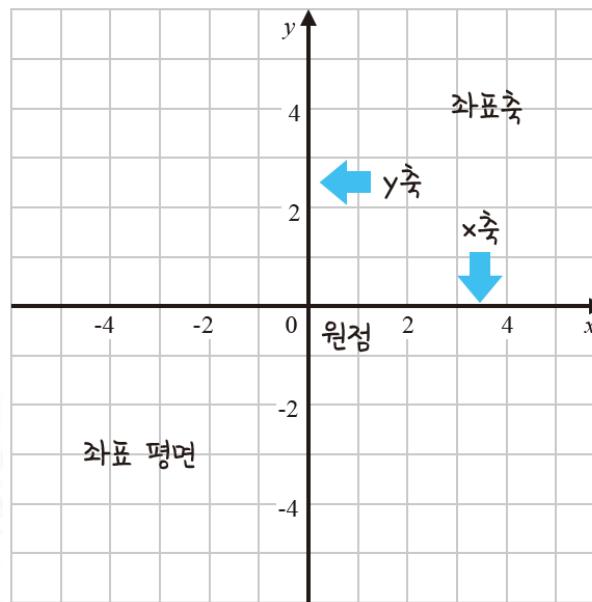


# **Straight and Slope & Exponential and Square Roots**

## ❖ Coordinate planes and quadrants

### Coordinate plane

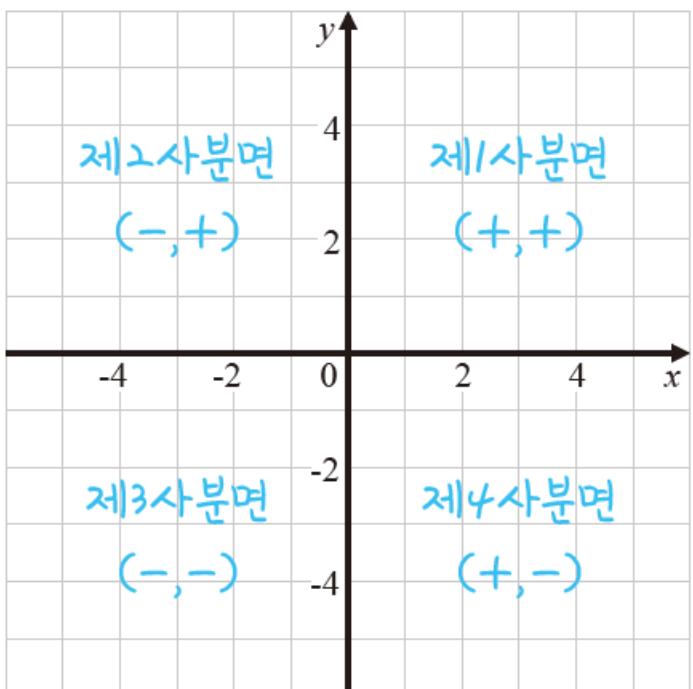
- A coordinate plane is a plane composed of two axes, the x-axis and the y-axis, and the x-axis and y-axis are collectively called the coordinate axes
- The horizontal line is called the x-axis and the vertical line is called the y-axis
- The intersection of the x-axis and y-axis is called the origin  $(0, 0)$  and is represented by the symbol O



## ❖ Coordinate planes and quadrants

### Quadrant

- The four areas that are divided into the x- and y-axis are called quadrants
- The quadrants are called the 1st, 2nd, 3rd, and 4th quadrants in counterclockwise order



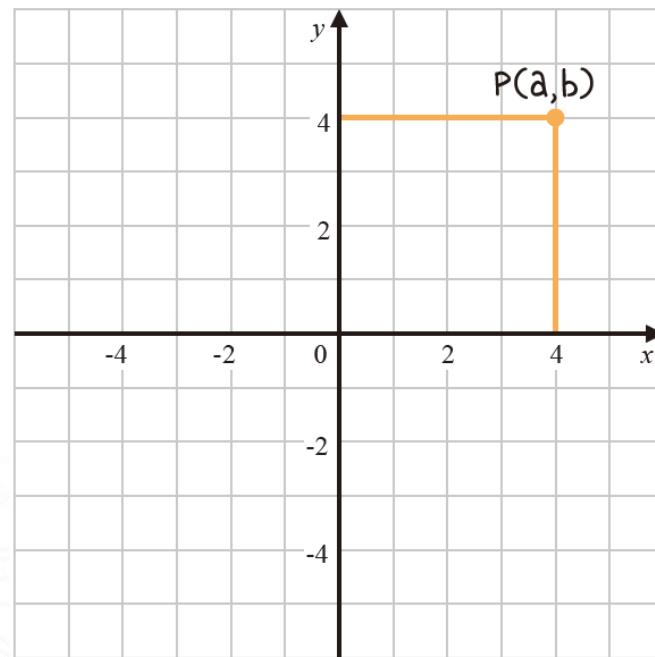
- 제1사분면:  $x > 0, y > 0$ 을 만족하는 영역
- 제2사분면:  $x < 0, y > 0$ 을 만족하는 영역
- 제3사분면:  $x < 0, y < 0$ 을 만족하는 영역
- 제4사분면:  $x > 0, y < 0$ 을 만족하는 영역

## ❖ Points above the coordinate plane

- In the coordinate plane, based on the origin  $(0, 0)$ , the x-axis is positive on the right and negative on the left
- The y-axis is positive if it is above the origin and negative if it is below the origin
  - In a vertical line, the points were only in the direction of right and left, but in the coordinate plane, the top and bottom were added, making a total of four
- In the coordinate plane, the position of the point is determined by the vertical line x-axis and y-axis, so the position is represented by symbols like  $(a, b)$
- When the point position is represented by  $(a, b)$ ,  $a, b$  are real numbers
- $a$  tells you where the dot is on the x-axis and  $b$  tells you where the dot is on the y-axis

## ❖ Points above the coordinate plane

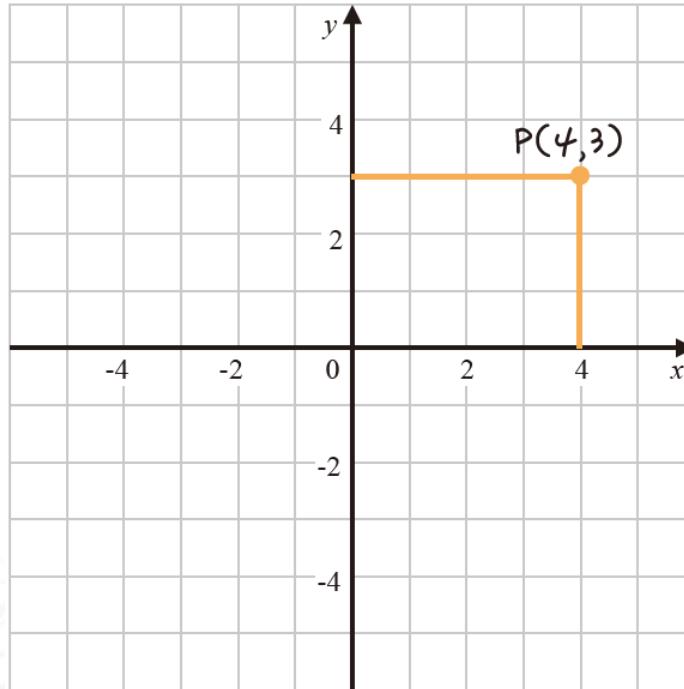
- $P(a, b)$  is a symbol indicating that the position of the point P is  $(a, b)$
- In this case, the  $(a, b)$  corresponding to the position of point P is called the coordinates of point P
- a is the x coordinate of the point P, and b is the y coordinate of the point P



## ❖ Points above the coordinate plane

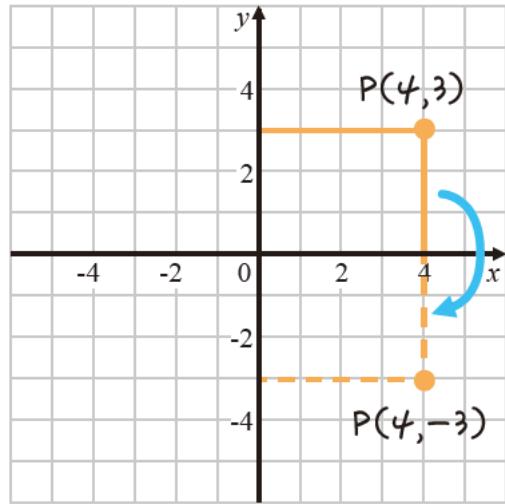
### Point symmetry over a coordinate plane

- If we mirror the x and y axes for points on the coordinate plane
- Suppose you have  $P(4, 3)$  where the x coordinate of the point P is 4 and the y coordinate is 3, as follows

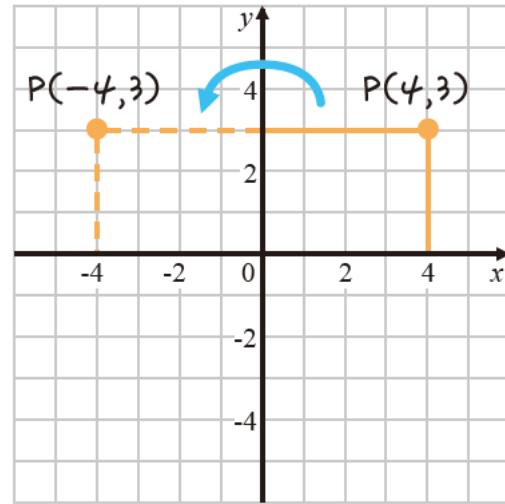


## ❖ Points above the coordinate plane

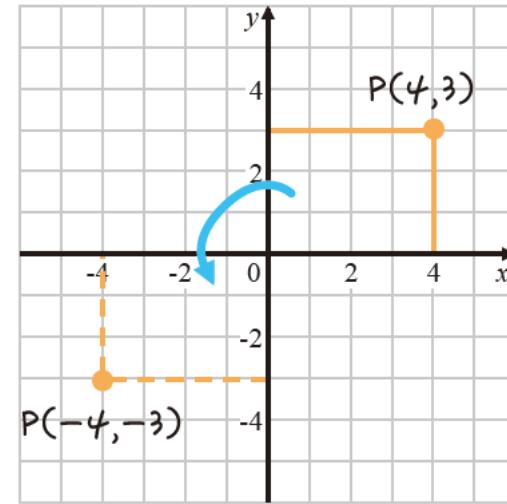
- Symmetric transposing  $P(4, 3)$  on the x-axis changes only the sign of the y-coordinates and flipping it on the y-axis reverses the sign of the x-coordinate
- When symmetric transposing on the origin, both the sign of the x coordinate and the sign of the y coordinate are reversed



