Trendlines and numerical issues in Google Spreadsheet

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Abstract

Google Spreadsheet is an online spreadsheet program in the style of Microsoft Excel. It provides much of Excel's data management and charting functionality, though not all, and also allows spreadsheets to be shared and edited cooperatively. Perhaps the most powerful feature of Google Spreadsheets is the API, which allows the owner of a spreadsheet to programatically modify and read it. Using that API, Google Spreadsheet can provide a turnkey and user friendly window into arbitrary data as it is gathered. The number of Google apps users is growing, with at least 40 million users[1] that have apps accounts, and some of those users will depend on Spreadsheet's accuracy for their business. This paper will examine Google Spreadsheet's accuracy at computing linear best fit lines with ill-conditioned data, comparing it with GNU Octave and Antony Kaplan's work on Excel [2] as well as some related quirks of Google Spreadsheet.

[1]Google Apps for Enterprise: http://www.google.com/enterprise/apps/business/

[2]Kaplan 2010 http://cs.nyu.edu/courses/spring12/CSCI-UA.0421-001/kaplanproject.pdf

Floating Point Numbers in Google Spreadsheet

It can be difficult to tell what sort of floating point math a spreadsheet program uses. Microsoft Excel, for instance, uses cosmetic rounding to make binary floating point numbers appear to be decimal floating point numbers [3]. Google, on the other hand, made no attempt to confirm or deny the floating-point truth. To the best of my knowledge, exhibited below, Google uses software implemented decimal floating point.

If we look at the edge of Google Spreadsheet's precision in binary, we see that $1 + 2^{-46} = 1 + 2^{-47} \neq 1$. This suggests that Google's floating point is not in the form $m * 2^x$.

The edge of Google Spreadsheet's precision with binary floating point:

i	2^{-i}	$1 + 2^{-i}$	$1 = 1 + 2^{-i}$	$1 + 2^{-i} = 1 + 2^{-(i+1)}$
44	0.000000000000057	1.00000000000006	FALSE	FALSE
45	0.000000000000028	1.00000000000003	FALSE	FALSE
46	0.000000000000014	1.00000000000001	FALSE	TRUE
47	0.000000000000007	1.00000000000001	FALSE	FALSE
48	0.000000000000004	1	TRUE	TRUE
49	0.000000000000002	1	TRUE	TRUE
50	0.000000000000001	1	TRUE	TRUE
51	0	1	TRUE	TRUE

In decimal we see no such irregularities:

The edge of Google Spreadsheet's precision with decimal floating point:

i	10^{-i}	$1 + 10^{-i}$	$1 = 1 + 10^{-i}$	$1 + 10^{-i} = 1 + 10^{-(i+1)}$
12	0.000000000001	1.000000000001	FALSE	FALSE
13	0.0000000000001	1.0000000000001	FALSE	FALSE
14	0.00000000000001	1.00000000000001	FALSE	FALSE
15	0.000000000000001	1	TRUE	TRUE

[3] How Futile are Mindless. . . : http://www.cs.berkeley.edu/ wkahan/Mindless.pdf

This gives us a few identities:

$$1 + 2^{-47} \neq 1$$
$$1 + 2^{-48} = 1$$
$$1 + 2^{-46} = 1 + 2^{-47} = 1 + 10^{-14}$$
$$1 = 1 + 10^{-15}$$

Where Excel used hardware binary floating point and hushed it up, Google seems to have implemented true decimal floating point.

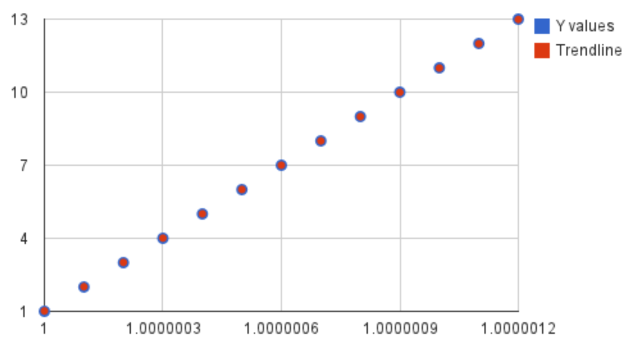
The TREND function

Google Spreadsheet's basic linear best fit function is TREND(data_X,data_Y,x). Given two vectors of data, it calculates the value of the linear best fit at x. However, as the data it is given becomes ill conditioned, the accuracy of TREND desintigrates. Here we examine data series' of the form $(1 + i/10^{-j}, i + 1)$, where i ranges from 0 to 12.

