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Computational Neuroscience: Homework 4

Due on April 25, 2012

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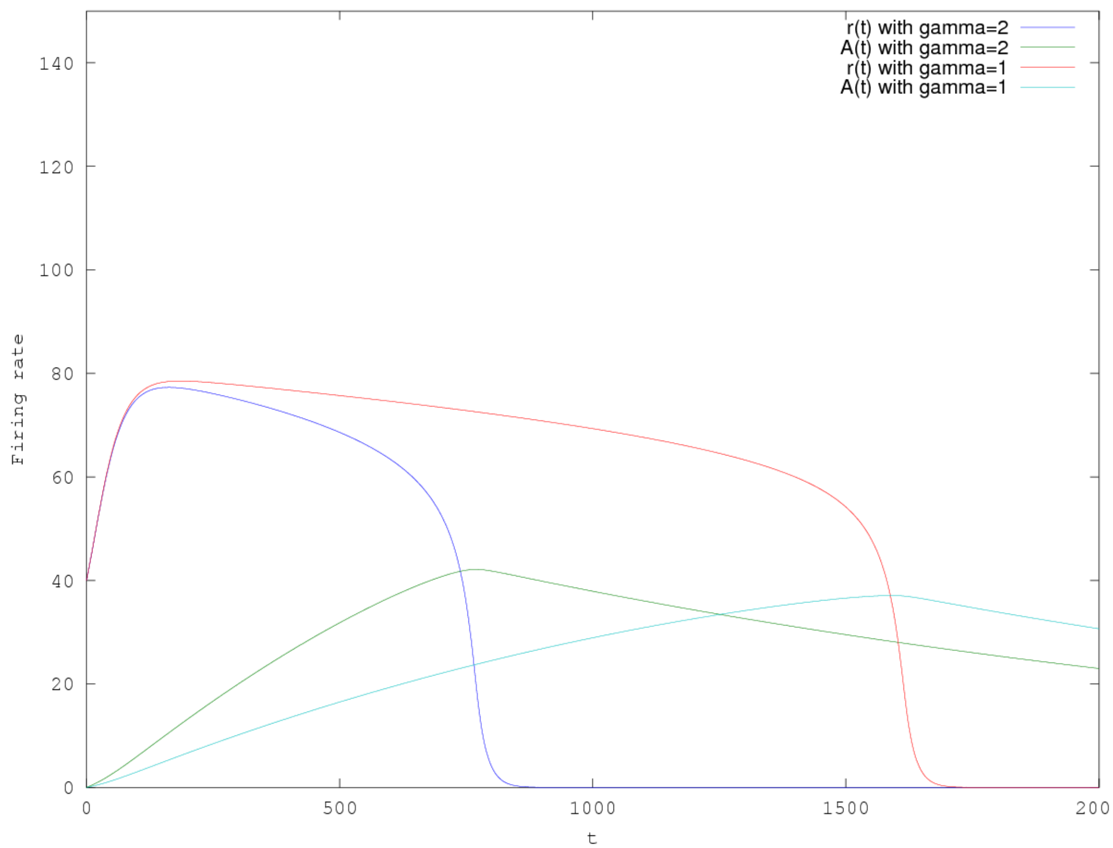
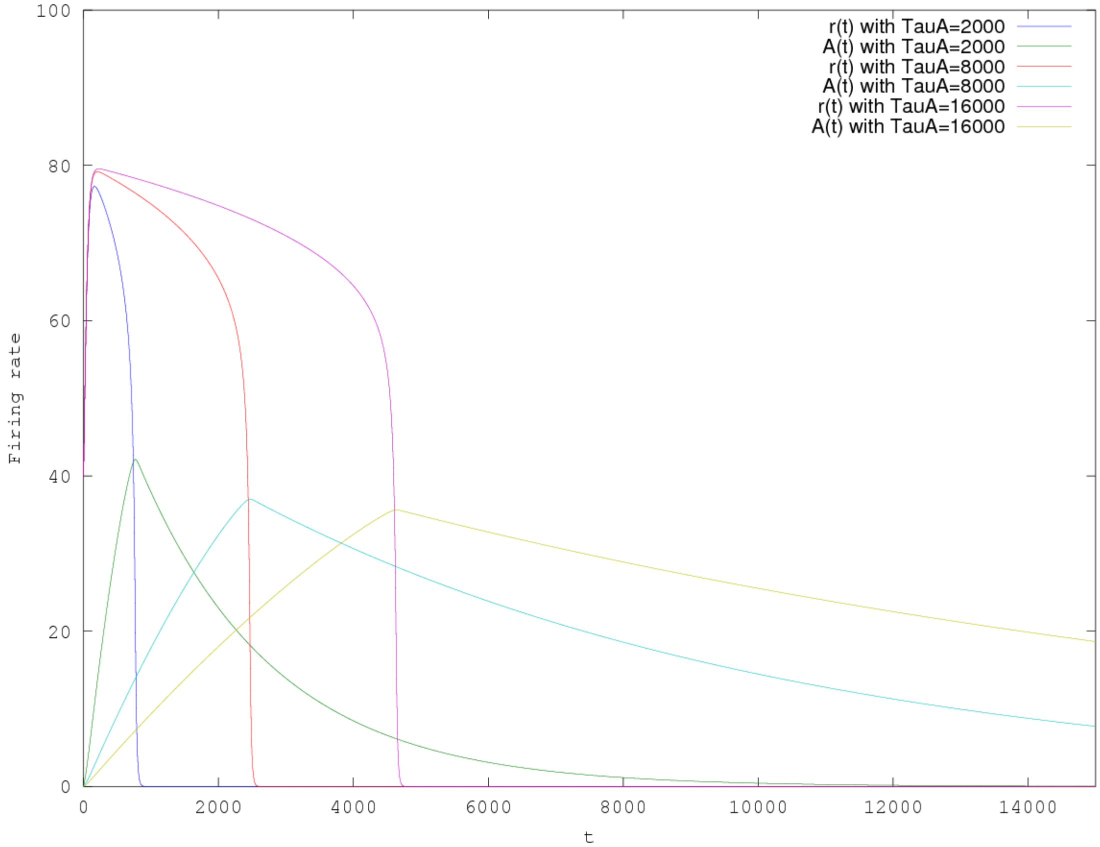
Short-Term Memory model with Adaptation: 1

