# Combinatorial Logic

from Prof. Joongnam Jeon's lecture slides

Combinatorial Logic Circuit

Combinational logic circuit
- ◦ the output of a combinatorial logic circuit is completely determined by given input values

Table 4-1. m×n combinatorial logic circuit

입력 $\xrightarrow{n}$ 조합 논리회로 {논리 게이트} $\xrightarrow{m}$ 출력

[그림 4-1] n×m 조합 논리회로

| No.	Input				Output			
	X_n	X_{n-1}	...	X_1	Y_m	Y_{m-1}	...	Y_1
0	0	0	0	0				
1	0	0	0	1				
2	0	0	1	0				
...								
2^n-1	1	1	1	1				

Logic Equation to Logic Circuit
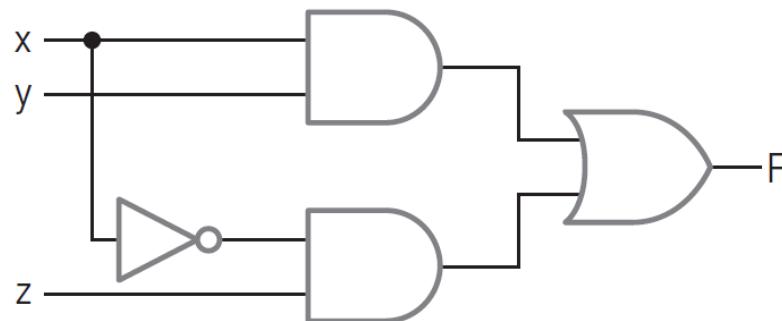
Precedence order

◦ OR < AND < NOT < Parentheses

Drawing Logic Circuit Diagram

1. Place input signals at the left-side, output signals at the right-side
2. Draw logic gates and connect their input and outputs considering precedence order

Ex. $F = x \cdot y + x' \cdot z$

Minterms

Generate logic formula from truth table
- sum of products
  - examples
    - F1(x,y,z) = x + y·z
    - F2(x,y,z) = x·y + x'·z
    - F3(x,y,z) = x'·y'·z' + x·y·z
- products of sums

Minterm: a product where each variable (or its negative) is connected as an operand
- a minterm evaluates to True only with one combination of input values
- ex. for three input variables x, y, z,
  - minterms: x'y'z', x'yz, xy'z, etc.

Minterms

Input			Minterms							
x	y	z	m_0 x'y'z'	m_1 x'y'z	m_2 x'yz'	m_3 x'yz	m_4 xy'z'	m_5 xy'z	m_6 xyz'	m_7 xyz
0	0	0								
0	0	1								
0	1	0								
0	1	1								
1	0	0								
1	0	1								
1	1	0								
1	1	1								

Sum of Minterms

| x	y	z	F	m_1	m_3	m_6	m_7
0	0	0	0				
0	0	1	1				
0	1	0	0				
0	1	1	1				
1	0	0	0				
1	0	1	0				
1	1	0	1				
1	1	1	1				

(1) Sum of Minterms
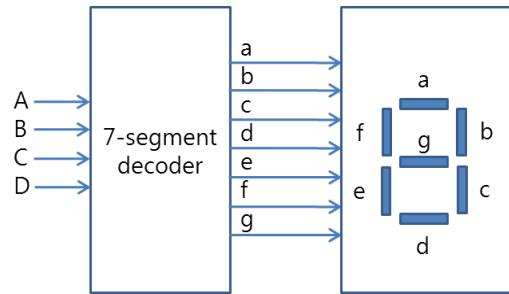
F(x,y,z) = _____

(2) Simplified Formula

F(x,y,z) = _____
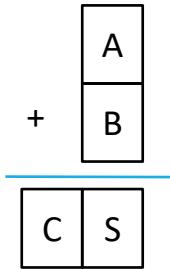
7-Segment Display

8421 BCD 7-segment Display Decoder
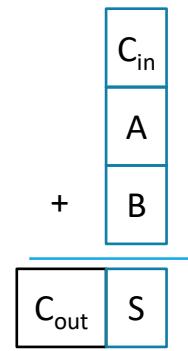
Hex. No	A	B	C	D	a	b	c	d	e	f	g
0	0	0	0	0	1	1	1	1	1	1	0
1	0	0	0	1							
2	0	0	1	0							
3	0	0	1	1							
4	0	1	0	0							
5	0	1	0	1							
6	0	1	1	0							
7	0	1	1	1							
8	1	0	0	0							
9	1	0	0	1							
A	1	0	1	0	x	x	x	x	x	x	x
B	1	0	1	1	x	x	x	x	x	x	x
C	1	1	0	0	x	x	x	x	x	x	x
D	1	1	0	1	x	x	x	x	x	x	x
E	1	1	1	0	x	x	x	x	x	x	x
F	1	1	1	1	x	x	x	x	x	x	x

