PHYS3451 - Lab 5 -March 31

1. (5 marks) Starting with the time-independent Schrödinger equation in spherical coordinates:
$$-\frac{\hbar^2}{2m}\nabla^2\psi(r,\theta,\phi) + V(r)\psi(r,\theta,\phi) = E\psi(r,\theta,\phi)$$
find the equation if the solution is of the form $\psi(r,\theta,\phi) = R(r)\Theta(\theta)\Phi(\phi)$.

2. (5 marks) Find the equation for the ϕ component. Use a separation constant of $-m_\ell^2$. What are the solutions to this equation? What are the values of the separation constant m_ℓ?

3. (5 marks) Now separate the r and θ components using a separation constant of $\ell(\ell+1)$.

4. (5 makrs) Find the solutions to $\Theta(\theta)$. What are the acceptable values of ℓ? You may find it helpful to make the substitution $z = \cos\theta$. (You should find that the solutions are associated Legendre Polynomials.)

5. (5 marks) To solve the radial equation, assume the potential has the form $V(r) = -Ze^2/r$. You may find the substitutions $\rho = 2\beta r$, $\beta^2 = -8mE/\hbar^2$ and $\gamma = 2mZe^2/\beta\hbar^2$. (Hint: We did a similar problem in class and found the solutions were associated Laguerre polynomials.)

6. (3 marks) What values of energy are allowed?

7. (9 marks) Plot the radial probability densities ($R^*R4\pi r^2$) for $(n,\ell) = (1,0), (2,1)$ and (3,1). If you do not have python installed, you can use the notebook at `https://www.kaggle.com/catherinelovekin/hatom`. You will need to make a copy of the notebook, and you can edit for 15 minutes without creating an account. (You can also create an account for free if you would like.) The notebook creates the required variables for you, and there are also some helpful python examples.

8. (3 marks) What is the probability density in the ϕ direction? Does this depend on m_ℓ?

9. (10 marks) The form of the θ component can be shown in polar diagrams. These show the value of the quantity $\Theta^*\Theta$ at each angle θ measured relative to the z axis. Make polar plots of the $\ell = 3$ case for all allowed values of m_ℓ.