3D Medusa

From sudokuwiki.org, the puzzle solver's site

3D Medusa extends Simple Colouring (or 'Single's chains') into a third dimension. Simple Colouring looked for pairs of X in rows, columns and boxes. Wherever the chains led they stuck to the same candidate number. This is good for tracking an elimination when you have made notes on a paper Sudoku for a particular number but it limits the scope of the strategy. The way we extend the search is up through the bi-value cells which contain two different numbers. You can think of the different candidate numbers as existing in a third dimension lifting up from the page with 1 at the bottom and 9 at the top.

The devastating effect of colouring is that we are showing that ALL of one colour will be the solution. We don't know which set yet - but if any one of those cells becomes the solution we can know for certain ALL the cells of the same colour

Rule 1 - Twice in a Cell

There are six different ways we eliminate - six contradictions. The first is in the example to the right. It doesn't matter where you start on the grid. In this example I've started with the 4s in row B. By colouring one green and the other yellow we mentally draw a line between them, done graphically on the diagram. Going into the third dimension in **B7** when we colour the 4 yellow we can colour the 9 green - since there are only two values left in the cell.

Continue to look for bi-value and bi-location candidates and you soon build up a web of connections. This is where the image of Medusa was perhaps attached to this strategy - her head being a tangle of snakes.

3D Medusa Rule 1 : Load Example or : From the Start

When you have built up a web of connections, alternating between two colours you might find a cell with the same colour set twice. This has been ringed in **H2**. Since we know that if yellow candidates have the potential to be ALL true we can't have a situation where two yellow numbers are competing for the same cell. This is a contradiction and therefore we can state that no yellow numbers can be the solution!

Rule 1 is: **If two candidates in a cell have the same colour - all of that colour can be removed - and the opposite colour are all solutions**

Update October 2015 : My old definition of Rule 1 only made a negative assertion about the yellow candidates - they can be removed. But **Steve Jacobs**, also programming a solver, alerted me to the fact that all green candidates MUST be the solutions to their cells - a positive assertion. This is self-evident for bi-value cells (where only green and yellow exist and green becomes a **Naked Single**) but also for cells like **H1** in the example. The 1 in **H1** becomes a **Hidden Single**. This is true because of the binary either/or connections in the Medusa web. My solver will continue to only remove the yellow candidates - hidden singles will be eliminated in the next step - but for pen and paper solvers - go fill in the 'green' solved cells. Jacobs' corollary also applies to Rule 2 and Rule 6.

Note: this rule does not exist in Simple Colouring since the same number does not appear twice in the same cell.

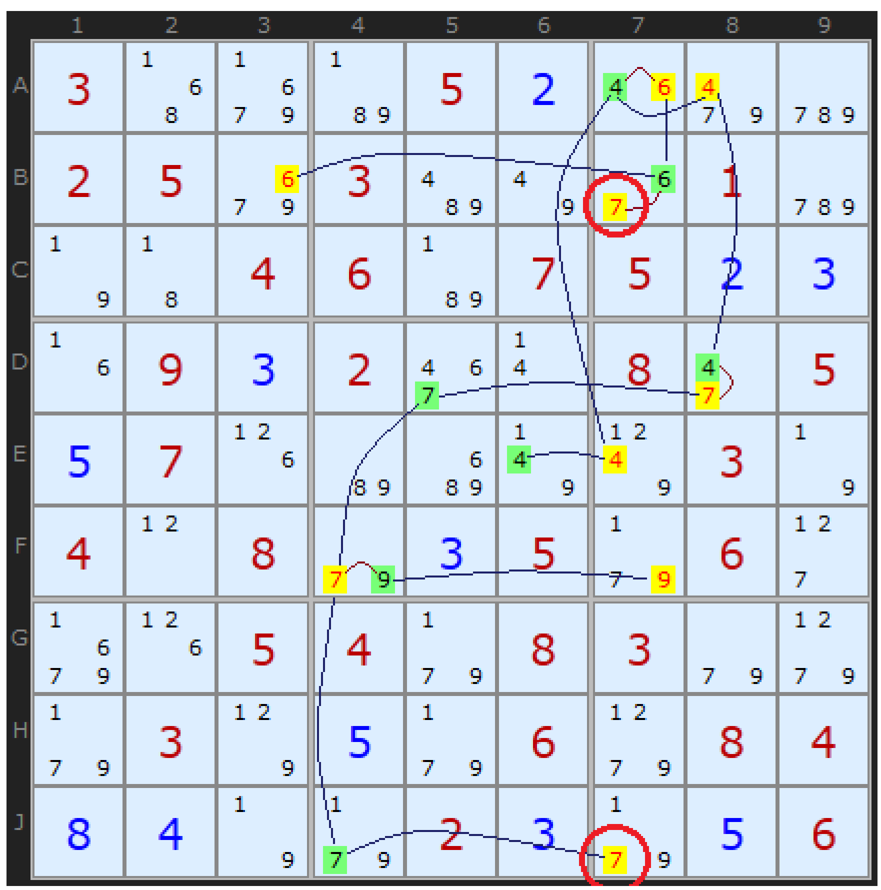
As an exercise, try colouring any of the highlighted cells starting from a different position. You may end up swapping the colours around and you may find some new connections. But eventually - in this example - you will get two of the same colour on **H2**. This is a very powerful yet simple strategy.