- X: Don't care
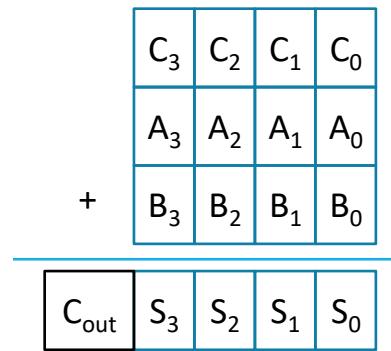
Adder

(a) Half adder

(b) Full adder

(c) 4-bit Adder

Half Adder

- Input: A, B

- Output: S(sum), C(carry)

- Truth Table
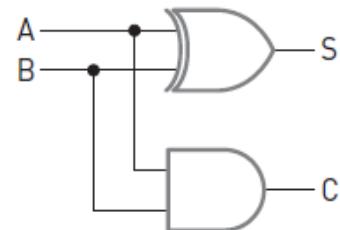
Input		Output		Note
A	B	C	S	
0	0			0 + 0 = 00
0	1			0 + 1 = 01
1	0			1 + 0 = 01
1	1			1 + 1 = 10

Logic Formula

S = _____

C = _____
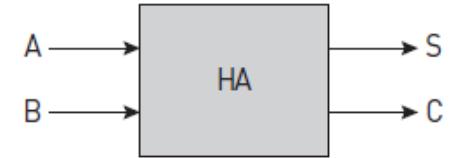
Logic Circuit

(a) 논리회로도   (b) 블록도

Full Adder

- Input: A, B, C_{in}(carry in)

- Output: S(sum), C_{out}(carry out)

- Truth Table
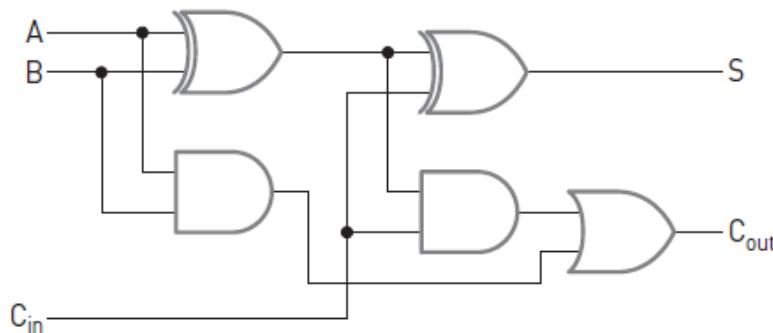
Input			Output		Note
A	B	C_{in}	C_{out}	S	
0	0	0			0 + 0 + 0 = 00
0	0	1			0 + 0 + 1 = 01
0	1	0			0 + 1 + 0 = 01
0	1	1			0 + 1 + 1 = 10
1	0	0			1 + 0 + 0 = 01
1	0	1			1 + 0 + 1 = 10
1	1	0			1 + 1 + 0 = 10
1	1	1			1 + 1 + 1 = 11