① x축 대칭

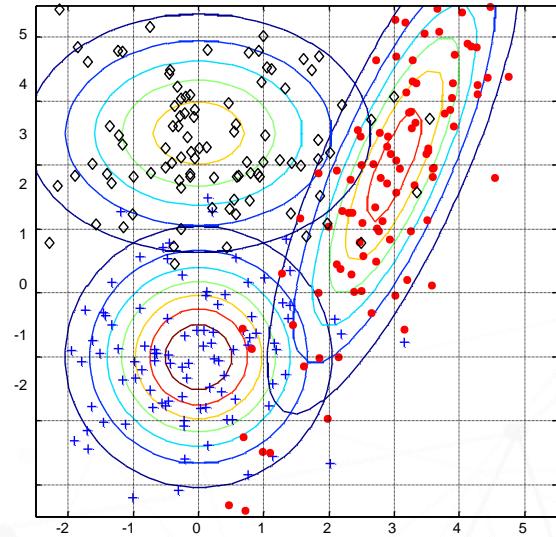
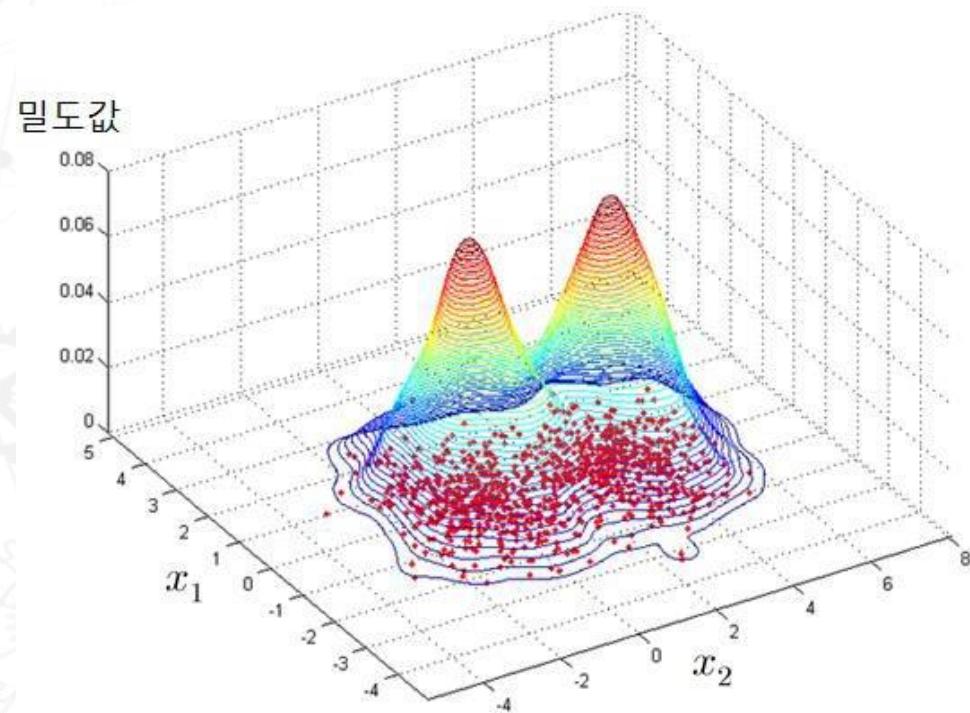


② y축 대칭



③ 원점 대칭

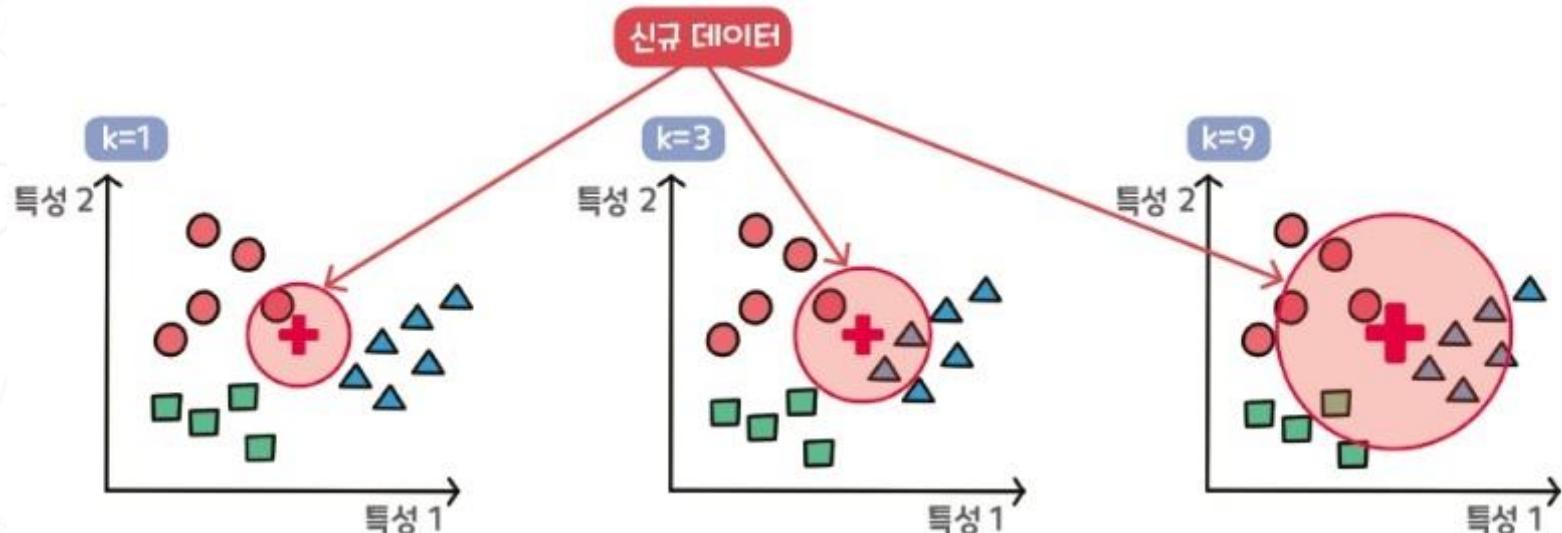
# Coordinate Plane for Data



# K-Nearest Neighbor

## ❖ Principles

- Algorithms to classify which of the existing groups of data (K groups) belongs to when new data comes in
- (Example) When new data is entered when K=1, new data is classified as a red circle, when K=3, and when K=9, it is classified as a blue triangle

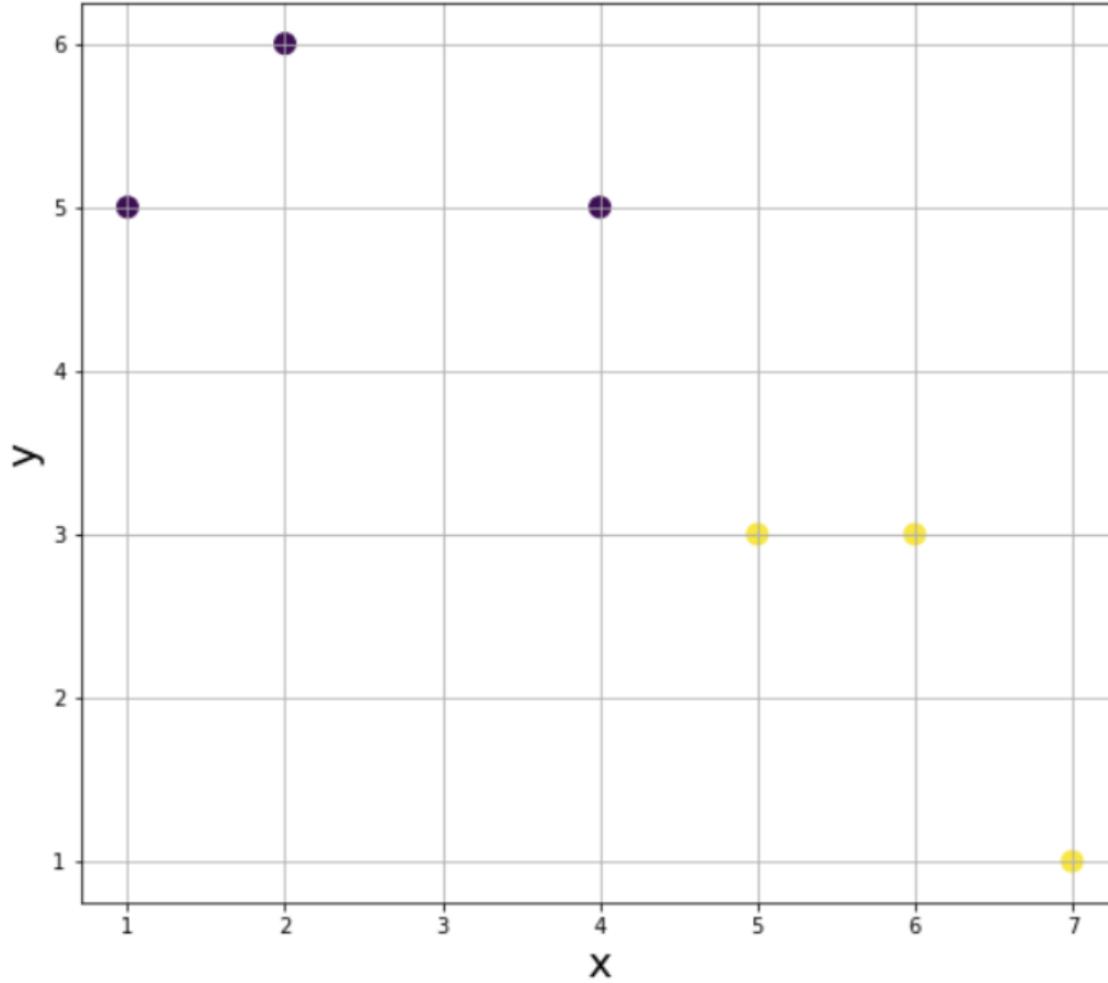


## ❖ Principles

- KNNs are not significantly affected by the noise present in the learning data and are quite effective when the number of learning data is large
- However, it is unclear which hyperparameters are suitable for analysis, so there is a disadvantage that researchers should randomly select according to each characteristic of the data

