

## Homework 4: Discrete Mathematics Spring 2021 – Proofs

Due Sunday February 14 @11:59pm

1. **Direct proof**

Using a **direct proof**, prove that for all integers  $n$ , 3 divides  $(3n + 1)(3n + 2)(3n + 3)$ .

2. **Proof by contrapositive**

Prove by **contrapositive** following statement:

For any integer  $n$ , if  $3n + 1$  is even, then  $n$  is odd.

3. **Proof of if and only if (IFF)**

Let  $x, y$  be integers. Prove that the product  $xy$  is odd if and only if  $x$  and  $y$  are both odd integers.

4. **Proof by cases**

Prove by cases that if  $n$  is an integer, then  $n^2 \geq n$ .

5. **Proof by Counter Example**

**Prove or disprove:** For every prime number  $p > 2$ , there exists a natural number  $n$  such that  $p = 2^n - 1$ .

6. **Proof by contradiction:** Prove by contradiction that:

For all integers  $n \in \mathbb{Z}$ , if  $n^2$  is odd, then  $n$  is odd.