Homework 8

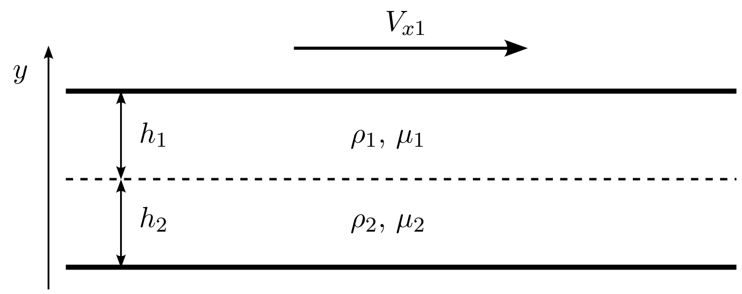
Robert Picardo

BIOENG 104 Biological Transport Phenomena | Aaron Streets

University of California, Berkeley

1 Problem 1

Imagine a scenario in which you have two different immiscible fluids sandwiched between two plates. The two fluids have densities and viscosities ρ_1 and μ_1, and ρ_2 and μ_2 respectively (where ρ_1 is slightly smaller than ρ_2). Flow is generated between the stationary plate on the bottom and a top plate moving with constant velocity V_{x1}. Assume we wait until this flow reaches steady state, $V_x(y) \neq f(t)$.



(a) Solve for the flow velocity, $V_x(y)$, in this system.

We make the following assumptions:

- fully-developed, laminar, and incompressible flow
- both fluids are Newtonian
- no edge effects

outside of the assumption that the system is at steady state. If we take a differential element of the fluid body, we can employ Newton's second law, which states that the change in momentum is equal to the sum of forces applied ($\sum F_x$) plus the momentum flow rate entering the element, as well as the momentum flow rate leaving it.

$$\frac{\mathrm{d}(mV_x)}{\mathrm{d}t} = \underbrace{\frac{mV_x}{\Delta x \Delta y \Delta z} V_x|_x \Delta y \Delta z}_{\text{momentum flow rate in}} - \underbrace{\frac{mV_x}{\Delta x \Delta y \Delta z} V_x|_{x+\Delta x} \Delta y \Delta z}_{\text{momentum flow rate out}} + \sum F_x$$

where m is the mass of the differential element. Rearranging the above equation, we get

$$\frac{\mathrm{d}(mV_x)}{\mathrm{d}t} = \frac{mV_x}{\Delta x}(V_x|_x - V_x|_{x+\Delta x}) + \sum F_x$$

From the assumption of fully-developed flow, the velocity field does not vary throughout the direction of flow (in the x-direction). Thus, $V_x|x = V_x|_{x+\Delta x}$, and the change in momentum is solely due to the forces applied on the element.

$$\frac{\mathrm{d}(mV_x)}{\mathrm{d}t} = \sum F_x$$

From the steady-state assumption, $\mathrm{d}mV_x/\mathrm{d}t$ equals zero. For the right side of the above equation, the forces applied to the differential element come from the shear stresses along the y-faces.

$$0 = \sum F_x = \tau_{yx}|_{y+\Delta y}\Delta x\Delta z - \tau_{yx}|_y\Delta x\Delta z$$

Dividing both sides by $\Delta x\Delta y\Delta z$,

$$0 = \frac{\tau_{yx}|_{y+\Delta y} - \tau_{yx}|_y}{\Delta y}$$

and taking the limit as Δy approaches zero,

$$0 = \frac{\mathrm{d}\tau_{yx}}{\mathrm{d}y}$$

Recall that, for a Newtonian fluid, the shear stress on the y-face of a differential volume is proportional to $\mathrm{d}V_x/\mathrm{d}y$.

$$0 = \frac{\mathrm{d}}{\mathrm{d}y}\left(\mu\frac{\mathrm{d}V_x}{\mathrm{d}y}\right) \tag{1.1}$$

where μ is the fluid viscosity. Integrating twice, we obtain a particular solution.

$$\begin{aligned}
\int \frac{\mathrm{d}}{\mathrm{d}y}\left(\mu\frac{\mathrm{d}V_x}{\mathrm{d}y}\right)\mathrm{d}y &= \int 0\,\mathrm{d}y \\
\mu\frac{\mathrm{d}V_x}{\mathrm{d}y} &= A\mu \\
\int \frac{\mathrm{d}V_x}{\mathrm{d}y}\,\mathrm{d}y &= \int A\,\mathrm{d}y \\
V_x(y) &= Ay + B
\end{aligned}$$

for A and B constants. Since there are two domains with different values of ρ and μ, we have to let $V_x(y)$ be a piecewise function defined as two linear functions, V_1 and V_2, for each domain.

$$\begin{aligned}
V_{x1}(y) &= A_1y + B_1 \qquad h_2 < x \leq h_1 + h_2 \\
V_{x2}(y) &= A_2y + B_2 \qquad 0 \leq x \leq h_2
\end{aligned}$$

To solve for the four variables, we need four boundary conditions.