I based my Matlab script (run in Octave) on STM_MF.m. For the first run, I used $\gamma = 2$ to match the graph in the homework description. With $\gamma = 1$, A(t) increased more slowly, and therefore took longer to grow large enough to drive down r(t).



2

For the second set of trials, I left γ at 2. As I increased τ_A, the system slowed down. A(t) changed more slowly, so it took longer for A(t) to increase to drive R(t) down, and it took longer for A(t) do decrease again after R(t) plummeted. The shape of the curves was qualitatively similar, however.

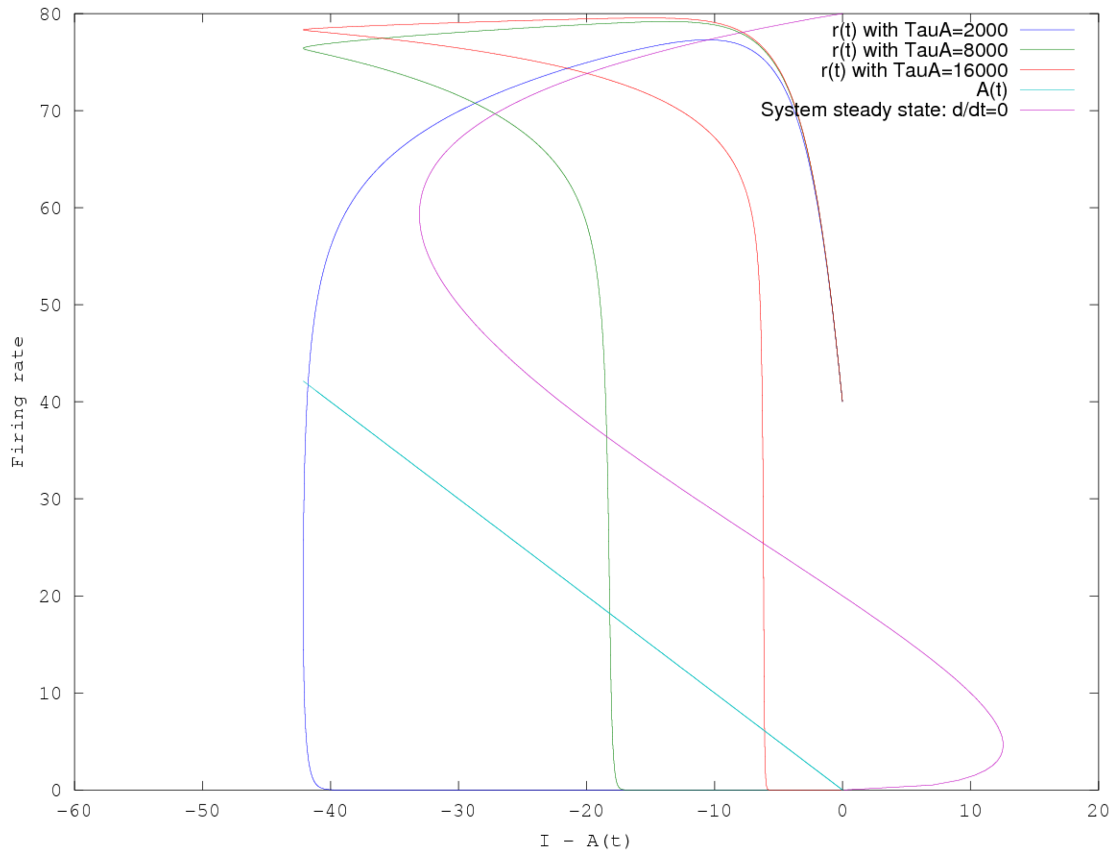


3

I approached finding the steady state curve analytically for individual values of r. Given: $\tau(dr/dt) = -r + S_{NR}(ar - A)$ (I = 0), we can set dr/dt to 0 and calculate from there.

$$\begin{aligned}
r &= S_{NR}(ar - A) \\
r &= \frac{M * (ar - A)^2}{\sigma^2 + (ar - A)^2} \\
\delta &= (ar - A)^2 \\
r &= \frac{M * \delta}{\sigma^2 + \delta} \\
r\sigma^2 + r\delta &= M * \delta \\
\frac{r\sigma^2}{\delta} &= M - r \\
\frac{1}{\delta} &= \frac{M - r}{r\sigma^2} \\
\delta &= \frac{r\sigma^2}{M - r} \\
A^2 - 2arA + a^2r^2 - \frac{r\sigma^2}{M - r} &= 0
\end{aligned}$$

