Equations and Inequalities

& Functions

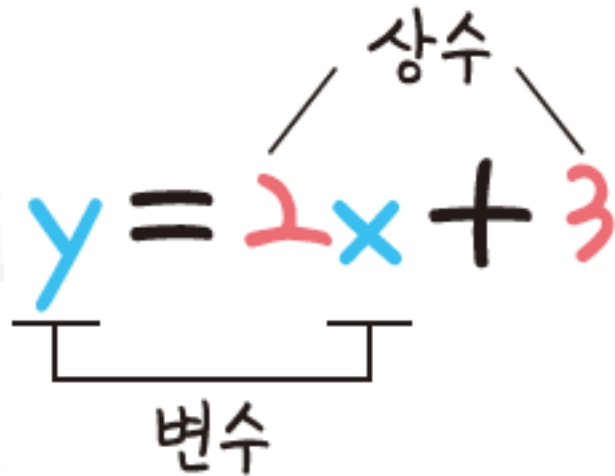# Variables and Equations

❖ **Variables**

Variables and Constants

- Variables mean 'changing'

- 'Number of changes' is called a variable

$$y = 2x + 3$$

상수

변수

x 값	1	2	3
y 값	5	7	9

Variables and Equations

❖ **Variables**

▪ Python creates variables in the form of 'variable name = value' to use them, while also assigning (storing) values

$$x = 5$$

변수 이름   값

Variables and Equations

❖ **Variables**

- When creating variable names, you must follow these rules

    - Characters and numbers, _(subline characters) are allowed

    - No space available

    - Distinguishing between uppercase and lowercase letters

    - Characters and numbers can be used interchangeably, but must start with letters

    - Special characters (+,-, @, %, etc.) are not allowed

Variables and Equations

❖ **Variables**

▪ Python declares variables as follows

```
In [1]:
# x 변수에 5 값을 저장합니다
x = 5
print(x)

5


In [2]:
# x, y, z 변수에 1, 2, 3 값을 각각 저장합니다
x, y, z = 1, 2, 3
print(x, y, z)

1, 2, 3
```

Variables and Equations

❖ **Variables**

Term, constant term, coefficient

- Term means an expression consisting of a product of numbers of letters

- In other words, multiplication of numbers and letters or multiplication of letters and letters becomes a term

- As in Equation 1.1, the result of multiplying numbers and letters is a term because it is a number

- Equation 1.2 with only characters multiplied the characters by 1, so this can also be called a term
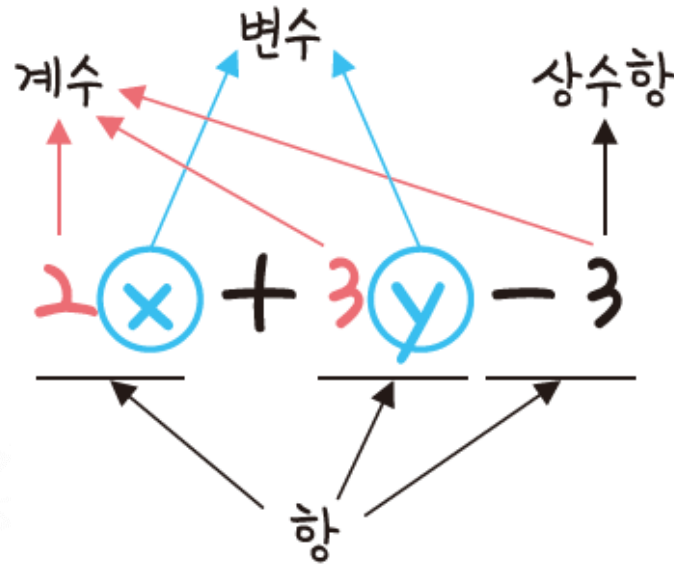
$$3a, \ 2a^2 \qquad\qquad\qquad \text{수식 1.1}$$

$$a, \ a^2 \qquad\qquad\qquad \text{수식 1.2}$$

Variables and Equations

❖ **Variables**

- Constant term means a term with only numbers among the terms

- For example, if you have the expression 2x+y+1, where 1 is a constant term

- Coefficient is a constant that is multiplied by a variable in a unary expression consisting of a constant and a variable

Variables and Equations

❖ **Variables**

- In the formula 2x+3y – 3, the terms are 2x and 3y, which are the product of numbers and letters

- -3 also becomes a term, but it is a term and a constant term because there are only numbers

- The coefficient is the number multiplied before the letter, so 2 for 2x and 3 for 3y

- Include the constant term -3 in the coefficient because the -3 constant can be viewed as the product of x^0 (x squared) and -3
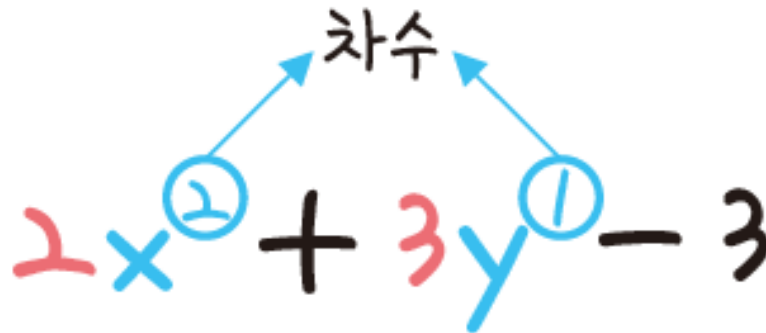
Variables and Equations

