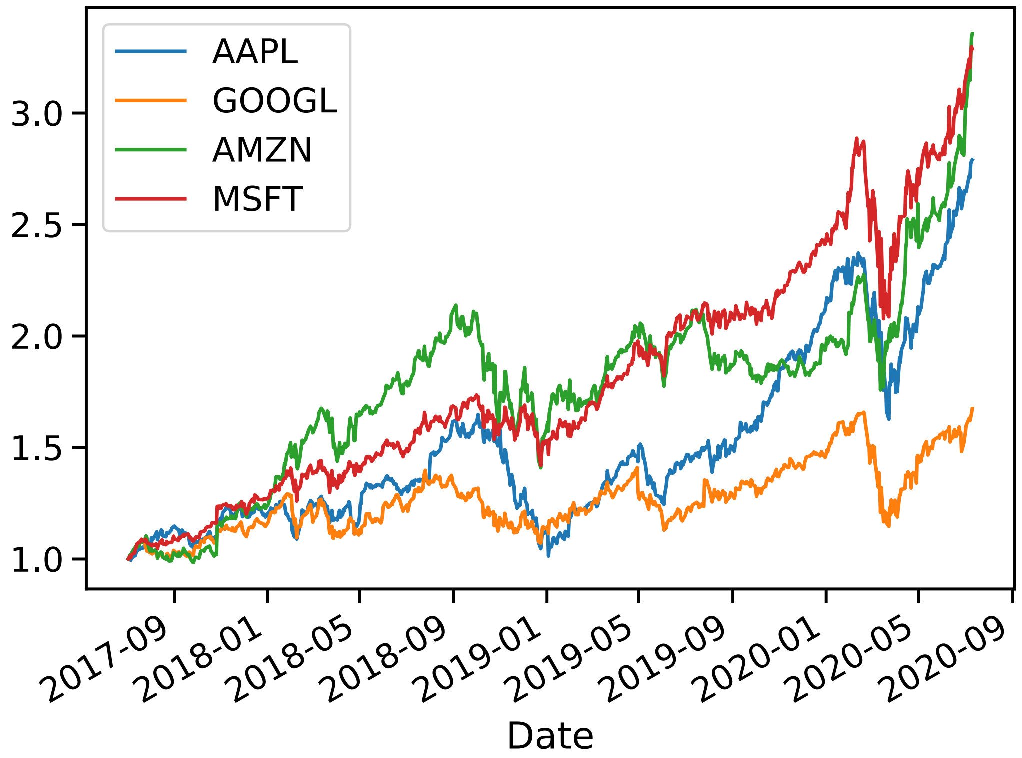
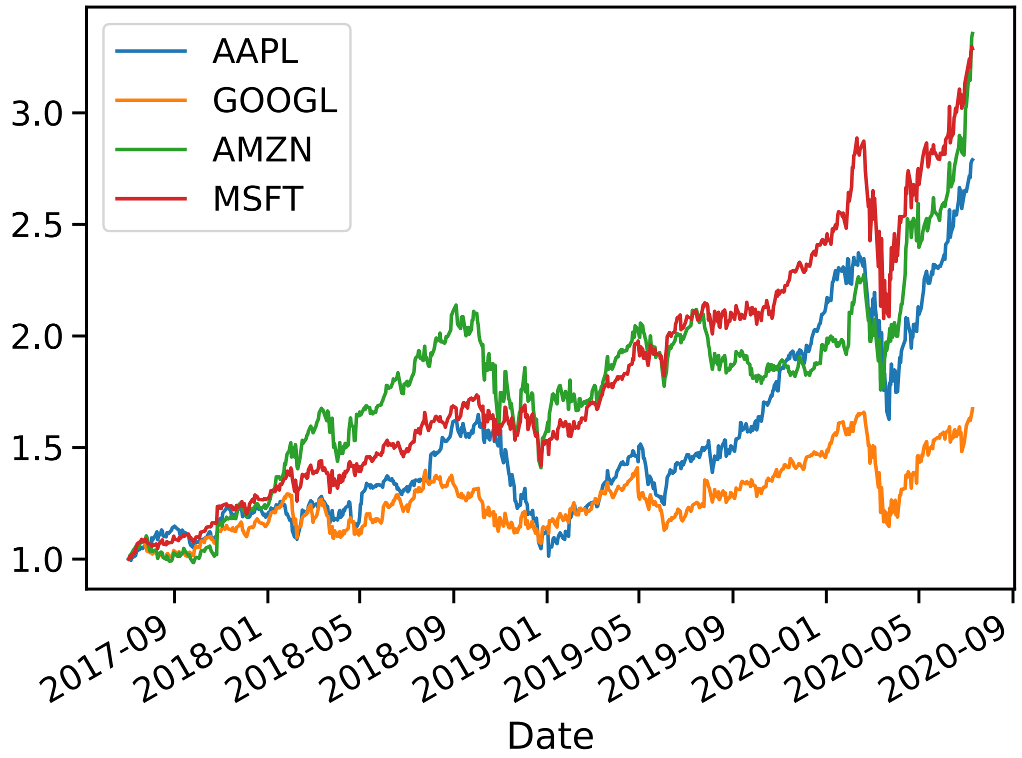
Gaussian Process Latent Variable Models in Finance

Rajbir-Singh Nirwan

July 15, 2020



AAPL
GOOGL
AMZN
MSFT
3.0
2.5
2.0
1.5
1.0
2017-09
2018-01
2018-05
2018-09
2019-01
2019-05
2019-09
2020-01
2020-05
2020-09
Date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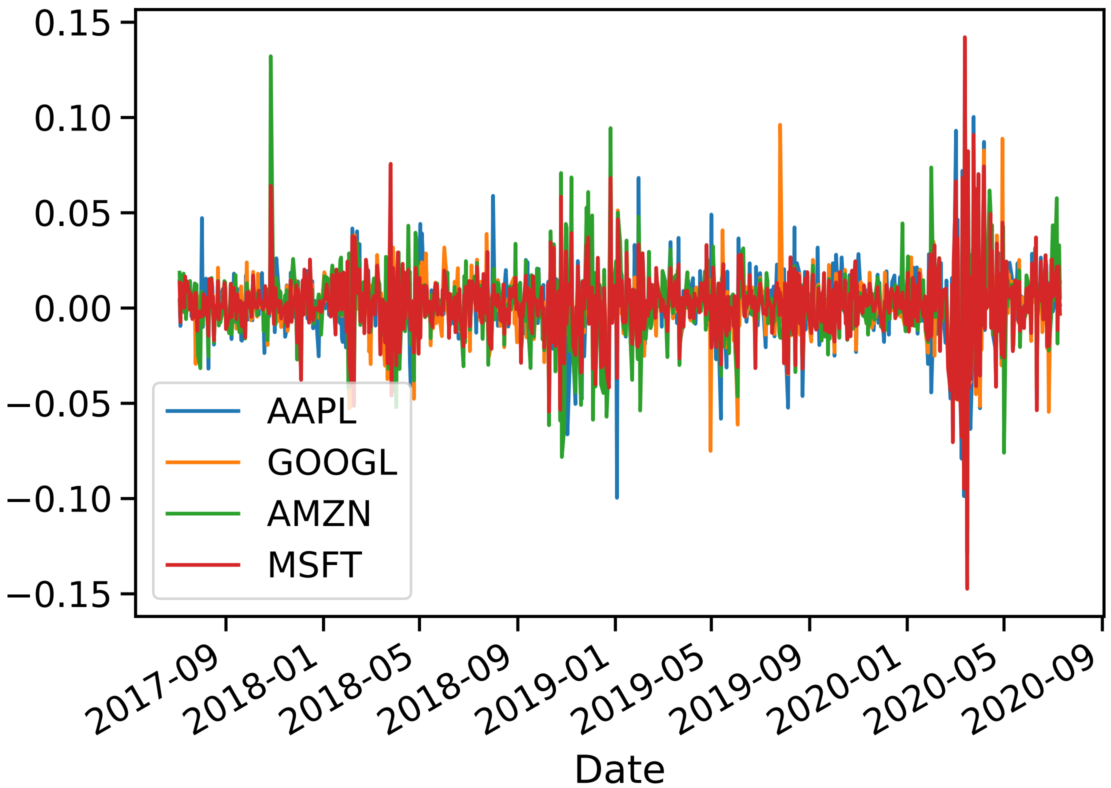
0.15
0.10
0.05
0.00
−0.05
−0.10
−0.15
AAPL
GOOGL
AMZN
MSFT
2017-09
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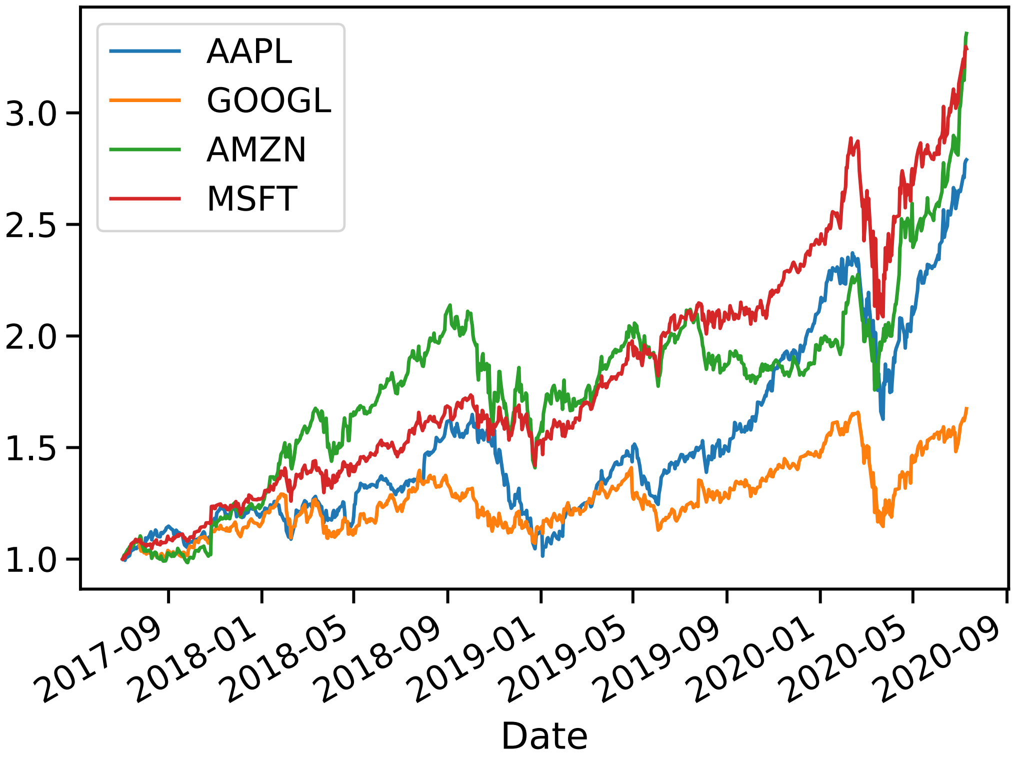
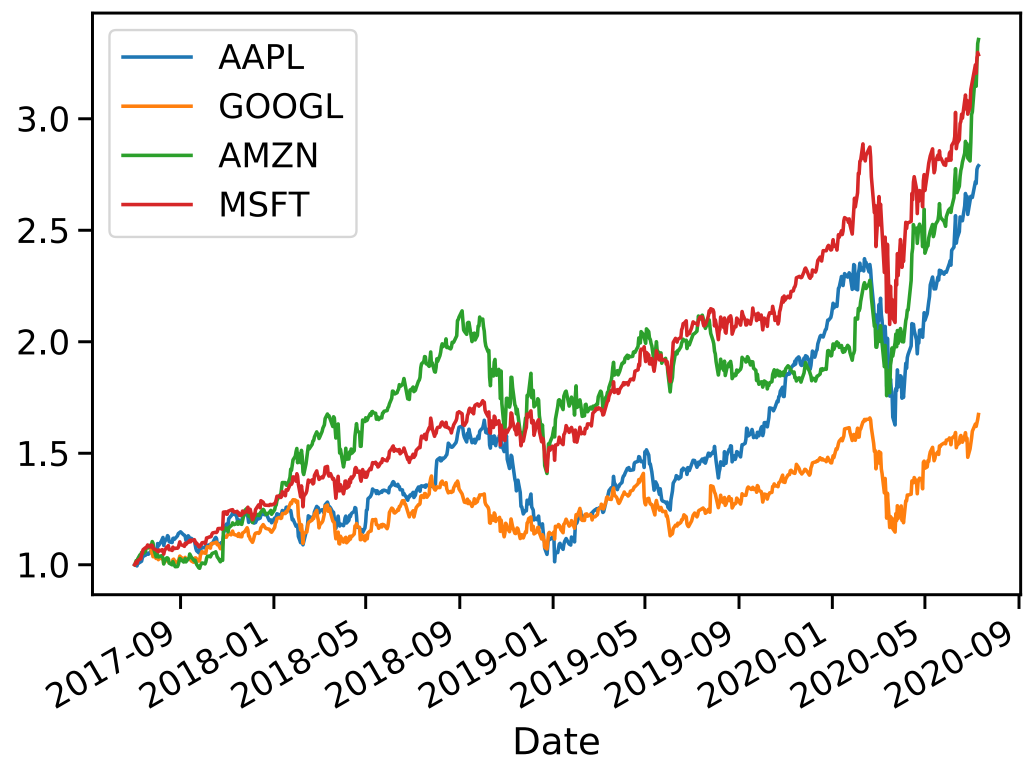


RETURNS	07.07.20	08.07.20	09.07.20	10.07.20
AAPL	-0.31	2.33	0.43	0.17
GOOGL	-0.64	0.92	1.00	1.34
AMZN	-1.86	2.70	3.29	0.55
MSFT	-1.16	2.20	0.70	-0.30
FOOD				
BANK				



