X-Wing Strategy

From sudokuwiki.org, the puzzle solver's site

This strategy is looking at single numbers in rows and columns. It should be easier to spot in a game as we can concentrate on just one number at a time.

The picture on the right shows a classic X-Wing, this example being based on the number **seven**. The X is formed from the diagonal correspondence of squares marked **A, B, C** and **D**. What's special about them?

X-Wing example 1 : Load Example or : From the Start

Well, **A** and **B** are a locked pair of **7**'s. So is **C** and **D**. They are locked because they are the only **7**'s in rows B and F. We know therefore that if **A** turns out to be a **7** then **B** cannot be a **7**, and vice versa. Likewise if **C** turns out to be a **7** then **D** cannot be, and vice versa.

What is interesting is the **7**'s present elsewhere in the fourth and eighth columns. These have been highlighted with green boxes.

Think about the example this way. **A, B, C** and **D** form a rectangle. If **A** turns out to be a **7** then it rules out a **7** at **C** as well as **B**. Because **A** and **CD** are 'locked' then **D** must be a **7** if **A** is. Or vice versa. So a 7 MUST be present at **AD** or **BC**. If this is the case then any other **7**'s along the edge of our rectangle are redundant. We can remove the **7**'s marked in the green squares.

The rule is

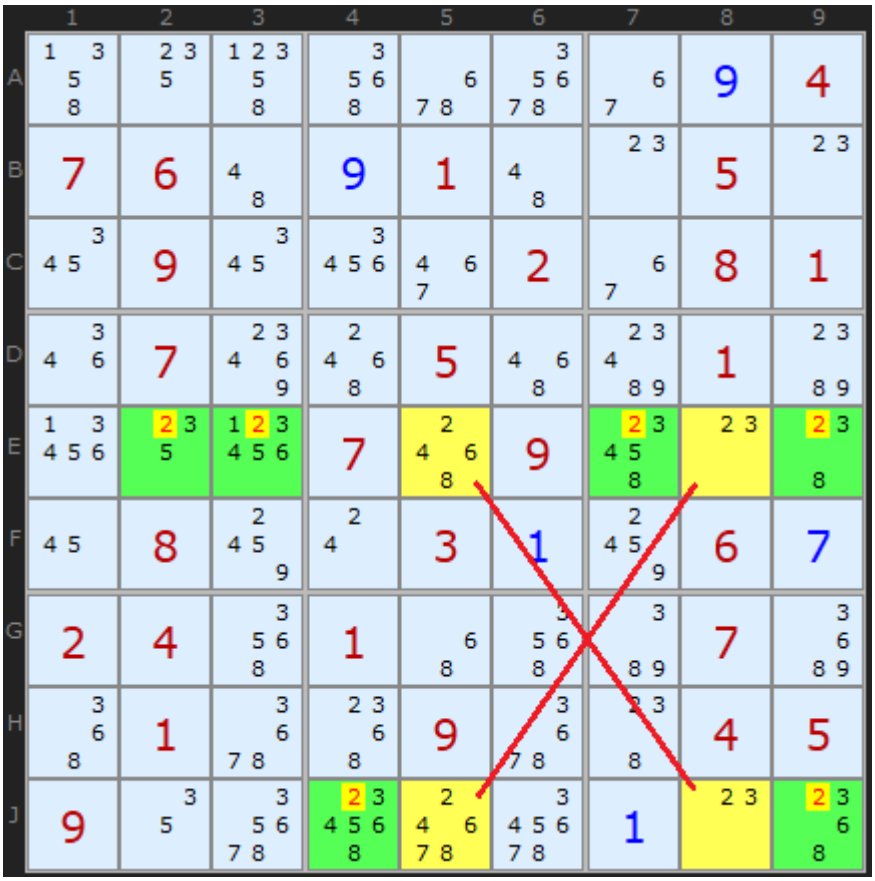
When there are

- *only two possible cells for a value in each of two different rows,*
- *and these candidates lie also in the same columns,*

then all other candidates for this value in the columns can be eliminated.

