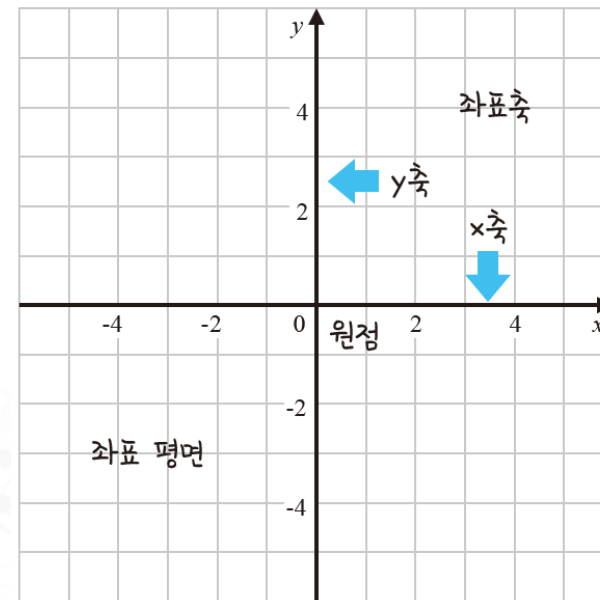


Straight and Slope & Exponential and Square Roots

❖ Coordinate planes and quadrants

Coordinate plane

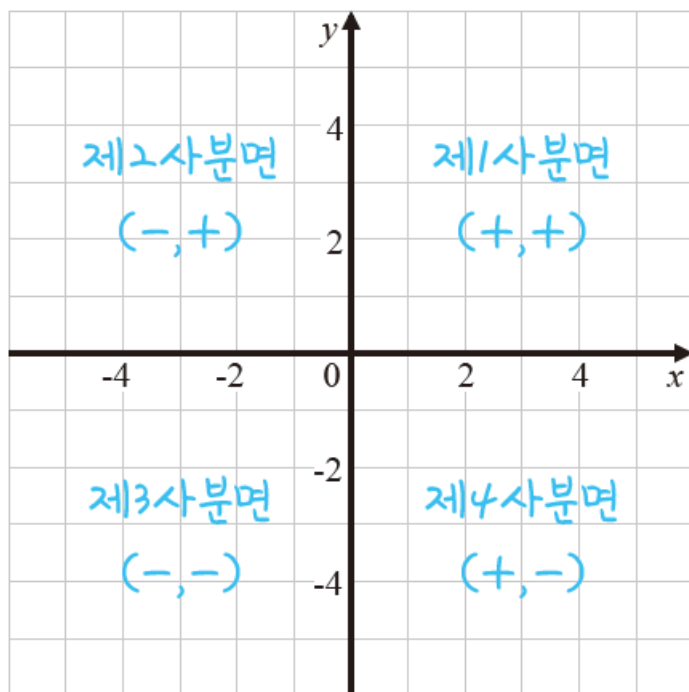
- A coordinate plane is a plane composed of two axes, the x-axis and the y-axis, and the x-axis and y-axis are collectively called the coordinate axes
- The horizontal line is called the x-axis and the vertical line is called the y-axis
- The intersection of the x-axis and y-axis is called the origin (0, 0) and is represented by the symbol O



❖ Coordinate planes and quadrants

Quadrant

- The four areas that are divided into the x- and y-axis are called quadrants
- The quadrants are called the 1st, 2nd, 3rd, and 4th quadrants in counterclockwise order



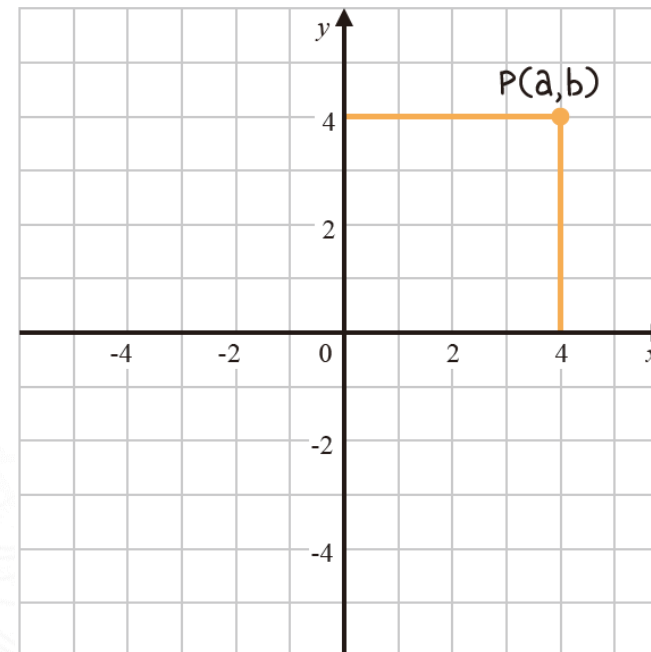
- 제1사분면: $x > 0, y > 0$ 을 만족하는 영역
- 제2사분면: $x < 0, y > 0$ 을 만족하는 영역
- 제3사분면: $x < 0, y < 0$ 을 만족하는 영역
- 제4사분면: $x > 0, y < 0$ 을 만족하는 영역

❖ Points above the coordinate plane

- In the coordinate plane, based on the origin $(0, 0)$, the x-axis is positive on the right and negative on the left
- The y-axis is positive if it is above the origin and negative if it is below the origin
 - In a vertical line, the points were only in the direction of right and left, but in the coordinate plane, the top and bottom were added, making a total of four
- In the coordinate plane, the position of the point is determined by the vertical line x-axis and y-axis, so the position is represented by symbols like (a, b)
- When the point position is represented by (a, b) , a, b are real numbers
- a tells you where the dot is on the x-axis and b tells you where the dot is on the y-axis

❖ Points above the coordinate plane

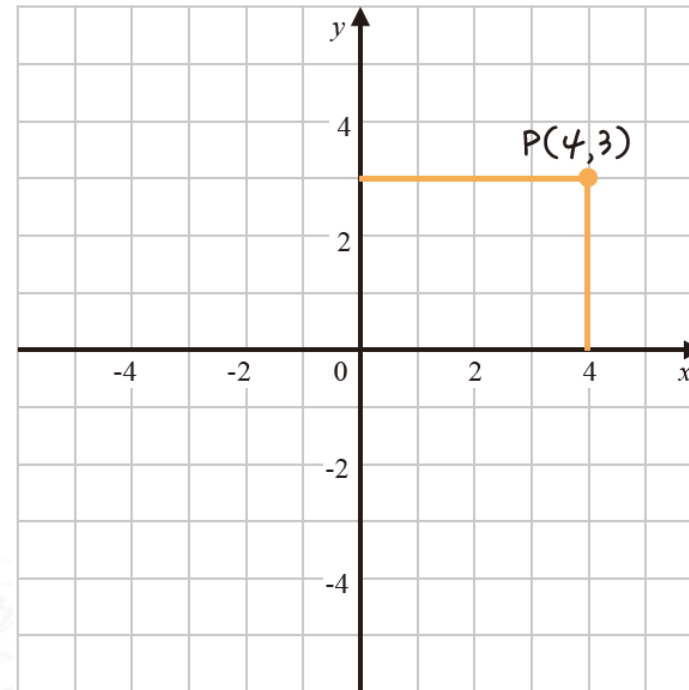
- $P(a, b)$ is a symbol indicating that the position of the point P is (a, b)
- In this case, the (a, b) corresponding to the position of point P is called the coordinates of point P
- a is the x coordinate of the point P , and b is the y coordinate of the point P