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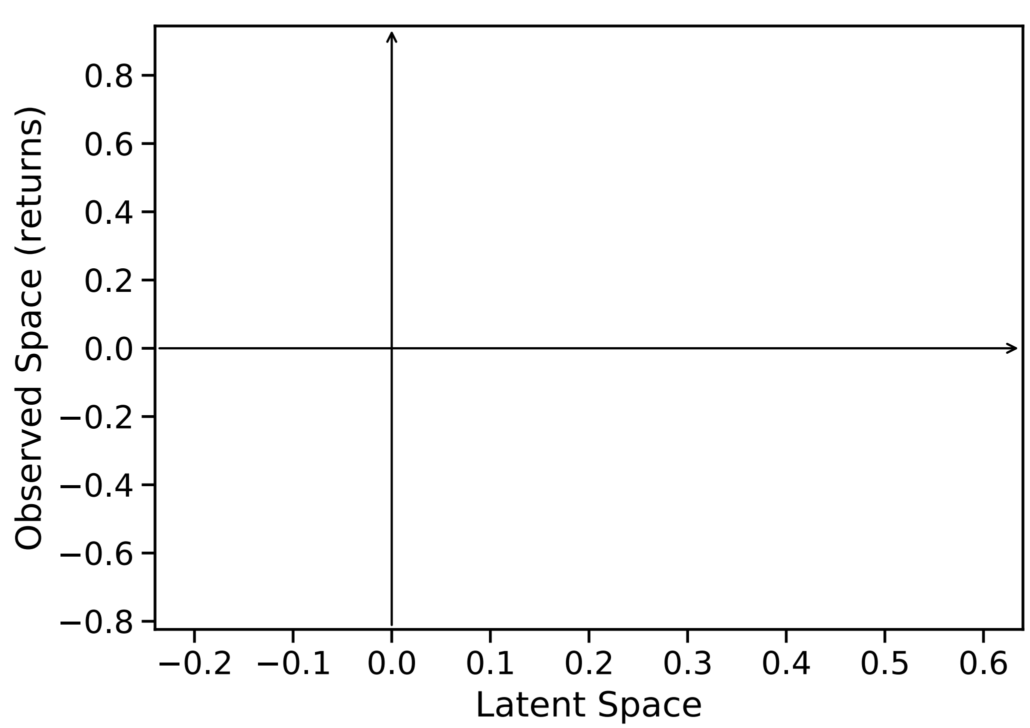
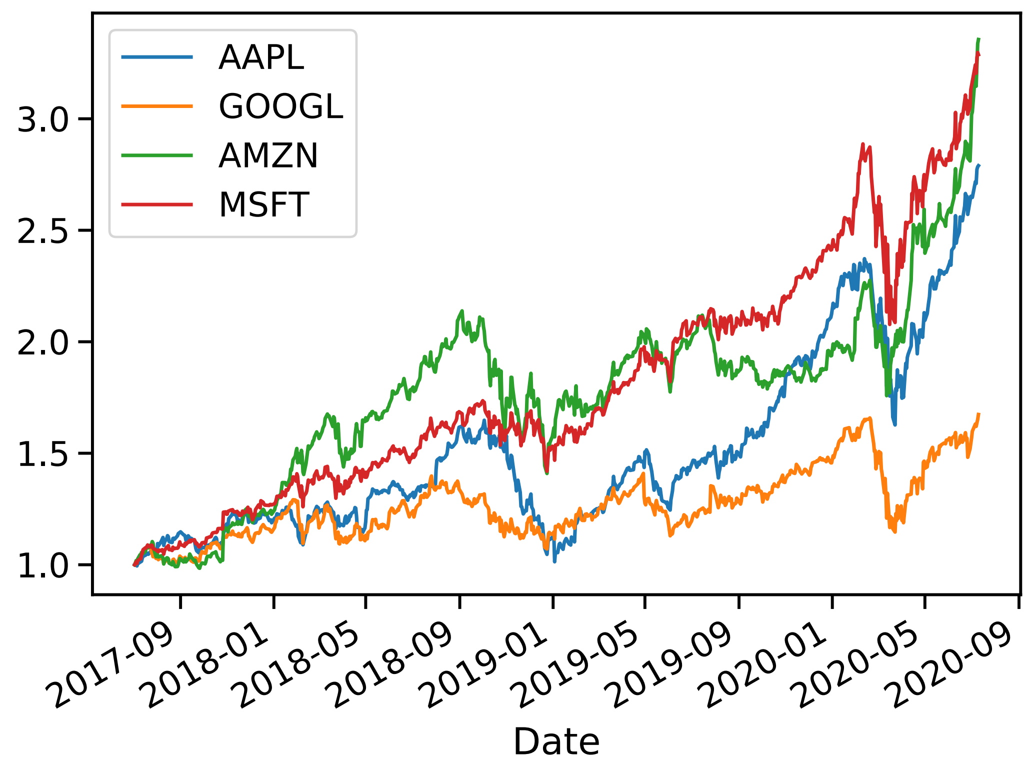
Generative Model





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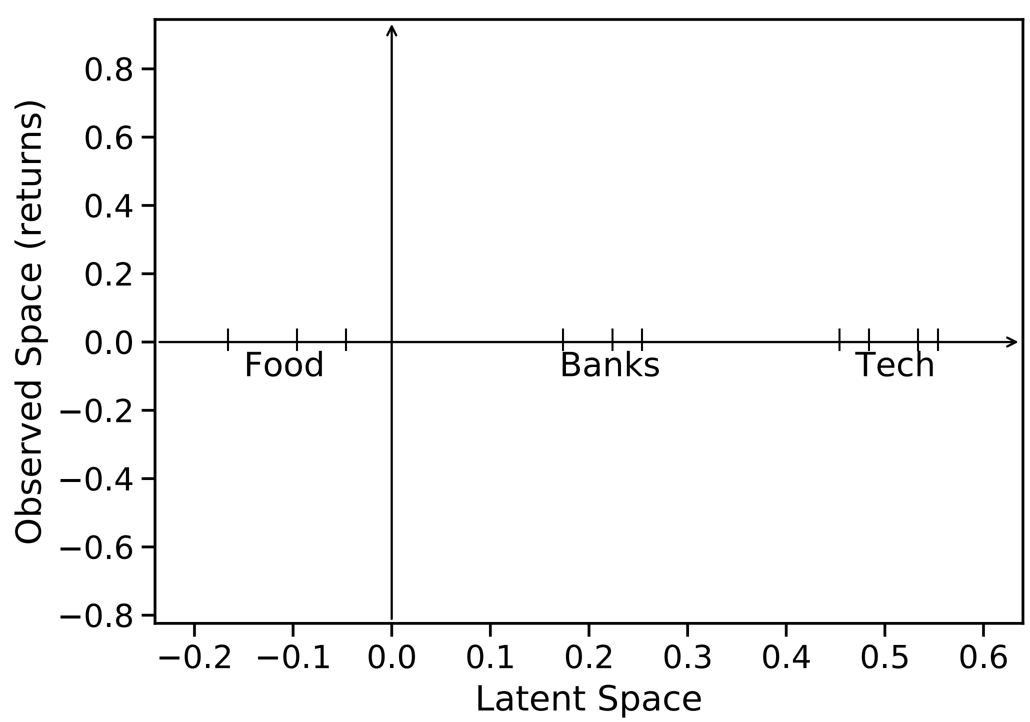

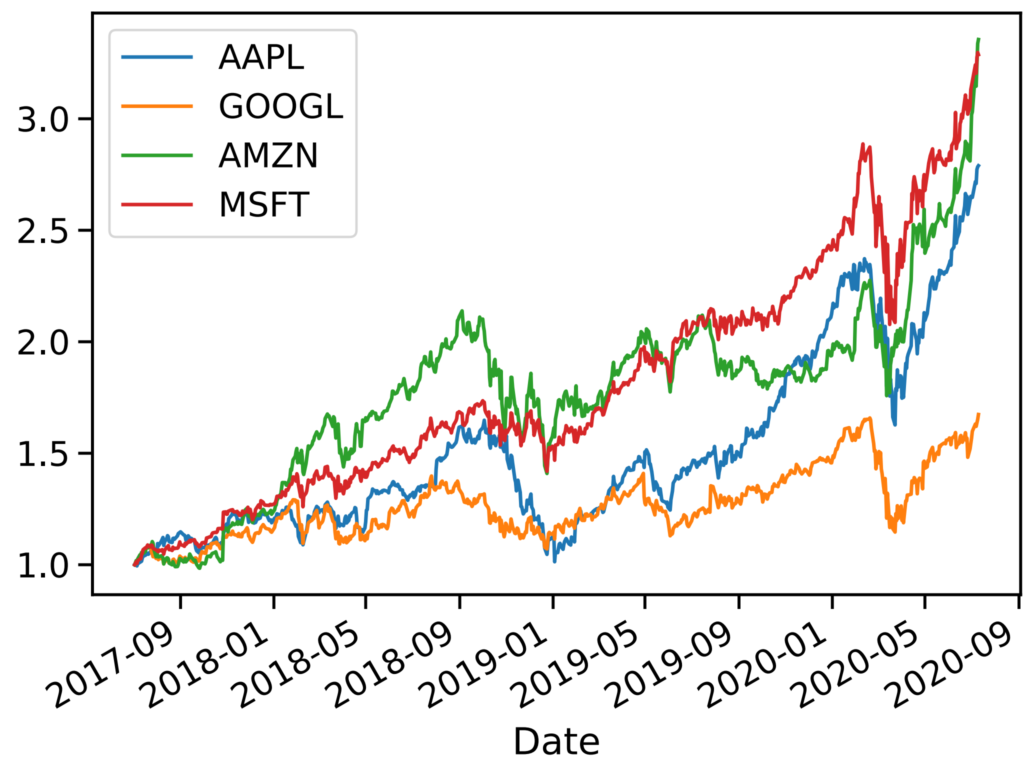

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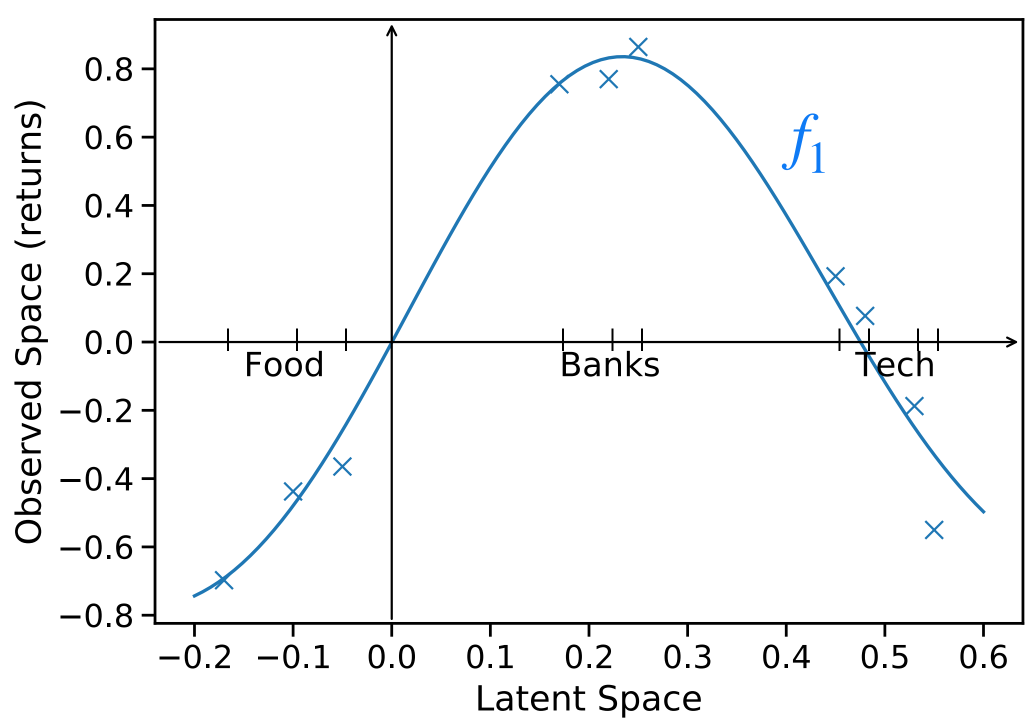

