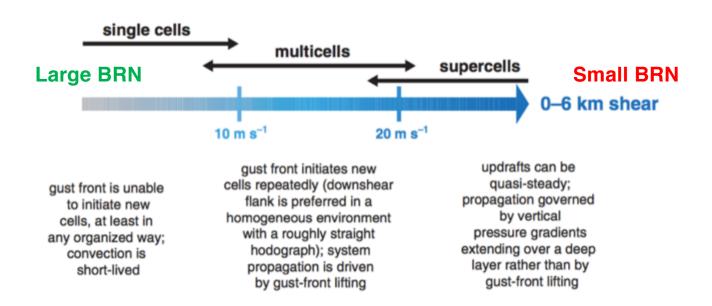
Distinguishing between single cell and multicell environments

Bulk Richardson Number (BRN) = CAPE / $(0.5*U^2)$

U – magnitude of vector difference between 0 – 6km mean wind and 0 – 500 m mean wind.

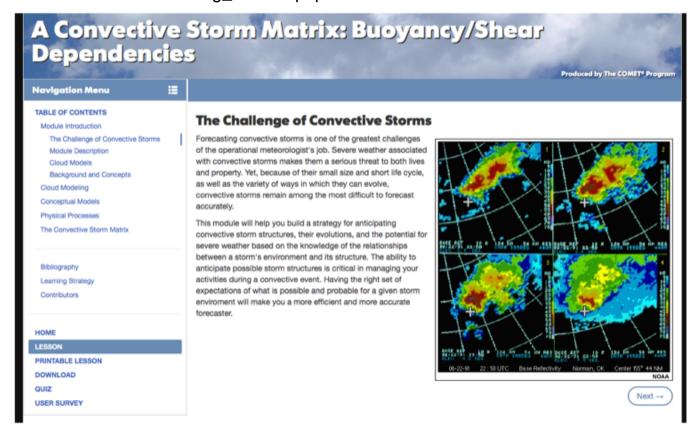


Three types of storms evident from WK82 simulations:

- 1) Single cell
 - short lifetime
 - driven by buoyancy
- 2) Multi-cell
 - longer lifetime
 - discrete propagation via gust front
 - · need some amount of shear
- 3) Supercell
 - longer lifetime
 - continuous propagation via VPPGF
 - storm splitting
 - large shear

Sign up for a free account on MetEd and go through...

https://www.meted.ucar.edu/training_module.php?id=22



Some convective updrafts rotate...



Read *Davies-Jones (1984)*: "Streamwise vorticity: The origin of updraft rotation in supercell storms"

(M&R Section 8.4.3)

Start with full vorticity equation...

$$\frac{\mathrm{d}(\boldsymbol{\omega} + f\mathbf{k})}{\mathrm{d}t} = \underbrace{\frac{\partial \boldsymbol{\omega}}{\partial t}}_{\text{local tendency}} \underbrace{+(\mathbf{v} \cdot \nabla)(\boldsymbol{\omega} + f\mathbf{k})}_{\text{advection}}$$

$$= \underbrace{[(\boldsymbol{\omega} + f\mathbf{k}) \cdot \nabla]\mathbf{v}}_{\text{tilting/stretching}} \underbrace{+\frac{1}{\rho^2} \nabla \rho \times \nabla p}_{\text{baroclinic generation}}$$

$$\underbrace{+\nabla \times \mathbf{F}.}_{\text{viscous effects}} (2.88)$$

Extract vertical vorticity equation...

$$\mathbf{k} \cdot \frac{\partial \boldsymbol{\omega}}{\partial t} = \frac{\partial \zeta}{\partial t} = -\mathbf{v} \cdot \nabla(\zeta + f) + \boldsymbol{\omega} \cdot \nabla w + f \frac{\partial w}{\partial z}$$

$$+ \frac{1}{\rho^2} \left(\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} - \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} \right) + \mathbf{k} \cdot \nabla \times \mathbf{F}. \quad (2.91)$$
Baroclinic effects

Vertical vorticity equation neglecting baroclinic, viscous, earth's rotation effects...

$$\frac{\partial \zeta}{\partial t} = -\mathbf{v} \cdot \nabla \zeta + \boldsymbol{\omega} \cdot \nabla w$$

$$=-u\frac{\partial \zeta}{\partial x}-v\frac{\partial \zeta}{\partial y}-w\frac{\partial \zeta}{\partial z}+\xi\frac{\partial w}{\partial x}+\eta\frac{\partial w}{\partial y}+\zeta\frac{\partial w}{\partial z}$$

Linearize vertical vorticity equation (mean + perturbation; mean = environment)

Neglect non-linear terms (products of perturbations)

