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A search for sparticles in zero lepton final states

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ABSTRACT

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A search for sparticles in zero lepton final states

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Russell W. Smith

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Acknowledgements

Dedication

Introduction

Particle physics is a remarkably successful field of scientific inquiry. The ability to precisely predict the properties of a exceedingly wide range of physical phenomena, such as the description of the cosmic microwave background [1, 2], the understanding of the anomalous magnetic dipole moment of the electron [3, 4], and the measurement of the number of weakly-interacting neutrino flavors [5] is truly amazing.

The theory that has allowed this range of predictions is the *Standard Model* of particle physics (SM). The Standard Model combines the electroweak theory of Glashow, Weinberg, and Salam [6–8] with the theory of the strong interactions, as first envisioned by Gell-Mann and Zweig [9, 10]. This quantum field theory (QFT) contains a tiny number of particles, whose interactions describe phenomena up to at least the TeV scale. These particles are manifestations of the fields of the Standard Model, after application of the Higgs Mechanism. The particle content of the SM consists only of the six quarks, the six leptons, the four gauge bosons, and the scalar Higgs boson.

Despite its impressive range of described phenomena, the Standard Model has some theoretical and experimental deficiencies. The SM contains 26 free parameters¹. It would be more theoretically pleasing to understand these free parameters in terms of a more fundamental theory. The major theoretical concern of the Standard Model, as it pertains to this thesis, is the *hierachy problem*[11–15]. The light mass

¹This is the Standard Model corrected to include neutrino masses. These parameters are the fermion masses (6 leptons, 6 quarks), CKM and PMNS mixing angles (8 angles, 2 CP-violating phases), W/Z/Higgs masses (3), the Higgs field expectation value, and the couplings of the strong, weak, and electromagnetic forces (3 α_{force}).

86 of the Higgs boson (125 GeV) should be quadratically dependent on the scale of UV
 87 physics, due to the quantum corrections from high-energy physics processes. The
 88 most perplexing experimental issue is the existence of *dark matter*, as demonstrated
 89 by galactic rotation curves [16–22]. This data has shown that there exists additional
 90 matter which has not yet been seen interacting with the particles of the Standard
 91 Model. There is no particle in the SM which can act as a candidate for dark matter.

92 Both of these major issues, as well as numerous others, can be solved by the
 93 introduction of *supersymmetry* (SUSY) [15, 23–35]. In supersymmetric theories, each
 94 SM particles has a so-called *superpartner*, or sparticle partner, differing from given SM
 95 particle by 1/2 in spin. These theories solve the hierarchy problem, since the quantum
 96 corrections induced from the superpartners exactly cancel those induced by the SM
 97 particles. In addition, these theories are usually constructed assuming R -parity,
 98 which can be thought of as the “charge” of supersymmetry, with SM particles having
 99 $R = 1$ and sparticles having $R = -1$. In collider experiments, since the incoming
 100 SM particles have total $R = 1$, the resulting sparticles are produced in pairs. This
 101 produces a rich phenomenology, which is characterized by significant hadronic activity
 102 and large missing transverse energy (E_T^{miss}), which provide significant discrimination
 103 against SM backgrounds [36].

104 Despite the power of searches for supersymmetry where E_T^{miss} is a primary
 105 discriminating variable, there has been significant interest in the use of other variables
 106 to discriminate against SM backgrounds. These include searches employing variables
 107 such as αT , $M_{T,2}$, and the razor variables (M_R, R^2) [37–47]. In this thesis, we
 108 will present the first search for supersymmetry using the novel Recursive Jigsaw
 109 Reconstruction (RJR) technique. RJR can be considered the conceptual successor
 110 of the razor variables. We impose a particular final state “decay tree” on an events,
 111 which roughly corresponds to a simplified Feynmann diagram in decays containing
 112 weakly-interacting particles. We account for the missing degrees of freedom associated

113 to the weakly-interacting particles by a series of simplifying assumptions, which allow
114 us to calculate our variables of interest at each step in the decay tree. This allows an
115 unprecedented understanding of the internal structure of the decay and the ability to
116 construct additional variables to reject Standard Model backgrounds.

117 This thesis details a search for the superpartners of the gluon and quarks, the
118 gluino and squarks, in final states with zero leptons, with 13.3 fb^{-1} of data using the
119 ATLAS detector. We organize the thesis as follows. The theoretical foundations of
120 the Standard Model and supersymmetry are described in Chapters 2 and 3. The
121 Large Hadron Collider and the ATLAS detector are presented in Chapters 4 and 5.
122 Chapter 5 provides a detailed description of Recursive Jigsaw Reconstruction and a
123 description of the variables used for the particular search presented in this thesis.
124 Chapter 6 presents the details of the analysis, including details of the dataset, object
125 reconstruction, and selections used. In Chapter 7, the final results are presented;
126 since there is no evidence of a supersymmetric signal in the analysis, we present the
127 final exclusion curves in simplified supersymmetric models.

The Standard Model

130 **2.1 Overview**

131 A Standard Model is another name for a theory of the internal symmetry group
 132 $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$, with its associated set of parameters. *The* Standard
 133 Model refers specifically to a Standard Model with the proper parameters to describe
 134 the universe. The SM is the culmination of years of work in both theoretical
 135 and experimental particle physics. In this thesis, we take the view that theorists
 136 construct a model with the field content and symmetries as inputs, and write down the
 137 most general Lagrangian consistent with those symmetries. Assuming this model is
 138 compatible with nature (in particular, the predictions of the model are consistent with
 139 previous experiments), experimentalists are responsible measuring the parameters of
 140 this model This will be applicable for this chapter and the following one.

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141 Additional theoretical background is in 9.6. The philosophy and notations are
 142 inspired by [48, 49].

143 **2.2 Field Content**

The Standard Model field content is

$$\begin{aligned}
 \text{Fermions} &: Q_L(3, 2)_{+1/3}, U_R(3, 1)_{+4/3}, D_R(3, 1)_{-2/3}, L_L(1, 2)_{-1}, E_R(1, 1)_{-2} \\
 \text{Scalar (Higgs)} &: \phi(1, 2)_{+1} \\
 \text{Vector Fields} &: G^\mu(8, 1)_0, W^\mu(1, 3)_0, B^\mu(1, 1)_0
 \end{aligned} \tag{2.1}$$

144 where the $(A, B)_Y$ notation represents the irreducible representation under $SU(3)$
 145 and $SU(2)$, with Y being the electroweak hypercharge. Each of these fermion fields
 146 has an additional index, representing the three generation of fermions.

147 We observed that Q_L, U_R , and D_R are triplets under $SU(3)_C$; these are the *quark*
 148 fields. The *color* group, $SU(3)_C$ is mediated by the *gluon* field $G^\mu(8, 1)_0$, which has
 149 8 degrees of freedom. The fermion fields $L_L(1, 2)_{-1}$ and $E_R(1, 1)_{-2}$ are singlets under
 150 $SU(3)_C$; we call them the *lepton* fields.

151 Next, we note the “left-handed” (“right-handed”) fermion fields, denoted by L (R)
 152 subscript, The left-handed fields form doublets under $SU(2)_L$. These are mediated
 153 by the three degrees of freedom of the “W” fields $W^\mu(1, 3)_0$. These fields only act
 154 on the left-handed particles of the Standard Model. This is the reflection of the
 155 “chirality” of the Standard Model; the left-handed and right-handed particles are
 156 treated differently by the electroweak forces. The right-handed fields, U_R, D_R , and
 157 E_R , are singlets under $SU(2)_L$.

158 The $U(1)_Y$ symmetry is associated to the $B^\mu(1, 1)_0$ boson with one degree of
 159 freedom. The charge Y is known as the electroweak hypercharge.

160 To better understand the phenomenology of the Standard Model, let us investigate
 161 each of the *sectors* of the Standard Model separately.

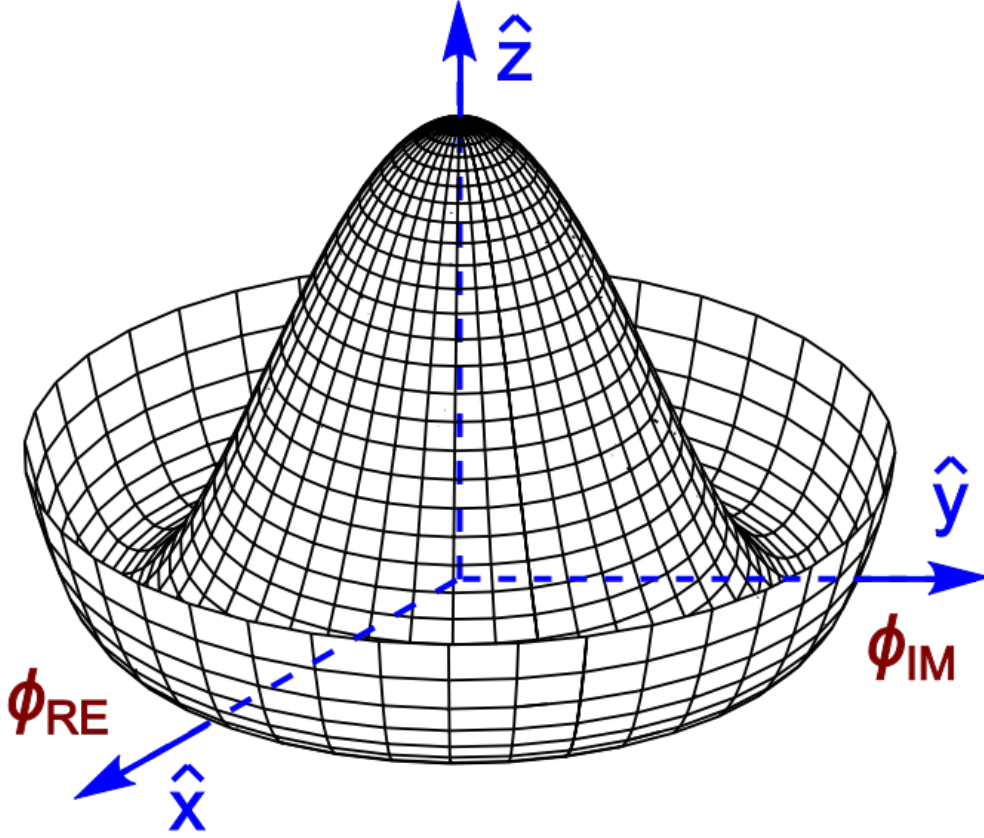
162 **Electroweak sector**

The electroweak sector refers to the $SU(2)_L \otimes U(1)_Y$ portion of the Standard
 Model gauge group. Following our philosophy of writing all gauge-invariant and
 renormalizable terms, the electroweak Lagrangian can be written as

$$\mathcal{L} = W_a^{\mu\nu} W_{\mu\nu}^a + B^{\mu\nu} B_{\mu\nu} + (D^\mu \phi)^\dagger D_\mu \phi - \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2. \quad (2.2)$$

where $W_a^{\mu\nu}$ are the three ($a = 1, 2, 3$) gauge bosons associated to the $SU(2)_L$ gauge
 group, $B^{\mu\nu}$ is the one gauge boson of the $U(1)_Y$ gauge group, and ϕ is the complex

Figure 2.1: Sombrero potential



Higgs multiplet. The covariant derivative D^μ is given by

$$D^\mu = \partial^\mu + \frac{ig}{2} W_a^\mu \sigma_a + \frac{ig'}{2} B^\mu \quad (2.3)$$

where $i\sigma_a$ are the Pauli matrices times the imaginary constant, which are the generators for $SU(2)_L$, and g and g' are the $SU(2)_L$ and $U(1)_Y$ coupling constants, respectively. The field strength tensors $W_a^{\mu\nu}$ and $B^{\mu\nu}$ are given by the commutator of the covariant derivative associated to each field

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \quad (2.4)$$

$$W_a^{\mu\nu} = \partial^\mu W_a^\nu - \partial^\nu W_a^\mu - g\epsilon_{abc} W_a^\mu W_b^\nu, \quad i = 1, 2, 3$$

164 The terms in the Lagrangian 2.2 proportional to μ^2 and λ make up the “Higgs
 165 potential” [50]. As normal (see Appendix 9.6), we restrict $\lambda > 0$ to guarantee our
 166 potential is bounded from below, and we also require $\mu^2 < 0$, which gives us the
 167 standard “sombbrero” potential shown in 2.1.

