Effect of Diversification

Systemic Risk Group at FIAS

Rajbir Singh Nirwan, September 28th, 2020

What is Diversification?

Diversification is a risk management strategy that mixes a wide variety of investments within a portfolio.

https://www.investopedia.com/

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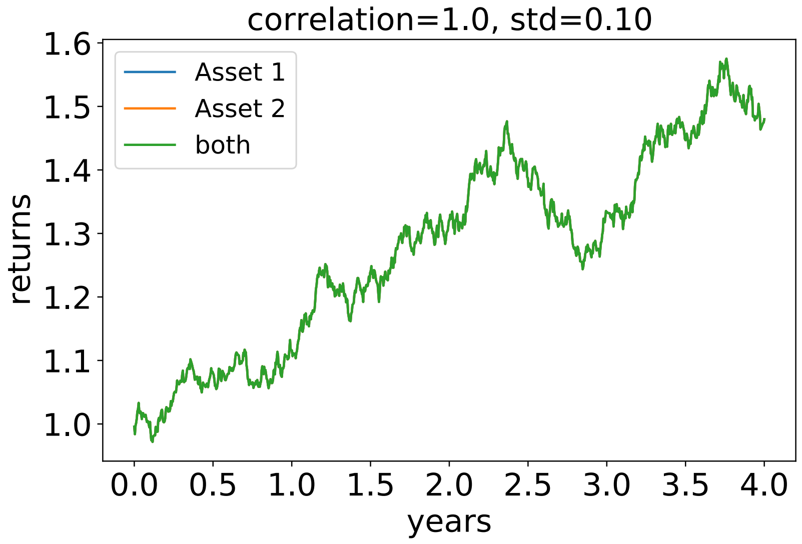

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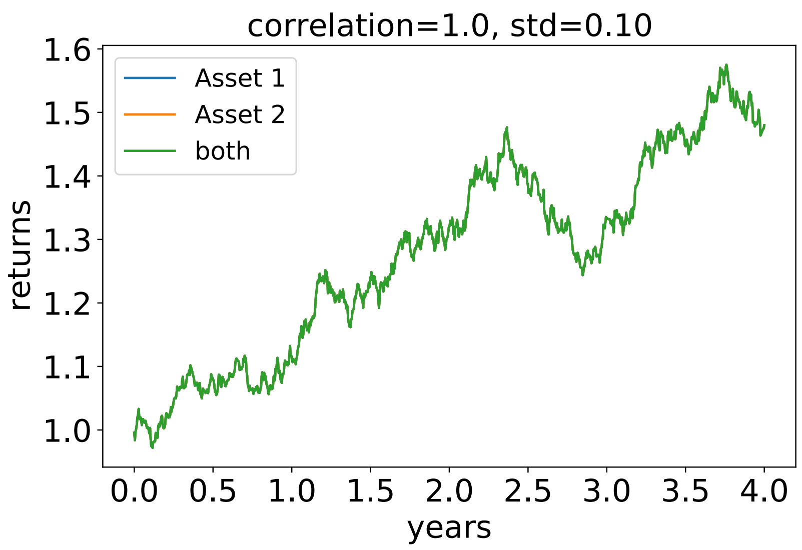