❖ Points above the coordinate plane

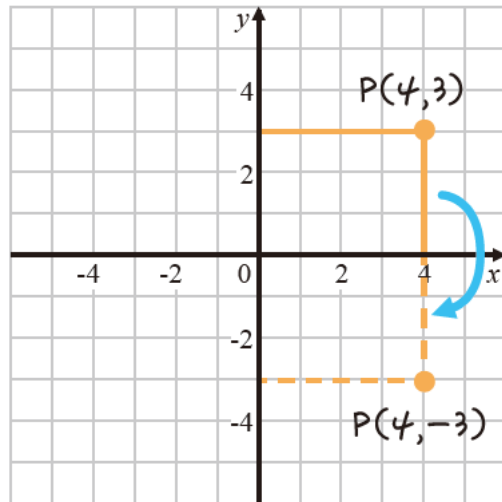
Point symmetry over a coordinate plane

- If we mirror the x and y axes for points on the coordinate plane
- Suppose you have $P(4, 3)$ where the x coordinate of the point P is 4 and the y coordinate is 3, as follows

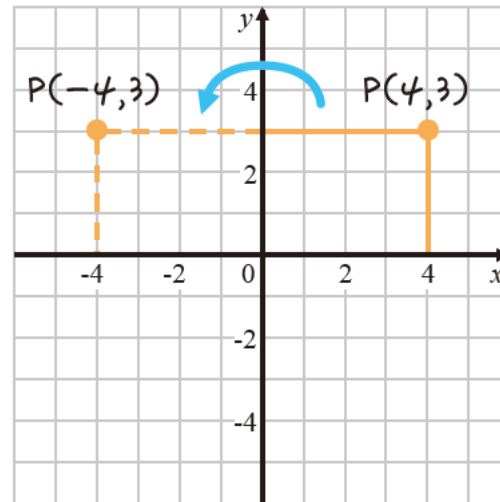


❖ Points above the coordinate plane

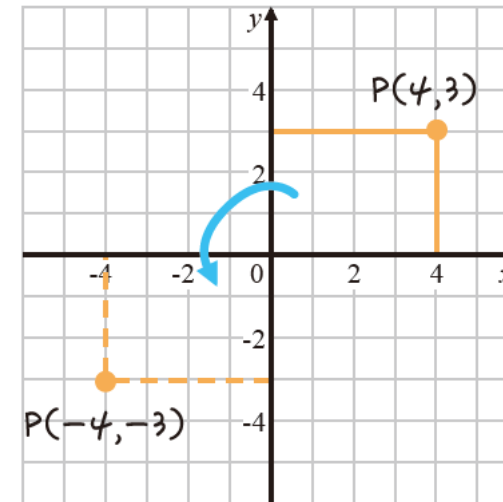
- Symmetric transposing $P(4, 3)$ on the x-axis changes only the sign of the y-coordinates and flipping it on the y-axis reverses the sign of the x-coordinate
- When symmetric transposing on the origin, both the sign of the x coordinate and the sign of the y coordinate are reversed



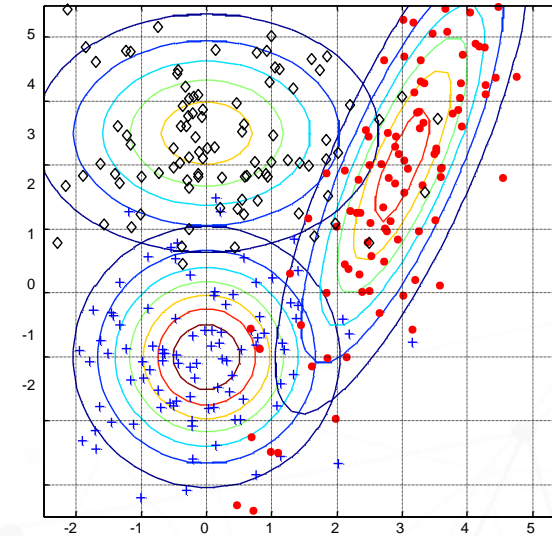
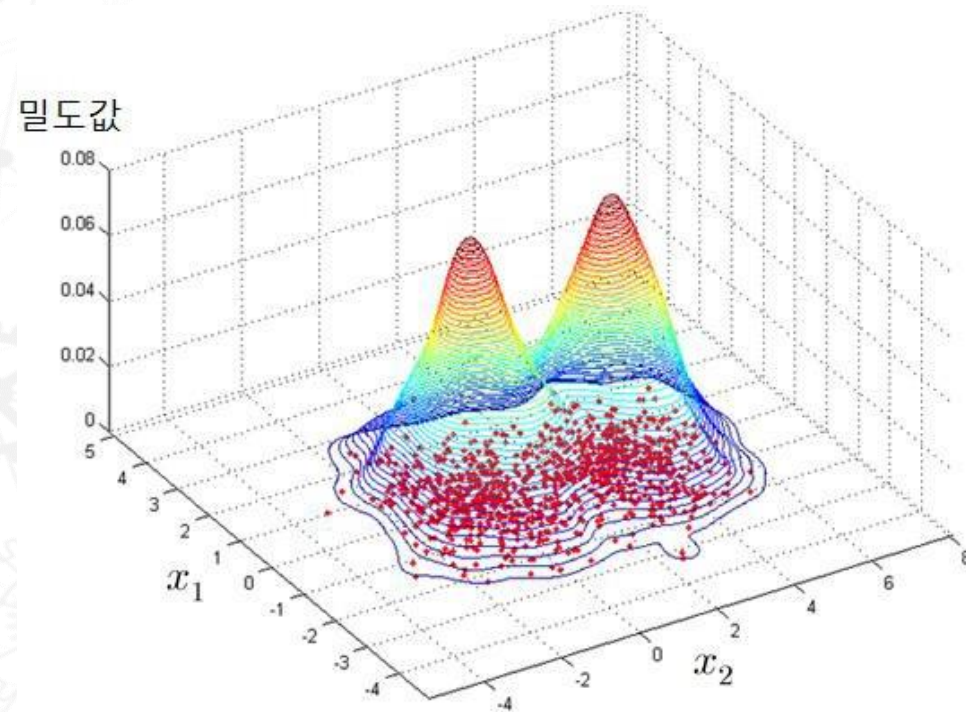
① x축 대칭



