

Homework 8: Discrete Mathematics Spring 2021 – Number Theory I

Due Sunday April 4th @11:59:00pm

Show all your work to receive full credit
Please note that we will make this a homework quiz in Gradescope

1. Proof by Strong induction

Let $P(n)$ the proposition that we can form n dollars using just \$2 and \$5 bills.

1. Is this proposition true to form \$1? \$2? \$3? \$4? \$5?
2. Prove by strong induction that $P(n)$ is true for all $n \geq 5$.

2. Divisibility

What are the quotient and remainder when

1. 44 is divided by 8
2. 777 is divided by 21
3. -123 is divided by 19
4. 789 is divided by 23

3. GCD

- (a) Find the greatest common divisor of 2328 and 440 by applying Euclid's algorithm. Show the steps of the algorithm.
- (b) A mother is organizing her child birthday party. She has 63 cupcakes and 84 cookies. She wants to include both snacks on each plate. Plates should be identical, that is, we want the same number of cupcakes in all plates and we want the same number of cookies in all plates. It is not necessary to have the same number of cupcakes and the same number of cookies in each plate. She wants to use all of the snacks. What is the greatest number of plates she can make using all snacks? How many of each type of snacks will be on each of these plates?

4. GCD

- (a) For which n does $\gcd(n, n+2) = 1$ hold?
- (b) Suppose $\gcd(a, b) = 1$ and $\gcd(b, c) = 1$. Does this imply that $\gcd(a, c) = 1$?

5. Prime factorization

- (a) Find the prime factorization of each of these integers.
 1. 39
 2. 108
 3. 10!
- (b) Consider two positive integers and their prime factorization:
 $a = 2^4 \times 3^4 \times 5^2 \times 13^4 \times 49$ and $b = 2^2 \times 5^4 \times 7^4 \times 11^2 \times 27$.
What is the greatest common divisor of a and b ?

6. GCD properties

Prove that:

$$\gcd(a, b, c) = \gcd(a, \gcd(b, c)).$$