

# MERGE SORT

RECURSION + MERGE SORT ALGORITHM

References:

<http://algs4.cs.princeton.edu/home/>

<http://homepages.math.uic.edu/~leon/cs-mcs401-s08/handouts/quicksort.pdf>

# RECURSIVE METHODS

## Properties of good recursive methods

- **Identify a **base case**:**
  - When should the recursion stop?
  - The base case is a condition that makes the method return without calling itself.
- **Method arguments**
  - Recursive methods must have at least one argument as a means of communication.
- **Recursive calls must lead to the base case.**
  - Each call to the same function must modify the argument such that reaching the base case is guaranteed.

# RECURSIVE ALGORITHMS

**How to write a recursive algorithm:**

- **Break down the original problem into smaller parts**
  - The parts should be of the same type as the original problem.
- **Identify the cases where the problem can be solved directly:**
  - When it can't get any simpler
- **Combine the solutions of the smaller problems.**
  - This should be the solution to the original problem

# MERGE SORT

- **Divide the array into two equal subarrays**
- **Recursively sort each subarray**
- **Merge the two sorted subarrays into one sorted array.**
  - Use an auxiliary array to hold the elements
  - Keep track of index in subarray 1 and 2 with  $i$  and  $j$  respectively
  - Compare elements at index  $i$  &  $j$
  - Copy smaller element to the array at index  $k$

# MERGE SORT

```
private static void mergeSort(int[] data, int lo, int hi) {  
    if (lo < hi) {  
        //find index of middle element  
        int mid = (lo + hi) / 2;  
        //recursive calls for first and second half of the array  
        mergeSort(data, lo, mid);  
        mergeSort(data, mid + 1, hi);  
        //merge the elements of each side of the array  
        merge(data, lo, mid, hi);  
    }  
}
```

# MERGE METHOD

**Merge (data, lo, mid, hi)**

**Create a temporary array**

**Set  $i=lo$ , and  $j=mid+1$**

**Loop while ( $i \leq mid$  and  $j \leq hi$ )**

**compare elements at  $i$  and  $j$**

**copy smaller element to temp and increment  $i$  or  $j$**

**while ( $i \leq mid$ )**

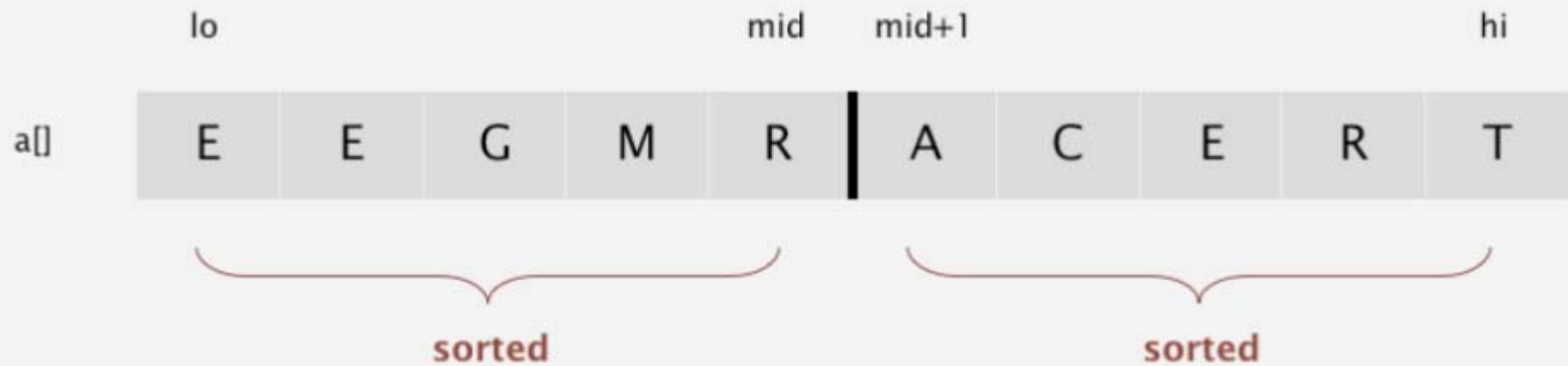
**copy element at  $i$  to temp, increment  $i$**

**While ( $j \leq hi$ )**

**copy element at  $j$  to temp, increment  $j$**

**Copy elements from temp back to data array**

# MERGE DEMO



# MERGE DEMO

a[]

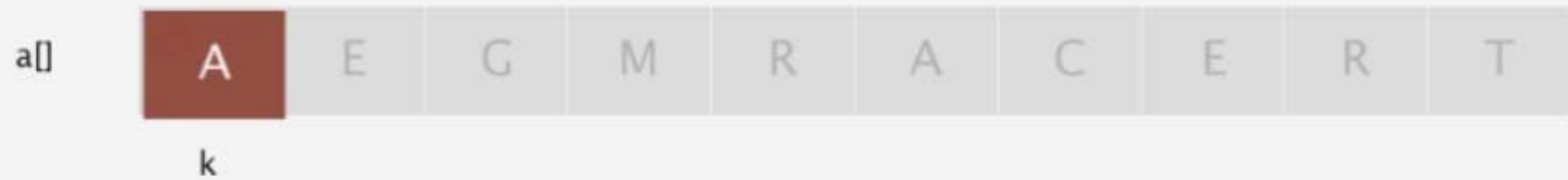
E	E	G	M	R	A	C	E	R	T
---	---	---	---	---	---	---	---	---	---

aux[]

E	E	G	M	R		A	C	E	R	T
---	---	---	---	---	--	---	---	---	---	---



# MERGE DEMO



compare minimum in each subarray



# MERGE DEMO



compare minimum in each subarray



# MERGE DEMO

a[]

A	C	E	M	R	A	C	E	R	T
---	---	---	---	---	---	---	---	---	---

k

compare minimum in each subarray

The diagram shows an array `aux[]` containing the characters E, E, G, M, R, A, C, E, R, T. A vertical line separates the array into two parts. The index `i` points to the first 'E' (index 1), and the index `j` points to the 'E' at index 8. The characters from index 6 to 7 (A, C) are shown in a lighter gray, indicating they are not part of the current comparison.

# MERGE DEMO



compare minimum in each subarray



# MERGE DEMO



compare minimum in each subarray



# MERGE DEMO

a[]

A

C

E

E

E

G

C

E

R

T

k

compare minimum in each subarray

aux[]

E

E

G

M

## R

A

C

E

R

T

i

j

# MERGE DEMO

a[]

A

C

E

E

E

G

M

E

R

T

k

compare minimum in each subarray

aux[]

E

E

G

M

R

A

C

C

E

R

T

i

j

# MERGE DEMO



compare minimum in each subarray





# MERGE DEMO



one subarray exhausted, take from other



# MERGE METHOD

**Merge (data, lo, mid, hi)**

**Create a temporary array**

**Set  $i=lo$ , and  $j=mid+1$**

**Loop while ( $i \leq mid$  and  $j \leq hi$ )**

**compare elements at  $i$  and  $j$**

**copy smaller element to temp and increment  $i$  or  $j$**

**while ( $i \leq mid$ )**

**copy element at  $i$  to temp, increment  $i$**

**While ( $j \leq hi$ )**

**copy element at  $j$  to temp, increment  $j$**

**Copy elements from temp back to data array**

# MERGE DEMO



one subarray exhausted, take from other



# MERGE DEMO



one subarray exhausted, take from other



# MERGE SORT

```
private static void mergeSort(int[] data, int lo, int hi) {  
    if (lo < hi) {  
        //find index of middle element  
        int mid = (lo + hi) / 2;  
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    }  
}
```

