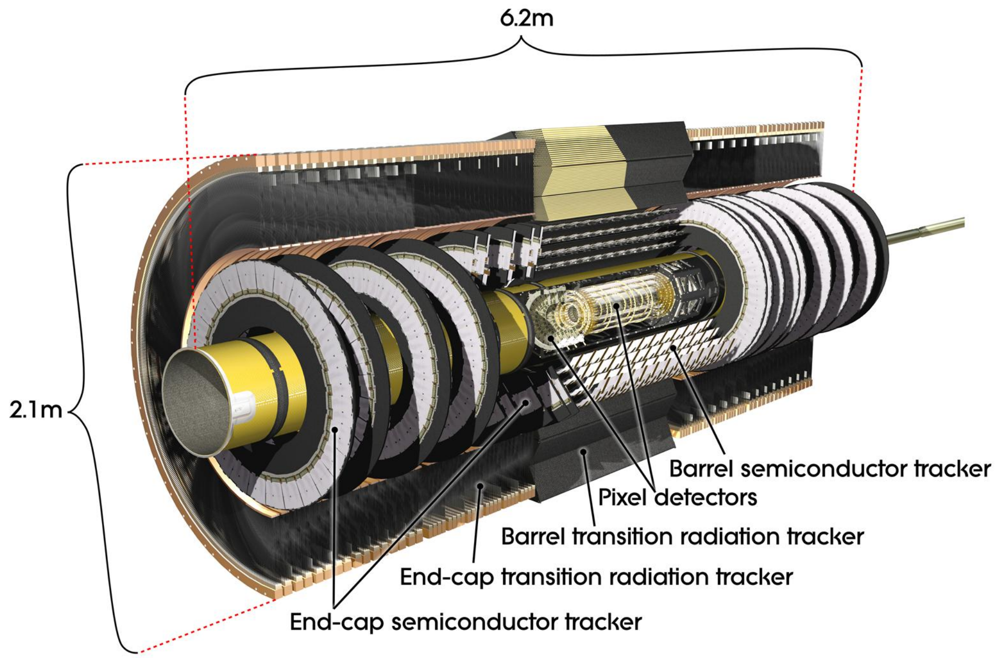
Figure 5.2: The ATLAS magnet system



charged particles under the Lorentz force, which subsequently allows one to measure their momentum.

The ATLAS central solenoid [89] is a 2.3 m diameter, 5.3 m long solenoid at the center of the ATLAS detector. It produces a uniform magnetic field of 2 T; this strong field is necessary to accurately measure the charged particles in this field. An important design constraint for the central solenoid was the decision to place it in between the inner detector and the calorimeters. To avoid excessive impacts on measurements in the calorimetry, the central solenoid must be as transparent as

Figure 5.3: The ATLAS inner detector



possible[2].

The toroid system consists of eight air-core superconducting barrel loops; these give ATLAS its distinctive shape. There are also two endcap air-core magnets. These produce a magnetic field in a region of approximately 26 m in length and 10 m of radius. The magnetic field in this region is non-uniform, due to the prohibitive costs of a solenoid magnet of that size.

5.2 Inner Detector

The ATLAS inner detector consists of three separate tracking detectors, which are known as, in order of increasing distance from the interaction point, the Pixel Detector [90], Semiconductor Tracker (SCT) , and the Transition Radiation Tracker (TRT) . When charged particles pass through these tracking layers, they produce *hits*, which

[2]This is also one of the biggest functional differences between ATLAS and CMS; in CMS, the solenoid is outside of the calorimeters.

using the known 2 T magnetic field, allows the reconstruction of *tracks*. Tracks are used as inputs for reconstruction of many higher-level physics objects, such as electrons, muons, photons, and $E_{\mathrm{T}}^{\mathrm{miss}}$. Accurate track reconstruction is thus crucial for precise measurements of charged particles.

Pixel Detector

schematic

The ATLAS pixel detector consists four layers of silicon "pixels". This refers to the segmentation of the active medium into the pixels; compare to the succeeding silicon detectors, which will use silicon "strips". This provides precise 3D hit locations. The layers are known as the "Insertable"[3]B-Layer (IBL), the B-Layer (or Layer-0), Layer-1, and Layer-2, in order of increasing distance from the interaction point. These layers are very close to the interaction point, and therefore experience a large amount of radiation.