❖ **Variables**

Monomials and Polynomials

- In polynomials, "多" means "many"

- Expressions with one term are monomials, and expressions that are the sum of two or more terms are polynomials

- Referring back to the formula, 2x + 3y -3 is a polynomial because it has three terms

- 2x, 3y, -3 are each monomials

Variables and Equations

❖ **Variables**

Degree

- Degree means the number of times the character is multiplied

- In the formula, $2x^2$ has degree 2 because it is multiplied by x twice, and 3y has degree 1 because it is multiplied by y once

- The constant term -3 is x^0 (or y^0), so it has order zero

Variables and Equations

❖ **Variables**

- Finding the quadratic with respect to x results in a quadratic expression with respect to x, and finding the quadratic with respect to y results in a quadratic expression with respect to y

- Polynomials of degree one are called first degree, and polynomials of degree two are called quadratic

Equations and Inequalities

❖ **Equation**

Equality

- To understand equations, you must first understand 'equality'

- Equality is a relational expression in which an expression consisting of numbers and letters on either side of an equals sign (=) means "equal to" each other

- For example, when calculating 2+2 = 4, the = sign is called an "equals sign", which means that the left and right sides are equal to each other

Equations and Inequalities

❖ **Equation**

Left and Right

- The left side of the equal sign is called the left side and the right side is called the right side, and together they are called both sides

$$2 + 1 = 3$$

좌변   우변

양변

Equations and Inequalities

❖ **Equation**

- The following are all examples of equations

  - 2 + 5 = 3 + 4

  - 6 - 2 = 2 + 2

  - 6 - 3 = 3

Equations and Inequalities

❖ **Equation**

  ▪ In this case, the expression is equal whether it is true or false

  ▪ An expression is said to be true if it is correct, and false if it is incorrect

    • Equation that is true : 9 = 6 + 3

    • Equation that is false : 7 + 2 = 6 - 4

Equations and Inequalities

❖ **Equation**

Equation

- An equation is an expression that can be true or false depending on an unknown, such as x

- Equations must have an **equal sign** and an **unknown variables**

- For example, say x + 2 = 6

  - When x is 4, both the left and right sides become 6, and so on

  - When x is 3, the left side becomes 5 and the right side becomes 6, which is a false equation

Equations and Inequalities

❖ **Equation**

- ▪ In this way, depending on the unknown x, it can be true or false, so x + 2 = 6 is an equation

- ▪ When the equation is true, the unknown is called the solution of the equation (or the root of the equation)

- ▪ Using the previous example again, the solution of the equation is 4 because the expression was true when x + 2 = 6 is 4

Equations and Inequalities

❖ **Equation**

Linear equation and quadratic equation

▪ A linear equation is an equation of degree 1

▪ Since it is difficult to determine the linear equation by itself, it is necessary to calculate all terms by transposition to the left side

▪ In other words, (linear equation) = 0 form must be created and calculated

▪ Transposition(이항): By moving a term, it means to change the sign of a term in an equation or inequality and move it to another side

Equations and Inequalities

❖ **Equation**

- For example, if x + 1 = 3 is transposition to the left side, then x + 1 - 3 = 0 is a linear equation because (linear equation) = 0

이항

$$(1)\ x + 1 = 3$$

$$(2)\ x + 1 - 3 = 0$$

이항하면 기호가 바뀜

$$(3)\ x - 2 = 0$$

(일차방정식) = 0 형태이므로 일차방정식임

❖ **Equation**

- In the case of 2(x + 1) = 3 + 2x, the left-side transposition becomes 2x + 2 - 3 - 2x = 0, but it is not a linear equation

  because the calculation shows that x is missing and there is no variable representing the degree

이항

$$(1) \ 2(x + 1) = \boxed{3 + 2x}$$

$$(2) \ 2(x + 1) \ \boxed{-3 \ -2x} = 0$$

이항하면 기호가 바뀜

$$(3) \ 2x + 2 - 3 - 2x = 0$$

$$(4) \ \underline{-1 = 0}$$

x가 사라지므로 일차방정식이 아님

Equations and Inequalities

❖ **Equation**

- After determining the linear equation, the solution is found in the following order:

  (1) All variables (x, y, etc.) are transpose to the left, and all constants are transpose to the right

  (2) Calculate each side

  (3) Divide both sides by the coefficient of x

For example, say you have 2x + 2 = 3 + 3x

  (1) If we transpose a variable to the left side and a constant to the right side,

     we get 2x - 3x = 3 - 2

  (2) If we calculate each side, we get -x = 1

  (3) If we divide both sides by the coefficient of x, -1, we get x = -1

Equations and Inequalities

❖ **Equation**

- ▪ If the linear equation has a degree of x of 1, then the quadratic equation is an equation with a degree of x of degree 2

- ▪ It can be expressed as follows Equation 1.4

$$ax^2 + bx + c = 0$$

수식 1.4

- ▪ Note that if a = 0, it cannot be a quadratic equation because the order of the highest order term is 1, so must be a ≠ 0

Equations and Inequalities

❖ **Equation**

- ▪ You can also solve equations in Python

  - In Python, to solve an equation, you use the SymPy library and the solve() function

- ▪ What is SymPy:

  - SymPy is a library for symbolic math in Python

  - It's all written in Python, and includes extensions for speed, visualization, and more

  - SymPy allows you to solve algebra problems with symbolic math

  - In SymPy, symbolic variables use the symbol() function, which must be predefined before using symbolic variables, such as from sympy import Symbol, solve

Equations and Inequalities

❖ **Equation**

- Python's SymPy library is used for the following mathematical solutions:

  - Finding the Solution of an Equation

  - Differential and Integration

Equations and Inequalities

❖ **Equation**

- Especially when solving equations, you need the solve() function

- To use the solve() function, you need to install the NumPy and SymPy libraries with the following command in the Anaconda Prompt window:

```
> pip install numpy   또는   conda install numpy
> pip install sympy   또는   conda install sympy
```

Equations and Inequalities

❖ **Equation**

▪ In Python, this is implemented as follows:

```python
In [4]:
# SymPy 라이브러리를 불러오고, 사용할 기호변수 x를 선언합니다
from sympy import Symbol, solve
x = Symbol('x')

# 방정식을 풀려면 "(일차방정식) = 0"으로 만들어 주어야 합니다
# 이를 위해 모든 식을 좌변으로 이항한 후 equation으로 변수화합니다
equation = 2 * x - 6
```

Equations and Inequalities

❖ **Equation**

```
# 방정식을 풀려면 SymPy에 내장된 solve() 함수를 사용합니다
# solve() 함수 안에 equation을 입력하면
# 방정식을 풀어서 결과를 반환합니다
solve(equation)

[3]
```

Equations and Inequalities

❖ **Equation**

연습 문제

다음 방정식의 해를 구하세요.

(1) $4 = k - 2$

(2) $10 = 2k$

(3) $\dfrac{k}{2} = 8$

Equations and Inequalities

❖ **Equation**

문제 풀이

$(1)\, 4 = k - 2$

$k = 4 + 2 = 6$

In [5]:

```python
from sympy import Symbol, solve
k = Symbol('k')
equation = k - 2 - 4
solve(equation)
```

[6]

Equations and Inequalities

❖ **Equation**

$$(2)\ 10 = 2k$$

$$k = \frac{10}{2} = 5$$

In [6]:

```python
from sympy import Symbol, solve
k = Symbol('k')
equation = 2 * k - 10
solve(equation)
```

[5]

Equations and Inequalities

❖ **Equation**

$$(3)\ \frac{k}{2} = 8$$

$$k = 2 \times 8 = 16$$

In [7]:

```python
from sympy import Symbol, solve
k = Symbol('k')
equation = k / 2 - 8
solve(equation)
```

[16]

Equations and Inequalities

❖ **Equation**

Identity (항등식)

- An identity is an expression that is always true no matter what number is assigned to an unknown

- For example, if 2x + 1 = 1 + 2x, then if you swap places 1 + 2x on the right side according to the law of exchange, it becomes 2x + 1, and so on, so it is an identity

- The law of exchange(교환법칙): Even if the number (or variable) on both sides of the operation symbol is swapped, it means that the result of the calculation is the same

구분	방정식	항등식
참인 수식일 조건	미지수가 특정한 값을 가질 때만 참인 수식	미지수가 어떤 값을 가져도 참인 수식
좌변과 우변의 조건	좌변 ≠ 우변	좌변 = 우변

Equations and Inequalities

❖ **Equation**

다음 중 방정식과 항등식을 모두 고르세요.

(1) $x + x = 2x$

(2) $2x + 1 < 6$

(3) $2x - x = x$

(4) $2 + 5 = 7$

Equations and Inequalities

❖ **Equation**

문제 풀이

(1) 좌변의 $x + x = 2x$가 되므로 우변 $2x$와 같아 항등식입니다.

(2) 등호가 없기 때문에 등식이 아닙니다.

(3) 좌변의 $2x - x = x$가 되므로 우변 x와 같아 항등식입니다.

(4) $2 + 5 = 7$에서 미지수가 없기 때문에 방정식도 아니고 항등식도 아닙니다.

Equations and Inequalities

❖ **Equation**

Simultaneous equation (연립방정식)

- A simultaneous equations is a group of equations that contain multiple unknowns

- Example

$$\begin{cases} 3x + y = 2 \quad \text{------} \ (1) \\ x - 2y = 3 \quad \text{------} \ (2) \end{cases}$$

수식 1.5

Equations and Inequalities

❖ **Equation**

- When an equation such as 1.5 involves variables x and y, it can be considered a simultaneous equations because it contains two unknowns

- In such cases, there should be at least two equations if there are two unknowns, and if there are three unknowns, there should be at least three equations given

$$\begin{cases} x + y = 2 \\ y + z = 5 \\ z - x = 3 \end{cases}$$

수식 1.6

Equations and Inequalities

❖ **Equation**

- Let's solve the problem of Equation 1.5

- Because the coefficients of x and y in (1) and (2) in Equation 1.5 are different, addition or subtraction between expressions (1) and (2) cannot reduce the unknown

- (1), (2) The expression must be multiplied by the appropriate number to match the coefficients of x or y

Equations and Inequalities

❖ **Equation**

▪ Multiply the (2) expression in Formula 1.5 by 3 to get x = 1, y = -1

$$3x + y = 2$$

×3 취함

$$-) \boxed{3x - 6y = 9}$$

$$7y = -7$$

▪ Since y is -1, we can substitute y in the expression (1) to get x.