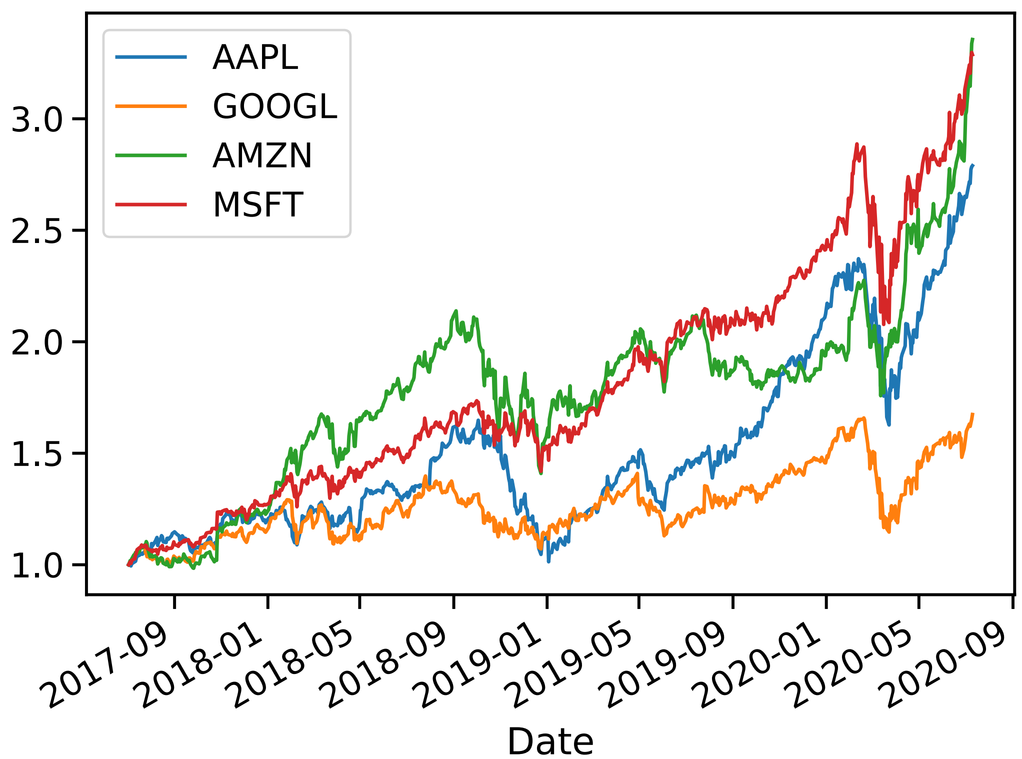

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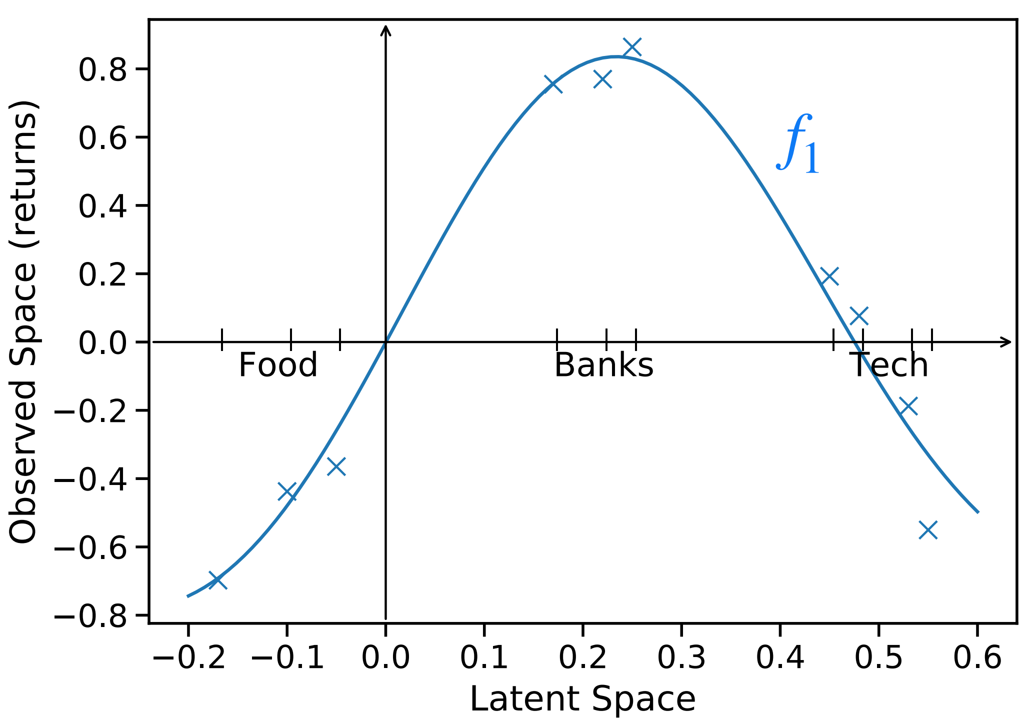
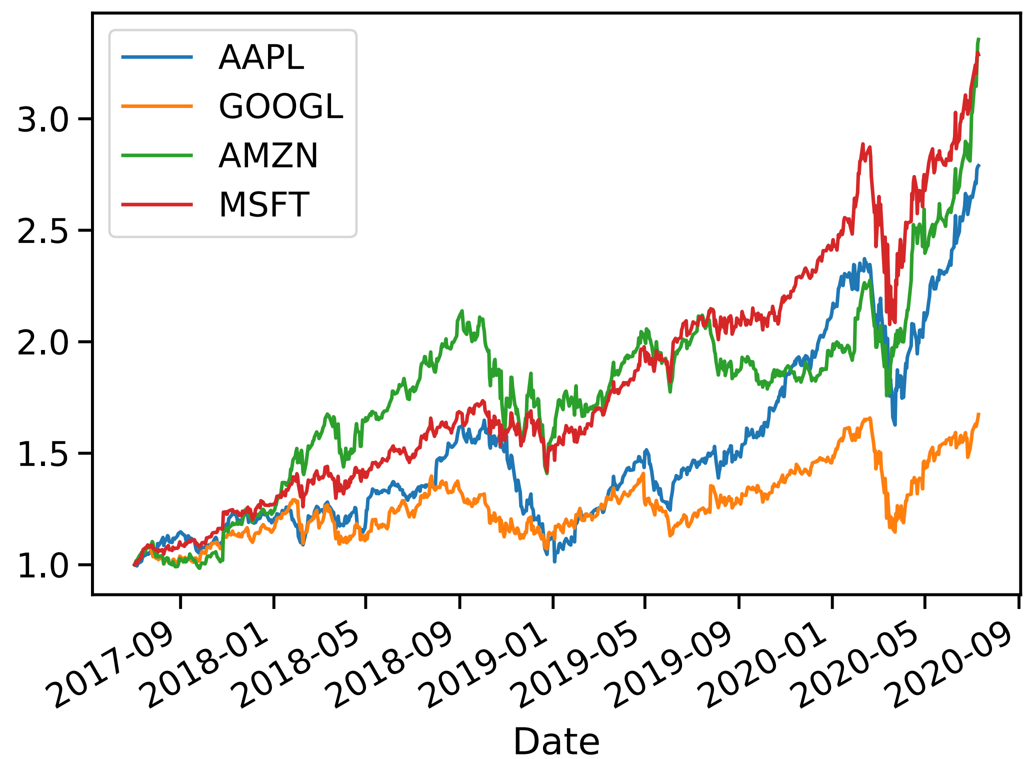


	Day1	Day2
Bank1	−0.70	
Bank2	−0.44	
Bank3	−0.36	
Food1	0.75	
Food2	0.77	
Food3	0.86	
Tech1	0.19	
Tech2	0.08	
Tech3	−0.19	
Tech4	−0.55	



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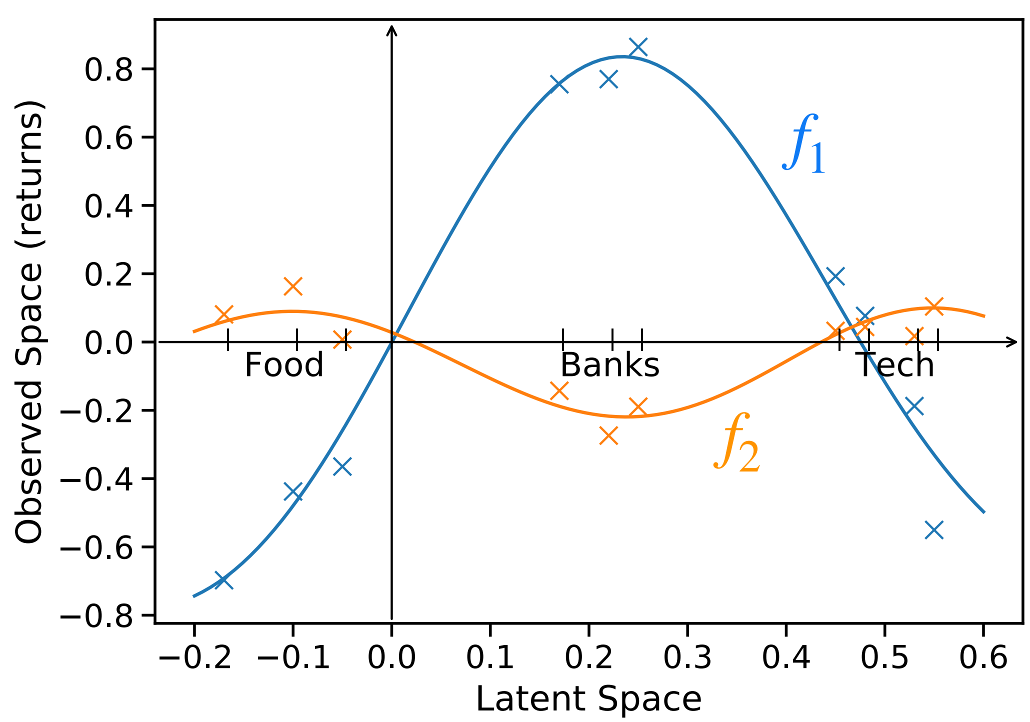

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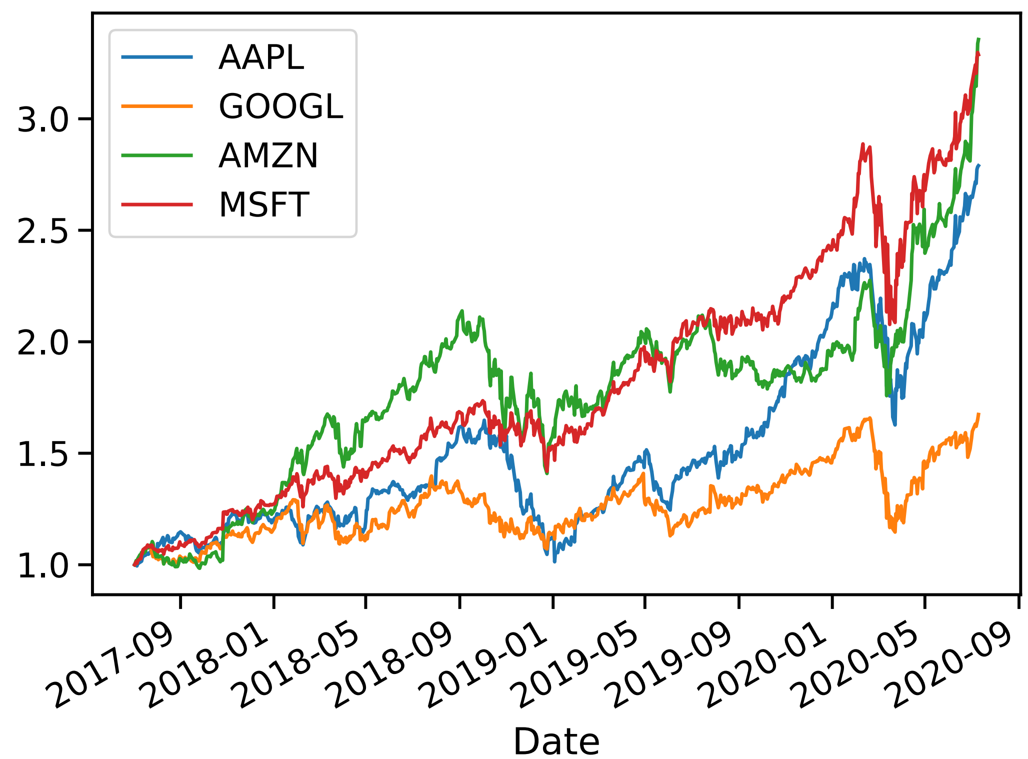

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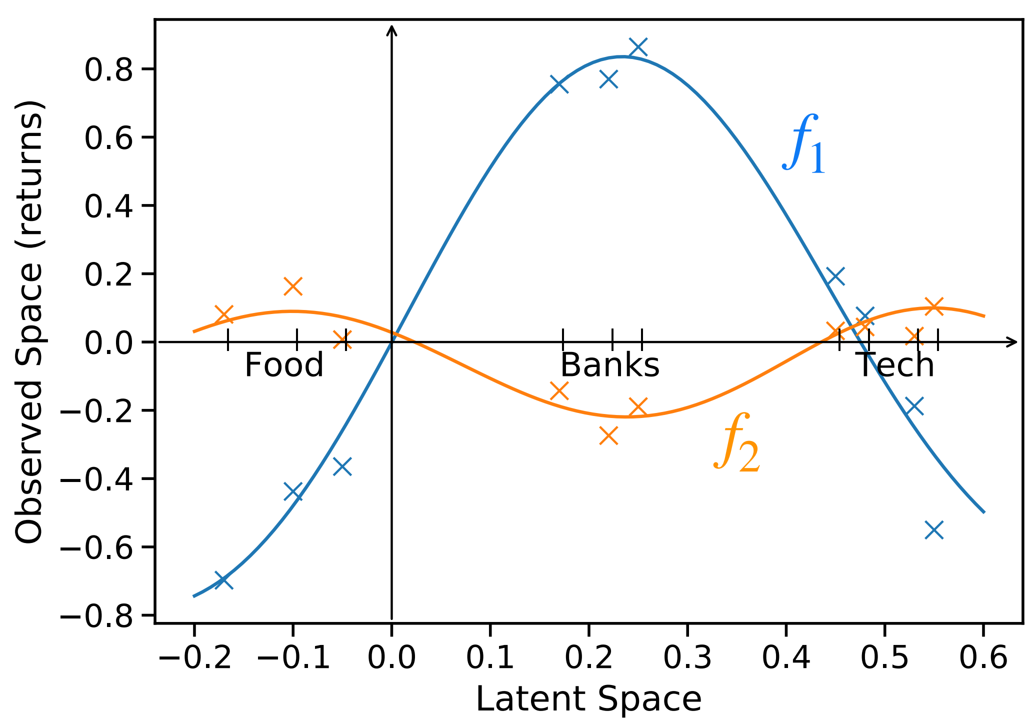

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Outline

- Gaussian Processes
- Latent Variable Models
- Applications
    - Portfolio Allocation
    - Predicting missing Values
    - Structure Identification

Gaussian Processes

- Non-Parametric Kernel based approach
- Utilize full power of Bayesian statistics
- Complexity increases with the number of data points

Gaussian Processes

Weight space view

$$\Phi : x \rightarrow (\phi_1(x), \phi_2(x), \ldots, \phi_D(x))$$

$$f(x) = \boldsymbol{w}^T \Phi(x)$$

Simple and easy to interpret
but limited flexibility

Gaussian Processes

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$$k : x, x' \to k(x, x')$$

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$$k(x, x') = \sum_d \lambda_d \phi_d(x) \phi_d(x')$$

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$$k(x, x') = (xx' + c)^d$$
$$\Phi(x) = \textit{polynomials up to order } d$$

$$k(x, x') = \exp(-0.5\ (x - x')^2 / \ell^2)$$
$$\Phi(x) = \textit{infinitly many basis functions}$$

Gaussian Processes

Any finite collection of function values at $x_1, x_2, \ldots, x_N$ is jointly Gaussian distributed

$$p\left(f\left(x_1\right), f\left(x_2\right), \ldots, f\left(x_N\right)\right) = \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{K})$$

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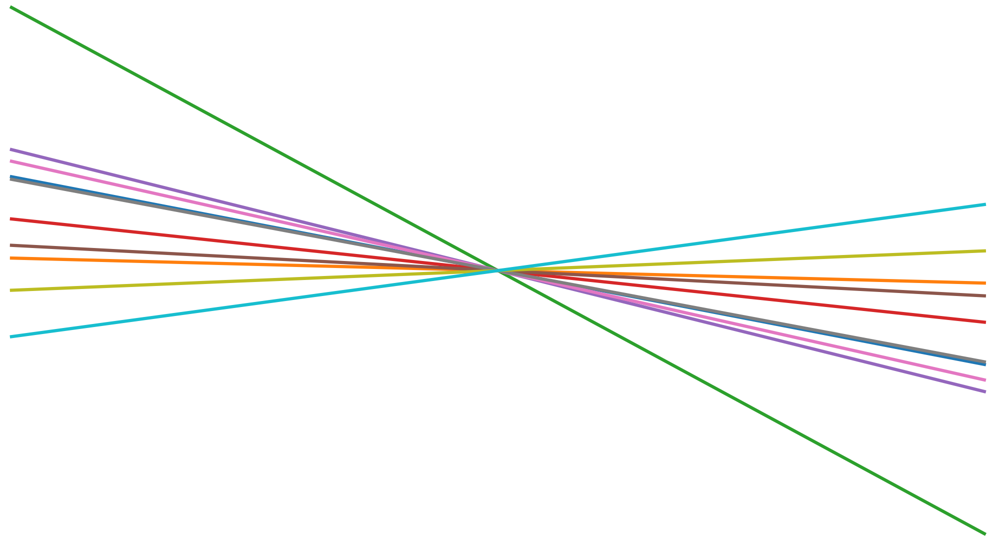
Common Kernel Functions

$$k_{linear}(x, x') = xx'$$



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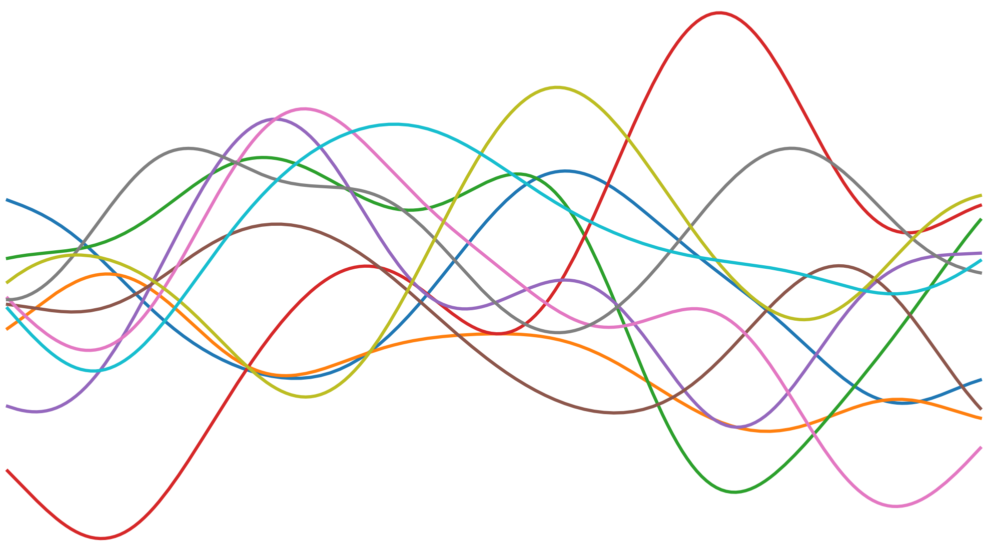

