Homework 3: Discrete Mathematics Spring 2021 – Logic II

Due Sunday February 7 @11:59:00pm

Show all your work to receive full credit

1. **Laws of PL**

   Consider the following proposition:

   $$\neg p \vee (q \wedge (r \vee p))$$

   Write the negation of the proposition above and simplify it using the laws of logic into an expression that can't be reduced further.

2. **Inference**

   Consider the following premises:

   1. $p \rightarrow (q \vee r)$
   2. $\neg q$
   3. $\neg r$

   Prove using inference rules that we can derive the conclusion:

   $$\neg p$$

   Make sure to cite the inference rule you used at each step.

3. **FOL**

   Consider the following statements:

   1. Some cats are lovely.
   2. All mice are smart.
   3. All cats are lovely and all mice are smart.
   4. All cats play with all mice.

   Let the domain be **Animals**. Consider the following predicates:

   1. $cat(x)$: x is a cat
   2. $mouse(x)$: x is a mouse
   3. $play(x, y)$: x plays with y
   4. $lovely(x)$: x is lovely
   5. $smart(x)$: x is smart.

   1. Translate the following statements in FOL. Use only the predicates above along with connectives and quantifiers. No justification needed.

4. **FOL**

   Same context as the previous question (cats and mice FOL).

   Write the negation of all FOL expressions your wrote for the statement:

   All cats are lovely and all mice are smart

5. **FOL**

Consider the following statement:

THERE IS NO SMALLEST POSITIVE RATIONAL NUMBER.

1. Is the statement true? Explain.

2. Translate the statement to FOL. You are required to:
   - Consider the domain to be the set of rational numbers $\mathbb{Q}$. Use the set $\mathbb{Q}$. Do not use $\mathbb{Q}^+$.
   - No need to define what a rational number q is, just use $q \in \mathbb{Q}$).
   - Invent as many variables as you need.
   - You cannot use predicates like "smallest(q)" or variants of it. You can only use comparison operators between numbers like $\leq, <, \geq, >, =$.