▪ In other words, if we substitute y = -1 for the (1) 3x + y = 2, then 3x - 1 = 2, so x = 1

▪ (2) equal to the x value found in the expression

Equations and Inequalities

❖ **Equation**

▪ In Python, a simultaneous equations is solved as follows

```
In [8]:
# SymPy 라이브러리를 불러오고, 사용할 기호변수 x, y를 선언합니다
from sympy import Symbol, solve
x = Symbol('x')
y = Symbol('y')


# 방정식을 풀려면 "(일차방정식) = 0"으로 만들어 주어야 합니다
# 이를 위해 모든 식을 좌변으로 이항한 후 equation1과 equation2로 변수화합니다
equation1 = 3 * x + y - 2
equation2 = x - 2 * y - 3
```

Equations and Inequalities

❖ **Equation**

```
# 방정식을 풀려면 SymPy에 내장된 solve() 함수를 사용합니다
# solve() 함수 안에 equation을 차례로 입력하면
# 방정식을 풀어서 결과를 반환합니다
solve((equation1, equation2), dict=True) # dict 옵션은 해를 딕셔너리
형태로 반환합니다

[{x: 1, y: -1}]
```

Equations and Inequalities

❖ **Inequality (부등식)**

- In an expression that includes an equal sign (=) and an unknown, the equation is true or false depending on the unknown

- Expressions expressed using inequality signs (‹, ≤, ›, ≥) are called inequalities

- Inequalities are of two types, depending on the condition

  - Absolute inequality: An inequality that always holds for all real values

  - Conditional inequality: An inequality that holds only for a real value

Equations and Inequalities

❖ **Inequality**

- Let's take a look at absolute and conditional inequalities with the following examples

  (1) $x + 2 \leq 7$

  (2) $x^2 + 5 \geq 0$

- (1) Solving the expression gives $x \leq 5$

  - A conditional equation where a value of x is true if it is less than or equal to 5, and false if it is bigger than 5

- (2) If you solve the expression, no matter what value you put in x, it will always be bigger than 0, so it will always be an absolute inequality that holds for all real numbers

Equations and Inequalities

❖ **Inequality**

- The nature of the inequality is:

- If you add the same number to both sides of an inequality, the direction of the inequality does not change

  - If we add 5 to both sides in the 5 > 4 inequality, 5 + 5 > becomes 4 + 5 and does not change the direction of the inequality sign

- If you subtract the same number from both sides of the inequality, the direction of the inequality does not change

  - If we subtract 2 from the 5 > 4 inequality, 5 – 2 > 4 - 2 and the inequality sign direction does not change

Equations and Inequalities

❖ **Inequality**

- When both sides of an inequality are multiplied by the positive number, the direction of the inequality remains the same, and when a negative number is multiplied, the direction of the inequality changes

- Equal sign orientation reversed

  - (1) If we multiply both sides by 2 in the 5 > 4 inequality, 5×2 > 4×2 and the inequality sign direction does not change

  - (2) If we multiply both sides by -2 in the 5 > 4 inequality, 5× (-2) becomes 4× > (-2), reversing the direction of the inequality sign

Equations and Inequalities

❖ **Inequality**

- When dividing the same number on both sides of an inequality, dividing by a positive number remains the same, and

    dividing by a negative number changes the direction of the inequality

    ① If we divide 5 > 4 by 2 on both sides in the inequality,

    and the inequality sign direction does not change   예시 $\dfrac{5}{2} > \dfrac{4}{2}$

    ② If we divide -2 on both sides in the 5 > 4 inequality,

    and the direction of the inequality sign is reversed $\dfrac{5}{(-2)} < \dfrac{4}{(-2)}$

Equations and Inequalities

❖ **Inequality**

▪ Finally, let's take a look at the types of inequality in the following table

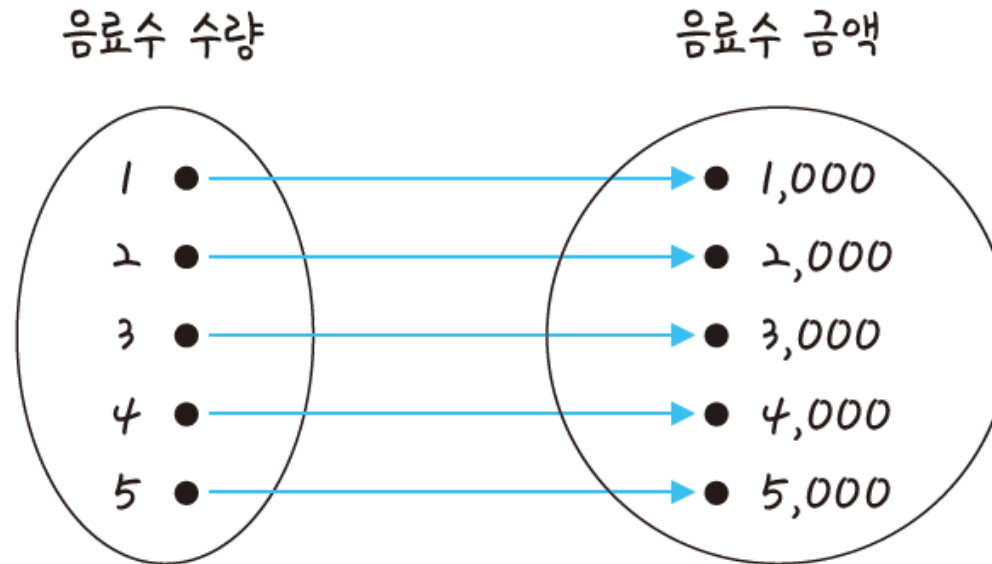| 부등호	부등식 예시	설명	그림으로 표현
>	x > a	x는 a보다 큽니다.	
<	x < a	x는 a보다 작습니다.	
≥	x ≥ a	x는 a보다 크거나 같습니다.	
≤	x ≤ a	x는 a보다 작거나 같습니다.	