② y축 대칭

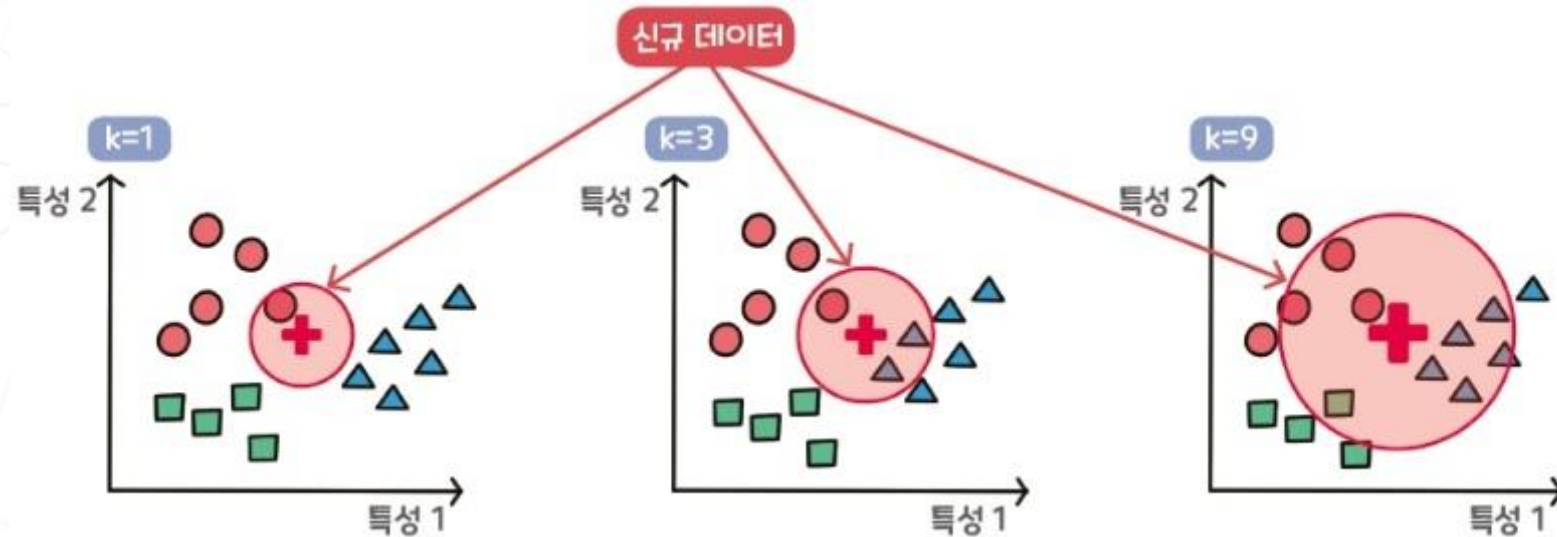


③ 원점 대칭



❖ Principles

- Algorithms to classify which of the existing groups of data (K groups) belongs to when new data comes in
- (Example) When new data is entered when $K=1$, new data is classified as a red circle, when $K=3$, and when $K=9$, it is classified as a blue triangle

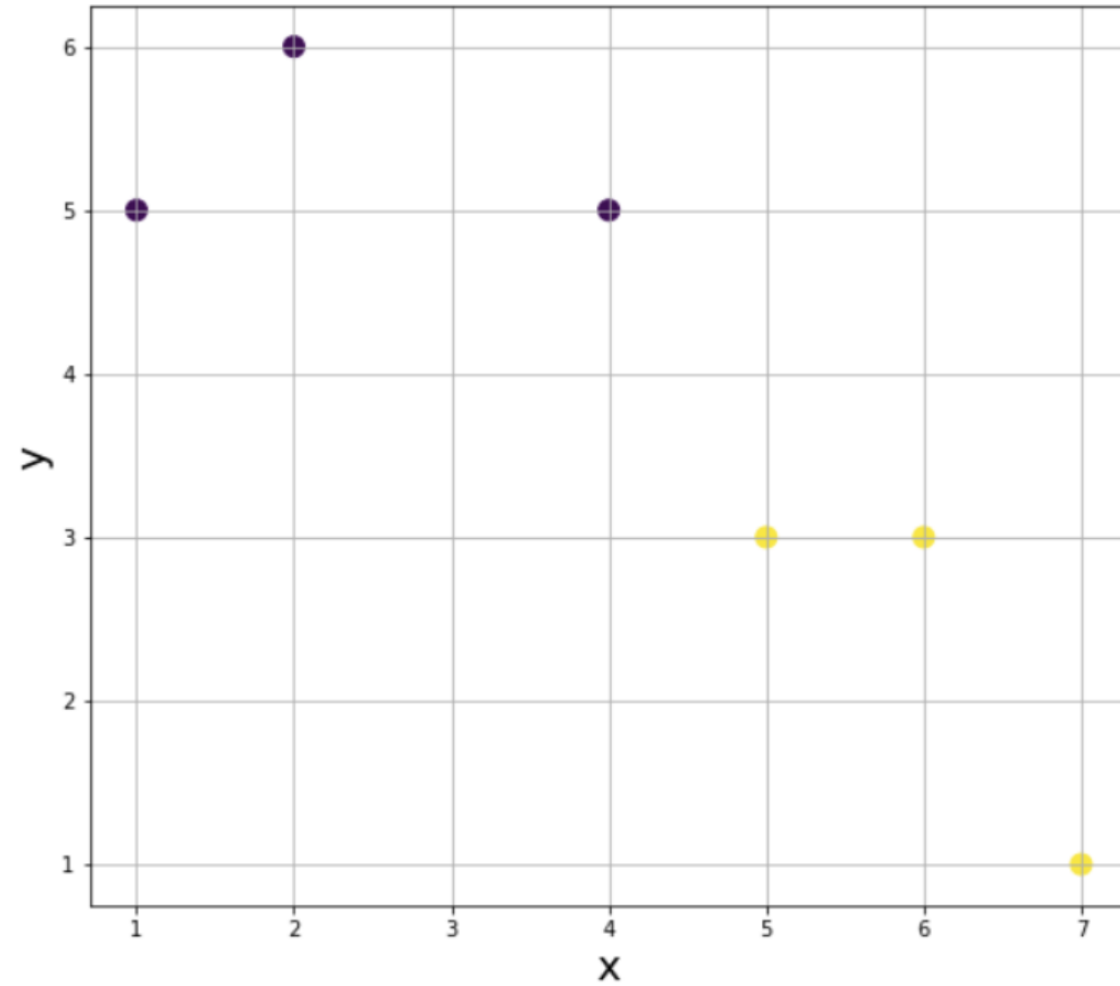


❖ Principles

- KNNs are not significantly affected by the noise present in the learning data and are quite effective when the number of learning data is large
- However, it is unclear which hyperparameters are suitable for analysis, so there is a disadvantage that researchers should randomly select according to each characteristic of the data

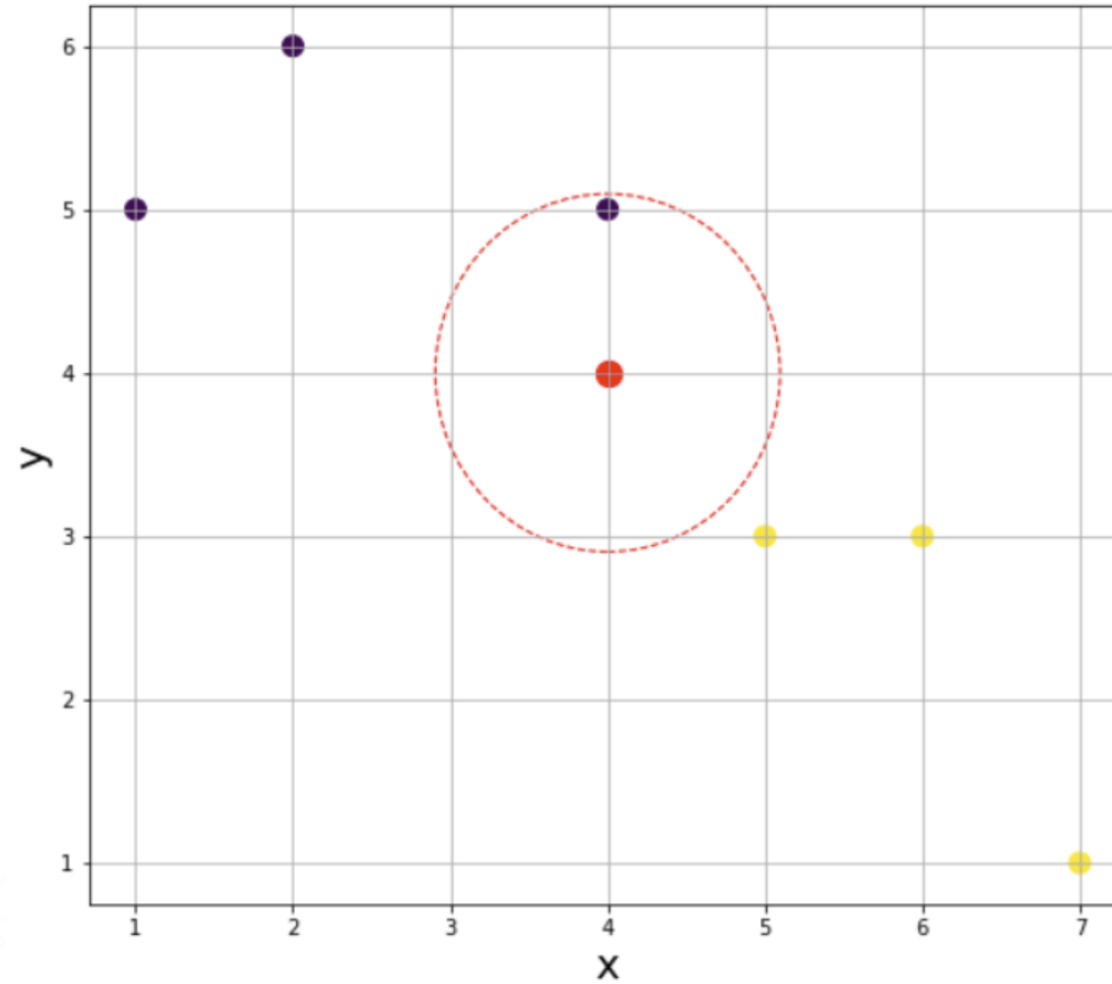
K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



