hypothesis testing for paired data

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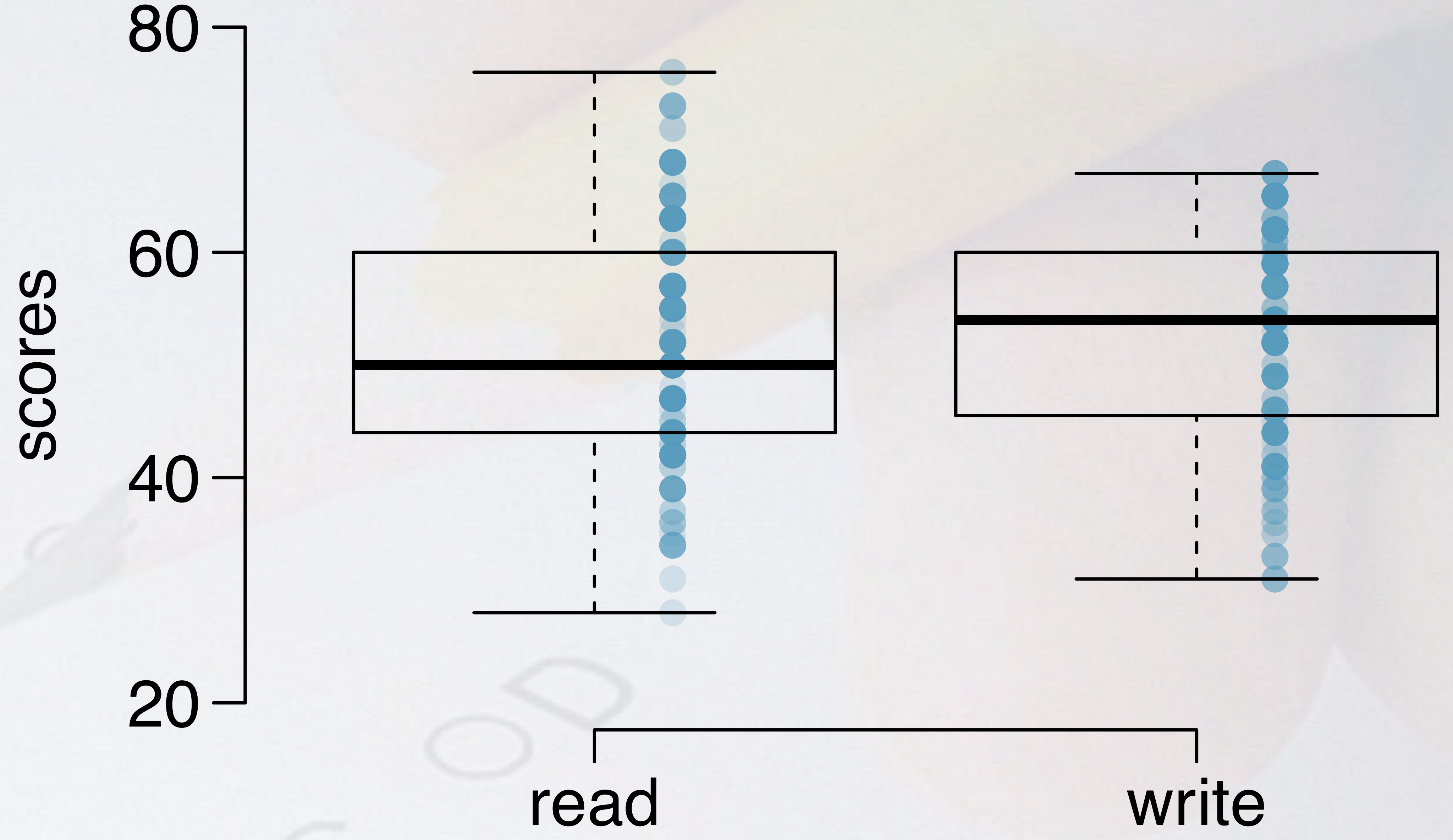
high school and beyond

200 observations were randomly sampled from the High School and Beyond survey. The same students took a reading and writing test. At a first glance, how are the distributions of reading and writing scores similar? How are they different?