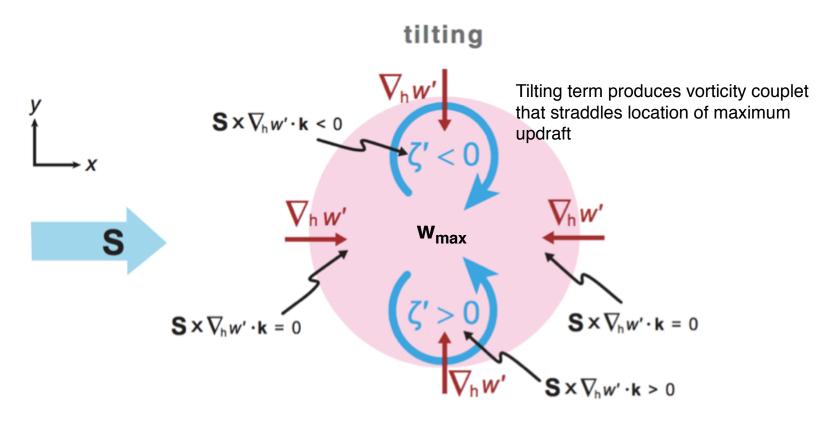
$$u = \bar{u} + u'$$
 $v = \bar{v} + v'$ $w = w'$ $\zeta = \zeta'$

$$\frac{\partial \zeta'}{\partial t} = -\overline{u} \frac{\partial \zeta'}{\partial x} - \overline{v} \frac{\partial \zeta'}{\partial y} + \frac{\partial \overline{u}}{\partial z} \frac{\partial w'}{\partial y} - \frac{\partial \overline{v}}{\partial z} \frac{\partial w'}{\partial x},$$
advection tilting

Remember: no nonlinear effects here (perturbations increasing perturbations)!

$$\frac{\partial \zeta'}{\partial t} = \underbrace{-\overline{\mathbf{v}} \cdot \nabla_{\mathbf{h}} \zeta'}_{\text{advection}} + \mathbf{S} \times \nabla_{\mathbf{h}} w' \cdot \mathbf{k},$$

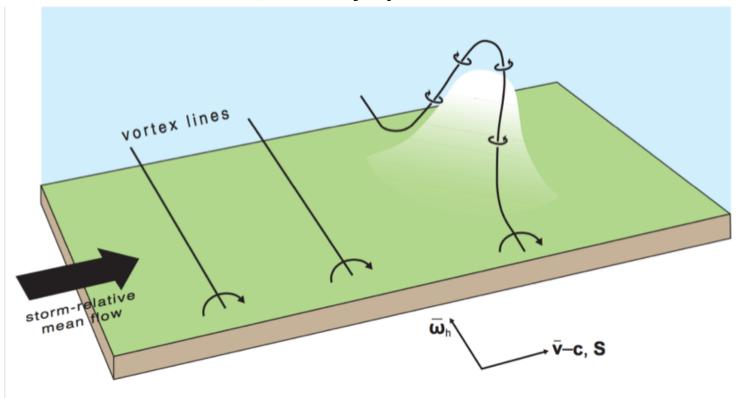
If vertical vorticity zero, no advection can occur, so tilting must first produce vertical vorticity, which can then be advected.

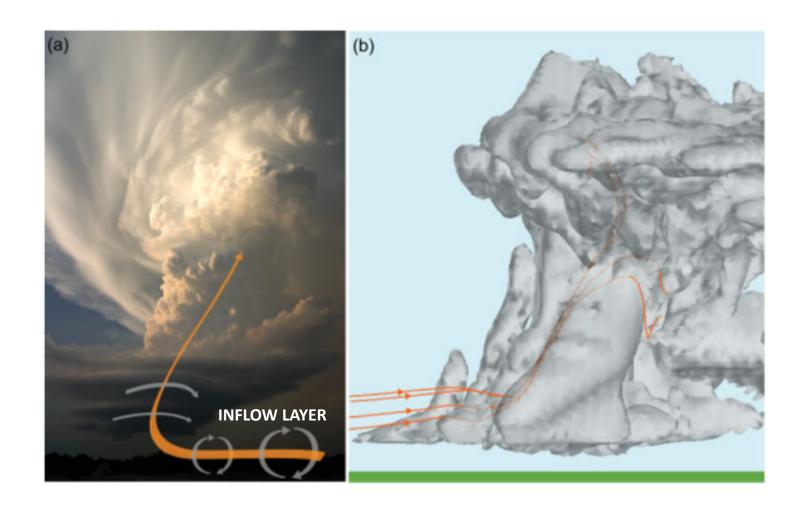


Vortex Lines

- Curve in fluid tangent to local vorticity vector (same relationship between vortex line/vorticity as streamline/velocity).
- Vortex lines cannot be broken and can only terminate at a surface (e.g., the ground)
- Vortex tubes are collections of vortex lines
- Helmholtz's theorem:
 - Vortex lines move as material lines in inviscid, barotopic flow (when friction and baroclinity can be neglected).