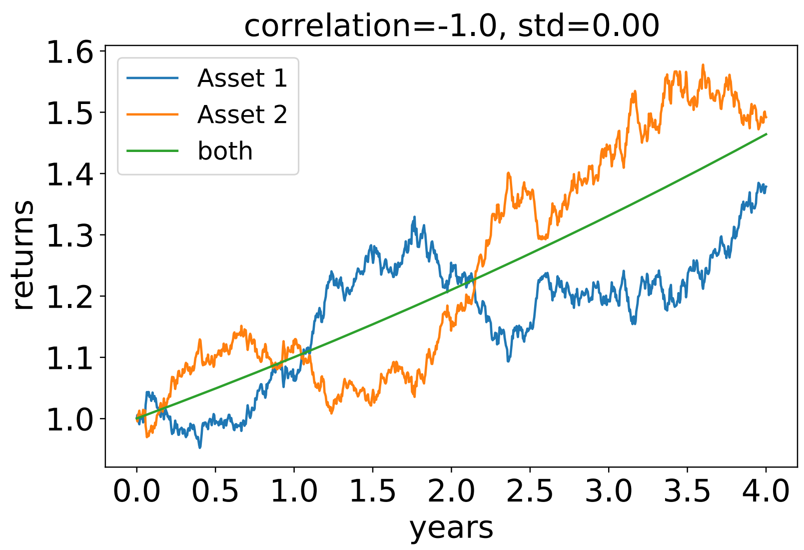

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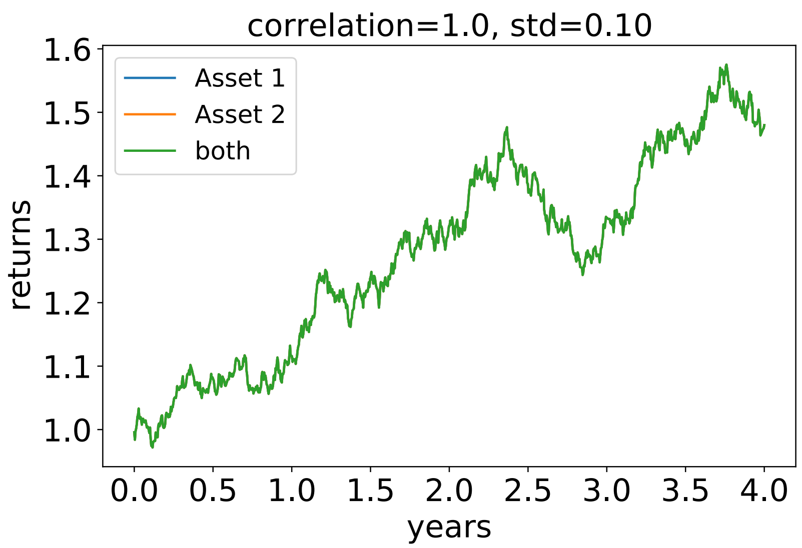