Basic Functions

❖ **Function**

What is a function?

- A function is a relationship that corresponds any element in the first set to one element in the second set

음료수 수량	1	2	3	4	5
음료수 금액	1,000	2,000	3,000	4,000	5,000

Basic Functions

❖ **Function**

▪ When the value of X is determined for the variables X and Y, when the value of Y is determined, Y is called a function of X and can be expressed as follows:

Basic Functions

❖ **Function**

▪ The correspondence is expressed as Y = 1000X.

▪ In this case, the relation between X and Y is called a function expression, and it is expressed as Equation 2.1

$$Y = 1000X$$

수식 2.1

▪ 1000X can be expressed as f(X) as a function like Equation 2.2

$$Y = f(X)$$

수식 2.2

Basic Functions

❖ **Function**

Function Values

- As previously defined as 'a function corresponds to only one value Y according to the value of X

- Here, the Y, which is determined by the X value, is called the function value

- For example, in Equation 2.1, if Y = 1000X (or f (X) = 1000X), then X = 1 equals Y = 1000, so the function value is 1000

$$f(1) = 1000$$

Basic Functions

❖ **Function**

$f(x) = aX + 2$ 일 때, $f(3) = 8$ 입니다. 다음을 구하세요.

(1) a 값은?

(2) $f(6) - f(2)$ 값은?

Basic Functions

❖ **Function**

문제 풀이

(1) $f(3) = 8$이라는 것은 $X = 3$일 때 함수 값이 8이라는 의미입니다. 따라서 $f(x)$ $= aX + 2$에서 X에 3를 대입하면 8이 됩니다.

$8 = 3 \times a + 2$

$6 = 3a$

$a = 2$가 되므로 $f(x) = 2X + 2$입니다.

(2) $f(x) = 2X + 2$ 식에서 X에 6과 2의 값을 대입하면 다음과 같이 8이 됩니다.

$f(6) - f(2) = ((2 \times 6) + 2) - ((2 \times 2) + 2) = 14 - 6 = 8$

Basic Functions

❖ **Function and Equation**

$$y = 2x + 3 \qquad\qquad 수식\,2.3$$

$$y - 2x - 3 = 0 \qquad\qquad 수식\,2.4$$

- The two expressions may look the same, **Equation 2.3 is a function** and **Equation 2.4 is an equation**

- When there are x and y variables, if the value of y is determined by the value of x, it is defined as a function

- In Equation 2.3, when x is 1, y is 5, and when x is 2, y is 7, so the value of y is determined by the value of x, so this is a function

Basic Functions

❖ **Function and Equation**

▪ The relationship between a function and an equation is as follows:

- Within the range of real numbers, **both functions and equations can be expressed in the coordinate plane**

- An equation is a concept that encompasses functions

- All functions can be expressed by turning them into equations

Advanced Functions

❖ **Linear function and graphs**

Linear function

▪ A linear function is a function with a degree of 1 in the highest order term

▪ For example, a function of degree 1 of x is a linear function, such as y = ax + b

일차함수 예		$y = ax + b,\ f(x) = ax + b$
일차함수가 아닌 예	분수함수	$y = \dfrac{1}{x} + 1$
	상수함수	$y = 2$
	일차방정식	$ax + b = 1$
	일차부등식	$ax + b > 0,\ ax + b \geq 0$

Advanced Functions

❖ **Linear function and graphs**

연습 문제

다음 중 일차함수를 모두 고르세요.

(1) $y = 0x + 1$

(2) $y = 2x + 8$

(3) $y = 6$

(4) $xy = 2$

Advanced Functions

❖ **Linear function and graphs**

문제 풀이

(1)과 (3)은 x에 계수가 없으므로 상수함수입니다.

(2)는 x의 계수가 1이므로 일차함수입니다.

(4)는 $y = \dfrac{2}{x}$ 가 되므로 분수함수입니다.