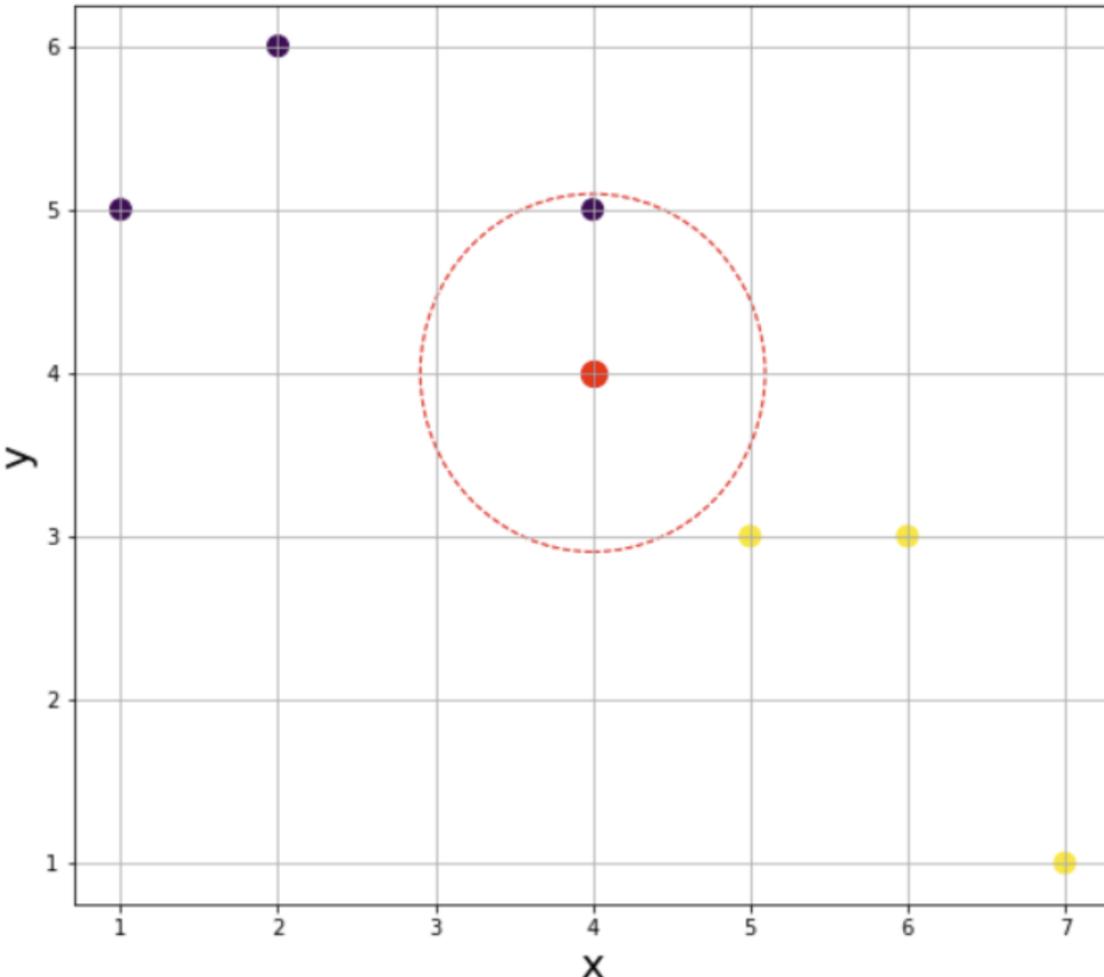
# K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



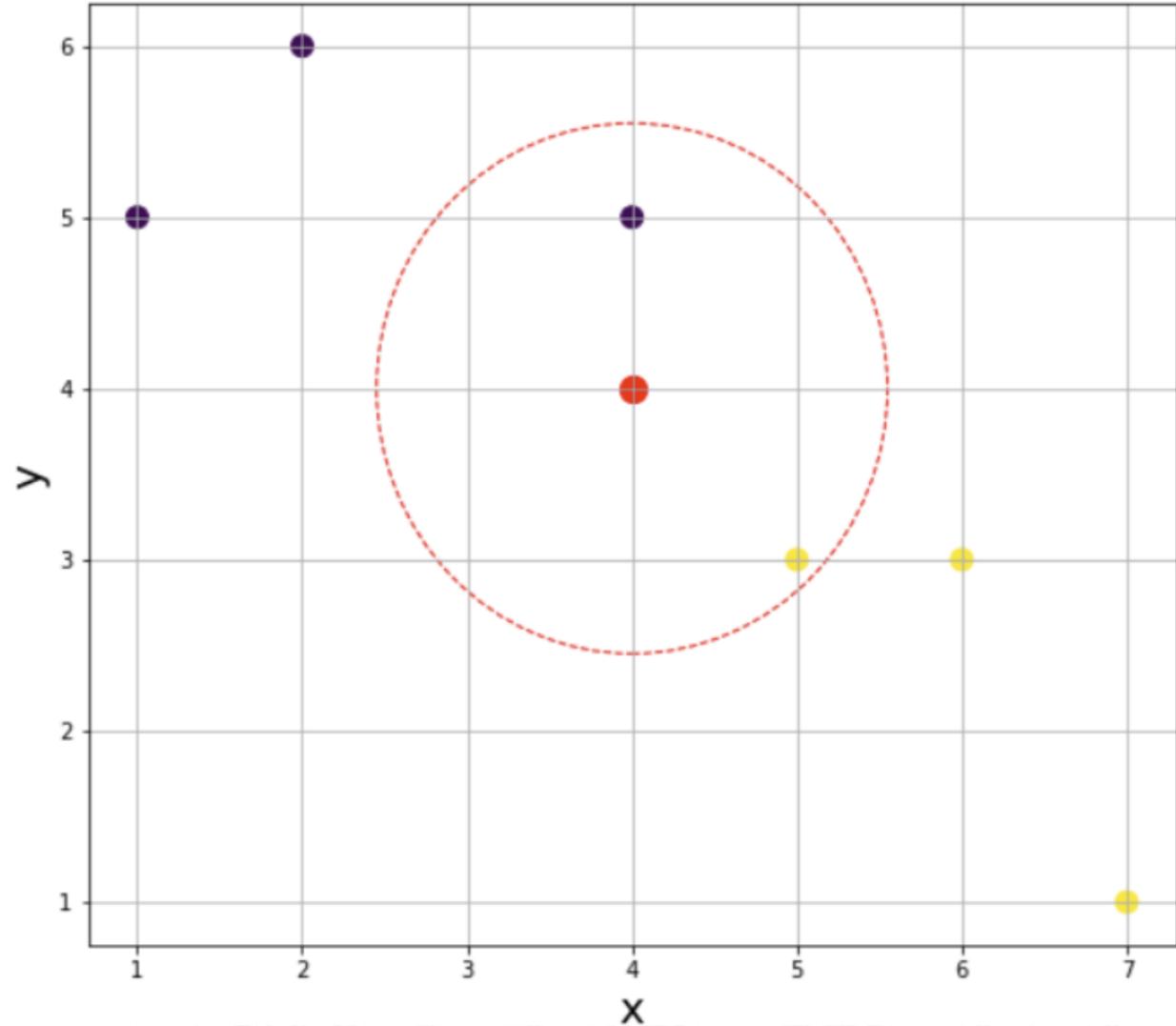
# K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
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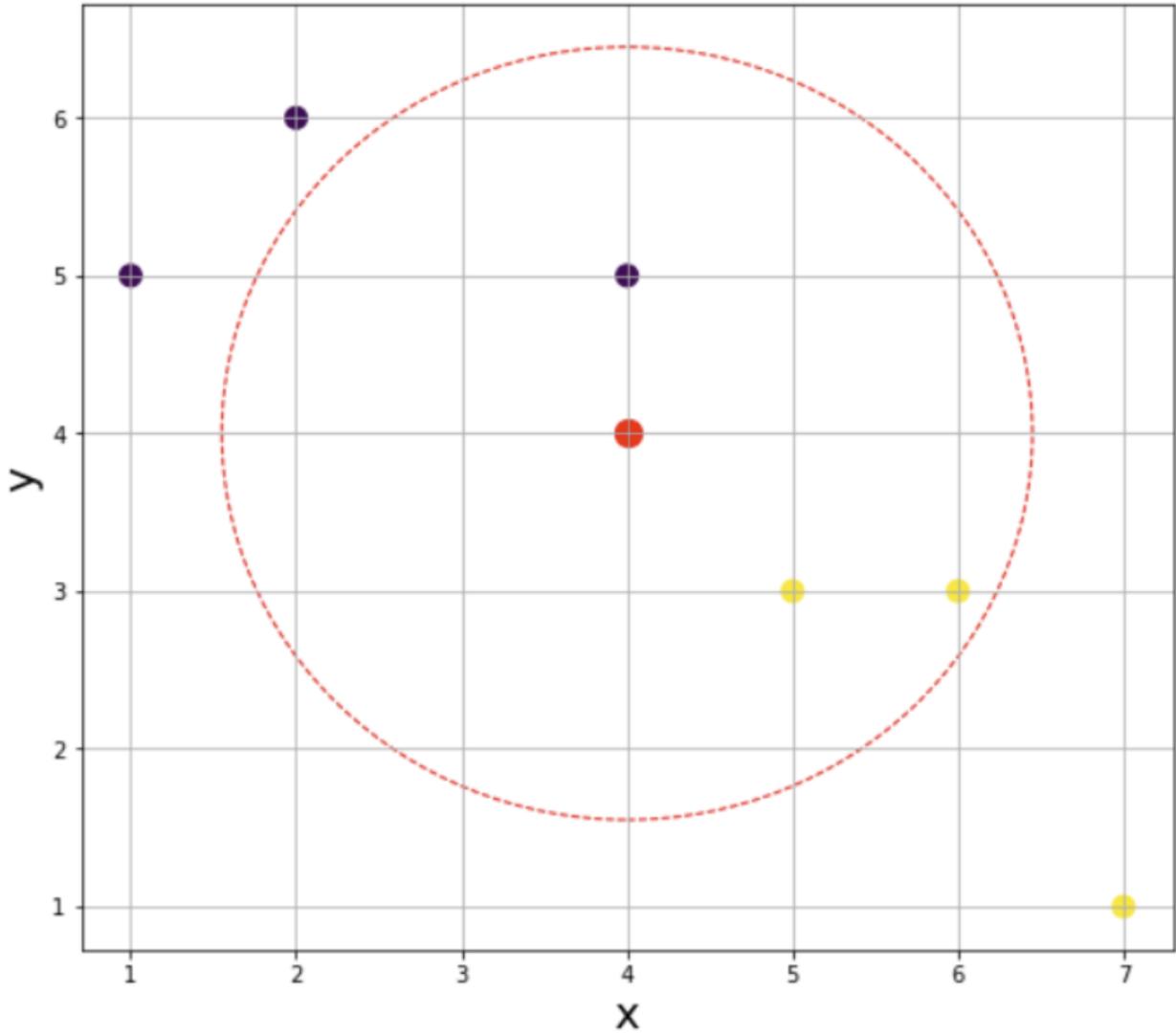
# K-Nearest Neighbor