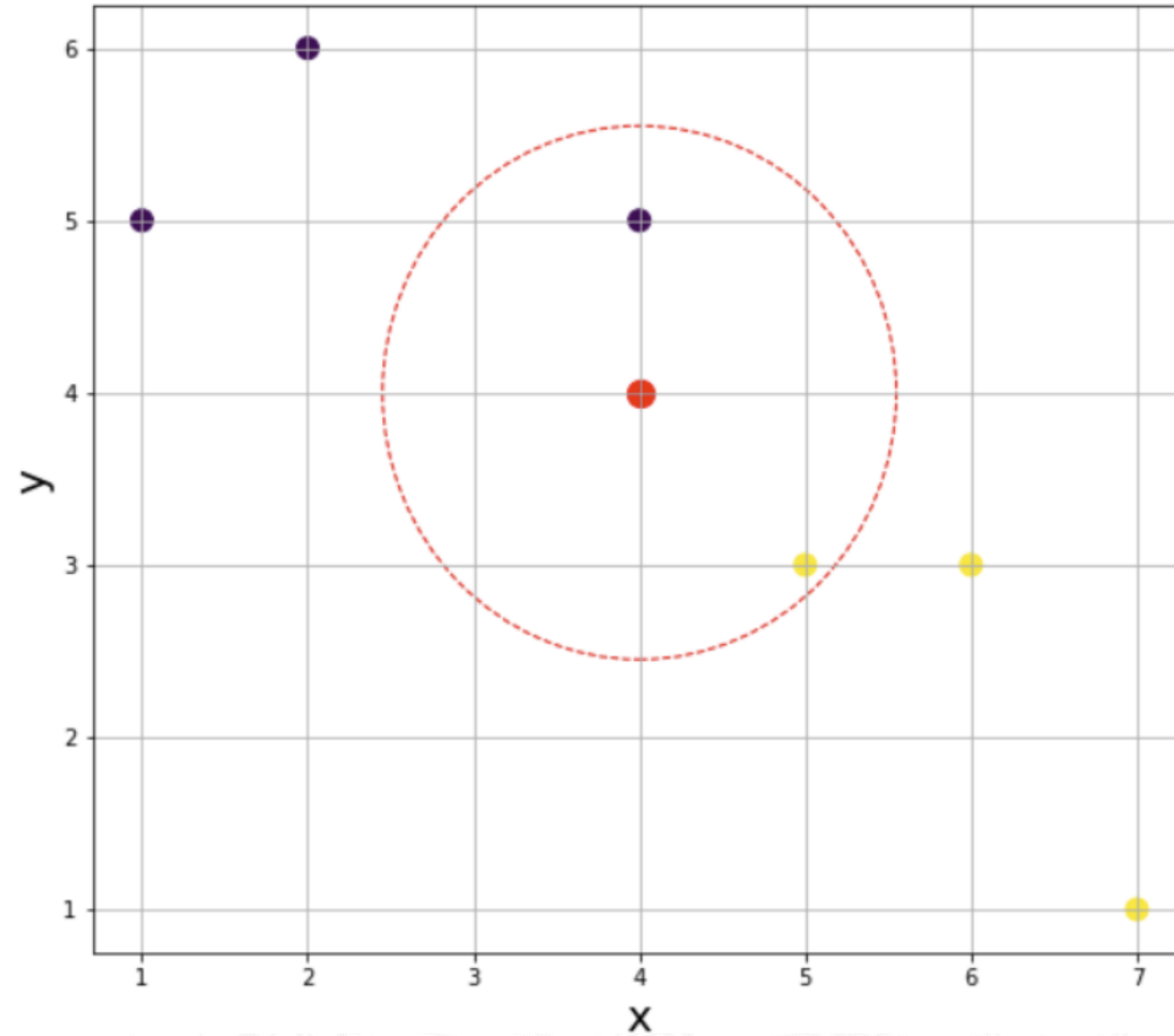
K-Nearest Neighbor

	x	y	label
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1	2	6	0
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3	5	3	1
4	6	3	1
5	7	1	1