Given that the same students took the reading and the writing tests, are the reading and writing scores of each student independent of each other?

	ID	read	write
1	70	57	52
2	86	44	33
3	141	63	44
4	172	47	52
…	…	…	…
200	137	63	65

analyzing paired data

- When two sets of observations have this special correspondence (not independent), they are said to be paired.
- To analyze paired data, it is often useful to look at the difference in outcomes of each pair of observations:

$$\text{diff} = \text{read} - \text{write}$$

- It is important that we always subtract using a consistent order.

	ID	read	write	diff
1	70	57	52	5
2	86	44	33	11
3	141	63	44	19
4	172	47	52	-5
...	...	...	...	...
200	137	63	65	-2

parameter of interest

Average difference between the reading and writing scores of **all** high school students.

μ_{diff}

point estimate

Average difference between the reading and writing scores of **sampled** high school students.

$\bar{x}_{diff}$

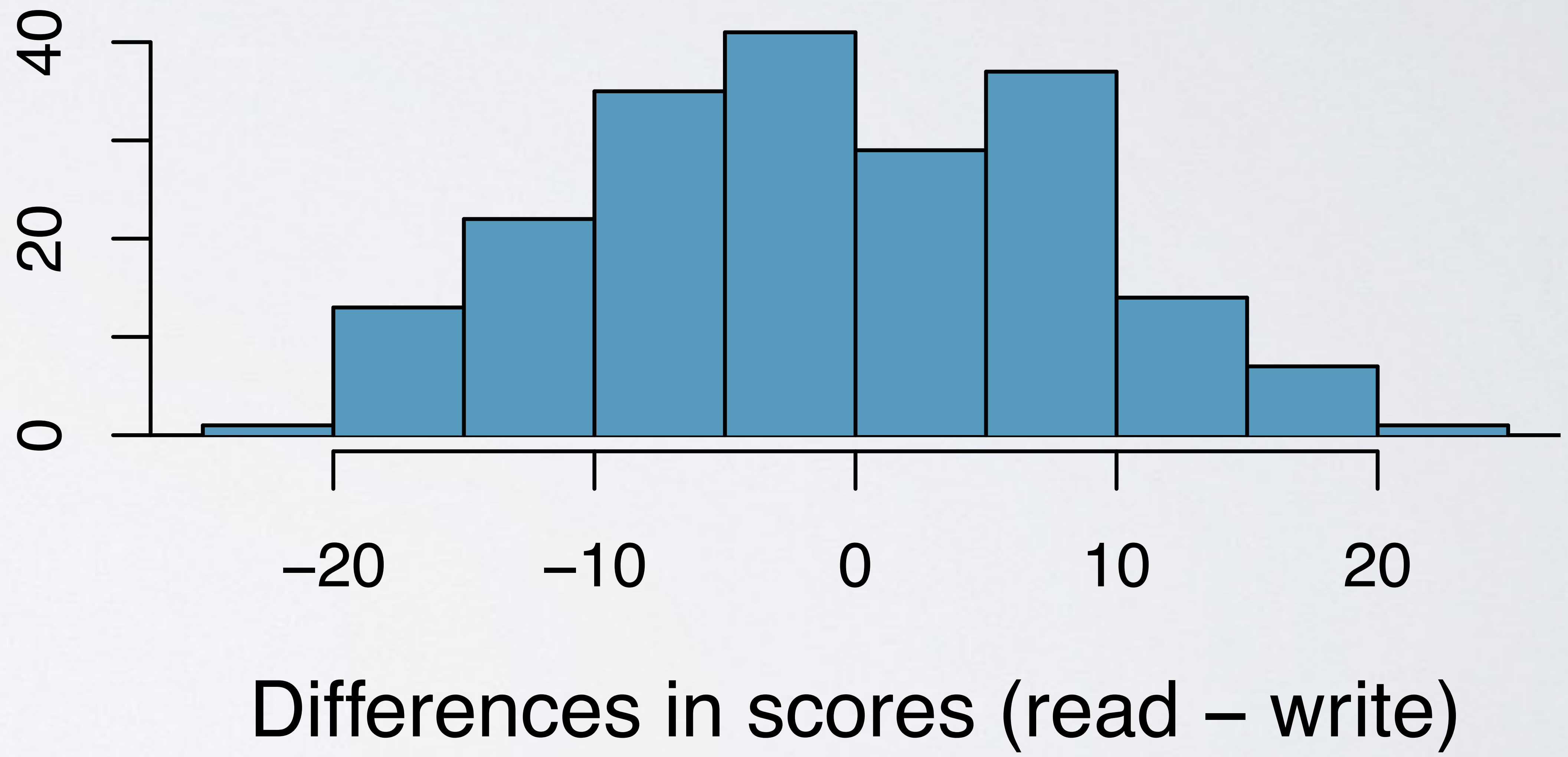
If in fact there was no difference between the scores on the reading and writing exams, what would you expect the average difference to be?