Layer-1, Layer-2, and Layer-3 were installed with the initial construction of ATLAS. They contain front-end integrated electronics (FEI3s) bump-bonded to 1744 silicon modules; each module is 250 μm in thickness and contains 47232 pixels. These pixels have planar sizes of 50 x 400 μ^2 or 50 x 600 μ^2, to provide highly accurate location information. The FEI3s are mounted on long rectangular structures known as staves, which encircle the beam pipe. A small tilt to each stave allows full coverage in ϕ even with readout systems which are installed. These layers are at radia of 50.5 mm, 88.5 mm, and 122.5 mm from the interaction point.

The IBL was added to ATLAS after Run1 in 2012 at a radius of 33 mm from the interaction point. The entire pixel detector was removed from the center of ATLAS to allow an additional pixel layer to be installed. The IBL was required to preserve

[3] *Very* often, the IBL is mistakenly called the Inner B-Layer, which would have been a much more sensible name.

the integrity of the pixel detector as radiation damage leads to inoperative pixels in the other layers. The IBL consists of 448 FEI4 chips, arranged onto 14 staves. Each FEI4 has 26880 pixels, of planar size 50 x 250 μm. This smaller granularity was required due to the smaller distance to the interaction point.

In total, a charged particle passing through the inner detector would expect to leave four hits in the pixel detector.

Semiconductor Tracker

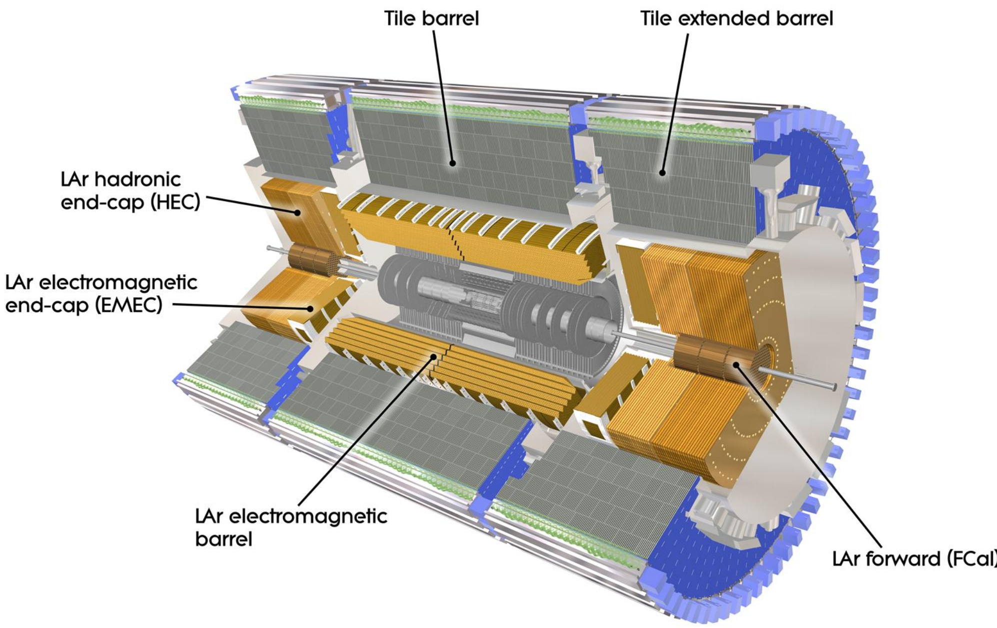
schematic

The SCT is directly beyond Layer-2 of the pixel detector. This is a silicon strip detector, which do not provide the full 3D information of the pixel detector. The dual-sensors of the SCT contain 2 x 768 individual strips; each strip has area 6.4 cm^2. The SCT dual-sensor is then double-layered, at a relative angle of 40 mrad; together these layers provide the necessary 3D information for track reconstruction. There are four of these double-layers, at radia of 284 mm, 355 mm, 427 mm, and 498 mm. These double-layers provide hits comparable to those of the pixel detector, and we have four additional hits to reconstruct tracks for each charged particle.

