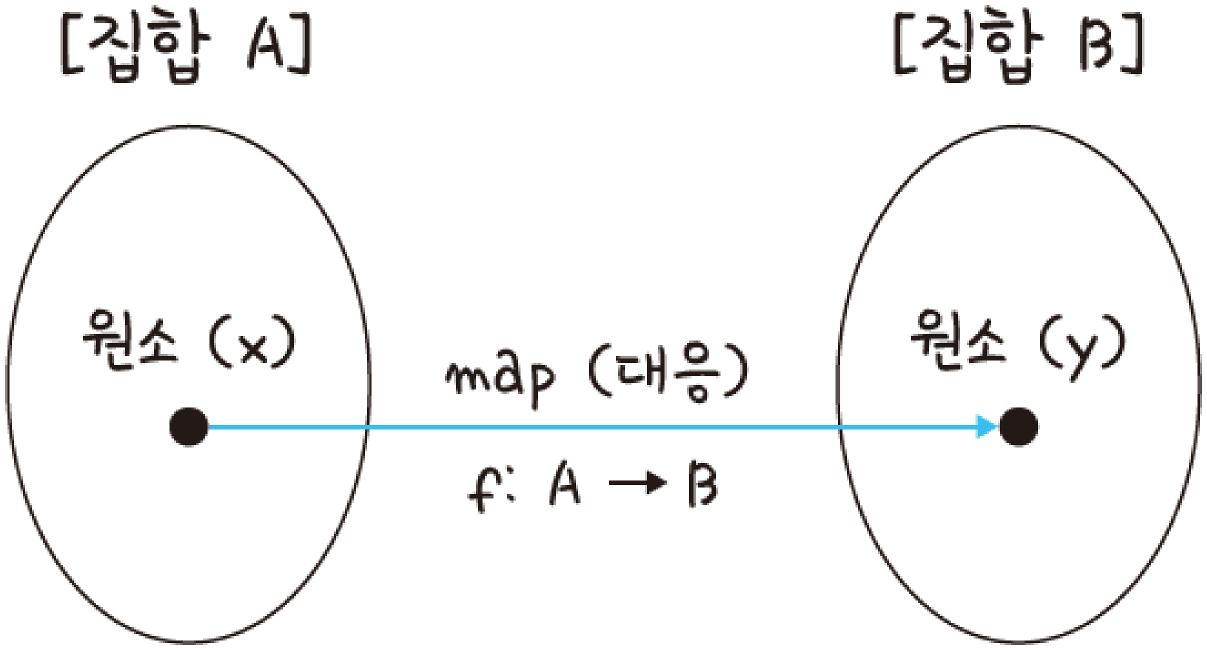
Matrix Transformation

Function and linear transformation

❖ Understanding functions

What is function

- The function is the correspondence between two sets
- In other words, the conversion of components of A into components of B is called a function



Function and linear transformation

❖ **Understanding functions**

- The mathematical expression of the function is as follows

$$f: X \rightarrow Y \text{ 혹은 } f: X \mapsto Y$$

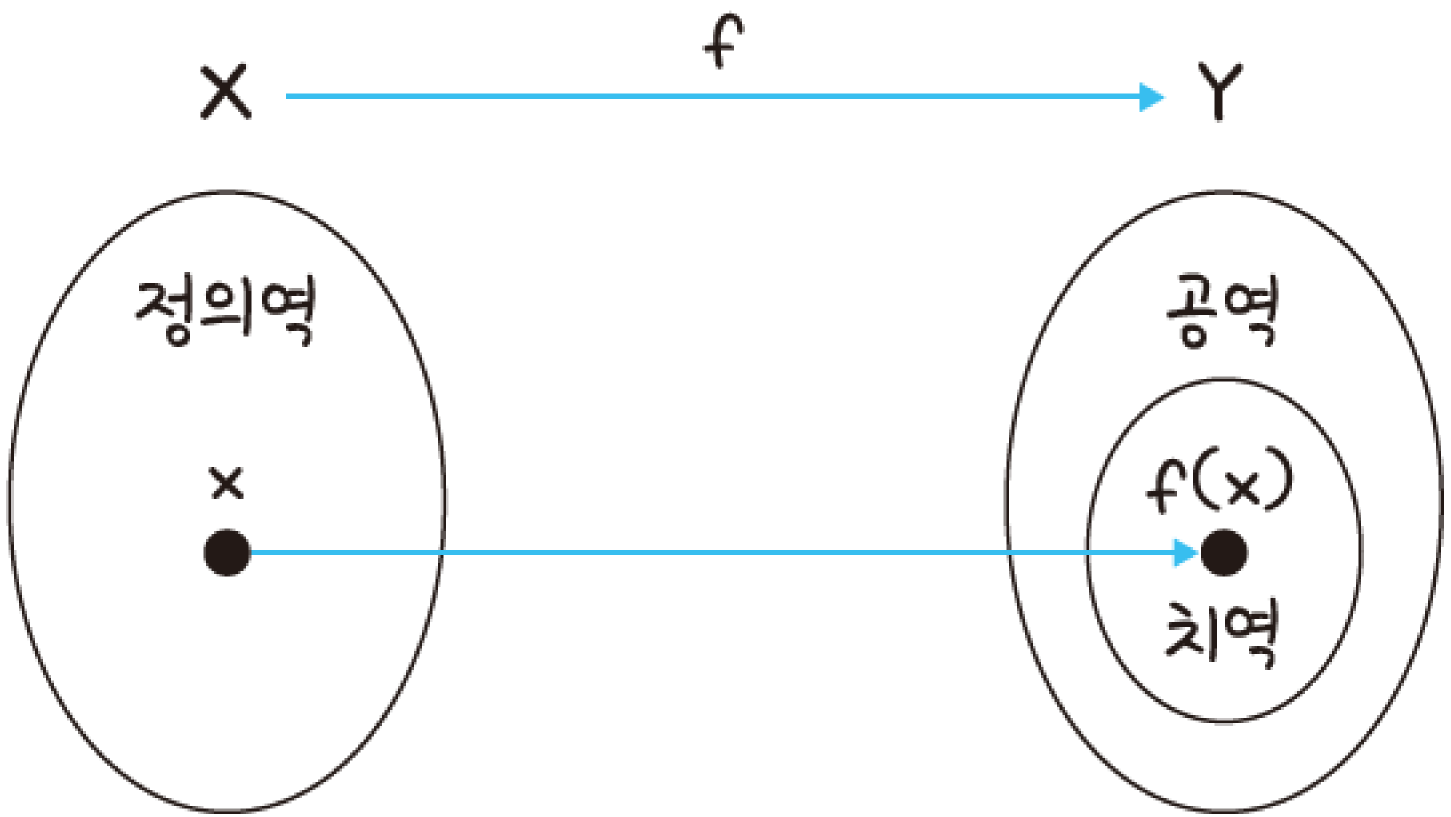
Function and linear transformation

❖ Understanding functions

Domain, co-domain, range

- Domain, co-domain, and range can be defined as follows
  - Domain means set X in sets X and Y related by function f
    - This means that the set X belongs to the domain at f: X → Y
  - The co-domain means set Y between sets X and Y
    - That is, the set Y belongs to airspace at f: X → Y
  - Range is a subset of the airspace and refers to the values associated with X among the Y values corresponding to the X value

Function and linear transformation



Function and linear transformation

MATHEMATIC

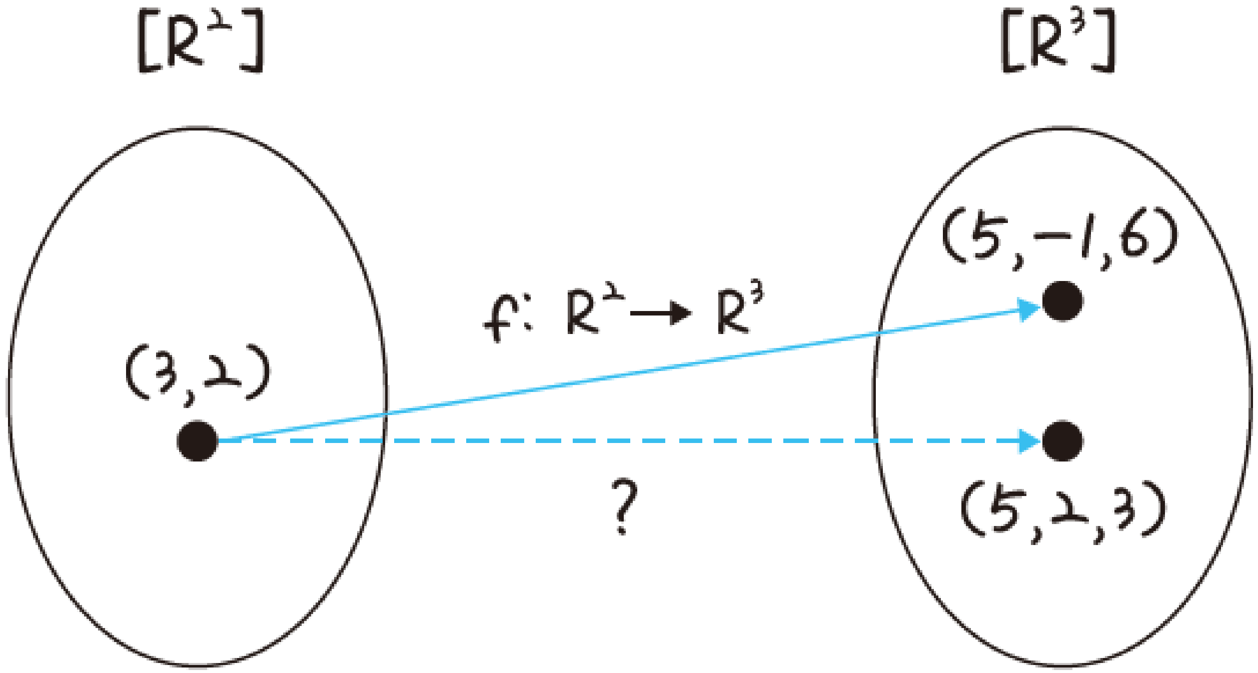
❖ **Understanding functions**

- For example, if f: $R^2 \rightarrow R$ is assumed to be $f(x_1, x_2) = 2$, then the expression f: $x_1, x_2 \rightarrow 2$ is also possible
- At this time, the domain is R^2, co-domain is R, and the range is 2
- As another example, if $f : R^2 \rightarrow R^3$ is assumed to be $f(x_1, x_2) = (x_1 + x_2, x_2 - x_1, 2x_1)$, we conclude that the domain is R^2, co-domain is R^3