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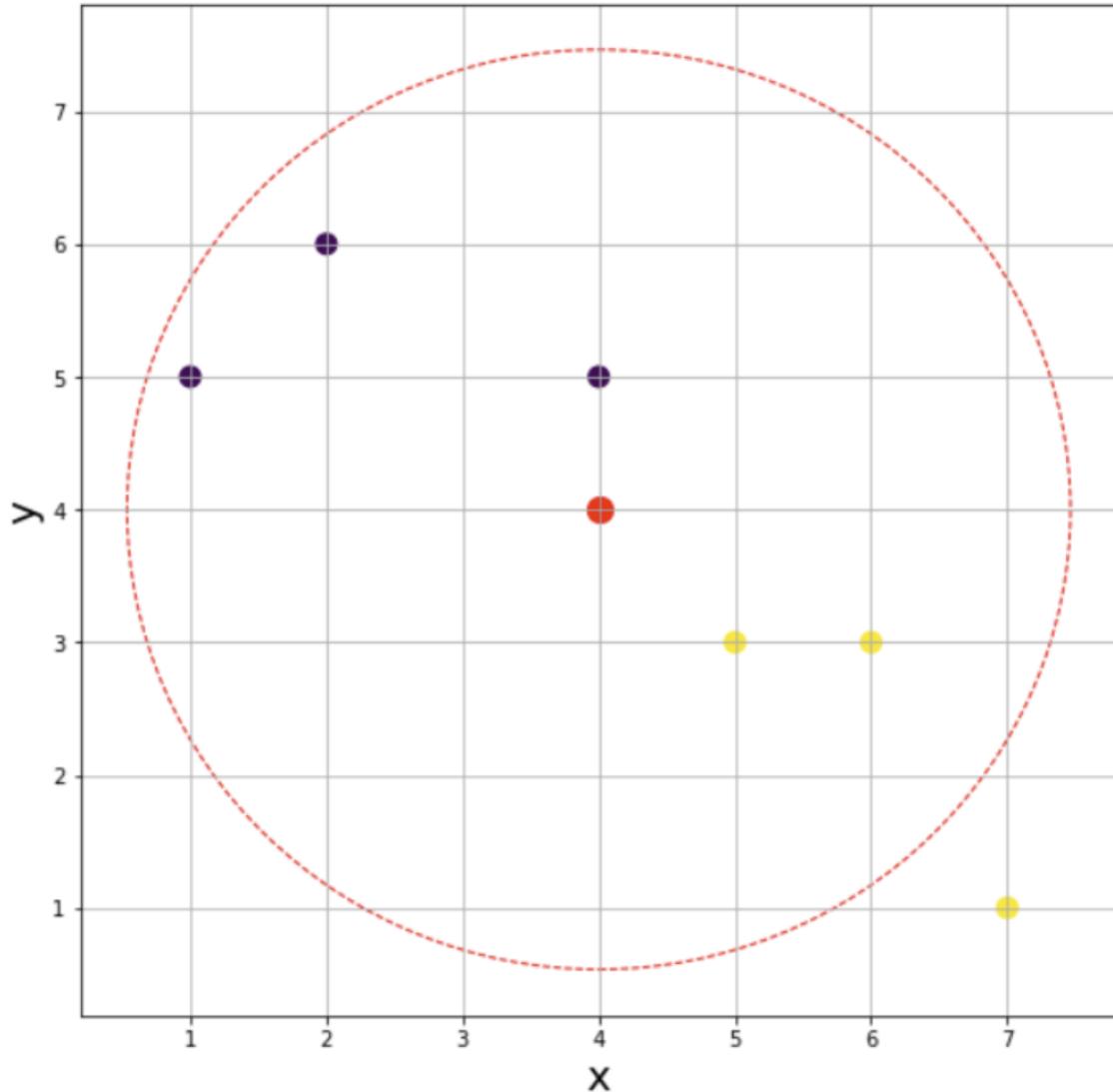
# K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



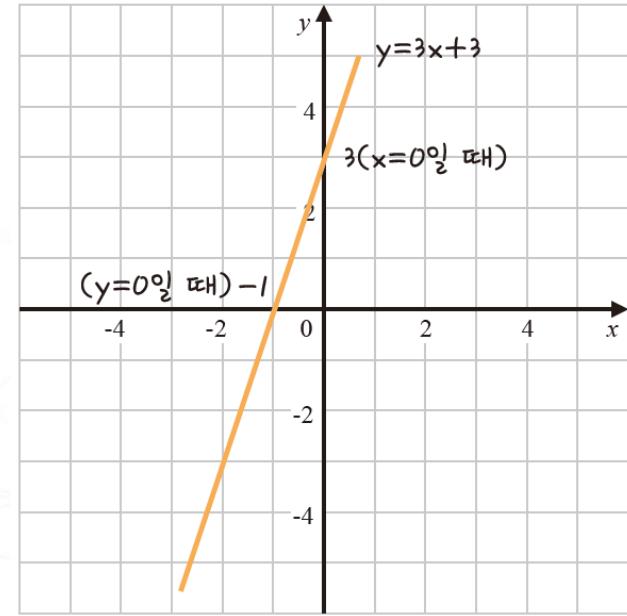
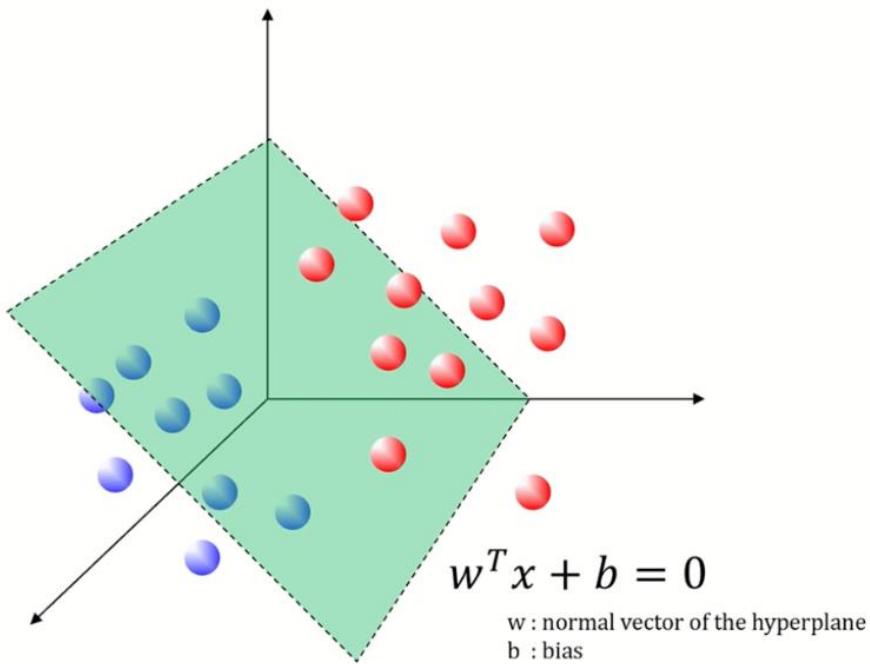
# K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



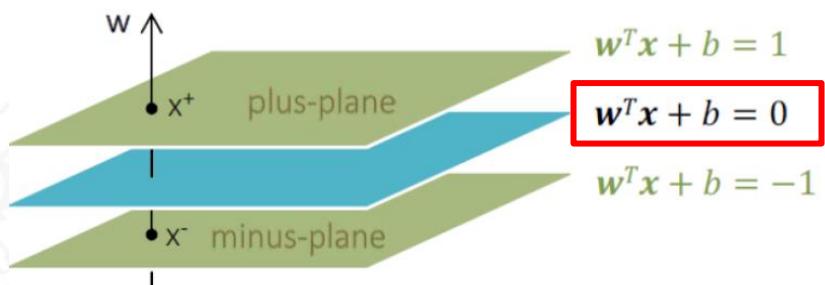
## ❖ x intercepts and y intercepts

- An intercept is a point where a straight line on a coordinate plane meets the x-axis (x-coordinate) and a point where it meets the y-axis (y-coordinate)
- The x intercept is the x coordinate of the point where it meets the x-axis, meaning the x value when  $y = 0$
- The y-intercept is the y-coordinate of the point where it meets the y-axis, meaning the y-value when  $x = 0$



## ❖ x and y intercepts

- (1) Finding the x intercept(절편)
  - The x intercept is found to be the value of x when  $y = 0$  in a given linear function.
  - If we substitute  $y = 0$  in  $y = 3x + 3$ , then  $0 = 3x + 3$ , so  $3x = -3$ , i.e.  $x = -1$
- (2) Finding the y intercept
  - The y intercept is a y value when  $x = 0$  in a given linear function
  - If we substitute  $x = 0$  in  $y = 3x + 3$ , then  $y = 3 \times 0 + 3$ , so  $y = 3$



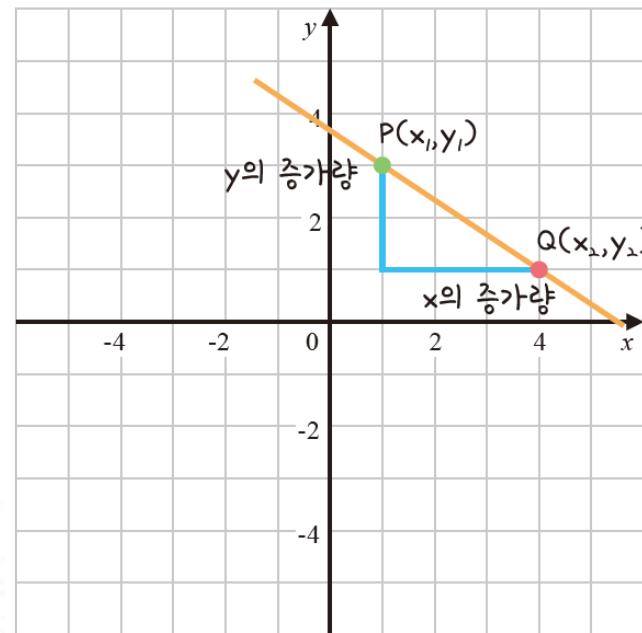
## ❖ Slope

- The slope indicates how skewed the graph is
- In this case, the degree of slope is expressed as a number, not an angle
- The formula to find the slope is:

$$\text{기울기} = \frac{y\text{의 증가량}}{x\text{의 증가량}} = \frac{y_2 - y_1}{x_2 - x_1}$$

## ❖ Slope

- Suppose there are any two points  $P(x_1, y_1)$ ,  $Q(x_2, y_2)$  in the graph
- The increase in  $x$  at two points becomes '(x coordinate of point Q) - (x coordinate of point P)', and the increase in  $y$  becomes '(y coordinate of point Q) - (y coordinate of point P)'

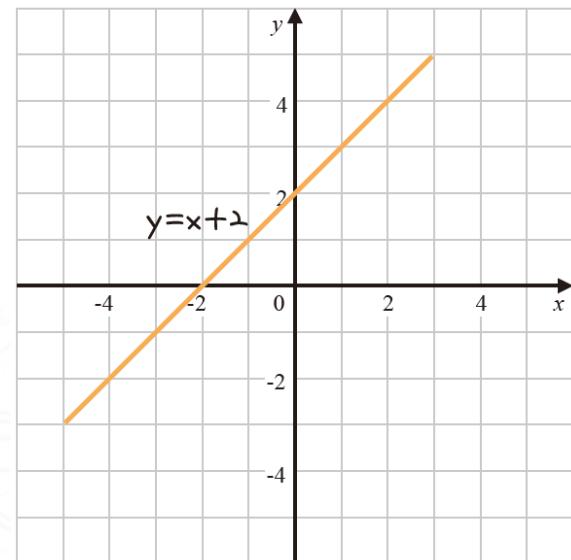


## ❖ Slope

(1) If we find the coordinates of the point where it meets the x-axis and the coordinates of the point where it meets the y-axis, we get  $(-2, 0), (0, 2)$