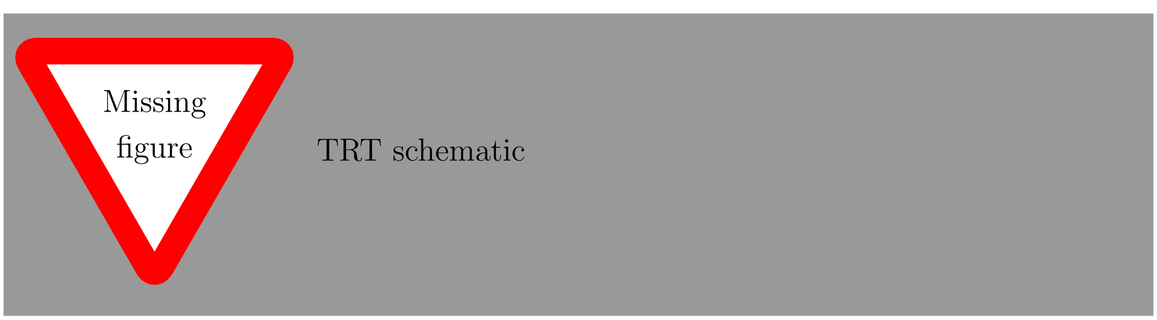
Transition Radiation Tracker

The Transition Radiation Tracker is the next detector radially outward from the SCT. It contains straw drift tubes; these contain a tungsten gold-plated wire of 32 μm diameter held under high voltage (-1530 V) with the edge of the Kapton-aluminum tube. They are filled with a gas mixture of primarily xenon that is ionized when a charged particle passes through the tube. The ions are collected by the "drift" due to the voltage inside the tubes, which is read out by the electronics. This gives so-called "continuous tracking" throughout the tube, due to the large number of ions produced.

Figure 5.4: The ATLAS calorimeter



The TRT is so-named due to the *transition radiation* (TR) it induces. Due to the dielectric difference between the gas and tubes, TR is induced. This is important for distinguishing electrons from their predominant background of minimum ionizing particles. Generally, electrons have a much larger Lorentz factor than minimum ionizing particles, which leads to additional TR. This can be used as an additional handle for electron reconstruction.



5.3 Calorimetry

The calorimetry of the ATLAS detector also includes multiple subdetectors; these subdetectors allow precise measurements of the electrons, photons, and hadrons produced by the ATLAS detector. Generically, calorimeters work by stopping particles in their material, and measuring the energy deposition. This energy is deposited as a cascade particles induce from interactions with the detector material known *showers*. ATLAS uses *sampling* calorimeters; these alternate a dense absorbing material, which induces showers, with an active layer which measures energy depositions by the induced showers. Since some energy is deposited into the absorption layers as well, the energy depositions must be properly calibrated for the detector.

Electromagnetic objects (electrons and photons) and hadrons have much different interaction properties, and thus we need different calorimeters to accurately measure these different classes of objects; we can speak of the *electromagnetic* and *hadronic* calorimeters. ATLAS contains four separate calorimeters : the liquid argon (LAr) electromagnetic barrel calorimeter, the Tile barrel hadronic calorimeter, the LAr endcap electromagnetic calorimeter, the LAr endcap hadronic calorimeter, and the LAr Forward Hadronic Calorimeter (FCal). Combined, these provide full coverage in ϕ up to $|\eta| < 4.9$, and can be seen in Fig.5.4.

Electromagnetic Calorimeters

The electromagnetic calorimeters of the ATLAS detector consist of the barrel and endcap LAr calorimeters. These are arranged into an ingenious "accordion" shape, shown in , which allows full coverage in ϕ and exceptional coverage in η while still allowing support structures for detector operation. The accordion is made of layers with liquid argon (active detection material) and lead (absorber) to induce electromagnetic showers. The LAr EM calorimeters are each more than 20 radiation lengths deep, which provides the high stopping power necessary to properly measure the electromagnetic showers.