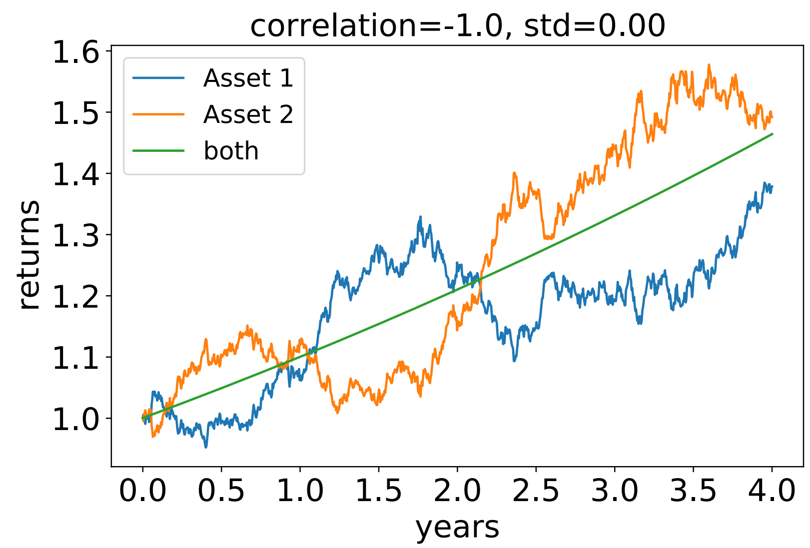

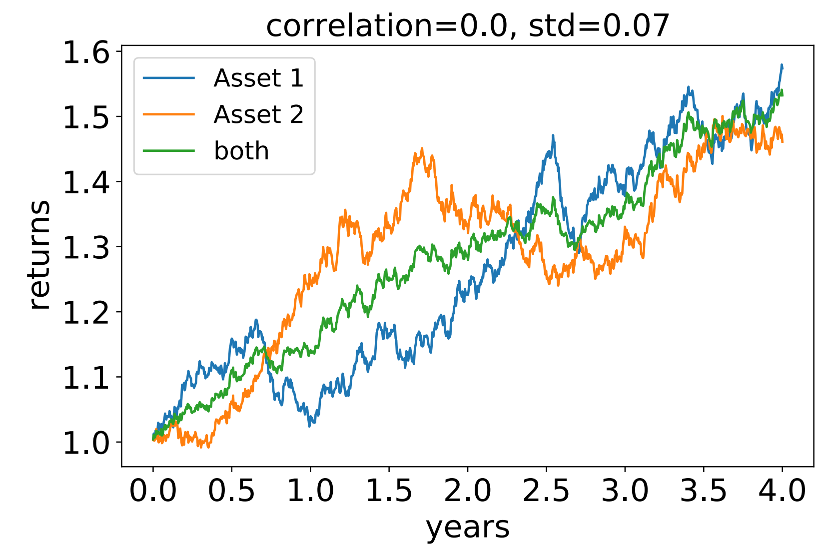

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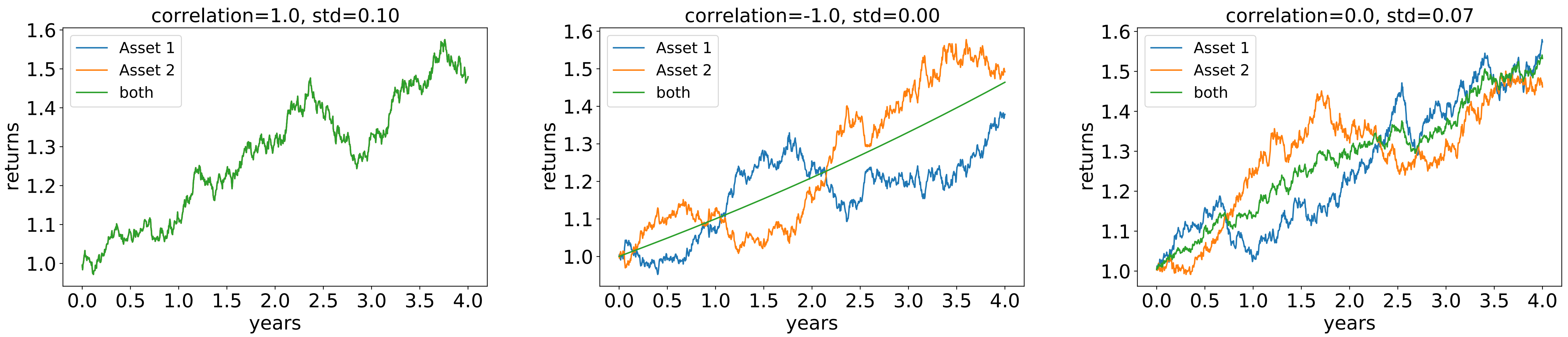
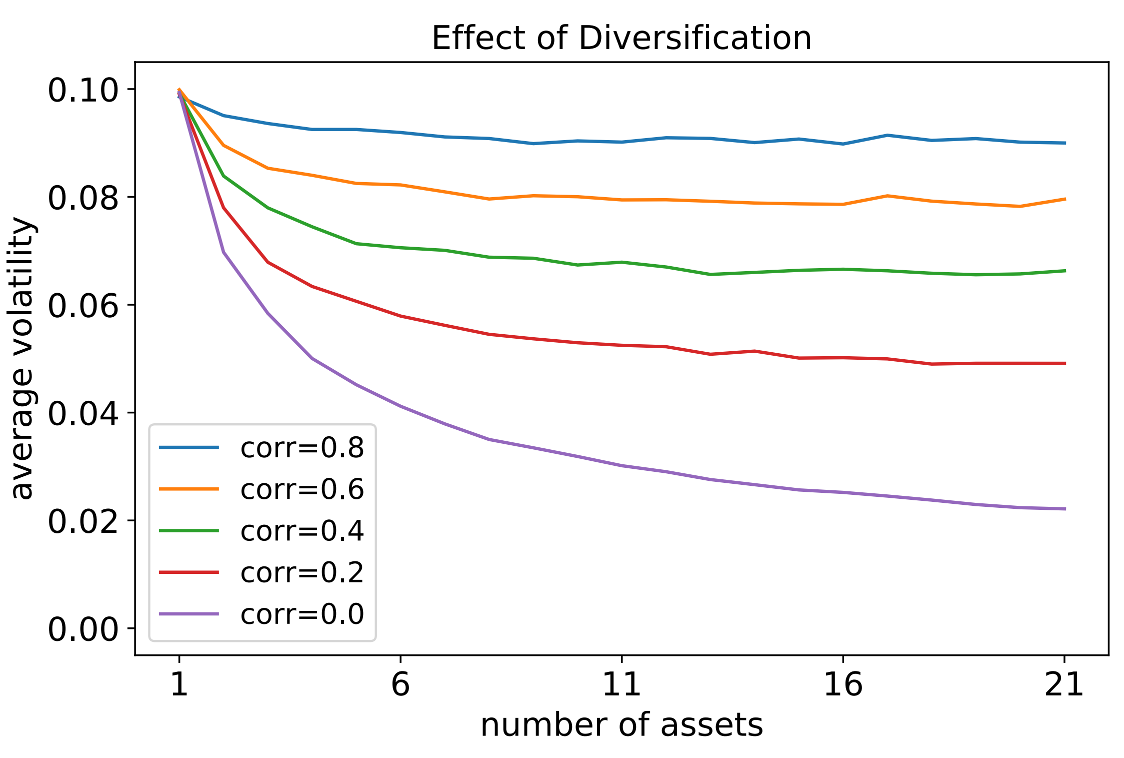


The more decorrelated assets we have in our portfolio, the lower the risk.



Correlation of different asset classes

Historical Correlation[1]: January 2010 - December 2019

Click Asset Class to Highlight RESET

	Positive	Negative
High	0.7-1.0	(0.7)-(1.0)
Moderate	0.4-0.7	(0.4)-(0.7)
Low	0.0-0.4	(0.0)-(0.4)

	Investment Grade Bonds	Cash	Commodities	Currencies	Equity Market Neutral	Event Driven	Global	Hedge Funds	International Equity	Long/Short Equity	Managed Futures	REITs	S&P 500®
Investment Grade Bonds	1.00												
Cash	0.11	1.00											
Commodities	(0.25)	0.07	1.00										
Currencies	(0.00)	(0.08)	(0.54)	1.00									
Equity Market Neutral	(0.03)	(0.04)	0.37	(0.64)	1.00								
Event Driven	(0.22)	(0.03)	0.57	(0.39)	0.41	1.00							
Global	(0.17)	0.01	0.61	(0.58)	0.47	0.80	1.00						
Hedge Funds	(0.02)	(0.03)	0.51	(0.42)	0.51	0.88	0.83	1.00					
International Equity	(0.11)	(0.00)	0.58	(0.66)	0.53	0.77	0.96	0.81	1.00				
Long/Short Equity	(0.18)	(0.03)	0.52	(0.49)	0.56	0.84	0.90	0.91	0.86	1.00			
Managed Futures	0.42	0.02	(0.07)	0.02	0.11	0.11	0.16	0.47	0.13	0.23	1.00		
REITs	0.30	0.02	0.25	(0.31)	0.28	0.46	0.65	0.54	0.58	0.56	0.29	1.00	
S&P 500®	(0.22)	(0.00)	0.57	(0.46)	0.40	0.77	0.97	0.79	0.85	0.87	0.16	0.65	1.00

https://www.guggenheiminvestments.com/

Estimation of correlation using Gaussian Processes

Any finite collection of function values at $x_1, x_2, \ldots, x_N$ is jointly Gaussian distributed

$$p\left(f\left(x_1\right), f\left(x_2\right), \ldots, f\left(x_N\right)\right) = \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{K}) = \mathcal{N}\left(\begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1N} \\ k_{21} & k_{22} & \cdots & k_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ k_{N1} & k_{N2} & \cdots & k_{NN} \end{pmatrix}\right) \qquad k_{ij} = k(x_i, x_j)$$

