

# Homework 1

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## Question 1

A Pythagorean triple consists of three positive integers  $a$ ,  $b$ , and  $c$ , such that

$$a^2 + b^2 = c^2$$

## Question 2

Given a quadratic equation  $ax^2 + bx + c = 0$  where  $x$  is an unknown variable,  $a$ ,  $b$ , and  $c$  are constants. The solution to the quadratic equation is called quadratic formula and is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

## Question 3

$$1 + 2 + 3 + 4 + \cdots + 98 + 99 + 100 = \sum_{x=1}^{100} x$$

$$x_1 + x_2 + x_3 + \cdots + x_n = \sum_{i=1}^n x_i$$

$$x_1 \times x_2 \times x_3 \times \cdots \times x_n = \prod_{i=1}^n x_i$$

$$f(x) = \int_a^b x^2 dx$$

## Question 4

$f: \mathbb{Z} \mapsto \mathbb{N}$  defined by

$$f(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x - 1 & \text{if } x < 0 \end{cases}$$

## Question 5

$$V = \{x \in \mathbb{Z} | x < 100\} \cap \{x \in \mathbb{Z} | x \text{ is prime}\}$$

$$V \subset W$$

## Question 6

$$((\alpha \rightarrow \beta) \wedge (\beta \rightarrow \gamma)) \rightarrow (\alpha \rightarrow \gamma)$$

## Question 7

Consider the following statements about integers:

1. For every  $x$ , there is a  $y$ , such that  $x + y = 0$
2. There is a  $y$ , such that for every  $x$ , we have  $x + y = 0$

In symbols, these statements are written respectively:

1.  $\forall x \exists y x + y = 0$
2.  $\exists y \forall x x + y = 0$

## Question 8

If  $x$  is even, then  $x^2$  is even

*Proof.*  $x$  is an even number.

$\exists a \in \mathbb{Z}$  such that  $x = 2a$

$$x^2 = (2a)^2 = 4a^2 = 2(2a^2)$$

Let  $c = 2a^2, c \in \mathbb{Z}$

$$x^2 = 2c$$

Therefore,  $x^2$  is even. □

## Question 9

$x$	$y$	$x \vee y$
TRUE	TRUE	TRUE
TRUE	FALSE	TRUE
FALSE	TRUE	TRUE
FALSE	FALSE	FALSE