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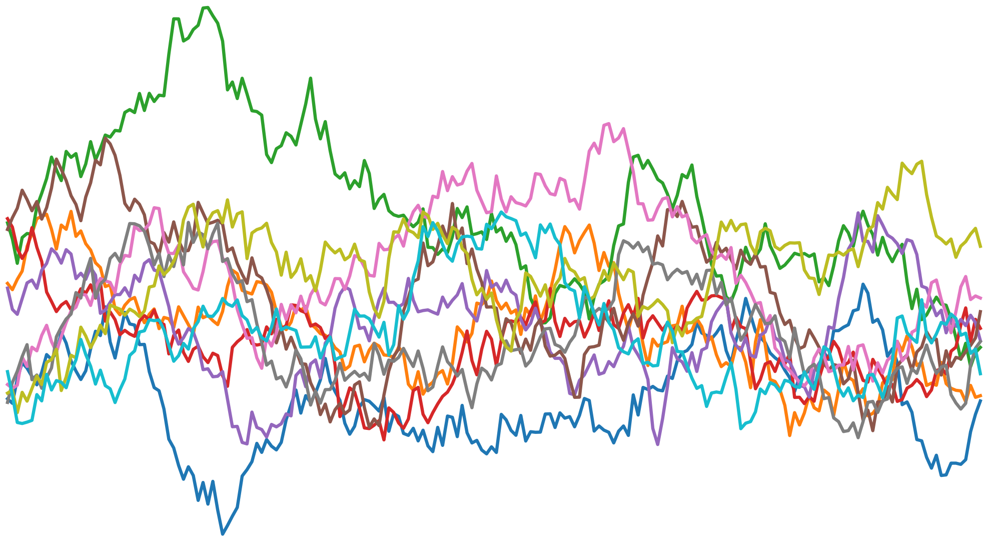

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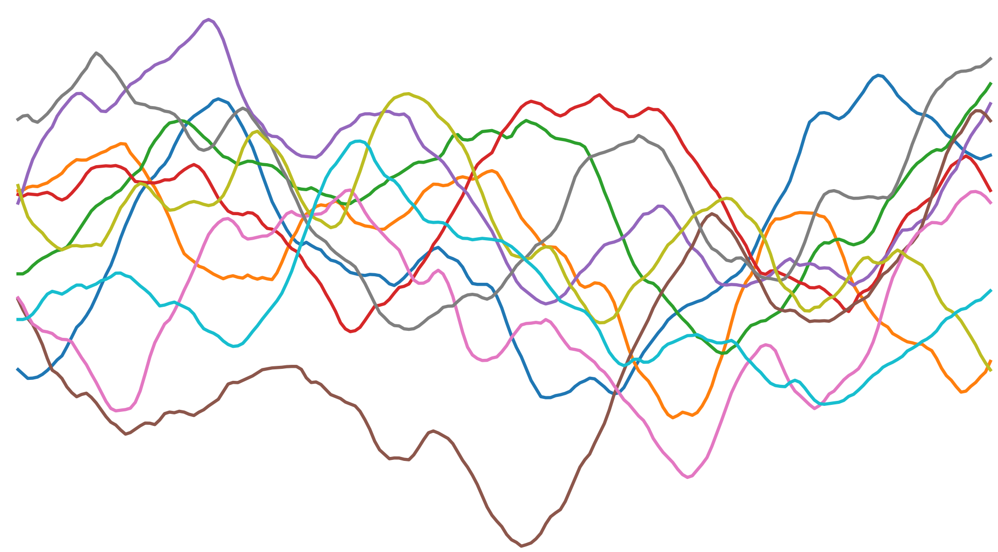

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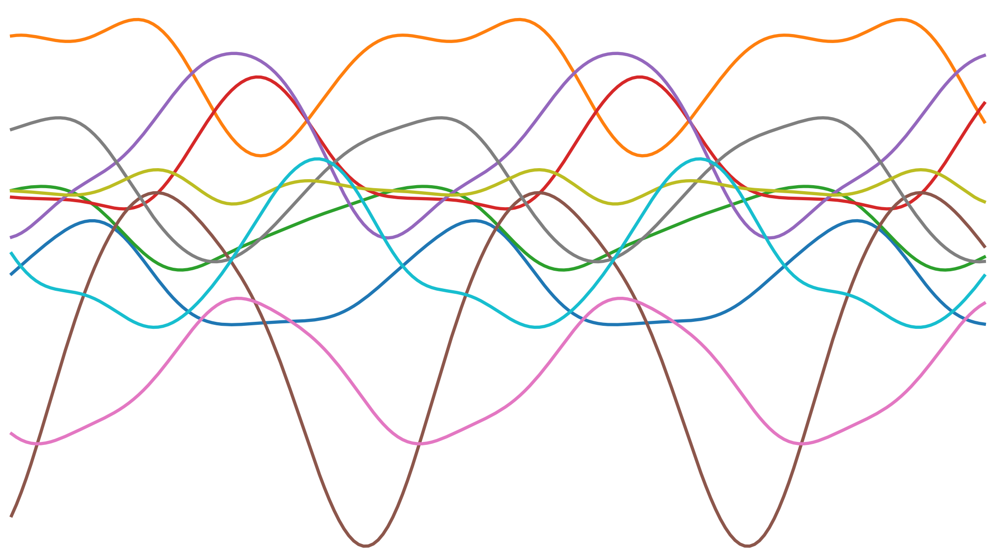

Gaussian Processes

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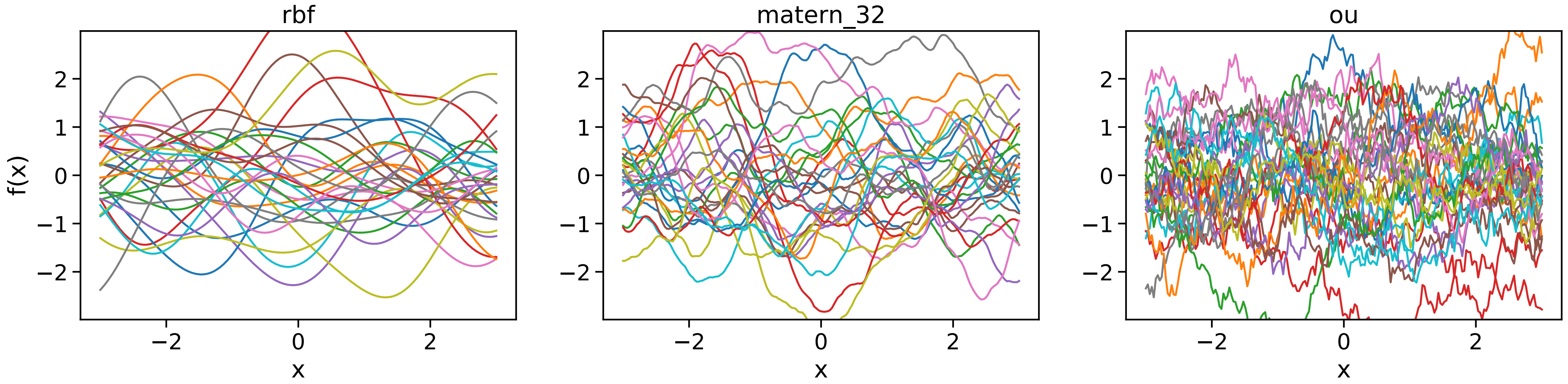

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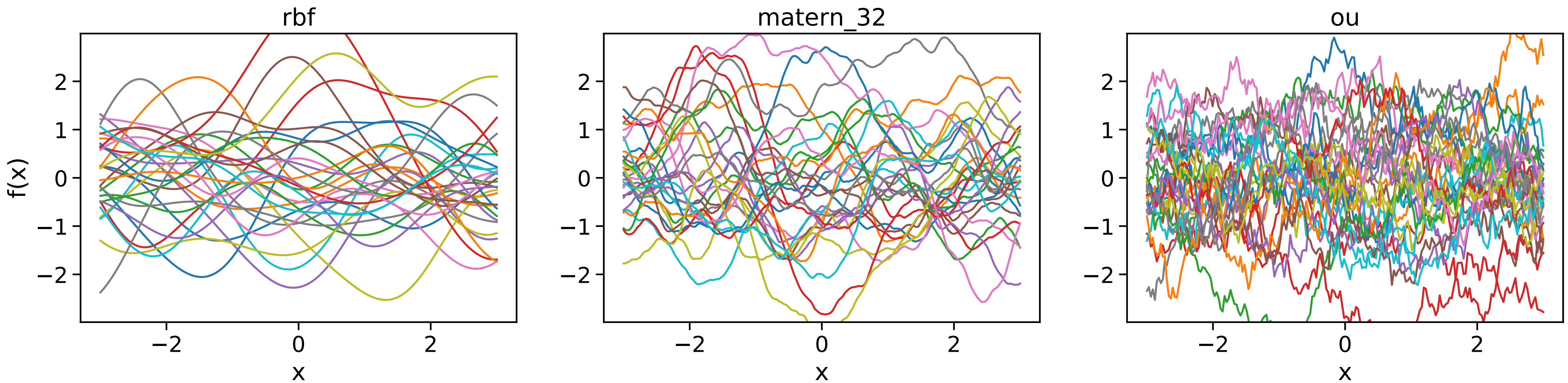
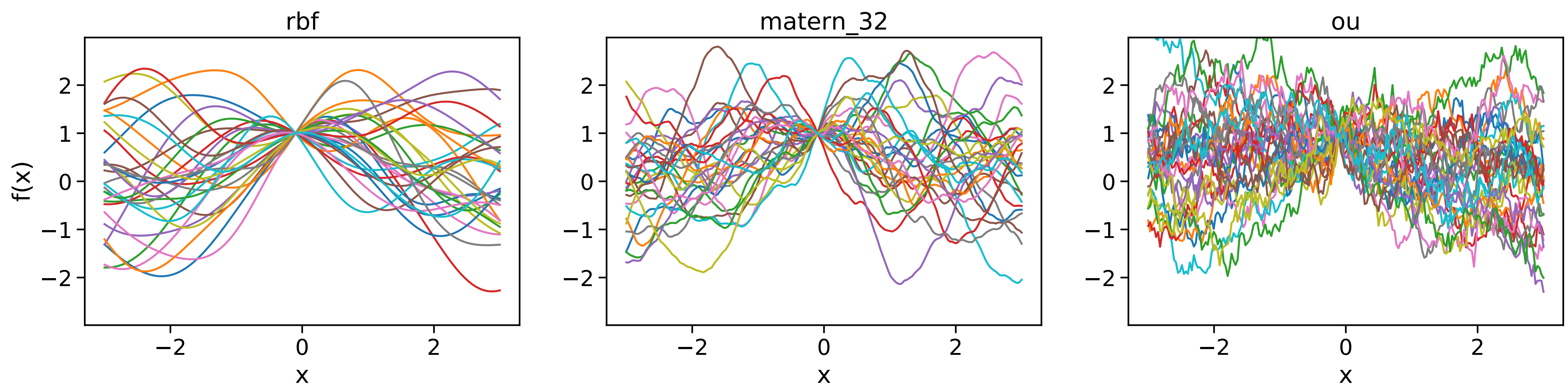


Posterior



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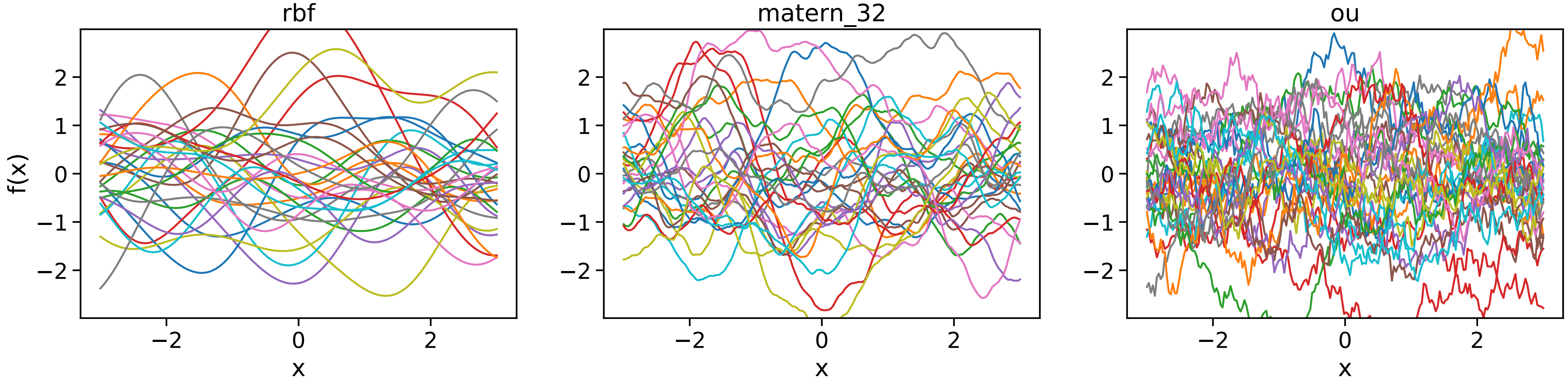

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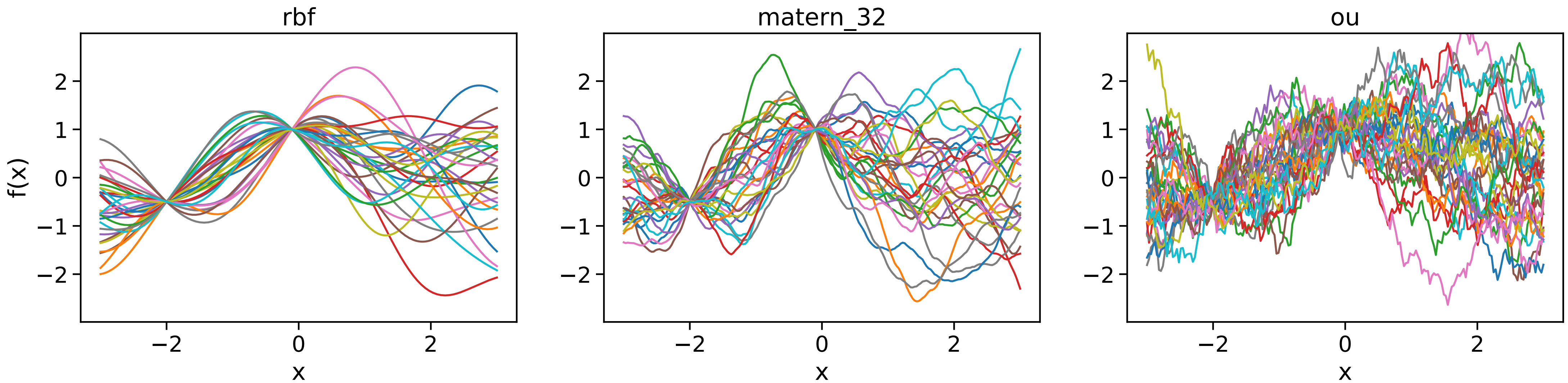