This potential has infinitely many minima at $\langle \phi \rangle = \sqrt{2m/\lambda}$; the ground state is *spontaneously* broken by the choice of ground state, which induces a vacuum expectation value (VEV). Without loss of generality, we can choose the Higgs field ϕ to point in the real direction, and write the Higgs field ϕ in the following form :

$$\phi = \frac{1}{\sqrt{2}} \exp\left(\frac{i}{v} \sigma_a \theta_a\right) \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \quad (2.5)$$

We choose a gauge to rotate away the dependence on θ_a , such that we can write simply

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}. \quad (2.6)$$

Now, we can see how the masses of the vector bosons are generated from the application of the Higgs mechanism. We plug Eq.2.6 back into the electroweak Lagrangian, and only showing the relevant mass terms in the vacuum state where $h(x) = 0$ see that (dropping the Lorentz indices) :

$$\begin{aligned} \mathcal{L}_M &= \frac{1}{8} \left| \begin{pmatrix} gW_3 + g'B & g(W_1 - iW_2) \\ g(W_1 + iW_2) & -gW_3 + g'B \end{pmatrix} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2 \\ &= \frac{g^2 v^2}{8} \left[W_1^2 + W_2^2 + \left(\frac{g'}{g} B - W_3\right)^2 \right] \end{aligned} \quad (2.7)$$

Defining the *Weinberg* angle $\tan(\theta_W) = g'/g$ and the following *physical* fields :

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \quad (2.8)$$

$$Z^0 = \cos \theta_W W_3 - \sin \theta_W B$$

$$A^0 = \sin \theta_W W_3 + \cos \theta_W B$$

we can write the piece of the Lagrangian associated to the vector boson masses as

$$\mathcal{L}_{M_V} = \frac{1}{4}g^2v^2W^+W^- + \frac{1}{8}(g^2 + g'^2)v^2Z^0Z^0. \quad (2.9)$$

and we have the following values of the masses for the vector bosons :

$$\begin{aligned} m_W^2 &= \frac{1}{4}v^2g^2 \\ m_Z^2 &= \frac{1}{4}v^2(g^2 + g'^2) \\ m_A^2 &= 0 \end{aligned} \quad (2.10)$$

168 We thus see how the Higgs mechanism gives rise to the masses of the W^\pm and Z
 169 boson in the Standard Model; the mass of the photon is zero, as expected. The
 170 $SU(2)_L \otimes U(1)_Y$ symmetry of the initially massless $W_{1,2,3}$ and B fields is broken to
 171 the $U(1)_{EM}$. Of the four degrees of freedom in the complex Higgs doublet, three are
 172 “eaten” when we give mass to the W^\pm and Z_0 , while the other degree of freedom is
 173 the Higgs particle, as found in 2012 by the ATLAS and CMS collaborations [51, 52].

174 Quantum Chromodynamics

Quantum chromodynamics (or the theory of the *strong* force) characterizes the behavior of *colored* particles, collectively known as *partons*. The partons of the Standard Model are the (fermionic) quarks, and the (bosonic) gluons. The strong force is governed by $SU(3)_C$, an unbroken symmetry in the Standard Model, which implies the gluon remains massless. Defining the covariant derivative for QCD as

$$D^\mu = \partial^\mu + ig_s G_a^\mu L_a, a = 1, \dots, 8 \quad (2.11)$$

where L_a are the generators of $SU(3)_C$, and g_s is the coupling constant of the strong force. The QCD Lagrangian then is given by

$$\mathcal{L}_{\text{QCD}} = i\bar{\psi}_f D_\mu \gamma^\mu \psi_f - \frac{1}{4}G_{a,\mu\nu}G_a^{\mu\nu} \quad (2.12)$$

where the summation over f is for quarks *families*, and $G_a^{\mu\nu}$ is the gluon field strength tensor, given by

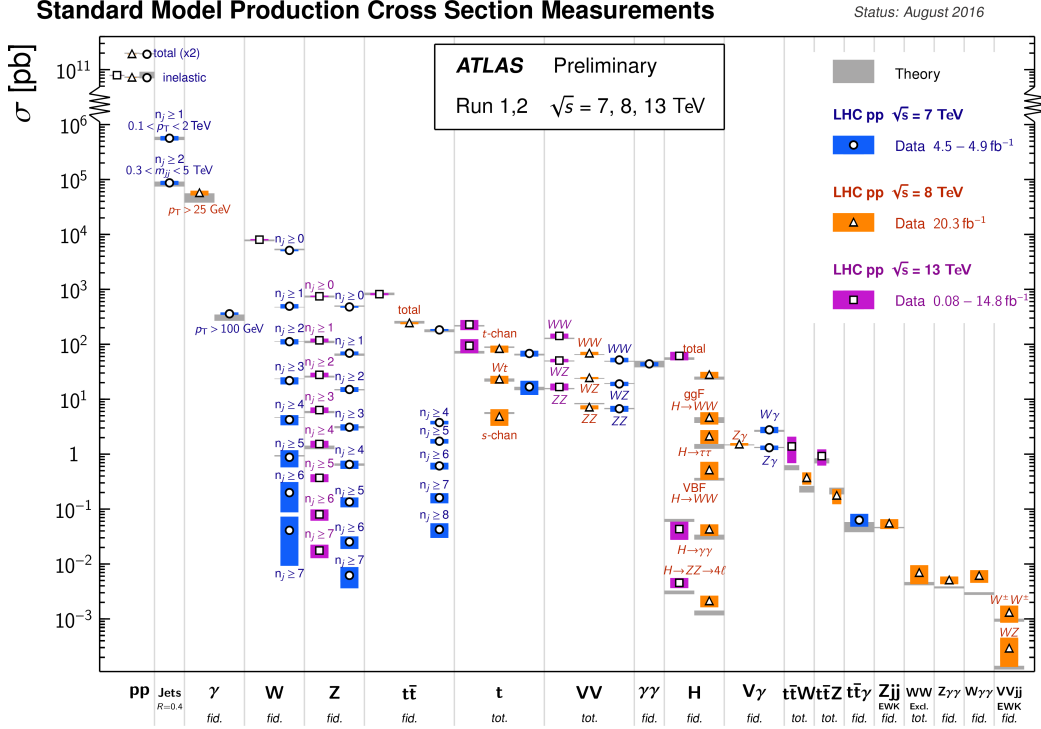
$$G_a^{\mu\nu} = \partial^\mu G_a^\nu - \partial^\nu G_a^\mu - g_s f^{abc} G_b^\mu G_c^\nu, a, b, c = 1, \dots, 8 \quad (2.13)$$

175 where f^{abc} are the structure constants of $SU(3)_C$, which are analogous to ϵ_{abc} for
 176 $SU(2)_L$. The kinetic term for the quarks is contained in the standard ∂_μ term, while
 177 the field strength term contains the interactions between the quarks and gluons, as
 178 well as the gluon self-interactions.

179 Written down in this simple form, the QCD Lagrangian does not seem much
 180 different from the QED Lagrangian, with the proper adjustments for the different
 181 group structures. The gluon is massless, like the photon, so one could naïvely expect
 182 an infinite range force, and it pays to understand why this is not the case. The
 183 reason for this fundamental difference is the gluon self-interactions arising in the
 184 field strength tensor term of the Lagrangian. This leads to the phenomena of *color*
 185 *confinement*, which describes how one only observes color-neutral particles alone in
 186 nature. In contrast to the electromagnetic force, particles which interact via the
 187 strong force experience a *greater* force as the distance between the particles increases.
 188 At long distances, the potential is given by $V(r) = -kr$. At some point, it is more
 189 energetically favorable to create additional partons out of the vacuum than continue
 190 pulling apart the existing partons, and the colored particles undergo *fragmentation*.
 191 This leads to *hadronization*. Bare quarks and gluons are actually observed as sprays
 192 of hadrons (primarily kaons and pions); these sprays are known as *jets*, which are
 193 what are observed by experiments.

194 It is important to recognize the importance of understanding these QCD inter-
 195 actions in high-energy hadron colliders such as the LHC. Since protons are hadrons,
 196 proton-proton collisions such as those produced by the LHC are primarily governed by
 197 the processes of QCD. In particular, by far the most frequent process observed in LHC
 198 experiments is dijet production from gluon-gluon interactions (see Fig.2.2). These

Figure 2.2: Cross-sections of various Standard Model processes



199 gluons that interact are part of the *sea* particles inside the proton; the simple $p = uud$
200 model does not apply. The main *valence* uud quarks are constantly interacting via
201 gluons, which can themselves radiate gluons or split into quarks, and so on. A more
202 useful understanding is given by the colloquially-known *bag* model [53, 54], where the
203 proton is seen as a “bag” of (in principle) infinitely many partons, each with energy
204 $E < \sqrt{s} = 6.5$ TeV. One then collides this (proton) bag with another, and views the
205 products of this very complicated collision, where calculations include many loops in
206 nonperturbative QCD calculations.

207 Fortunately, we are generally saved by the QCD factorization theorems [55]. This
208 allows one to understand the hard (i.e. short distance or high energy) $2 \rightarrow 2$ parton
209 process using the tools of perturbative QCD, while making series of approximations
210 known as a *parton shower* model to understand the additional corrections from
211 nonperturbative QCD. We will discuss the reconstruction of jets by experiments in
212 Ch.5.

213 Fermions

214 We will now look more closely at the fermions in the Standard Model [56].

215 As noted earlier in Sec.2.2, the fermions of the Standard Model can be first
216 distinguished between those that interact via the strong force (quarks) and those
217 which do not (leptons).

There are six leptons in the Standard Model, which can be placed into three *generations*.

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}, \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}, \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix} \quad (2.14)$$

218 There is the electron (e), muon (μ), and tau (τ), each of which has an associated
219 neutrino (ν_e, ν_μ, ν_τ). Each of the so-called charged (“electron-like”) leptons has
220 electromagnetic charge -1 , while the neutrinos all have $q_{EM} = 0$.

221 Often in an experimental context, lepton is used to denote the stable electron
222 and metastable muon, due to their striking experimental signatures. Taus are often
223 treated separately, due to their much shorter lifetime of $\tau_\tau \sim 10^{-13}s$; these decay
224 through hadrons or the other leptons, so often physics analyses at the LHC treat
225 them as jets or leptons, as will be done in this thesis.

226 As the neutrinos are electrically neutral, nearly massless, and only interact via the
227 weak force, it is quite difficult to observe them directly. Since LHC experiments rely
228 overwhelmingly on electromagnetic interactions to observe particles, the presence of
229 neutrinos is not observed directly. Neutrinos are instead observed by the conservation
230 of four-momentum in the plane transverse to the proton-proton collisions, known as
231 *missing transverse energy*.

There are six quarks in the Standard Model : up, down, charm, strange, top, and bottom. Quarks are similar organized into three generations :

$$\begin{pmatrix} u \\ d \end{pmatrix}, \begin{pmatrix} c \\ s \end{pmatrix}, \begin{pmatrix} t \\ b \end{pmatrix} \quad (2.15)$$

232 where we speak of “up-like” quarks and “down-like” quarks.

233 Each up-like quark has charge $q_{up} = 2/3$, while the down-like quarks have $q_{down} =$
234 $-1/3$. At the high energies of the LHC, one often makes the distinction between
235 the light quarks (u, d, c, s), the bottom quark, and top quark. In general, due to
236 the hadronization process described above, the light quarks, with masses $m_q < \sim$
237 1.5GeV are indistinguishable by LHC experiments. Their hadronic decay products
238 generally have long lifetimes and they are reconstructed as jets.¹ The bottom quark
239 hadronizes primarily through the B -mesons, which generally travels a short distance
240 before decaying to other hadrons. This allows one to distinguish decays via b -quarks
241 from other jets; this procedure is known as *b-tagging* and will be discussed more in
242 Ch.5. Due to its large mass, the top quark decays before it can hadronize; there
243 are no bound states associated to the top quark. The top is of particular interest at
244 the LHC; it has a striking signature through its most common decay mode $t \rightarrow Wb$.
245 Decays via tops, especially $t\bar{t}$ are frequently an important signal decay mode, or an
246 important background process.

