

Homework 1

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Question 1

A Pythagorean triple consists of three positive integers a , b , and c , such that

$$a^2 + b^2 = c^2$$

Question 2

Given a quadratic equation $ax^2 + bx + c = 0$ where x is an unknown variable, a , b , and c are constants. The solution to the quadratic equation is called quadratic formula and is given by:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Question 3

$$1 + 2 + 3 + 4 + \cdots + 98 + 99 + 100 = \sum_{x=1}^{100} x$$

$$x_1 + x_2 + x_3 + \cdots + x_n = \sum_{i=1}^n x_i$$

$$x_1 \times x_2 \times x_3 \times \cdots \times x_n = \prod_{i=1}^n x_i$$

$$f(x) = \int_a^b x^2 dx$$

Question 4

$f: \mathbb{Z} \mapsto \mathbb{N}$ defined by

$$f(x) = \begin{cases} 2x & \text{if } x \geq 0 \\ -2x - 1 & \text{if } x < 0 \end{cases}$$

Question 5

$$V = \{x \in \mathbb{Z} | x < 100\} \cap \{x \in \mathbb{Z} | x \text{ is prime}\}$$

$$V \subset W$$

Question 6

$$((\alpha \rightarrow \beta) \wedge (\beta \rightarrow \gamma)) \rightarrow (\alpha \rightarrow \gamma)$$

Question 7

Consider the following statements about integers:

1. For every x , there is a y , such that $x + y = 0$
2. There is a y , such that for every x , we have $x + y = 0$

In symbols, these statements are written respectively:

1. $\forall x \exists y x + y = 0$
2. $\exists y \forall x x + y = 0$

Question 8

If x is even, then x^2 is even

Proof. x is an even number.

$\exists a \in \mathbb{Z}$ such that $x = 2a$

$$x^2 = (2a)^2 = 4a^2 = 2(2a^2)$$

Let $c = 2a^2, c \in \mathbb{Z}$

$$x^2 = 2c$$

Therefore, x^2 is even. □

Question 9

| x | y | $x \vee y$ |
|-------|-------|------------|
| TRUE | TRUE | TRUE |
| TRUE | FALSE | TRUE |
| FALSE | TRUE | TRUE |
| FALSE | FALSE | FALSE |