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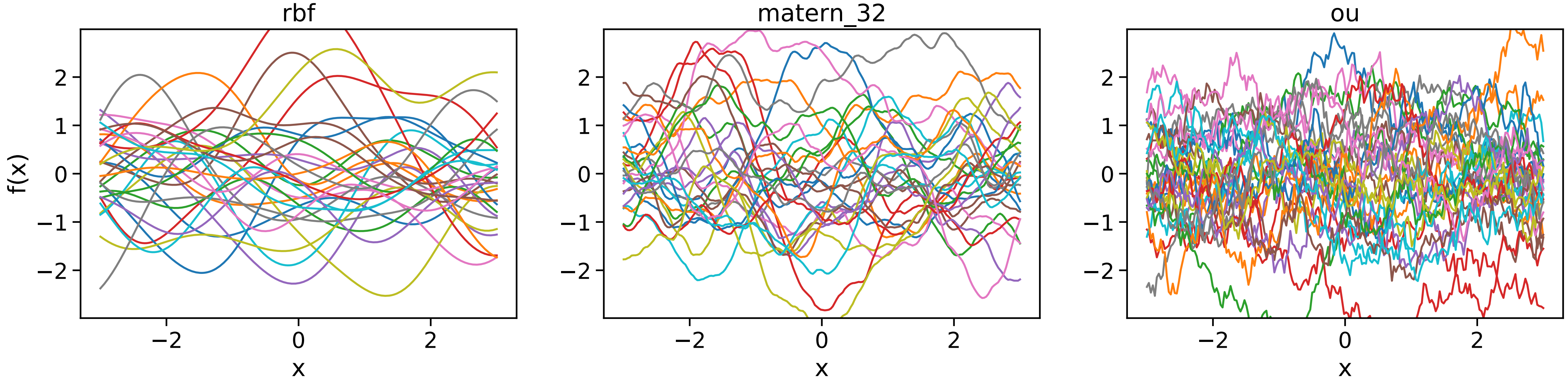
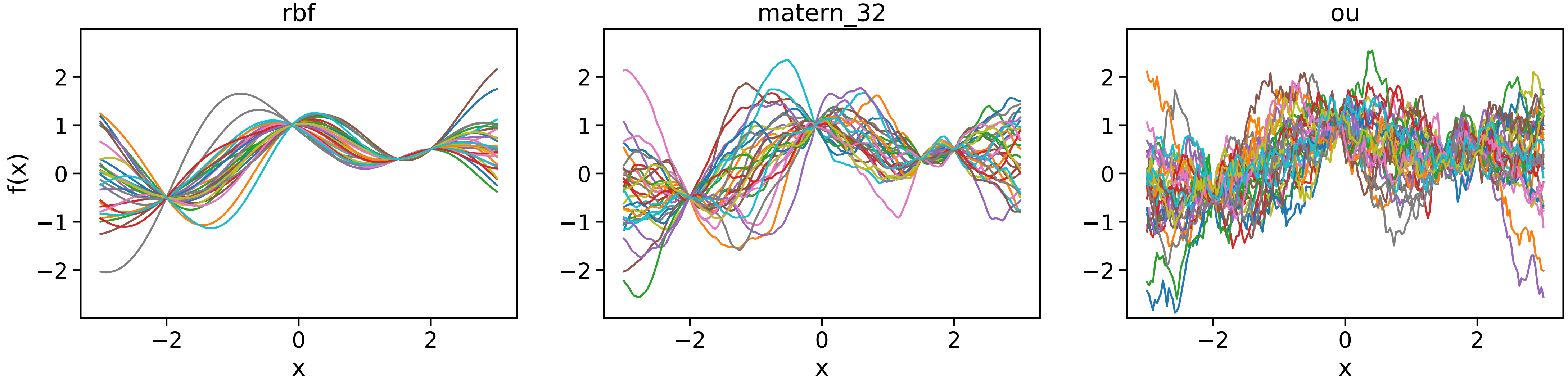
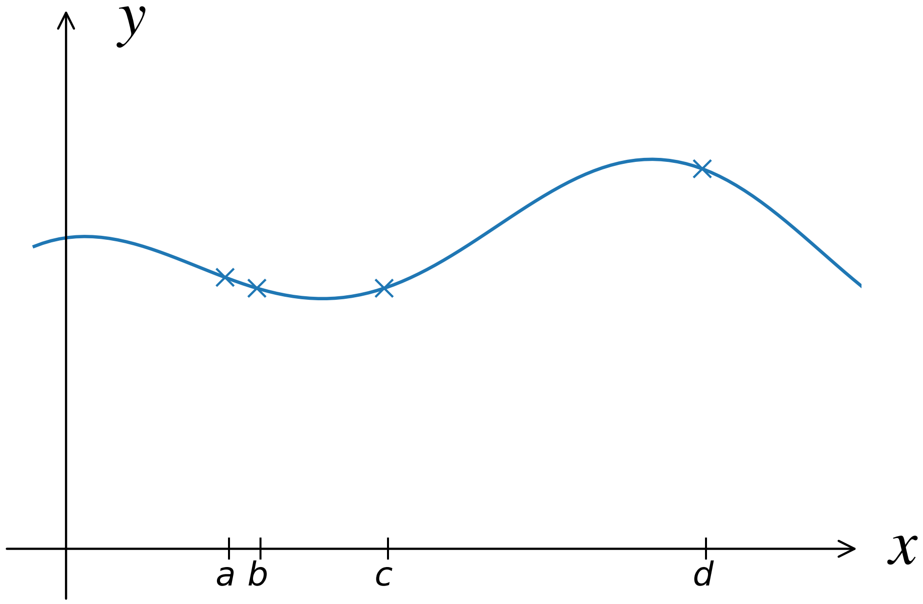


Posterior



Latent Variable Models

$$\boldsymbol{X} \in \mathbb{R}^{N \times Q} \xrightarrow{f} \boldsymbol{Y} \in \mathbb{R}^{N \times D}$$



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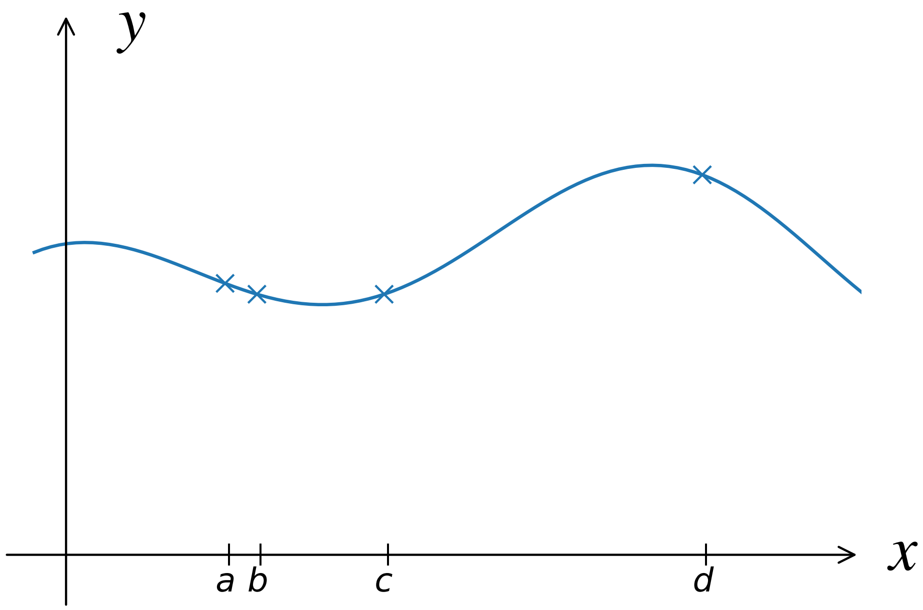

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \xrightarrow{f_1} \begin{pmatrix} a_1 & & & \\ b_1 & & & \\ c_1 & & & \\ d_1 & & & \end{pmatrix}$$

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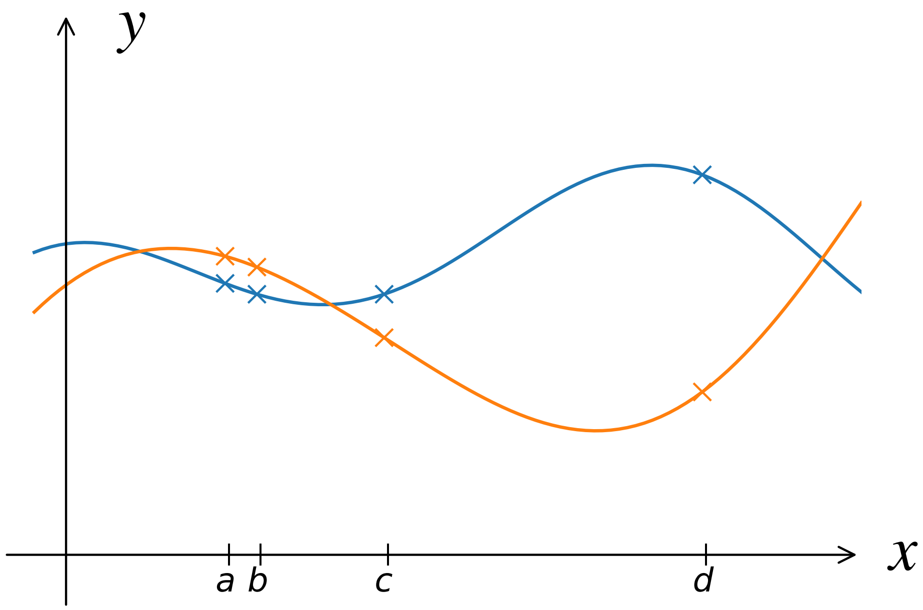

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \xrightarrow{f_2} \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \\ c_1 & c_2 \\ d_1 & d_2 \end{pmatrix}$$

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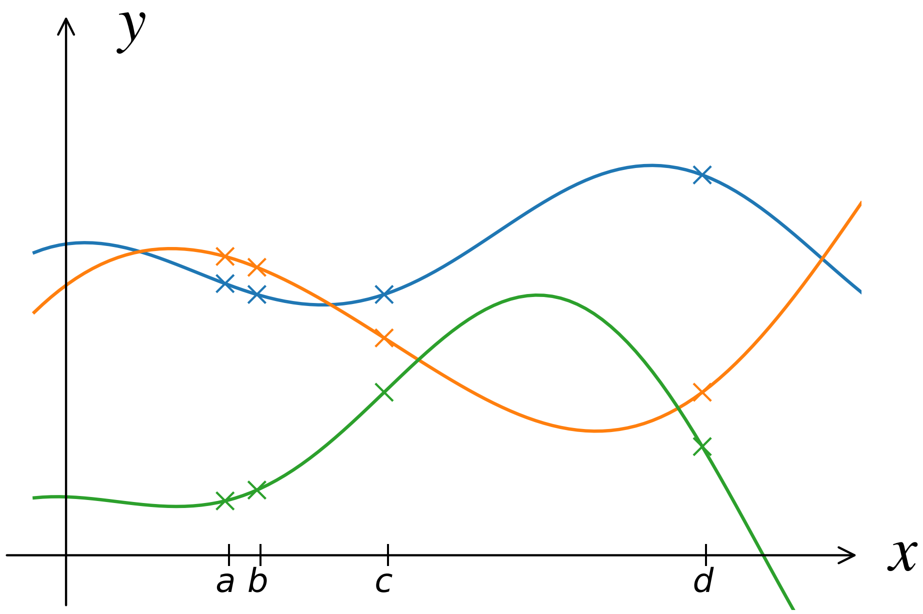

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \xrightarrow{f_3} \begin{pmatrix} a_1 & a_2 & a_3 & \\ b_1 & b_2 & b_3 & \cdots \\ c_1 & c_2 & c_3 & \\ d_1 & d_2 & d_3 & \end{pmatrix}$$

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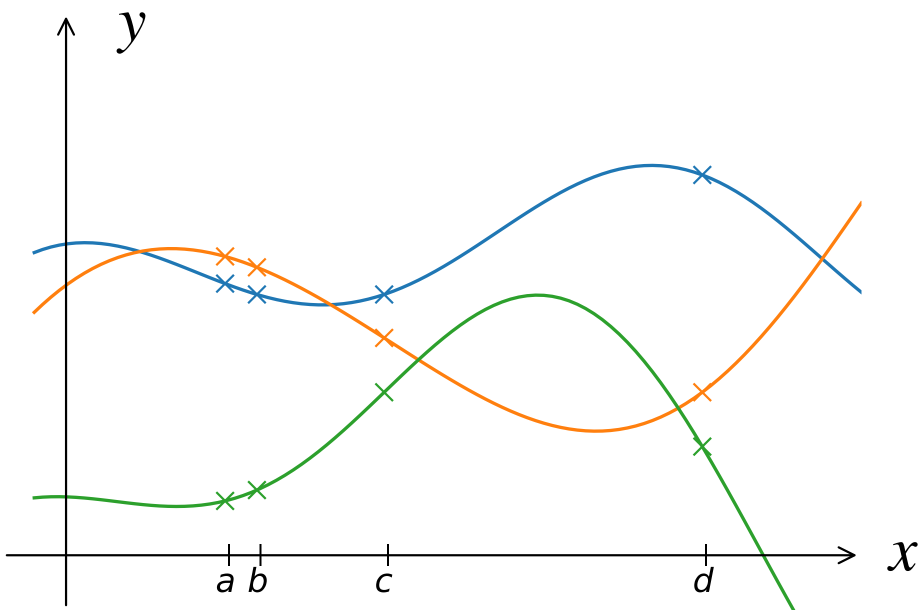
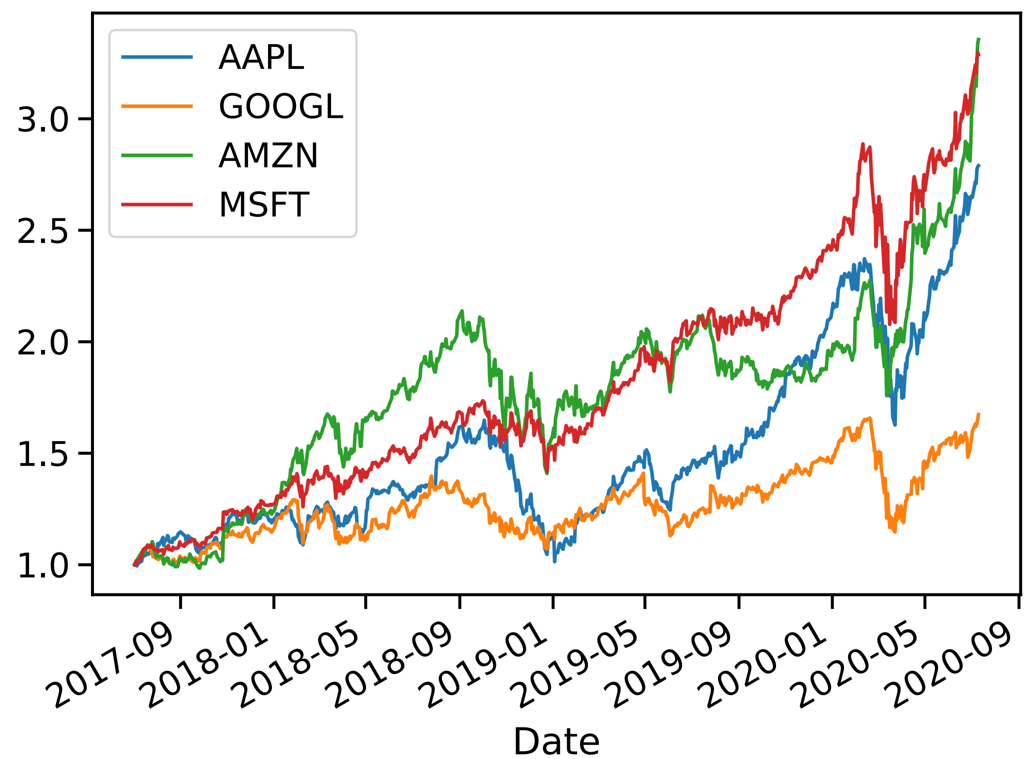