Rule 2 - Twice in a Unit

This rule **is** shared with Simple Colouring. Its the same principle as the first rule but we are looking for two coloured occurrences of X in the same unit (row, column or box) as opposed the two of the colour in the same cell.

The example shows most of the links between bi-value and bi-location candidates, coloured between green and yellow. Ringed in red are two 7s in column 7. Since both cannot be true neither can be true and all yellow coloured candidates can be removed - and all green coloured candidates are solutions to their cells.

(Example requires three Medusa Rules 6 before Rules 2 comes into play)

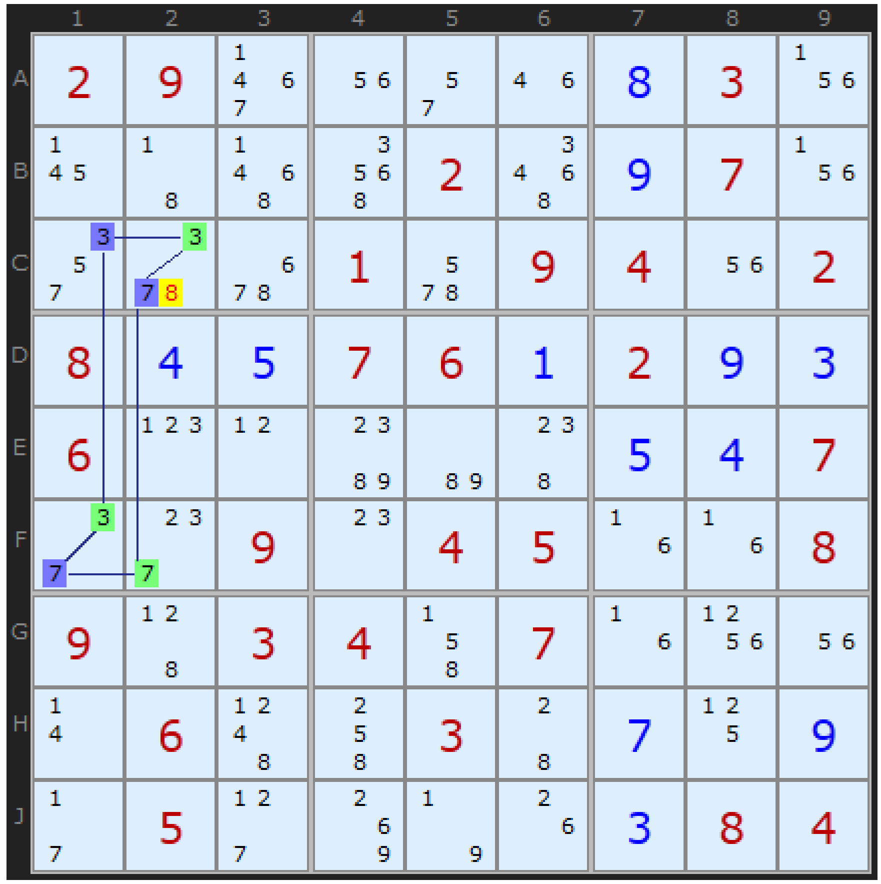


3D Medusa Rule 2 : Load Example *or :* From the Start

Rule 3 - Two colours in a cell

If you had unticked 3D Medusa in the solver this example would have been found by a number of later strategies, particularly Alternating Inference Chains as the pattern is a classic **Nice Loop**. 3 and 7 alternate. It doesn't matter where you start in a Nice Loop but you can trace the on / off or green/blue round the loop. 3s and 7s neatly occur twice in units and cells.

But 3D Medusa is not about loops, its about the network of links. This example just happens to be the same formation. We know that either ALL the blue candidates will be true, or ALL the green ones. If there are any another candidates in any cell with two colours, they cannot be solutions. Hence the 8 can be removed from **C2**. In Nice Loop terms, this is an *off-chain* elimination.



