PHYS3451 - Lab 2 - Feb 10

To do this lab, we will use SymPy, which is a python library that allows us to do symbolic operations. If you would like, you can install python and SymPy on your computer following the instructions at `https://docs.sympy.org/latest/install.html#anaconda`. Or, you can use the online version at `https://live.sympy.org/`. See the SymPy Commands sheet for instructions on how to use SymPy. There is also a tutorial available at `https://docs.sympy.org/latest/index.html`.

1. Given the two vectors

$$\vec{A} = 2\hat{e}_1 + \hat{e}_2 - \hat{e}_3, \qquad \vec{B} = \hat{e}_1 - \hat{e}_2 + \hat{e}_3$$

   (a) Find a unit vector that is perpendicular to both $\vec{A}$ and $\vec{B}$.

   (b) Prove that your unit vector is orthogonal to *both* $\vec{A}$ and $\vec{B}$ using dot products.

2. Given the matrix

$$A = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \sqrt{3} & 0 & 0 \\ \sqrt{3} & 0 & 2 & 0 \\ 0 & 2 & 0 & \sqrt{3} \\ 0 & 0 & \sqrt{3} & 0 \end{pmatrix}$$

   (a) Find $det(A)$.

   (b) Does A^{-1} exist? If yes, find A^{-1}.

   (c) A symmetric matrix is one where $A^T = A^{-1}$, and an anti-symmetric matrix is one where $A^T = -A$. Is A symmetric, anti-symmetric, or neither?

3. Solve the following system of equations using row-reduction (Gauss-Jordan elimination).

$$\begin{cases} w \phantom{{}+3x} + 10y - 4z = 1 \\ w + x + 4y - z = 2 \\ 2w + 3x + 2y + z = 5 \\ -2w - 2x - 8y + 2z = -4 \\ x - 6y + 3z = 1 \end{cases} \tag{1}$$

4. Find the eigenvalues and eigenvectors for the following matrix:

$$\begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$