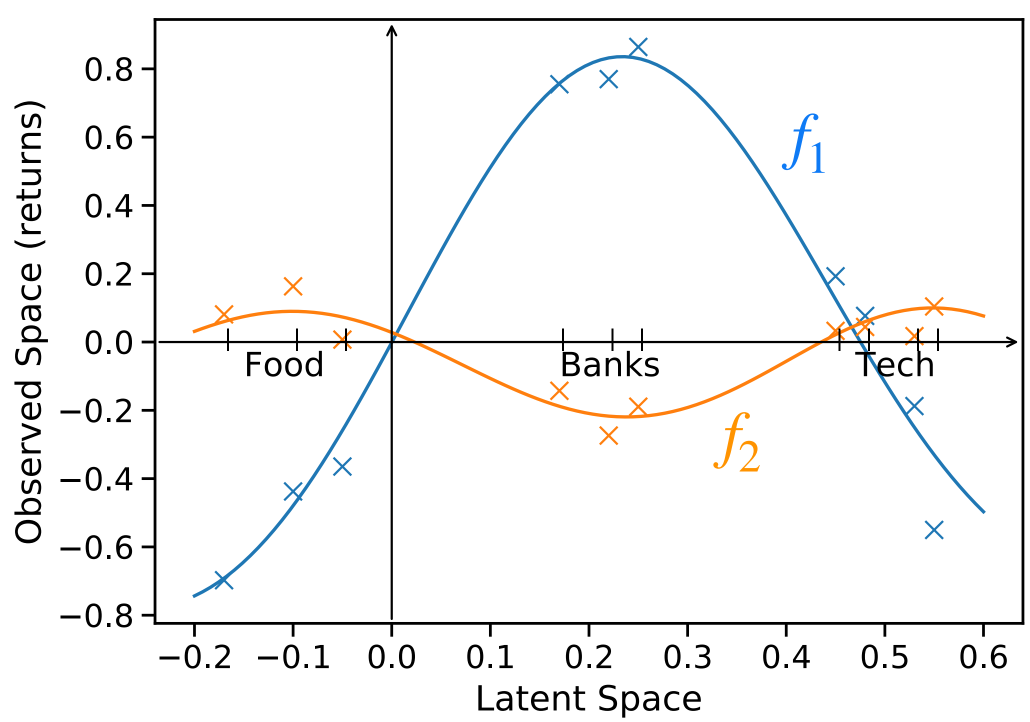
$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} \xrightarrow{f_i} \begin{pmatrix} a_1 & a_2 & a_3 & \\ b_1 & b_2 & b_3 & \\ c_1 & c_2 & c_3 & \cdots \\ d_1 & d_2 & d_3 & \end{pmatrix}$$

- Can we infer the hidden state $\boldsymbol{X}$ only by looking at $\boldsymbol{Y}$? Yes
- Inference using GPs also gives us the covariance $\boldsymbol{K}$ between different points



RETURNS	07.07.20	08.07.20	09.07.20	10.07.20
AAPL	−0.31	2.33	0.43	0.17
GOOGL	−0.64	0.92	1.00	1.34
AMZN	−1.86	2.70	3.29	0.55
MSFT	−1.16	2.20	0.70	−0.30
FOOD				
BANK				

Generative Model



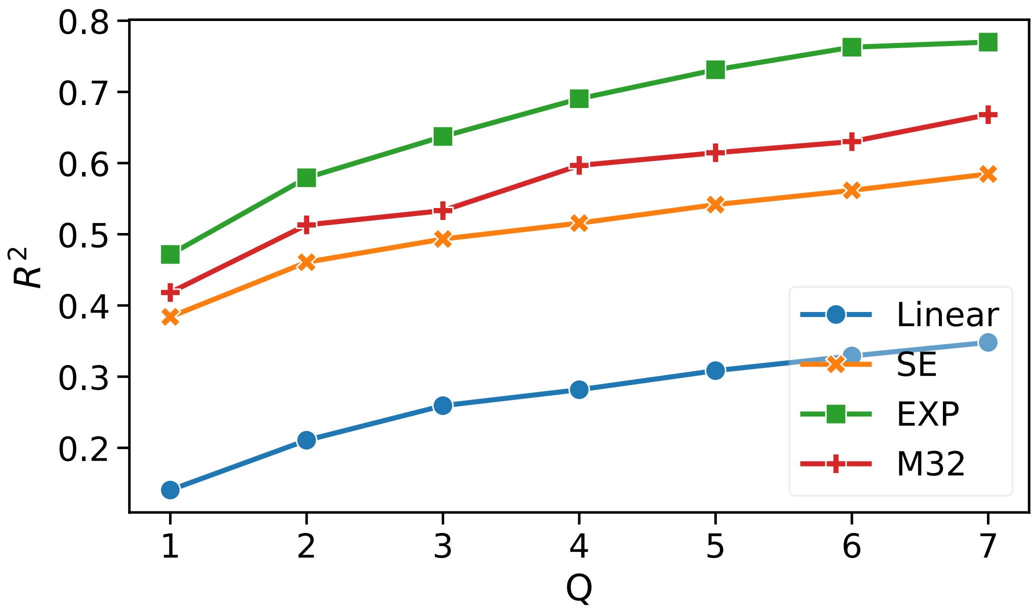
	Day1	Day2
Bank1	−0.70	0.08
Bank2	−0.44	0.16
Bank3	−0.36	0.01
Food1	0.75	−0.14
Food2	0.77	−0.27
Food3	0.86	−0.19
Tech1	0.19	0.03
Tech2	0.08	0.04
Tech3	−0.19	0.02
Tech4	−0.55	0.10

Experiments

Use Variational Bayes for the inference - data $Y \in \mathbb{R}^{N \times D}$

Approximate the true posterior $p(\boldsymbol{\theta}, \boldsymbol{X} \mid \boldsymbol{Y})$ with a simple distribution $q_{\boldsymbol{\phi}}(\boldsymbol{\theta}, \boldsymbol{X})$

R^2 - Variance of the data captured by the model



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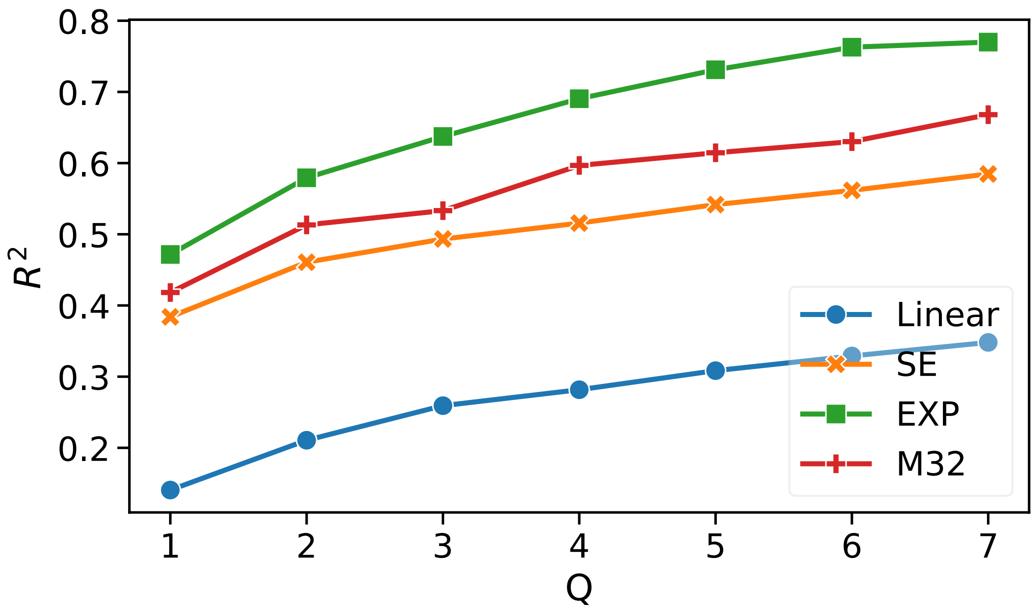
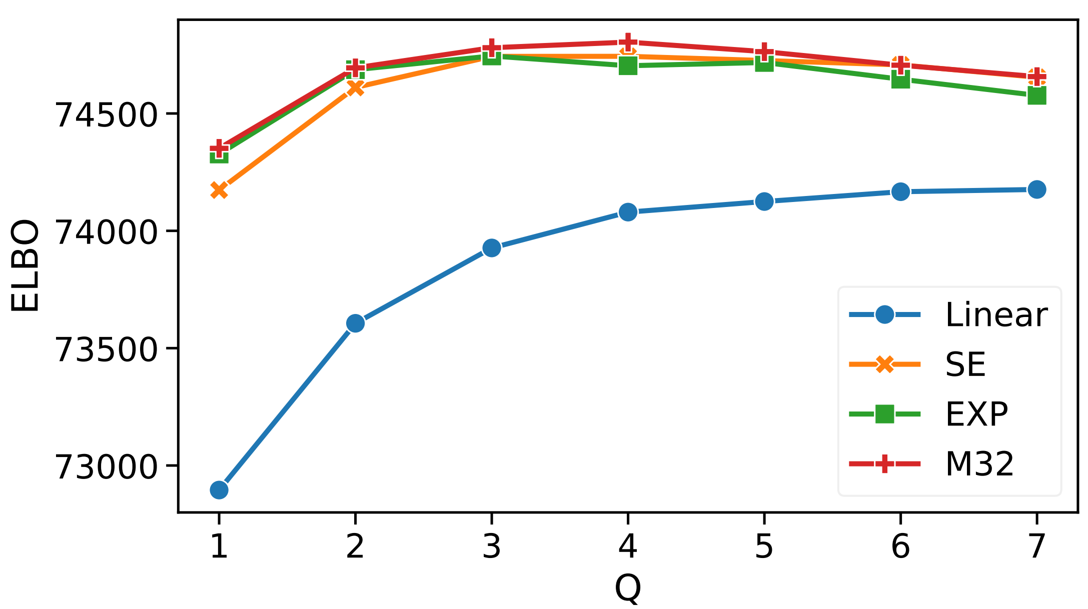


ELBO - Lower bound to the marginal likelihood



Portfolio Allocation

Given N stocks, how should I weight them to get an optimal portfolio?

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$$\boldsymbol{w}_{opt} = \min_{\boldsymbol{w}} \left(\boldsymbol{w}^T \boldsymbol{K} \boldsymbol{w} - q \boldsymbol{w}^T \boldsymbol{\mu} \right)$$

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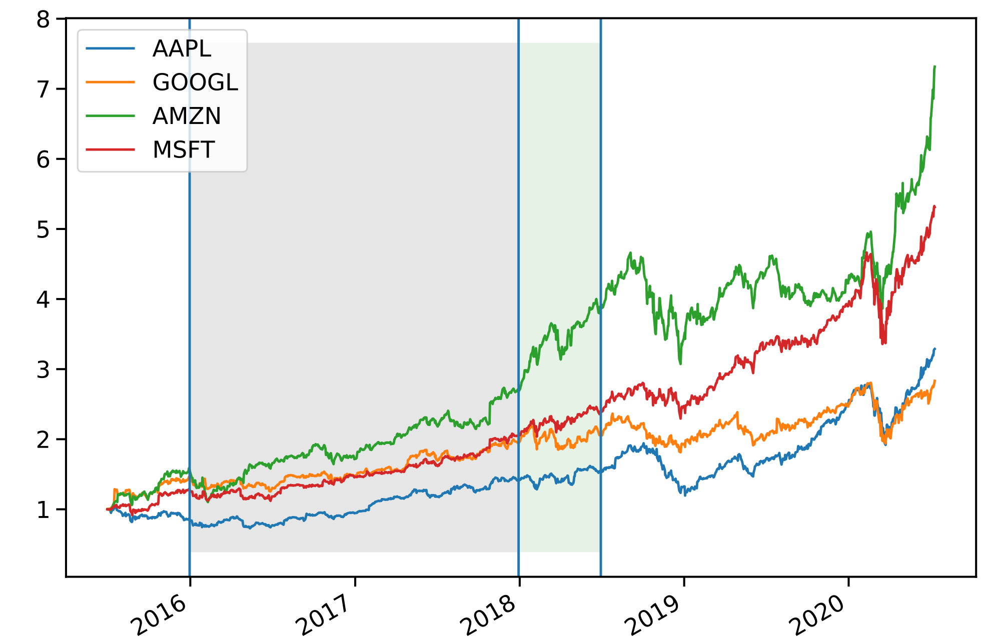

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Hold portfolio for next 6 months

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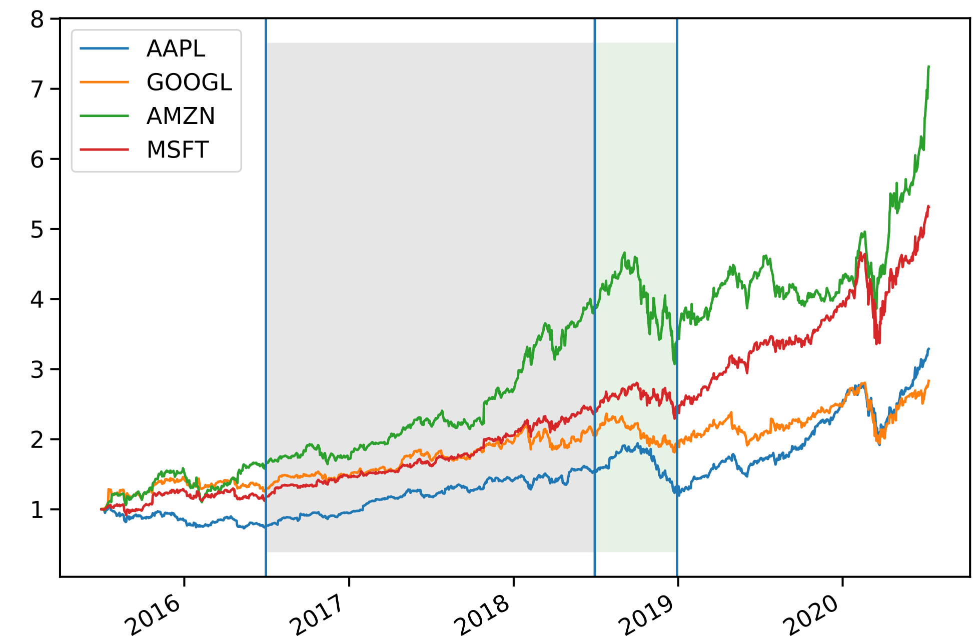

