Homework 5: Discrete Mathematics Spring 2021 – Sets

Due Sunday February 21 @11:59:00pm

Unless stated otherwise, show all your work to receive full credit

This homework is shorter than usual; Please note there is no late date. The latest you can submit your homework is Monday Feb 22; we will post the solution on Tue Feb 23 morning.

1. **Set builders**

   Rewrite the following sets using set builder notation. Use only mathematical notations, no English. Make sure your set builder is not ambiguous, that is it generates all the elements in the set, and it does not generate elements not in the set. Feel free to use connectives, and quantifiers to express the predicates in the set builder. No need to justify your work.

   1. $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
   2. $\{-10, -8, -6, -4, -2, 0, 2, 4, 6, 8, 10\}$
   3. $\{1, 3, 5, 7, 9, 11, 13, ...\}$
   4. $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

2. **Set builders**

   Using enumeration, write the sets corresponding to the following set builders. Use ellipsis if needed. No justification needed.

   1. $A = \{x | x \in \mathbb{Z}^{+} \wedge 5|(x-1)\}$
   2. $B = \{x | x \in \mathbb{Z} \wedge x = (-1)^{k} \text{ for some integer k}\}$

3. **Is in ($\in$) and is a subset of ($\subseteq$)**

   Let:
   $$S = \{\emptyset, \{4, 5\}, 1, 3, 6, \{2\}\}$$

   Which of the following are true and which are False? Note: You don?t need to justify your answer, but make sure you really understand.

   1. $\emptyset \notin S$
   2. $\emptyset \subseteq S$
   3. $6 \in S$
   4. $\{6\} \in S$
   5. $\{6\} \subseteq S$
   6. $2 \notin S$
   7. $2 \nsubseteq S$
   8. $\{2\} \in S$
   9. $\{2\} \nsubseteq S$
   10. $\{\{2\}\} \nsubseteq S$
   11. $\emptyset \in P(S)$
   12. $\{\emptyset\} \in P(S)$

   Note:

   - $\emptyset$ denotes empty set.
   - $P(S)$ denotes powerset of S.
   - There is no need to justify your answers, but make sure you understand.

4. Using the operation on sets and the sets A, B, C, express the shaded area. No justification needed.
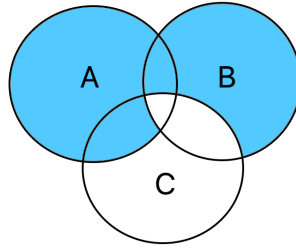
5. **Operations on sets**

Let $A = \{a, b, 2, 3\}$, $B = \{2, 3, 4\}$. Please calculate:

1. $A \cup B$
2. $A \cap B$
3. $A - B$
4. $B - A$
5. $A \times B$
6. $P(B)$

7. $\overline{B}$. For this set, please write it in two different ways (set of elements, and then with a set builder). Let $\mathbb{U} = \{x \mid x \in \mathbb{Z} \wedge x \geq 0\}$ be the universal set.

No need to justify your work.