247 Interactions in the Standard Model

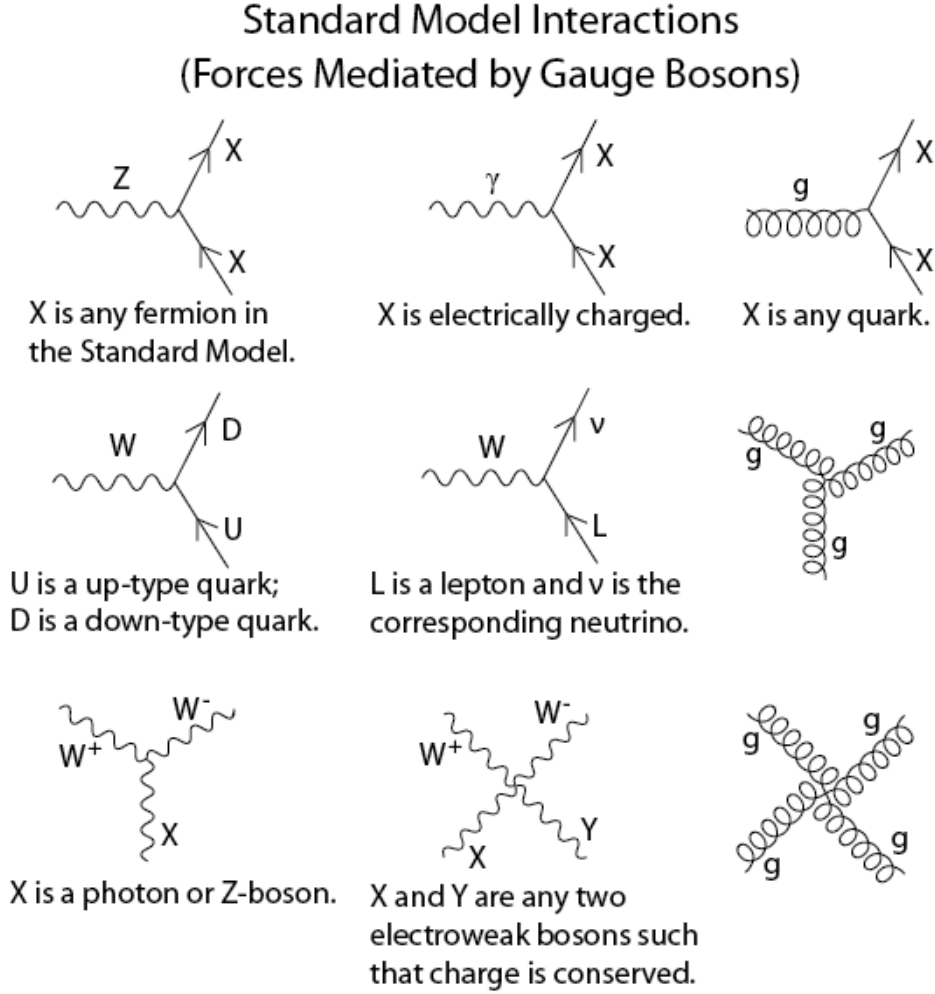
248 We briefly overview the entirety of the fundamental interactions of the Standard
249 Model; these can also be found in 2.3.

250 The electromagnetic force, mediated by the photon, interacts with via a three-
251 point coupling all charged particles in the Standard Model. The photon thus interacts
252 with all the quarks, the charged leptons, and the charged W^\pm bosons.

253 The weak force is mediated by three particles : the W^\pm and the Z^0 . The Z^0 can
254 interact with all fermions via a three-point coupling. A real Z_0 can thus decay to
255 a fermion-antifermion pair of all SM fermions except the top quark, due to its large

¹In some contexts, charm quarks are also treated as a separate category, although it is quite difficult to distinguish charm quarks from the other light quarks.

Figure 2.3: The interactions of the Standard Model



256 mass. The W^\pm has two important three-point interactions with fermions. First, the
 257 W^\pm can interact with an up-like quark and a down-like quark; an important example
 258 in LHC experiments is $t \rightarrow Wb$. The coupling constants for these interactions are
 259 encoded in the unitary matrix known as the Cabibbo–Kobayashi–Maskawa (CKM)
 260 matrix [57, 58], and are generally known as flavor-changing interactions. Secondly,
 261 the W^\pm interacts with a charged lepton and its corresponding neutrino. In this case,
 262 the unitary matrix that corresponds to CKM matrix for quarks is the identity matrix,
 263 which forbids (fundamental) vertices such as $\mu \rightarrow We$. For leptons, instead this is
 264 a two-step process : $\mu \rightarrow \nu_\mu u W \rightarrow \nu_\mu u \bar{\nu}_e e$. Finally, there are the self-interactions

265 of the weak gauge bosons. There is a three-point and four-point interaction; all
 266 combinations are allowed which conserve electric charge.

267 The strong force is mediated by the gluon, which as discussed above also carries
 268 the strong color charge. There is the fundamental three-point interaction, where a
 269 quark radiates a gluon. Additionally, there are the three-point and four-point gluon-
 270 only interactions.

271 2.3 Deficiencies of the Standard Model

272 At this point, it is quite easy to simply rest on our laurels. This relatively simple
 273 theory is capable of explaining a very wide range of phenomenom, which ultimately
 274 break down only to combinations of nine diagrams shown in Fig.2.3. Unfortunately,
 275 there are some unexplained problems with the Standard Model. We cannot go
 276 through all of the potential issues in this thesis, but we will motivate the primary
 277 issues which naturally lead one to *supersymmetry*, as we will see in Ch.3.

The Standard Model has many free paramaters; see Table 2.1 In general, we prefer
 models with less free parameters. A great example of this fact, and the primary
 experimental evidence for EWSB, is the relationship between the couplings of the
 weak force and the masses of the gauge bosons of the weak force :

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} \stackrel{?}{=} 1 \quad (2.16)$$

278 where ? indicates that this is a testable prediction of the Standard Model (in
 279 particular, that the gauge bosons gain mass through EWSB). This relationship has
 280 been measured within experimental and theoretical predictions. We would like to
 281 produce additional such relationships, which would exist if the Standard Model is a
 282 low-energy approximation of some other theory.

283 An additional issue is the lack of *gauge coupling unification*. The couplings of
 284 any quantum field theory “run” as a function of the distance scales (or inversely,

Table 2.1: Parameters of the Standard Model. For values dependent on the renormalization scheme, we use a combination of the on-shell normalization scheme [59–62] and modified minimal subtraction scheme with $m_{\bar{M}S}$ as indicated in the table[63]

m_e	Electron mass	511 keV
m_μ	Muon mass	105.7 MeV
m_τ	Tau mass	1.78 GeV
m_u	Up quark mass	1.9 MeV ($m_{\bar{M}S} = 2GeV$)
m_d	Down quark mass	4.4 MeV ($m_{\bar{M}S} = 2GeV$)
m_s	Strange quark mass	87 MeV ($m_{\bar{M}S} = 2GeV$)
m_c	Charm quark mass	1.32 GeV ($m_{\bar{M}S} = m_c$)
m_b	Bottom quark mass	4.24 GeV ($m_{\bar{M}S} = m_b$)
m_t	Top quark mass	172.7 GeV (on-shell renormalization)
θ_{12} CKM	12-mixing angle	13.1°
θ_{23} CKM	23-mixing angle	2.4°
θ_{13} CKM	13-mixing angle	0.2°
δ CKM	CP-violating Phase	0.995
g'	U(1) gauge coupling	0.357 ($m_{\bar{M}S} = m_Z$)
g	SU(2) gauge coupling	0.652 ($m_{\bar{M}S} = m_Z$)
g_s	SU(3) gauge coupling	1.221 ($m_{\bar{M}S} = m_Z$)
θ_{QCD}	QCD vacuum angle	~ 0
VEV	Higgs vacuum expectation value	246 GeV
m_H	Higgs mass	125 GeV

energy scales) of the theory. The idea is closely related to the unification of the electromagnetic and weak forces at the so-called *electroweak scale* of $O(100 \text{ GeV})$. One would hope this behavior was repeated between the electroweak forces and the strong force at some suitable energy scale. The Standard Model does automatically not exhibit this behavior, as we can see in Fig.2.4.

The most significant problem with the Standard Model is the *hierarchy problem*. In its most straightforward incarnation, the Higgs scalar field is subject to quantum corrections through loop diagrams, as shown in Fig.2.5. For demonstration, we use the contributions from the top quark, since the top quark has the largest Higgs Yukawa coupling due to its large mass. In general, we should expect these corrections to quadratically dependent on the scale of the ultraviolet physics, Λ . Briefly assume there is no new physics before the Planck scale of gravity, $\Lambda_{\text{Planck}} = 10^{19} \text{ GeV}$. In this

Figure 2.4: The running of Standard Model gauge couplings. The Standard Model couplings do not unify at high energies, which indicates it cannot completely describe nature through the Planck scale.

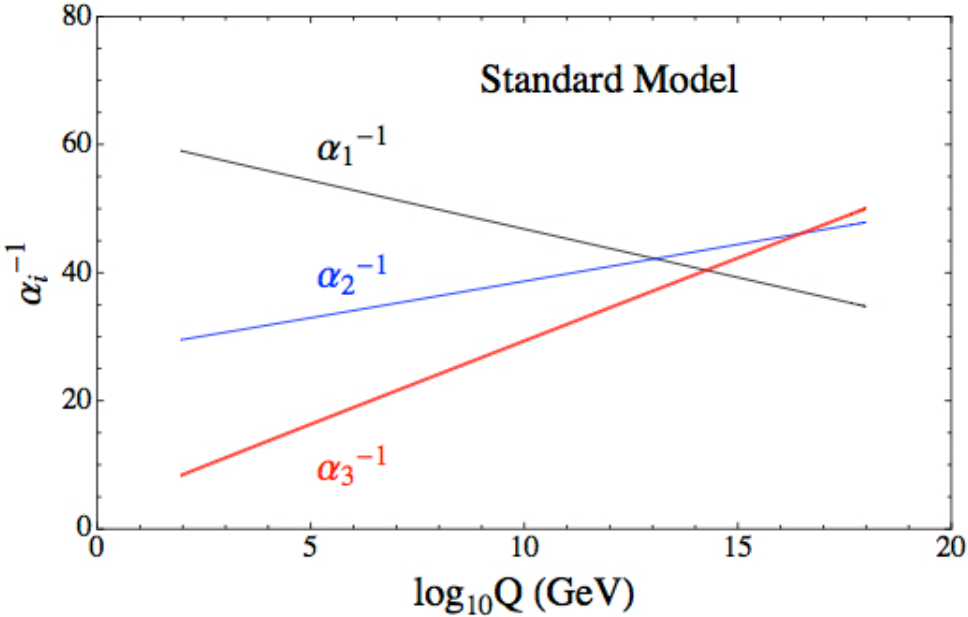
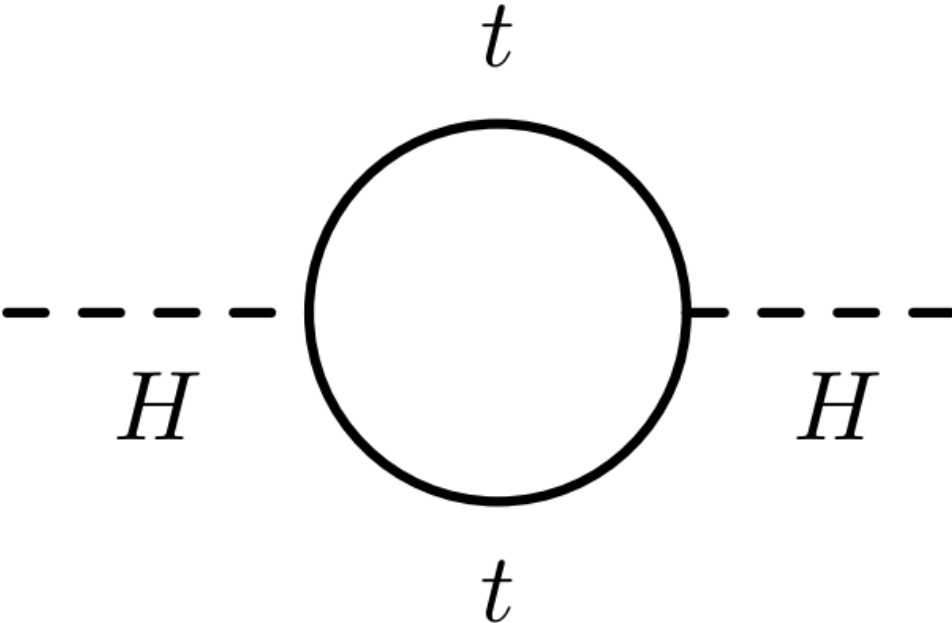