Function and linear transformation

❖ Understanding functions

- Let's check the range with a specific case
- For example, F: (3, 2) → (5, -1, 6) can be expressed as



Function and linear transformation

❖ **Understanding functions**

- Could (5, 2, 3) be the range in the set R^3?
  - In conclusion, only (5, -1, 6) can be in range
- The range is defined as the values associated with X among the Y values corresponding to the X value
- For function f (x1, x2) = (x1 + x2, x2 – x1, 2 x1), the result of the Y value is

$$f(x_1, x_2) = (x_1 + x_2,\ x_2 - x_1,\ 2x_1) \ \rightarrow\ f(3, 2) = (3+2,\ 2-3,\ 2\times 3) = (5, -1, 6)$$

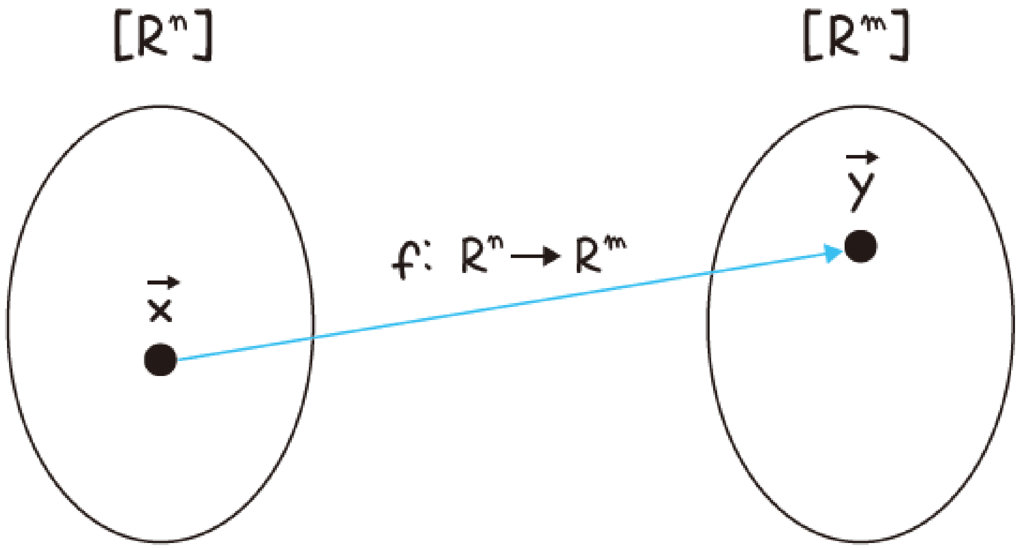
- For Y values (5, -1, 6), range is possible
  - (5, 2, 3) cannot be range because (5, 2, 3) cannot come out as a result of function f(x1, x2) = (x1 + x2, x2 – x1, 2 x1)
- That is, (5, 2, 3) may be airspace, but not range

Function and linear transformation

❖ Transformation of vector

- Functions refer to the relationship between elements in one set and elements in another set
- The elements of the set here are called vectors
- Vectors can be expressed as

$$\vec{x} \in R^n \; (R^n = \{x_1, x_2, \cdots, x_n \mid n-\text{튜플},\ x_1, x_2, \cdots, x_n \in R^n\})$$



Function and linear transformation

MATHEMATIC

❖ **Transformation of vector**

- Transformation of vector is to perform various transformations on vectors in 3D space
- Let's check the exact meaning with an example
- Can be expressed as $$f\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} x_1 + 3x_2 \\ 2x_3 \end{bmatrix}$$

when $f(x_1, x_2, x_3) = (x_1 + 3x_2, 2x_3)$ in transform to $f: R^3 \mapsto R^2$

Function and linear transformation

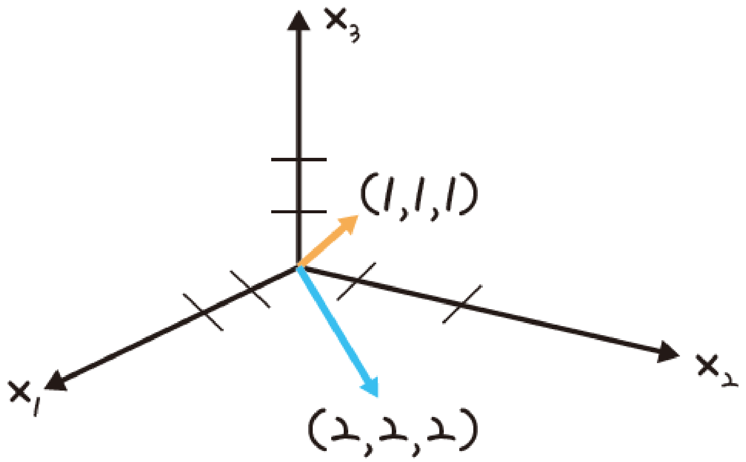
❖ Transformation of vector

- if (1) $x_1 = x_2 = x_3 = 1$, it becomes

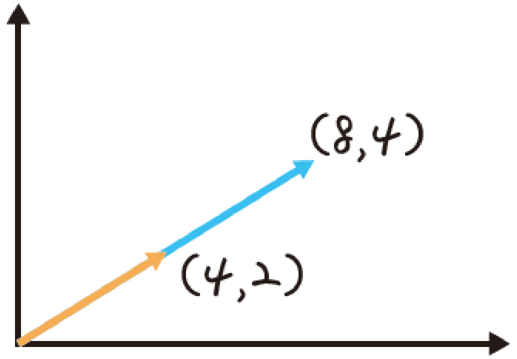
$$f\begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{bmatrix}1+(3\times 1)\\ 2\times 1\end{bmatrix} = \begin{bmatrix}4\\2\end{bmatrix}$$

- (2) Assuming that $x_1 = x_2 = x_3 = 2$, it becomes

$$f\begin{pmatrix}2\\2\\2\end{pmatrix} = \begin{bmatrix}2+(3\times 2)\\ 2\times 2\end{bmatrix} = \begin{bmatrix}8\\4\end{bmatrix}$$



① f: R³ 그래프



② f: R² 그래프

MATHEMATIC

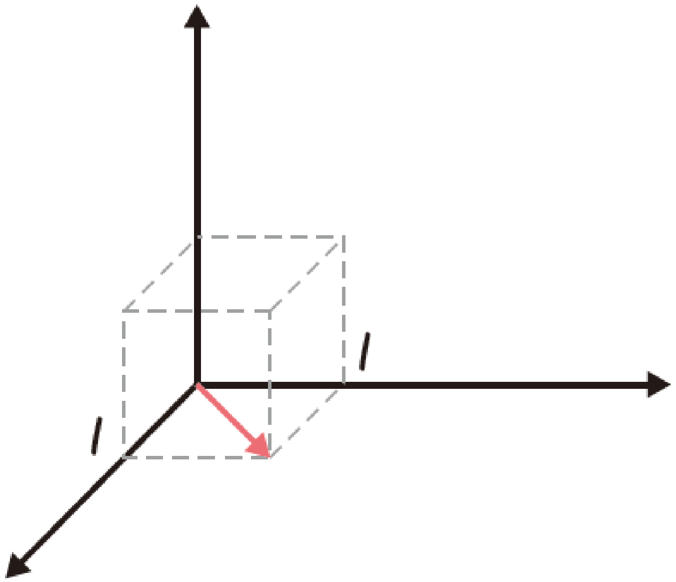
❖ Transformation of vector

- The conversion from (1) to (2) is called ***transformation of vector***
- This means that it can change in various forms depending on the values of x_1, x_2, and x_3
- Calculating and graphing the $M\vec{v}$ when there is a vector $\vec{v}$ and set M as follows