- no-slip condition 1: $V_{x2}(0) = 0$
- no-slip condition 2: $V_{x1}(h_1 + h_2) = V_{x1}$
- interface condition 1: $V_{x1}(h_2) = V_{x2}(h_2)$
- interface condition 2: $\tau^1_{yx}(y = h_2) = \tau^2_{yx}(y = h_2)$

From no-slip condition 1, $B_2 = 0$. From no-slip condition 2,

$$A_1(h_1 + h_2) + B_1 = V_{x1} \tag{1.2}$$

and from interface condition 1,

$$A_1 h_2 + B_1 = A_2 h_2 \tag{1.3}$$

Using the definition of a Newtonian fluid—that $\tau_{yx} = \mu\, \mathrm{d}V_x/\mathrm{d}y$—we can transform the second interface condition as follows:

$$\mu_1 A_1 = \mu_2 A_2 \tag{1.4}$$

We can solve for A_2 and B_1 in terms of A_1 from Equations (1.2) and (1.4)...

$$\begin{aligned} B_1 &= V_{x1} - A_1(h_1 + h_2) \\ A_2 &= \mu_1 A_1/\mu_2 \end{aligned}$$

and substitute them to Equation (1.3).

$$\begin{aligned} A_1 h_2 + V_{x1} - A_1(h_1 + h_2) &= \mu_1 A_1 h_2/\mu_2 \\ A_1 &= \frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} \end{aligned}$$

So, our unknown constants are

$$\begin{aligned} A_1 &= \frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} \\ A_2 &= \frac{\mu_1}{\mu_2}\frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} = \frac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} \\ B_1 &= V_{x1} - \frac{V_{x1}(h_1 + h_2)}{h_1 + (\mu_1/\mu_2)h_2} \\ B_2 &= 0 \end{aligned}$$

and so the velocity profiles for each domain are

$$V_{x1}(y) = \frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} y + V_{x1} - \frac{V_{x1}(h_1 + h_2)}{h_1 + (\mu_1/\mu_2)h_2} \qquad V_{x2}(y) = \frac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} y$$

$$\boxed{V_x(y) = \begin{cases} \dfrac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} y & 0 \le y < h_2 \\ \dfrac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} y + V_{x1} - \dfrac{V_{x1}(h_1+h_2)}{h_1 + (\mu_1/\mu_2)h_2} & h_2 \le y \le h_1 + h_2 \end{cases}} \tag{1.5}$$

> (b) Show that if $\rho_1 = \rho_2$, and $\mu_1 = \mu_2$, then the solution simplifies to the solution we had in class.

Substituting $\mu_1 = \mu_2$ into Equation (1.5) gives

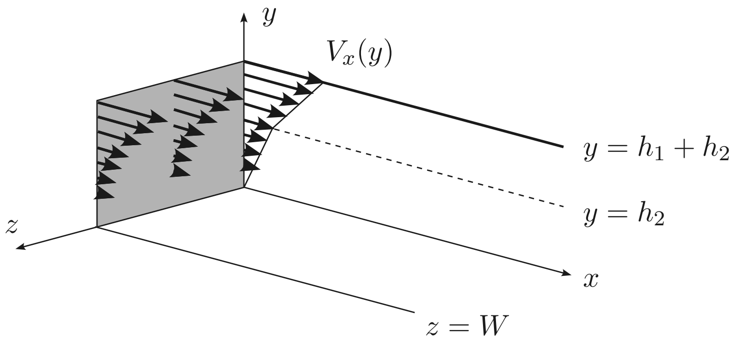
$$V_{x2}(y) = \frac{V_{x1}}{h_1 + h_2} y$$
$$V_{x1}(y) = \frac{V_{x1}}{h_1 + h_2} y + V_{x1} - \frac{V_{x1}(h_1+h_2)}{h_1+h_2} = \frac{V_{x1}}{h_1+h_2} y + (V_{x1} - V_{x1}) = \frac{V_{x1}}{h_1+h_2} y$$

so V_x simplifies to

$$V_x = \frac{V_{x1}}{h} y \qquad \forall y \in [0, h]$$

where $h = h_1 + h_2$.