Figure 2.5: The dominant quantum loop correction to the Higgs mass in the Standard Model.



case, we expect the corrections to the Higgs mass like

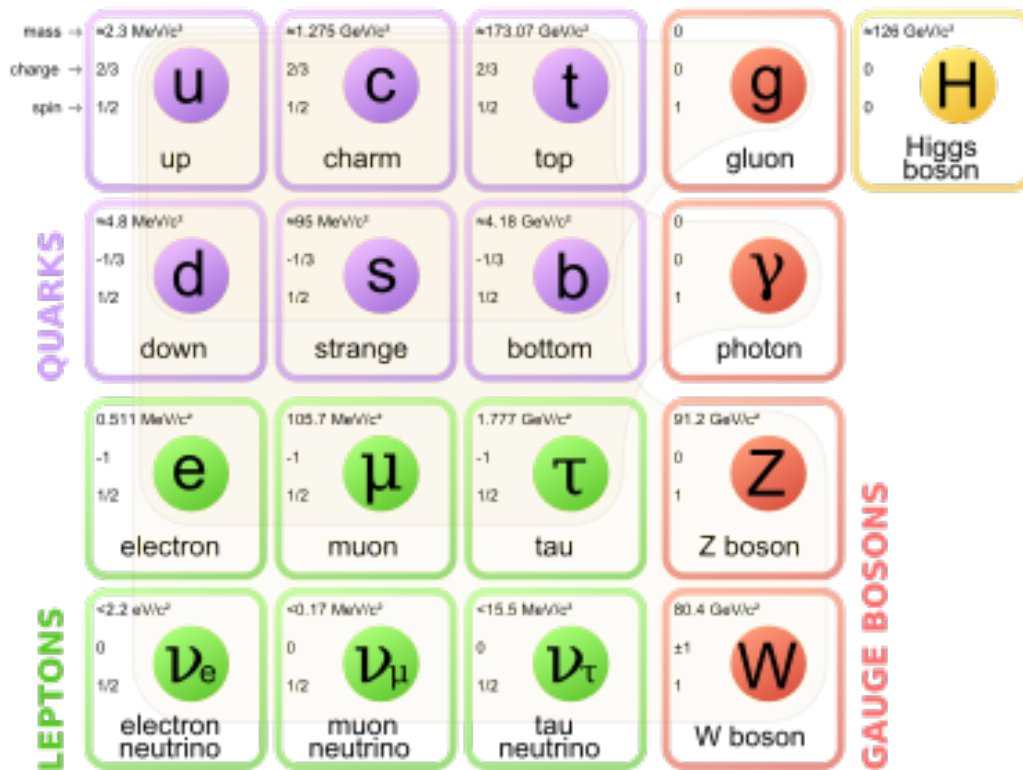
$$\delta m_H^2 \approx \left(\frac{m_t}{8\pi^2 \langle \phi \rangle_{VEV}} \right)^2 \Lambda_{Planck}^2. \quad (2.17)$$

290 To achieve the miraculous cancellation required to get the observed Higgs mass of
 291 125 GeV, one needs to then set the bare Higgs mass m_0 , our input to the Standard
 292 Model Lagrangian, itself to a *precise* value $\sim 10^{19}$ GeV. This extraordinary level of
 293 parameter finetuning is quite undesirable, and within the framework of the Standard
 294 Model, there is little that can be done to alleviate this issue.

295 An additional concern, of a different nature, is the lack of a *dark matter* candidate
 296 in the Standard Model. Dark matter was discovered by observing galactic rotation
 297 curves, which showed that much of the matter that interacted gravitationally was
 298 invisible to our (electromagnetic) telescopes [16–22]. The postulation of the existence
 299 of dark matter, which interacts at least through gravity, allows one to understand
 300 these galactic rotation curves. Unfortunately, no particle in the Standard Model could
 301 possibly be the dark matter particle. The only candidate truly worth another look is
 302 the neutrino, but it has been shown that the neutrino content of the universe is simply
 303 too small to explain the galactic rotation curves [22, 64]. The experimental evidence
 304 from the galactic rotations curves thus show there *must* be additional physics beyond
 305 the Standard Model, which is yet to be understood.

306 In the next chapter, we will see how these problems can be alleviated by the theory
 307 of supersymmetry.

Figure 2.6: Particles of the Standard Model



Chapter 3

Supersymmetry

This chapter will introduce supersymmetry (SUSY) [15, 23–35]. We will begin by introducing the concept of a *superspace*, and discuss some general ingredients of supersymmetric theories. This will include a discussion of how the problems with the Standard Model described in Ch.2 are naturally fixed by these theories.

The next step is to discuss the particle content of the *Minimally Supersymmetric Standard Model* (MSSM). As its name implies, this theory contains the minimal additional particle content to make Standard Model supersymmetric. We then discuss the important phenomenological consequences of this theory, especially as it would be observed in experiments at the LHC.

3.1 Supersymmetric theories : from space to superspace

Coleman-Mandula “no-go” theorem

We begin the theoretical motivation for supersymmetry by citing the “no-go” theorem of Coleman and Mandula [65]. This theorem forbids *spin-charge unification*; it states that all quantum field theories which contain nontrivial interactions must be a direct product of the Poincaré group of Lorentz symmetries, the internal product from of gauge symmetries, and the discrete symmetries of parity, charge conjugation, and time reversal. The assumptions which go into building the Coleman-Mandula

theorem are quite restrictive, but there is one unique way out, which has become known as *supersymmetry* [26, 66]. In particular, we must introduce a *spinorial* group generator Q . Alternatively, and equivalently, this can be viewed as the addition of anti-commuting coordinates; space plus these new anti-commuting coordinates is then called *superspace* [67]. We will not investigate this view in detail, but it is also a quite intuitive and beautiful way to construct supersymmetry[15].

Supersymmetry transformations

A *supersymmetric* transformation Q transforms a bosonic state into a fermionic state, and vice versa :

$$Q |\text{Fermion}\rangle = |\text{Boson}\rangle \quad (3.1)$$

$$Q |\text{Boson}\rangle = |\text{Fermion}\rangle \quad (3.2)$$

To ensure this relation holds, Q must be an anticommuting spinor. Additionally, since spinors are inherently complex, Q^\dagger must also be a generator of the supersymmetry transformation. Since Q and Q^\dagger are spinor objects (with $s = 1/2$), we can see that supersymmetry must be a spacetime symmetry. The Haag-Lopuszanski-Sohnius extension [66] of the Coleman-Mandula theorem [65] is quite restrictive about the forms of such a symmetry. Here, we simply write the (anti-) commutation relations [15] :

$$Q_\alpha, Q_{\dot{\alpha}}^\dagger = -2\sigma_{\alpha\dot{\alpha}\mu} P_\mu \quad (3.3)$$

$$Q_\alpha, Q_{\dot{\beta}} = Q_{\dot{\alpha}}^\dagger, Q_{\dot{\beta}}^\dagger = 0 \quad (3.4)$$

$$[P^\mu, Q_\alpha] = [P^\mu, Q_{\dot{\alpha}}^\dagger] = 0 \quad (3.5)$$

337 Supermultiplets

338 In a supersymmetric theory, we organize single-particle states into irreducible
339 representations of the supersymmetric algebra which are known as *supermultiplets*.
340 Each supermultiplet contains a fermion state $|F\rangle$ and a boson state $|B\rangle$; these two
341 states are known as *superpartners*. These are related by some combination of
342 Q and Q^\dagger , up to a spacetime transformation. Q and Q^\dagger commute with the mass-
343 squared operator $-P^2$ and the operators corresponding to the gauge transformations
344 [15]; in particular, the gauge interactions of the Standard Model. In an unbroken
345 supersymmetric theory, this means the states $|F\rangle$ and $|B\rangle$ have exactly the same mass,
346 electromagnetic charge, electroweak isospin, and color charges. One can also prove
347 [15] that each supermultiplet contains the exact same number of bosonic (n_B) and
348 fermion (n_F) degrees of freedom. We now explore the possible types of supermultiples
349 one can find in a renormalizable supersymmetric theory.

350 Since each supermultiplet must contain a fermion state, the simplest type of
351 supermultiplet contains a single Weyl fermion state ($n_F = 2$) which is paired with
352 $n_B = 2$ scalar bosonic degrees of freedom. This is most conveniently constructed as
353 single complex scalar field. We call this construction a *scalar supermultiplet* or *chiral*
354 *supermultiplet*. The second name is indicative; only chiral supermultiplets can contain
355 fermions whose right-handed and left-handed components transform differently under
356 the gauge interactions (as of course happens in the Standard Model).

357 The second type of supermultiplet we construct is known as a *gauge* supermul-
358 tiplet. We take a spin-1 gauge boson (which must be massless due to the gauge
359 symmetry, so $n_B = 2$) and pair this with a single massless Weyl spinor¹. The gauge
360 bosons transform as the adjoint representation of their respective gauge groups;
361 their fermionic partners, which are known as gauginos, must also. In particular,
362 the left-handed and right-handed components of the gaugino fermions have the same

¹Choosing an $s = 3/2$ massless fermion leads to nonrenormalizable interactions.

363 gauge transformation properties.

364 Excluding gravity, this is the entire list of supermultiplets which can participate
365 in renormalizable interactions in what is known as $N = 1$ supersymmetry. This
366 means there is only one copy of the supersymmetry generators Q and Q^\dagger . This is
367 essentially the only “easy” phenomenological choice, since it is the only choice in four
368 dimensions which allows for the chiral fermions and parity violations built into the
369 Standard Model, and we will not look further into $N > 1$ supersymmetry in this thesis.

370 The primary goal, after understanding the possible structures of the multiplets
371 above, is to fit the Standard Model particles into a multiplet, and therefore make
372 predictions about their supersymmetric partners. We explore this in the next section.

373 3.2 Minimally Supersymmetric Standard Model

374 To construct what is known as the MSSM [[susyPrimer](#), [68–71](#)], we need a few
375 ingredients and assumptions. First, we match the Standard Model particles with
376 their corresponding superpartners of the MSSM. We will also introduce the naming
377 of the superpartners (also known as *sparticles*). We discuss a very common additional
378 restraint imposed on the MSSM, known as R -parity. We also discuss the concept of
379 soft supersymmetry breaking and how it manifests itself in the MSSM.

380 Chiral supermultiplets

381 The first thing we deduce is directly from Sec.???. The bosonic superpartners
382 associated to the quarks and leptons *must* be spin 0, since the quarks and leptons must
383 be arranged in a chiral supermultiplet. This is essentially the note above, since the
384 chiral supermultiplet is the only one which can distinguish between the left-handed
385 and right-handed components of the Standard Model particles. The superpartners of
386 the quarks and leptons are known as *squarks* and *sleptons*, or *sfermions* in aggregate.

387 (for “scalar quarks”, “scalar leptons”, and “scalar fermion”²). The “s-” prefix
388 can also be added to the individual quarks i.e. *selectron*, *sneutrino*, and *stop*. The
389 notation is to add a \sim over the corresponding Standard Model particle i.e. \tilde{e} , the
390 selectron is the superpartner of the electron. The two-component Weyl spinors of the
391 Standard Model must each have their own (complex scalar) partner i.e. e_L, e_R have
392 two distinct partners : \tilde{e}_L, \tilde{e}_R . As noted above, the gauge interactions of any of the
393 sfermions are identical to those of their Standard Model partners.