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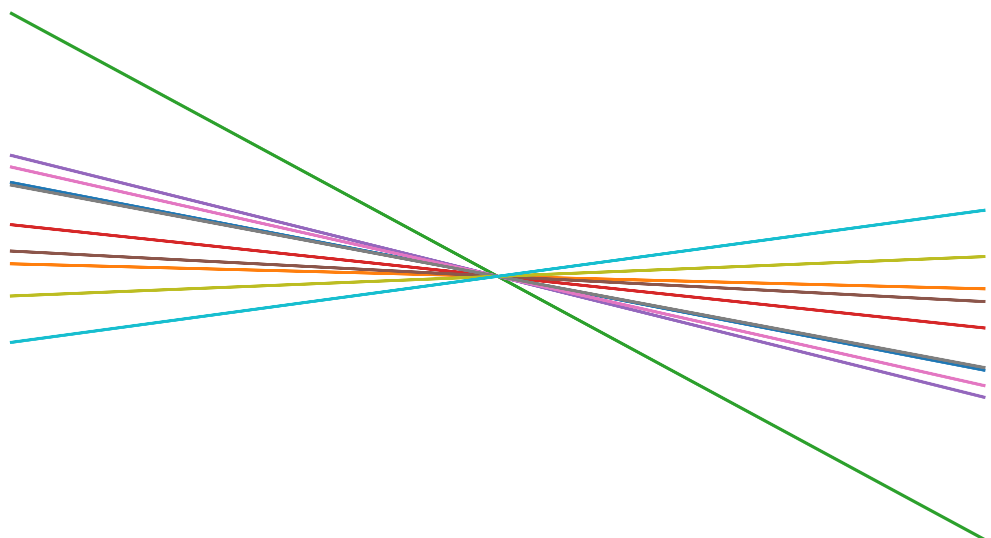
Common Kernel Functions

$$k_{linear}(x, x') = xx'$$



$$k_{rbf}(x, x') = \exp\left(-\frac{1}{2\ell^2}(x - x')^2\right)$$

$$k_{ou}(x, x') = \exp\left(-\frac{1}{\ell}|x - x'|\right)$$

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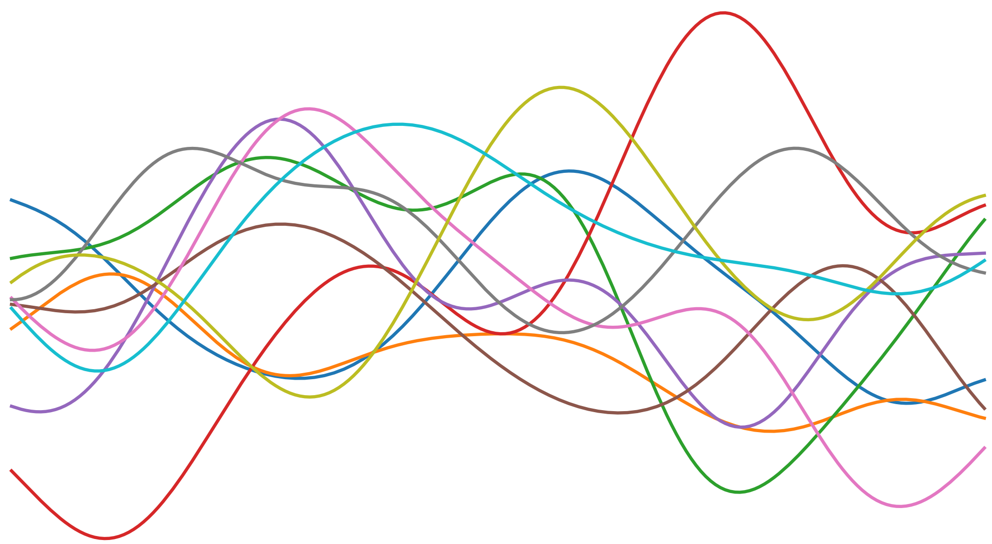

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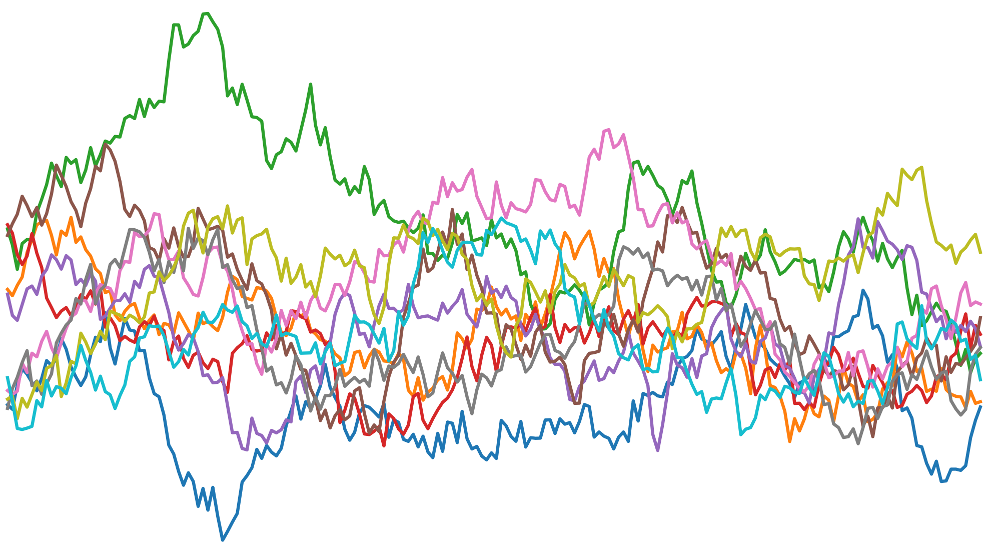