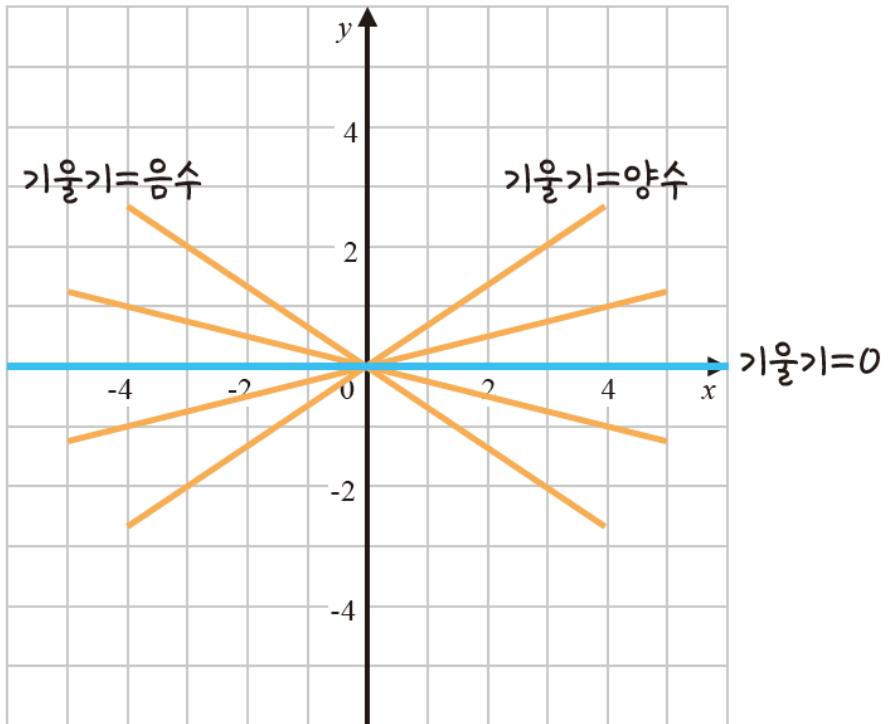
(2) Applying  $\frac{y_2 - y_1}{x_2 - x_1}$ , we get  $\frac{2 - 0}{0 - (-2)} = 1$

- Slope is 1
- Also consistent with the slope of the  $y = x + 2$  graph, which is 1



## ❖ Positive and negative slopes

- The slope is 0 if it is parallel to the x-axis, and it has a positive slope if the right end of the straight line is above the x-axis
- If the right end of a straight line is lower than the x-axis, it has a negative slope



## ❖ Positive and negative slopes

- In other words, if the value of  $y$  also increases when the value of  $x$  increases, it has a positive slope, and if the value of  $y$  decreases when the value of  $x$  increases, it has a negative slope
- If the value of  $y$  does not change when the value of  $x$  increases, the slope is 0

## ❖ Exponent

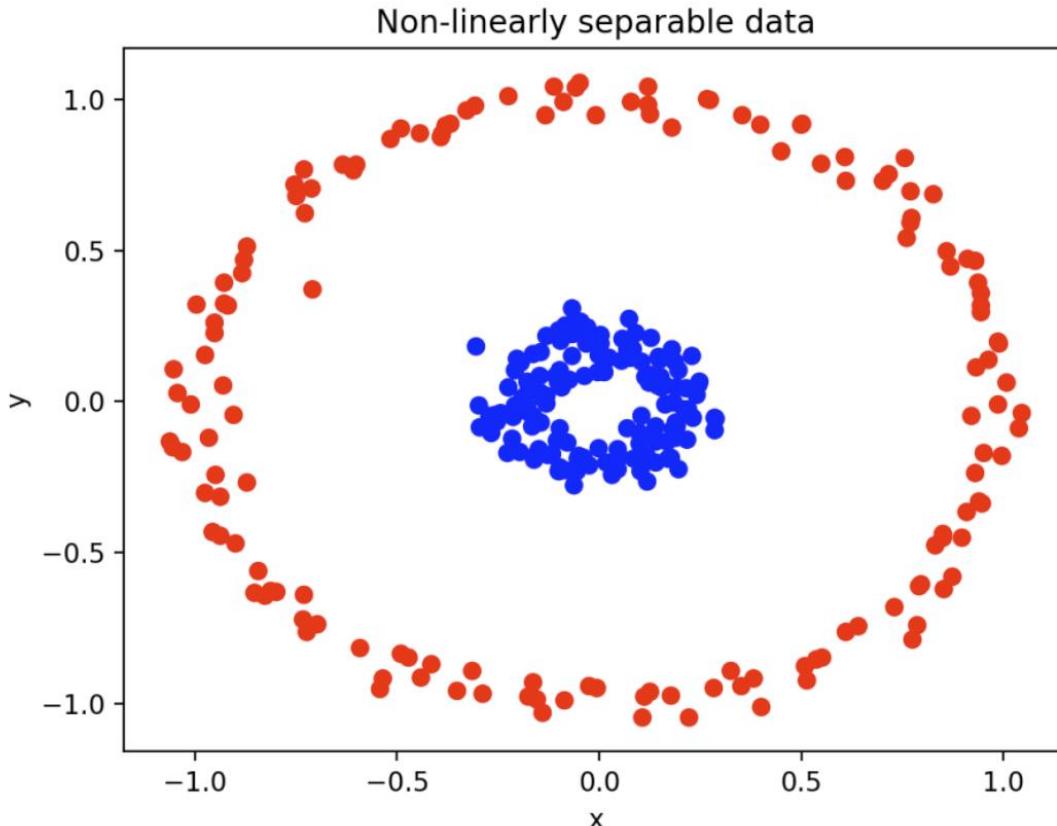
- An exponent is a letter or number that is added to the upper right of a number or letter to indicate the number of powers

$$y = a^n$$

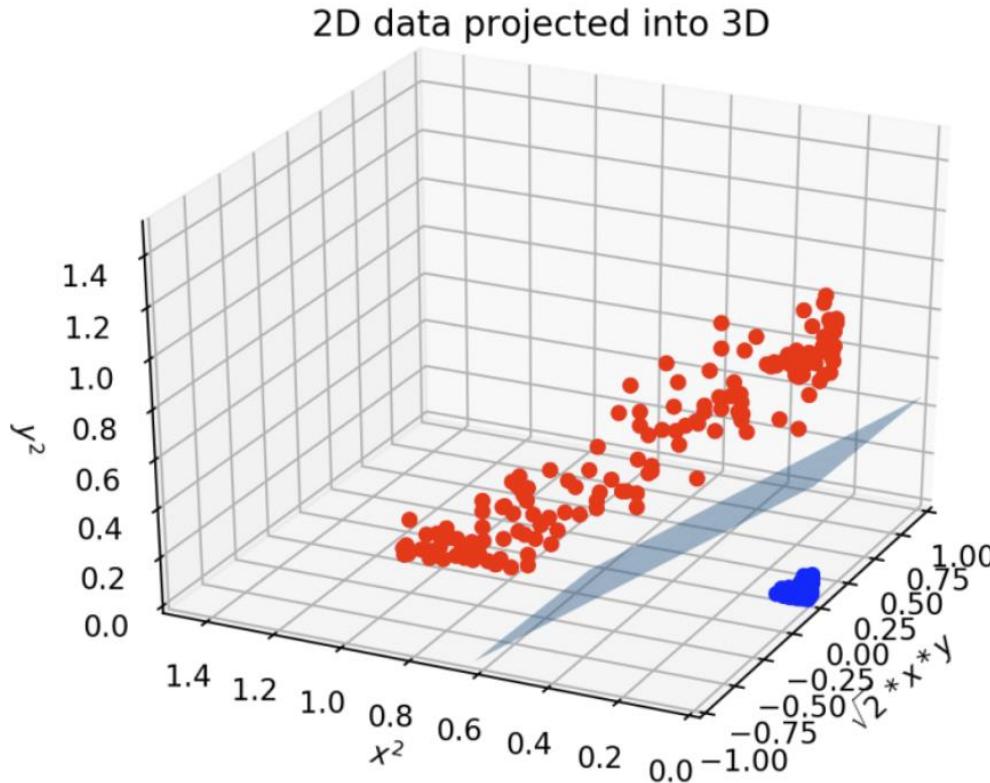
↑  
밑

← 지수

# Exponential



# Exponential



$$(x, y) \rightarrow (\sqrt{2} \cdot x \cdot y, x^2, y^2)$$

$$(1, 2) \rightarrow (2\sqrt{2}, 1, 4)$$

## ❖ The law of exponent

- The law of exponent includes the product rule, quotient rule, and the power of a power rule
- Product rule
  - When  $m = 3, n = 4$ , as:

$$(1) \text{ 지수의 합: } a^m \times a^n = a^{m+n}$$

$$a^3 \times a^4$$

$$= (a \times a \times a) \times (a \times a \times a \times a)$$

$$= a^7 = a^{3+4}$$

예시  $2^3 \times 2^5 = 2^{3+5} = 2^8$

## ❖ The law of exponent

- Quotient rule

### (2) 지수의 차

$$\textcircled{1} \quad a^m \div a^n = a^{m-n} \quad (m \geq n \text{ 일 때})$$

예시  $2^5 \div 2^3 = 2^{5-3} = 2^2$

$$\textcircled{2} \quad a^m \div a^n = \frac{1}{a^{n-m}} \quad (m < n \text{ 일 때})$$

예시  $2^3 \div 2^5 = \frac{1}{2^{5-3}} = \frac{1}{2^2}$

## ❖ The law of exponent

- Power of a power rule

(3) 지수의 곱:  $(a^m)^n = a^{mn}$  ( $m, n$ 은 양의 정수)

$$(a^2)^3$$

$$= a^2 \times a^2 \times a^2$$

$$= a \times a \times a \times a \times a \times a$$

$$= a^6 = a^{2 \times 3}$$

예시  $(2^4)^6 = 2^{4 \times 6} = 2^{24}$

# Power and power root

## ❖ Power

- A power is a calculation that multiplies the same number several times
- For example,  $5 \times 5 \times 5 \times 5 \times 5$  is expressed as  $5^5$  and read as 5 to the power of 5
- Common expressions include:

$$a^n = \underbrace{a \times a \times a \times \cdots \times a}_{n \text{번}}$$

## ❖ Power

- Example

$(2 \times 2) 2^2 \rightarrow 2$  squared (when you multiply twice, it just reads as squared)

$(2 \times 2 \times 2 \times 2 \times 2) 2^5 \rightarrow 2$  to the power of 5

$(2 \times 2 \times 2 \times 2 \times 2 \times 2) 2^6 \rightarrow 2$  to the power of 6

$(a \times a \times a \times a \times a \times a \times a) a^7 \rightarrow a$  to the powers of 7

## ❖ Power

- There are caveats to this
  - (1) Like Equation 4.1, it can only be used with the same numbers and letters

$$\frac{3 \times 3 \times 3}{3^3} \times \frac{6 \times 6}{6^2} \times \frac{9}{9}, \quad \frac{a \times a}{a^2} \times \frac{b \times b \times b \times b}{b^4}$$

