Homework 3 - Logic II (Spring 2021)

Ryan, Soeyadi rs4163

February 7, 2021

Question 1

$\neg p \vee (q \wedge (r \vee p))$

$\neg(\neg p \vee (q \wedge (r \vee p)))$
$p \wedge \neg(q \wedge (r \vee p))$
$p \wedge (\neg q \vee \neg(r \vee p))$
$p \wedge (\neg q \vee (\neg r) \wedge (\neg p))$
$p \wedge (\neg q \vee (\neg r \wedge \neg p))$
$p \wedge (\neg q \vee \neg r) \wedge (\neg q \vee \neg p)$
$p \wedge (\neg q \vee \neg p) \wedge (\neg q \vee \neg r)$
$p \wedge (\neg p \vee \neg q) \wedge (\neg q \vee \neg r)$
$(p \wedge \neg p) \vee (p \wedge \neg q) \wedge (\neg q \vee \neg r)$
$False \vee (p \wedge \neg q) \wedge (\neg q \vee \neg r)$
$(p \wedge \neg q) \wedge (\neg q \vee \neg r)$
$p \wedge \neg q \wedge (\neg q \vee \neg r)$
$p \wedge \neg q$

Question 2

1.$p \implies (q \vee r)$
2.$\neg q$
3.$\neg r$
4.$\neg q \wedge \neg r$ (2, 3 - Conjunction inference rule)
5.$\neg(q \vee r)$ (4 - DeMorgan's Law)
6.$\neg p$ (1, 5 - Modus tollens)

Question 3

Domain: A (for Animals)

1. $\exists x\ \mathcal{E}A\ cat(x) \land lovely(x)$
2. $\forall x\ \mathcal{E}A\ mouse(x) \implies smart(x)$
3. $\forall x\ \mathcal{E}A(cat(x) \implies lovely(x)) \land (mouse(x) \implies smart(x))$
4. $\forall x\ \forall y\ \mathcal{E}A\ cat(x) \land mouse(y) \implies play(x, y)$

Question 4

Domain: A (for Animals)

1. $\neg(\exists x\ \mathcal{E}A\ cat(x) \wedge lovely(x))$
   $\forall x\ \mathcal{E}A\ \neg cat(x) \vee \neg lovely(x))$

2. $\neg(\forall x\ \mathcal{E}A\ \ mouse(x) \implies smart(x))$
   $\neg(\forall x\ \mathcal{E}A\ \neg mouse(x) \vee smart(x))$
   $\exists x\ \mathcal{E}A\ mouse(x) \wedge \neg smart(x)$

3. $\neg(\forall x\ \mathcal{E}A\ (cat(x) \implies lovely(x)) \wedge (mouse(x) \implies smart(x)))$
   $\neg(\forall x\ \mathcal{E}A\ (\neg cat(x) \vee lovely(x)) \wedge (\neg mouse(x) \vee smart(x)))$
   $\exists x\ \mathcal{E}A\ \neg(\neg cat(x) \vee lovely(x)) \vee \neg(\neg mouse(x) \vee smart(x)))$
   $\exists x\ \mathcal{E}A\ (cat(x) \wedge \neg lovely(x)) \vee (mouse(x) \wedge \neg smart(x))$

4. $\neg(\forall x\ \forall y\ \mathcal{E}A\ cat(x) \wedge mouse(y) \implies play(x,y))$
   $\neg(\forall x\ \forall y\ \mathcal{E}A\ \neg(cat(x) \wedge mouse(y)) \vee play(x,y))$
   $\neg(\forall x\ \forall y\ \mathcal{E}A\ (\neg cat(x) \vee \neg mouse(y)) \vee play(x,y))$
   $\exists x \exists y\ \mathcal{E}A\ \neg(\neg cat(x) \vee \neg mouse(y)) \wedge \neg play(x,y)$
   $\exists x \exists y\ \mathcal{E}A\ (cat(x) \wedge mouse(y)) \wedge \neg play(x,y)$

Question 5

1. This is True. If we have a rational number, meaning that it is the quotient of two integers a and b, and this number is positive, or > 0, we can always multiply the divisor b by 2 to get an even smaller positive rational number N.

2. Domain: Q
Variables: Rational number, q
An integer, b

$\forall q \; \exists Q \; \frac{q}{b} > \frac{q}{2b} > 0$