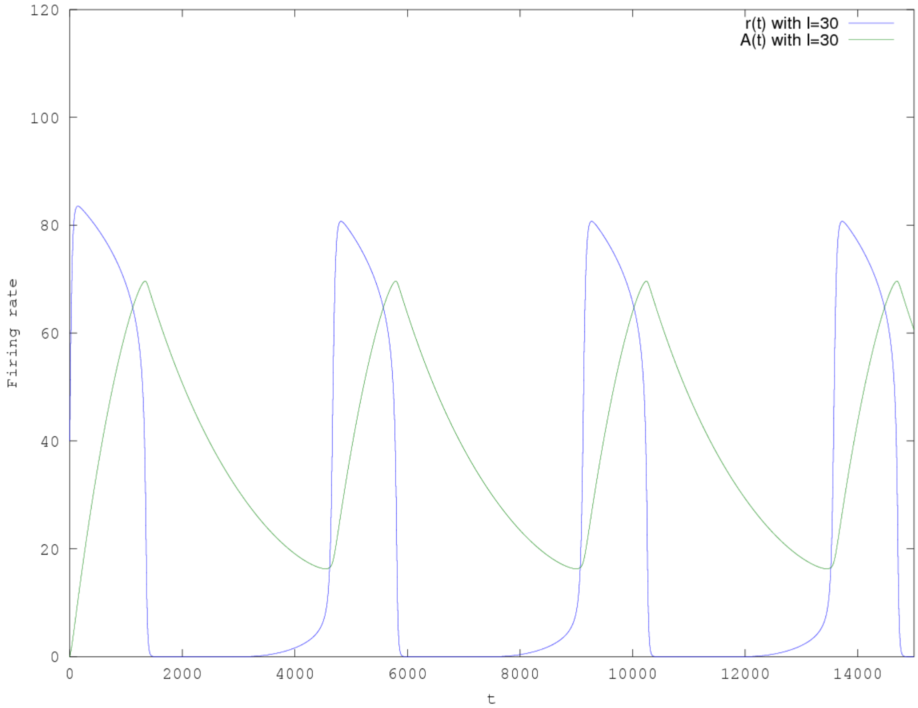
Since everything other than A is constant for a given r, this is a quadratic in A, easily solved. In the graphs below, we can see R jump to the upper part of the steady state, stay there for a little while as A(t) grows (STM), then fall to the bottom as A(t) dominates the input. When τ_A is lowest, we can see A(t) immediately forcing down r(t), so it doesn't spend much time at the top of the graph, but as we raise τ_A, r(t) remains for much of A(t)'s rise.