수식 4.1

- In Equation 4.1, you can see that only the same numbers or letters are grouped together and expressed as a power  
(if the numbers or letters are different, they cannot be grouped together)
  - (2) Be careful when calculating fractions

# Power and power root

## ❖ Power

- Use parentheses when expressing fractions as powers as follows
- Not to use parentheses because it can give the wrong answer

$$\frac{1}{2} \times \frac{1}{2} = \frac{1^2}{2} = \frac{1}{2}$$

괄호를 사용하지 않은 예시

$$\frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

괄호를 사용한 예시

 Power

- Power has three characteristics
  - 0 and 1, when a negative number squared, is characterized by:

The zero squared of a non-zero real number is always 1 :  $a^0 = 1$

예시  $10^0 = 1$ ,  $1000^0 = 1$

1 square of a non-zero real number equals the real number :  $a^1 = a$

예시  $10^1 = 10$ ,  $1000^1 = 1000$

The negative square of a non-zero real number is equal to :  $a^{-n} = \frac{1}{a^n}$        $\frac{1}{\text{실수의 양의 제곱}}$

**예시**  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

# Power and power root

## ❖ Power root

- Square and square root are opposite concepts
- In the quadratic equation  $x^2 = a$ , a is called the square of x, and x is called the square root of 'a'
- 4 is a power of 2, and 2 is a square root of 4

$$(\pm 2)^2 \quad \begin{array}{c} \xrightarrow{\text{제곱}} \\ \xleftarrow{\text{제곱근}} \end{array} \quad 4$$

## ❖ Power root

- There are two square roots, a positive square root and a negative square root, and the positive one is called the positive square root, and the negative one is called the negative square root
- -2 is the negative square root of 4, and 2 is the positive square root of 4

# Power and power root

## ❖ Power root

- In  $x^2 = a$ , the positive  $x$  is the square root of  $a$ , so in  $x^n = a$ ,  $a$  is the power of  $x$ , and  $x$  is the power root of  $a$  (same as the square root principle of quadratic equations)

$$(\pm 2)^3 \xrightleftharpoons[\text{거듭제곱근}]{\text{거듭제곱}} 8$$

## ❖ Power root

### Power root properties

- The most important concept in the property of the power root is the root ( $\sqrt{\cdot}$ )
- A root is 'to square a certain number so that the number in the root comes out'

$$2^2 = 4, \sqrt{4} = \pm 2$$

# Power and power root

## ❖ Power root

- The properties of the power root are as follows:

$\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$ : (1) If the power root is equal, the multiplication can be grouped into one power root

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$ : (2) If the power root is equal, the division can be grouped into one power root

$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$ : (3) The power can go into the power root

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$ : (4) Multiplying by power roots

$\sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$ : (5) The power root and the power are reducible

## ❖ Power root

- **Reduction of fraction**
  - Simple by dividing the denominator and numerator of a fraction by a common divisor
  - In other words, the denominator and numerator are no longer separated from each other
- AI does not directly solve using the properties of power root

# Power and power root

## ❖ Power root

- To calculate the power root in Python, use the math library to implement

In [9]:

```
# 거듭제곱의 표현은 **으로 합니다  
2**5
```

32

In [10]:

```
# math.sqrt() 함수를 사용하여 거듭제곱근을 구합니다  
import math  
math.sqrt(2)
```

1.4142135623730951

## ❖ Power root

In [11]:

```
math.sqrt(9)
```

3.0

## ❖ Factorization

- Factorization is the process of expressing complex expressions as products by grouping them into common factors
- In other words, the purpose of factorization is to decompose the equation into more basic and simpler pieces

$$mx + my = m(x+y), \quad mx - my = m(x-y)$$

T공통인수T      T공통인수T

Data -> pre-processing -> labeling, noise reduction,outliner detection,

FE + CLS  
regression

## ❖ Factorization

- Another definition is the expression of a polynomial in the form of a product of a monomial and a polynomial, or a product of multiple polynomials, called factorization
- A monomial or polynomial multiplied is called a factor of the first expression

$$x^2 + 6x + 5 \quad \begin{array}{c} \xrightarrow{\text{인수분해}} \\ \xleftarrow{\text{전개}} \end{array} \quad \begin{array}{c} (x+1)(x+5) \\ \hline \text{인수} \quad \text{인수} \end{array}$$

## ❖ Factorization

- **Expansion** is the modification of an expression in the form of a product into the form of a sum
- Expressing it in the form of a monomial sum

### 인수분해 공식

$$(1) x^2 + 2xy + y^2 = (x + y)^2$$

$$(2) x^2 - 2xy + y^2 = (x - y)^2$$

$$(3) x^2 - y^2 = (x + y)(x - y)$$

$$(4) x^2 + (a + b)x + ab = (x + a)(x + b)$$

Data -> pre-processing -> Training -> classification

Training      model training      .

- For reference, an expression in the square of a polynomial or an expression that is multiplied by a constant, i.e., an expression that can be expressed in the form of  $(ax + b)^2$  or  $k(ax + b)^2$  (where  $k$  is a constant) is called a perfect square expression

## ❖ Factorization

- Use the Sympy library to implement in Python as follows

In [12]:

```
# 파이썬 SymPy의 expand, factor, Symbol을 호출하고 기호변수 x를 선언합니다
from sympy import expand, factor, Symbol
x = Symbol('x')
```

In [13]:

```
# expand()는 수식을 (x + 1) * (x + 5)로 전개합니다
expand((x + 1) * (x + 5))
```

$x^2+6x+5$

## ❖ Factorization

In [14]:

```
# factor()는 인수분해하는 함수로, x2 + 6x + 5를 인수분해합니다
```

```
factor(x**2 + 6*x + 5)
```

$(x+1)(x+5)$

## ❖ Factorization

### 연습 문제

(1)  $x^3y - xy^3$ 을 인수분해하세요.

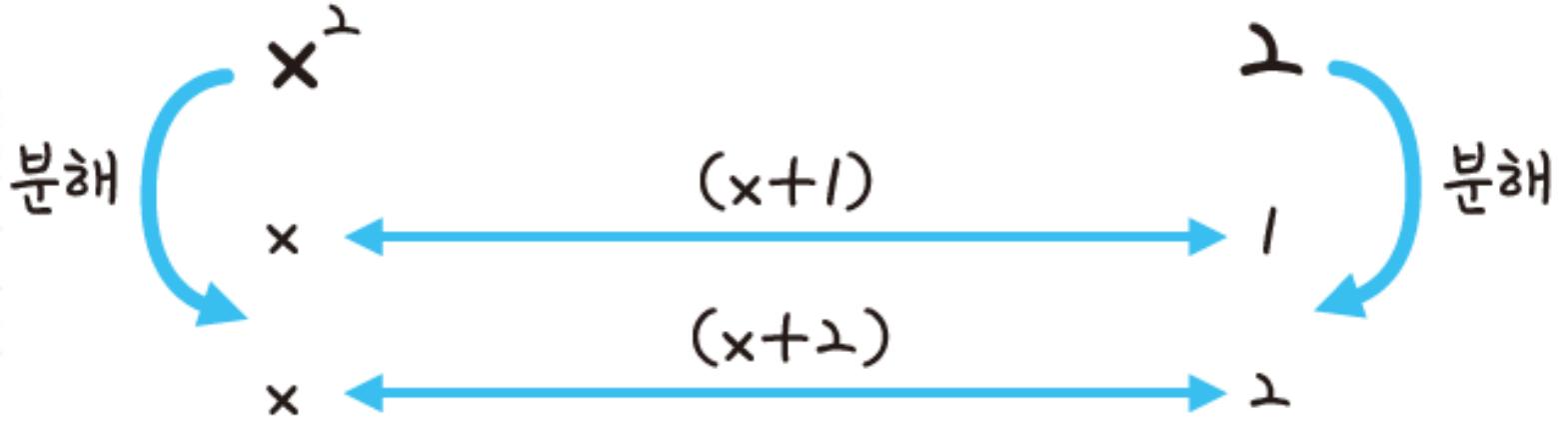
(2)  $x^2 + 3x + 2$ 를 인수분해하세요.

### 문제 풀이

(1)  $x^3y - xy^3$ 을 공통인수  $xy$ 로 묶어 주면  $xy(x^2 - y^2)$ 이 됩니다. 인수분해 공식 (3)을 적용하면  $xy(x + y)(x - y)$ 가 됩니다.

(2)  $x^2 + 3x + 2$ 에는 공통인수가 없으므로 식을 간단히 만들면 다음과 같습니다.

$x^2 + 3x + 2$  식에서



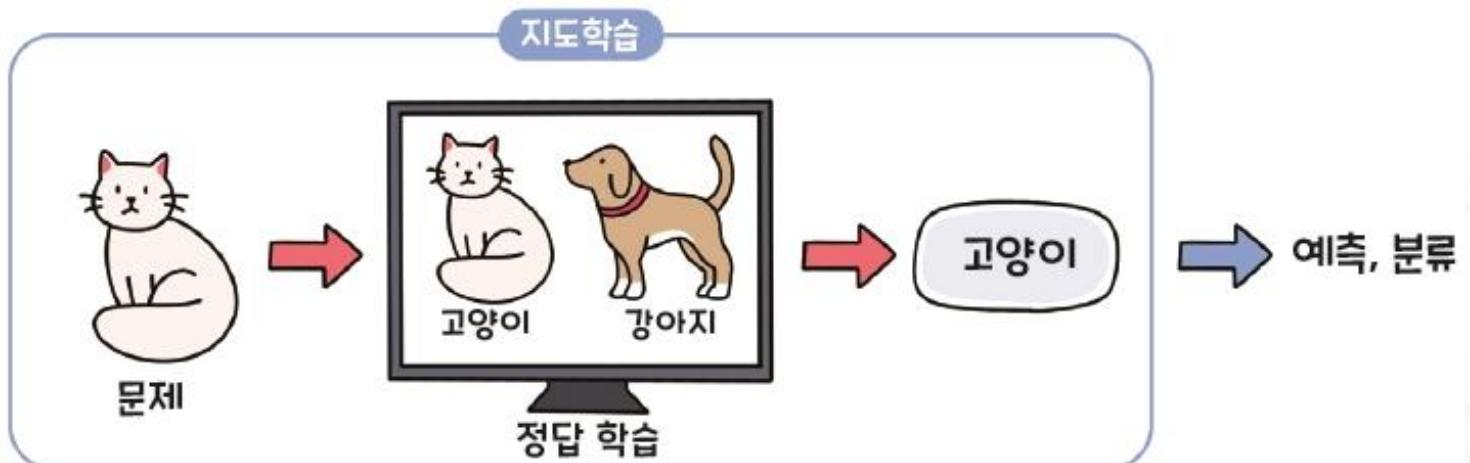
## ❖ Categorization for AI training

- Supervised learning : classification and regression      $x = \text{data}$       $y = \text{class label}$
- Unsupervised learning : clustering
- Reinforcement learning : use rewards for actions taken in the environment to conduct learning