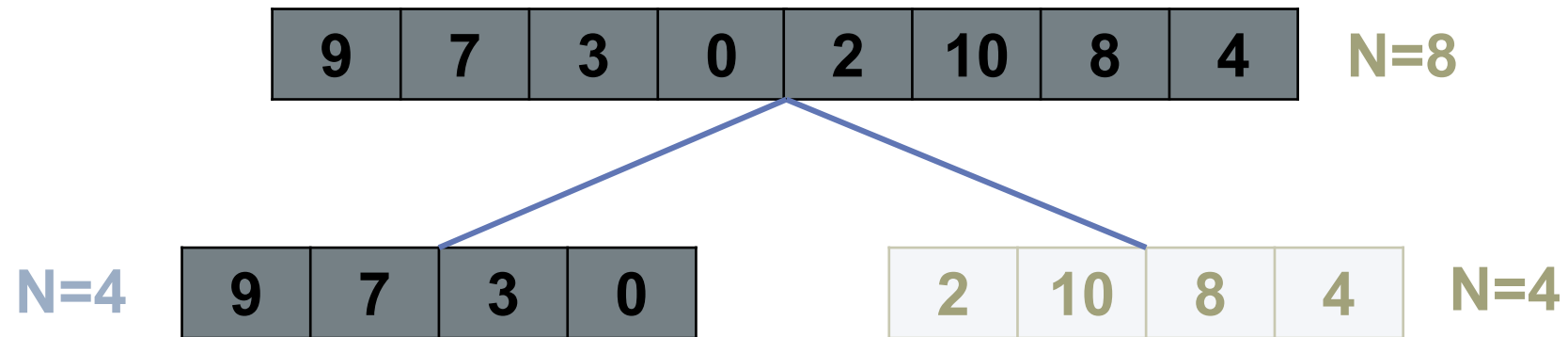
# MERGE SORT

9	7	3	0	2	10	8	4
---	---	---	---	---	----	---	---

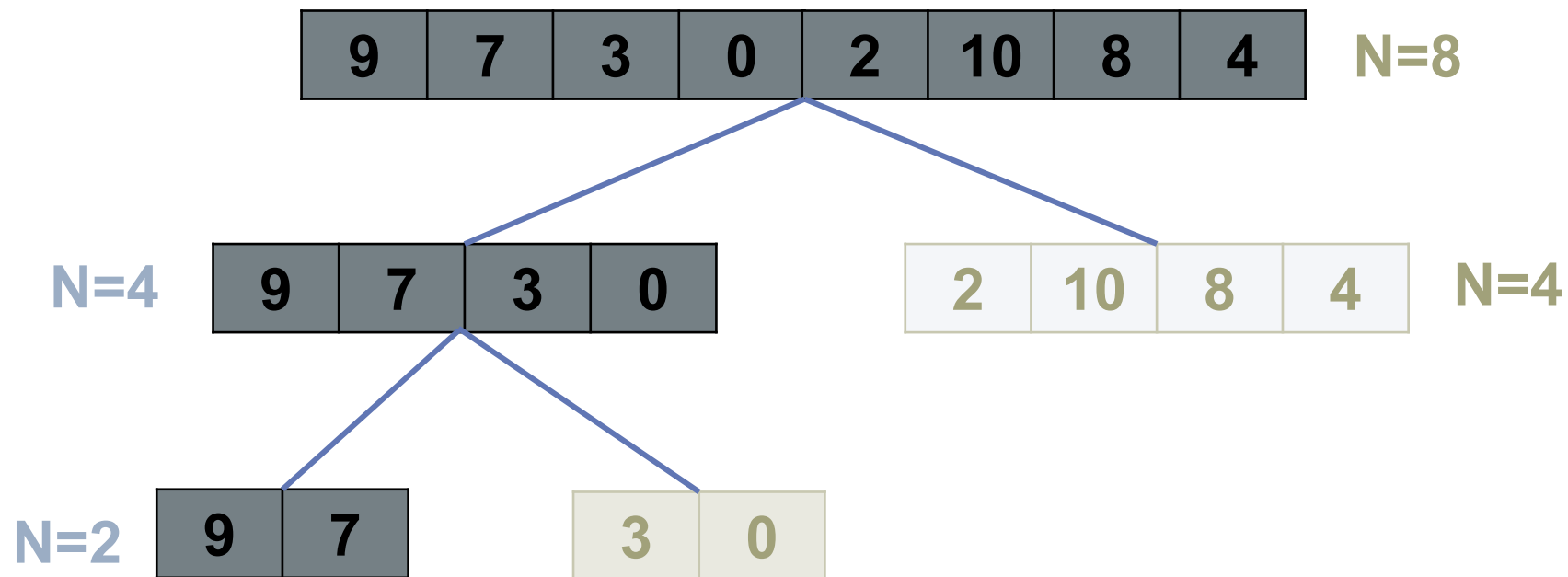
**N=8**



# MERGE SORT

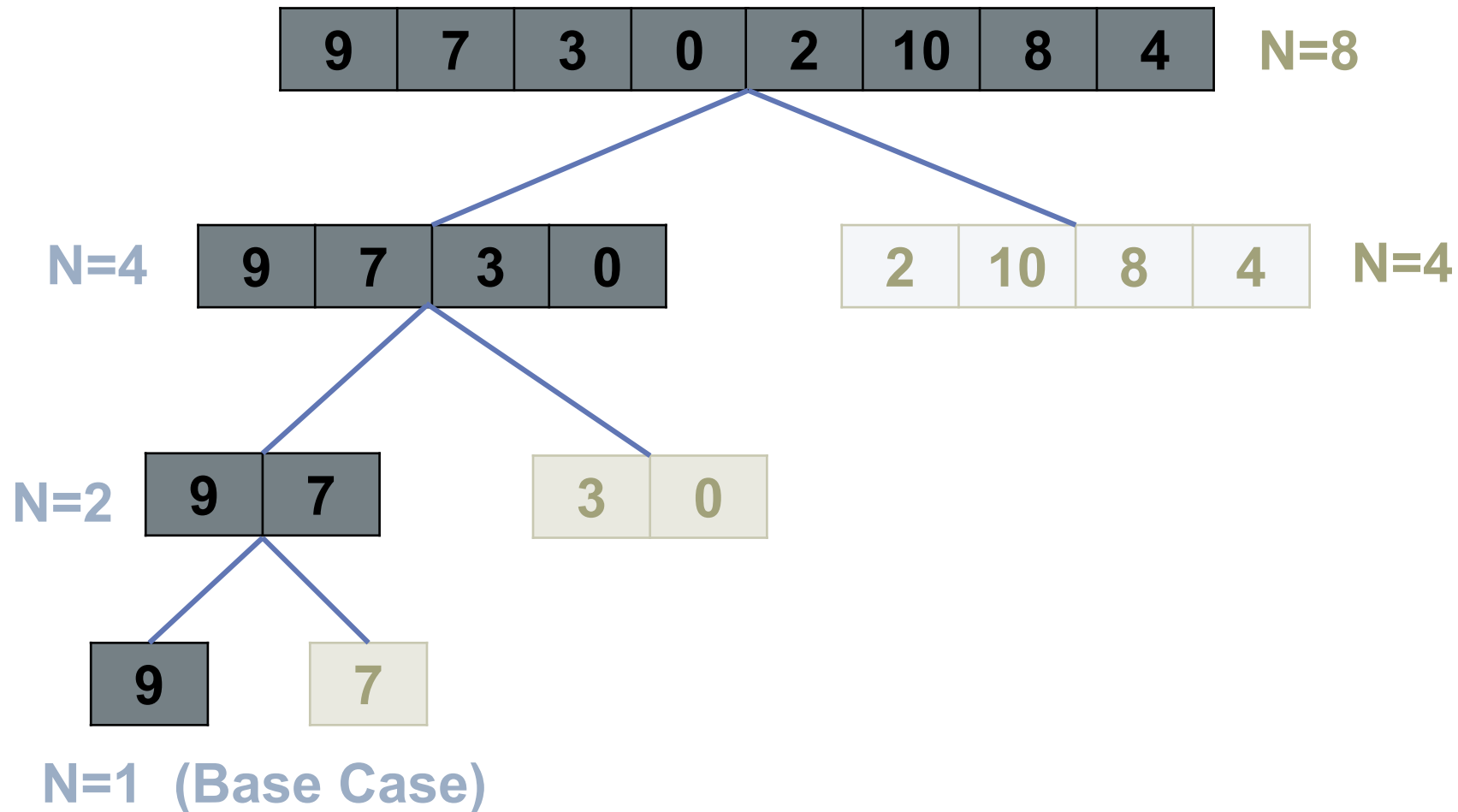


# MERGE SORT





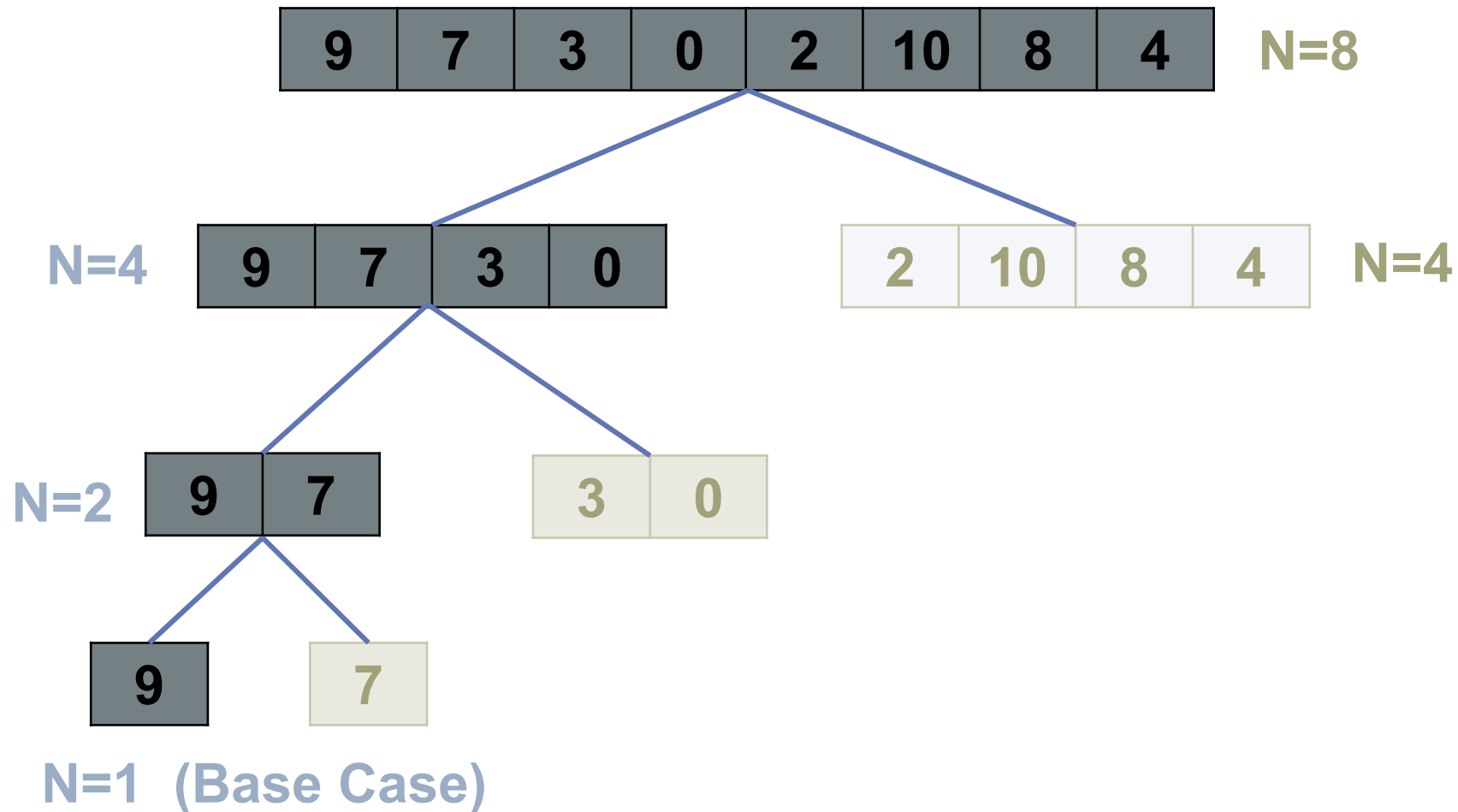
# MERGE SORT



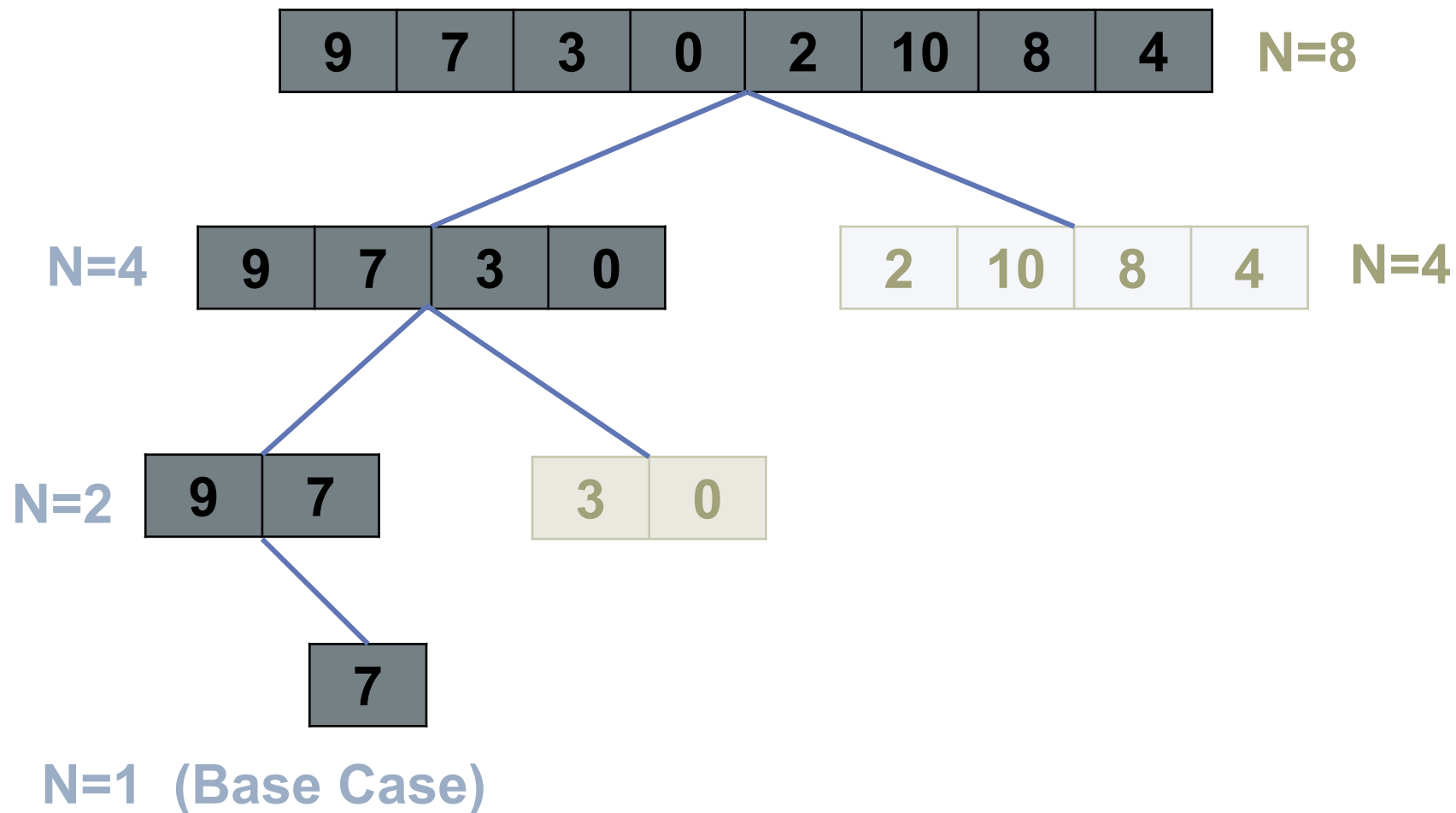