Vortex lines in sheared flow, tilted by updraft





To assess impacts of advection, need to look at changes in storm-relative frame of reference.

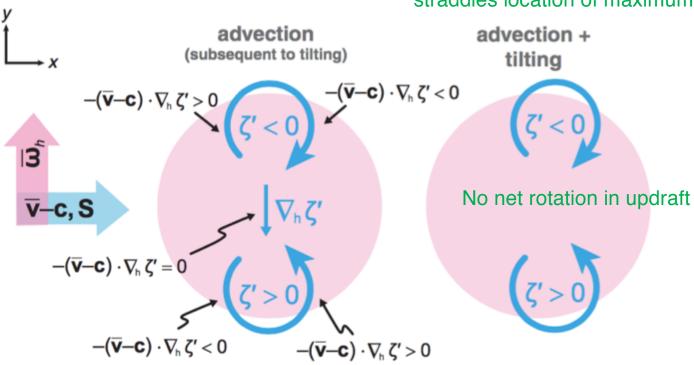
That is, how does vertical vorticity change in the reference frame of the storm?

$$\left(\frac{\partial \zeta'}{\partial t}\right)_{\rm sr} = \underbrace{-(\overline{\mathbf{v}} - \mathbf{c}) \cdot \nabla_{\rm h} \zeta'}_{\rm advection} \underbrace{+\mathbf{S} \times \nabla_{\rm h} w' \cdot \mathbf{k}}_{\rm tilting},$$

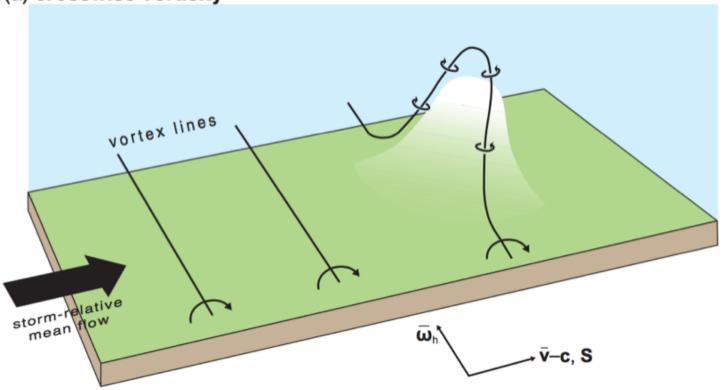
Tilted vorticity couplets can be advected in various directions...

(a) crosswise vorticity

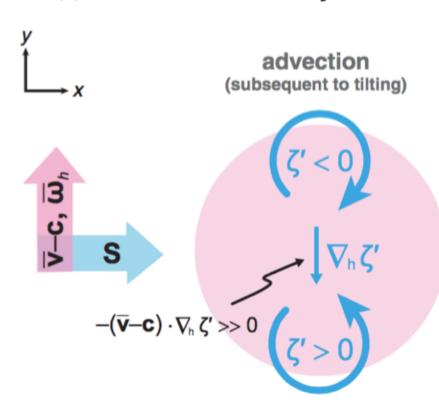
Tilting term produces vorticity couplet that straddles location of maximum updraft



(a) crosswise vorticity



(b) streamwise vorticity



advection + tilting

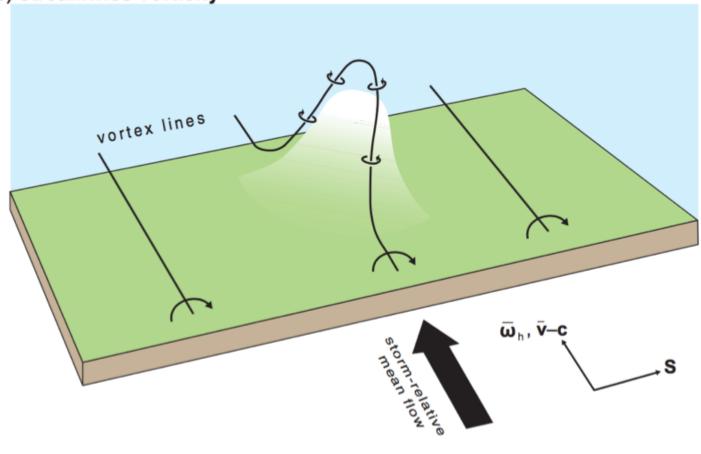


Net cyclonic rotation in updraft

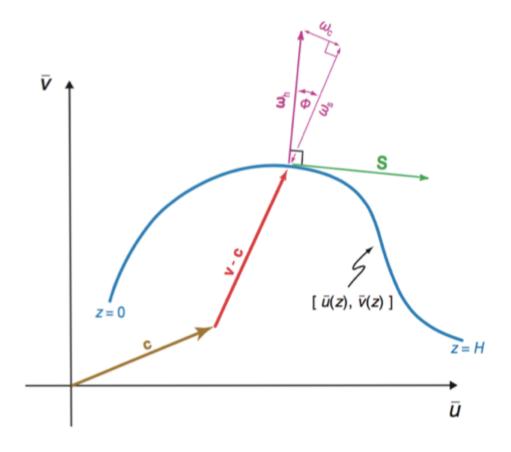


Downward shift of couplet is larger since gradient of vorticity is larger, positive vorticity shifted toward w_{max}