> (c) Find the average velocity, $\langle V \rangle$, between the plates through a cross-sectional area defined by the total height of the channel and a width, W (into and out of the plane of the paper). (You can ignore edge effects and assume that the velocity profile is not changing with "z", into and out of the page.



The average velocity can be calculated through

$$\langle V \rangle = \frac{1}{\int_{\mathcal{A}} \mathrm{d}A} \int_{\mathcal{A}} (\mathbf{V} \cdot \hat{\mathbf{n}})\, \mathrm{d}A = \frac{1}{A} \int_{\mathcal{A}} (\mathbf{V} \cdot \hat{\mathbf{x}})\, \mathrm{d}A = \frac{1}{A} \int_{\mathcal{A}} V_x \, \mathrm{d}a \tag{1.6}$$

where $A = (h_1 + h_2)W$ is the cross-sectional area and $\mathrm{d}a = \mathrm{d}y\,\mathrm{d}z$. We know that

$$\int_{\mathcal{A}} V_x \, \mathrm{d}a = \int_0^W \int_0^{h_1+h_2} V_x(y)\, \mathrm{d}y\, \mathrm{d}z \tag{1.7}$$

Evaluating the inside integral with respect to y,

$$\begin{aligned}
\int_0^{h_1+h_2} V_x(y)\,\mathrm{d}y &= \int_0^{h_2} V_{x2}(y)\,\mathrm{d}y + \int_{h_2}^{h_1+h_2} V_{x1}(y)\,\mathrm{d}y \\
&= \int_0^{h_2} A_2 y\ \mathrm{d}y + \int_{h_2}^{h_1+h_2} (A_1 y + B_1)\,\mathrm{d}y \\
&= \frac{A_2}{2} y^2 \Big|_0^{h_2} + \left(\frac{A_1}{2} y^2 + B_1 y\right)\Big|_{h_2}^{h_1+h_2} \\
&= \frac{A_2}{2} h_2^2 + \left[\frac{A_1}{2}(h_1+h_2)^2 + B_1(h_1+h_2)\right] - \left[\frac{A_1}{2} h_2^2 + B_1 h_2\right] \\
&= \frac{1}{2} h_2^2 (A_2 - A_1) + \frac{A_1}{2}(h_1+h_2)^2 + B_1 h_1
\end{aligned}$$

where $A_1 = V_{x1}/[h_1 + (\mu_1/\mu_2)h_2]$. So, from Equation (1.7),

$$\begin{aligned}
\int_{\mathcal{A}} V_x\,\mathrm{d}a &= \int_0^W \left[\frac{1}{2} h_2^2 (A_2 - A_1) + \frac{A_1}{2}(h_1+h_2)^2 + B_1 h_1\right] \mathrm{d}z \\
&= W\left[\frac{1}{2} h_2^2 (A_2 - A_1) + \frac{A_1}{2}(h_1+h_2)^2 + B_1 h_1\right]
\end{aligned}$$

and from Equation (1.6),

$$\begin{aligned}
\langle V \rangle &= \frac{1}{(h_1+h_2)W} \cdot W\left[\frac{1}{2} h_2^2 (A_2 - A_1) + \frac{A_1}{2}(h_1+h_2)^2 + B_1 h_1\right] \\
&= \frac{1}{h_1+h_2}\left[\frac{1}{2} h_2^2 (A_2 - A_1) + \frac{A_1}{2}(h_1+h_2)^2 + B_1 h_1\right]
\end{aligned}$$

Recall from part (a) that our values of A_i and B_i are

$$A_1 = \frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} \qquad A_2 = \frac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} \qquad B_1 = V_{x1} - \frac{V_{x1}(h_1+h_2)}{h_1 + (\mu_1/\mu_2)h_2}$$