	ID	read	write	diff
1	70	57	52	5
2	86	44	33	11
3	141	63	44	19
4	172	47	52	-5
...	...	...	...	...
200	137	63	65	-2

$$\bar{x}_{diff} = -0.545$$

$$s_{diff} = 8.887$$

$$n_{diff} = 200$$



hypotheses for paired means

$H_0 : \mu_{diff} = 0$ There is no difference between the average reading and writing scores.

$H_A : \mu_{diff} \neq 0$ There is a difference between the average reading and writing scores.

nothing new!

one numerical variable

diff
5
11
19
-5
…
-2

hypothesis about the mean

$$H_0 : \mu_{diff} = 0$$
$$H_A : \mu_{diff} \neq 0$$

Hypothesis testing for a ~~single mean~~: difference between paired means

1. Set the hypotheses: $H_0 : \cancel{\mu}\ \mu_{diff} = null\ value$

   $H_A : \cancel{\mu}\ \mu_{diff} < or > or \neq null\ value$

2. Calculate the point estimate: $\cancel{\bar{x}}\ \bar{x}_{diff}$

3. Check conditions:
   1. ***Independence:*** Sampled observations must be independent (random sample/assignment & if sampling without replacement, $\cancel{n}$ < 10% of population)
   2. ***Sample size/skew:*** $\cancel{n}\ n_{diff} \geq 30$, larger if the population distribution is very skewed.

4. Draw sampling distribution, shade p-value, calculate test statistic

$$Z = \frac{x_{diff} - \mu_{diff}}{SE_{\bar{x}_{diff}}}$$

5. Make a decision, and interpret it in context of the research question:

practice

Describe the sampling distribution of the differences between the paired means of reading and writing scores.

$$H_0 : \mu_{diff} = 0$$

$$H_A : \mu_{diff} \neq 0$$

$$\bar{x}_{diff} = -0.545$$

$$s_{diff} = 8.887$$

$$n_{diff} = 200$$

$$\bar{x}_{diff} \sim N\left(mean = 0, SE = \frac{8.887}{\sqrt{200}} \approx 0.628\right)$$

practice

Calculate the test statistic and the p-value for this hypothesis test.