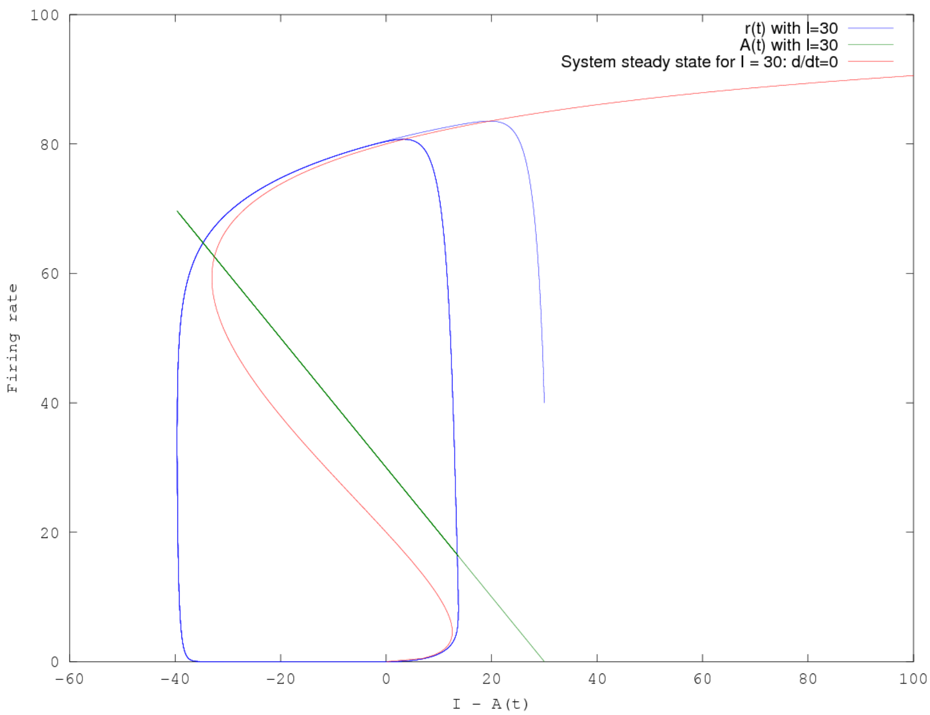


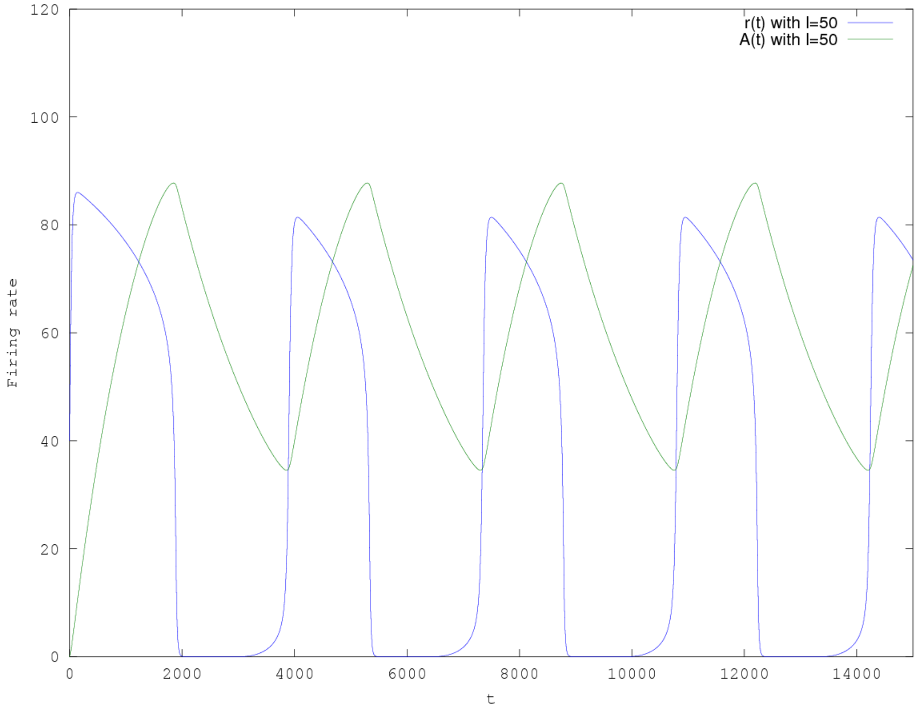
4

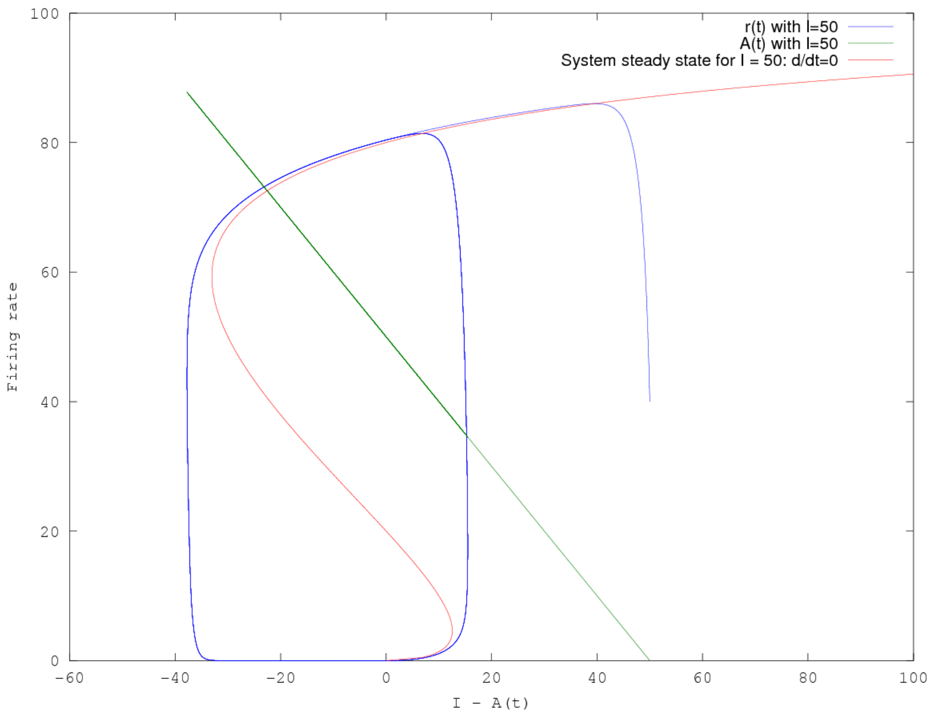
For varying the values of I, I set τ_A back to 2000. In these time courses, instead of the firing rate falling to nothing, the constant input current causes the neuron to reach a steady state solid firing rate. The steady state is about 62 for I=30, 72 for I=50, and 78 for I=70. For any given I, we could find what steady state it would go to by first checking the sign of dr/dt, to know whether it would go to a zero or positive steady state, then if $dr/dt > 0$, we can solve for r in $r = S_{NA}(ar - A(r))$.

