الجامعة السورية الخاصة
SYRIAN PRIVATE UNIVERSITY

مقرر تعلم الآلة | كلية الهندسة المعلوماتية | المحاضرة التاسعة

Support Vector Machine (SVM) 2

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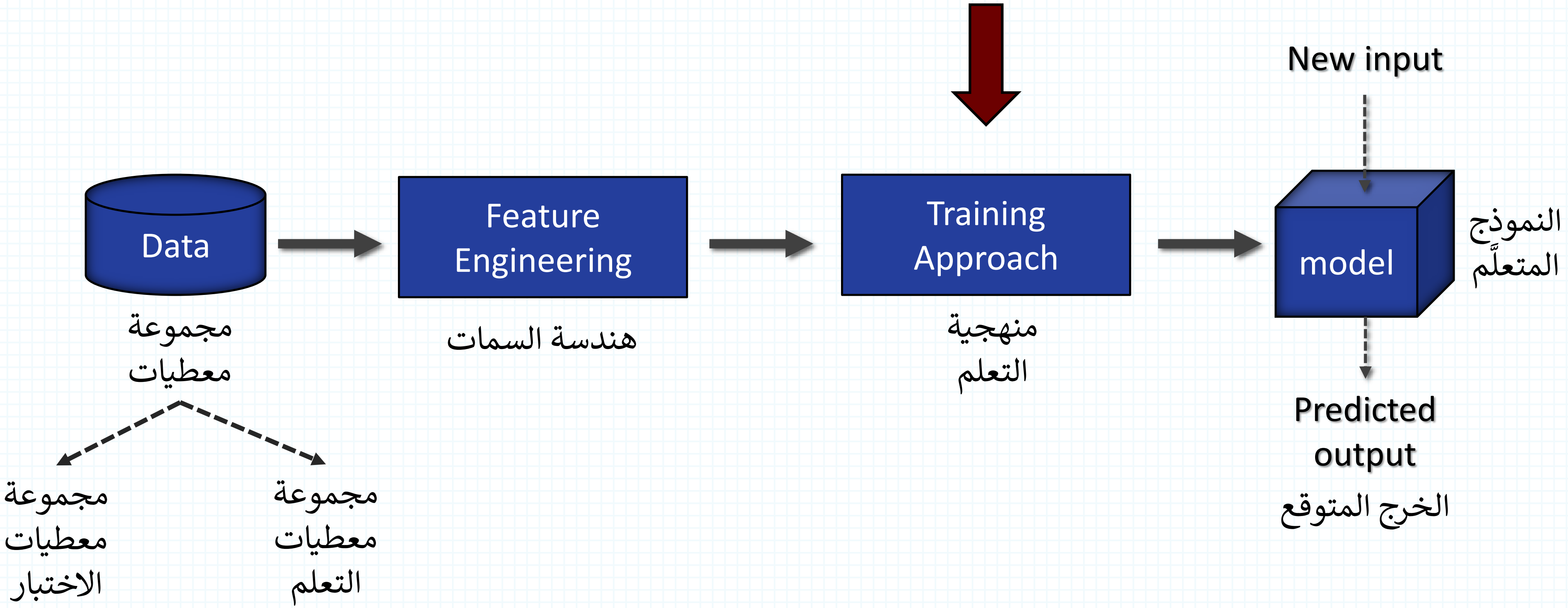
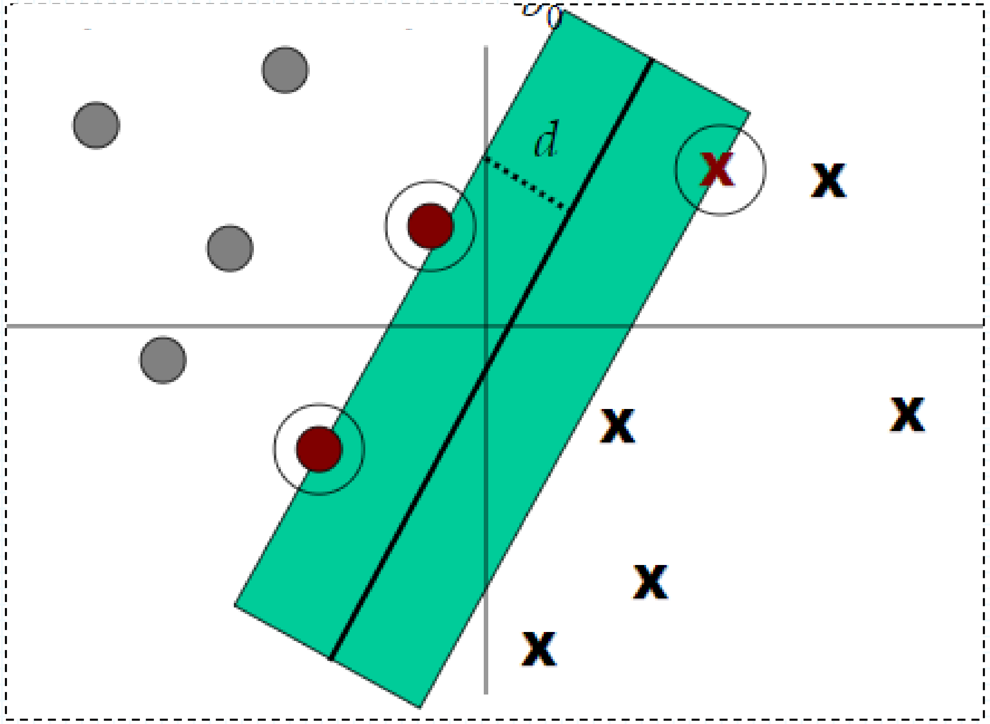
ML Pipeline


New input
Data
Feature Engineering
Training Approach
model
النموذج المتعلَّم
مجموعة معطيات
هندسة السمات
منهجية التعلم
Predicted output
الخرج المتوقع
مجموعة معطيات الاختبار
مجموعة معطيات التعلم

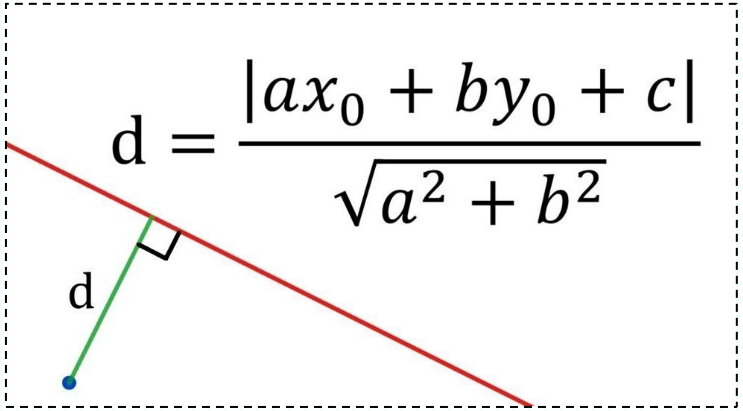
Intuition

- **Our problem:**

Maximizing the shortest distance to the closest positive or negative point.



The distance between a point (X_0,Y_0) and a line aX+bY+c=0



So.. What is our optimization problem?

- **Our problem:**

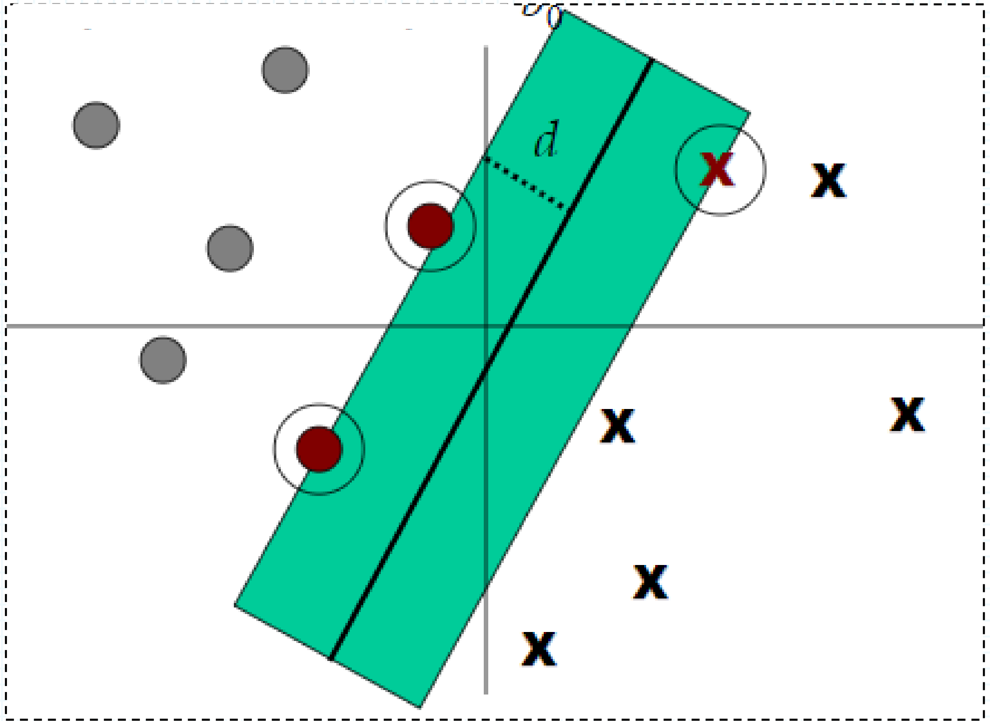
Maximizing the shortest distance to the closest positive or negative point.

In our case

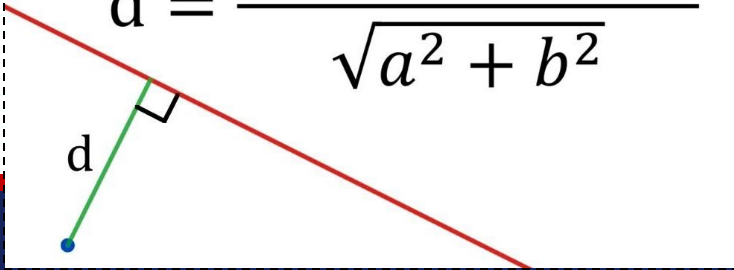
the distance of a hyperplane equation: $w^T\Phi(x) + b = 0$ from a given point vector $\Phi(x_0)$ can be easily written as :

$$d_H(\phi(x_0)) = \frac{|w^T(\phi(x_0)) + b|}{||w||_2}$$

$$||w||_2 =: \sqrt{w_1^2 + w_2^2 + w_3^2 + \dots w_n^2}$$



$$\mathrm{d} = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

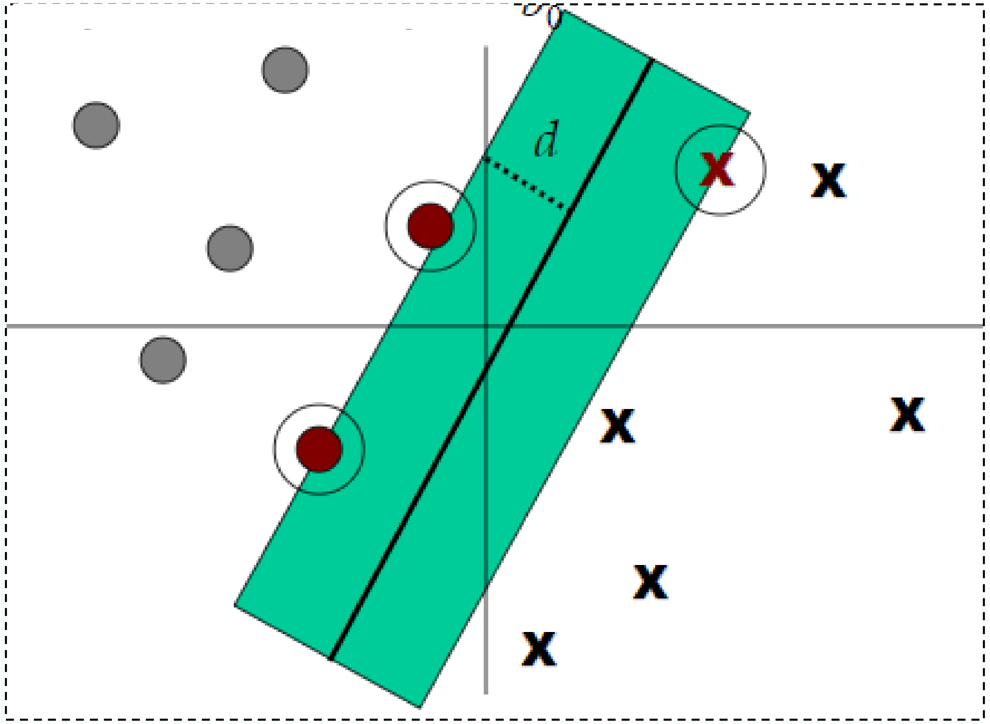


So.. What is our optimization problem?

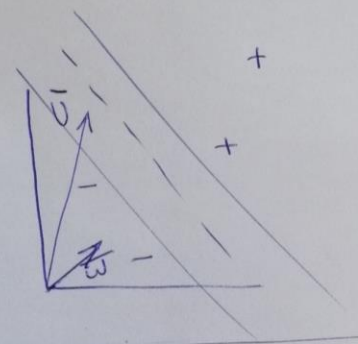
- **Our problem:**

Maximizing the shortest distance to the closest positive or negative point.

$$w^* = arg_w max\, [min_n\, d_H(\phi(x_n))]$$



Note that W represents all parameters i.e. w and b



$\bar{w}\cdot\bar{U} \geq C$ the N^+

$\bar{w}\cdot\bar{U} + b \geq \phi$

$\bar{w}\, x_+^* + b \geq 1$

$w\, x_- + b \leq -1$

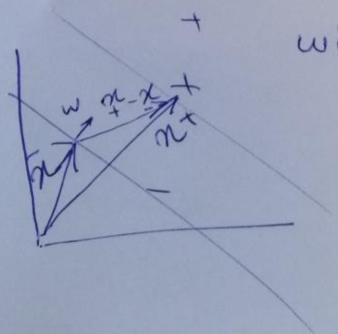
$y_i = +1$ for + samples

$\quad\; -1$ for − samples

constrait

$y_i(\bar{w}\cdot\bar{x}_i + b) \geq 1$ for all points

(*) $y_i(\bar{w}\,\bar{x}_i + b) = 1$ for support vector



width $= (\bar{x}_+ - \bar{x}_-)\cdot\dfrac{\bar{w}}{\|w\|}$

support vector

$= (\bar{x}_+\bar{w} - \bar{x}_-\bar{w})\dfrac{1}{\|w\|}$ (*)

from (*)

$x_+ \rightarrow \bar{w}x_+ + b = 1 \rightarrow \bar{w}x_+ = 1 - b$

$x_- \rightarrow \bar{w}x_- + b = -1 \rightarrow -\bar{w}x_- = 1 + b$

$\Downarrow$

width $= \dfrac{2}{\|w\|}$

Goal Maximize width $\dfrac{2}{\|w\|}$

$\Downarrow$

Minimize $\|w\|$

$\Downarrow$

Goal $\boxed{\text{Minimize } \frac{1}{2}\|w\|^2}$

true only if the constrait is satisfied

$\Downarrow$

use lagrange Multiplier

$L = \frac{1}{2}\|\bar{w}\|^2 - \sum \alpha_i\left[y_i(\bar{w}\,\bar{x}_i + b) - 1\right]$

Simplified Objective

$$\boxed{w^*, b^* = \arg\underset{w,b}{Min}\frac{1}{2}\|w\|^2\ ,\quad s.t.\ \ y_n\left(w^T\left(\emptyset(x_n)\right) + b\right) \geq 1 \quad \forall n}$$

How to find the optimum w and b?

SVM Optimization

$$w^*, b^* = \arg\underset{w,b}{Min} \frac{1}{2}\|w\|^2 , \quad s.t. \;\; y_n\left(w^T\big(\emptyset(x_n)\big) + b\right) \geq 1 \quad \forall n$$

Solved by Lagrange multiplier method:

$$L(w, b, \alpha) = \frac{1}{2}\|w\|^2 \; - \sum_n \alpha_n [y_n\big(w^T\big(\emptyset(x_n)\big) + b\big) - 1]$$

where $\boldsymbol{\alpha}$ is the Lagrange multiplier

The optimization problem can be solved by setting **derivatives** of *Lagrangian* to 0

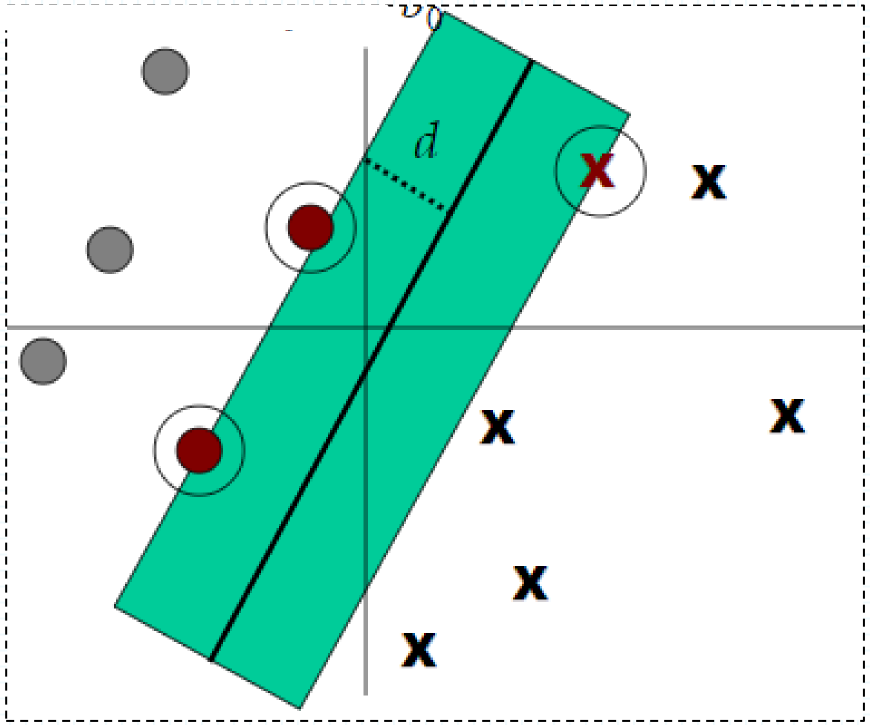
SVM Optimization

$$L(w,b,\alpha) = \frac{1}{2}\|w\|^2 \; - \sum_{n} \alpha_n [y_n\big(w^T\big(\emptyset(x_n)\big) + b\big) - 1]$$

$$\begin{cases} \dfrac{\partial L}{\partial w} = w \; - \displaystyle\sum_{n} \alpha_n y_n \emptyset(x_n) \quad = 0 \;\; \Rightarrow \;\; w = \displaystyle\sum_{n} \alpha_n y_n \emptyset(x_n) \\ \dfrac{\partial L}{\partial b} = \displaystyle\sum_{n} \alpha_n y_n = 0 \;\; \Rightarrow \displaystyle\sum_{n} \alpha_n y_n \; = 0 \end{cases}$$

SVM Optimization

$$L = \sum_{n} \alpha_n + \frac{1}{2} \sum_{n,m} \alpha_n \alpha_m y_n y_m \emptyset^T(x_m) \emptyset(x_n)$$



The Boundary:

$$Y = w^T\big(\emptyset(x)\big) + b = \sum_n \alpha_n y_n \emptyset^T(x_n)\,\emptyset(x)$$

Only depend on inner products

SVM Optimization

$$w^*, b^* = \arg\underset{w,b}{Min}\frac{1}{2}\|w\|^2, \quad s.t.\ \ y_n(w^T(\emptyset(x_n)) + b) \geq 1 \quad \forall n$$

$$Y = w^T(\emptyset(x)) + b = \sum_n \alpha_n y_n \emptyset^T(x_n)\,\emptyset(x)$$

The decision rule in SVMs only depends on the dot product with support vectors

several important implications

Computational efficiency
Memory efficiency
Robustness to noise and outliers