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private static void mergeSort(int[] data, int lo, int hi) {  
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        mergeSort(data, mid + 1, hi);  
        //merge the elements of each side of the array  
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    }  
}
```

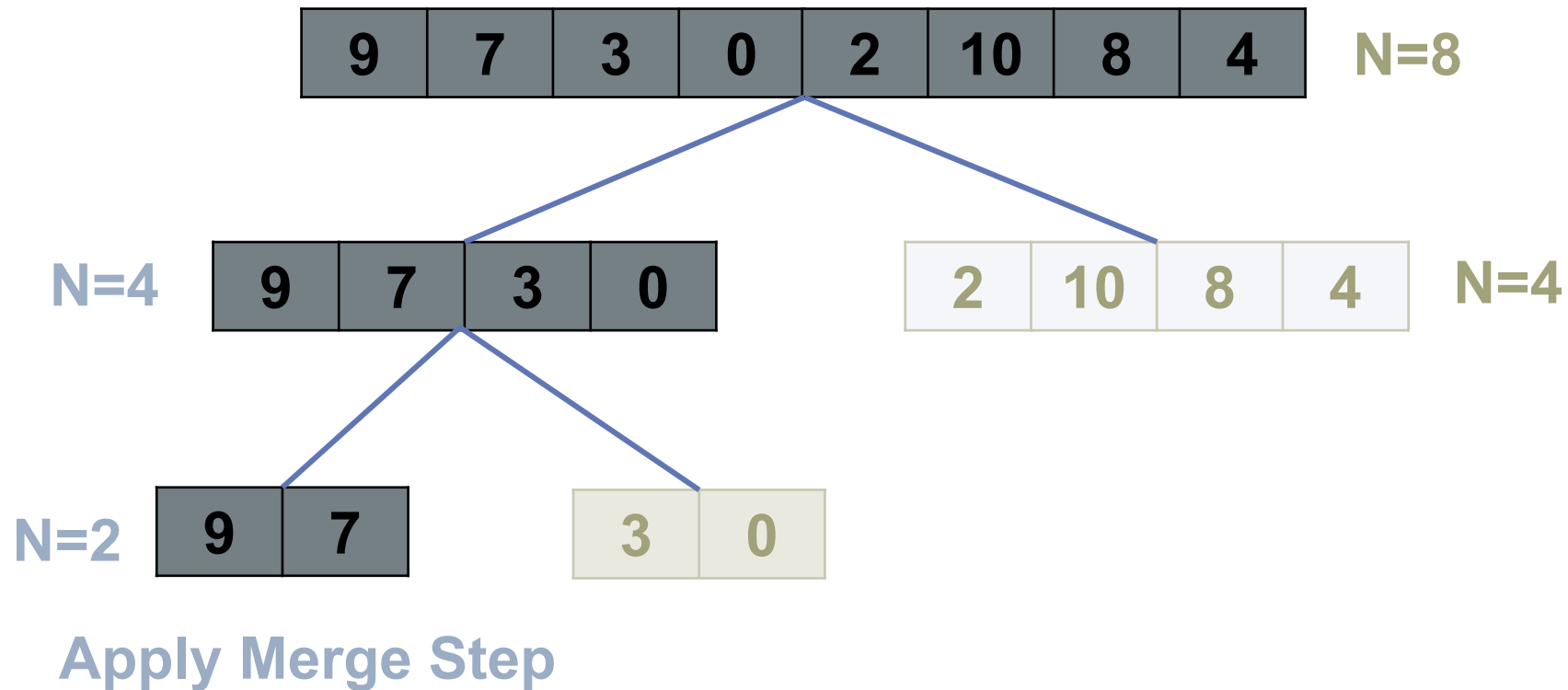
# MERGE SORT



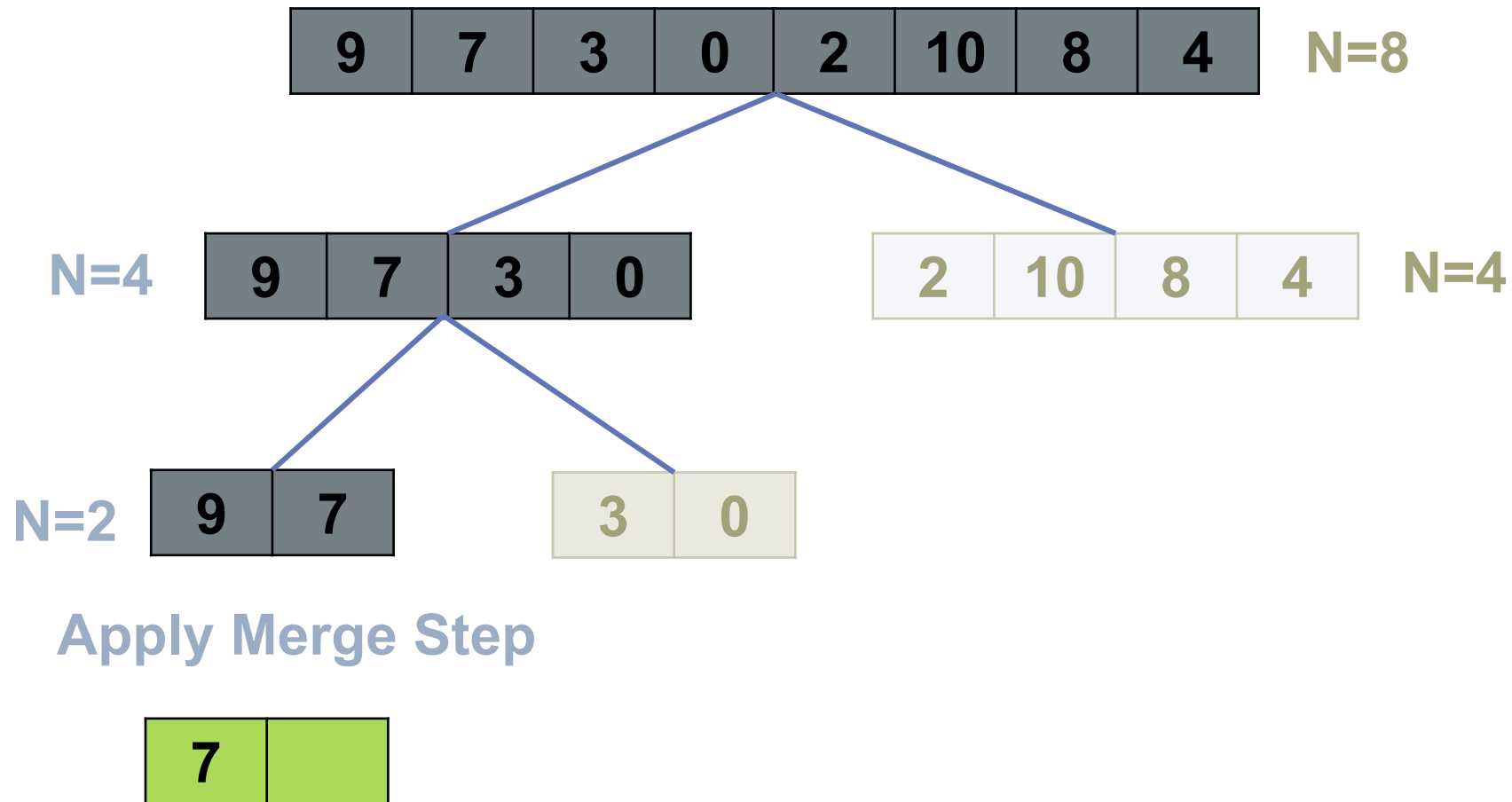
# MERGE SORT



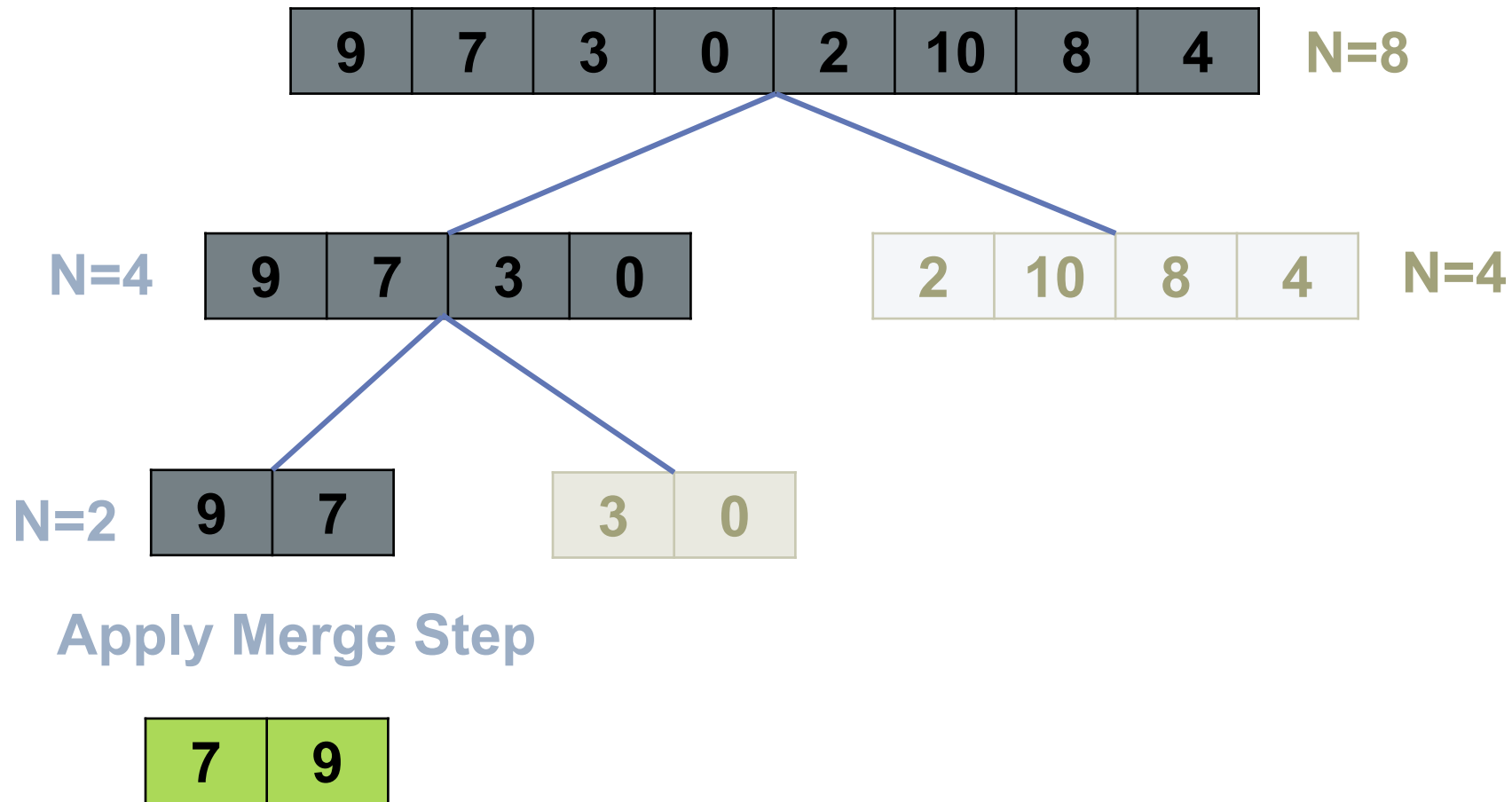
# MERGE SORT



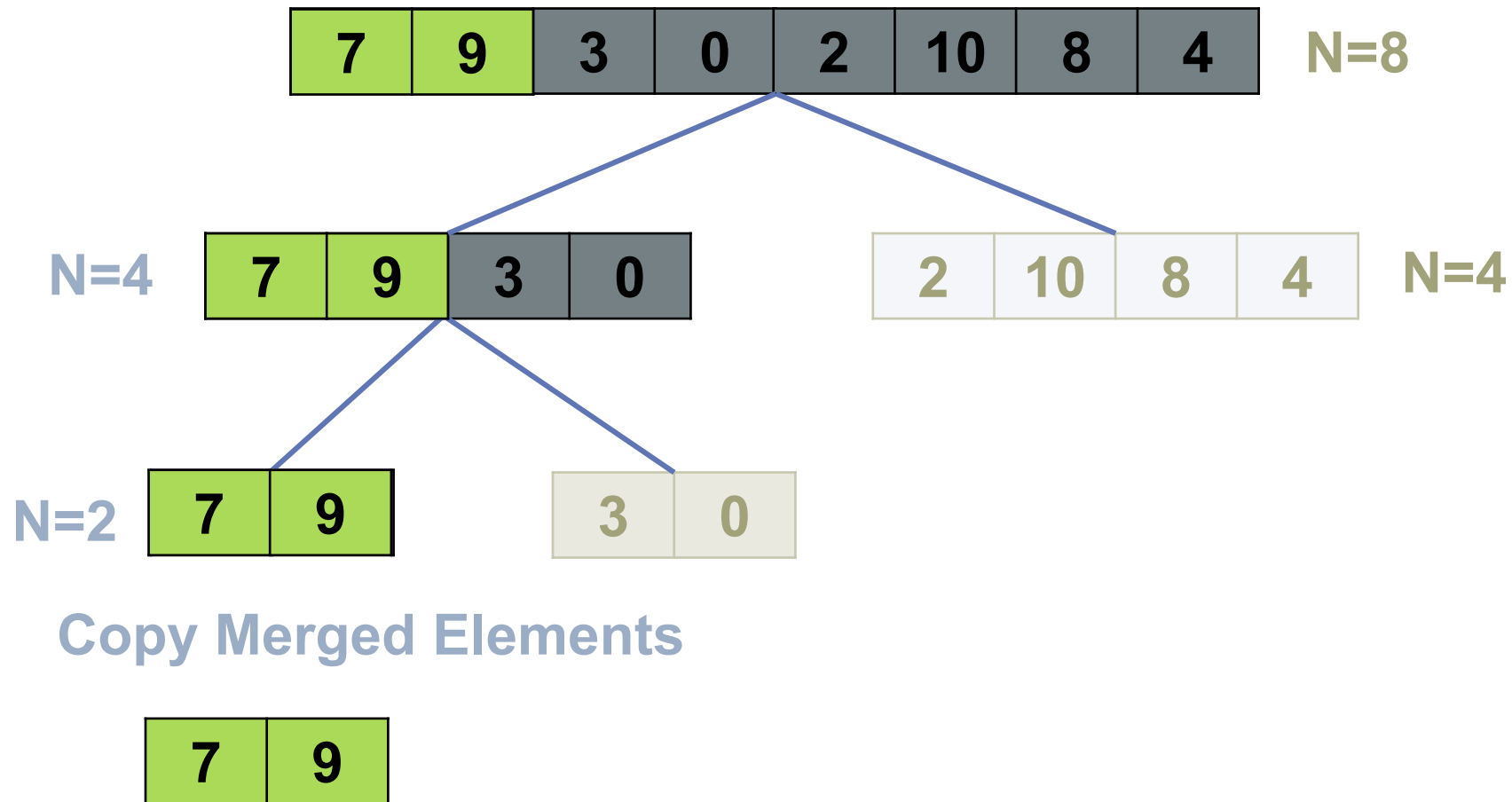
# MERGE SORT



# MERGE SORT

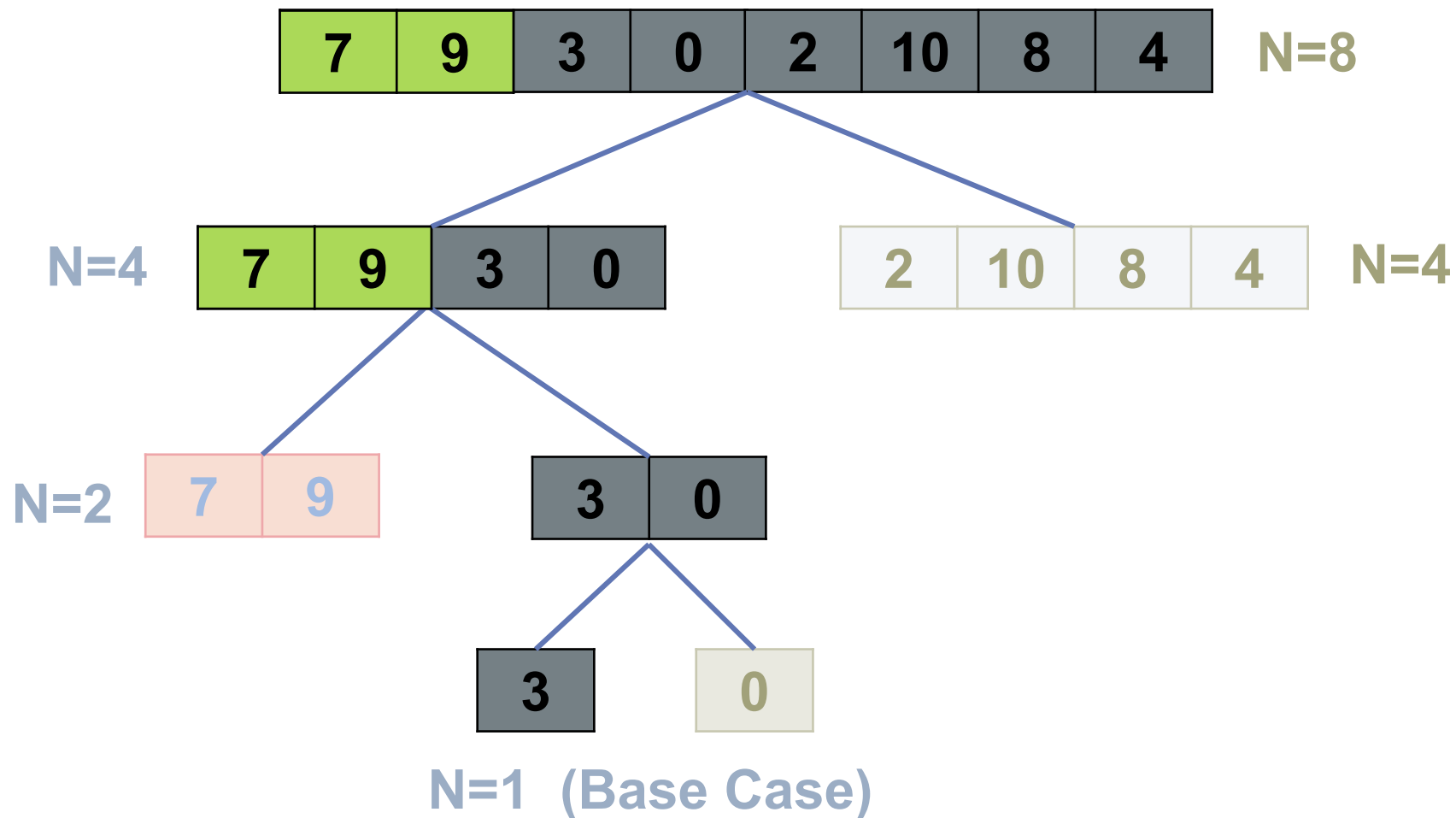