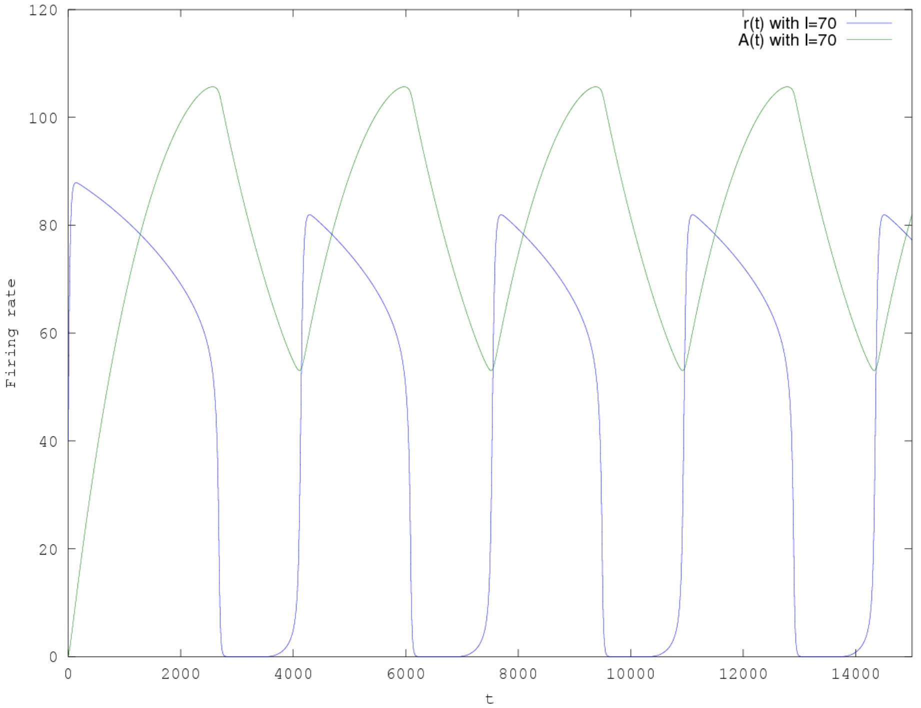
In the six graphs below, we can see that the system oscillates for each of these values of I, fastest for I=50, slightly slower for I=70 since A is greater, and slowest for I=30. In the phase planes, we can see that for each input value, the firing rate first jumps upwards, then goes into a cycle of slowly decreasing as the input decreases, falling fast as A(t) rises, then jumping again as A(t) falls.