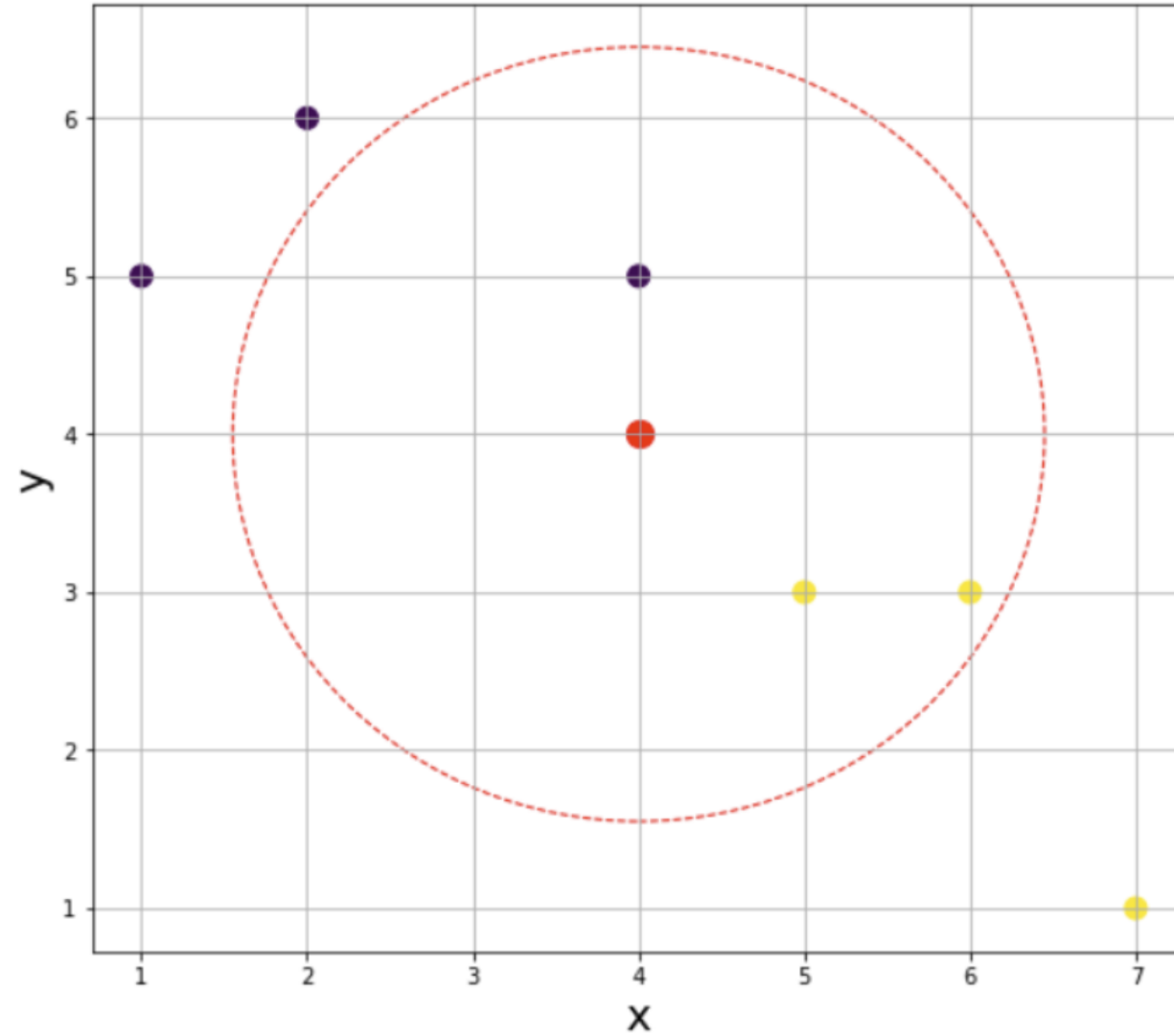
K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



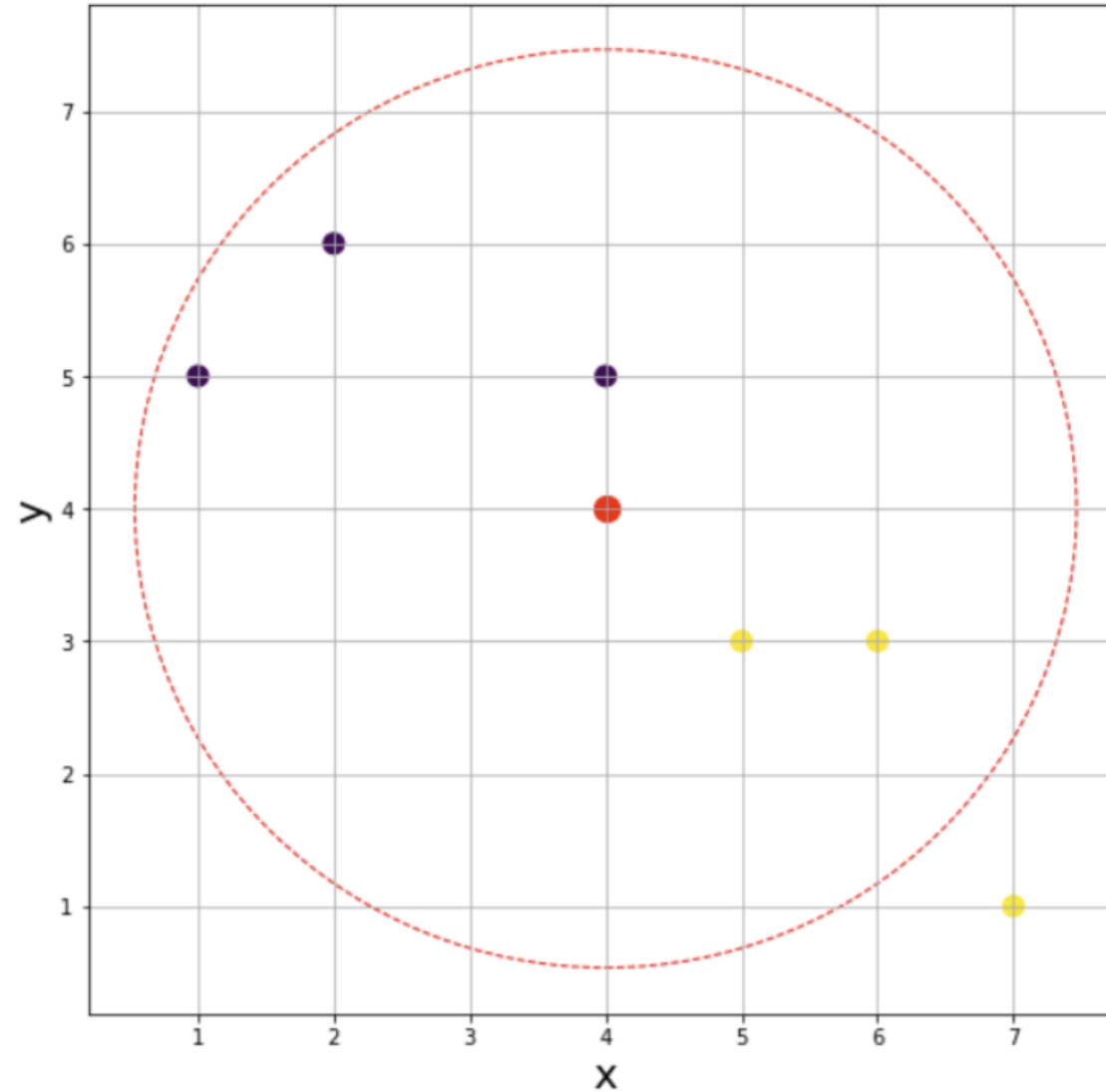
K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



