This is a first-order difference equation in $\partial v_{t-1}/\partial \phi_1$ with parameter ψ_1. If the MA term is not invertible, $|\psi_1| > 1$, then this equation may eventually explode resulting in the algorithm failing.

Example 13.17 Estimating a VARMA(1,1) Model

Consider the VARMA(1,1) model

$$y_t = \mu + \Phi_1 y_{t-1} + v_t + \Psi_1 v_{t-1}, \qquad v_t \sim iid\, N(0, V),$$

where y_t is of dimension N. The conditional log-likelihood function with $s = 1$ in (13.23) is

$$\begin{aligned}\ln L_T(\theta) &= \frac{1}{T-1}\sum_{t=2}^{T} \ln f(y_t|y_{t-1}, ..., y_1;\theta)\\ &= -\frac{N}{2}\ln 2\pi - \frac{1}{2}\ln|V| - \frac{1}{2(T-1)}\sum_{t=2}^{T} v_t' V^{-1} v_t,\end{aligned}$$

where the $(N \times 1)$ disturbance vector at time t is $v_t = y_t - \mu - \Phi_1 y_{t-1} - \Psi_1 v_{t-1}$. Table 13.2 gives the maximum likelihood estimates from estimating the VARMA(1,1)

$$\begin{aligned}y_{1,t} &= \mu_1 + \phi_{1,1,1} y_{1,t-1} + v_{1,t} + \psi_{1,1,1} v_{1,t-1} + \psi_{1,1,2} v_{2,t-1}\\ y_{2,t} &= \mu_2 + \phi_{1,2,2} y_{2,t-1} + v_{2,t} + \psi_{1,2,1} v_{1,t-1} + \psi_{1,2,2} v_{2,t-1}.\end{aligned}$$

The $T = 500$ observations are generated by simulating the model with the true parameters given in Table 13.2. The disturbance term, v_t, has zero mean and covariance matrix given by the identity matrix. There is good agreement between the parameter estimates and the true population parameters. Also reported are the quasi-maximum likelihood standard errors and t statistics that allow for heteroskedasticity. The residual covariance matrix is

$$\widehat{V} = \frac{1}{499}\sum_{t=2}^{500} \widehat{v}_t \widehat{v}_t' = \begin{bmatrix} 0.956 & 0.040\\ 0.040 & 0.966\end{bmatrix},$$

which is obtained at the final iteration. □

Example 13.18 Estimating a VAR(p) Model

The conditional log-likelihood function of the N dimensional VAR(p) model in (13.23) is

$$\begin{aligned}\ln L_T(\theta) &= \frac{1}{T-p}\sum_{t=p+1}^{T} \ln f(y_t|y_{t-1}, ..., y_{t-p};\theta)\\ &= -\frac{N}{2}\ln 2\pi - \frac{1}{2}\ln|V| - \frac{1}{2(T-p)}\sum_{t=p+1}^{T} v_t' V^{-1} v_t,\end{aligned}$$