# MERGE SORT

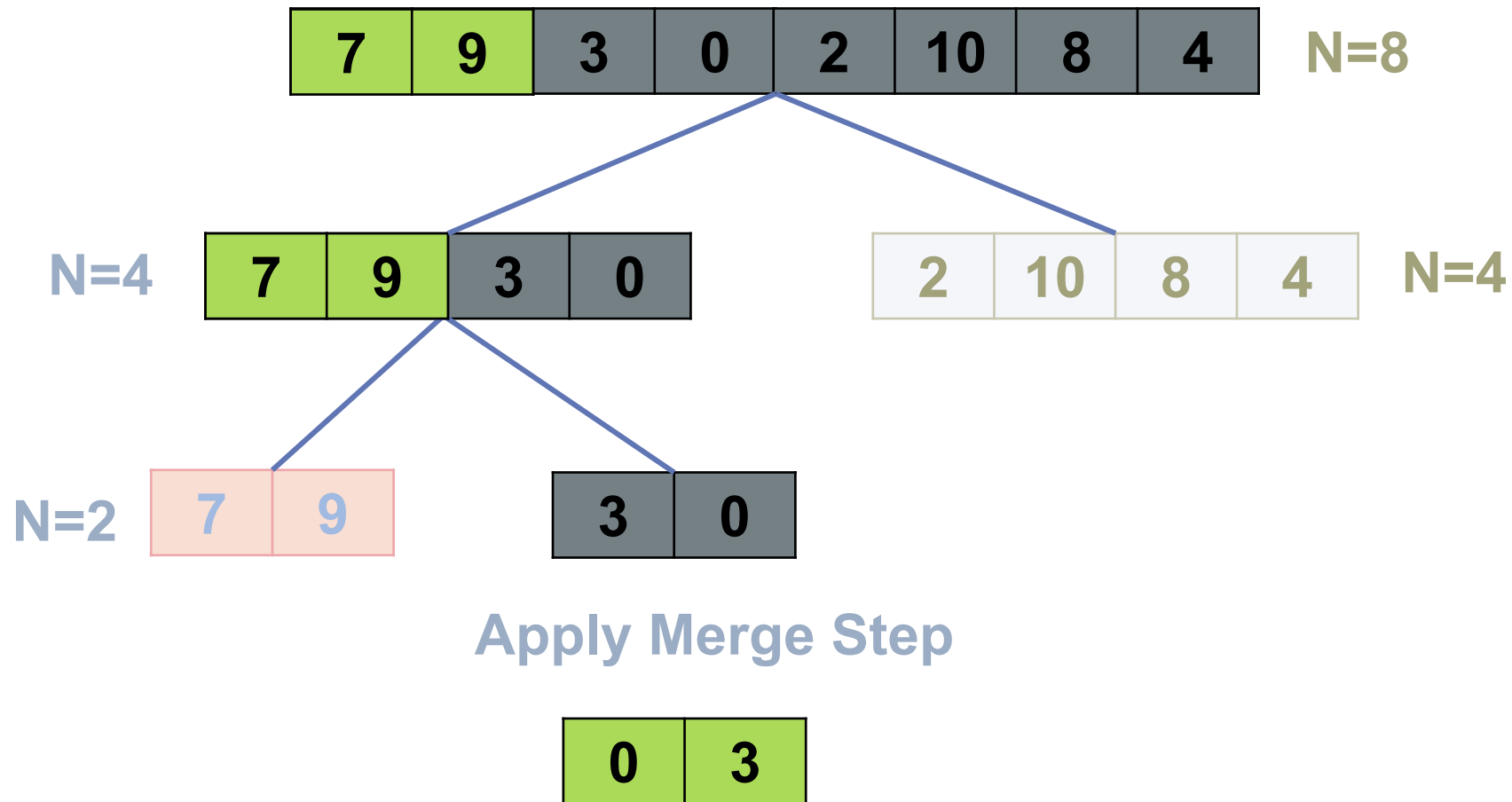




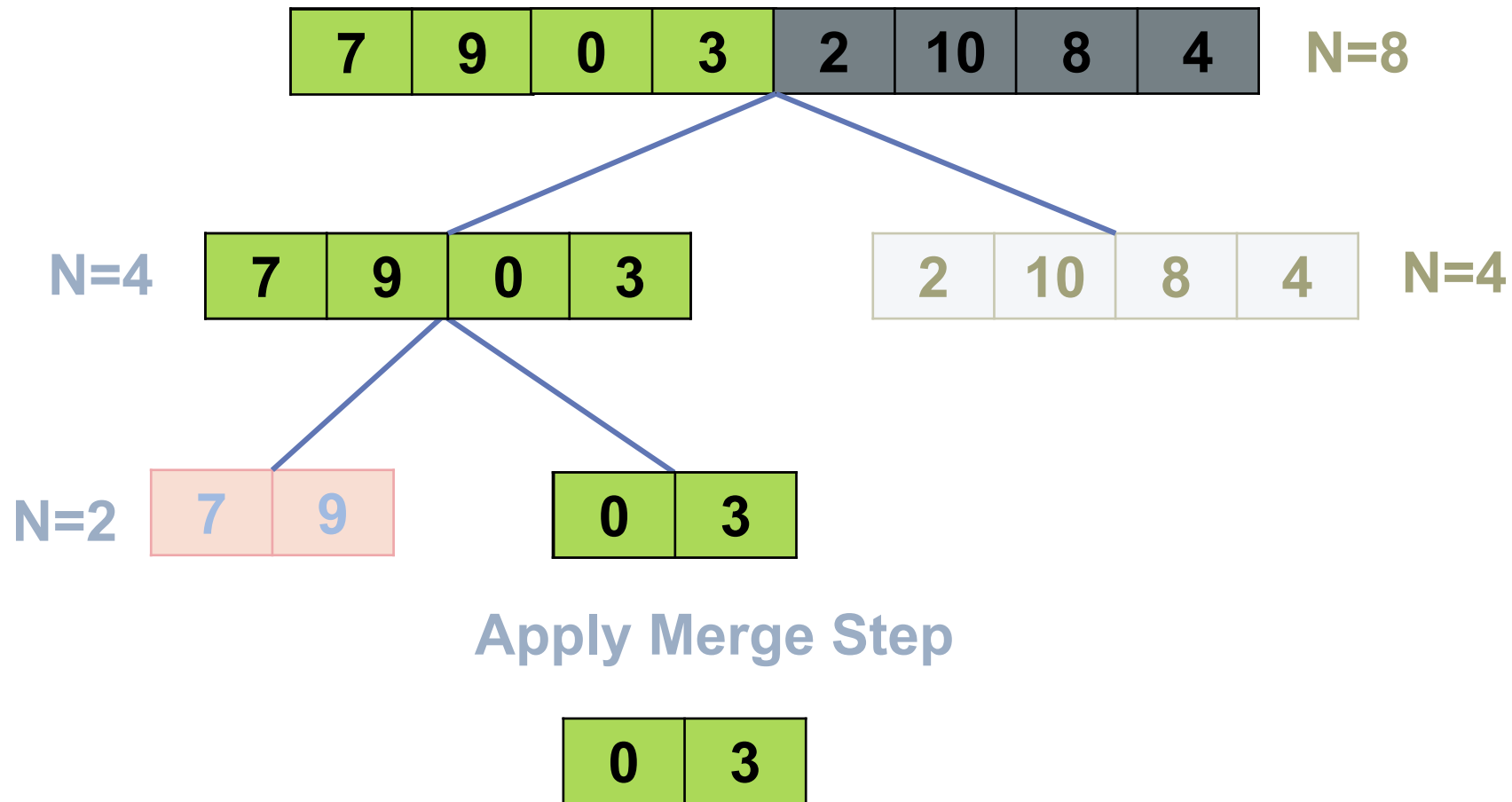
# MERGE SORT



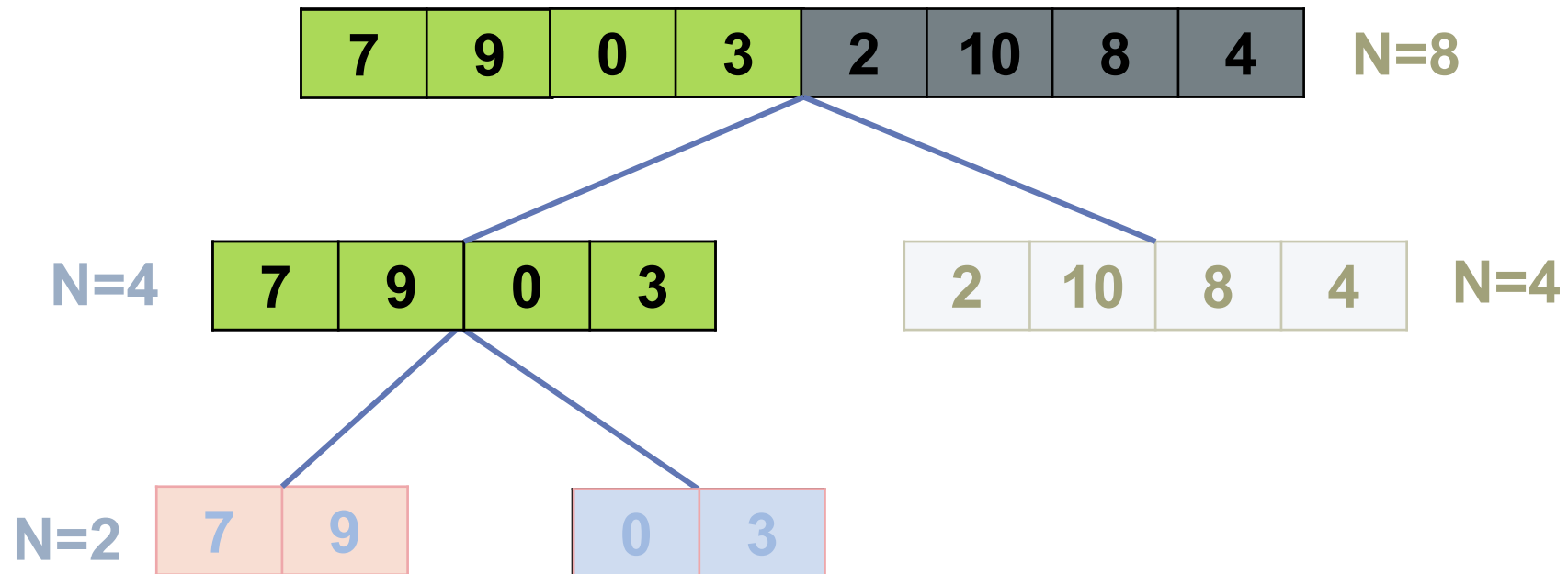
# MERGE SORT



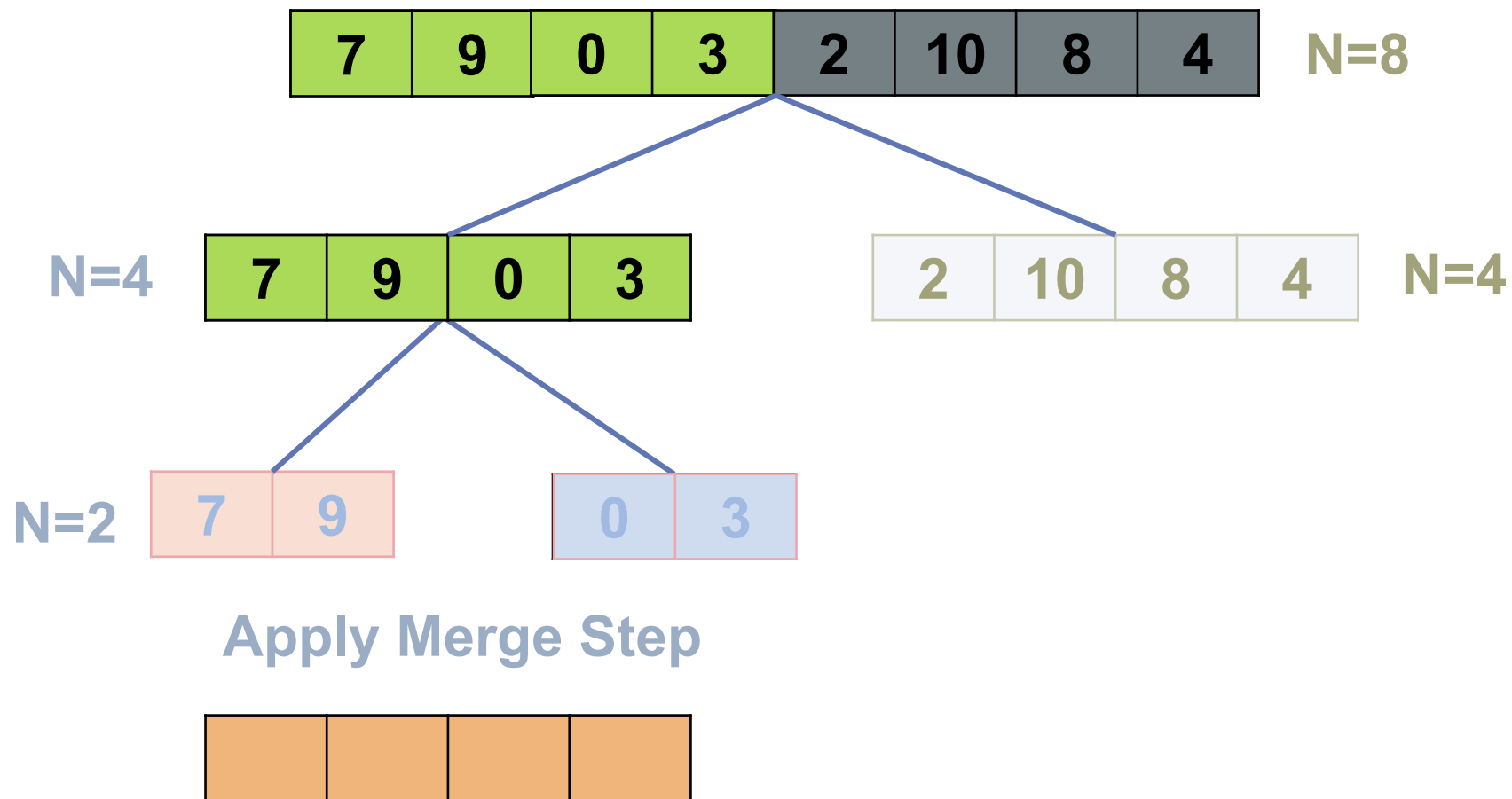
# MERGE SORT



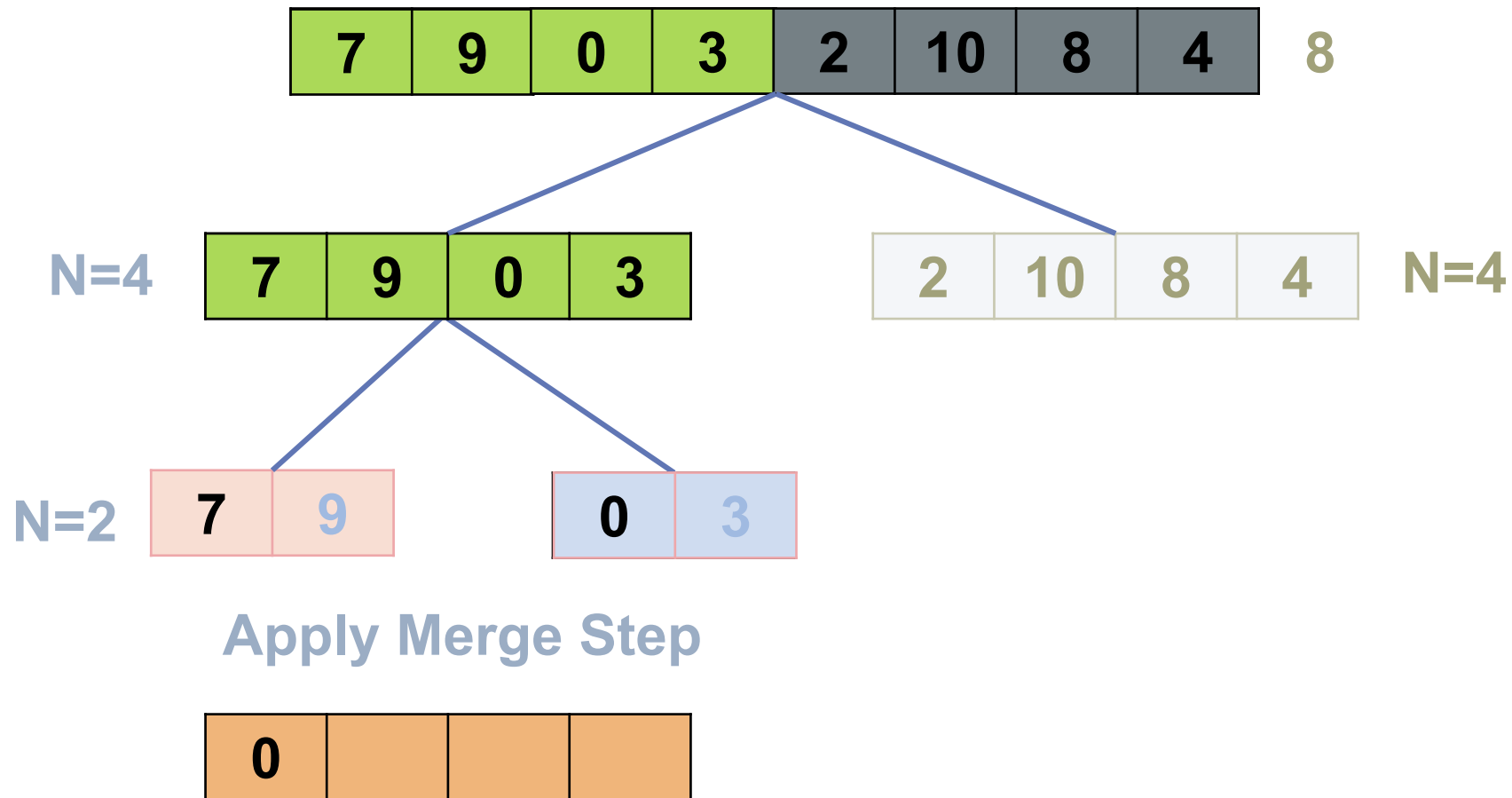
# MERGE SORT



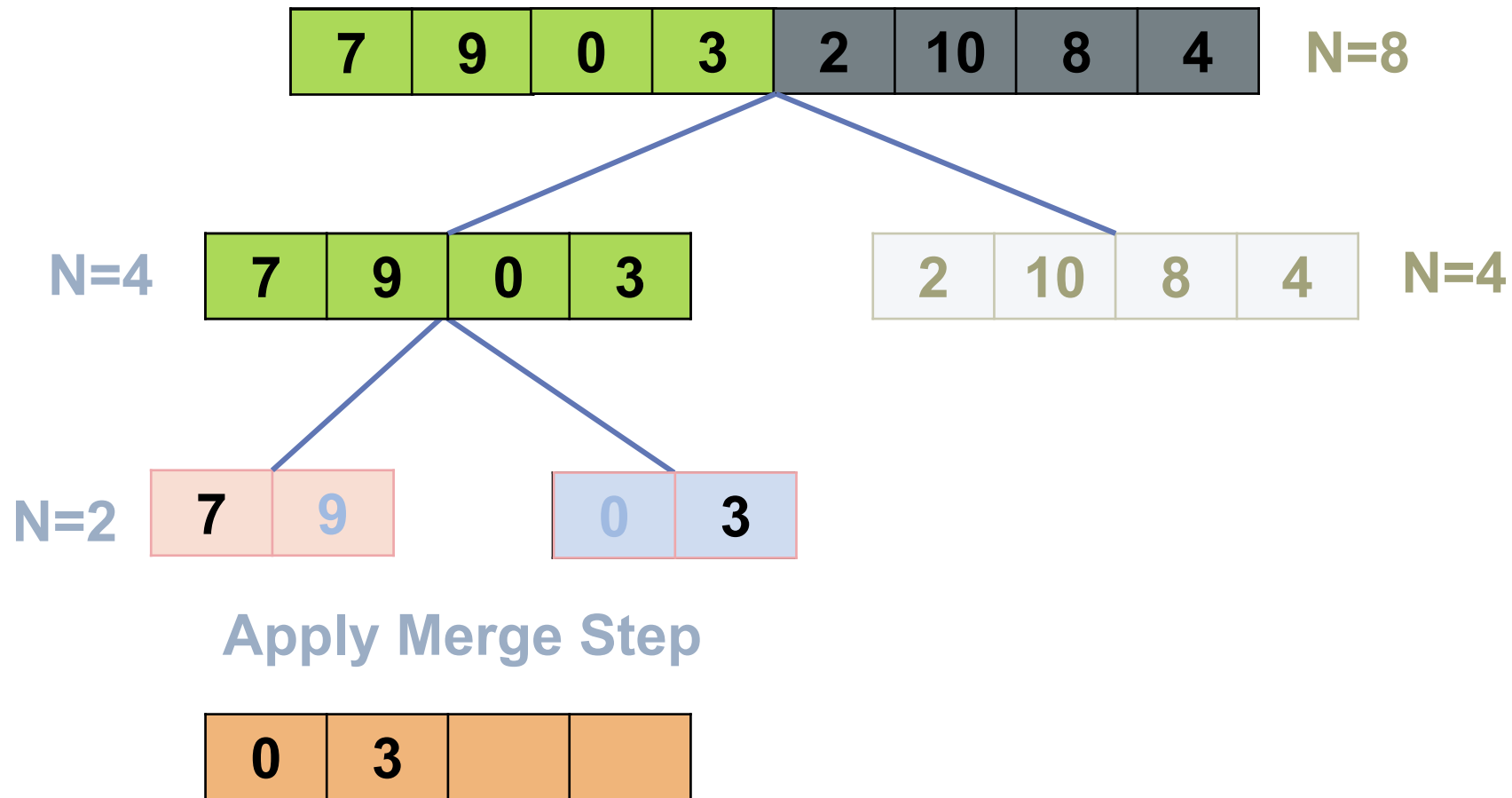