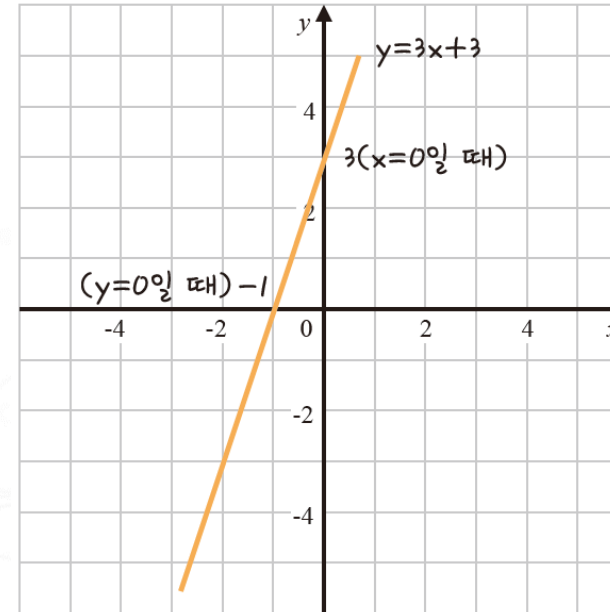
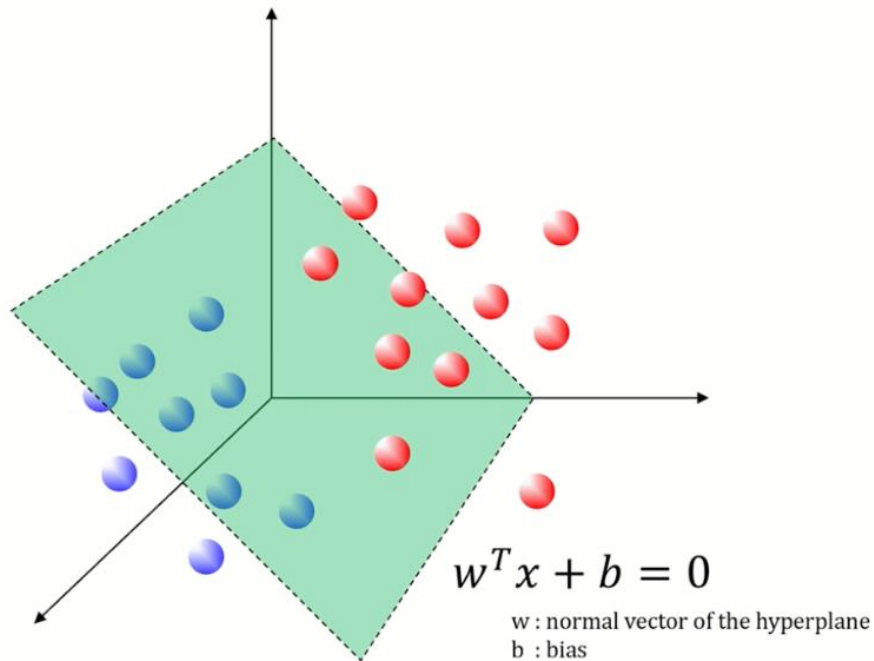
K-Nearest Neighbor

	x	y	label
0	1	5	0
1	2	6	0
2	4	5	0
3	5	3	1
4	6	3	1
5	7	1	1



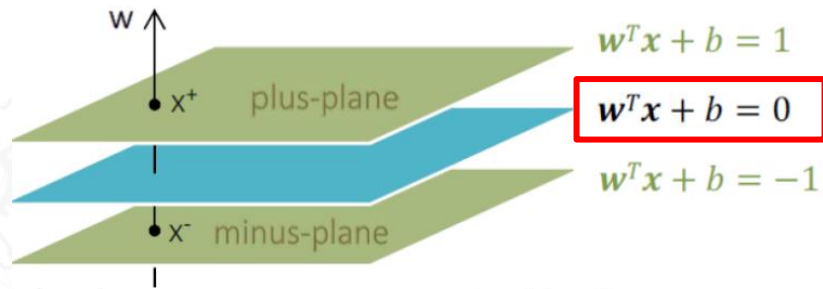
❖ x intercepts and y intercepts

- An intercept is a point where a straight line on a coordinate plane meets the x-axis (x-coordinate) and a point where it meets the y-axis (y-coordinate)
- The x intercept is the x coordinate of the point where it meets the x-axis, meaning the x value when $y = 0$
- The y-intercept is the y-coordinate of the point where it meets the y-axis, meaning the y-value when $x = 0$



❖ x and y intercepts

- (1) Finding the x intercept(절편)
 - The x intercept is found to be the value of x when $y = 0$ in a given linear function.
 - If we substitute $y = 0$ in $y = 3x + 3$, then $0 = 3x + 3$, so $3x = -3$, i.e. $x = -1$
- (2) Finding the y intercept
 - The y intercept is a y value when $x = 0$ in a given linear function
 - If we substitute $x = 0$ in $y = 3x + 3$, then $y = 3 \times 0 + 3$, so $y = 3$



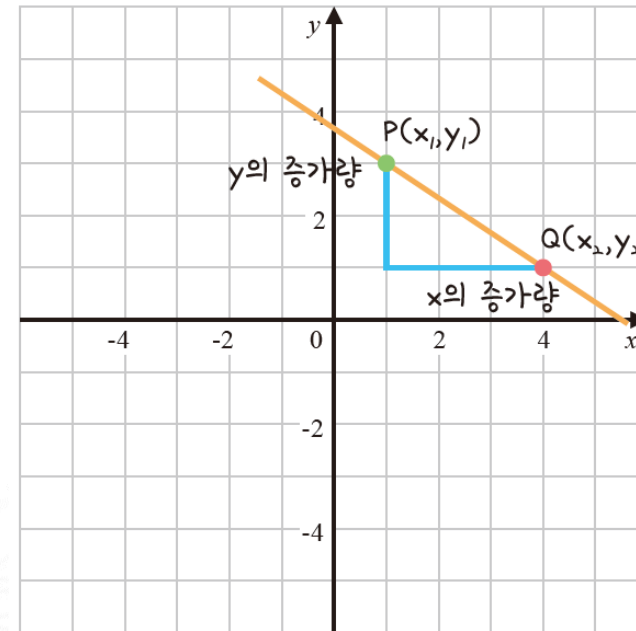
❖ Slope

- The slope indicates how skewed the graph is
- In this case, the degree of slope is expressed as a number, not an angle
- The formula to find the slope is:

$$\text{기울기} = \frac{y\text{의 증가량}}{x\text{의 증가량}} = \frac{y_2 - y_1}{x_2 - x_1}$$

❖ Slope

- Suppose there are any two points $P(x_1, y_1)$, $Q(x_2, y_2)$ in the graph
- The increase in x at two points becomes '(x coordinate of point Q) - (x coordinate of point P)', and the increase in y becomes '(y coordinate of point Q) - (y coordinate of point P)'

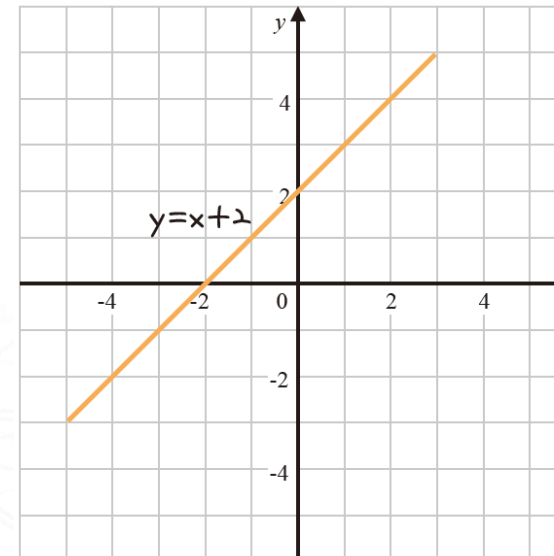


❖ Slope

(1) If we find the coordinates of the point where it meets the x-axis and the coordinates of the point where it meets the y-axis, we get $(-2, 0)$, $(0, 2)$