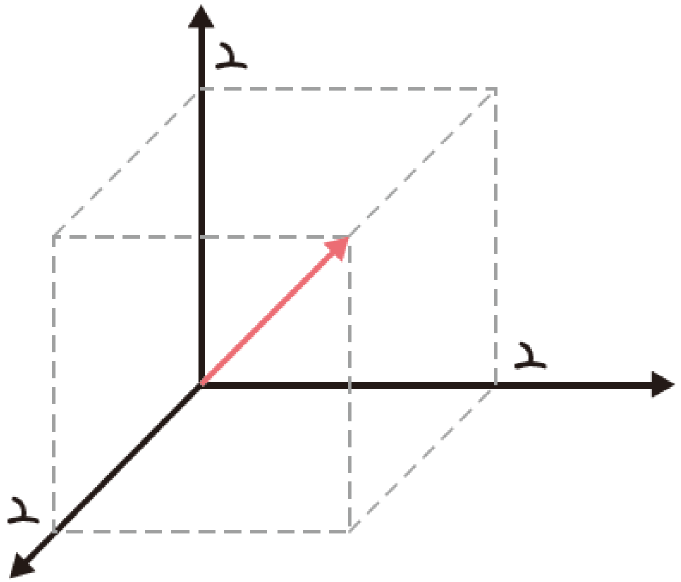
$$\vec{v} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \ M = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix} \text{일 때,}$$

$$M\vec{v} = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 3 \\ 0 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} (2\times 1)+(0\times 1)+(1\times 0) \\ (1\times 1)+(1\times 1)+(3\times 0) \\ (0\times 1)+(2\times 1)+(2\times 0) \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \vec{v'}$$

Function and linear transformation



① 벡터 $\vec{v}$



② 벡터 $\vec{v}'$

Function and linear transformation

MATHEMATIC

❖ Transformation of vector

- A vector $\vec{v}$ transformed into a $\overrightarrow{v'}$ (multiplication of vector $\vec{v}$ and set M) is also called a "transformation of vector"
- Transformation of vector is written in capital ***T*** as follows

$$T: R^n \rightarrow R^m$$

- Transformation of vector is used to rotate and move points or vectors in 3D spaces

Function and linear transformation

❖ Linear transformation

- Linear is an adjective form of line, and 'linear' can be inferred to mean that a certain property changes in the end, and the variable is one-dimensional
  (결국 어떤 성질이 변하는 데 그 변수가 1차원적이라는 의미로 유추할 수 있음)
- In other words, linearity means a straight line-like figure or an object with similar properties, and the function uses the shape as a 'straight line.'
  (선형성이란 직선과 같은 도형 또는 그와 성질이 비슷한 대상이라는 것으로, 함수에서는 그 모양이 '직선'이라는 의미로 사용함)
- In mathematics, linearity must satisfy the following two conditions simultaneously for any number x, y and function f
  - **Superposition**(중첩성): $f(x + y) = f(x) + f(y)$
  - **Homogeneity**(동질성): $f(ax) = af(x)$ for any number a

Function and linear transformation

❖ Linear transformation

- Linear transformations are functions between ***two vector spaces that preserve linear combination***
- In other words, we need to understand the implications of (1) the function between two vector spaces, which preserves linear coupling
- Linear combination means that the following two conditions must be met
  - Addition principle(합의 법칙): $T(\overrightarrow{a}+\overrightarrow{b}) = T(\overrightarrow{a}) + T(\overrightarrow{b})$
  - Scalar product principle(스칼라 곱의 법칙): $T(c\overrightarrow{a}) = cT(\overrightarrow{a})$

Function and linear transformation

❖ Linear transformation

- Let's look at the meaning of linear transformation as an example
- Suppose you have matrices A and B

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 4 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

Function and linear transformation

❖ Linear transformation

- Multiplying matrices A and B is as follows:

$$AB = \begin{bmatrix} (1\times 5)+(2\times 6)+(3\times 7) \\ (3\times 5)+(4\times 6)+(5\times 7) \end{bmatrix}$$

$$= \begin{bmatrix} 5+12+21 \\ 15+24+35 \end{bmatrix}$$

$$= \begin{bmatrix} 38 \\ 74 \end{bmatrix}$$

Function and linear transformation

❖ **Linear transformation**

- AB, which multiplies a matrix A by an arbitrary matrix B, means transforming vector B from three dimensions to two dimensions
(행렬 A에 임의의 행렬 B를 곱하는 AB는 벡터 B를 3차원에서 2차원으로 변환시키는 것을 의미)

- In other words, linear transformation is understood as a rule of ***moving from one vector space to another***, such as 3D to 2D transformation
(즉, 선형 변환은 3차원에서 2차원으로 변환처럼 한 벡터 공간에서 다른 벡터 공간으로 이동하는 규칙 정도로 이해함)

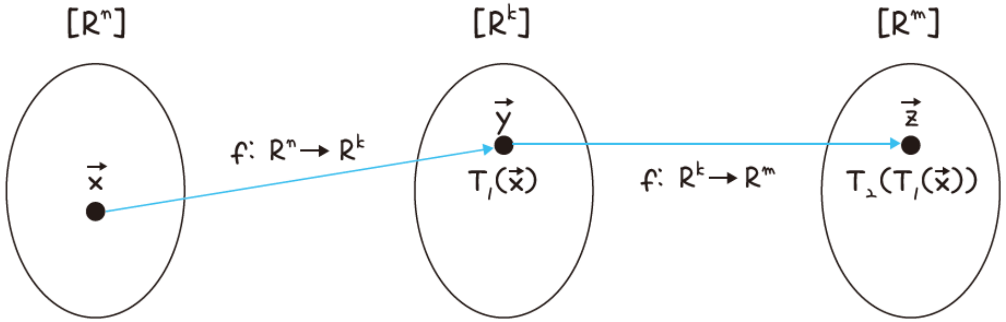
Function and linear transformation

❖ Linear transformation

- Since linear transformation is also a function, linear transformation can also be understood as a composite(합성) concept
- Assume the following linear transformation (the co-domain of T_1 is the same as the domain of T_2)

$$T_1: R^n \rightarrow R^k,\ T_2: R^k \rightarrow R^m$$

- Here, $T_1(\vec{x})$ is calculated for vector $\vec{x}$ on R^n
- Then let's take the vector $T_1(\vec{x})$ in R^k and calculate the final vector $T_2(T_1(\vec{x}))$
- In other words, the following linear transformation can be synthesized



Function and linear transformation

❖ Linear transformation

- The calculation process is the process of applying T_1 to the vector on R^n and then applying T_2, and in the end, the whole process can be viewed as a transformation with $\mathbf{R^n \rightarrow R^m}$
- This conversion to $R^n \rightarrow R^m$ is called **a combination of T_2 and T_1**, and is written as follows (read as T_2 circle T_1)

$$T_2 \circ T_1$$

- Using the previous function can draw the following conclusions

$$(T_2 \circ T_1)(\vec{x}) = T_2(T_1(\vec{x}))$$

- At this time, the order of T_2 and T_1 is very important
- This is because if the order changes, it can be a different transformation
- $T_2 \circ T_1$ satisfies the following equation $\quad T_2 \circ T_1 : R^n \rightarrow R^m$

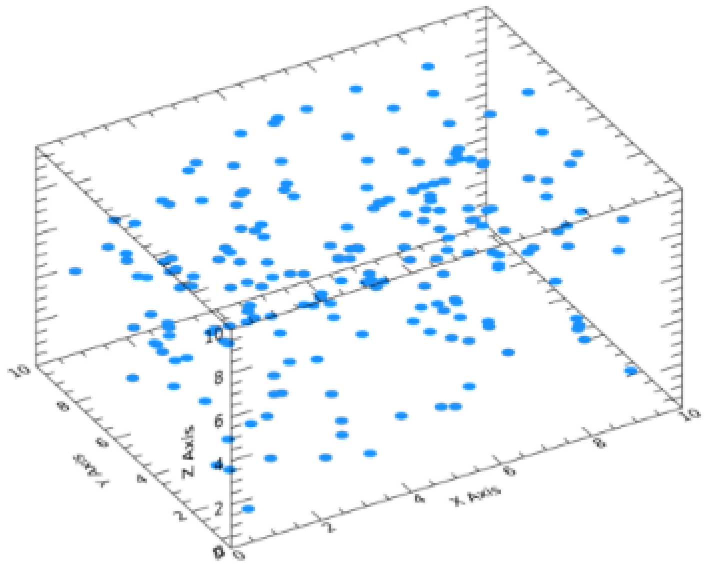
Function and linear transformation

❖ **Linear transformation**

$$\begin{bmatrix} 9 & 13 & 5 & 2 \\ 1 & 11 & 7 & 6 \\ 3 & 7 & 4 & 1 \\ 6 & 0 & 7 & 10 \end{bmatrix}$$