The reverse is also true for 2 columns with 2 common rows. Since this strategy works in the other direction as well, we will look at an example next.

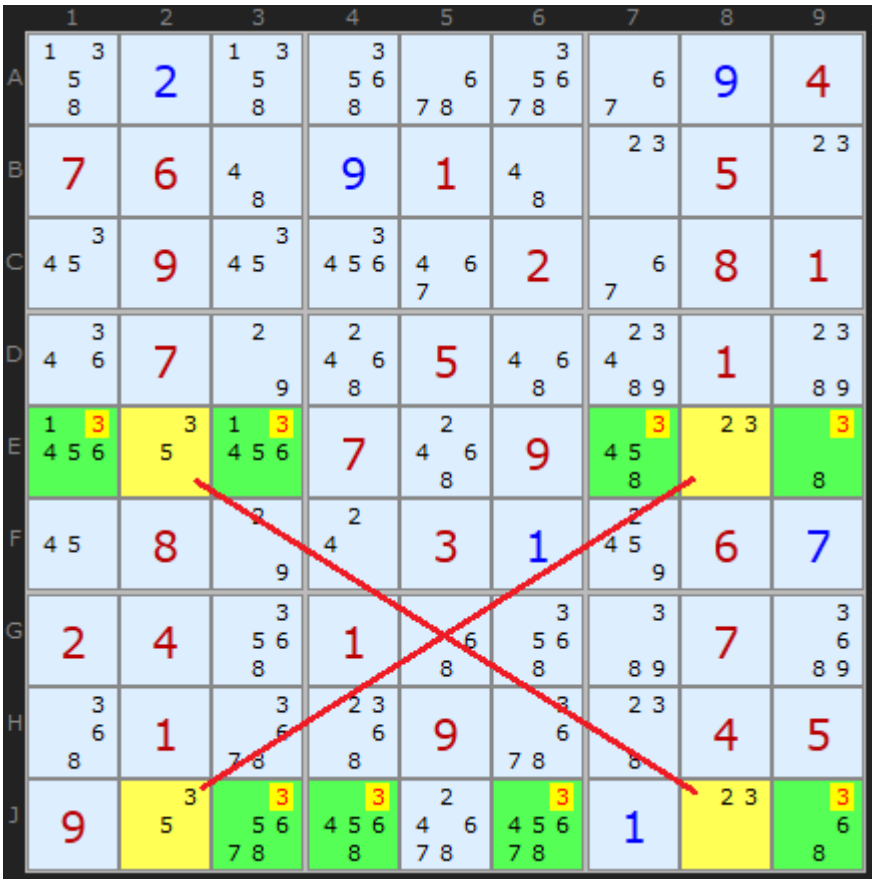
In this second example I've chosen a Sudoku puzzle where an enormous number of candidates can be removed using two X-Wings. The first is a '2-Wing'. The yellow high lighted cells show the X-Wing formation. Note that the orientation is in the columns this time, as opposed to rows as above. Looking at columns we can see that candidate 2 only occurs twice - in the yellow cells. Which ever way the 2s could be placed (**E5/J8** or **E8/J5**) six other 2s in the rows can be removed - the green highlighted cells.



X-Wing example 2 : Load Example or : From the Start

A few steps later the second X-Wing is found on candidate 3 in the same rows. Whichever way round the 3 can be placed in those rows (**E2/J8** or **E8/J2**) there can be no other 3 in rows E and J except in those yellow cells.

Generalising X-Wing



X-Wing example 3

X-Wing is not restricted to rows and columns. We can also extend the idea to boxes as well.
If we generalise the rule above we get:

When there are

- *only 2 candidates for a value, in each of 2 different units of the same kind,*
- *and these candidates lie also on 2 other units of the same kind,*

then all other candidates for that value can be eliminated from the latter two units.