The barrel component of the LAr EM calorimeter extends from the center of the detector out to $|\eta| < 1.475$. The calorimeter has a presampler, which measures the energy of any EM shower induced before the calorimeter. This has segmentation of $\Delta\eta = 0.025, \Delta\phi = .01$ There are three "standard" layers in the barrel, which have decreasing segmentation into calorimeter *cells* as one travels radially outward from the interaction point. The first layer has segmentation of $\Delta\eta = 0.003, \Delta\phi = .1$, and is quite thin relative to the other layers at only 4 radiation lengths deep. It provides precise η and ϕ measurements for incoming EM objects. The second layer is the deepest at 16 radiation lengths, with a segmentation of $\Delta\eta = 0.025, \Delta\phi = 0.025$. It is primarily responsible for stopping the incoming EM particles, which dictates its large relative thickness, and measures most of the energy of the incoming particles. The third layer is only 2 radiation lengths deep, with a rough segmentation of $\Delta\eta = 0.05, \Delta\phi = .025$. The deposition in this layer is primarily used to distinguish hadrons interacting electromagnetically and entering the hadronic calorimeter from the strictly EM objects which are stopped in the second layer.

The barrel EM calorimeter has a similar overall structure, but extends from $1.4 < |\eta| < 3.2$. The segmentation in η is better in the endcap than the barrel; the ϕ segmentation is the same. In total, the EM calorimeters contain about 190000 individual calorimeter cells.

Hadronic Calorimeters

The hadronic calorimetry of ATLAS sits directly outside the EM calorimetry. It contains three subdetectors : the barrel Tile calorimeter, the endcap LAr calorimeter, and the Forward (Hadronic) LAr Calorimeter. Similar to the EM calorimeters, these are sampling calorimeters that alternate steel (dense material) with an active layer (plastic scintillator).

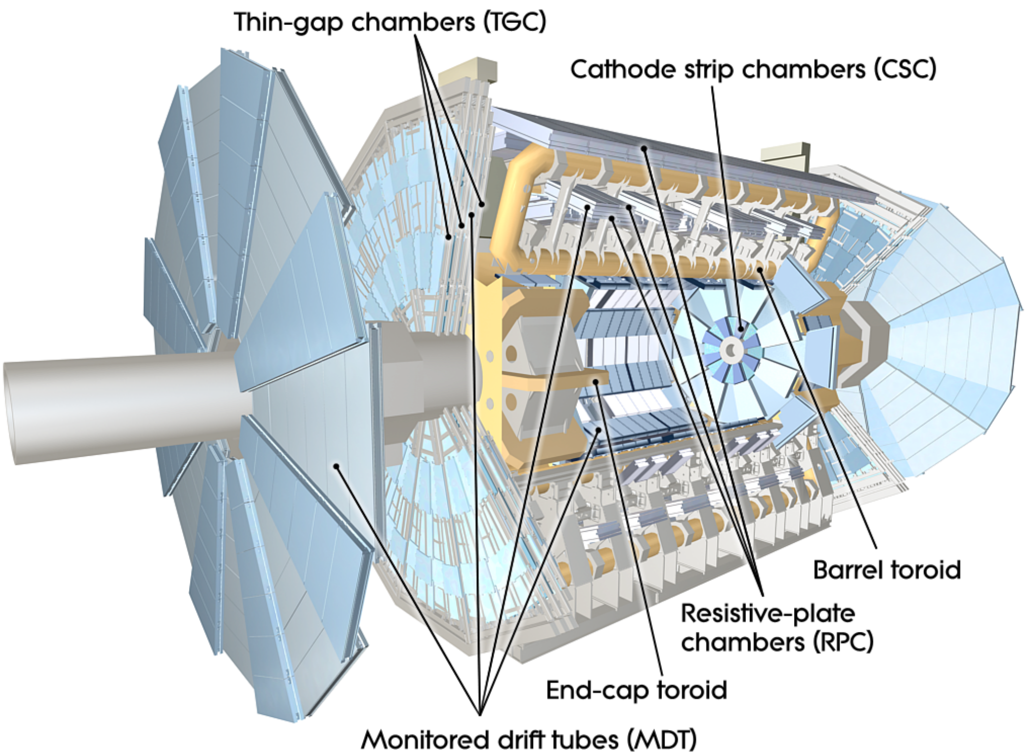
The barrel Tile calorimeter extends out to $|\eta| < 1.7$. There are again three layers, which combined give about 10 interactions length of distance, which provides excellent stopping power for hadrons. This is critical to avoid excess *punchthrough* to the muon spectrometer beyond the hadronic calorimeters. The first layer has a depth of 1.5 interaction lengths. The second layer is again the thickest at a depth of 4.1 interaction lengths; most of the energy of incoming particle is deposited here. Both the first and second layer have segmentation of about $\Delta\eta = 0.1, \Delta\phi = 0.1$. Generally, one does not need as fine of granularity in the hadronic calorimeter, since the energy depositions in the hadronic calorimeters will be summed into the composite objects we know as jets. The third layer has a thickness of 1.8 interaction lengths, with a segmentation of $\Delta\eta = 0.2, \Delta\phi = 0.1$. The use of multiple layers allows one to understand the induced hadronic shower as it propagates through the detector material.