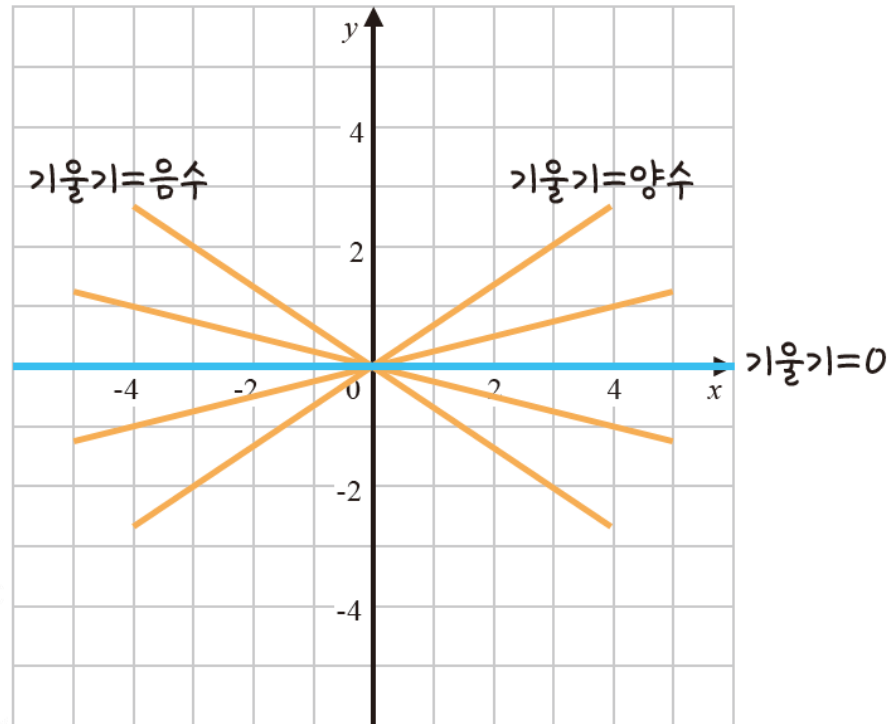
(2) Applying $\frac{y_2 - y_1}{x_2 - x_1}$, we get $\frac{2 - 0}{0 - (-2)} = 1$

- Slope is 1
- Also consistent with the slope of the $y = x + 2$ graph, which is 1



❖ Positive and negative slopes

- The slope is 0 if it is parallel to the x-axis, and it has a positive slope if the right end of the straight line is above the x-axis
- If the right end of a straight line is lower than the x-axis, it has a negative slope



❖ Positive and negative slopes

- In other words, if the value of y also increases when the value of x increases, it has a positive slope, and if the value of y decreases when the value of x increases, it has a negative slope
- If the value of y does not change when the value of x increases, the slope is 0

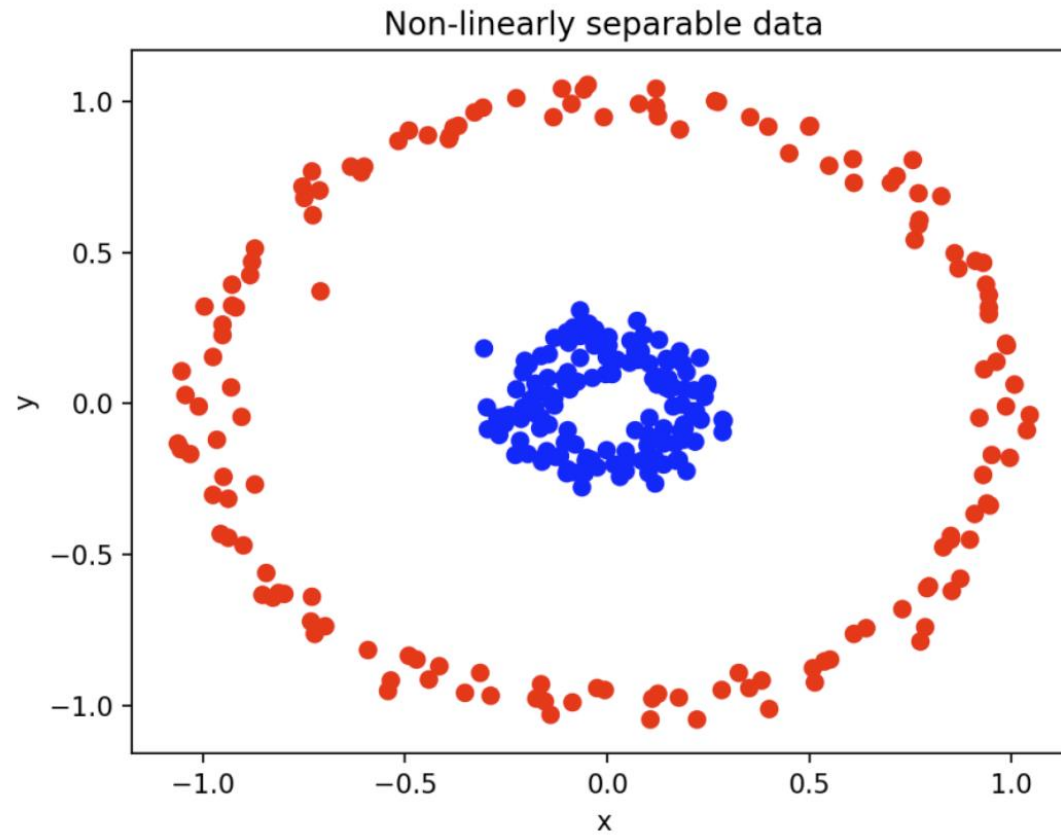
❖ Exponent

- An exponent is a letter or number that is added to the upper right of a number or letter to indicate the number of powers

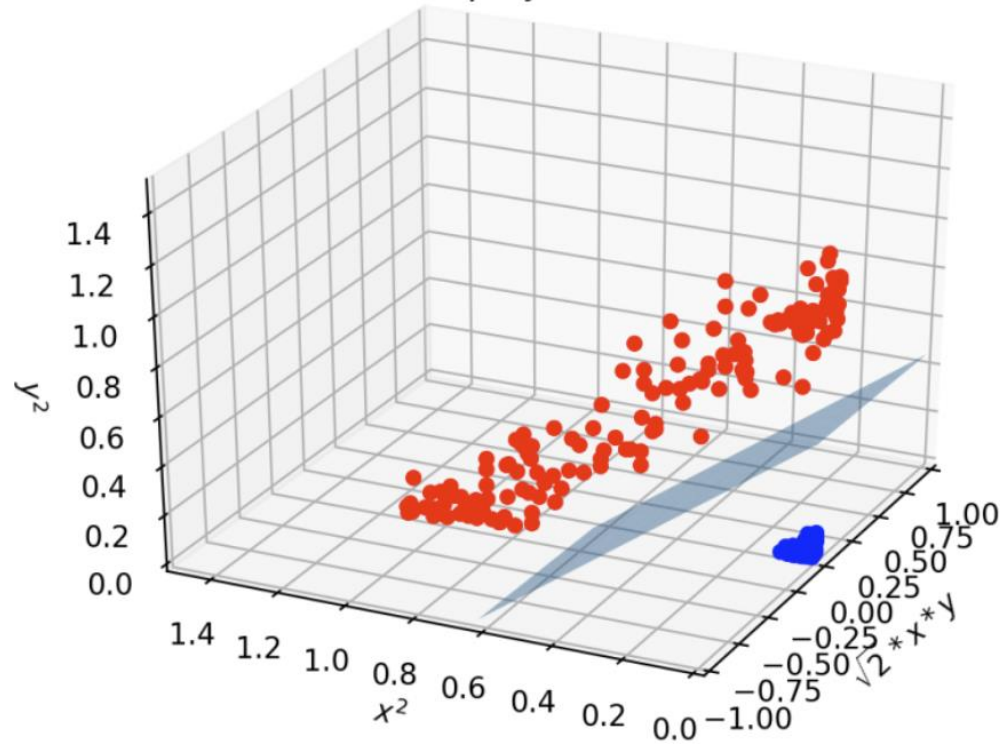
$$y = a^n$$

Diagram illustrating the components of the exponential equation $y = a^n$:

- The base a is labeled with the Korean word "밑" (bottom) and an upward-pointing blue arrow.
- The exponent n is labeled with the Korean word "지수" (exponent) and a leftward-pointing blue arrow.