# MERGE SORT



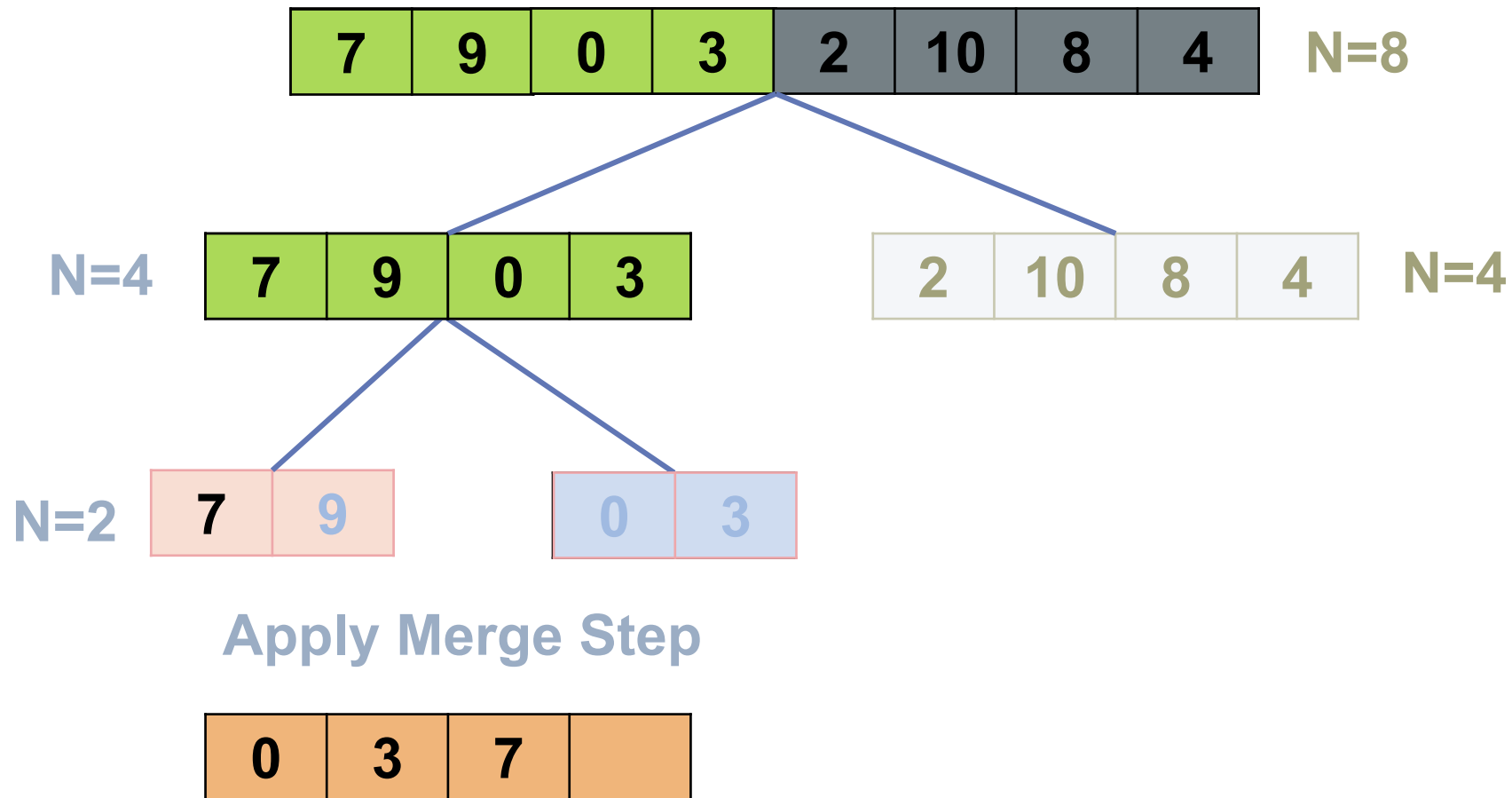
# MERGE SORT



# MERGE SORT

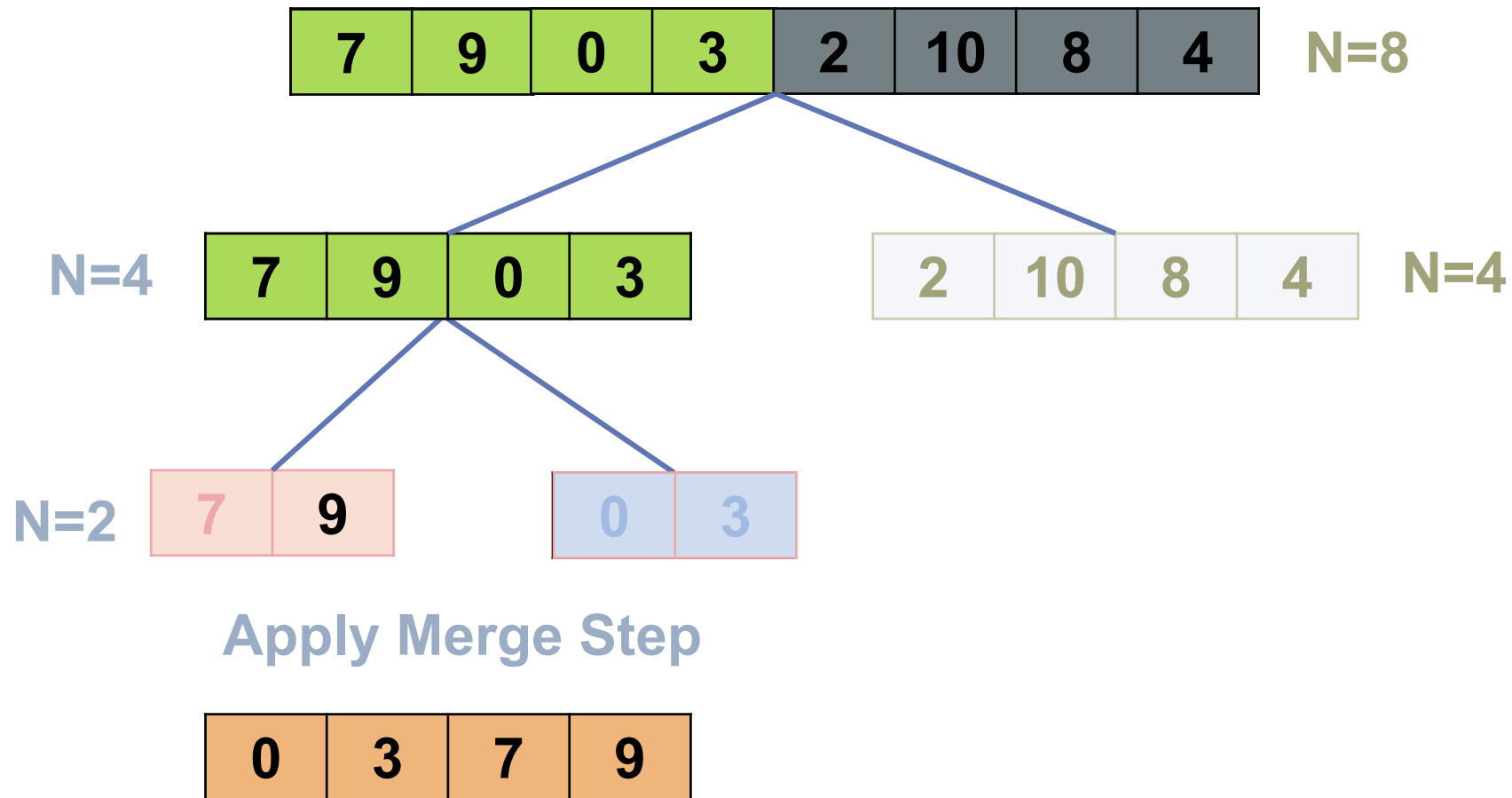


# MERGE SORT

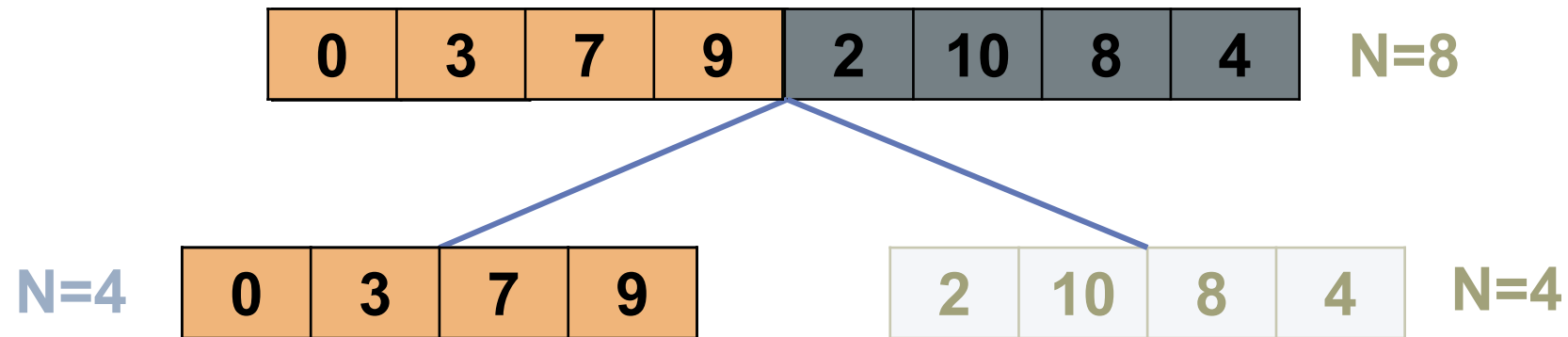




# MERGE SORT



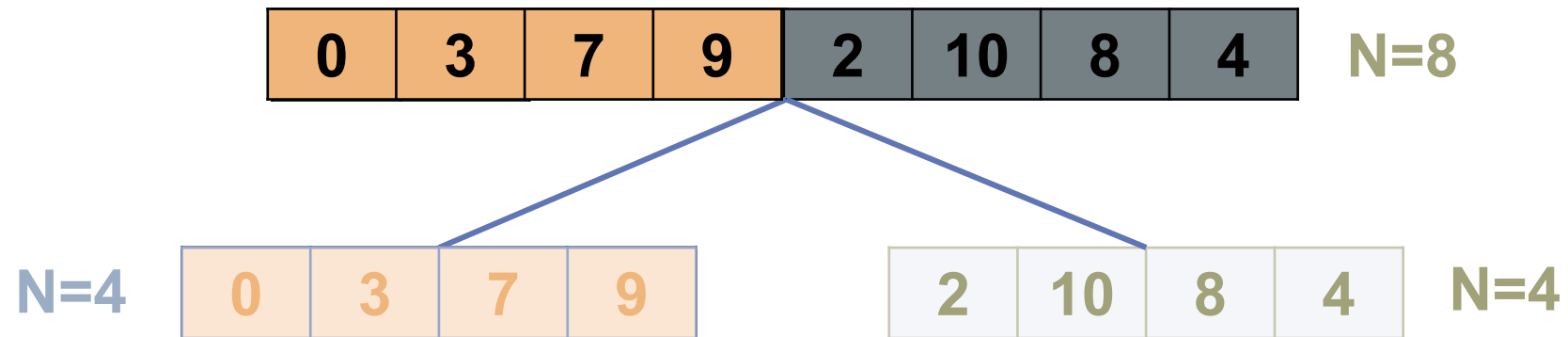
# MERGE SORT



Copy Merged Elements

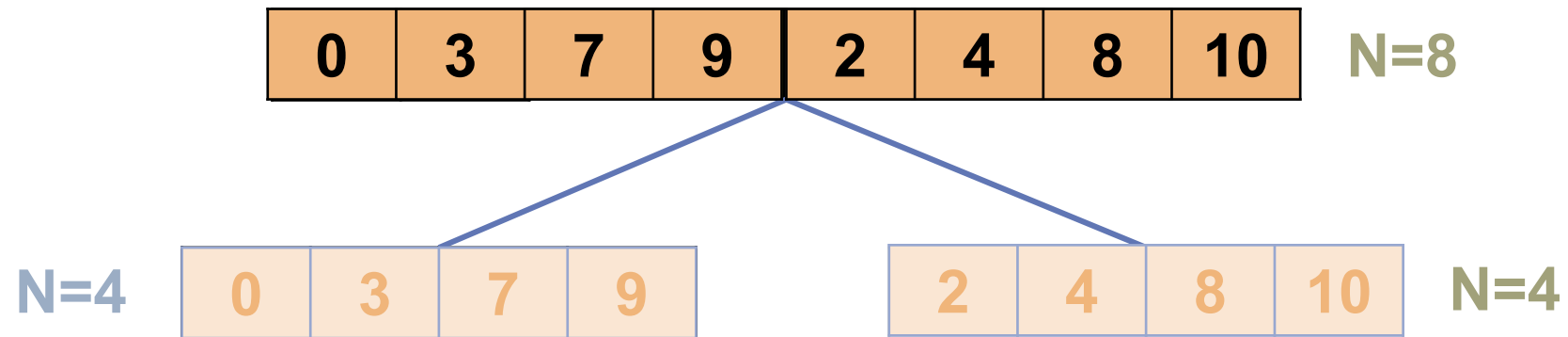
0	3	7	9
---	---	---	---