The endcap LAr hadronic calorimeter covers the region $1.5 < |\eta| < 3.2$. It is again a sampling calorimeter; the active material is LAr with a copper absorbed. It does not use the accordion shape of the other calorimeters; it has a "standard" flat shape perpendicular to the interaction point. The segmentation varies with η. For $1.5 < |\eta| < 2.5$, the cells are $\Delta\eta = 0.1, \Delta\phi = 0.1$; in the region $2.5 < |\eta| < 3.2$, the cells are $\Delta\eta = 0.2, \Delta\phi = 0.2$ in size.

5.4 Muon Spectrometer

5.5 Trigger System

Figure 5.5: The ATLAS muon spectrometer



Chapter 6

The Recursive Jigsaw Technique

Here you can write some introductory remarks about your chapter. I like to give each sentence its own line.

When you need a new paragraph, just skip an extra line.

6.1 Razor variables

By using the asterisk to start a new section, I keep the section from appearing in the table of contents. If you want your sections to be numbered and to appear in the table of contents, remove the asterisk.

6.2 SuperRazor variables

6.3 The Recursive Jigsaw Technique

6.4 Variables used in the search for zero lepton SUSY

Chapter 7

Title of Chapter 1

Chapter 8

Title of Chapter 1

Here you can write some introductory remarks about your chapter. I like to give each sentence its own line.

When you need a new paragraph, just skip an extra line.

8.1 Object reconstruction

Photons, Muons, and Electrons

Jets

Missing transverse momentum

Probably longer, show some plots from the PUB note that we worked on

8.2 Signal regions

Gluino signal regions

Squark signal regions

Compressed signal regions

8.3 Background estimation

Z vv

W ev

ttbar

Chapter 9

Title of Chapter 1

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9.1 Statistical Analysis

maybe to be moved to an appendix

9.2 Signal Region distributions

9.3 Pull Plots

9.4 Systematic Uncertainties

9.5 Exclusion plots

Conclusion

Here you can write some introductory remarks about your chapter. I like to give each sentence its own line.

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9.6 New Section

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The Standard Model

In this appendix, we provide a brief overview of the basic ingredients involved in construction of the Standard Model Lagrangian : quantum field theory, symmetries, and symmetry breaking.

Quantum Field Theory

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In this section, we provide a brief overview of the necessary concepts from Quantum Field Theory (QFT).

In modern physics, the laws of nature are described by the "action" S, with the imposition of the principle of minimum action. The action is the integral over the spacetime coordinates of the "Lagrangian density" $\mathcal{L}$, or Lagrangian for short. The Lagrangian is a function of "fields"; general fields will be called $\phi(x^\mu)$, where the indices μ run over the space-time coordinates. We can then write the action S as

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$$S = \int d^4x \mathcal{L}[\phi_i(x^\mu), \partial_\mu \phi_i(x^\mu)] \tag{9.1}$$

where we have an additional summation over i (of the different fields). Generally, we impose the following constraints on the Lagrangian :

1. Translational invariance - The Lagrangian is only a function of the fields ϕ and their derivatives $\partial_\mu \phi$

2. Locality - The Lagrangian is only a function of one point x_μ in spacetime.

3. Reality condition - The Lagrangian is real to conserve probability.

4. Lorentz invariance - The Lagrangian is invariant under the Poincarégroup of spacetime.

5. Analyticity - The Lagrangian is an analytical function of the fields; this is to allow the use of pertubation theory.

6. Invariance and Naturalness - The Lagrangian is invariant under some internal symmetry groups; in fact, the Lagrangian will have *all* terms allowed by the imposed symmetry groups.