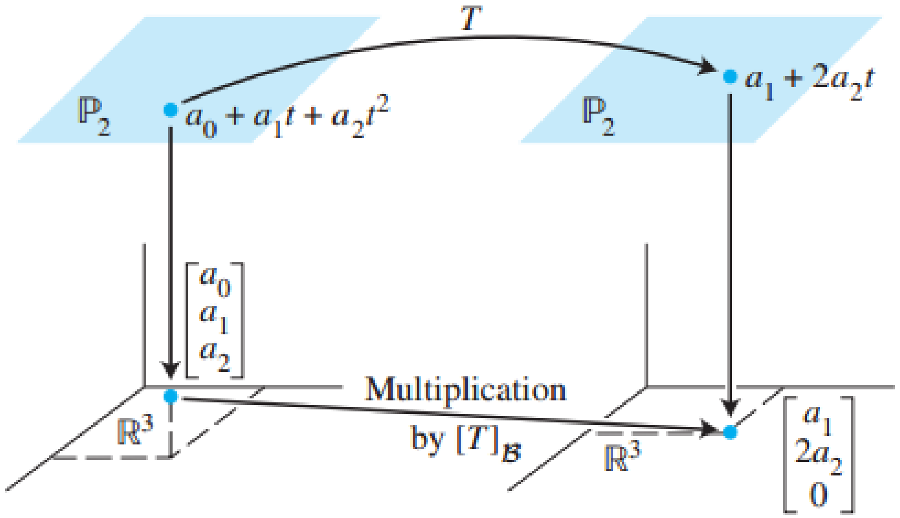
$$\begin{bmatrix} 9 & \boxed{13} & \boxed{5} & \boxed{2} \\ 1 & \boxed{11} & \boxed{7} & \boxed{6} \\ 3 & \boxed{7} & \boxed{4} & \boxed{1} \\ 6 & \boxed{0} & \boxed{7} & \boxed{10} \end{bmatrix}$$

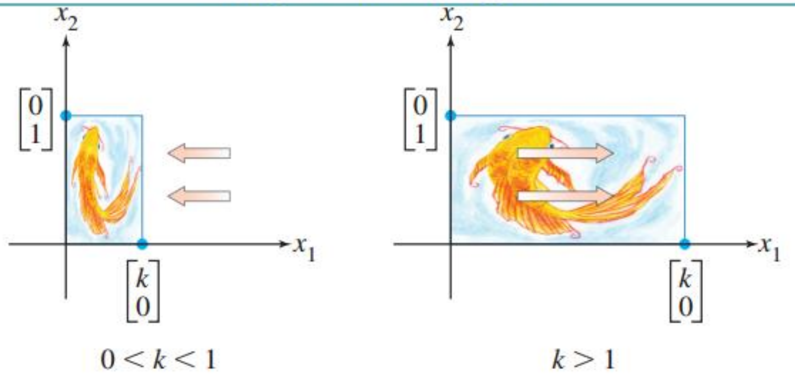
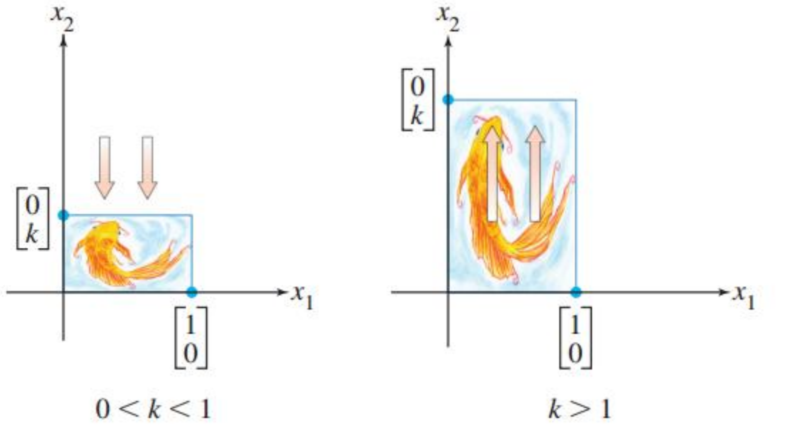
$$\begin{bmatrix} 9 & 13 & 5 & 2 \\ \boxed{1 \quad 11 \quad 7 \quad 6} \\ \boxed{3 \quad 7 \quad 4 \quad 1} \\ \boxed{6 \quad 0 \quad 7 \quad 10} \end{bmatrix}$$



Function and linear transformation

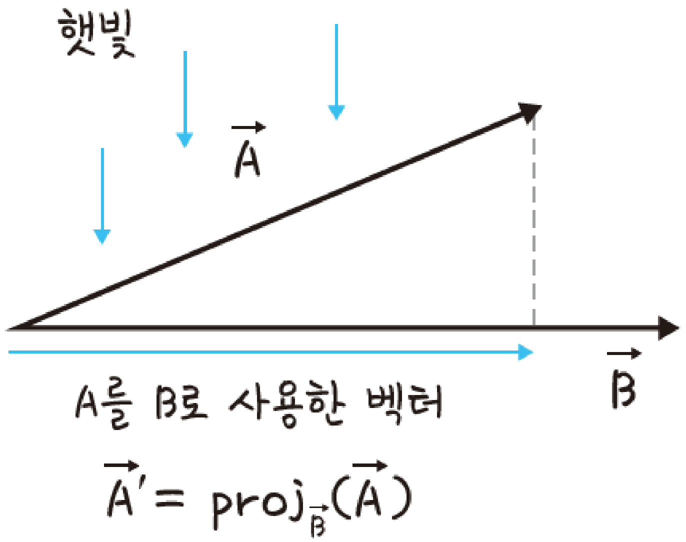
❖ Linear transformation



Transformation	Image of the Unit Square	Standard Matrix
Horizontal contraction and expansion		$\begin{bmatrix} k & 0 \\ 0 & 1 \end{bmatrix}$
Vertical contraction and expansion		$\begin{bmatrix} 1 & 0 \\ 0 & k \end{bmatrix}$

❖ Orthogonal projection (정사영)

- orthogonal projection refers to the shadow that occurs in a plane perpendicular to a particular figure (or line segment) when light is illuminated
- The $\overrightarrow{A'}$ is called the $\vec{A}$ projection



※ *Proj*는 projection의 약자로 '사영'을 뜻하며, 프로젝션 벡터라고 읽습니다.

Function and linear transformation

MATHEMATIC

❖ Orthogonal projection

- The mathematical expression of the orthogonal projection is as follows
- Orthogonal projection with $\vec{A}$ as $\vec{B}$ for two vector $\vec{A}$ and $\vec{B}$ can be expressed by the following formula

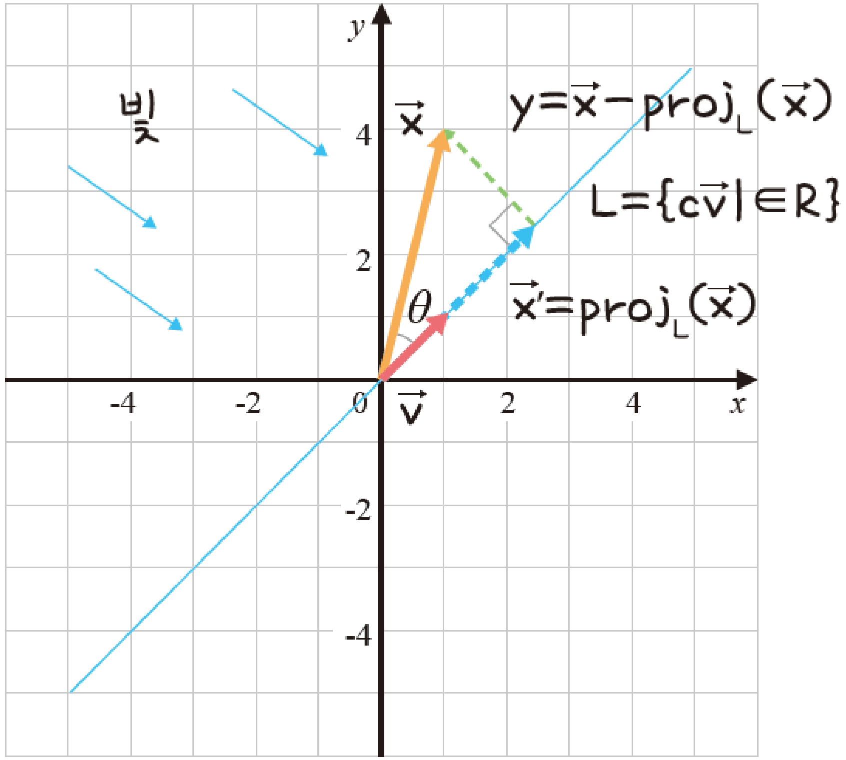
$$proj_{\vec{B}}(\overrightarrow{A}) = \frac{\overrightarrow{A} \cdot \overrightarrow{B}}{|\overrightarrow{A}|}\overrightarrow{A}$$

- Since the **Orthogonal projection is also a vector**, the previous equation can be derived using the size and direction of the vector

Function and linear transformation

❖ Orthogonal projection

- The vector $\vec{v}$ multiplied by an arbitrary scalar (c) is called a straight line L
- At this point, let's define the orthogonal projection $(proj_L(\overrightarrow{x}))$ of the vector $\vec{x}$ to the straight line L

