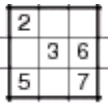
Extended Unique Rectangles

From sudokuwiki.org, the puzzle solver's site



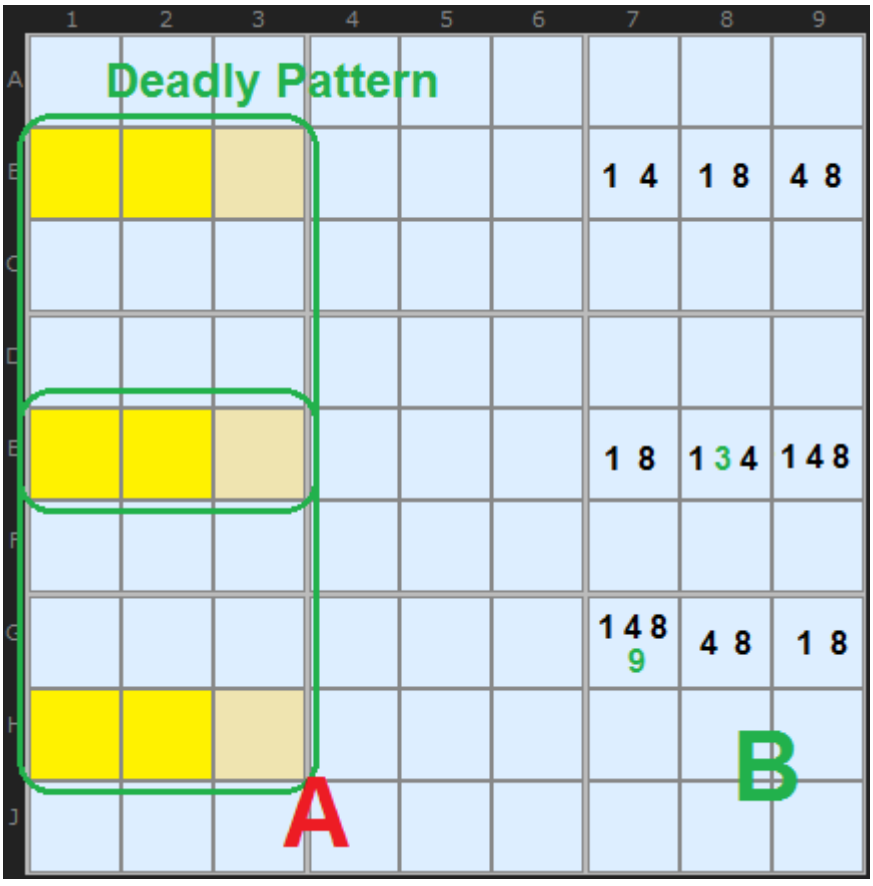
Unique Rectangles can be larger structures than simple 2 by 2.
In this extension I show how the solver can now detect 2 by 3.
Just as a Naked Pair or Hidden Pair can be extended to a Triple, so can the cells that form Unique Rectangles.

A **Triple** is any three cells with *a total* of three candidates left. That doesn't always mean
[1,2,3] + [1,2,3] + [1,2,3]
Which is exactly three candidates in three cells. Other Triples will have some digits missing, like
[1,2,3] + [1,2,3] + [2,3]
[1,2] + [1,2,3] + [1,2,3]
[1,2,3] + [1,2] + [2,3]
And the ultimate skinny triple will have two out of three in each cell:
[1,2] + [1,3] + [2,3]
But all of these have - in total - three candidates in three cells.

Now, 2 by 3 is a bit of an odd shape. Other extensions, like X-Wing to Swordfish jump from 2 by 2 (four cells) to 3 by 3 (nine cells). We don't need to search for a nine-cell 3 by 3 although they do theoretically exist. Take a look at the diagram to the right. In the left hand side marked **A** I've drawn the full 3 by 3 Deadly Rectangle.

Note: it must occupy exactly three rows, three columns and three boxes.

To the right labelled B I've marked out a candidate distribution of 1s, 4s and 8s. For each row and each column you'll see a Triple. If we've arrived at this state we're in trouble because there are many ways to settle those cells in all combinations of 1, 4 and 8. The only thing that saves us are the extra candidates marked in green.