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7. Renormalizabilty - The Lagrangian will be renormalizable - in practice, this means there will not be terms with more than power 4 in the fields.

The key item from the point of view of this thesis is that of "Invariance and Natural". We impose a set of "symmetries" and then our Lagragian is the most general which is allowed by those symmetries.

Symmetries

Symmetries can be seen as the fundamental guiding concept of modern physics. cite? Symmetries are described by "groups". . To illustrate the importance of symmetries and their mathematical description, groups, we start here with two of the simplest and most useful examples : $\mathbb{Z}_2$and $U(1)$.

$\mathbb{Z}_2$symmetry

$\mathbb{Z}_2$symmetry is the simplest example of a "discrete" symmetry. Consider the most general Lagrangian of a single real scalar field $\phi(x_\mu)$

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\mu}{2\sqrt{2}}\phi^3 - \lambda\phi^4 \tag{9.2}$$

Now we *impose* the symmetry

$$\mathcal{L}(\phi) = \mathcal{L}(-\phi) \tag{9.3}$$

This has the effect of restricting the allowed terms of the Lagrangian. In particular, we can see the term $\phi^3 \to -\phi^3$ under the symmetry transformation, and thus must be disallowed by this symmetry. This means under the imposition of this particular symmetry, our Lagrangian should be rewritten as

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \lambda\phi^4 \tag{9.4}$$

The effect of this symmetry is that the total number of ϕ particles can only change by even numbers, since the only interaction term $\lambda\phi^4$ is an even power of the field. This symmetry is often imposed in supersymmetric theories, as we will see in Chapter 3.

$U(1)$ symmetry

$U(1)$ is the simplest example of a continuous (or *Lie*) group. Now consider a theory with a single complex scalar field $\phi = \operatorname{Re}\phi + i \operatorname{Im}\phi$

$$\mathcal{L}_\phi = \delta_{i,j}\frac{1}{2}\partial_\mu\phi_i\partial^\mu\phi_j - \frac{m^2}{2}\phi_i\phi_j - \frac{\mu}{2\sqrt{2}}\phi_i\phi_j\phi_k - \lambda\phi_i\phi_j\phi_k\phi_l \tag{9.5}$$

where $i, j, k, l = Re, Im$. In this case, we impose the following $U(1)$ symmetry : $\phi \to e^{i\theta}, \phi^* \to e^{-i\theta}$. We see immediately that this again disallows the third-order terms, and we can write a theory of a complex scalar field with $U(1)$ symmetry as

$$\mathcal{L}_\phi = \partial_\mu\phi\partial^\mu\phi^* - \frac{m^2}{2}\phi\phi^* - -\lambda(\phi\phi^*)^2 \tag{9.6}$$

Local symmetries

The two examples considered above are "global" symmetries in the sense that the symmetry transformation does not depends on the spacetime coordinate x_μ. We know look at local symmetries; in this case, for example with a local $U(1)$ symmetry, the transformation has the form $\phi(x_\mu) \rightarrow e^{i\theta(x_m u)}\phi(x_\mu)$. These symmetries are also known as "gauge" symmetries; all symmetries of the Standard Model are gauge symmetries.

There are wide-ranging consequences to the imposition of local symmetries. To begin, we note that the derivative terms of the Lagrangian 9.2 are *not* invariant under a local symmetry transformation

$$\partial_\mu \phi(x_\mu) \rightarrow \partial_\mu(e^{(}i\theta(x_\mu)\phi(x_\mu)) = (1 + i\theta(x_\mu))e^{(}i\theta(x_\mu)\phi(x_\mu) \tag{9.7}$$

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This leads us to note that the kinetic terms of the Lagrangian are also not invariant under a gauge symmetry. This would lead to a model with no dynamics, which is clearly unsatisfactory.