Advanced Functions

❖ **Linear function and graphs**

Linear function graphs

Graph of the linear function y = ax

- This graph goes past the origin (0, 0) because if x = 0, then y = 0

- Let's see how x and y change depending on the value of the constant
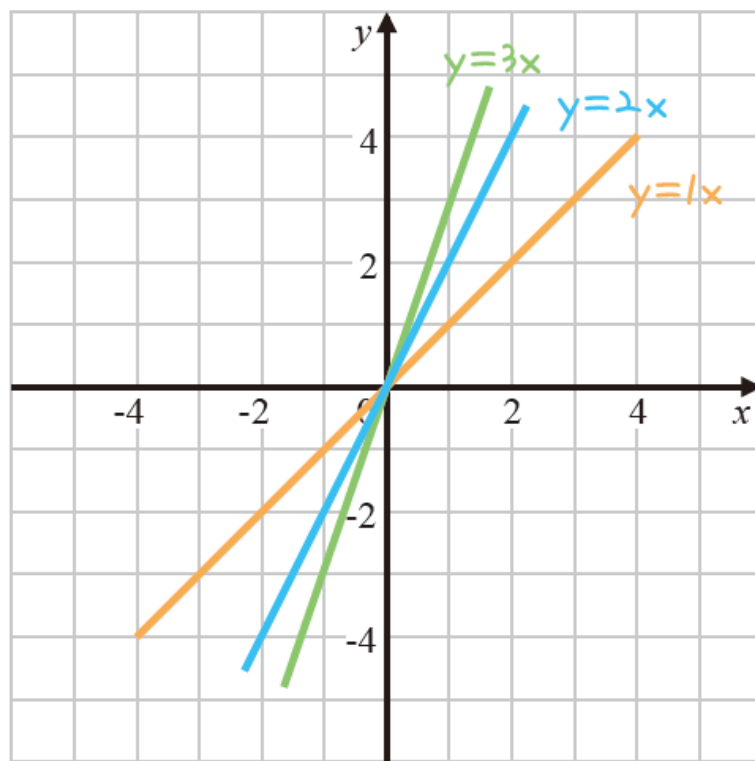
- (1) When a is positive, the change is:

a 값	1	2	3
y 값	1x	2x	3x

- In other words, the larger the number A, the closer the graph is to the y-axis

Advanced Functions

❖ **Linear function and graphs**

(2) When a is negative, the change is:

a 값	−1	−2	−3
y 값	−1x	−2x	−3x

- In other words, the smaller the number A, the closer the graph is to the y-axis

① y=ax 그래프(a가 양수)   ② y=ax 그래프(a가 음수)

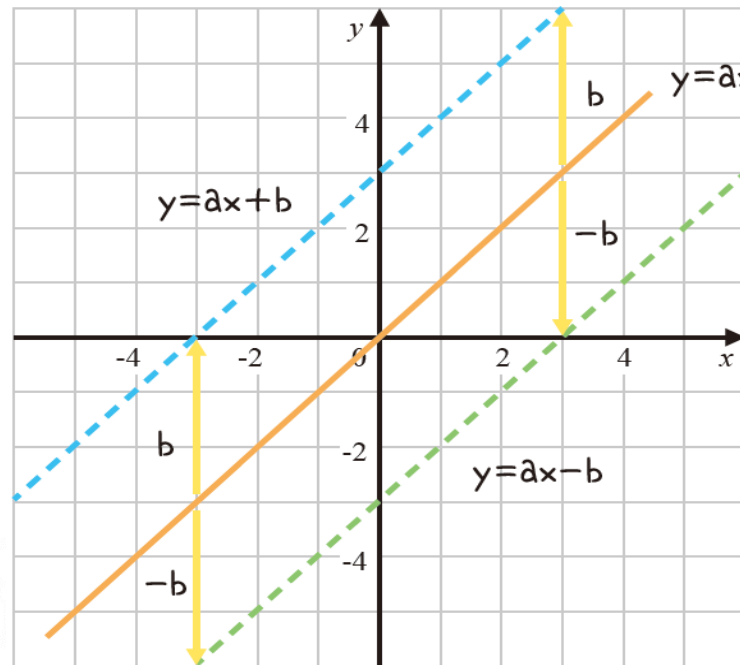# Advanced Functions

❖ **Linear function and graphs**

- To sum up, a › 0 means that bigger 'a' value is closer to the y axis, and a ‹ 0 the smaller 'a' value is closer to the y axis

- a › 0, as the x value increases, so does the y value, and the graph becomes a straight line pointing right upward

- On the other hand, at a ‹ 0, y decreases as x increases, and the graph becomes a straight line pointing downward to the right

Advanced Functions

❖ **Linear functions and graphs**

Graph of the linear function y = ax + b

- y = ax + b graph is a parallel translation of a y = ax graph by b

- Parallel translate the y = ax graph by b in the y-axis

Advanced Functions

❖ **Linear functions and graphs**

- If b is positive, then y = ax translates the graph in the positive direction (upwards) of the y-axis

- If b is negative, then y = ax translates the graph to the negative direction (downward) of the y-axis

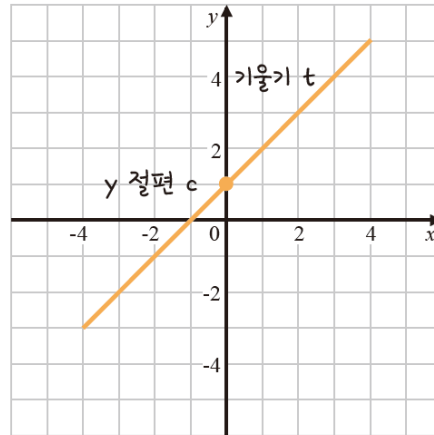# Advanced Functions

❖ **Linear functions and graphs**

Equations of a straight line

- In the coordinate plane, the linear function graph shape is called a **equation of a straight line** because it is a straight line

- The equation of a straight line is in the form y = ax + b, which is the same shape as the linear function

- 'a' is called the slope of the straight line, and 'b' is called the y intercept

- Intercept: It means the coordinates of the point where the graph of the function meets the x- or y-axis