Here's how we go from the 3 by 3 *ideal* pattern to the much more likely 2 by 3 version: Take away any one of the columns with numbers and you still have a multiple solution problem. For example, if we took away column 8 (and ignored the green extra candidates for a moment), we could still fill column 7 with 1 + 8 + 4 (going down the rows), 4 + 1 + 8 and 4 + 8 + 1. Same for column 9. We could fill it with 4 + 1 + 8 or 4 + 8 + 1 or 8 + 4 + 1. If you take any column away it looks like the problem stays a problem. So conclusion - don't worry about looking for 3 by 3 Deadly Patterns - they will come out in the wash when you find 2 by 3.

In this first example we have an Extended Deadly Pattern based on the three candidates {1,3,5}. The yellow cells mark the pattern. Notice this pattern is vertical like the theory example above - but as we are not looking for (and don't have) a 3 by 3, we need to look out for the pattern working horizontally as well. One of these is below.

	1	2	3	4	5	6	7	8	9
A	9	1 6	7	1 8	2	4	3	6 8	5
B	8	4	2	3	6	5	9	1	7
C	1 5 6	3	1 5	9	1 8	7	4	2 6 8	2 8
D	1 2 5	1 5 7 8	4	5 7	1 3 8	9	6	2 5 8	2 3 8
E	1 3 5	1 7 8	1 3	2	4	6	1 7	5 8	9
F	2 5 6	1 5 6 7	9	5 7	1 3 8	1 8	1 7	4	2 3 8
G	1 3 5	1 5	1 3 5	4	9	2	8	7	6
H	7	9	6	1 8	5	1 8	2	3	4
J	4	2	8	6	7	3	5	9	1

Example Extended UR : Load Example *or :* From the Start

The only type of UR I've implemented in the solver, so far, is **Unique Rectangle Type 1**.

This says that the odd cell out - the cell with an extra candidate or two - is the key. Those extra candidates must be part of the solution. If they didn't exist we've have a full Deadly Pattern and we need to avoid that at all costs. So 6 in **C1** is the extra candidate - it must be the solution, so we can remove 1 and 5 from **C1** and progress.

Here is a similar Extended Unique Rectangle based on the cells AEH78 (which just means cells on rows A, E and H and columns 7 and 8). The Triple is {4,7,9} and we have 1 and 6 - on cell **E8**. That's the get-out-of jail card avoiding the bigger Deadly Pattern. Take away the 4 and 7 from **E8** and we're back into single solution territory.

	1	2	3	4	5	6	7	8	9
A	4 5	6	3	8	2	1 4 5	7 9	7 9	1 4 5
B	7	4 8	5 8	1 3 4 5	1 3 5	9	2	1 4	6
C	2	1	9	4 5 6	5 6	7	3	8	4 5
D	4 5 6	3	2	1 4 5 6 7	9	1 4 5 6	8	1 4 6 7	1 4
E	9	4 7 8	5 8	1 3 4 5 6 7	1 3 5 6 8	1 3 4 5 6	4 7	1 4 7 6	2
F	4 6	4 7 8	1	4 6 7	6 8	2	5	3	9
G	1	2	4	9	7	8	6	5	3
H	8	5	6	2	1 3	1 3	4 9	4 9	7
J	3	9	7	5 6	4	5 6	1	2	8

Example Extended UR 2 : Load Example or : From the Start

Finally a horizontal example.

The Triple {3,7,8} sits over the two rows D and E. There is a mixture of bi-value and tri-value cells but we clearly have two triples with the same values very close to each other. We have to use the extra candidates provided. There is a lone 1 in **D2** that gets us out of trouble.

	1	2	3	4	5	6	7	8	9
A	3 6 7	9	3 4 7	8	4 6	1	5	2	3 7
B	8	5	1	3 7	9	2	6	3 7	4
C	3 6 7	2	3 4 7	3 7	4 6	5	9	1	8
D	1 2 7	1 3 8	9	6	5	3 7	3 7 8	4	2 3 7
E	4	3 8	2 5	1 2 9	1 2	3 7	3 7 8	5 9	6
F	2 5 7	6	2 3 5 7	2 9	8	4	1	3 5 7 9	2 3 7 9
G	2 3 5 9	7	2 3 5	1 2	1 2 3	6	4	8	1 3 9
H	1 3 9	1 3	6	4	7	8	2	3 9	5
J	2 3	4	8	5	1 2 3	9	3 7	6	1 3 7

Example Extended UR 3 : Load Example or : From the Start

I will admit, instances of the Extended Unique Rectangle are not common, but if you are just getting into Unique Rectangle you'll be looking for the very common Type 1. I think it's worth keeping an eye out for this formation at the same time. You will notice a similar cluster of common candidates and it's just possible - if you remember the three boxes spread rule - that you'll find one when you're looking for the 2 by 2 version.

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