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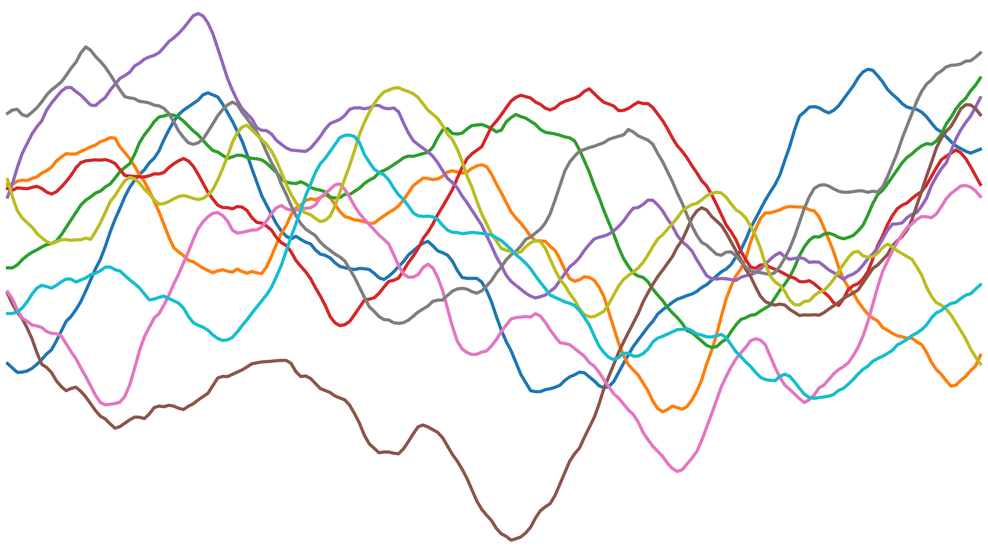

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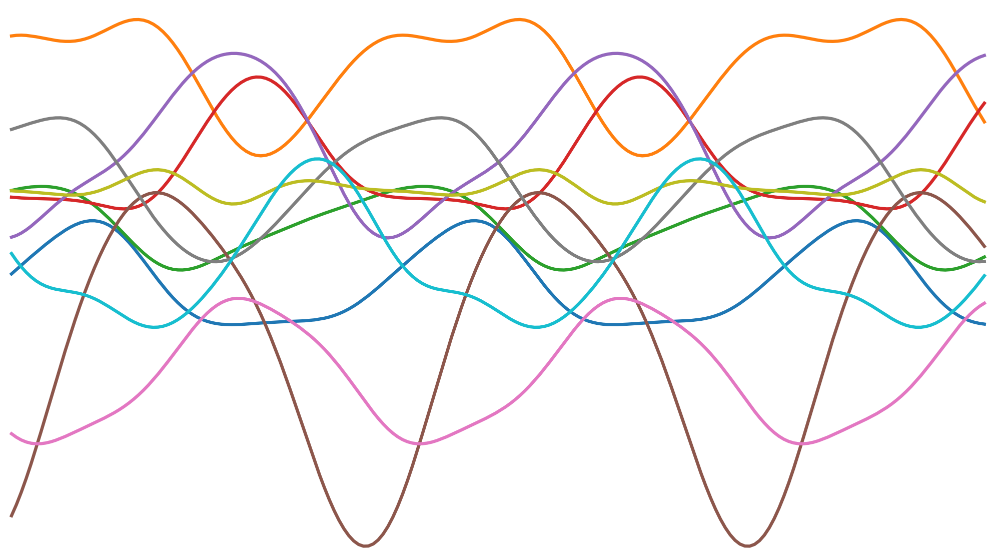