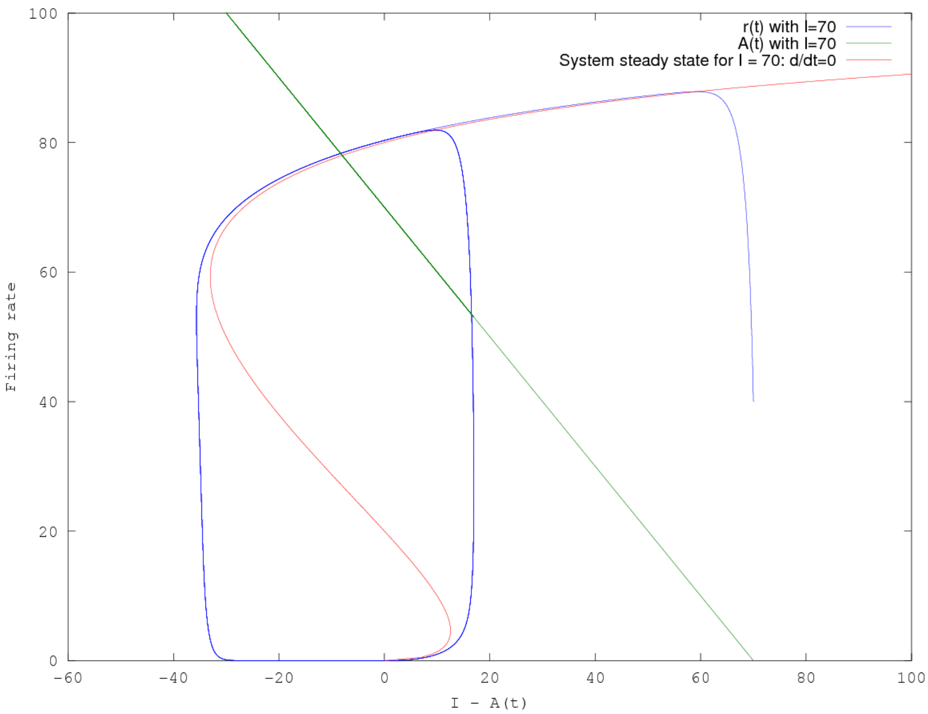

r(t) with I=30
A(t) with I=30
Firing rate
t
0
20
40
60
80
100
120
0
2000
4000
6000
8000
10000
12000
14000


r(t) with I=30
A(t) with I=30
System steady state for I = 30: d/dt=0
Firing rate
I - A(t)
0
20
40
60
80
100
-60
-40
-20
0
20
40
60
80
100


r(t) with I=50
A(t) with I=50
Firing rate
t
0
20
40
60
80
100
120
0
2000
4000
6000
8000
10000
12000
14000


r(t) with I=50
A(t) with I=50
System steady state for I = 50: d/dt=0
Firing rate
I - A(t)
0
20
40
60
80
100
-60
-40
-20
0
20
40
60
80
100