Full Adder
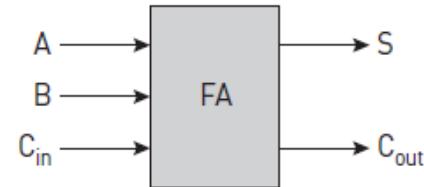
Logic Formula

- S = A⊕B⊕Cin = (A⊕B)⊕Cin
- Cout(carry out) = A·B + B·Cin = A·B + (A⊕B)·Cin

Logic Circuit
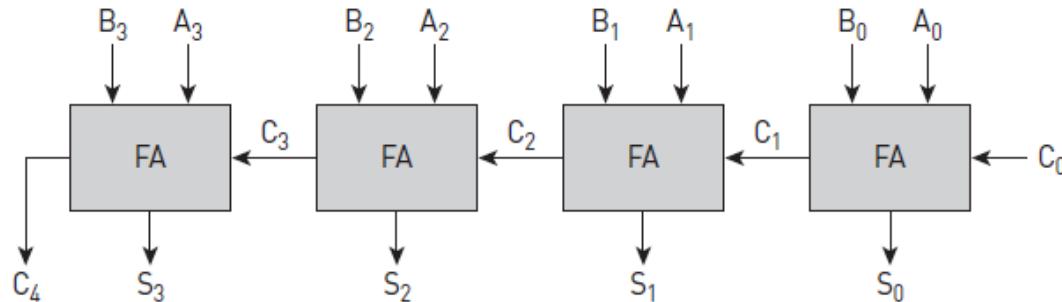

(a) 반가산기 2개와 OR 게이트로 구현

(b) 블록도

Parallel Adder

4-bit parallel adder

Ex. C_0=0, X=0011, Y=1110

```
X:              0 0 1 1
Y:               1 1 1 0
S:
C:
```
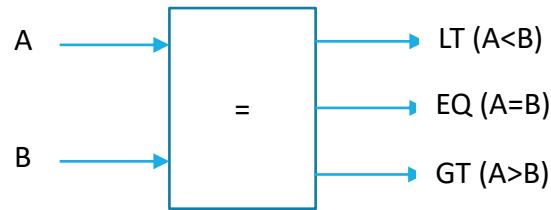
Comparator

Input and output

A → [=] → LT (A<B)

B → [=] → EQ (A=B)

→ GT (A>B)
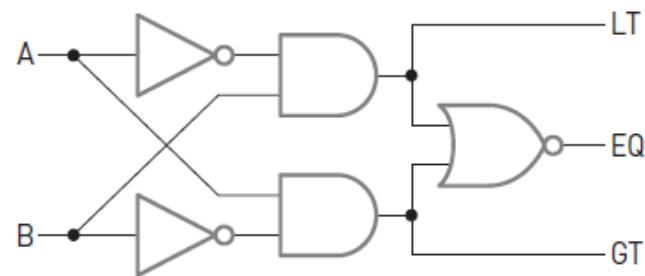
Truth table

Input		Output			Note
A	B	LT	EQ	GT	
0	0				0 = 0
0	1				0 < 1
1	0				1 > 0
1	1				1 = 1

Logic Formula

LT = _____ // A=0, B = 1

GT = _____ // A=1, B=0

EQ = _____ // AB = 00 or 11

  = (LT + GT)'
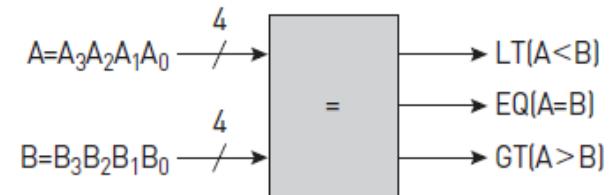
Logic Circuit

4-bit Comparator

Compare two 4-bit numbers: either GT, EQ, LT

입력		출력			비고
$A_3A_2A_1A_0$	$B_3B_2B_1B_0$	LT	EQ	GT	
$X_3X_2X_1 0$	$X_3X_2X_1 1$				A < B
$X_3X_2 0\,d$	$X_3X_2 1\,d$				
$X_3 0\,d\,d$	$X_3 1\,d\,d$				
$0\,d\,d\,d$	$1\,d\,d\,d$				
$X_3X_2X_1X_0$	$X_3X_2X_1X_0$				A=B
$X_3X_2X_1 1$	$X_3X_2X_1 0$				A > B
$X_3X_2 1\,d$	$X_3X_2 0\,d$				
$X_3 1\,d\,d$	$X_3 0\,d\,d$				
$1\,d\,d\,d$	$0\,d\,d\,d$				

$A=A_3A_2A_1A_0 \xrightarrow{4}$ = $\rightarrow$ LT(A<B)

$\rightarrow$ EQ(A=B)

$B=B_3B_2B_1B_0 \xrightarrow{4}$ $\rightarrow$ GT(A>B)

Example
1) A = 1010, B = 1001
   LT=__, EQ=__, GT=__

2) A = 0101, B = 0101
   LT=__, EQ=__, GT=__

3) A = 0001, B = 0010
   LT=__, EQ=__, GT=__