(b) streamwise vorticity



Streamwise and crosswise vorticity at particular level on hodograph

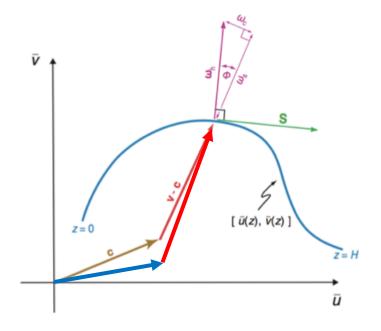


Streamwise and crosswise vorticity at particular level on hodograph

Correlation of updraft speed with positive vertical vorticity depends on magnitude of environmental crosswise vs. streamwise vorticity in storm inflow.

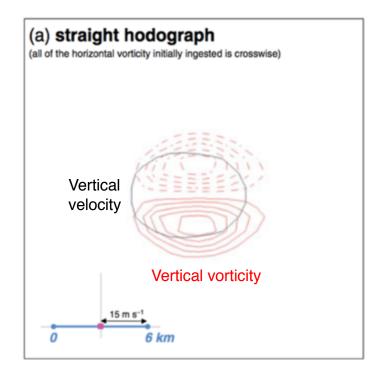
Storm motions that deviate to right of mean wind usually result in larger streamwise

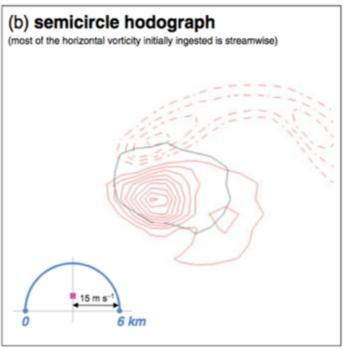
vorticity:



Summary of development of mid-level rotation in an updraft

- vorticity couplet develops due to tilting of environmental horizontal vorticity into the vertical
- couplets immediately advected by storm-relative winds in a way that depends on horizontal streamwise vs. crosswise vorticity
- if largely streamwise, updraft maximum and vorticity maximum can become co-located, leading to non-linear effects.





Helicity

Helicity is a measure of the degree to which the direction of fluid motion is aligned with the vorticity of the fluid (related to streamwise vorticity).

$$\mathcal{H} = \mathbf{v} \cdot \boldsymbol{\omega} = \mathbf{v} \cdot \nabla \times \mathbf{v}$$

Dot product of velocity vs. vorticity vector (0 if perpendicular; Beltrami flows)

Before, we were really just assessing streamwise vorticity at one level within the storm. Need to look at depth that matters to storm (inflow)...

$$\mathcal{H} = \int_0^d \overline{\mathbf{v}} \cdot \overline{\boldsymbol{\omega}}_{\mathrm{h}} \, \mathrm{d}z \qquad \overline{\boldsymbol{\omega}}_{\mathrm{h}} = \left(-\frac{\partial \overline{\boldsymbol{v}}}{\partial z}, \frac{\partial \overline{\boldsymbol{u}}}{\partial z}\right) = \mathbf{k} \times \mathbf{S}$$

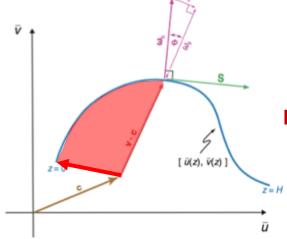
Overbars indicate environmental wind

Helicity

Again, only helicity in storm-relative reference frame is relevant...

Storm-relative helicity (or storm-relative environmental helicity)

$$SRH = \int_0^d (\overline{\mathbf{v}} - \mathbf{c}) \cdot \overline{\boldsymbol{\omega}}_h \, dz = \int_0^d |\overline{\mathbf{v}} - \mathbf{c}| \, \omega_s \, dz$$



Magnitude of SRH is twice shaded area.

Helicity

SRH is positive if area is associated with streamwise vorticity (negative if associated with antistreamwise vorticity).

Can also compute with a set of wind observations...

$$SRH = \sum_{n=1}^{N-1} [(u_{n+1} - c_x)(v_n - c_y) - (u_n - c_x)(v_{n+1} - c_y)]$$