Due to the scalar nature of the Higgs, it must obviously lie in a chiral supermultiplet. To avoid gauge anomalies and ensure the correct Yukawa couplings to the quarks and leptons[15], we must add additional Higgs bosons to any supersymmetric theory. In the MSSM, we have two chiral supermultiplets. The SM (SUSY) parts of the multiplets are denoted $H_u(\tilde{H}_u)$ and $H_d(\tilde{H}_d)$. Writing out H_u and H_d explicitly:

$$H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} \quad (3.6)$$

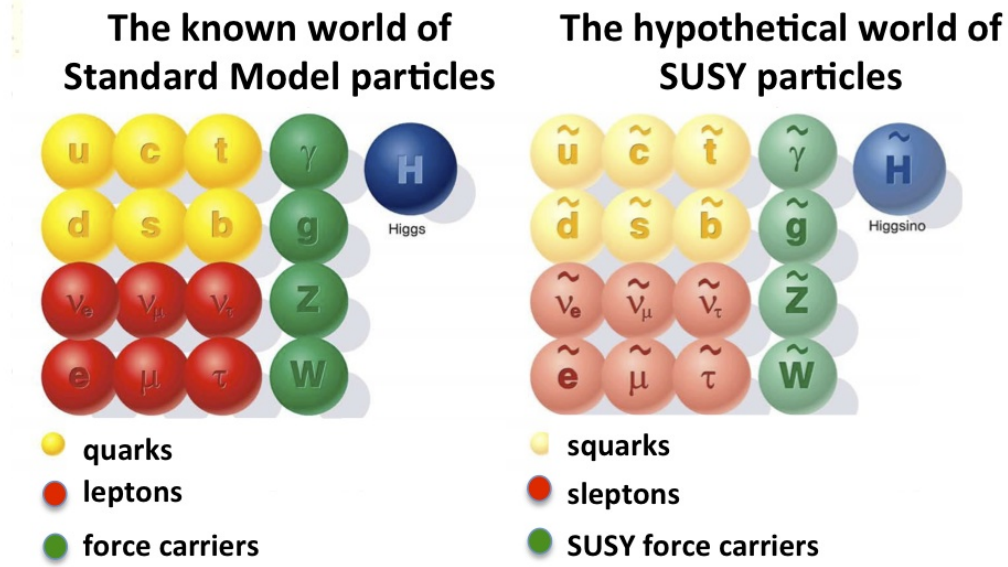
$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \quad (3.7)$$

$$(3.8)$$

394 we see that H_u looks very similar to the SM Higgs with $Y = 1$, and H_d is symmetric
395 to this with $+$ \rightarrow $-$, with $Y = -1$. The SM Higgs boson, h_0 , is a linear superposition
396 of the neutral components of these two doublets. The SUSY parts of the Higgs
397 multiplets, \tilde{H}_u and \tilde{H}_d , are each left-handed Weyl spinors. For generic spin-1/2
398 sparticles, we add the “-ino” suffix. We then call the partners of the two Higgs
399 collectively the *Higgsinos*.

²The last one should probably have bigger scare quotes.

Figure 3.1: Particles of the MSSM



400 Gauge supermultiplets

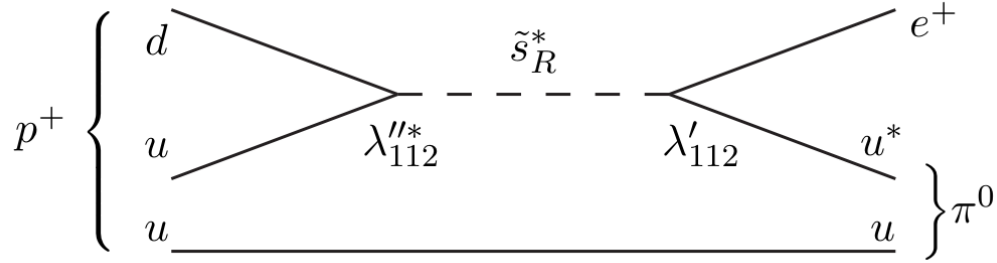
401 The superpartners of the gauge bosons must all be in gauge supermultiplets since
 402 they contain a spin-1 particle. Collectively, we refer to the superpartners of the
 403 gauge bosons as the gauginos.

404 The first gauge supermultiplet contains the gluon, and its superpartner, which is
 405 known as the *gluino*, denoted \tilde{g} . The gluon is of course the SM mediator of $SU(3)_C$;
 406 the gluino is also a colored particle, subject to $SU(3)_C$. From the SM before EWSB,
 407 we have the four gauge bosons of the electroweak symmetry group $SU(2)_L \otimes U(1)_Y$:
 408 $W^{1,2,3}$ and B^0 . The superpartners of these particles are thus the *winos* $\tilde{W}^{1,2,3}$ and
 409 *bino* \tilde{B}^0 , where each is placed in another gauge supermultiplet with its corresponding
 410 SM particle. After EWSB, without breaking supersymmetry, we would also have the
 411 zino \tilde{Z}^0 and photino $\tilde{\gamma}$.

412 The entire particle content of the MSSM can be seen in Fig.3.1.

413 At this point, it's important to take a step back. Where are these particles?
 414 As stated above, supersymmetric theories require that the masses and all quantum

Figure 3.2: This Feynmann diagram shows how proton decay is induced in the MSSM, if one does not impose R -parity.



415 numbers of the SM particle and its corresponding sparticle are the same. Of course,
 416 we have not observed a selectron, squark, or wino. The answer, as it often is, is that
 417 supersymmetry is *broken* by the vacuum state of nature [15].

418 R -parity

This section is a quick aside to the general story. R - *parity* refers to an additional discrete symmetry which is often imposed on supersymmetric models. For a given particle state, we define

$$R = (-1)^{3(B-L)+2s} \quad (3.9)$$

419 where B, L is the baryon (lepton) number and s is the spin. The imposition of
 420 this symmetry forbids certain terms from the MSSM Lagrangian that would violate
 421 baryon and/or lepton number. This is required in order to prevent proton decay, as
 422 shown in Fig.3.2³. .

423 In supersymmetric models, this is a \mathbb{Z}_2 symmetry, where SM particles have $R = 1$
 424 and sparticles have $R = -1$. We will take R - *parity* as part of the definition of
 425 the MSSM. We will discuss later the *drastic* consequences of this symmetry on SUSY
 426 phenomenology

³Proton decay can actually be prevented by allowing only one of the four potential R-parity violating terms to survive.

427 Soft supersymmetry breaking

The fundamental idea of *soft* supersymmetry breaking[15, 34, 35, 72, 73] is that we would like to break supersymmetry without reintroducing the quadratic divergences we discussed at the end of Chapter 2. We write the Lagrangian in a form :

$$\mathcal{L}_{\text{MSSM}} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{soft}} \quad (3.10)$$

428 In this sense, the symmetry breaking is “soft”, since we have separated out the
429 completely symmetric terms from those soft terms which will not allow the quadratic
430 divergences to the Higgs mass.

431 The explicitly allowed terms in the soft-breaking Lagrangian are [35].

- 432 • Mass terms for the scalar components of the chiral supermultipletss
- 433 • Mass terms for the Weyl spinor components of the gauge supermultipletss
- 434 • Trilinear couplings of scalar components of chiral supermultiplets

In particular, using the field content described above for the MSSM, the softly-broken portion of the MSSM Lagrangian can be written

$$\mathcal{L}_{\text{soft}} = -\frac{1}{2} \left(M_3 \tilde{g} \tilde{g} + M_2 \tilde{W} \tilde{W} + M_1 \tilde{B} \tilde{B} + c.c. \right) \quad (3.11)$$

$$- \left(\tilde{u} a_u \tilde{Q} H_u - \tilde{d} a_d \tilde{Q} H_d - \tilde{e} a_e \tilde{L} H_d + c.c. \right) \quad (3.12)$$

$$- \tilde{Q}^\dagger m_Q^2 \tilde{Q} - \tilde{L}^\dagger m_L^2 \tilde{L} - \tilde{u} m_u^2 \tilde{u}^\dagger - \tilde{d} m_d^2 \tilde{d}^\dagger - \tilde{e} m_e^2 \tilde{e}^\dagger \quad (3.13)$$

$$- m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (b H_u H_d + cc). \quad (3.14)$$

435 where we have introduced the following notations :

- 436 1. M_3, M_2, M_1 are the gluino, wino, and bino masses.
- 437 2. a_u, a_d, a_e are complex 3×3 matrices in family space.
- 438 3. $m_Q^2, m_u^2, m_d^2, m_L^2, m_e^2$ are hermitian 3×3 matrices in family space.

439 4. $m_{H_u}^2, m_{H_d}^2, b$ are the SUSY-breaking contributions to the Higgs potential.

440 We have written matrix terms without any sort of additional notational decoration
 441 to indicate their matrix nature, and we now show why. The first term 1 are
 442 straightforward; these are just the straightforward mass terms for these fields. There
 443 are strong constraints on the off-diagonal terms for the matrices of 2 [74, 75]; for
 444 simplicity, we will assume that each $a_i, i = u, d, e$ is proportional to the Yukawa
 445 coupling matrix : $a_i = A_{i0} y_i$. The matrices in ?? can be similarly constrained by
 446 experiments [68, 75–82] Finally, we assume that the elements 4 contributing to the
 447 Higgs potential as well as all of the 1 terms must be real, which limits the possible
 448 CP-violating interactions to those of the Standard Model. We thus only consider
 449 flavor-blind, CP-conserving interactions within the MSSM.

The important mixing for mass and gauge interaction eigenstates in the MSSM occurs within electroweak sector, in a process akin to EWSB in the Standard Model. The neutral portions of the Higgsinos doublets and the neutral gauginos ($\tilde{H}_u^0, \tilde{H}_d^0, \tilde{B}^0, \tilde{W}^0$ of the gauge interaction basis mix to form what are known as the *neutralinos* of mass basis :

$$M_{\tilde{\chi}} = \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \quad (3.15)$$

450 where $s(c)$ are the sine and cosine of angles related to EWSB, which introduced
 451 masses to the gauginos and higgsinos. Diagonalization of this matrix gives the four
 452 neutralino mass states, listed without loss of generality in order of increasing mass :
 453 $\chi_{1,2,3,4}^0$.

454 The neutralinos, especially the lightest neutralino $\tilde{\chi}_1^0$, are important ingredients
 455 in SUSY phenomenology.

456 The same process can be done for the electrically charged gauginos with
 457 the charged portions of the Higgsino doublets along with the charged winos
 458 $(\tilde{H}_u^+, \tilde{H}_d^+, \tilde{W}^+, \tilde{W}^-)$. This leads to the *charginos*, again in order of increasing mass
 459 : $\tilde{\chi}_{1,2}^\pm$.

460 3.3 Phenomenology

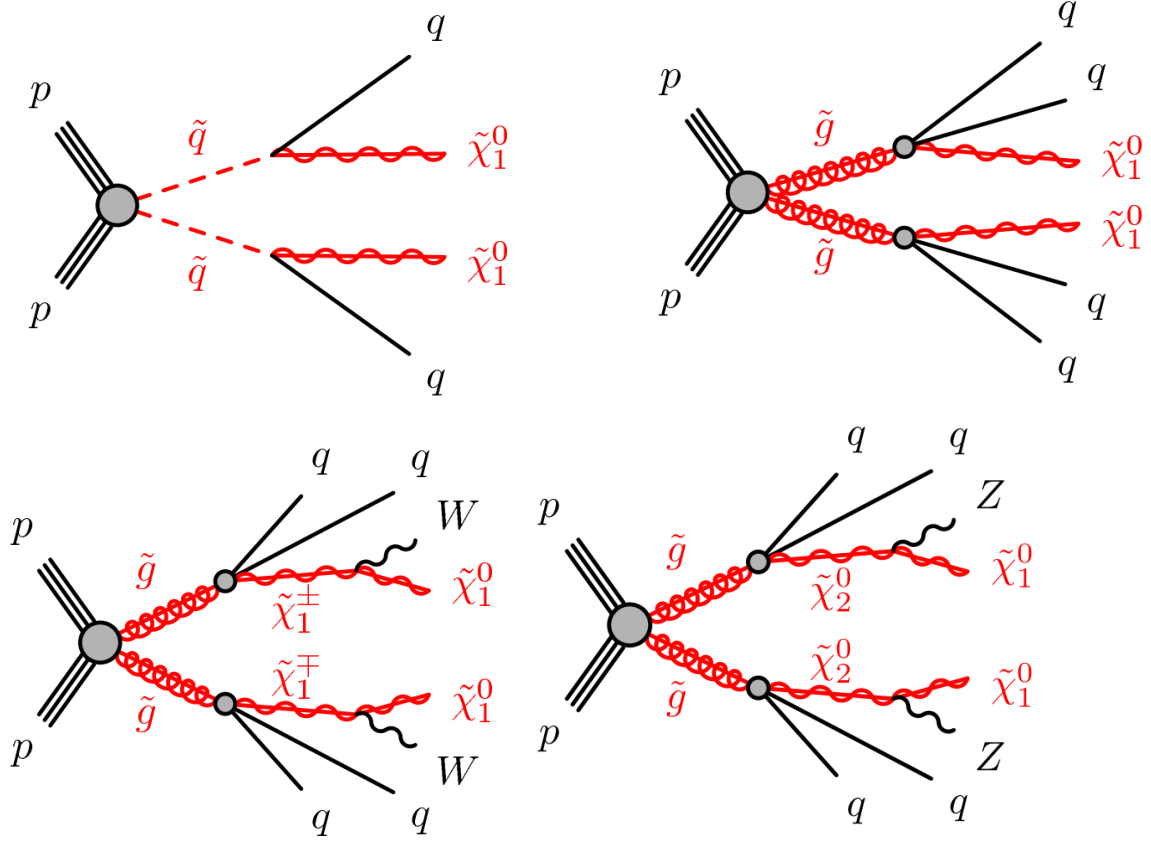
461 We are finally at the point where we can discuss the phenomenology of the MSSM,
 462 in particular as it manifests itself at the energy scales of the LHC.