Portfolio Allocation

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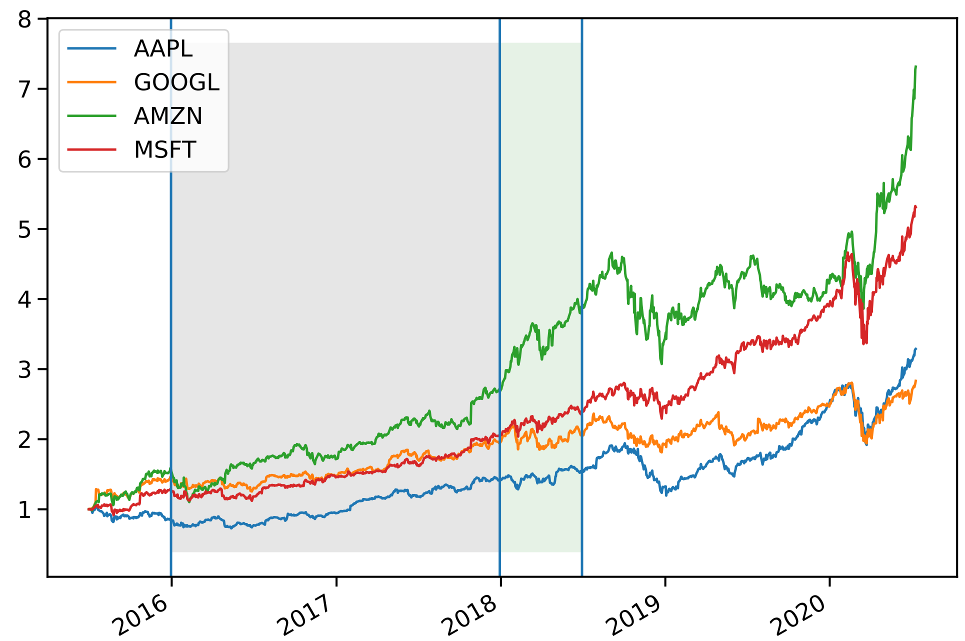
Learn weights on previous 2 years
Hold portfolio for next 6 months



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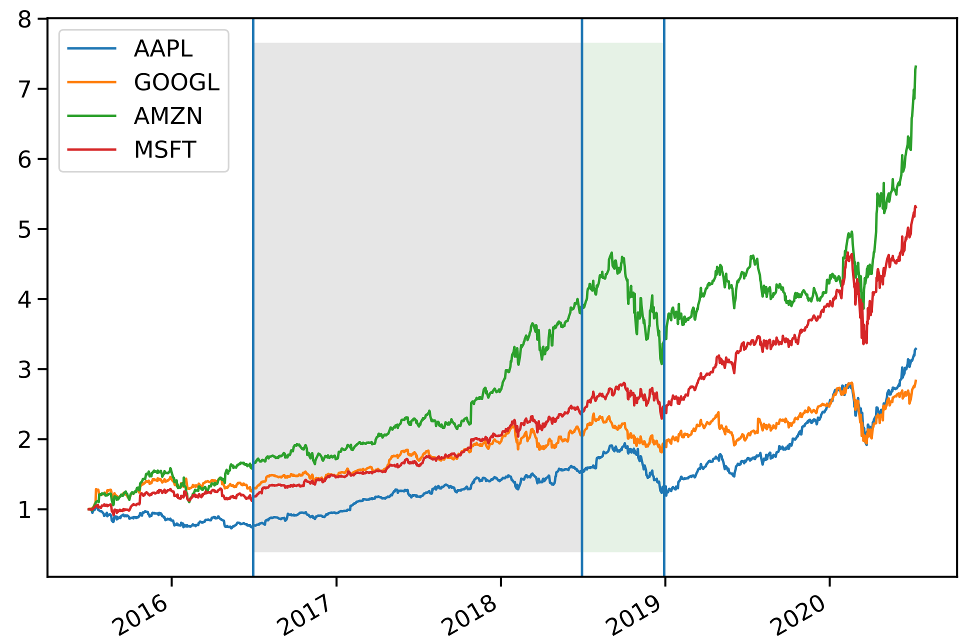

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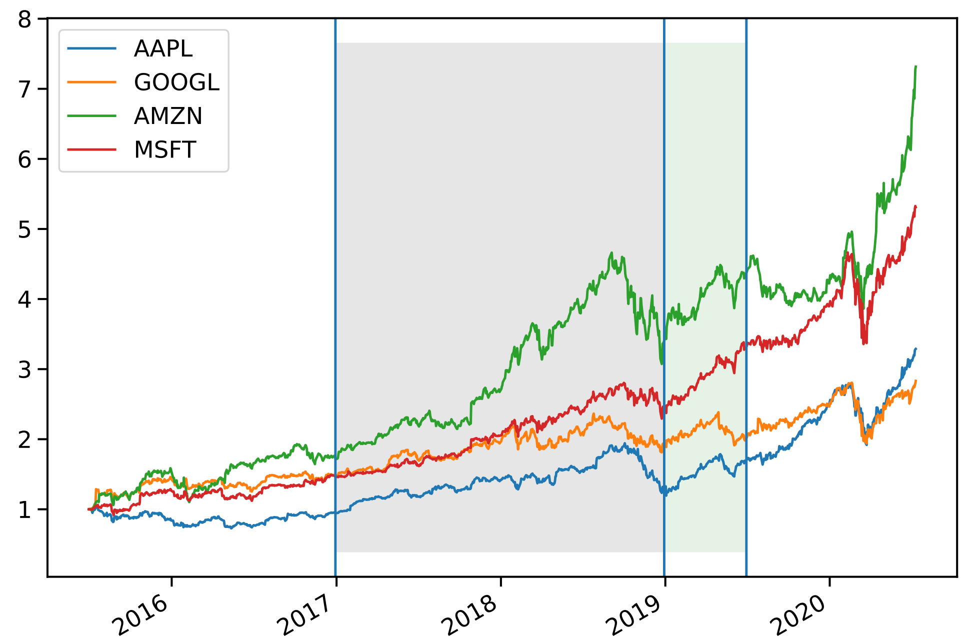