  • When it meets the X axis, it is called an x intercept, and when it meets the y axis, it is called a y intercept
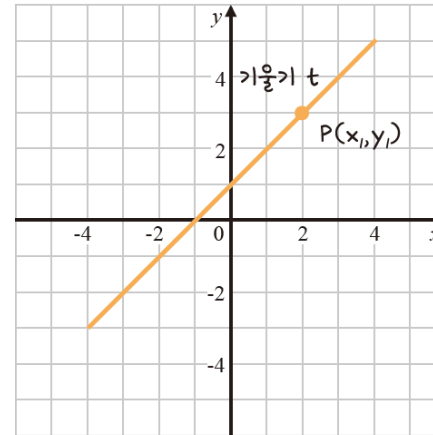
$$y = ax + b$$

직선의 기울기   y 절편

Advanced Functions

❖ **Linear functions and graphs**

- ▪ Let's find the slope of a straight line

  - • (1) Finding the equation of a straight line given a slope and a y intercept. If the slope is t, and the y intercept is c, then the equation of a straight line is y = tx + c

  - • (2) Given the coordinates of a slope and a point, find the equation of a straight line

  - • It should be a straight line with a slope t and a coordinate of a point P(x1, y1)

① 기울기와 y 절편이 주어졌을 때   ② 기울기와 한 점의 좌표가 주어졌을 때

Advanced Functions

MATHEMATIC

❖ **Linear functions and graphs**

- By substituting y = ax + b, we can derive as follows:

$$y_1 = tx_1 + b \quad \because 기울기 \ t와 \ 점의 \ 좌표(x_1, y_1) \ 대입$$

$$b = y_1 - tx_1$$

- Substituting the values t and b for y = ax + b as:

$$y = tx + (y_1 - tx_1) \quad \because 기울기 \ t와 \ b = y_1 - tx_1 \ 대입$$

$$y - y_1 = t(x - x_1)$$

- The equation of a straight line with a slope t and a point (x1, y1) is y - y1 = t(x - x1)

Advanced Functions

❖ **Linear functions and graphs**

연습 문제

(1) 기울기가 2고 y 절편이 5인 직선의 방정식을 구하세요.

(2) 기울기가 2고 점 (2, 5)를 지나는 직선의 방정식을 구하세요.

문제 풀이

(1) $y = ax + b$에서 기울기와 y 절편을 대입하면 $y = 2x + 5$입니다.

(2) $y - y_1 = t(x - x_1)$에서 기울기와 좌표를 대입하면 $y - 5 = 2(x - 2)$로 $y = 2(x - 2) + 5$가 되고, 풀이하면 $y = 2x + 1$입니다.

❖ **Quadratic functions and graphs**

Quadratic functions

- In a function y = f (x), when f (x) is quadratic with respect to x, it is called a quadratic function

- In other words, it has the same form as Equation 2.5

$$y = f(x) = ax^2 + bx + c$$

수식 2.5

- In this case, the x intercept is fired at ax^2 + bx + c, and the y intercept becomes c

$$y = ax^2 + bx + c$$

y 절편

이 수식의 해, x 절편

Advanced Functions

❖ **Quadratic functions and graphs**

Quadratic function graph

Quadratic function y = ax^2 graph

- In the first-order function, a in y = ax is called the gradient, and in the quadratic function it is called the coefficient of the quadratic term

- If the coefficient a of the quadratic term is bigger than 0 (a › 0), then the graph is convex downward, and if a is smaller than 0 (a ‹ 0), the graph is convex upwards

- Both sides are symmetrical to each other based on the origin (0,0)

Advanced Functions

❖ **Quadratic functions and graphs**

▪ As with the first-order function, let's see how the graph changes according to the value of the A constant in the quadratic function
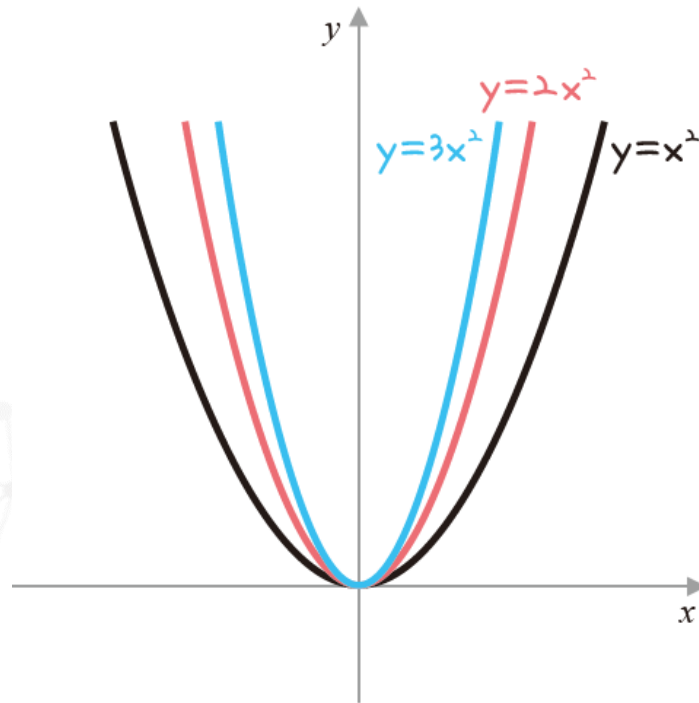
- (1) When a is positive, the change is:

a 값	1	2	3
y 값	$1x^2$	$2x^2$	$3x^2$

▪ The width of the graph narrows as the number 'a' increases

- (2) When a is negative, the change is:

