PHYS3451 - Lab 3 -March 3

To do this lab, we will use SymPy, which is a python library that allows us to do symbolic operations. If you would like, you can install python and SymPy on your computer following the instructions at `https://docs.sympy.org/latest/install.html#anaconda`. Or, you can use the online version at `https://live.sympy.org/`. See the SymPy Commands sheet for instructions on how to use SymPy. There is also a tutorial available at `https://docs.sympy.org/latest/index.html`.

At the end of lab, please submit the answers to your questions and a list of the commands you used to find your solutions (a text file is fine).

1. Given the function $f(x, y, z) = x^2 + xy + y^2 + z$, calculate:
   (a) $\vec{\nabla} f$
   (b) The line integral
   $$\int_{\vec{r_1}=(0,0,0)}^{\vec{r_2}=(1,1,1)} \vec{\nabla} f \cdot d\vec{r}$$
   using the paths:
      i. $(0,0,0) \to (1,0,0) \to (1,1,0) \to (1,1,1)$
      ii. $(0,0,0) \to (0,1,0) \to (1,1,0) \to (1,1,1)$
      iii. $(0,0,0) \to (1,1,1)$ along the line $x = y = z$.
   (c) Find the same line integrals for the vector $\vec{u} = (x - y)\hat{i} + (x + y)\hat{j} + z\hat{k}$.
2. Calculate the integral:
   $$\oint_S \vec{r} \cdot d\vec{\sigma}$$
   where S is the surface of a unit cube and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.
3. Show that the result of the previous question is equal to $\int_V \vec{\nabla} \cdot \vec{r} dr$, where V is the volume of the unit cube.
4. For a sawtooth wave given by
   $$f(t) = \begin{cases} t + \pi & |t| \leq \pi \\ 0 & |t| > \pi \end{cases}$$
   (a) Find the series coefficients.
   (b) Verify your coefficients by hand.