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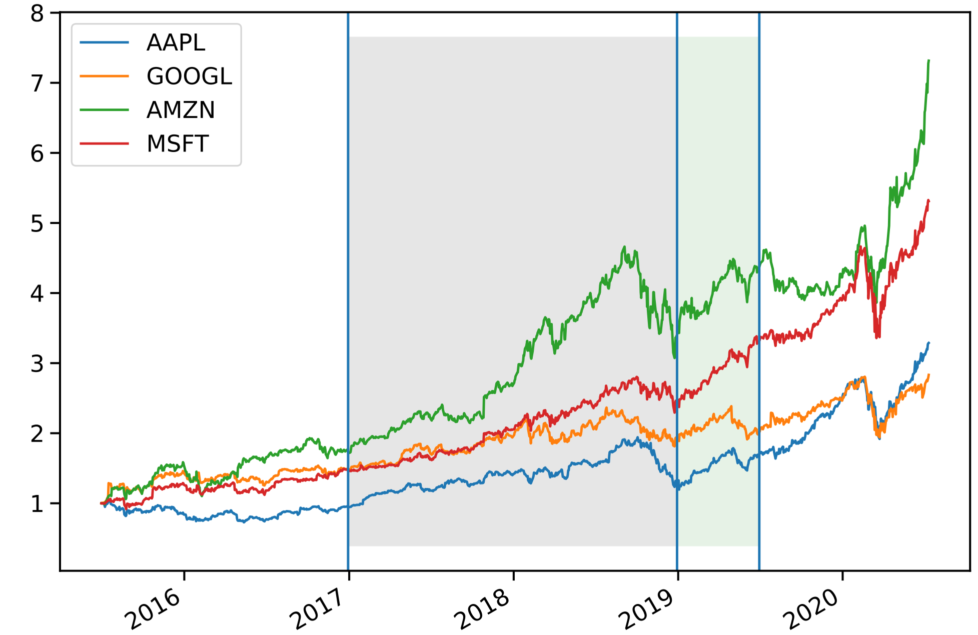

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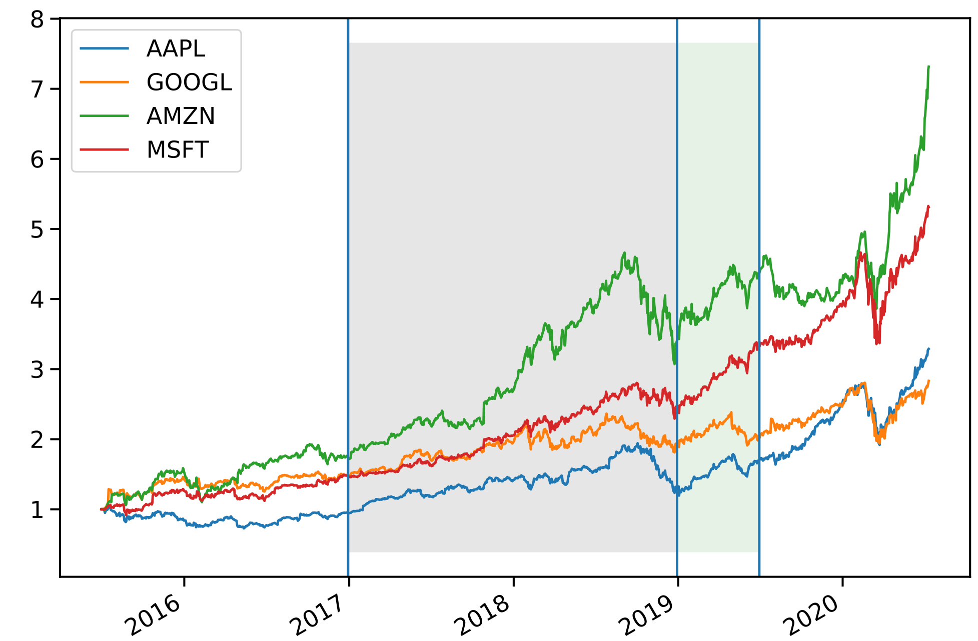


Backtesting on S&P500 from 2002 to 2018

Model	Linear	SE	EXP	M32	Sample Cov	Ledoit Wolf	Eq. Weighted
Mean	0.142	0.151	0.155	0.158	0.149	0.148	0.182
Std	0.158	0.156	0.154	0.153	0.159	0.159	0.232
Sharpe ratio	0.901	0.969	1.008	1.029	0.934	0.931	0.786

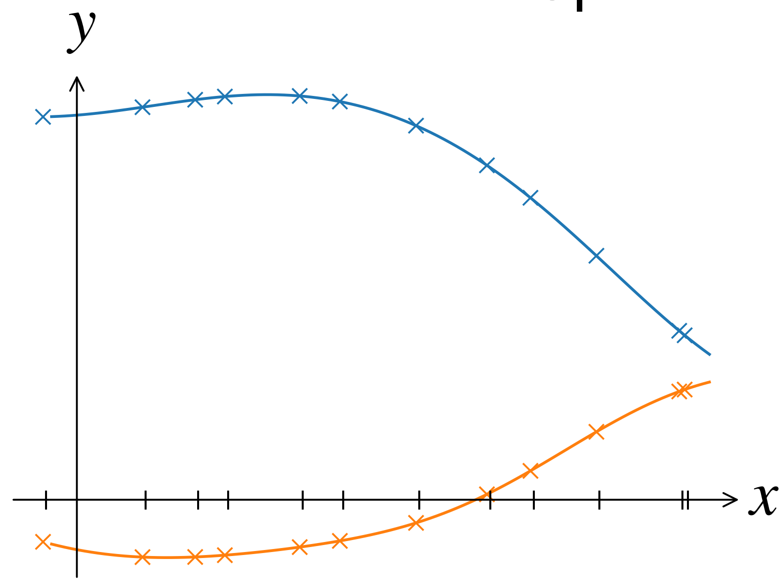
Predict Missing Values

$$\begin{pmatrix} & a_1 & a_2 & a_3 \\ & b_1 & b_2 & NaN \\ \cdots & c_1 & c_2 & c_3 \\ & d_1 & d_2 & d_3 \end{pmatrix}$$

Predict Missing Values

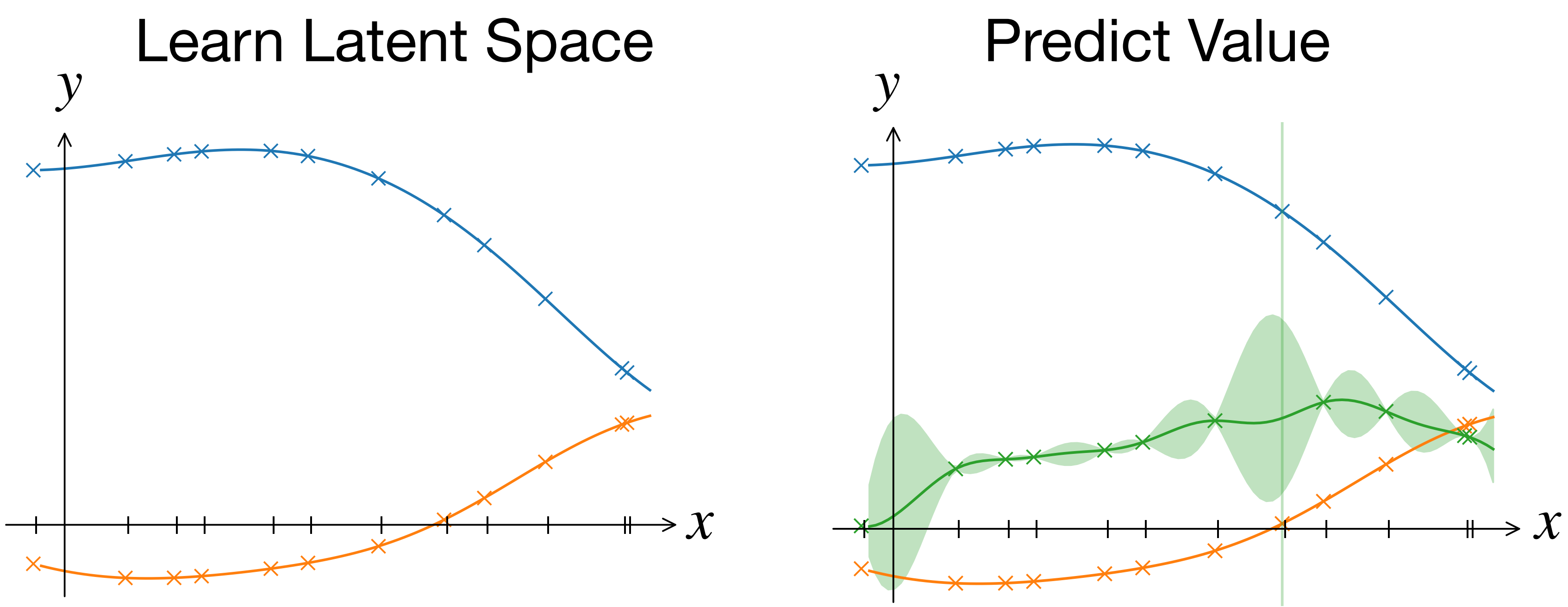
$$\begin{pmatrix} & a_1 & a_2 & a_3 \\ \cdots & b_1 & b_2 & NaN \\ & c_1 & c_2 & c_3 \\ & d_1 & d_2 & d_3 \end{pmatrix}$$

Learn Latent Space



Predict Missing Values

$$\left(\begin{array}{cccc} & a_1 & a_2 & a_3 \\ & b_1 & b_2 & NaN \\ \cdots & c_1 & c_2 & c_3 \\ & d_1 & d_2 & d_3 \end{array}\right)$$



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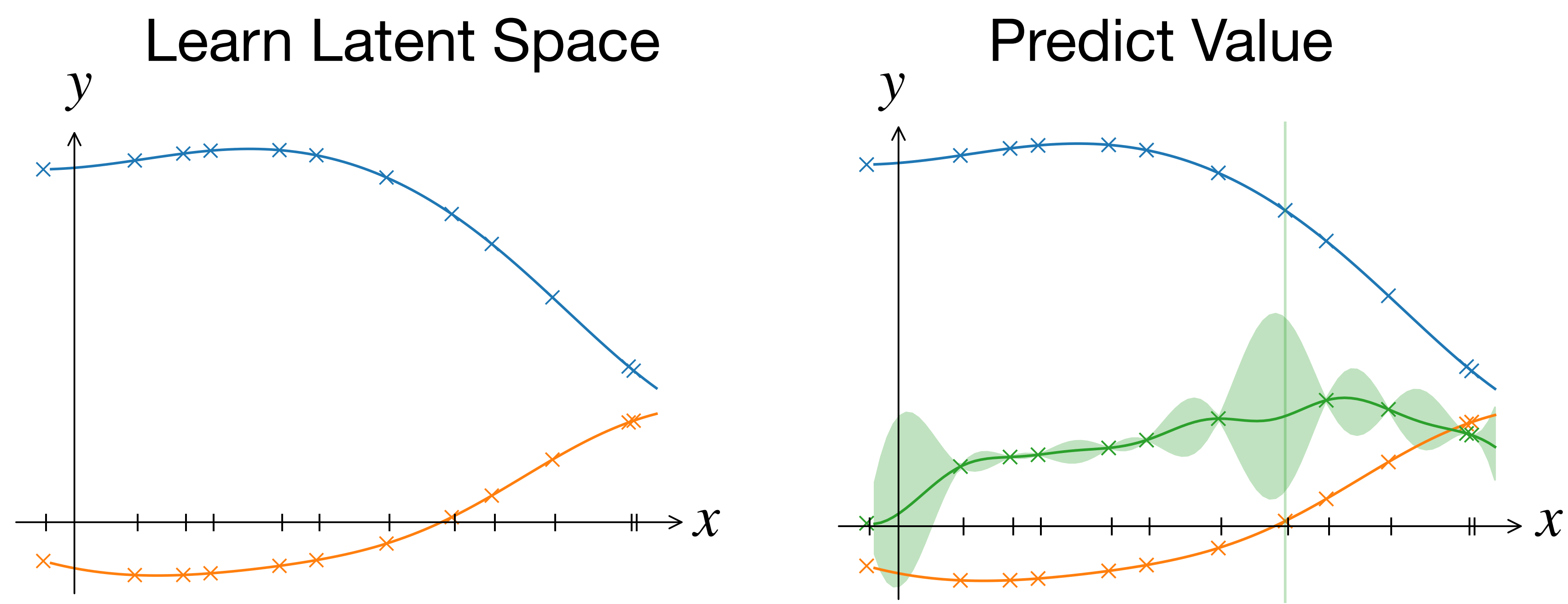
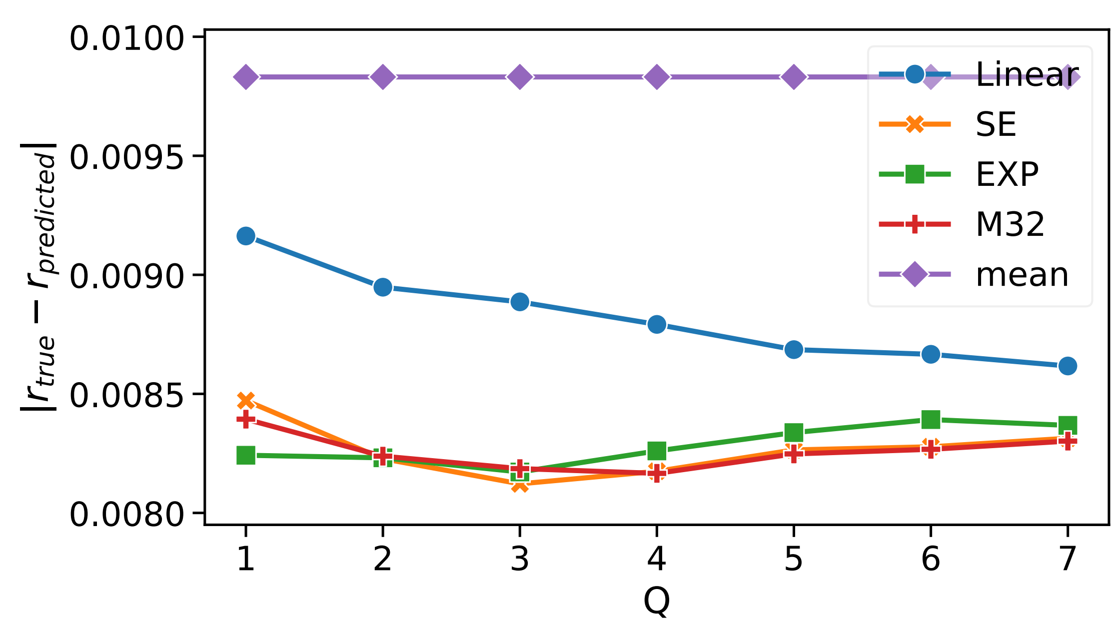
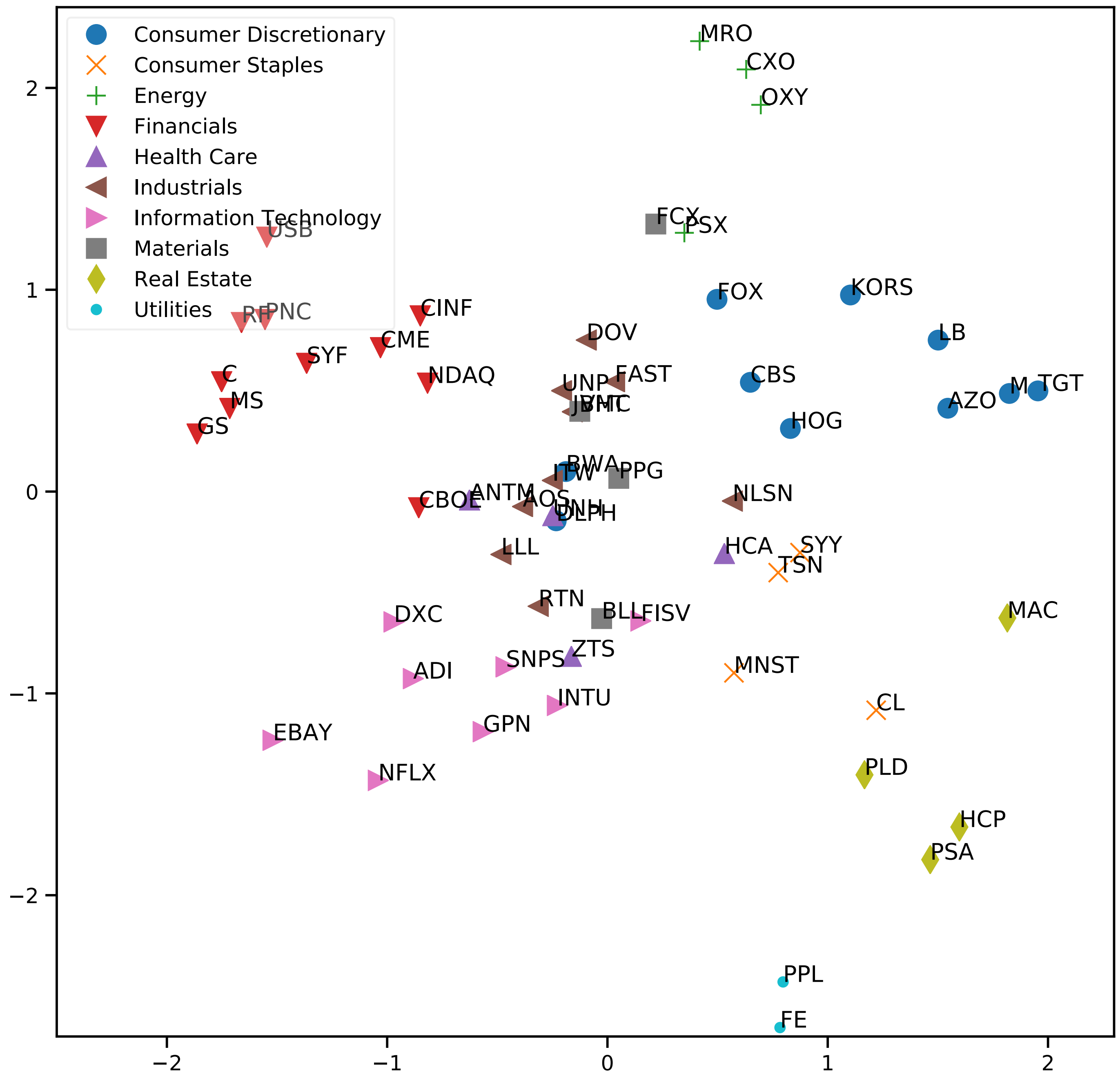