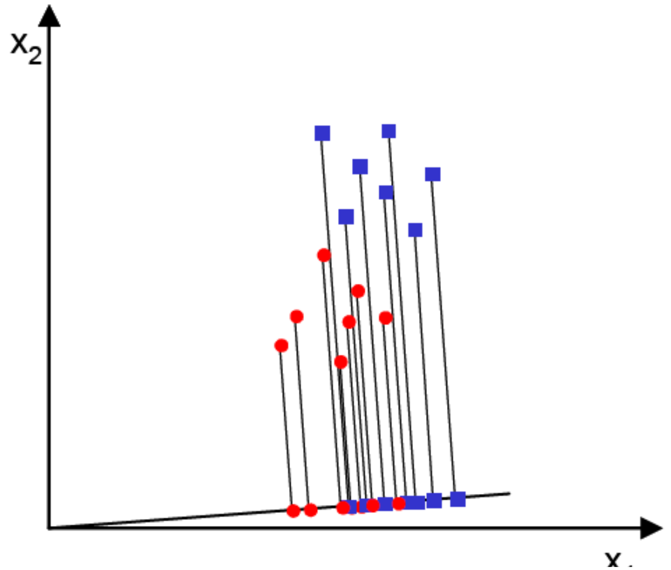


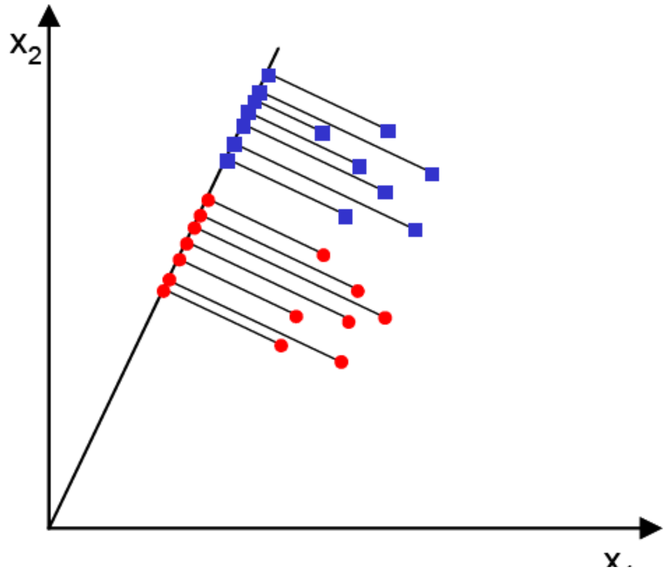
❖ Orthogonal projection

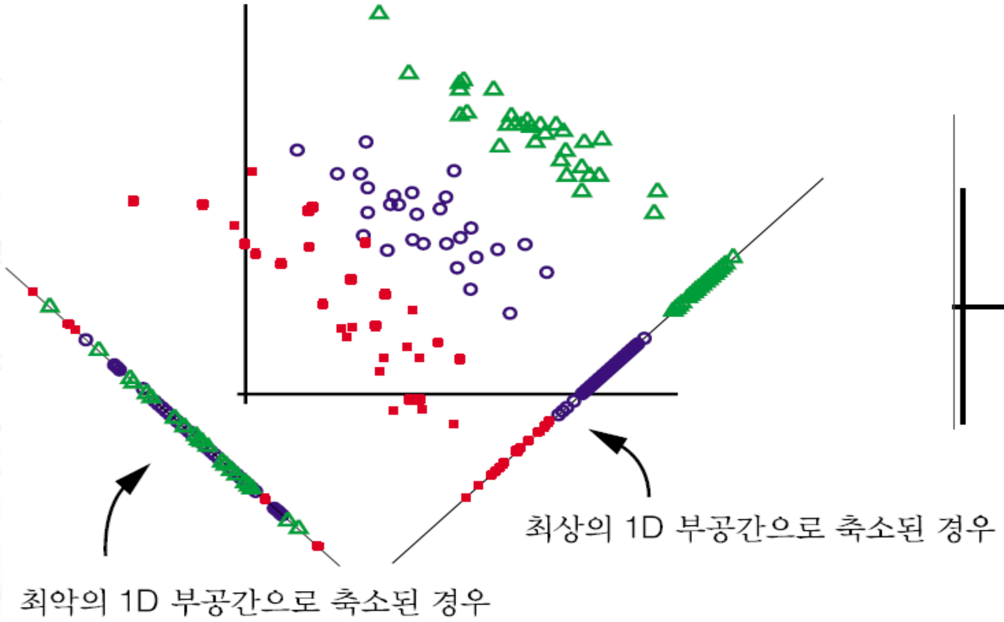
- Since light and L are orthogonal when light is reflected perpendicular to the straight line L, the $\vec{x}$ of the orthogonal projection on L can be called $\overrightarrow{x'}$
- That is, the vector $\overrightarrow{x'}$ is the orthogonal projection of the $\vec{x}$ with respect to the straight line L
- A vector y perpendicular to a straight line means that it is perpendicular to all vectors on the straight line
- Can be expressed as follows

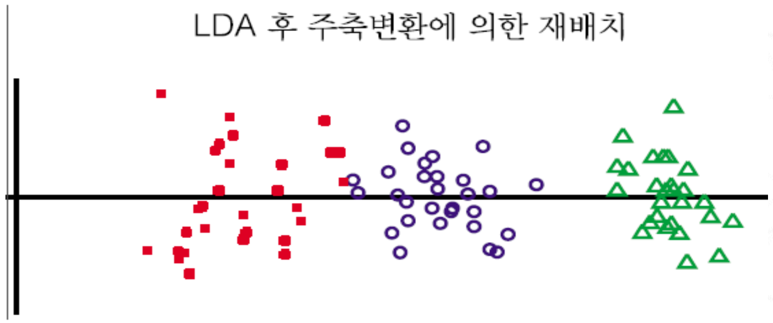
$$\vec{x} - proj_L(\vec{x}) = \frac{\text{직선에 수직}}{\text{직교}}$$

Function and linear transformation







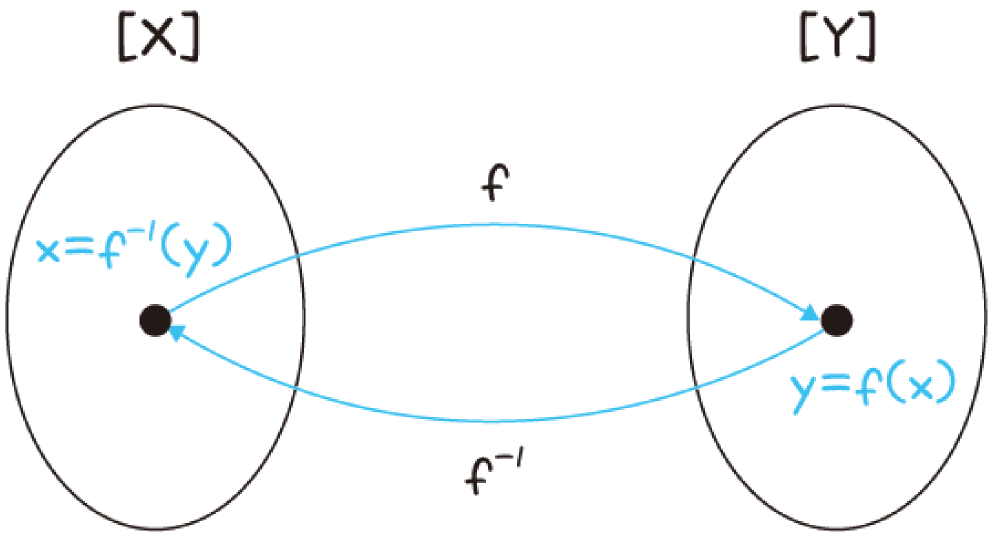


Inverse function and inverse transformation

❖ What is the inverse of function

Inverse function (역함수)

- Inverse function is a function that is obtained by inverting the values of variables and functions
- Inverse function reads $f^{-1}(x)$ as f Inverse function x or inverse f

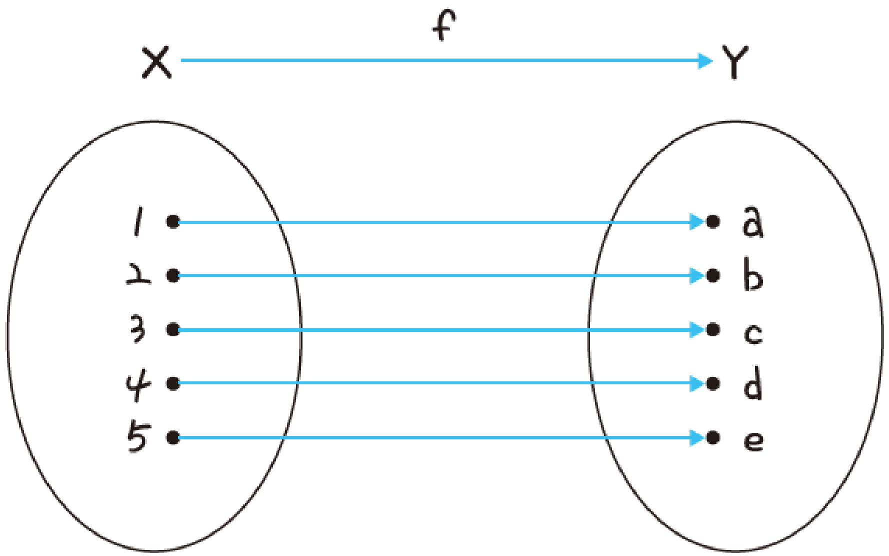


Inverse function and inverse transformation

❖ What is the inverse of function

- Since f and f^{-1} are functions that change the domain and airspace in one-to-one correspondence, they become each other's inverse function
- In other words, it can be expressed as follows $(f^{-1})^{-1} = f$