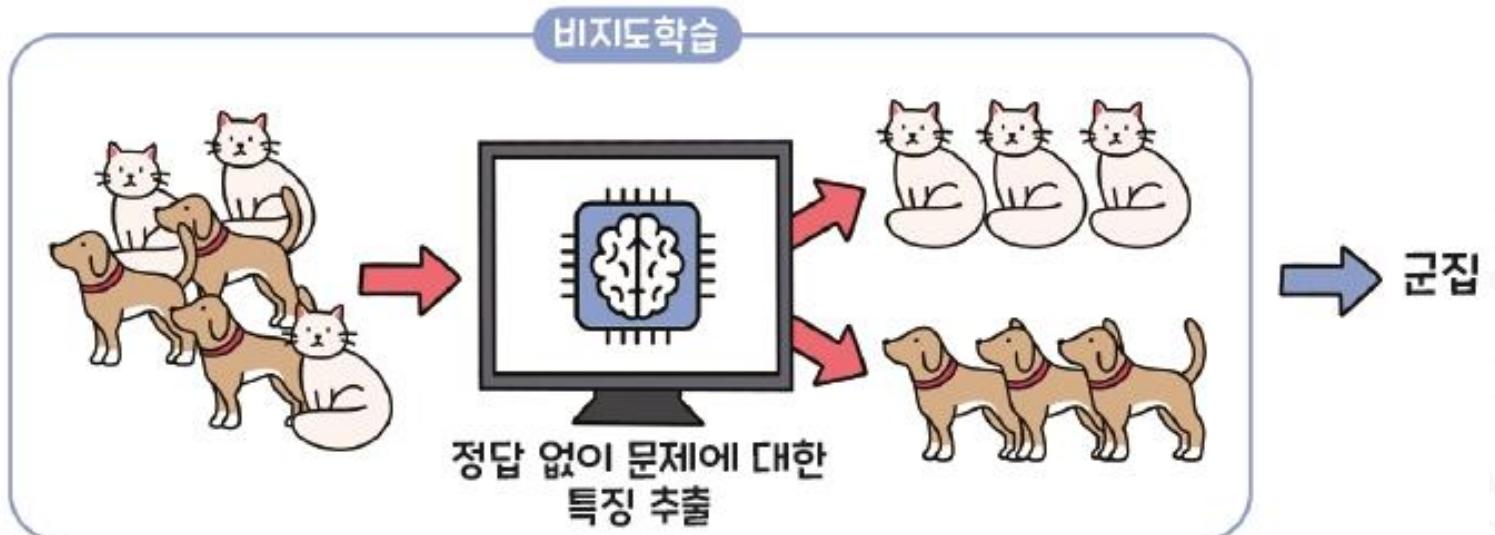
## ❖ Supervised learning

- Learning to predict the right answer to an unknown problem by learning questions and answers together
- The models used in supervised learning include prediction and classification



## ❖ Unsupervised learning

- A form of computer learning without the help
- Computer uses training data to find regularity between data



## ❖ Unsupervised learning

- Unlike supervised learning, which identified the relationship between  $x$  (input data) and  $y$  (labels in supervised learning),
- Unsupervised learning identifies the relationship between  $x$  by itself
- In other words, the difference between  $y$  (label)
  - Clustering is a model used in unsupervised learning

구분	지도학습	비지도학습
필요한 데이터 종류	$x$ (학습 데이터), $y$ (레이블)	$x$ (학습 데이터)

## ❖ Reinforcement learning

- Learning to be rewarded for what you've done
- How computers learn to choose the best behavior for a given state



## ❖ Reinforcement learning

- Agent: Subject to act in a given problem situation
- State : Current situation
- Action: Options that the player can take
- Rewards: Benefits that follow when a player does something
- Environment: means the problem itself
- Observation : Information about the collected by the agent

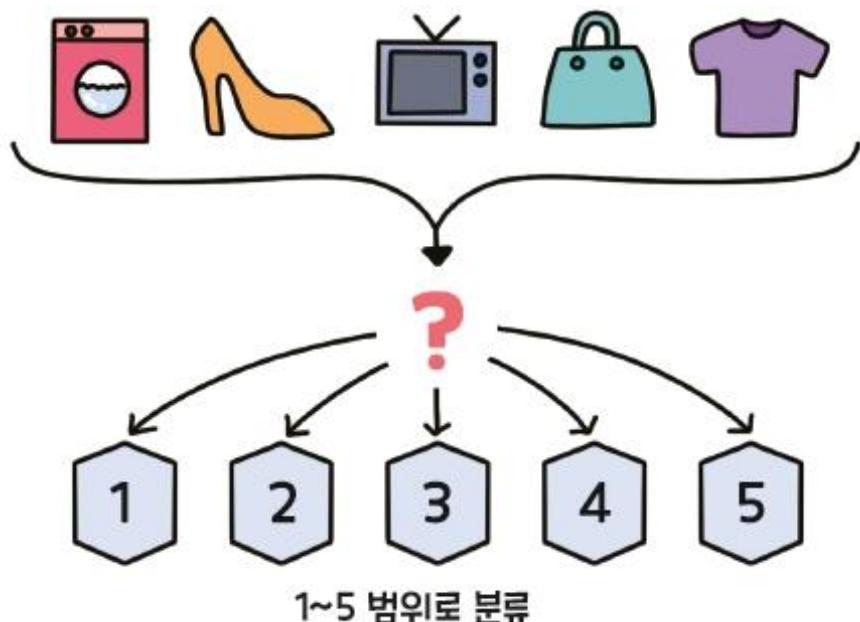
## ❖ Reinforcement learning

- Depending on the behavior chosen by the agent in a given environment, you are rewarded if the behavior is the right choice, and punished if the behavior is the wrong choice
- Reinforcement learning allows the agent to keep an eye on the status and learn (behavior) toward higher rewards



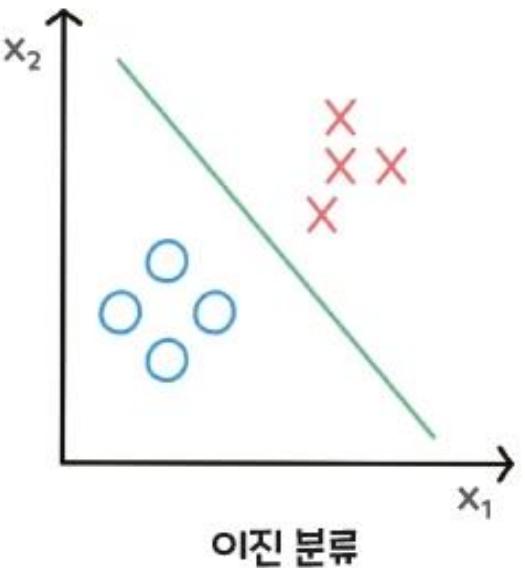
## ❖ Classification

- A technique for learning labeled data, classifying data with similar properties, and finding out which group the newly entered data

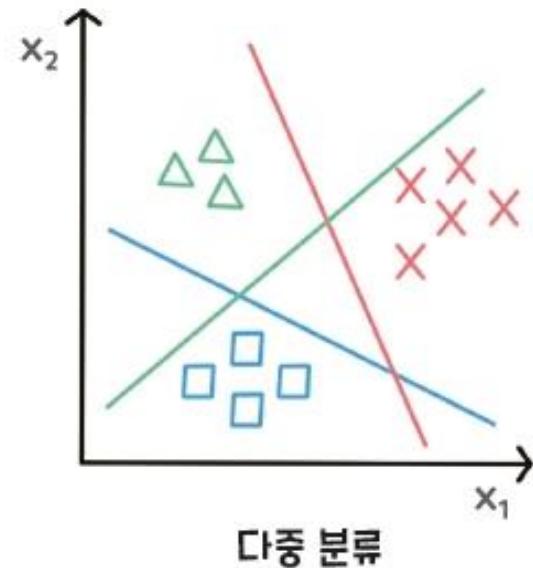


## ❖ Classification

- Binary classification : categorize data into 2 groups
- Multiclass classification : categorize data into 3 or more groups



이진 분류



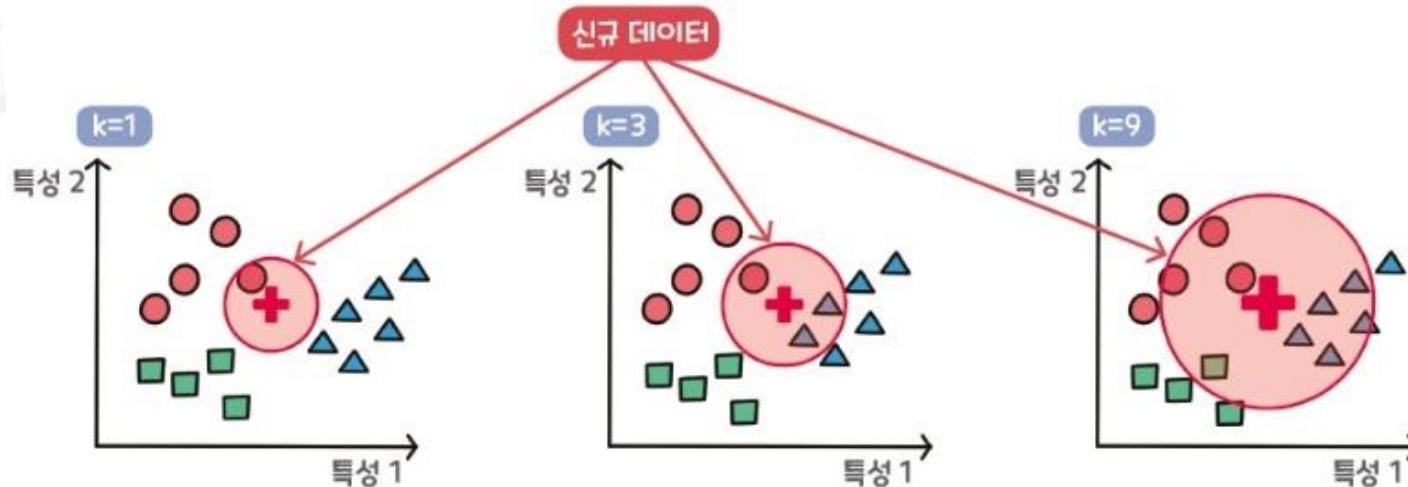
다중 분류

# Types of Machine Learning Algorithm

## ❖ Classification (Algorithm)

### ▪ K-neighbor nearest

- Algorithms to classify which of the existing groups of data (K groups) belongs to when new data comes in
- (Example) When new data is entered when  $K=1$ , new data is classified as a red circle, when  $K=3$ , and when  $K=9$ , it is classified as a blue triangle