So, in an attempt to simplify $\langle V \rangle$ a bit, we can calculate

$$\begin{aligned}
A_2 - A_1 &= \frac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} - \frac{V_{x1}}{h_1 + (\mu_1/\mu_2)h_2} \\
&= V_{x1}\left(\frac{\mu_1}{\mu_2 h_1 + \mu_1 h_2} - \frac{\mu_2}{\mu_2 h_1 + \mu_1 h_2}\right) = \frac{V_{x1}(\mu_1 - \mu_2)}{\mu_2 h_1 + \mu_1 h_2}
\end{aligned}$$

resulting in the average velocity to be

$$\begin{aligned}
\langle V \rangle &= \frac{1}{h_1+h_2}\left[\frac{V_{x1} h_2^2 (\mu_1 - \mu_2)}{2(\mu_2 h_1 + \mu_1 h_2)} + \frac{V_{x1}(h_1+h_2)^2}{2[h_1 + (\mu_1/\mu_2)h_2)]} + V_{x1} h_1 \left(1 - \frac{h_1+h_2}{h_1 + (\mu_1/\mu_2)h_2}\right)\right] \\
&= \boxed{\frac{V_{x1}}{h_1+h_2}\left[\frac{h_2^2(\mu_1 - \mu_2)}{2(\mu_2 h_1 + \mu_1 h_2)} + \frac{(h_1+h_2)^2}{2[h_1 + (\mu_1/\mu_2)h_2)]} + h_1 h_2 \frac{(\mu_1/\mu_2) - 1}{h_1 + (\mu_1/\mu_2)h_2}\right]}
\end{aligned}$$

(d) Across a distance, L, in the x-direction, how much force (in Newtons) is applied to the bottom plate? (Again assuming a width W.)

From the assumption that the fluid with density ρ_2 and viscosity μ_2 is Newtonian,

$$\tau_{yx} = \mu_2 \frac{\mathrm{d}V_x}{\mathrm{d}y} = \mu_2 \frac{\mathrm{d}V_{x2}}{\mathrm{d}y}$$

Since $V_{x2} = A_2 y + B_2$ from part (a), where $A_2 = V_{x1}/[(\mu_2/\mu_1)h_1 + h_2]$,

$$\tau_{yx} = \mu_2 \frac{V_{x1}}{(\mu_2/\mu_1)h_1 + h_2}$$

the force applied to the plate is the integral of the shear stress throughout the whole bottom plate region.

$$\mathbf{F}_{\text{applied to plate}} = \iint_{\mathcal{A}} \tau_{yx} \, \mathrm{d}a \, \hat{\mathbf{x}}$$

Since τ_{yx} is constant with respect to x and z, we can pull out τ_{yx} out of the integral.

$$\mathbf{F}_{\text{applied to plate}} = \tau_{yx} \int_0^W \int_0^L \mathrm{d}x \, \mathrm{d}z \, \hat{\mathbf{x}} = \tau_{yx} LW \, \hat{\mathbf{x}} = \boxed{\frac{\mu_2 V_{x1}}{(\mu_2/\mu_1)h_1 + h_2} LW \hat{\mathbf{x}}}$$

(e) Now let's assume that the bottom plate is moving in the negative x-direction with a velocity magnitude, V_{x2}. Solve for the steady-state flow velocity, $V_x(y)$, in this system.

Like in part (a), we make the following assumptions:

- fully-developed, laminar, and incompressible flow
- both fluids are Newtonian
- no edge effects

The same derivation from part (a) follows for part (e), leading to Equation (1.1):

$$\mu \frac{\mathrm{d}^2 V_x}{\mathrm{d}y^2} = 0$$

the solution of which is a linear function $V_x = Ay + B$ for A, B constants. Assuming the bottom plate is moving in the negative x-direction with speed V_{x2}, we have that

$$\begin{aligned} V_{x1}(y) &= A_1 y + B_1 \quad (h_2 \le y \le h_1 + h_2) \\ V_{x2}(y) &= A_2 y + B_2 \quad (0 \le y \le h_2) \end{aligned}$$

with boundary conditions

- no-slip condition 1: $V_{x1}(h_1 + h_2) = V_{x1}$
- no-slip condition 2: $V_{x2}(0) = -V_{x2}$
- interface condition 1: $V_{x1}(h_2) = V_{x2}(h_2)$
- interface condition 2: $\tau^1_{yx}(h_2) = \tau^2_{yx}(h_2)$

From the four boundary conditions, we have

- (no-slip condition 1) $A_1(h_1 + h_2) + B_1 = V_{x1}$ implies $B_1 = V_{x1} - A_1(h_1 + h_2)$
- (no-slip condition 2) $\boxed{B_2 = -V_{x2}}$
- (interface condition 2) $\mu_1 A_1 = \mu_2 A_2$ implies $A_2 = (\mu_1/\mu_2)A_1$