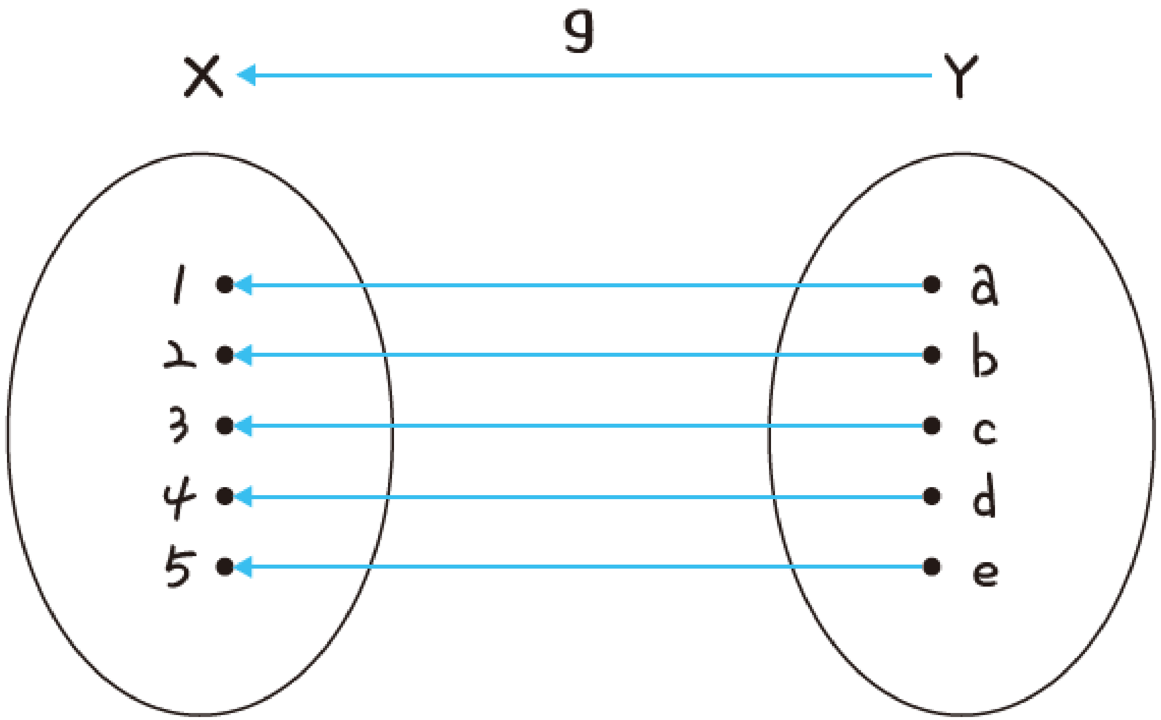
- For example, if you have two sets X = {1, 2, 3, 4, 5}, Y = {a, b, c, d, e}, then f is one-to-one correspondence if you have a function f of Y for X as follows



Inverse function and inverse transformation

❖ **What is the inverse of function**

- We can also think of a function where Y is the domain and X is the co-domain as follows
- Assuming this function is g, it is also a one-to-one correspondence



Inverse function and inverse transformation

❖ What is the inverse of function

- If function f: X → Y is ***one-to-one correspondence***(일대일대응), there is only one element x of X with y = f(x) for any element y of Y
- At this time, by corresponding x to y, a new function can be created in which Y is the domain and X is the domain, which is called the inverse function of f and is expressed as follows

$$f^{-1}: Y \rightarrow X$$

- The following formula is established

$$y = f(x) \Leftrightarrow x = g(y) \Leftrightarrow x = f^{-1}(y)$$

Inverse function and inverse transformation

❖ What is the inverse of function

- It is meaningless to distinguish (2) and (3) when domain X and co-domain Y are sets of whole real numbers
- $f \circ f^{-1} = f^{-1} \circ f = I$ (I is an expression of an identity function(항등함수))
- Not all functions have an inverse function
- When the previous property is satisfied, it has an inverse function, at which time the function is called an inverse function(가역함수)

일대일 대응 $\neq$ 일대일 함수

Inverse function and inverse transformation

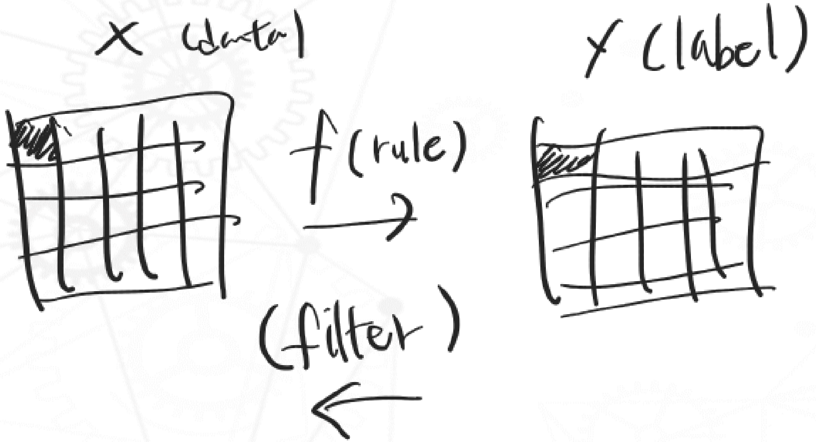
❖ **What is the inverse of function**

One-to-one function and one-to-one correspondence

- One-to-one function: a function that is $f(x_1) \neq f(x_2)$ when $x_1 \neq x_2$ for any element x_1, x_2 of the set X
- One-to-one correspondence: one-to-one function + (co-domain=range)

  - If $f(x_1) \in Y$ for element x_1 of set X
  - At this time, if $f(x_1) \neq f(x_2)$ is a one-to-one function when $x_1 \neq x_2$ it is a one-to-one function, and if co-domain = range, it is a one-to-one correspondence

x = data   f = rule   y = label



Inverse function and inverse transformation

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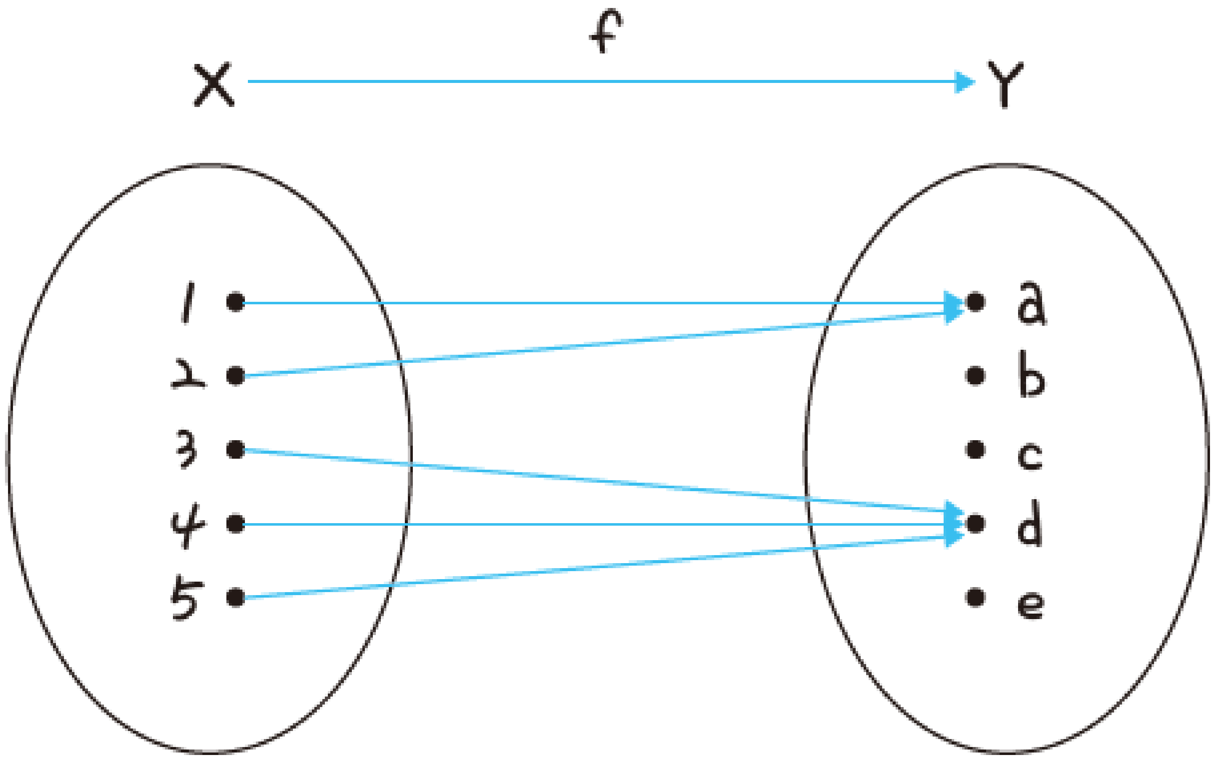
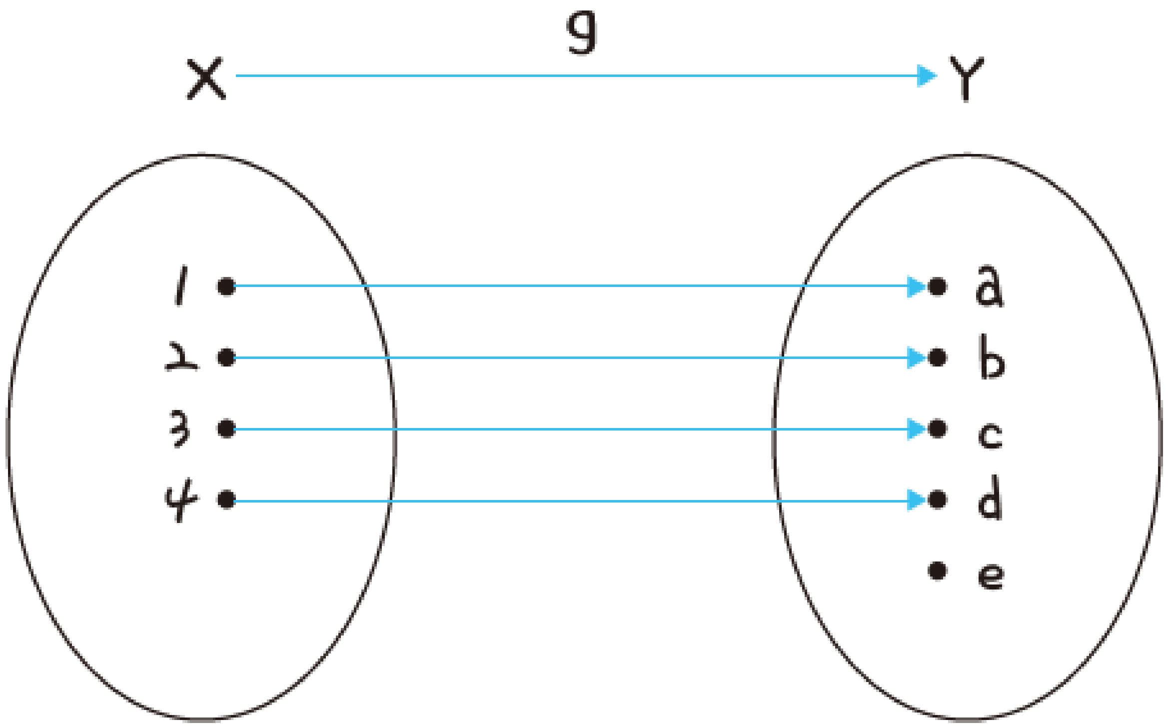