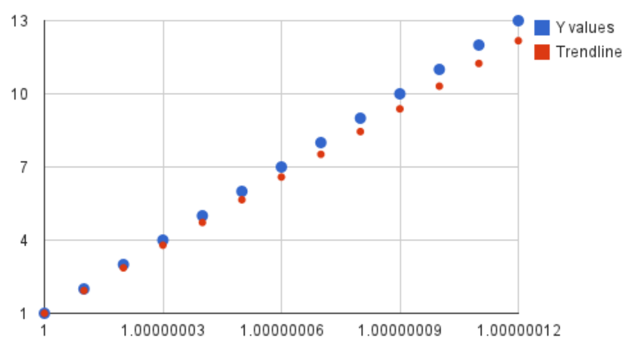
Data series for j=6

X Values	Y values	Trendline
1	1	1
1.00000001	2	1.93142622709274
1.00000002	3	2.86285246908665
1.00000003	4	3.79427869617939
1.00000004	5	4.72570492327213
1.00000005	6	5.65713113546372
1.00000006	7	6.58855739235878
1.00000007	8	7.51998360455036
1.00000008	9	8.4514098316431
1.00000009	10	9.38283605873585
1.0000001	11	10.3142623007298
1.00000011	12	11.2456885278225
1.00000012	13	12.1771147549152

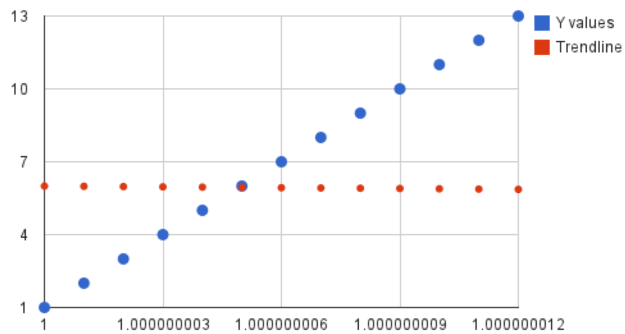
When $j = 0$, there is no numerical error ($R^2 = 1$). However, by the time $j = 5$, we begin to see error creep into the numbers ($R^2 = 0.999999999999965$). By the time $j = 7$, only one correct decimal place remains, creating barely visible error ($R^2 = 0.99999438640566$).



When $j = 8$, the trendline has ventured out alone, with an R^2 value of 0.983604355317896.

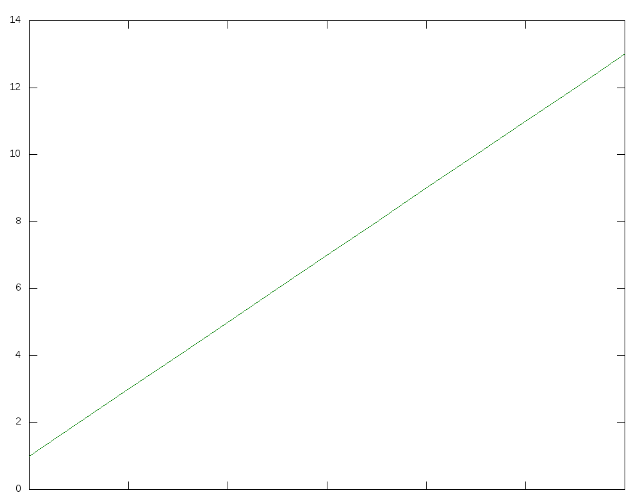


At $j = 9$, there is only madness. Rather than a slope of 10^j, the trendline has a small negative slope, and $R^2 = -0.078210891033562$.



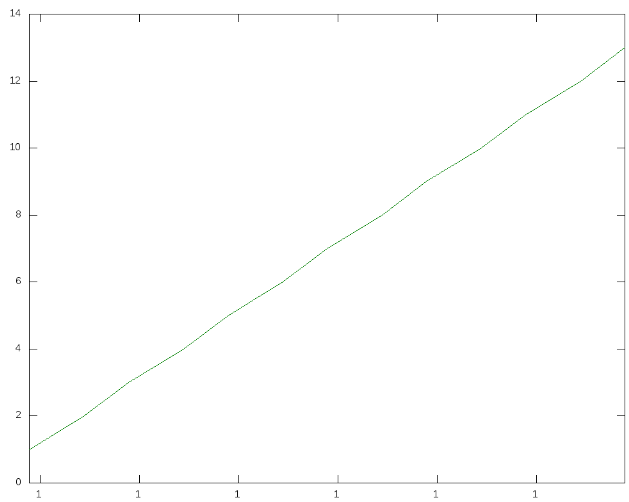
At $j = 10$, the spreadsheet crashes, and the page reloads, with the change forgotten. This is a consistent result.

By contrast, GNU Octave performs perfectly [4] up to $j = 14$, the edge of its numerical precision.



[4] Code used: ex = j; x = 1+0*10ˆex:10ˆex:1+12*10ˆex; y = 1:13;[b,s,r] = ols(y,x);plot(x,y,x,x*b);axis([1 x(size(x)(2)) 0 14]);

Only when $j = 15$ does Octave begin to reveal the underlying discrete nature of floating point.



At $j = 16$, Octave finally has an error and displays nothing, but there is no crash, and the failure to produce an answer makes sense, as 10^{-16}, the spacing of the x coordinates, is $\approx \epsilon/2$, where ϵ is provided in Octave as eps.

For better comparison with Kaplan[

Conclusion