Prediction of missing values for held out dataset



Visualization of Latent Space



Summary

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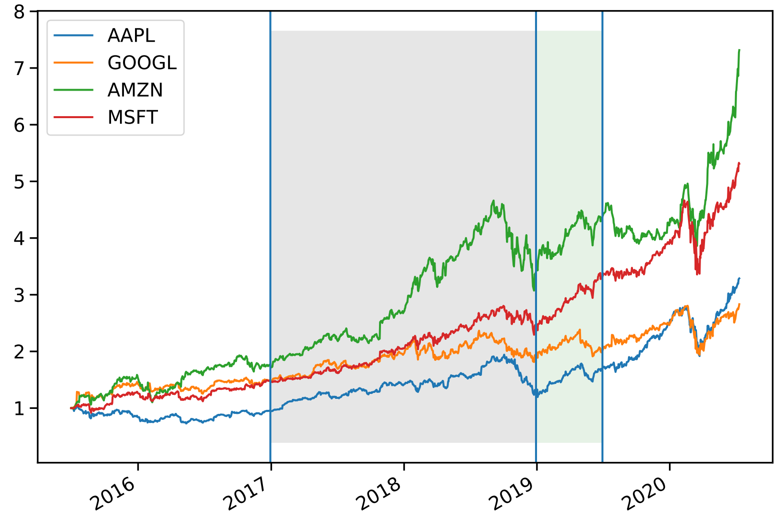

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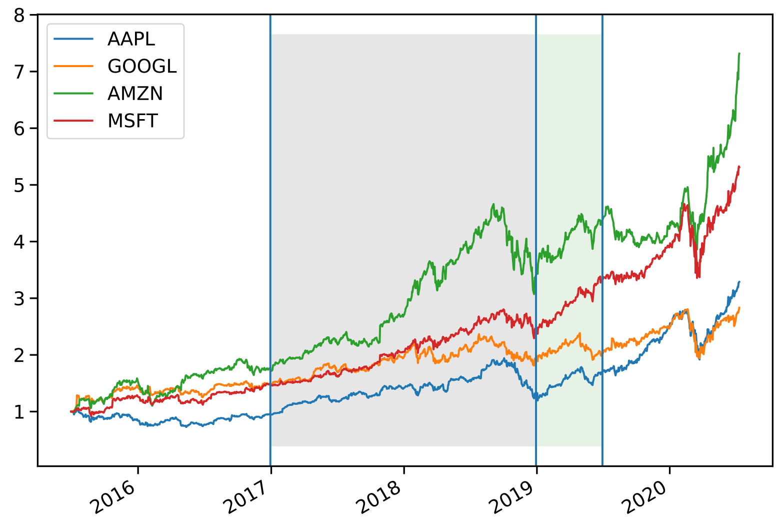

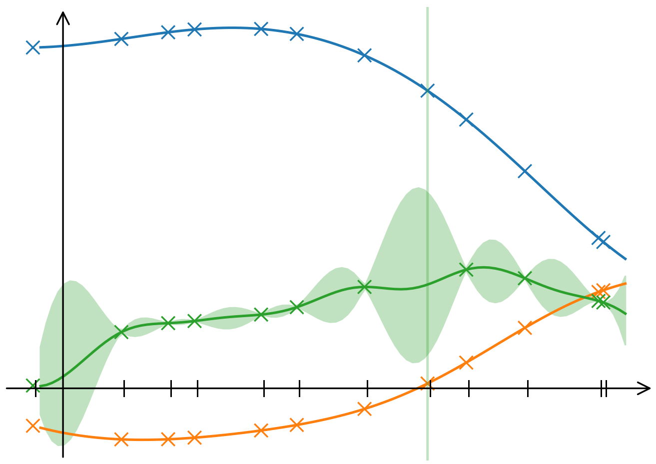

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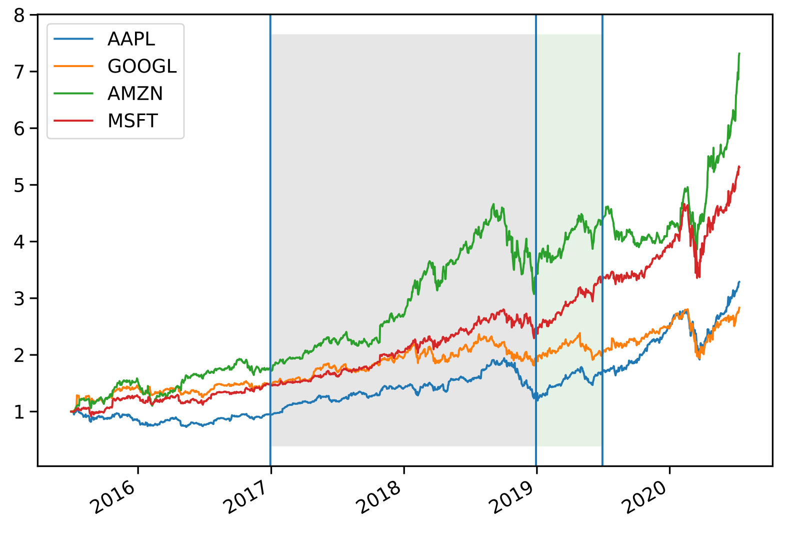

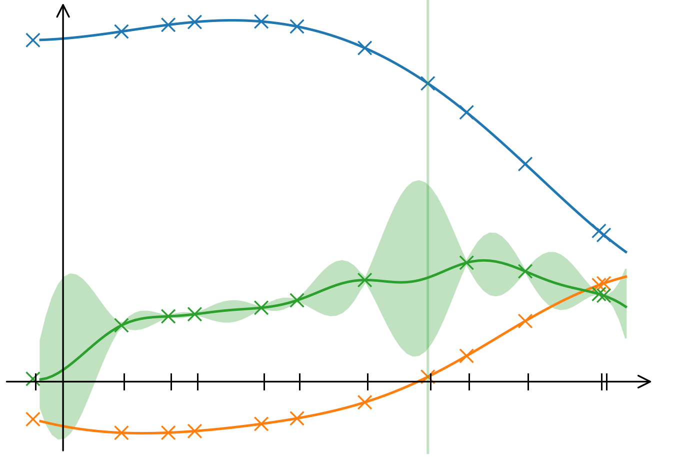

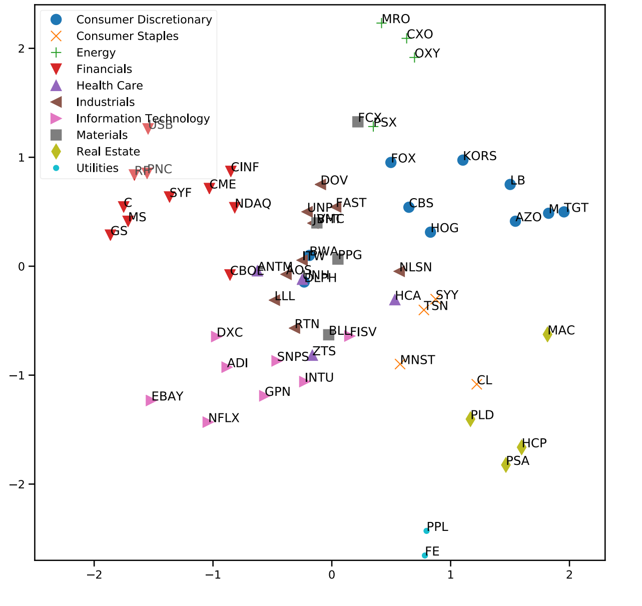

https://github.com/RSNirwan/GPLVMsInFinance

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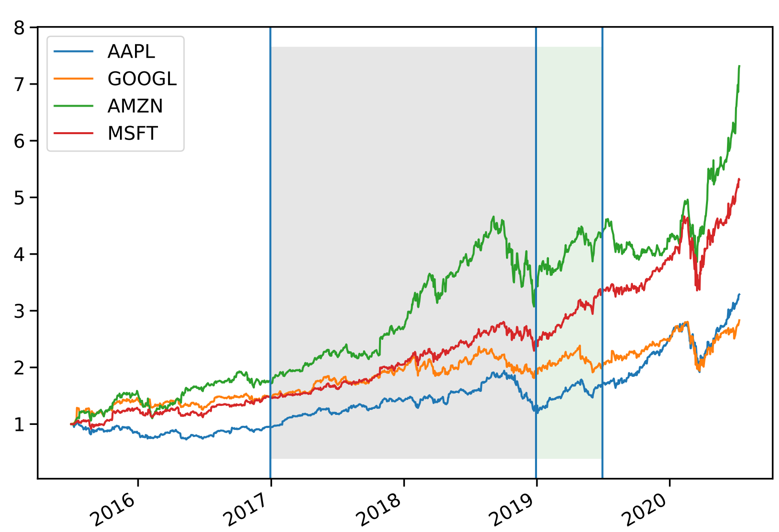

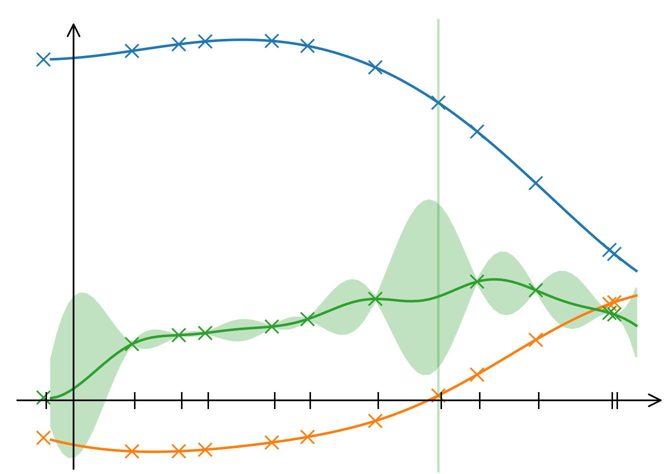

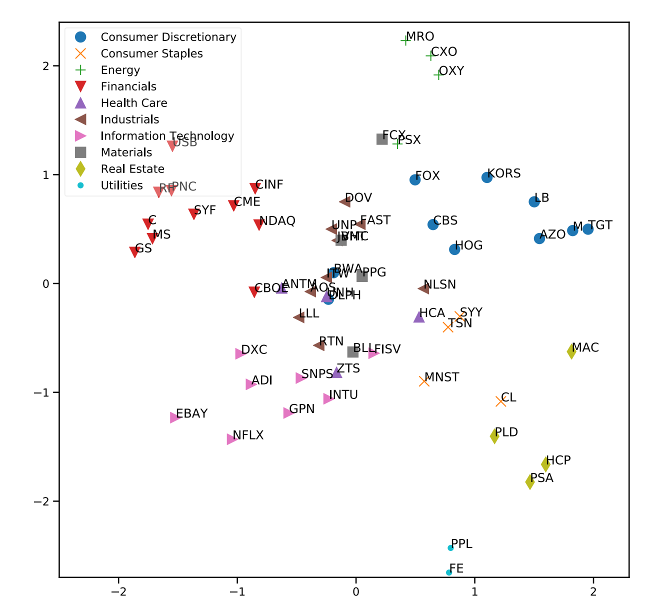

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