Inverse function and inverse transformation

❖ **What is the inverse of function**

- Different elements of set Y corresponding to the elements of set X
- One-to-one function because the co-domain is {a, b, c, d, e} and the co-domain =/ range is {a, b, c, d}



Co-domain ≠ range.

Inverse function and inverse transformation

❖ Inverse matrix (역행렬)

- If a number has an inverse, a matrix has an inverse matrix
- A matrix that yields a unit matrix E that is identity to multiplication when multiplied by a matrix A is called an inverse matrix of matrix A

$$AB = BA = E$$

- The matrix B, which is multiplied by A to produce E as in the previous equation, is called the inverse matrix of A and is written as A^{-1}

  (read as A inverse)

$$B = A^{-1}$$

Inverse function and inverse transformation

❖ Inverse matrix

- Let's look at the inverse matrix in the next example

$$A = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}$$

$$AB = \begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} (4\times(-2))+(3\times 3) & (4\times 3)+(3\times(-4)) \\ (3\times(-2))+(2\times 3) & (3\times 3)+(2\times(-4)) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$BA = \begin{bmatrix} -2 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} 4 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} ((-2)\times 4)+(3\times 3) & ((-2)\times 3)+(3\times 2) \\ (3\times 4)+((-4)\times 3) & (3\times 3)+((-4)\times 2) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- The unit matrix AB = E was derived, and BA = E was also derived by the unit matrix
- B is the inverse of A because the unit matrix was derived when the two matrices were multiplied

디지털 → Fusion → Fuzzy

(불확실성) (증명)

Inverse function and inverse transformation

❖ Inverse matrix

- For reference, $a^{-1} = \frac{1}{a}$ is established in the number, but $A^{-1} = \frac{1}{A}$ is not established in the matrix

- Typically, the property for matrix multiplication is the AB $\neq$ BA
- For the matrix and its inverse, $AA^{-1} = A^{-1}A$ is established ($AA^{-1} = A^{-1}A$ is established because the unit matrix is derived in the same way)
- If matrix A is 2×3 and A^{-1} is 3×2, then $AA^{-1} = E$, where E becomes a quadratic square matrix

$$\underset{2\times 3}{A}\quad \underset{3\times 2}{A^{-1}} = \underset{\boxed{2\times 2}}{E} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Inverse function and inverse transformation

❖ Inverse matrix

- Conversely, $A^{-1}A = E$ becomes a ***three-dimensional square matrix***
- Both AA^{-1} and $A^{-1}A$ have unit matrices(단위행렬) E but are different matrices
- For multiplication results to be the same nth unit matrix, A and A^{-1} must also be nth square matrices

MATHEMATIC

❖ Inverse matrix

Inverse matrix formula

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix} = (ad-bc)\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \quad (ad-bc \neq 0)$$

Inverse function and inverse transformation

❖ **Inverse matrix**

- If ad - bc = 0, there is no inverse matrix of A
- For example, does the inverse of matrix $A = \begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix}$ exist?

- Inverse matrix does not exist because ad - bc = 24 - 24 = 0

[역행렬의 공식]

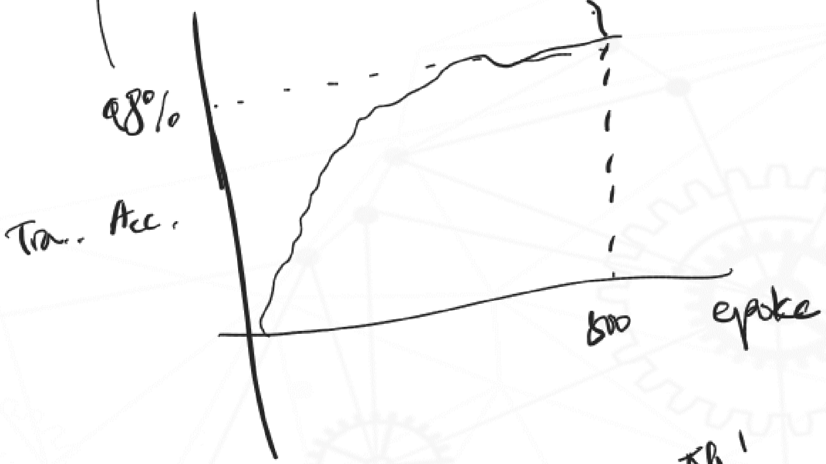
- For a quadratic square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

ad - bc ≠ 0 is $$A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

If ad - bc = 0, no inverse matrix of matrix A

epoke = 반복 횟수.

setration...?



training validation?

↑ ↑ ↑

Tr Va Ts

generalization 중요!

Inverse function and inverse transformation

❖ **Inverse matrix**

[Properties of inverse matrix]

$$(1)\ (A^{-1})^{-1} = A$$

$$(2)\ (kA)^{-1} = \frac{1}{k}A^{-1}$$

$$(3)\ (AB)^{-1} = B^{-1}A^{-1}$$

Dropout.

Batch Norm

Normalizarin

Overfitting

Underfiting

Inverse function and inverse transformation

❖ Inverse matrix

[Using the inverse matrix]

- Inverse matrix is useful for solving linear equations
- The following expression exists

$$\begin{aligned}
a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\
&\vdots \\
a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n &= b_n
\end{aligned}$$

Inverse function and inverse transformation

❖ Inverse matrix

- If the linear equation for x is expressed as a matrix (a, b is a constant)

$$\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}$$

wait matrix data output

Inverse function and inverse transformation

❖ **Inverse matrix**

- To put it more simply, it's as follows

$$AX = B$$

- If only the inverse matrix of A can be calculated, the solution of this simultaneous equations can be easily calculated as follows

$$X = A^{-1}B$$

Inverse function and inverse transformation

MATHEMATIC

❖ Inverse matrix

- Even in Python, a system of equations can be implemented by calculating inverse matrices as follows

In [56]:

```
# NumPy 라이브러리를 호출합니다
import numpy as np

# A에 4x4 행렬을 배치합니다
A = np.matrix([[1, 0, 0, 0], [2, 1, 0, 0], [3, 0, 1, 0], [4, 0, 0, 1]])

# A 행렬을 역행렬로 변환하기 위해 numpy.linalg.inv()를 사용합니다
print(np.linalg.inv(A))
```