We've expressed both B_1 and A_2 in terms of A_1. Plugging these values into interface condition 1,

$$\begin{aligned}
A_1 h_2 + V_{x1} - A_1(h_1 + h_2) &= (\mu_1/\mu_2)A_1 h_2 - V_{x2} \\
A_1\big[h_2(1 - \mu_1/\mu_2) - (h_1 + h_2)\big] &= -V_{x1} - V_{x2} \\
A_1\big[-h_1 - (\mu_1/\mu_2)h_2\big] &= -V_{x1} - V_{x2} \\
&\boxed{A_1 = \frac{V_{x1} + V_{x2}}{h_1 + (\mu_1/\mu_2)h_2}}
\end{aligned}$$

From the value of A_1, we can determine the values of the remaining unknowns.

$$\boxed{A_2 = \frac{V_{x1} + V_{x2}}{(\mu_2/\mu_1)h_1 + h_2}} \qquad \boxed{B_1 = V_{x1} - \frac{V_{x1} + V_{x2}}{h_1 + (\mu_1/\mu_2)h_2}(h_1 + h_2)}$$

Therefore, the velocity profile is

$$\boxed{V_x(y) = \begin{cases} \dfrac{V_{x1} + V_{x2}}{(\mu_2/\mu_1)h_1 + h_2} y - V_{x2} & 0 \le y < h_2 \\ \dfrac{V_{x1} + V_{x2}}{h_1 + (\mu_1/\mu_2)h_2} y + V_{x1} - \dfrac{V_{x1} + V_{x2}}{h_1 + (\mu_1/\mu_2)h_2}(h_1 + h_2) & h_2 \le y \le h_1 + h_2 \end{cases}}$$

(f) Assuming $\rho_1 = \rho_2$, and $\mu_1 = \mu_2$, and that $V_{x2} = -2V_{x1}$, is there any point at which velocity is zero? If so where?

If $\mu_1 = \mu_2$ and $V_{x2} = 2V_{x1}$ in the negative direction, then

$$\begin{aligned}
V_{x1}(y) &= \frac{V_{x1} + 2V_{x1}}{h_1 + h_2} y + V_{x1} - (V_{x1} + 2V_{x1}) = \frac{3V_{x1}}{h_1 + h_2} y - 2V_{x1} \\
V_{x2}(y) &= \frac{V_{x1} + 2V_{x1}}{h_1 + h_2} y - 2V_{x1} = \frac{3V_{x1}}{h_1 + h_2} y - 2V_{x1}
\end{aligned}$$

so really the velocity profile is a singular linear function from $0 \le y \le h_1 + h_2$. If the velocity profile were to equal zero, then

$$0 = \frac{3V_{x1}}{h_1 + h_2} y - 2V_{x1} \quad \text{then} \quad 2 = \frac{3y}{h_1 + h_2} \quad \text{then} \quad y = \frac{2}{3}(h_1 + h_2)$$

so the fluid has zero velocity at two-thirds the total height from the bottom.

2 Problem 2

Derive the average flow velocity $\langle v_z \rangle$ for steady-state, incompressible, fully-developed flow of a power law fluid (non-Newtonian fluid) through a pipe of radius R given that:

$$v_z(r) = \left(\frac{\Delta p}{2mL}\right)^{1/n} \left(\frac{R^{1+1/n}}{1+1/n}\right) \left[1 - \left(\frac{r}{R}\right)^{1+1/n}\right]$$

Assume that you are given the pressure difference across the pipe, Δp, the length of the pipe, L, the radius of the pipe R, and the fluid parameters m and n.

The average velocity $\langle v \rangle$ is, by definition, equal to

$$\langle v \rangle = \frac{1}{\int_{\mathcal{A}} \mathrm{d}A} \int_{\mathcal{A}} \mathbf{v} \cdot \mathrm{d}\mathbf{A} = \frac{1}{\int_{\mathcal{A}} \mathrm{d}A} \int_{\mathcal{A}} (\mathbf{v} \cdot \hat{\mathbf{n}})\, \mathrm{d}A$$

where $\mathbf{A} = A\hat{\mathbf{n}}$. Since the pipe has cross-sectional area πR^2 and we are only looking at the axial component of $\mathbf{v}$, then the dot product is simply $\mathbf{v}_z \cdot \hat{\mathbf{n}} = (v_z \hat{\mathbf{z}}) \cdot \hat{\mathbf{z}} = v_z$. In cylindrical coordinates, $\mathrm{d}A = r\, \mathrm{d}r\, \mathrm{d}\theta$, so

$$\begin{aligned} \langle v_z \rangle &= \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R v_z(r) r\, \mathrm{d}r\, \mathrm{d}\theta \\ &= \frac{1}{\pi R^2}(2\pi) \int_0^R v_z(r) r\, \mathrm{d}r \\ &= \frac{2}{R^2} \int_0^R v_z(r) r\, \mathrm{d}r \end{aligned}$$