463 As noted above in Sec.3.2, the assumption of R -parity has important conse-
 464 quences for MSSM phenomenology. The SM particles have $R = 1$, while the sparticles
 465 all have $R = -1$. Simply, this is the “charge” of supersymmetry. Since the particles of
 466 LHC collisions (pp) have total incoming $R = 1$, we must expect that all sparticles will
 467 be produced in *pairs*. An additional consequence of this symmetry is the fact that the
 468 lightest supersymmetric particle (LSP) is *stable*. Off each branch of the Feynmann
 469 diagram shown in Fig., we have $R = -1$, and this can only decay to another sparticle
 470 and a SM particle. Once we reach the lightest sparticle in the decay, it is absolutely
 471 stable. This leads to the common signature E_T^{miss} for a generic SUSY signal.

472 For this thesis, we will be presenting an inclusive search for squarks and gluinos
 473 with zero leptons in the final state. This is a very interesting decay channel⁴, due
 474 to the high cross-sections of $\tilde{g}\tilde{g}$ and $\tilde{q}\tilde{q}$ decays, as can be seen in Fig.?? [83]. This
 475 is a direct consequence of the fact that these are the colored particles of the MSSM.
 476 Since the sparticles interact with the gauge groups of the SM in the same way as their
 477 SM partners, the colored sparticles, the squarks and gluinos, are produced and decay
 478 as governed by the color group $SU(3)_C$ with the strong coupling g_s . The digluino
 479 production is particularly copious, due to color factor corresponding to the color octet

⁴Prior to Run1, probably the most *most* interesting SUSY decay channel.

Figure 3.3: SUSY signals considered in this thesis



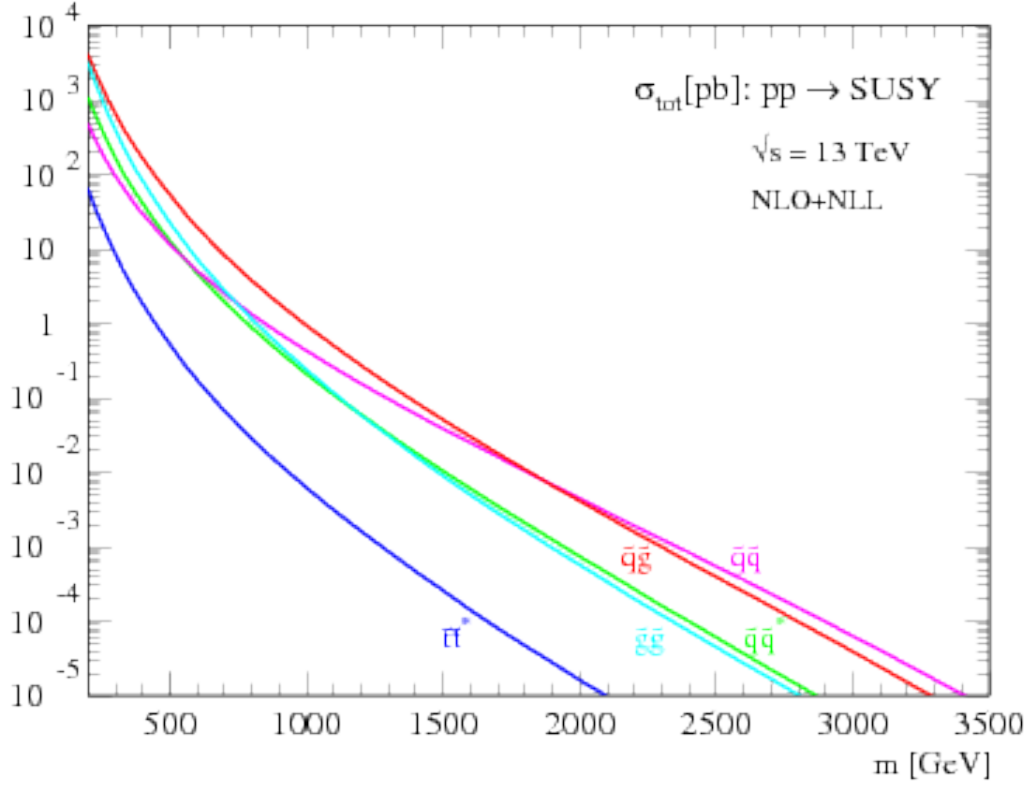
of $SU(3)C$.

In the case of disquark production, the most common decay mode of the squark in the MSSM is a decay directly to the LSP plus a single SM quark [15]. This means the basic search strategy of disquark production is two jets from the final state quarks, plus missing transverse energy for the LSPs. There are also cascade decays, the most common of which, and the only one considered in this thesis, is $\tilde{q} \rightarrow q\tilde{\chi}^\pm \rightarrow qW^\pm\chi^0$.

For digluino production, the most common decay is $\tilde{g} \rightarrow g\tilde{q}$, due to the large g_s coupling. The squark then decays as listed above. In this case, we generically search for four jets and missing transverse energy from the LSPs. We can also have the squark decay in association with a W^\pm or Z^0 ; in this thesis, we are interested in those cases where this vector boson goes hadronically.

In the context of experimental searches for SUSY, we often consider *simplified*

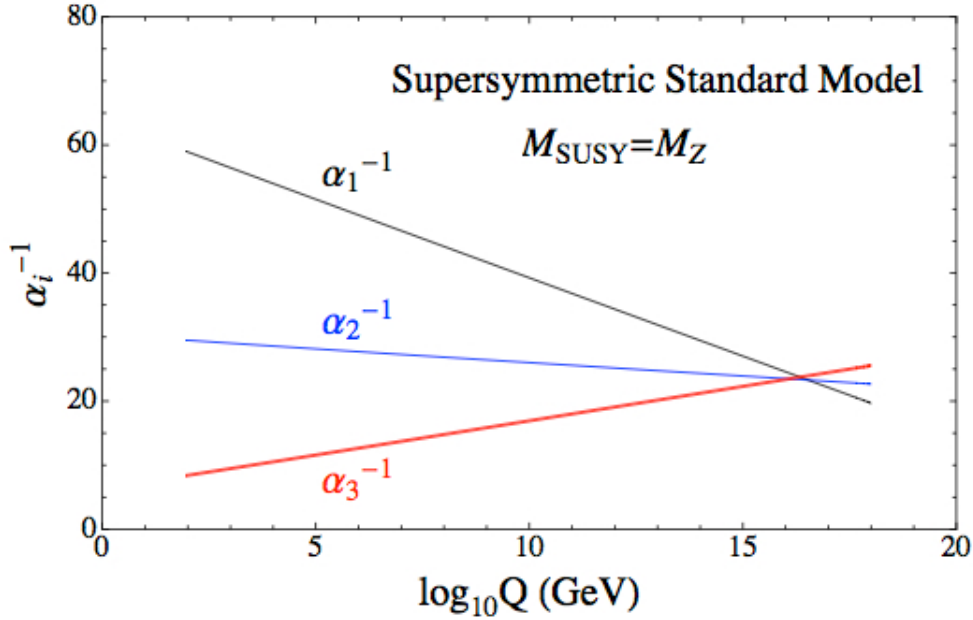
Figure 3.4: SUSY production cross-sections as a function of sparticle mass at $\sqrt{s} = 13$ TeV.



models. These models make certain assumptions which allow easy comparisons of results by theorists and rival experimentalists. In the context of this thesis, the simplified models will make assumptions about the branching ratios described in the preceding paragraphs. In particular, we will often choose a model where the decay of interest occurs with 100% branching ratio. This is entirely for ease of interpretation by other physicists⁵, but it is important to recognize that these are more a useful comparison tool, especially with limits, than a strict statement about the potential masses of sought-after beyond the Standard Model particle.

⁵In the author's opinion, this often leads to more confusion than comprehension. We will revisit the shortcomings of simplified models in the Conclusion to this thesis.

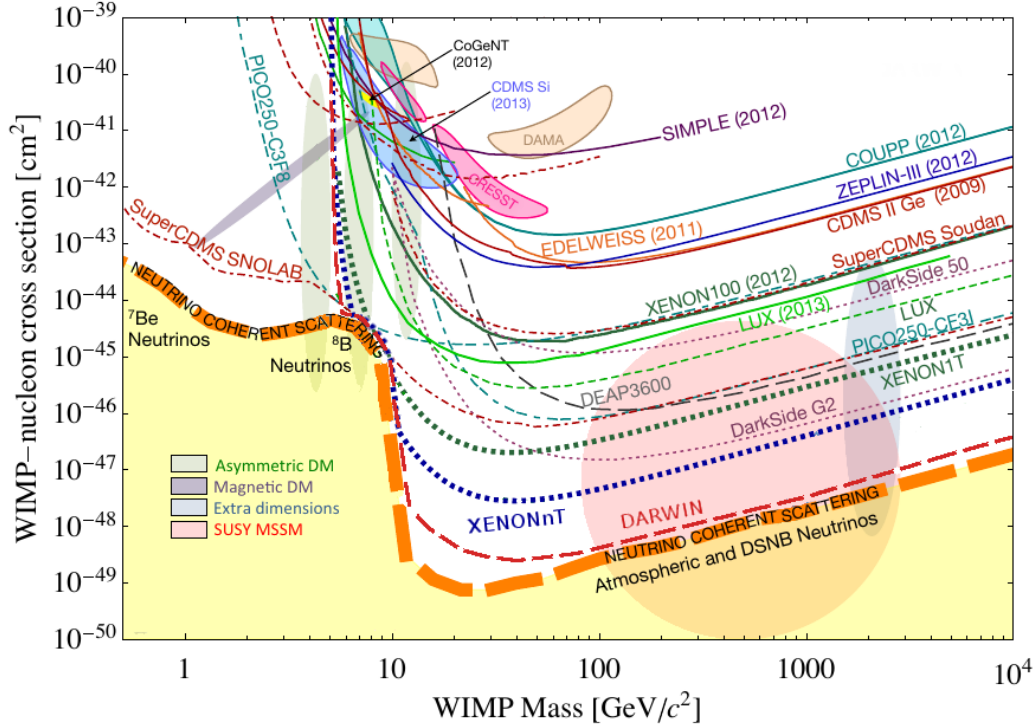
Figure 3.6: The running of Standard Model gauge couplings; compare to Fig.2.4. The MSSM gauge couplings nearly intersect at high energies.



515 Dark matter

516 As we discussed previously, the lack of any dark matter candidate in the Standard
517 Model naturally leads to beyond the Standard Model theories. In the Standard Model,
518 there is a natural dark matter candidate in the lightest supersymmetric particle[15]
519 The LSP would in dark matter experiments be called a *weakly-interacting massive*
520 *particle* (WIMP), which is a type of cold dark matter [22, 84]. These WIMPS would
521 only interact through the weak force and gravity, which is exactly as a model like the
522 MSSM predicts for the neutralino. In Fig.3.7, we can see the current WIMP exclusions
523 for a given mass. The range of allowed masses which have not been excluded for LSPs
524 and WIMPs have significant overlap. This provides additional motivation outside of
525 the context of theoretical details.

Figure 3.7: WIMP exclusions from direct dark matter detection experiments.



3.5 Conclusions

Supersymmetry is the most well-motivated theory for physics beyond the Standard Model. It provides a solution to the hierarchy problem, leads to gauge coupling unification, and provides a dark matter candidate consistent with galactic rotation curves. As noted in this chapter, due to the LSPs in the final state, most SUSY searches require a significant amount of missing transverse energy in combination with jets of high transverse momentum. However, there is some opportunity to do better than this, especially in final states where one has two weakly-interacting LSPs on opposite sides of some potentially complicated decay tree. We will see how this is done in Ch.??.

The Large Hadron Collider

538 This brief chapter will summarize the very basics of accelerator physics. We will
 539 describe the CERN accelerator complex, with particular focus on the Large Hadron
 540 Collider (LHC).