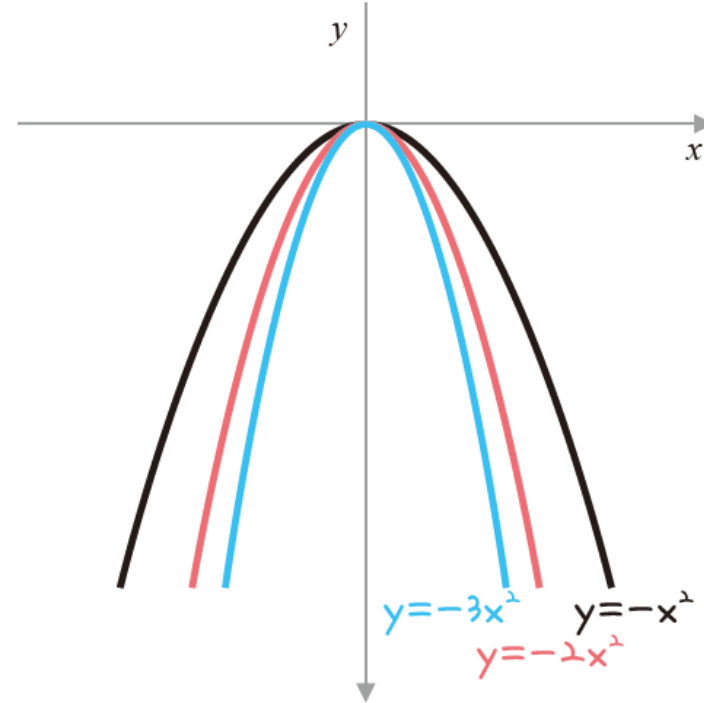| a 값	−1	−2	−3
y 값	$−1x^2$	$−2x^2$	$−3x^2$

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Advanced Functions

❖ **Quadratic functions and graphs**

▪ The width of the graph narrows as the number a gets smaller

① $y=ax^2$ ($a>0$)

② $y=ax^2$ ($a<0$)

Advanced Functions
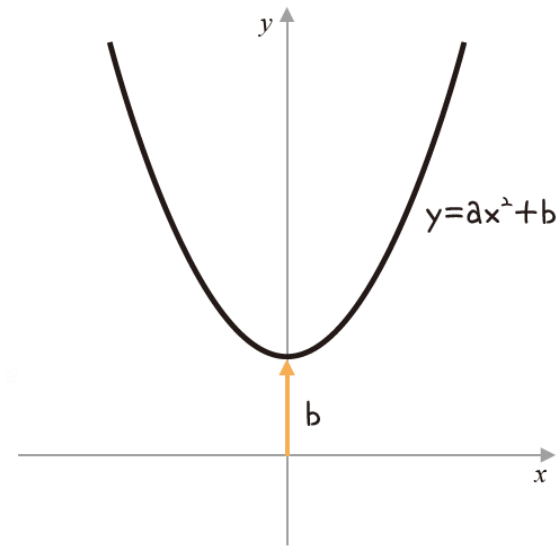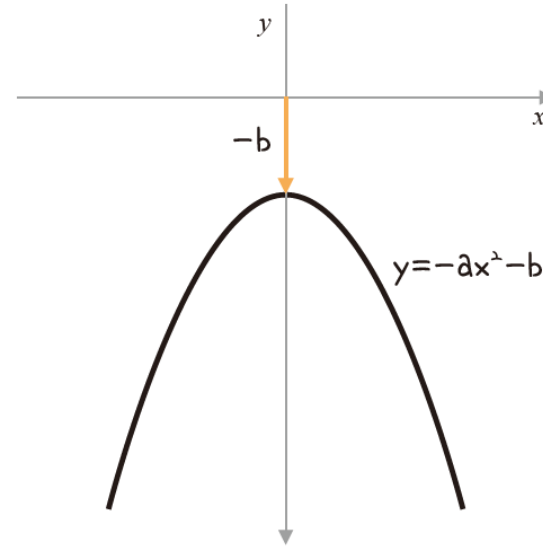
❖ **Quadratic functions and graphs**

Quadratic function y = ax² + b graph

- The $y = ax^2 + b$ graph is the $y = ax^2$ graph that is parallel translated by b

- In other words, if b is positive, then the $y = ax^2$ graph is paralleled by b in the positive direction (upwards) of the y-axis

- If both a and b are negative, then the $y = ax^2$ graph is flipped upside down, and b is parallel translated by b in the negative direction (downward) of the y axis

① $y=ax^2+b$ ② $y=-ax^2-b$

❖ **Quadratic functions and graphs**

▪ For example, let's draw a graph of the quadratic function

y = 2$(x - 3)^2$ + 5

- (1) At y = $2x^2$, the coefficients are positive, so it becomes a downward convex graph

- (2) Parallel translating y = $2x^2$ by 3 in the positive direction (right) of the x-axis and 5 in the positive direction (upwards) of the y-axis gives y - 5 = 2 $(x - 3)^2$

- (3) y = 2 $(x - 3)^2$ + 5 The graph is: