Homework 6 - Relations (Spring 2021)

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Question 1

4, 9, 5, 8, 7, 1, 3, 6, 2

Question 2

Set $A = \{10, 20, 30, 40, 50\}$
Let R be a relation defined on A

1. R is reflexive would require that the pairs

   $\{(10, 10), (20, 20), (30, 30), (40, 40), (50, 50)\}$ must be in R

2. R is symmetric would require no pairs, based on the fact that the empty set is vacuously symmetric.

3. If R is antisymmetric, and (x, y) and (y, x) belong to R
   if R is in fact antisymmetric
   $x = y$, since antisymmetry exists even if we have a pair such as $(3, 3)$ or $(5, 5) \in R$

4. Let $R = \{(10, 20)\}$
   R is antisymmetric and transitive

Question 3

1. S as a Boolean matrix

	0	1	2	3
0	1	1	0	1
1	1	1	0	0
2	0	0	1	0
3	1	0	0	1

2. $S^{-1} = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$

3. $S = \{(0,0), (0,1), (0,3), (1,0), (1,1), (2,2), (3,0), (3,3)\}$

   S is symmetrical and reflexive
   S^{-1} is symmetrical and reflexive
   R is irreflexive and antisymmetric and transitive
   T is non-reflexive and antisymmetric

Question 4

1. Let $x, y \in \mathbb{R}$
   $xRy\ IFF\ (x - y) \in \mathbb{Z}$

   Reflexivity: $\forall x \in A\ x\ R\ x$
   $x - x = 0$
   $0 = 0$
   $0\ \in \mathbb{Z}$

   Symmetric: $x - y \in \mathbb{Z} \implies y - x \in \mathbb{Z}$
   $x - y \implies (-x + y)$
   $x - y \implies -(x - y)$
   Since $x - y \in \mathbb{Z}$, the negation of it is also an integer

   Transitive: $\forall x, y, z \in A(\ xRy \wedge yRz \implies\ xRz)$
   $x - y + y - z$
   $x - z \in \mathbb{Z}$

2. $[1.75]$
$\{... - 3.25, -2.25, -1.25...\} \cup \{...0.75, 1.75, 2.75...\}$

Question 5

1.

$[1] = \{\pm 1, \pm 3, \pm 5...\}$
$[2] = \{\pm 2, \pm 4, \pm 6...\}$
$[3] = [1]$
$[4] = [2]$
$[5] = [1]$
$[6] = [2]$

2. There are 2 distinct equivalence classes; once we go past [2] and so on, we repeat the evens and the odds.

Question 6

1.

$[0] = \{0, -4, 4, -8, 8, -12, 12...\}$
$[1] = \{1, 1 \pm 4, 1 \pm 8, 1 \pm 12...\}$
$[2] = \{2, 2 \pm 4, 2 \pm 8, 2 \pm 12...\}$
$[3] = \{3, 3 \pm 4, 3 \pm 8, 3 \pm 12...\}$
$[4] = [0]$
$[5] = [1]$

2. There are four distinct equivalence classes; once we reach [4], we start repeating the sets from [0] and so on; the remainders are found from the same integers.

3. The equivalence classes are disjoint since they are not equal, and they make up all of the integers.