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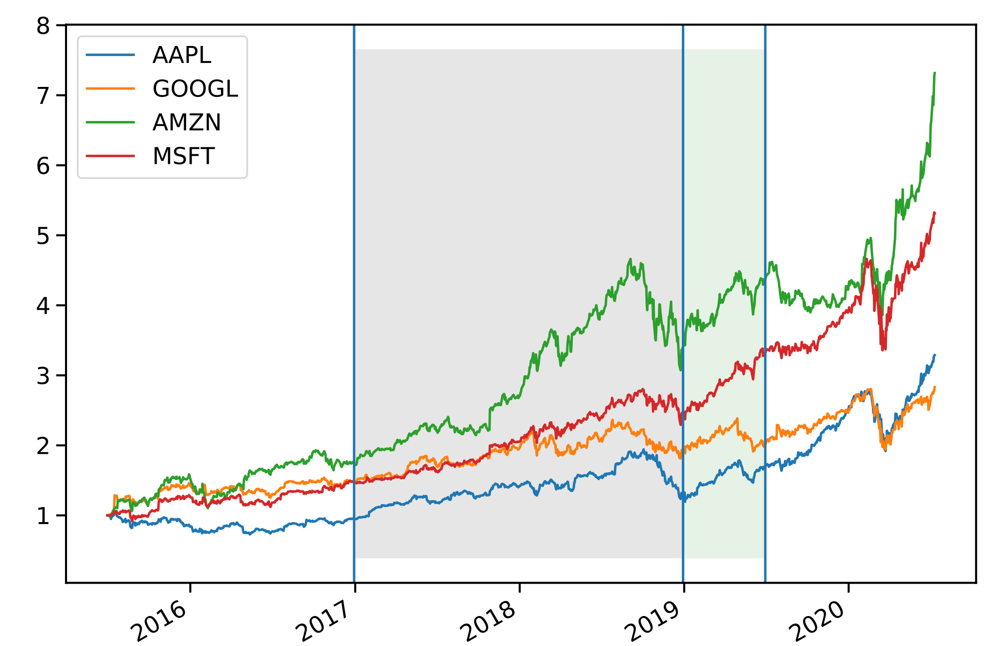

Backtesting on S&P500 from 2002 to 2018

Model	Linear	SE	EXP	M32	Sample Cov	Ledoit Wolf	Eq. Weighted
Mean	0.142	0.151	0.155	0.158	0.149	0.148	0.182
Std	0.158	0.156	0.154	0.153	0.159	0.159	0.232
Sharpe ratio	0.901	0.969	1.008	1.029	0.934	0.931	0.786

Bayesian Quantile Matching Estimation

Country	Sample Size	25	50	75
EL	12918	4930	7500	11000
ES	19177	8803	13681	20413
FR	21325	16185	21713	29008
IT	24969	10699	16247	22944
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```python
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from bqme.models import NormalQM

N, q, X = 100, [0.25, 0.5, 0.75], [-0.1, 0.3, 0.8]

# define priors
mu = Normal(0, 1, name='mu')
sigma = Gamma(1, 1, name='sigma')

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model = NormalQM(mu, sigma)