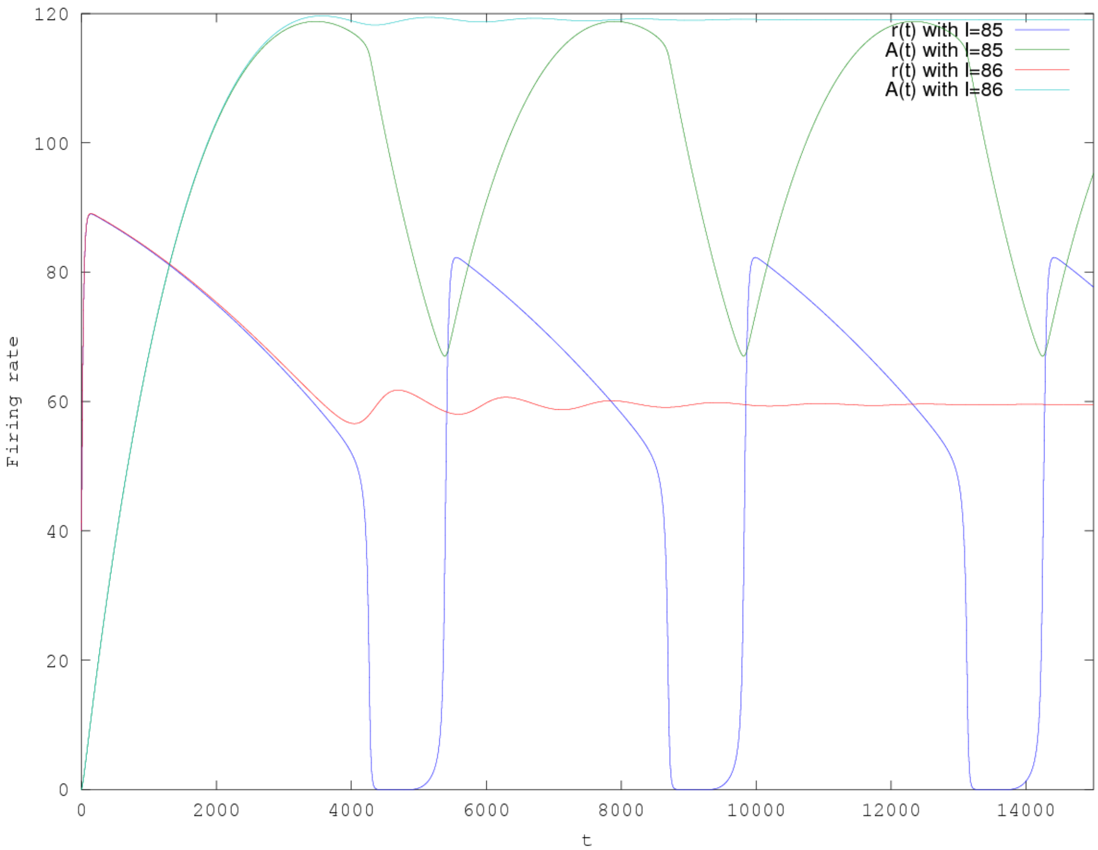

r(t) with I=70
A(t) with I=70
Firing rate
t
0
20
40
60
80
100
120
0
2000
4000
6000
8000
10000
12000
14000


r(t) with I=70
A(t) with I=70
System steady state for I = 70: d/dt=0
Firing rate
I - A(t)
0
20
40
60
80
100
-60
-40
-20
0
20
40
60
80
100

In order to find the threshold where the oscillating regime ends and a forced constant firing rate begins, I hypothesized that r(t) would stabilize at the left side of the upper part of the steady state curve. Then, to stop oscillation, I tried to choose an I such that $dA/dt = 0$ when $r(t) \approx 60$. Taking $dA/dt = 0$ gives $A = \gamma r = 2r$, so A should stabilize at about $A = 120$. Then, since we know $dr/dt = 0$, we can look at that equation.

$$\begin{aligned}
r &= S_{NA}(ar + I - A) \\
60 &= S_{NA}(180 + I - 120) \\
60 &= S_{NA}(60 + I) \\
60 &= S_{NA}(60 + I) \\
60 &= \frac{M * (60 + I)^2}{\sigma^2 + (60 + I)^2} \\
60 &= \frac{100 * (60 + I)^2}{120^2 + (60 + I)^2} \\
60 &= \frac{100 * (60 + I)^2}{120^2 + (60 + I)^2} \\
0 &= 40I^2 + 4800I - 720000 \\
I &\approx 87
\end{aligned}$$

I looked in the region around $I = 87$, and found that the regime change actually happens between $I = 85$ and $I = 86$.



r(t) with I=85
A(t) with I=85
r(t) with I=86
A(t) with I=86
Firing rate
t
0
20
40
60
80
100
120
0
2000
4000
6000
8000
10000
12000
14000