An integer is even if and only if it is divisible by 2

$$\forall x \in \mathbb{Z} \; even(x) \longleftrightarrow divisible\_by\_2(x)$$

$$\forall x \in \mathbb{Z} \; even(x) \longleftrightarrow 2 \mid x$$

2 divides x

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to prove this, you need to show that it works for all values of x

	$\forall x \; P(x)$	$\exists x \; P(x)$
Prove	Proof - induction - direct proof	Provide one example
Disprove	Provide a counter example	Same as proving $\forall x \; \neg P(x)$ Proof

Proving negation $(\exists x \; \neg P(x))$

All students in 3203 have red hair

A counter example would be
"Ryan does not have red hair"

There exists a natural number that is both prime and even.

Translate to FOL, the Golbach Conjecture

sum_of_two_primes(n)
even(n)

$\forall n \in \mathbb{N}\ \text{even}(n) \land n > 3 \implies \text{sum\_...}(n)$

Don't use sum_of_two_primes(n) as a predicate

(this means you need to compose it as it would be w/o predicates, using variables)

$p \to q \equiv \neg q \to \neg p$ Contrapositive

Proof by enumeration

When the domain is small,

$\forall x \in D \; p(x)$

Example: $\forall x \in \mathbb{Z} \; \underbrace{1 < x < 5}_{\text{Predicate}} \Rightarrow 2 | x \vee 3 | x$

Proof by Enumeration

$x = 2$ $\quad 2 | x$ ✓

$x = 3$ $\quad 3 | x$ ✓

$x = 4$ $\quad 2 | x$ ✓

end of Proof

$\square$

Prove that for any $n \leq 3$ and n is a positive integer that $n! < 2^n$

$\forall n \in \mathbb{Z} \quad 0 < n \leq 3 \Rightarrow n! < 2^n$

$n = 1 \Rightarrow n! \; 2^n$ ✓

$n = 2 \Rightarrow n! < 2^n$ ✓

$n = 3 \Rightarrow n! < 2^n$ ✓ $\square$

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1. <u>Direct Proof</u>

$$\begin{cases} \forall x \in D \quad p(x) \Rightarrow q(x) \\ \forall x \; \forall y \quad p(x,y) \Rightarrow q(x,y) \end{cases}$$

The sum of two even integers is an even integer

(invent variables)

Ex. Proof

$\forall x \in \mathbb{Z} \ \forall y \in \mathbb{Z}$

$\text{even}(x) \land \text{even}(y) \implies \text{even}(x+y)$