3D Medusa Rule 3 : Load Example *or :* From the Start

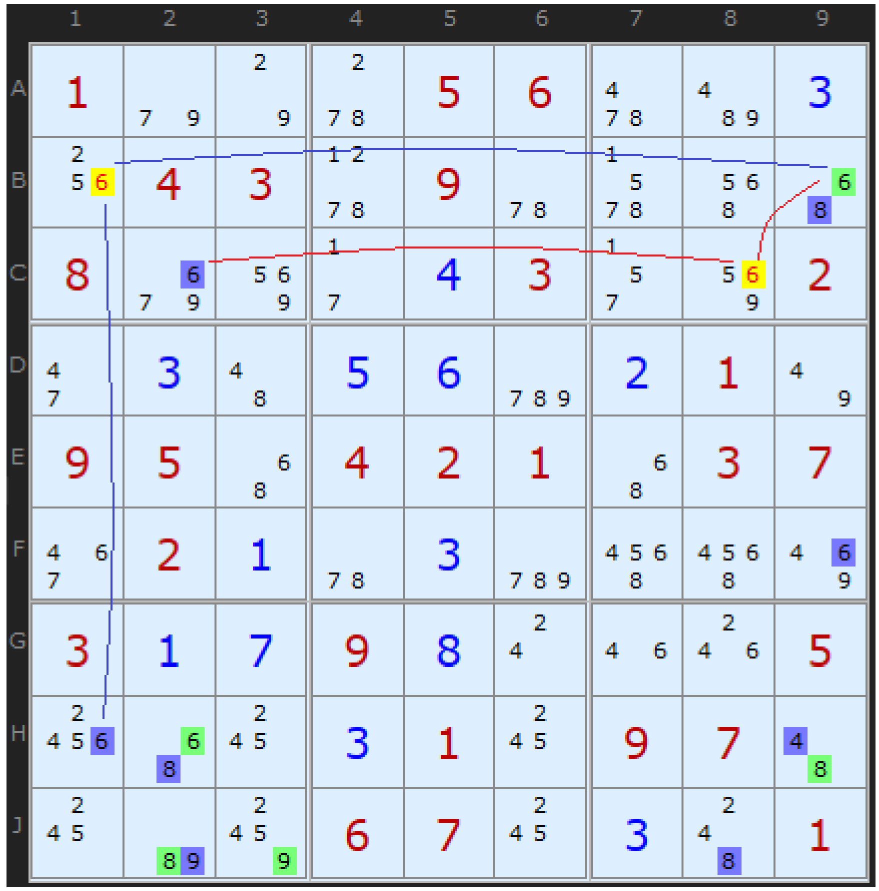
Simple Colouring cannot produce this elimination since it is restricted to a single candidate number.

Rule 4 - Two colours 'elsewhere'

If we can eliminate "off chain" in a cell we can certainly do so off-chain in a unit. In this example there are two sets of eliminations (blue and red lines) that point to 6s. We are certain than ALL blues are the solution or ALL greens. Therefore where there are candidates that can see both colours they can be removed. By 'see' we mean any candidates that are the same number as members of the blue/green links.

The 6 in **B1** is removed because of the coloured 6s along the row in **B9** and down the column in **H1**. In a similar way the blue 6 in **C2** and the green 6 in **B9** point to 6 in **C8**

This rule is shared with Simple Colouring.



3D Medusa Rule 4 : *Load Example* *or :* *From the Start*

A few steps later in the same puzzle we get a cluster of 4s, 6s and an 8 using the same observation.

Since February 2015 I have combined Rule 4 with the old Rule 5. I want to thank a reader going by the name **FallsOffRocks** for pointing out that the old Rule exactly covered all the eliminations that Rules 4 did and was redundant. I've folded Rule 5 ('elsewhere') into 4 ('along a unit') and decremented Rules 6 and 7. A simplification! So from now on there are only 6 Medusa Rules. This also affects Simple Colouring.

3D Medusa Rule 4 : *Load Example* *or :* *From the Start*

Rule 5 - Two colours Unit + Cell

This type of elimination looks to be the most complex - but inconveniently it is the most common. It's well worth looking out for. The rule says

If an uncoloured candidate can see a coloured candidate elsewhere (it shares a unit) and an opposite coloured candidate in its own cell, it can be removed..

So its a combination of unit and cell - the colours green and blue are found looking along a unit and within the same cell. The example to the right demonstrates this with four eliminations.

The logic is very appealing. Consider 1 in **E5**. If 1 were the solution to the cell it would remove a green 1 from **E6** AND a blue 7 from its own cell in **E5**. Since we know ALL blue or ALL green must be solutions we have a contradiction.

3D Medusa Rule 5 : Load Example or : From the Start

Rule 6 - Cell Emptied by Color

Anton Delprado in the comments below has discovered another way we can use 3D Medusa and I'm pleased to include it in the solver. It's almost a reflection of Rule 5. Take any cell that doesn't have any colors from the coloring and see if all the candidates can see the same color. If that color were the solution (and remember, all of one or all of the other will be) then all the candidates in that cell would be removed - leaving an empty cell!