Let us take inspiration from the case of global symmetries. We need to define a so-called "covariant" derivative D^μ such that

$$D^\mu \phi \rightarrow e^{iq\theta(x^\mu)D^\mu \phi} \tag{9.8}$$

$$D^\mu \phi^* \rightarrow e^{-iq\theta(x^\mu)D^\mu \phi} \tag{9.9}$$

$$\tag{9.10}$$

Since ϕ andϕ^* transforms with the opposite phase, this will lead the invariance of the Lagrangian under our local gauge transformation. This D^μ is of the following form

$$D^\mu = \partial_\mu - igqA^\mu \tag{9.11}$$

where A^μ is a vector field we introduce with the transformation law

$$A^\mu \to A^\mu - \frac{1}{g}\partial_\mu \theta \tag{9.12}$$

and g is the coupling constant associated to vector field. This vector field A^μ is also known as a “gauge” field.

Since we need to add all allowed terms to the Lagrangian, we define

$$F^{\mu\nu} = A^\mu A^\nu - A^\nu A^\mu \tag{9.13}$$

and then we must also add the kinetic term :

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} \tag{9.14}$$

The most general renormalizable Lagrangian with fermion and scalar fields can be written in the following form

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}Yukawa \tag{9.15}$$

Symmetry breaking and the Higgs mechanism

Here we view some examples of symmetry breaking. We investigate breaking of a global $U(1)$ symmetry and a local $U(1)$ symmetry. The SM will break the electroweak symmetry $SU(2)xU(1)$, and in Chapter 3 we will see how supersymmetry must also be broken.

There are two ideas of symmetry breaking

- Explicit symmetry breaking by a small parameter - in this case, we have a small parameter which breaks an “approximate” symmetry of our Lagrangian. An example would be the theory of the single scalar field 9.2, when $\mu << m^2$ and

$\mu << \lambda$. In this case, we can often ignore the small term when considering low-energy processes.

- Spontaneous symmetry breaking (SSB) - spontaneous symmetry breaking occurs when the Lagrangian is symmetric with respect to a given symmetry transformation, but the ground state of the theory is *not* symmetric with respect to that transformation. This can have some fascintating consequences, as we will see in the following examples

Symmetry breaking a

U(1) global symmetry breaking

Consider the theory of a complex scalar field under the $U(1)$ symmetry, or the transformation

$$\phi \to e^{i\theta}\phi \tag{9.16}$$

The Lagrangian for this theory is

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + \frac{\mu^2}{2}\phi^\dagger\phi + \frac{\lambda}{4}(\phi^\dagger\phi)^2 \tag{9.17}$$

Let us write this theory in terms of two scalar fields, h and ξ : $\phi = (h + i\xi)/\sqrt(2)$. The Lagrangian can then be written as

$$\mathcal{L} = \partial^\mu h \partial_\mu h + \partial^\mu \xi dmu \xi - \frac{\mu^2}{2}(h^2 + \xi^2) - \frac{\lambda}{4}(h^2 + \xi^2)^2 \tag{9.18}$$

First, note that the theory is only stable when $\lambda > 0$. To understand the effect of SSB, we now enforce that $\mu^2 < 0$, and define $v^2 = -\mu^2/\lambda$. We can then write the scalar potential of this theory as :

$$V(\phi) = \lambda(\phi^\dagger\phi - v^2/2)^2 \tag{9.19}$$

Minimizing this equation with respect to ϕ, we can see that the "vacuum expectation value" of the theory is

$$2 < \phi^\dagger\phi >=< h^2 + \xi^2 >= v^2 \tag{9.20}$$

We now reach the "breaking" point of this procedure. In the (h, ξ) plane, the minima form a circle of radius v. We are free to choose any of these minima to expand our Lagrangian around; the physics is not affected by this choice. For convenience, choose $< h >= v, < \xi^2 >= 0$.

Now, let us define $h' = h - v, \xi' = \xi$ with VEVs $< h' >= 0, < \xi' >= 0$. We can then write our spontaneously broken Lagrangian in the form

$$\mathcal{L} = \frac{1}{2}\partial_\mu h' \partial^\mu h' + \frac{1}{2}\partial_\mu \xi' \partial^\mu \xi' - \lambda v^2 h'^2 - \lambda v h'(h'^2 + \xi'^2) - \lambda(h'^2 + \xi'^2)^2 \tag{9.21}$$