Homework 10: Discrete Mathematics Spring 2021 – Probability

Due April 18 @11:59:00pm

As always, show your work.
Note: You don't need to calculate the exact number,
you can leave your response as a mathematical expression.

1. **Sample Space**

   You flip a coin consecutive times. The outcome of each flip is either Heads (H) or Tails (T). You observe the outcomes and note them. You stop the flipping if you obtain two consecutive Tails or after four flips, whichever comes first. Write the sample space for your experiment.

2. **Axioms of probability**

   In a summer camp, $\frac{3}{4}$ of kids are doing science, $\frac{1}{2}$ are doing art, and $\frac{1}{8}$ are doing neither. Please answer the following questions:

   1. What is the probability that a kid is in science or arts. Hint: Use the axioms of probability.
   2. What is the probability that a kid is in science and arts. Hint: Use the axioms of probability.
   3. What is the probability that a kid is in art but not science. Hint: Draw a table dividing the sample space into four mutually exclusive events.

3. **Probability calculation**

   Suppose you are choosing a letter at random from the word DISCRETE and your friend chooses a letter at random from the word ALGEBRA . What is the probability that you choose the same letter?

4. **Probability calculation**

   An urn contains five blue, six green and seven red balls. You choose five balls at random from the urn, without replacement (so you do not put a ball back in the urn after you pick it), what is the probability that you chose at least one ball of each color?
   (Hint: Consider the events: B, G, and R, denoting respectively that there are no blue, no green and no red balls chosen.)

5. **Conditional probability Bayes rule**

   Assume that 5% of males and 0.25% of female play soccer.

   If a soccer player is chosen at random, what is the probability of that person being male?

6. **Conditional probability Bayes rule**

   An unfair coin shows HEADS with probability p and TAILS with probability $1 - p$. Suppose this coin is tossed twice. Let A be the event that the coin comes up first HEADS and then TAILS, and let B be the event that the coin comes up first TAILS and then HEADS.

   (a) Calculate $P(A)$ and $P(B)$.

   (b) Same context as the previous question. Calculate $P(A|A \cup B)$ and $P(B|A \cup B)$
   Please show your work.