# MERGE SORT

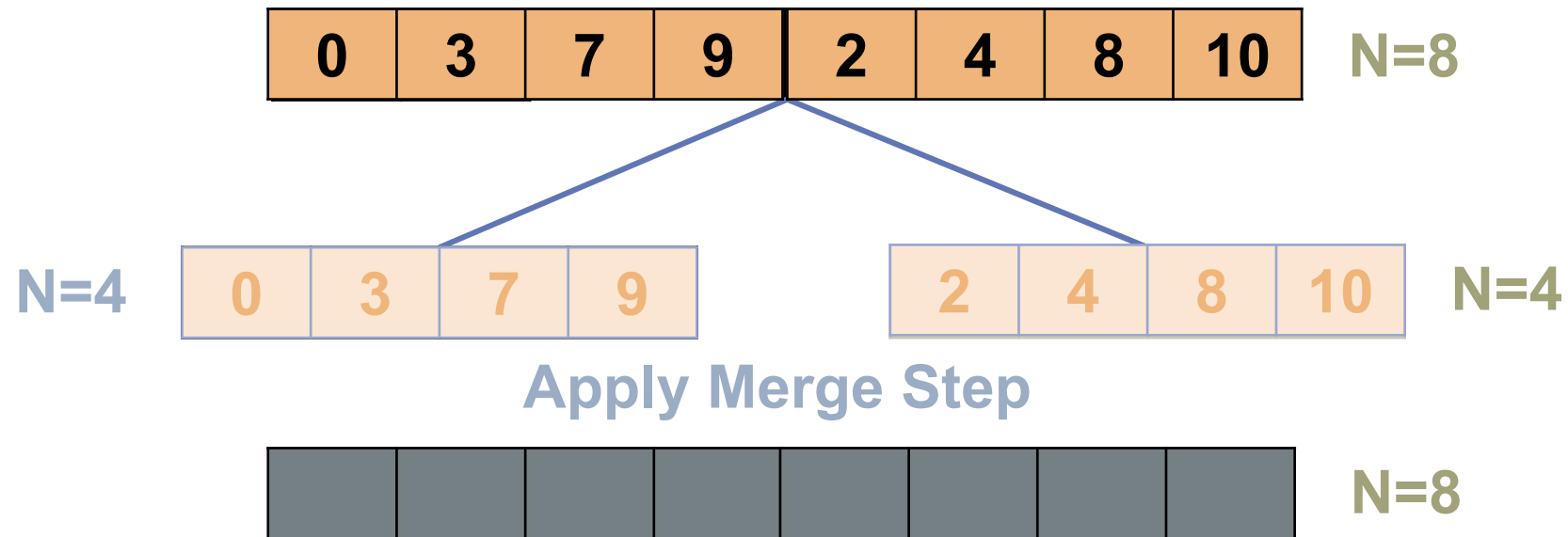


Repeat the same steps  
On this side

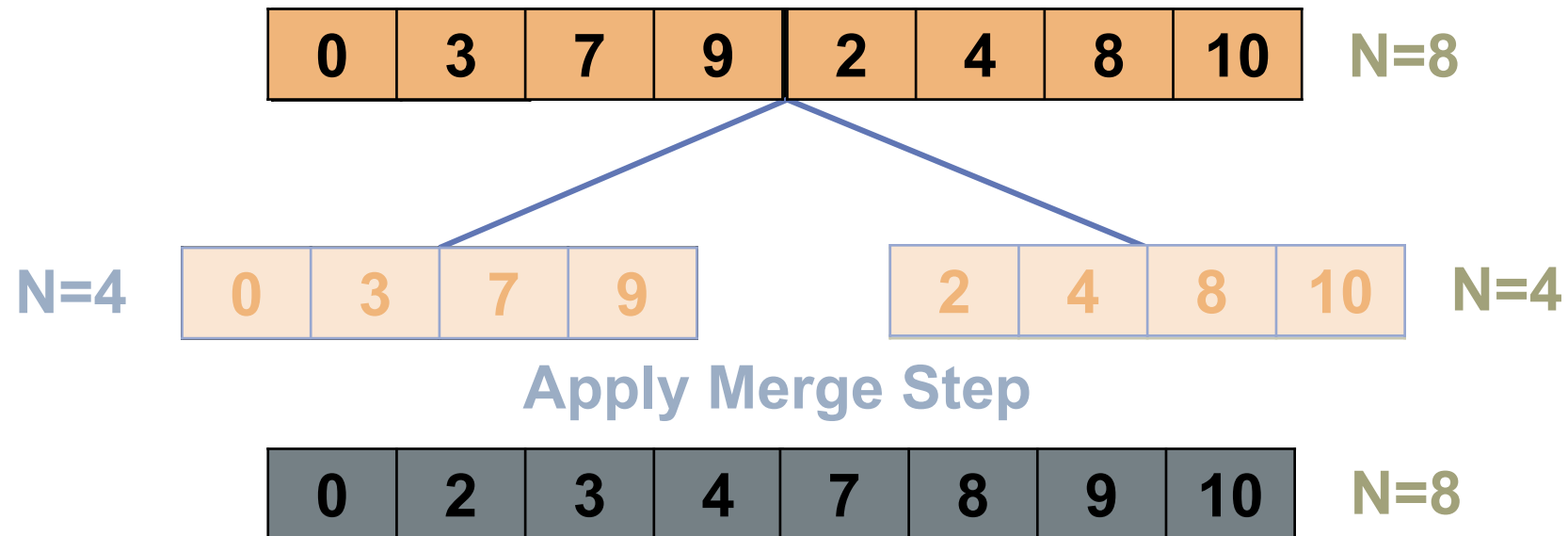
# MERGE SORT



# MERGE SORT



# MERGE SORT



# MERGE SORT

0	2	3	4	7	8	9	10	N=8
---	---	---	---	---	---	---	----	-----

Copy Merged Elements

0	2	3	4	7	8	9	10	N=8
---	---	---	---	---	---	---	----	-----

# IMPROVING MERGE SORT

- **Stop if array is already sorted.**
  - If largest item in first half  $\leq$  smallest item in second half
    - Array is already sorted, return.

A	B	C	D	E	F	G	H	I	J	M	N	O	P	Q	R	S	T	U	V
A	B	C	D	E	F	G	H	I	J	M	N	O	P	Q	R	S	T	U	V



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