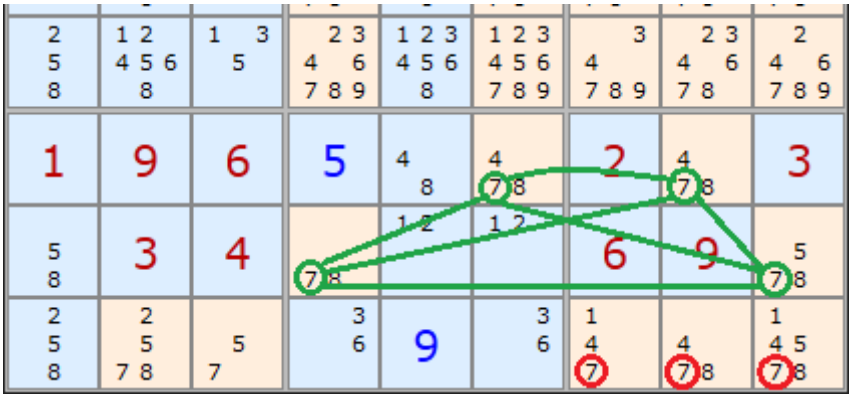
Now we have 6 combinations:

- Starting from 2 rows and eliminating in 2 columns
- Starting from 2 columns and eliminating in 2 rows
- Starting from 2 boxes and eliminating in 2 rows
- Starting from 2 boxes and eliminating in 2 columns
- Starting from 2 rows and eliminating in 2 boxes
- Starting from 2 columns and eliminating in 2 boxes

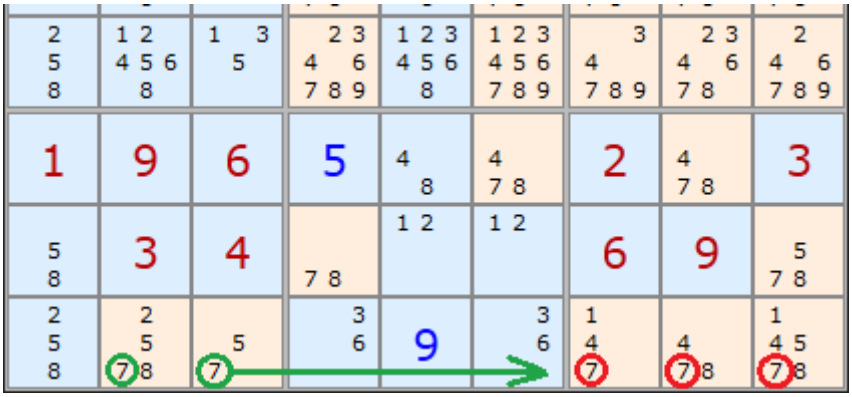
- Classic X-Wing
- Classic X-Wing
- Same effect as line/box reduction
- Same effect as line/box reduction
- Same effect as pointing pairs
- Same effect as pointing pairs

Here is an example of combination 5. Starting from 2 rows and eliminating in 2 boxes, in this case the last two boxes in the Sudoku. The rows are 7 and 8 and they each have two **7**s. Our x-Wing is now a trapezoid but the logic is the same.

We can be certain that 7 can be eliminated at **X**, **Y** and **Z**



But HOLD UP one moment. There is a simpler strategy that does the same job!



A and **B** above are a pointing pair. This removes the same 7s in the same place. Combination 6 is also the complement of a pointing pair. Combinations 3 and 4 are also complements of the Line/Box Reduction. Our generalisation of X-Wing to boxes hasn't profited us at all. We learn that

X-Wings containing boxes are the inverse of the Intersection Removal strategies

X-Wing Exemplars

These puzzles require the X-Wing strategy at some point but are otherwise trivial.
They make good practice puzzles.

- Exemplar 1, x1 (score 80)
- Exemplar 2, x1 (score 82)
- Exemplar 3, x2 (score 174, +1 Naked Pair)
- Exemplar 4, x2 (score 178, +1 Naked Pair)

Go back to Intersection Removal Continue to Singles Chains