Rule 6 highlights in cyan the cell that catches this. You can see the 2 and 5 in cell **C1** can see the yellow 2 in **C9** and 5 in **H1**. (Yellow is used to show eliminated cells). All green coloured candidates are solutions to their cells.

Rule 6 : Load Example or : From the Start

Rule 6 using 3 candidates in a cell : Load Example or : From the Start

This puzzle has an amazing series of Medusa calls, using many different rules. It ends with Rule 6. I wanted to show a second puzzle to emphasize that the cell we are comparing the Medusa net to can have any number of candidates. These are 4,6 and 9 in **C6**. Green candidates have been turned yellow because they are eliminated, but you can see that the 4, 6 and 9 can all see the same color somewhere along the column or row.

You can be certain that it will be one color or the other, never equally both. Because this strategy is easier to spot and somewhat follows on from Rule 4, the solver looks for it before Rule 5. But too late to re-number them now. Well spotted Anton!

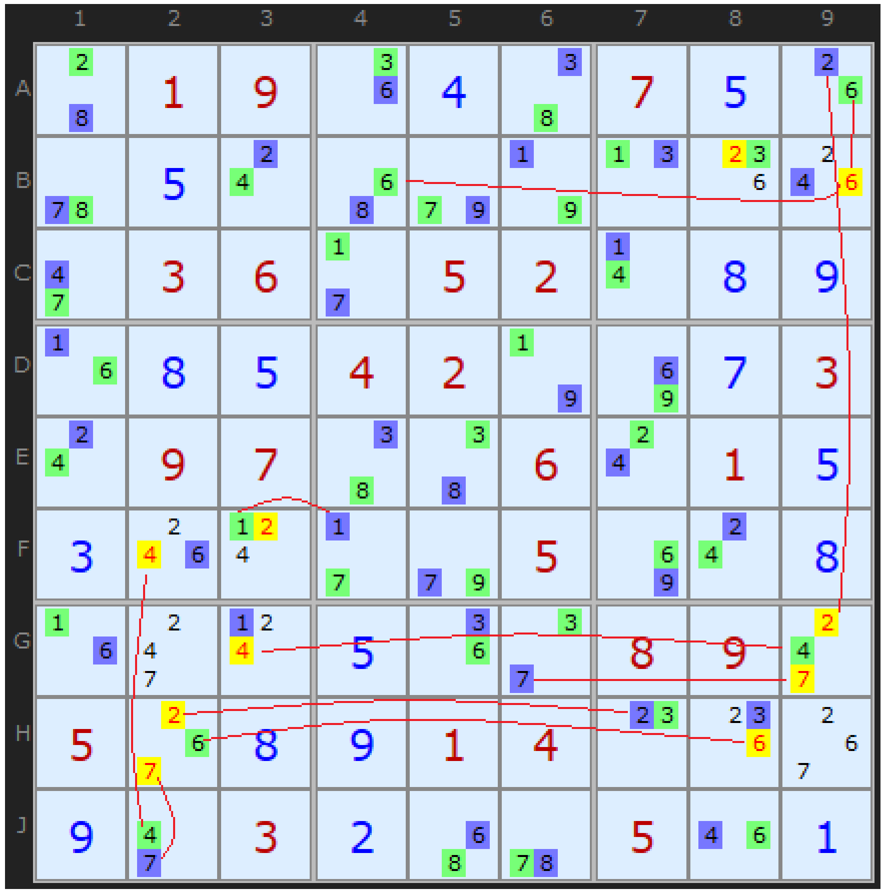
To end this article I want to show you some special puzzles discovered by Klaus Brenner starting with this 37 elimination Rule 1 Medusa that completely solves the puzzle from that point. We go from 35 known numbers to 70 and the rest is trivial!

There are two candidates in **A1** with the same color, 5 and 7. So All of those of that color can be removed.

However, the initial puzzle is not trivial and a very large number of steps are required before this mega medusa. Certainly an extreme grade.

37 Eliminations by Rule 1 : Load Example or : From the Start

A slightly different puzzle, 22 clues, gives us these 10 eliminations using Rule 5. I don't know another with more in one step.



Rule 5 : Load Example

3D Medusa Exemplars

These puzzles require the 3D Medusa strategy at some point but are otherwise trivial.
They make good practice puzzles.

- Exemplar 1, x1 (score 196)
- Exemplar 1, x1 (score 226)
- Exemplar 1, x2 (score 352, +1 Naked Pair)

Go back to XY-Chains Continue to Remote Pairs