# fit model
fit = model.sampling(N, q, X)
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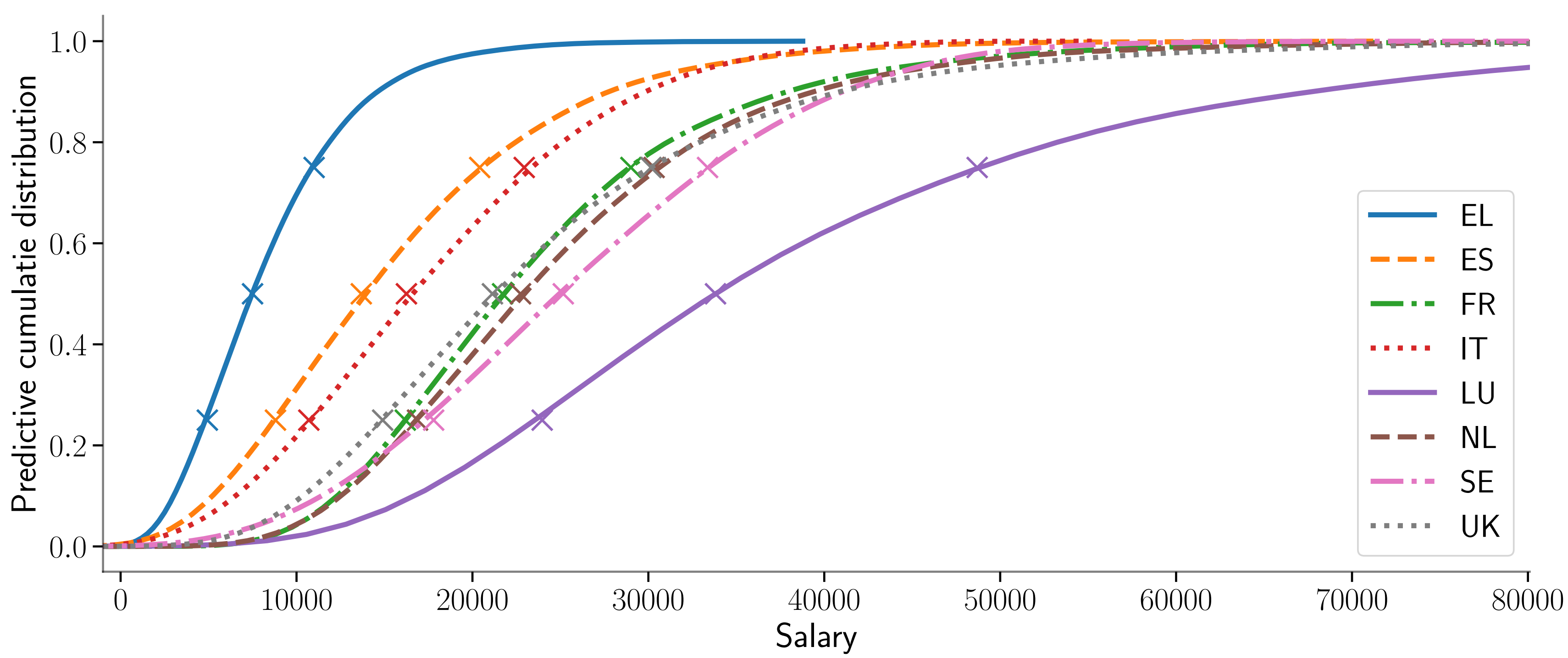
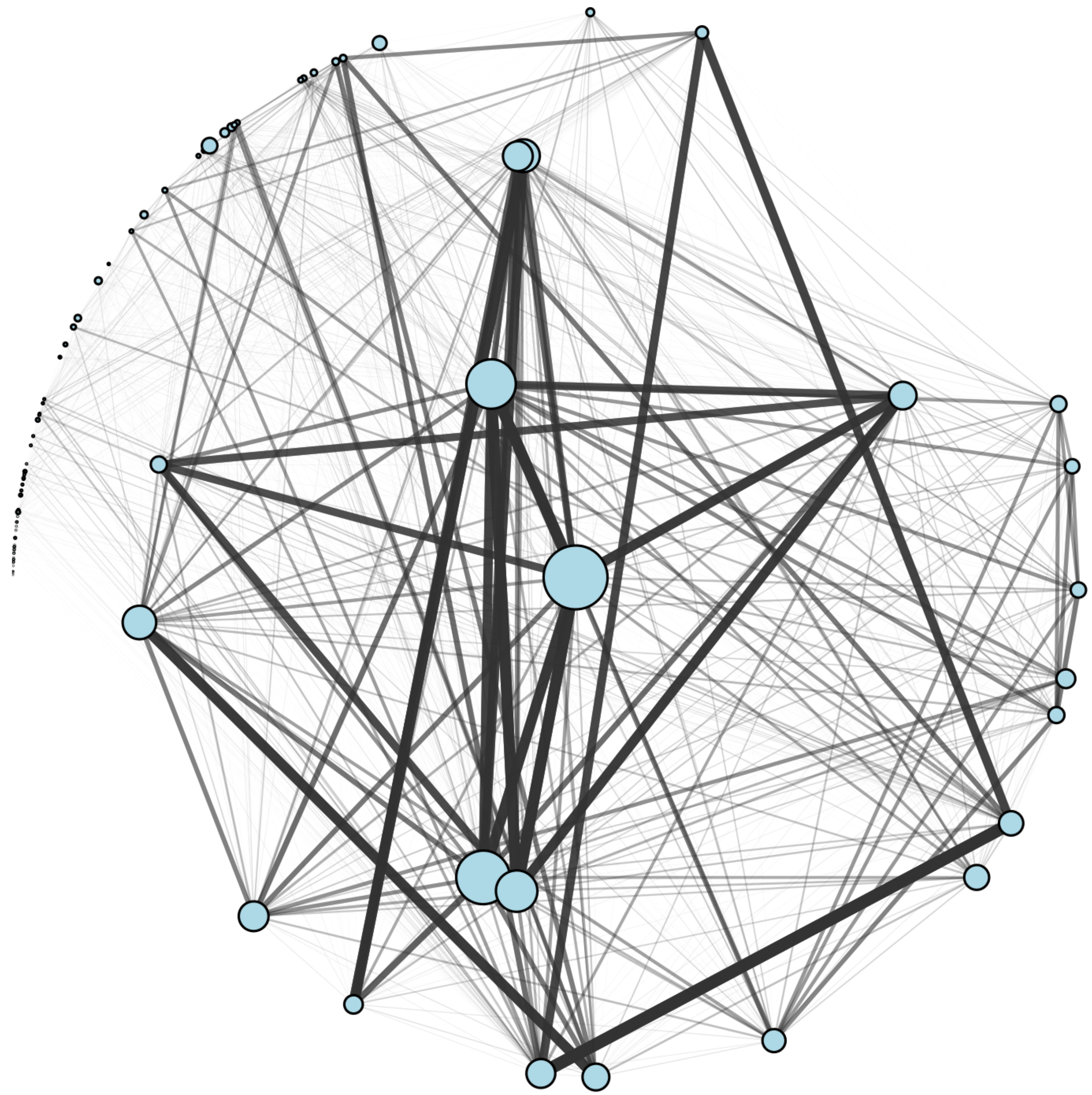


Network Analysis



https://github.com/RSNirwan

Summary

- Use of Gaussian processes in Finance
- Bayesian quantile matching estimation
- Network Analysis