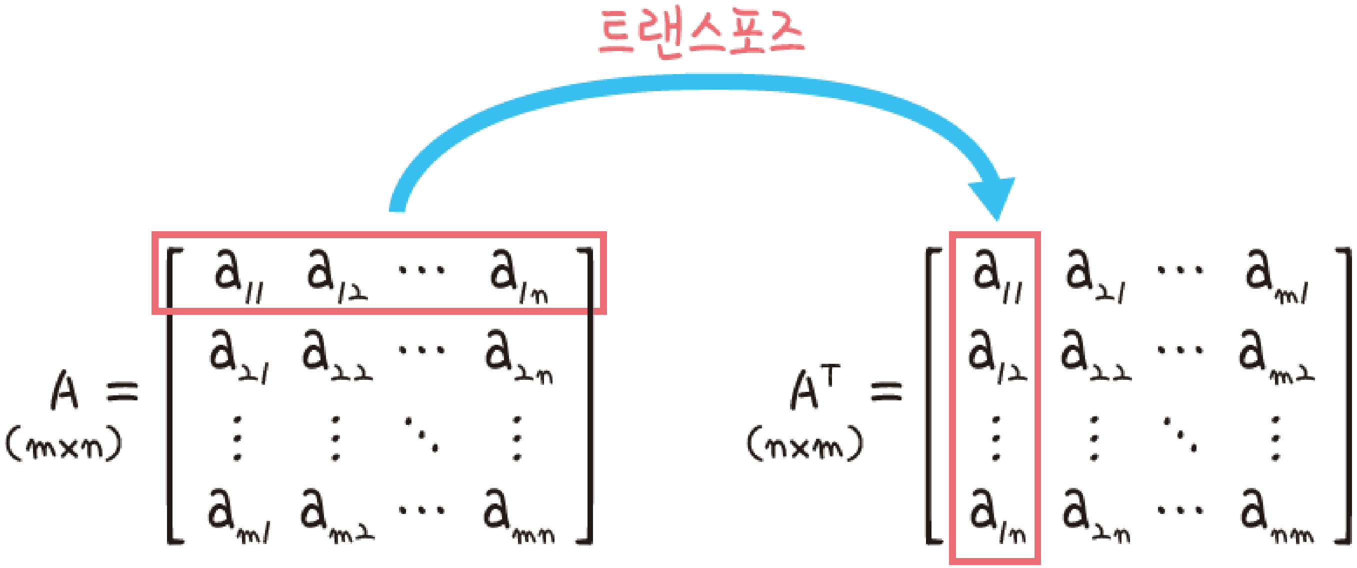
```
[[ 1,  0,  0,  0]
 [-2,  1,  0,  0]
 [-3,  0,  1,  0]
 [-4, -0, -0,  1]]
```

4x4 손연습 필요.

Transpose matrix

❖ What is transpose matrix

- Transpose matrix is a matrix that changes columns and rows
- In other words, an m×n matrix was changed to an n×m matrix



$$A_{(m\times n)} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad A^T_{(n\times m)} = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{m1} \\ a_{12} & a_{22} & \cdots & a_{m2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \cdots & a_{nm} \end{bmatrix}$$

① 행렬 A　　② 행렬 A^T

Transpose matrix

❖ What is transpose matrix

- Matrix A of $m \times n$ becomes $n \times m$ matrix in A^T
- The transpose matrix is written as follows and read as Transpose

$$A_{m \times n} = A^T{}_{n \times m}$$

Transpose matrix

MATHEMATIC

❖ What is transpose matrix

- Here are various examples of Transpose matrix

$$(1)\ \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \end{bmatrix}^T$$

$$(2)\ \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}^T$$

$$(3)\ \begin{bmatrix} 2 & 4 & 5 \\ 3 & 1 & 6 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 5 & 6 \end{bmatrix}^T$$

Transpose matrix

MATHEMATIC

❖ What is transpose matrix

- For reference, Transpose matrix has the following properties

[Properties of transpose matrix]

(1) $(A^T)^T = A$

(2) $(A+B)^T = A^T + B^T$

(3) $(AB)^T = B^TA^T$

(4) $(kA)^T = kA^T$ (k is any real number)

- Transpose matrix is used in a variety of places, including column and row vector conversion, and photo conversion in image processing

Transpose matrix

❖ What is transpose matrix

- Python uses NumPy to calculate the Transpose matrix, which uses the following three methods to calculate the Transpose matrix
  - **a.T**
  - **np.transpose(a)**
  - **np.swapaxes(a, 0, 1):** the factors 0 and 1 after a represent the axis
    - 0 means 2D, the highest-order axis, and 1 means 1D, the next highest-order axis
    - In other words, it means to change the rows and columns of the elements ((1,3) → (3,1))

Transpose matrix

❖ Transpose matrix

In [59]:

```python
# NumPy 라이브러리를 호출합니다
import numpy as np
# 원소가 총 15개 들어 있는 배열 a를 3x5로 배치합니다
a = np.arange(15).reshape(3, 5)
print(a)
```

```
[[ 0  1  2  3  4],
 [ 5  6  7  8  9],
 [10 11 12 13 14]]
```

행 기준 순서 반전 → 오른쪽으로 90° 회전

$$\rightarrow \begin{matrix} 10 & 11 & 12 & 13 & 14 \\ 5 & 6 & 7 & 8 & 9 \\ 0 & 1 & 2 & 3 & 4 \end{matrix} \rightarrow \begin{matrix} 0 & 5 & 10 \\ 1 & 6 & 11 \\ 2 & 7 & 12 \\ 3 & 8 & 13 \\ 4 & 9 & 14 \end{matrix}$$

A^T 구하기.

Transpose matrix

❖ Transpose matrix

In [60]:

```
# a 행렬을 전치행렬로 변환합니다
np.transpose(a)
```

Out [60]:

```
array([[ 0,  5, 10],
       [ 1,  6, 11],
       [ 2,  7, 12],
       [ 3,  8, 13],
       [ 4,  9, 14]])
```

Transpose matrix

MATHEMATIC

❖ Determinant of transpose matrix

- A determinant is a function that corresponds numbers to a matrix with the same number of rows and columns, i.e., a square matrix
- The determinant for matrix is A $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and det A
- If the determinant A = ad - bc is 0, the inverse of the matrix does not exist
- In an m×n matrix, the determinant can be organized as follows

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + \cdots + (-1)^{1+n} \det A_{1n}$$
$$= \sum_{j=1}^{n} (-1)^{1+j} a_{1j} \det A_{1j}$$

하나의 스칼라 값으로 내기 위해 determinant 사용.

Transpose matrix

❖ Determinant of transpose matrix

- Using this theorem, the determinant for square matrices of size 1, 2, and 3 is

$$1\times 1 \text{ 행렬의 행렬식: } \det[a] = a$$

$$2\times 2 \text{ 행렬의 행렬식: } \det\begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc$$

$$3\times 3 \text{ 행렬의 행렬식: } \det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = aei + bfg + cdh - ceg - bdi - afh$$

Transpose matrix

❖ Determinant of transpose matrix

- The formula may look difficult, so let's take an example
- Matrix A as follows

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & 5 & 0 \end{bmatrix} \begin{matrix} 1 & 2 \\ 2 & 3 \\ 0 & 5 \end{matrix}$$

- det A can be calculated as follows

$$\begin{aligned} \det A &= aei + bfg + cdh - ceg - bdi - afh \\ &= (1\times3\times0)+(2\times(-1)\times0)+(0\times2\times5)-(0\times3\times0)-(2\times2\times0)-(1\times(-1)\times5) \\ &= 0+0+0-0-0+5 \\ &= 5 \end{aligned}$$

Transpose matrix

❖ Determinant of transpose matrix

- Transpose matrix and determinant can be used to derive the following Transpose matrix properties

$$\det(A^T) = \det(A)$$

- For example, if the matrix A and A^T are as follows:

$$A = \begin{bmatrix} 2 & 7 \\ 5 & -3 \end{bmatrix} \qquad A^{-T} = \begin{bmatrix} 2 & 5 \\ 7 & -3 \end{bmatrix}$$

- det(A) = ad - bc = -6 - 35 = -41, det(A^T) = ad - bc = -6 - 35 = -41
- det(A) = det(A^T) is established
- Matrix expressions are mainly used to express the degree to which the linear transformation represented by the square matrix expands the volume

Transpose matrix

❖ Inverse of the transpose matrix

- If any matrix is reversible in the relationship between the Transpose matrix and the inverse matrix, then the Transpose matrix is also reversible
- The following properties are established

$$(A^T)^{-1} = (A^{-1})^T$$

Transpose matrix

- **Inverse of the transpose matrix**
  - Let's look at the proof of this as an example
    - When there is $A = \begin{bmatrix} 5 & 7 \\ -4 & 3 \end{bmatrix}$

(1) $(A^T)^{-1}$ value goes through the process twice as follows

$$A^T = \begin{bmatrix} 5 & -4 \\ 7 & 3 \end{bmatrix}$$

$$(A^T)^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{15 - (-28)}\begin{bmatrix} 3 & 4 \\ -7 & 5 \end{bmatrix} = \frac{1}{43}\begin{bmatrix} 3 & 4 \\ -7 & 5 \end{bmatrix}$$

Transpose matrix

❖ Inverse of the transpose matrix

(2) $(A^{-1})^T$ value also goes through the process twice as follows

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$= \frac{1}{15-(-28)}\begin{bmatrix} 3 & -7 \\ 4 & 5 \end{bmatrix} = \frac{1}{43}\begin{bmatrix} 3 & -7 \\ 4 & 5 \end{bmatrix}$$

$$(A^{-1})^T = \frac{1}{43}\begin{bmatrix} 3 & 4 \\ -7 & 5 \end{bmatrix}$$

$(A^T)^{-1} = (A^{-1})^T$ is established