541 4.1 Basics of Accelerator Physics

542 This section follows closely the presentation of [85].

Simple particle accelerators simply rely on the acceleration of charged particles in a static electric field. Given a field of strength E , charge q , and mass m , this is simply

$$a = \frac{qE}{m}. \quad (4.1)$$

543 This was used for many early accelerators For a given particle with a given mass and cite some?
 544 charge, this is of course limited by the static electric field which can be produced.
 545 This is limited by the electric breakdown at high voltages.

There are two complementary solutions to this issue. First, we use the *radio frequency acceleration* technique. This consist of using a time-varied electric field. We call the devices used for this *RF cavities*. Second, one bends the particles in a magnetic field, which allows them to pass through the same RF electric field over and over. This second process is limited by *synchrotron radiation*, which describes the radiation produced when a charged particle is accelerated. The power radiated is

$$P \sim \frac{1}{r^2} \left(E/m \right)^4 \quad (4.2)$$

546 where r is the radius of curvature and E, m is the energy (mass) of the charged
 547 particle. Given an energy which can be produced by a given set of RF cavities (which
 548 is *not* limited by the mass of the particle), one then has two options to increase the
 549 actual collision energy : increase the radius of curvature or use a heavier particle.
 550 Practically speaking, the easiest options for particles in a collider are protons and
 551 electrons, since they are (obviously) copious in nature and do not decay¹. Given the
 552 dependence on mass, we can see why protons are used to reach the highest energies.
 553 The tradeoff for this is that protons are not point particles, and we thus we don't
 554 know the exact incoming four-vectors of the protons, as discussed in Ch.2.

The primary “unit” of a proton collider is the (proton) *bunch*. Bunches of protons are induced by the RF cavities; particles are accelerated or decelerated by the cavities, and pushed together into bunches, which eventually pass through the RF cavities at the frequency of the cavity. Besides the energy of the beam, the most important quantity to characterize a beam is known as the *emittance*. The emittance is a description of the size of the bunch ellipse. The emittance is important mostly due to its influence on the *instantaneous luminosity*, which directly effects the rate of a given physics process. For process of cross-section σ , the rate is given by

add fig of
emittance

$$R = L\sigma \tag{4.3}$$

where L is the instantaneous luminosity, given by:

$$L = \frac{f_{\text{rev}} N_b^2 R}{4\pi\sigma^2} = \frac{f_{\text{rev}} n N_b^2 R}{4\pi\beta^* \beta_{\text{rel}} \epsilon} \tag{4.4}$$

555 4.2 Accelerator Complex

556 4.3 Large Hadron Collider

¹Muon colliders are a really cool option at high energies, since the relativistic γ factor gives them a relatively long lifetime in the lab frame.

557

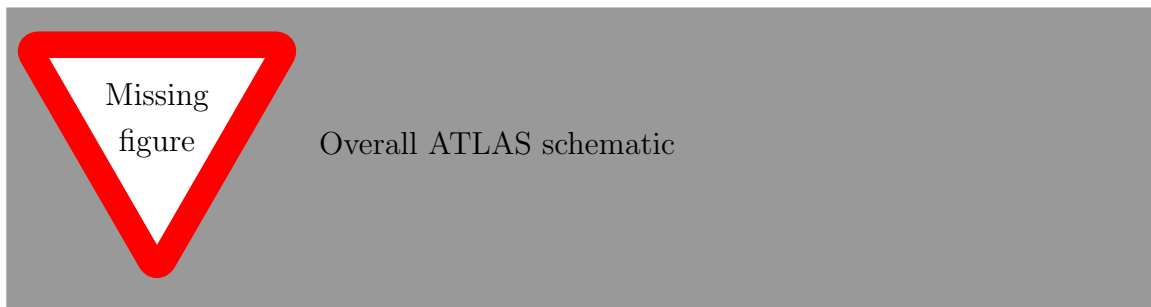
Chapter 5

558

The ATLAS detector

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560 sentence its own line.

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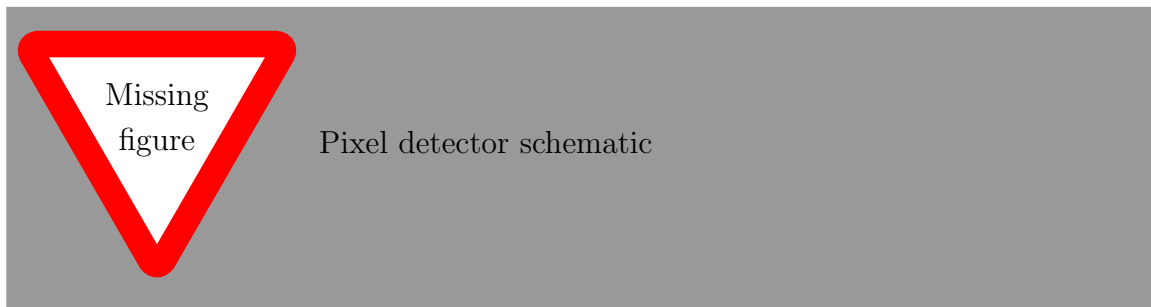
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563

5.1 Inner Detector

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566 table of contents. If you want your sections to be numbered and to appear in the
567 table of contents, remove the asterisk.

568 **Pixel Detector**

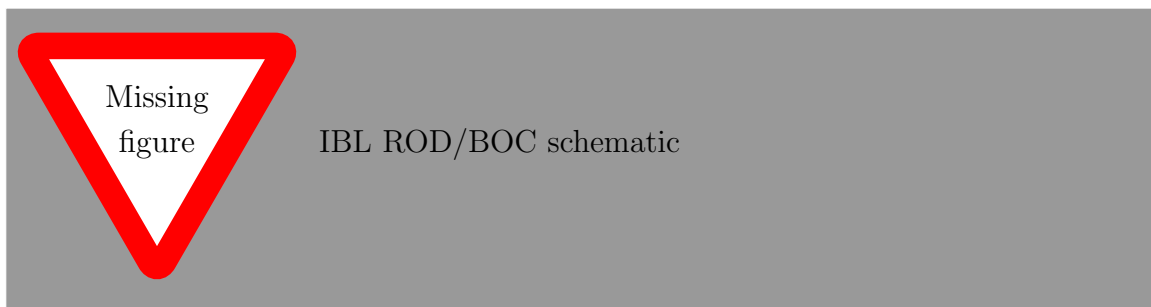


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570

571 **Insertable B-Layer**

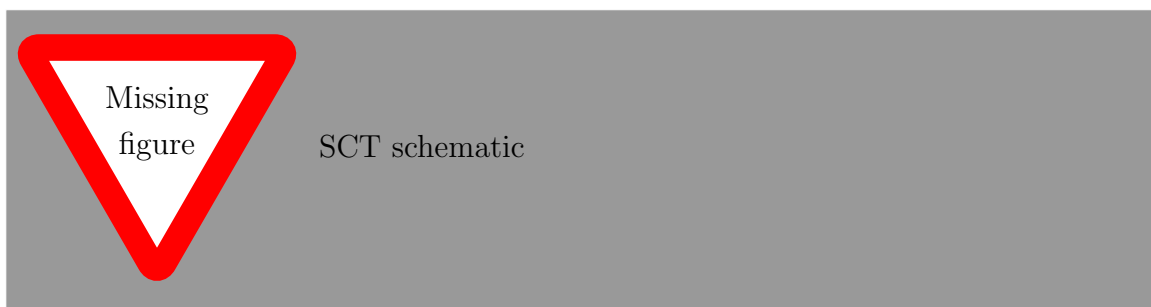
572 Qualification task, so add a bit more.



573

574

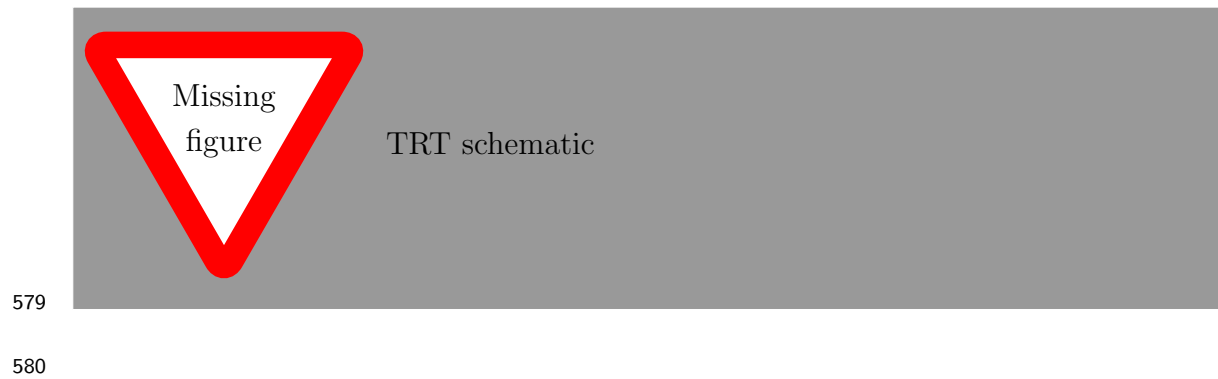
575 **Semiconductor Tracker**



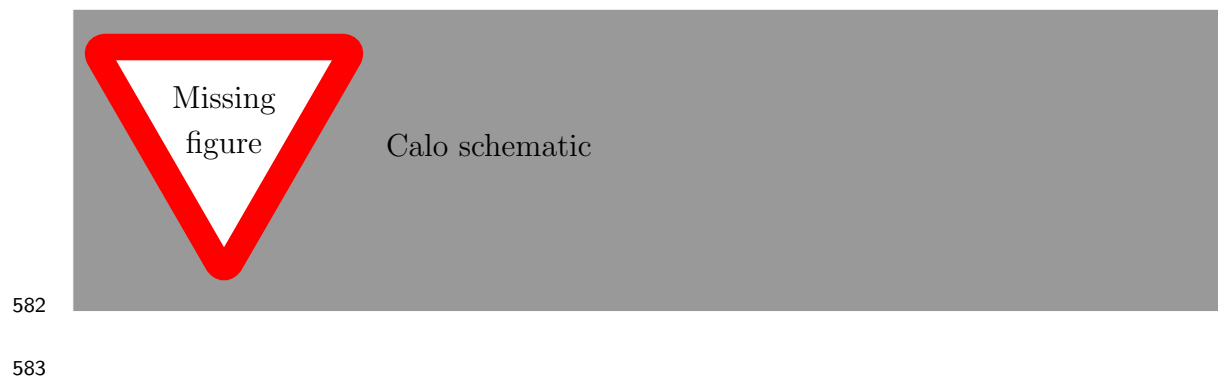
576

577

578 **Transition Radiation Tracker**



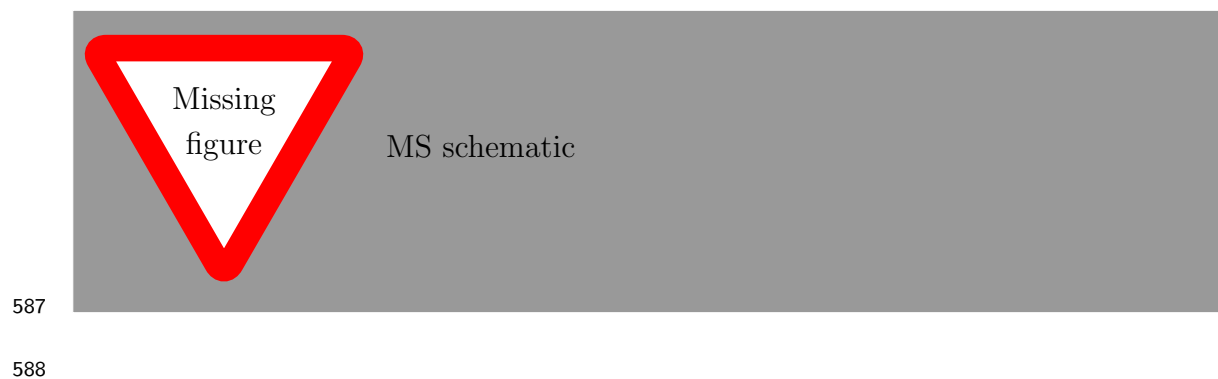
581 **5.2 Calorimeter**



584 **Electromagnetic Calorimeter**

585 **Hadronic Calorimeter**

586 **5.3 Muon Spectrometer**



The Recursive Jigsaw Technique

591 Here you can write some introductory remarks about your chapter. I like to give each
592 sentence its own line.

593 When you need a new paragraph, just skip an extra line.

594 **6.1 Razor variables**

595 By using the asterisk to start a new section, I keep the section from appearing in the
596 table of contents. If you want your sections to be numbered and to appear in the
597 table of contents, remove the asterisk.

598 **6.2 SuperRazor variables**

599 **6.3 The Recursive Jigsaw Technique**

600 **6.4 Variables used in the search for zero lepton**

601 **SUSY**

602

Chapter 7

603

Title of Chapter 1

604

Chapter 8

605

Title of Chapter 1

606 Here you can write some introductory remarks about your chapter. I like to give each
607 sentence its own line.

608 When you need a new paragraph, just skip an extra line.

609 **8.1 Object reconstruction**

610 **Photons, Muons, and Electrons**

611 **Jets**

612 **Missing transverse momentum**

613 Probably longer, show some plots from the PUB note that we worked on

614 **8.2 Signal regions**

615 **Gluino signal regions**

616 **Squark signal regions**

617 **Compressed signal regions**

618 **8.3 Background estimation**

619 **Z $\nu\nu$**

620 **W $e\nu$**

621 **$t\bar{t}$**

622

Chapter 9

623

Title of Chapter 1

624 Here you can write some introductory remarks about your chapter. I like to give each
625 sentence its own line.

626 When you need a new paragraph, just skip an extra line.

627 **9.1 Statistical Analysis**

628 maybe to be moved to an appendix

629 **9.2 Signal Region distributions**

630 **9.3 Pull Plots**

631 **9.4 Systematic Uncertainties**

632 **9.5 Exclusion plots**

633

Conclusion

634 Here you can write some introductory remarks about your chapter. I like to give each
635 sentence its own line.

636 When you need a new paragraph, just skip an extra line.

637 **9.6 New Section**

638 By using the asterisk to start a new section, I keep the section from appearing in the
639 table of contents. If you want your sections to be numbered and to appear in the
640 table of contents, remove the asterisk.

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866 [confId=11505#20160811](https://indico.fnal.gov/sessionDisplay.py?sessionId=3&confId=11505#20160811).

The Standard Model

In this appendix, we provide a brief overview of the basic ingredients involved in construction of the Standard Model Lagrangian : quantum field theory, symmetries, and symmetry breaking.

Quantum Field Theory

In this section, we provide a brief overview of the necessary concepts from Quantum Field Theory (QFT).

In modern physics, the laws of nature are described by the “action” S , with the imposition of the principle of minimum action. The action is the integral over the spacetime coordinates of the “Lagrangian density” \mathcal{L} , or Lagrangian for short. The Lagrangian is a function of “fields”; general fields will be called $\phi(x^\mu)$, where the indices μ run over the space-time coordinates. We can then write the action S as

$$S = \int d^4x \mathcal{L}[\phi_i(x^\mu), \partial_\mu \phi_i(x^\mu)] \quad (9.1)$$

where we have an additional summation over i (of the different fields). Generally, we impose the following constraints on the Lagrangian :

1. Translational invariance - The Lagrangian is only a function of the fields ϕ and their derivatives $\partial_\mu \phi$
2. Locality - The Lagrangian is only a function of one point x_μ in spacetime.

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and notes
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- 885 3. Reality condition - The Lagrangian is real to conserve probability.
- 886 4. Lorentz invariance - The Lagrangian is invariant under the Poincaré group of
887 spacetime.
- 888 5. Analyticity - The Lagrangian is an analytical function of the fields; this is to
889 allow the use of perturbation theory.
- 890 6. Invariance and Naturalness - The Lagrangian is invariant under some internal
891 symmetry groups; in fact, the Lagrangian will have *all* terms allowed by the
892 imposed symmetry groups.
893 7. Renormalizability - The Lagrangian will be renormalizable - in practice, this
894 means there will not be terms with more than power 4 in the fields.

895 The key item from the point of view of this thesis is that of “Invariance and
896 Natural”. We impose a set of “symmetries” and then our Lagrangian is the most
897 general which is allowed by those symmetries.

898 Symmetries

899 Symmetries can be seen as the fundamental guiding concept of modern physics.
900 Symmetries are described by “groups”. . To illustrate the importance of symmetries
901 and their mathematical description, groups, we start here with two of the simplest
902 and most useful examples : \mathbb{Z}_2 and $U(1)$.

903 \mathbb{Z}_2 symmetry

904 \mathbb{Z}_2 symmetry is the simplest example of a “discrete” symmetry. Consider the most
905 general Lagrangian of a single real scalar field $\phi(x_\mu)$

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \frac{\mu}{2\sqrt{2}}\phi^3 - \lambda\phi^4 \quad (9.2)$$

Now we *impose* the symmetry

$$\mathcal{L}(\phi) = \mathcal{L}(-\phi) \quad (9.3)$$

906 This has the effect of restricting the allowed terms of the Lagrangian. In particular,
 907 we can see the term $\phi^3 \rightarrow -\phi^3$ under the symmetry transformation, and thus must
 908 be disallowed by this symmetry. This means under the imposition of this particular
 909 symmetry, our Lagrangian should be rewritten as

$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{m^2}{2}\phi^2 - \lambda\phi^4 \quad (9.4)$$

910 The effect of this symmetry is that the total number of ϕ particles can only change
 911 by even numbers, since the only interaction term $\lambda\phi^4$ is an even power of the field.
 912 This symmetry is often imposed in supersymmetric theories, as we will see in Chapter
 913 3.

914 **$U(1)$ symmetry**

915 $U(1)$ is the simplest example of a continuous (or *Lie*) group. Now consider a theory
 916 with a single complex scalar field $\phi = \text{Re } \phi + i \text{Im } \phi$

$$\mathcal{L}_\phi = \delta_{i,j} \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_j - \frac{m^2}{2} \phi_i \phi_j - \frac{\mu}{2\sqrt{2}} \phi_i \phi_j \phi_k - \lambda \phi_i \phi_j \phi_k \phi_l \quad (9.5)$$

917 where $i, j, k, l = \text{Re}, \text{Im}$. In this case, we impose the following $U(1)$ symmetry
 918 : $\phi \rightarrow e^{i\theta} \phi, \phi^* \rightarrow e^{-i\theta} \phi^*$. We see immediately that this again disallows the third-order
 919 terms, and we can write a theory of a complex scalar field with $U(1)$ symmetry as

$$\mathcal{L}_\phi = \partial_\mu \phi \partial^\mu \phi^* - \frac{m^2}{2} \phi \phi^* - \lambda (\phi \phi^*)^2 \quad (9.6)$$

920 Local symmetries

921 The two examples considered above are “global” symmetries in the sense that the
922 symmetry transformation does not depend on the spacetime coordinate x_μ . We know
923 look at local symmetries; in this case, for example with a local $U(1)$ symmetry, the
924 transformation has the form $\phi(x_\mu) \rightarrow e^{i\theta(x_\mu)}\phi(x_\mu)$. These symmetries are also known
925 as “gauge” symmetries; all symmetries of the Standard Model are gauge symmetries.

There are wide-ranging consequences to the imposition of local symmetries. To begin, we note that the derivative terms of the Lagrangian 9.2 are *not* invariant under a local symmetry transformation

$$\partial_\mu \phi(x_\mu) \rightarrow \partial_\mu (e^{i\theta(x_\mu)} \phi(x_\mu)) = (1 + i\partial_\mu \theta(x_\mu)) e^{i\theta(x_\mu)} \phi(x_\mu) \quad (9.7)$$

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927 This leads us to note that the kinetic terms of the Lagrangian are also not invariant
928 under a gauge symmetry. This would lead to a model with no dynamics, which is
929 clearly unsatisfactory.

930 Let us take inspiration from the case of global symmetries. We need to define a
931 so-called “covariant” derivative D^μ such that

$$D^\mu \phi \rightarrow e^{iq\theta(x_\mu)} D^\mu \phi \quad (9.8)$$

$$D^\mu \phi^* \rightarrow e^{-iq\theta(x_\mu)} D^\mu \phi^* \quad (9.9)$$

$$(9.10)$$

932 Since ϕ and ϕ^* transform with the opposite phase, this will lead to the invariance
933 of the Lagrangian under our local gauge transformation. This D^μ is of the following
934 form

$$D^\mu = \partial_\mu - igqA^\mu \quad (9.11)$$

935 where A^μ is a vector field we introduce with the transformation law

$$A^\mu \rightarrow A^\mu - \frac{1}{g} \partial_\mu \theta \quad (9.12)$$

936 and g is the coupling constant associated to vector field. This vector field A^μ is
937 also known as a “gauge” field.

938 Since we need to add all allowed terms to the Lagrangian, we define

$$F^{\mu\nu} = A^\mu A^\nu - A^\nu A^\mu \quad (9.13)$$

939 and then we must also add the kinetic term :

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \quad (9.14)$$

940 The most general renormalizable Lagrangian with fermion and scalar fields can
941 be written in the following form

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_\phi + \mathcal{L}_\psi + \mathcal{L}_{Yukawa} \quad (9.15)$$

942 Symmetry breaking and the Higgs mechanism

943 Here we view some examples of symmetry breaking. We investigate breaking of a
944 global $U(1)$ symmetry and a local $U(1)$ symmetry. The SM will break the electroweak
945 symmetry $SU(2) \times U(1)$, and in Chapter 3 we will see how supersymmetry must also
946 be broken.

947 There are two ideas of symmetry breaking

- 948 • Explicit symmetry breaking by a small parameter - in this case, we have a small
949 parameter which breaks an “approximate” symmetry of our Lagrangian. An
950 example would be the theory of the single scalar field [9.2](#), when $\mu \ll m^2$ and

951 $\mu \ll \lambda$. In this case, we can often ignore the small term when considering
 952 low-energy processes.

953 • Spontaneous symmetry breaking (SSB) - spontaneous symmetry breaking
 954 occurs when the Lagrangian is symmetric with respect to a given symmetry
 955 transformation, but the ground state of the theory is *not* symmetric with respect
 956 to that transformation. This can have some fascinating consequences, as we
 957 will see in the following examples

958 Symmetry breaking a

959 **U(1) global symmetry breaking**

Consider the theory of a complex scalar field under the $U(1)$ symmetry, or the transformation

$$\phi \rightarrow e^{i\theta} \phi \quad (9.16)$$

The Lagrangian for this theory is

$$\mathcal{L} = \partial^\mu \phi^\dagger \partial_\mu \phi + \frac{\mu^2}{2} \phi^\dagger \phi + \frac{\lambda}{4} (\phi^\dagger \phi)^2 \quad (9.17)$$

Let us write this theory in terms of two scalar fields, h and ξ : $\phi = (h + i\xi)/\sqrt{2}$.

The Lagrangian can then be written as

$$\mathcal{L} = \partial^\mu h \partial_\mu h + \partial^\mu \xi \partial_\mu \xi - \frac{\mu^2}{2} (h^2 + \xi^2) - \frac{\lambda}{4} (h^2 + \xi^2)^2 \quad (9.18)$$

First, note that the theory is only stable when $\lambda > 0$. To understand the effect of SSB, we now enforce that $\mu^2 < 0$, and define $v^2 = -\mu^2/\lambda$. We can then write the scalar potential of this theory as :

$$V(\phi) = \lambda(\phi^\dagger \phi - v^2/2)^2 \quad (9.19)$$

Minimizing this equation with respect to ϕ , we can see that the “vacuum expectation value” of the theory is

$$2 \langle \phi^\dagger \phi \rangle = \langle h^2 + \xi^2 \rangle = v^2 \quad (9.20)$$

960 We now reach the “breaking” point of this procedure. In the (h, ξ) plane, the
 961 minima form a circle of radius v . We are free to choose any of these minima to expand
 962 our Lagrangian around; the physics is not affected by this choice. For convenience,
 963 choose $\langle h \rangle = v, \langle \xi^2 \rangle = 0$.

Now, let us define $h' = h - v, \xi' = \xi$ with VEVs $\langle h' \rangle = 0, \langle \xi' \rangle = 0$. We can then write our spontaneously broken Lagrangian in the form

$$\mathcal{L} = \frac{1}{2} \partial_\mu h' \partial^\mu h' + \frac{1}{2} \partial_\mu \xi' \partial^\mu \xi' - \lambda v^2 h'^2 - \lambda v h' (h'^2 + \xi'^2) - \lambda (h'^2 + \xi'^2)^2 \quad (9.21)$$