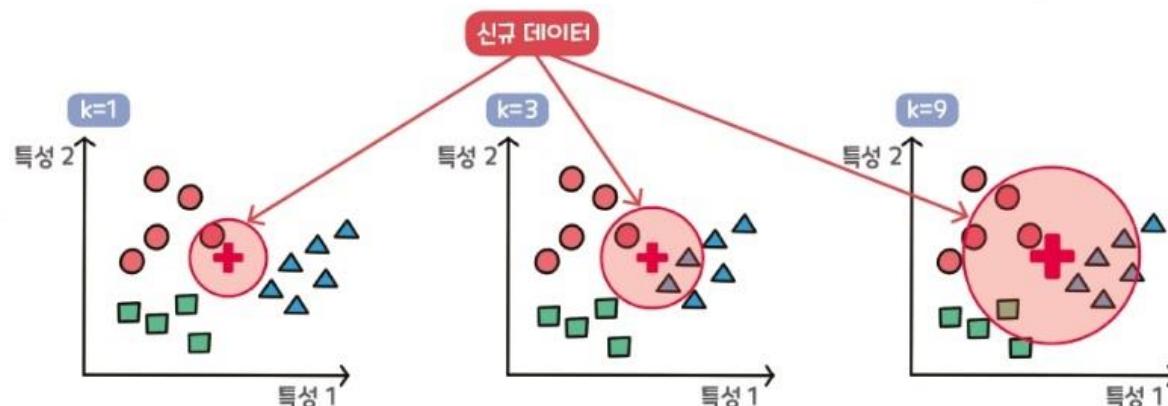


# Types of Machine Learning Algorithm

## ❖ Classification (Algorithm)

### ▪ K-neighbor nearest

- KNNs are not significantly affected by the noise present in the learning data and are quite effective when the number of learning data is large
- However, it is unclear which hyperparameters are suitable for analysis, so there is a disadvantage that researchers should randomly select according to each characteristic of the data

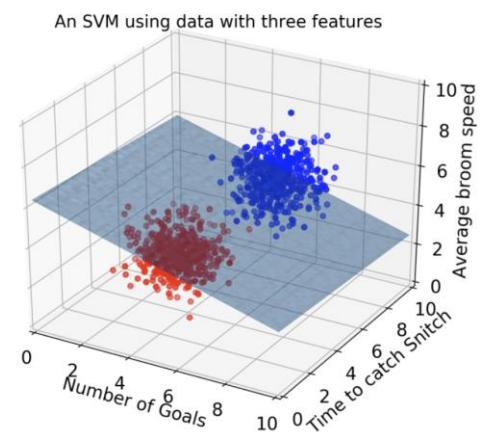
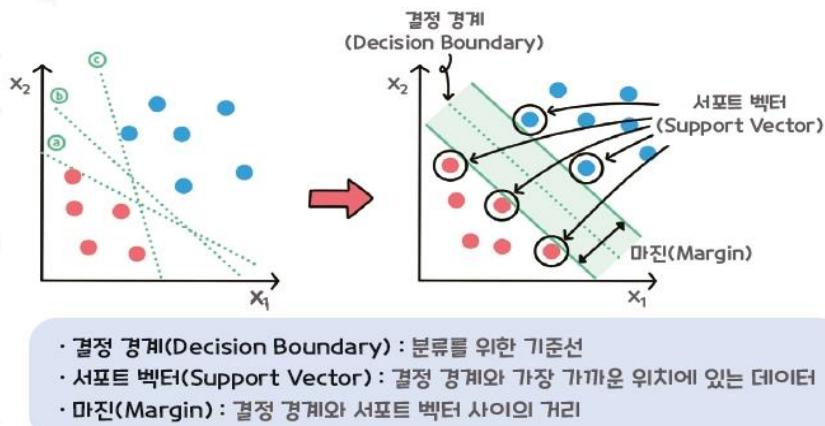


# Types of Machine Learning Algorithm

## ❖ Classification (Algorithm)

### ▪ Support vector machine

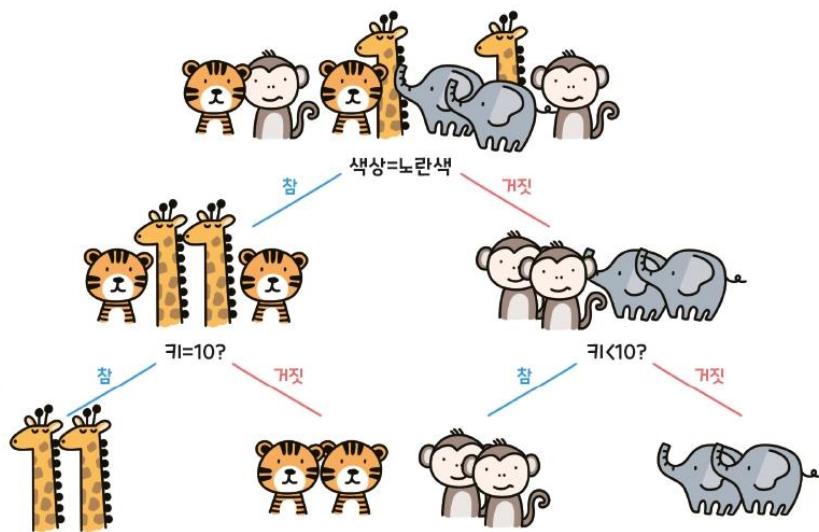
- Categorize data in the direction of maximizing margin, which means margin between two categories
- SVMs find and classify lines that maximize margins, so larger margins are more likely to be classified even if new data comes in
- SVM is easy to use and highly predictive
  - However, it takes time to build a model and the results are less descriptive



## ❖ Classification (Algorithm)

### ▪ Decision tree

- An analysis method for classifying decision-making rules into tree forms
- It is called 'decision tree' because the method of starting from the upper node and expanding to the lower node according to the classification criteria resembles 'tree'



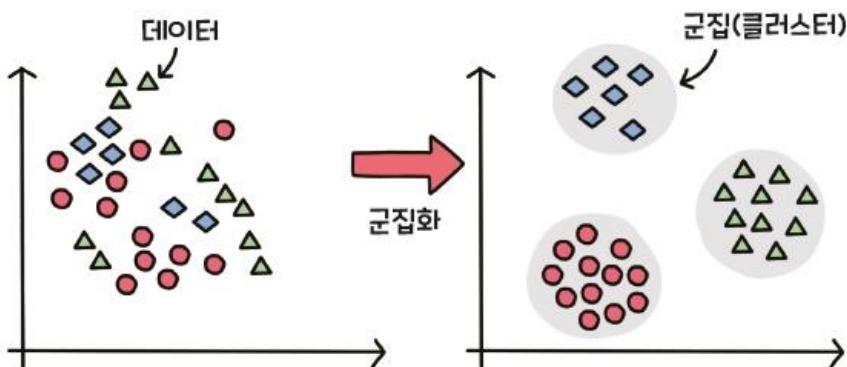
## ❖ Classification (Algorithm)

- Decision tree

- Decision Tree is intuitive and easy to understand the analysis process
- In the case of artificial neural networks, it is a black box model that is difficult to explain the analysis results, while decision trees can observe the analysis process with their eyes
- Need for a clear explanation of the results

## ❖ Unsupervised learning

- Cluster
  - A group of data with similar characteristics
- Clustering
  - Classifying the data into clusters according to a similar degree when given the data
  - Various data are mixed together, but the clustering process groups similar data as shown in the graph on the right

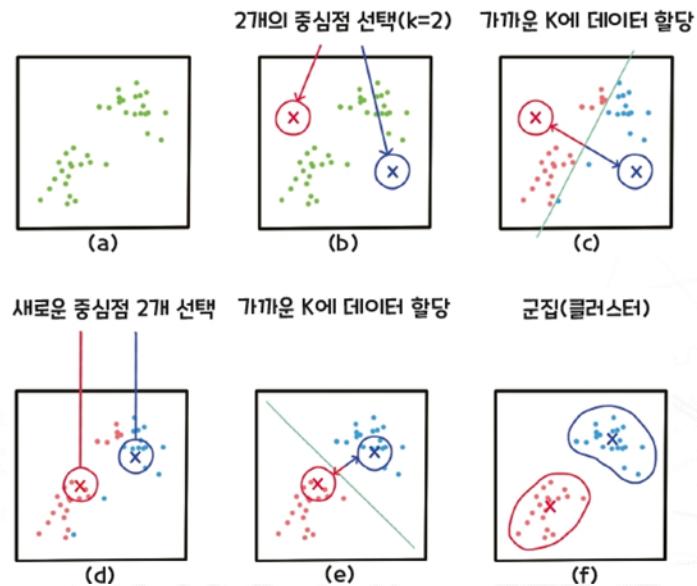


# Types of Machine Learning Algorithm

## ❖ Unsupervised learning

### ▪ K-means clustering

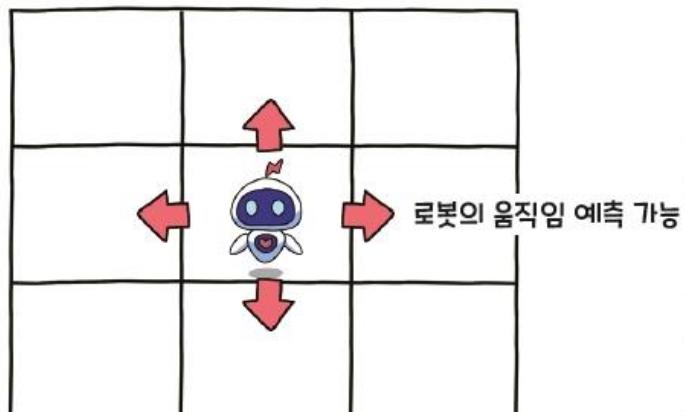
- 'K' is the number of groups to be grouped from the given data
- 'Means' means the average distance between the center of each cluster and the data
- The center of the cluster is called centroids



## ❖ Reinforcement learning

### ▪ Algorithm

- Model-based algorithms refer to the probability that an action in the current state will result in the next state
- Intuitive visibility of the robot's next state as it moves up, down, left, and right in a grid space
  - Model-based algorithms can predict changes in state according to behavior, resulting in optimal solutions



# Assignment #1

❖ 1-(1) 다음 연립방정식의 해를 python을 활용하여 구하여라

- $2x+3y=6$
- $3x+2y=12$

❖ 1-(2) 다음 연립방정식의 해를 python을 활용하여 구하여라

- $x+y+2z=9$
- $2x+4y-3z=1$
- $3x+6y-5z=0$

❖ Python을 활용하여 답을 구하여라

- 2-(1) 두 점  $(1, -2)$   $(3, 0)$ 을 지나는 직선의 방정식은?
- 2-(2) 두 점  $(3, 5)$   $(0, -4)$ 을 지나는 직선의 방정식은?
- 2-(3) 기울기가 3이고 점  $(5, 4)$ 를 지나는 직선의 방정식은?
- 2-(4) 기울기가 -2이고 점  $(3, 2)$ 를 지나는 직선의 방정식은?

# Assignment #3

❖ 다음 식을 python을 활용하여 인수분해 하여라

- $3xy - 6y^2 - x + 2y$
- $9x^2 - 4y^2 + 16y - 16$
- $x^2 + xy - x - 2y - 2$