$H_0 : \mu_{diff} = 0$

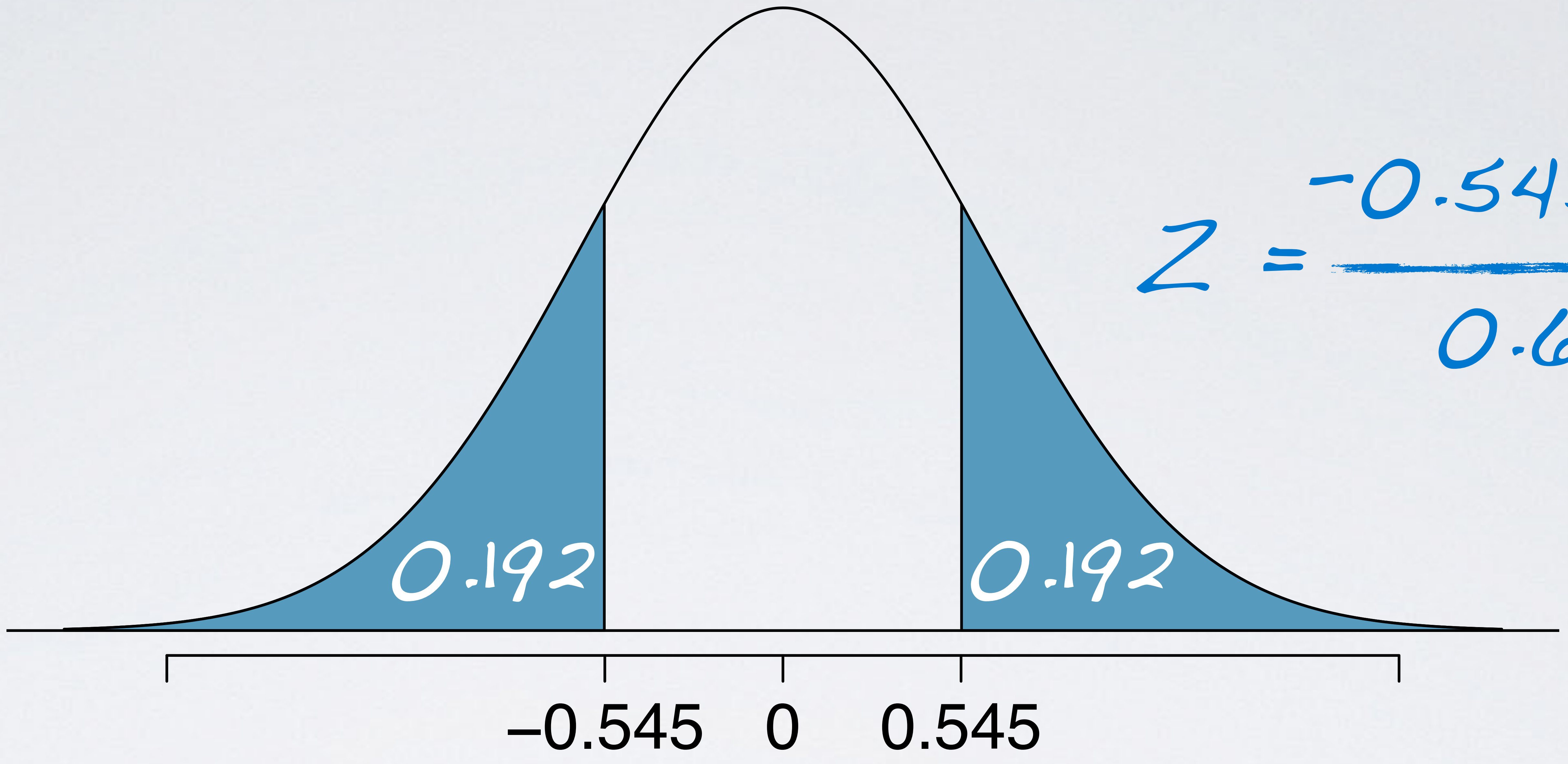
$H_A : \mu_{diff} \neq 0$

$\bar{x}_{diff} = -0.545$

$s_{diff} = 8.887$

$n_{diff} = 200$

$\bar{x}_{diff} \sim N(mean = 0, SE = 0.628)$



$Z = \dfrac{-0.545 - 0}{0.628} = -0.87$

p-value $= 0.192 \times 2$

$= 0.384$

https://bitly.com/dist_calc

practice

Which of the following is the correct interpretation of the p-value?

~~(a)~~ Probability that the average scores on the reading and writing exams are equal.

$P(H_0 \text{ is true})$

~~(b)~~ Probability that the average scores on the reading and writing exams are different.

$P(H_A \text{ is true})$

✔(c) Probability of obtaining a random sample of 200 students where the average difference between the reading and writing scores is at least 0.545 (in either direction), if in fact the true average difference between the scores is 0.

$P(\text{observed or more extreme outcome} \mid H_0 \text{ is true})$

~~(d)~~ Probability of incorrectly rejecting the null hypothesis if in fact the null hypothesis is true. $P(\text{reject} \mid H_0 \text{ is true}) = P(\text{Type 1 error})$

summary

- paired data (2 vars.) → differences (1 var.)
- most often $H_0 : \mu_{diff} = 0$
- same individuals: pre-post studies, repeated measures, etc.
- different (but dependent) individuals: twins, partners, etc.