Proofs on Sets

Proof 1: Inclusion of two sets

$A \subseteq B$

*basically show how an element in A is equivalent to what you'd get in B

To show this, let $x \in A$, show $x \in B$

Therefore, $A \subseteq B$

*If you take an element in A and you can show how it's in B

Proof 2: Equality of two sets

$$A = B$$

* you're showing how an element in A is equivalent to an element in B and vice versa

To show this, let $x \in A$, show $x \in B$

AND

let $x \in B$, show $x \in A$

Therefore, $A = B$

Example: $E = \{x \in \mathbb{Z} \mid x \text{ is even}\}$

$F = \{y \in \mathbb{Z} \mid y = a + b \text{ for some odd integers } a, b\}$

Because this is an equality set, you must show $E \subseteq F$ and $F \subseteq E$

$E \subseteq F$ (show even int is product of two odds)

$x \in \mathbb{Z}$ $x = 2a,\ a \in \mathbb{Z}$

$x = 2a + 1 - 1$

$2a+1$ is odd

-1 is odd

$2a+1-1$ is the sum of two odds

Therefore, $E \subseteq F$

$F \subseteq E$ (show sum of two odds is an even int)

$y \in \mathbb{Z}$ $y = 2b+1 + 2c+1,\ b, c \in \mathbb{Z}$

$y = 2b + 2c + 2$

$y = 2(b+c+1)$

$a = b+c+1$

$y = 2a$, the form of an even int

Therefore, $F \subseteq E$

4.3 Problems

1. Prove DeMorgan's law of intersection

$$\overline{(A\cap B)} = \bar{A} \vee \bar{B}$$

This is Proof 2's template

$\overline{A\cap B} \subseteq \bar{A} \vee \bar{B}$, $x \in \overline{A\cap B}$	$\bar{A} \cup \bar{B} \subseteq \overline{A\cap B}$, $x \in \bar{A} \cup \bar{B}$
$x \in \bar{A} \cup \bar{B} \equiv x \in \overline{A \cap B}$	$x \in \overline{A \cap B} \equiv x \in \bar{A} \vee \bar{B}$
$\equiv x \notin (A \cap B)$	$\equiv x \in \bar{A} \cup x \in \bar{B}$
$\equiv x \notin A \cap x \notin B$	$\equiv x \in (\bar{A} \vee \bar{B})$
$\equiv x \notin A \wedge x \notin B$	$\equiv x \notin (A \wedge B)$
$\equiv x \in \bar{A} \wedge x \in \bar{B}$	$\equiv x \in \overline{(A \wedge B)}$
$\equiv x \in \bar{A} \cup \bar{B}$	

Why can we make "union" from "And"?

2. Prove that if $A \subseteq B$, then $A \cup B = B$

equals, template 2

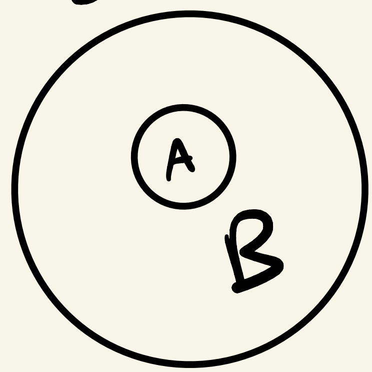
$A \cup B$

why "or"?

$\{x \mid x \in A \vee x \in B\}$

$\equiv \{x \mid x \in A \cup x \in B\}$

then $x \in B$ b/c $A \subseteq B$



$B \subseteq A \cup B$

$\{x \mid x \in B\}$

$\equiv \{x \mid x \in A \vee x \in B\}$

$\equiv \{x \mid x \in A \cup B\}$ b/c $A \subseteq B$

3. Let $A = \{1, \{2\}, 3, \{4,5\}, \{3\}, 6\}$

1. $3 \in A$ T
2. $\{3\} \in A$ F
3. $\{3\} \subseteq A$ T
4. $2 \notin A$ T
5. $2 \not\subseteq A$ T
6. $\{2\} \in A$ T
7. $\{2\} \not\subseteq A$ T
8. $\{\{2\}\} \not\subset A$ F
9. $\{\} \not\subset A$ F
10. $\{\} \subseteq A$ T