Letting C be

$$C = \left(\frac{\Delta p}{2mL}\right)^{1/n} \left(\frac{R^{1+1/n}}{1+1/n}\right)$$

we can evaluate the average velocity easily.

$$\begin{aligned} \langle v_z \rangle &= \frac{2C}{R^2} \int_0^R \left[1 - \frac{r^{1+1/n}}{R^{1+1/n}}\right] r\, \mathrm{d}r = \frac{2C}{R^2} \int_0^R \left[r - \frac{r^{2+1/n}}{R^{1+1/n}}\right] \mathrm{d}r \\ &= \frac{2C}{R^2} \left[\frac{1}{2}R^2 - \frac{\cancelto{R^2}{R^{3+1/n}}}{(3+1/n)(\cancel{R^{1+1/n}})}\right] = C\left(1 - \frac{2}{3+1/n}\right) = C\frac{1+1/n}{3+1/n} \end{aligned}$$

Substituting C back in,

$$\langle v_z \rangle = \left(\frac{\Delta p}{2mL}\right)^{1/n} \left(\frac{R^{1+1/n}}{1+1/n}\right)\left(\frac{1+1/n}{3+1/n}\right) = \boxed{\left(\frac{\Delta p}{2mL}\right)^{1/n} \left(\frac{R^{1+1/n}}{3+1/n}\right)}$$

3 Lab Assignment – Group Project Proposal

(1) Find **two** 2-hour time slots that at least two group members can meet per week. At least one time slot should overlap with one of the lab sections.

The time slots we chose are **Tuesday**, 7–9 pm, and **Wednesday**, 4–6 pm.

(2) With your group, brainstorm and come up with two possible ideas for your group project, a primary idea and a secondary idea. As a group write two draft proposals based on these ideas. Each draft proposal, in no more than 250 words, should introduce your model, describe the background, or context to explain why your question is important, and briefly discuss possible challenges in the proposed project. Your proposals should each contain at least one primary reference describing a similar technical study and one reference addressing the background of the question or application your study will address. Each group member should submit one of the two proposals.

Over the past decades, several pharmaceutical drugs have been developed to treat brain diseases, such as Alzheimer's, Parkinson's, and brain cancers. However, the major limitation is the blood brain barrier (BBB), due to the tight junctions of the endothelial cells of the capillary network that makes up this barrier. This lack of permeability makes it difficult for drugs to reach their intended site.[1] We aim to develop a computational model of the BBB that can help determine the necessary dosage of Memantine, a drug used to treat Alzheimer's, to achieve therapeutic concentration in the brain.

The importance of this goal is that Alzheimer's is a disease that affects millions; thus, accurate modeling of BBB drug delivery can help determine the proper dosage levels for optimal medicinal treatment against the disease. The model will reconstruct a small capillary network representative of the blood brain barrier. Given the capillaries' dimensions, blood flow, molecular drug properties, necessary concentrations, cell properties of the barrier, and any other necessary properties, this model can help answer our question.[2] Furthermore, taking into account known dimensions of the entire BBB network can scale our results into a realistic solution.

However, since the BBB is a heterogeneous network of capillaries, endothelial cells, and junctional proteins, balancing the computational expense of the model and its accuracy to real-life phenomena can be a challenge. Additionally, the non-static nature of the BBB's physiological conditions is difficult to account for in a numerical simulation.

References

[1] Langhoff W, Riggs A, Hinow P. Scaling behavior of drug transport and absorption in *in silico* cerebral capillary networks. PLOS One. 2018 07;13(7):1-14. Available from: https://doi.org/10.1371/journal.pone.0200266.

[2] Sefidgar M, Soltani M, Raahemifar K, Sadeghi M, Bazmara H, Bazargan M, et al. Numerical modeling of drug delivery in a dynamic solid tumor microvasculature. Microvascular Research. 2015;99:43-56. Available from: https://doi.org/10.1016/j.mvr.2015.02.007.