2D data projected into 3D



$$(x, y) \rightarrow (\sqrt{2} \cdot x \cdot y, x^2, y^2)$$

$$(1, 2) \rightarrow (2\sqrt{2}, 1, 4)$$

❖ The law of exponent

- The law of exponent includes the product rule, quotient rule, and the power of a power rule
- Product rule
 - When $m = 3$, $n = 4$, as:

(1) 지수의 합: $a^m \times a^n = a^{m+n}$

$$a^3 \times a^4$$

$$= (a \times a \times a) \times (a \times a \times a \times a)$$

$$= a^7 = a^{3+4}$$

예시 $2^3 \times 2^5 = 2^{3+5} = 2^8$

❖ The law of exponent

- Quotient rule

(2) 지수의 차

$$\textcircled{1} a^m \div a^n = a^{m-n} \text{ (} m \geq n \text{일 때)}$$

예시 $2^5 \div 2^3 = 2^{5-3} = 2^2$

$$\textcircled{2} a^m \div a^n = \frac{1}{a^{n-m}} \text{ (} m < n \text{일 때)}$$

예시 $2^3 \div 2^5 = \frac{1}{2^{5-3}} = \frac{1}{2^2}$

❖ The law of exponent

- Power of a power rule

(3) 지수의 곱: $(a^m)^n = a^{mn}$ (m, n 은 양의 정수)

$$(a^2)^3$$

$$= a^2 \times a^2 \times a^2$$

$$= a \times a \times a \times a \times a \times a$$

$$= a^6 = a^{2 \times 3}$$

예시 $(2^4)^6 = 2^{4 \times 6} = 2^{24}$

❖ Power

- A power is a calculation that multiplies the same number several times
- For example, $5 \times 5 \times 5 \times 5 \times 5$ is expressed as 5^5 and read as 5 to the power of 5
- Common expressions include:

$$a^n = \underbrace{a \times a \times a \times \cdots \times a}_{n \text{ times}}$$

❖ Power

- Example

$(2 \times 2) 2^2 \rightarrow 2$ squared (when you multiply twice, it just reads as squared)

$(2 \times 2 \times 2 \times 2 \times 2) 2^5 \rightarrow 2$ to the power of 5

$(2 \times 2 \times 2 \times 2 \times 2 \times 2) 2^6 \rightarrow 2$ to the power of 6

$(a \times a \times a \times a \times a \times a \times a) a^7 \rightarrow a$ to the powers of 7

❖ Power

- There are caveats to this

(1) Like Equation 4.1, it can only be used with the same numbers and letters

$$\frac{3 \times 3 \times 3}{3^3} \times \frac{6 \times 6}{6^2} \times 9, \quad \frac{a \times a}{a^2} \times \frac{b \times b \times b \times b}{b^4}$$

수식 4.1

- In Equation 4.1, you can see that only the same numbers or letters are grouped together and expressed as a power (if the numbers or letters are different, they cannot be grouped together)
- (2) Be careful when calculating fractions

❖ Power

- Use parentheses when expressing fractions as powers as follows
- Not to use parentheses because it can give the wrong answer

$$\frac{1}{2} \times \frac{1}{2} = \frac{1^2}{2} = \frac{1}{2}$$

괄호를 사용하지 않은 예시

$$\frac{1}{2} \times \frac{1}{2} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

괄호를 사용한 예시

❖ Power

- Power has three characteristics
- 0 and 1, when a negative number squared, is characterized by:

The zero squared of a non-zero real number is always 1 : $a^0 = 1$

예시 $10^0 = 1, 1000^0 = 1$

1 square of a non-zero real number equals the real number : $a^1 = a$

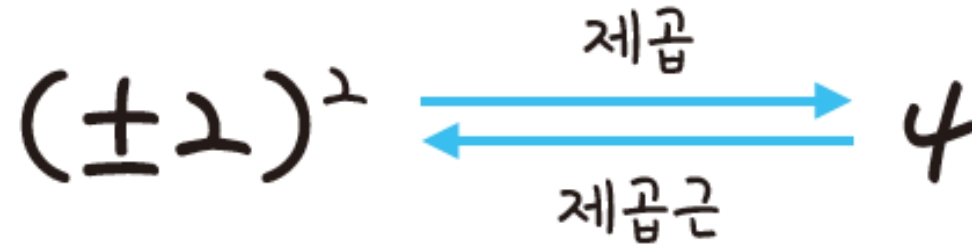
예시 $10^1 = 10, 1000^1 = 1000$

The negative square of a non-zero real number is equal to : $a^{-n} = \frac{1}{a^n}$ $\frac{1}{\text{실수의 양의 제곱}}$

예시 $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

❖ Power root

- Square and square root are opposite concepts
- In the quadratic equation $x^2 = a$, a is called the square of x , and x is called the square root of ' a '
- 4 is a power of 2, and 2 is a square root of 4


$$(\pm 2)^2 \begin{array}{c} \xrightarrow{\text{제공}} \\ \xleftarrow{\text{제공근}} \end{array} 4$$

❖ Power root

- There are two square roots, a positive square root and a negative square root, and the positive one is called the positive square root, and the negative one is called the negative square root
- -2 is the negative square root of 4, and 2 is the positive square root of 4

❖ Power root

- In $x^2 = a$, the positive x is the square root of a , so in $x^n = a$, a is the power of x , and x is the power root of a (same as the square root principle of quadratic equations)

$$(\pm 2)^3 \begin{array}{c} \xrightarrow{\text{거듭제곱}} \\ \xleftarrow{\text{거듭제곱근}} \end{array} 8$$

❖ Power root

Power root properties

- The most important concept in the property of the power root is the root ($\sqrt{\quad}$)
- A root is 'to square a certain number so that the number in the root comes out'

$$2^2 = 4, \sqrt{4} = \pm 2$$

❖ Power root

- The properties of the power root are as follows:

$\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$: (1) If the power root is equal, the multiplication can be grouped into one power root

$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$: (2) If the power root is equal, the division can be grouped into one power root

$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$: (3) The power can go into the power root

$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$: (4) Multiplying by power roots

$\sqrt[np]{a^{mp}} = \sqrt[n]{a^m}$: (5) The power root and the power are reducible

❖ Power root

- **Reduction of fraction**
 - Simple by dividing the denominator and numerator of a fraction by a common divisor
 - In other words, the denominator and numerator are no longer separated from each other
- AI does not directly solve using the properties of power root

❖ Power root

- To calculate the power root in Python, use the math library to implement

In [9]:

```
# 거듭제곱의 표현은 **으로 합니다  
2**5
```

32

In [10]:

```
# math.sqrt() 함수를 사용하여 거듭제곱근을 구합니다  
import math  
math.sqrt(2)
```

1.4142135623730951

❖ Power root

In [11]:

```
math.sqrt(9)
```

3.0

❖ Factorization

- Factorization is the process of expressing complex expressions as products by grouping them into common factors
- In other words, the purpose of factorization is to decompose the equation into more basic and simpler pieces

$$\underbrace{mx + my}_{\text{공통인수}} = m(x+y), \quad \underbrace{mx - my}_{\text{공통인수}} = m(x-y)$$

Data -> pre-processing -> labeling, noise reduction, outlier detection,

FE + CLS
regression

❖ Factorization

- Another definition is the expression of a polynomial in the form of a product of a monomial and a polynomial, or a product of multiple polynomials, called factorization
- A monomial or polynomial multiplied is called a factor of the first expression

$$x^2 + 6x + 5 \begin{array}{c} \xrightarrow{\text{인수분해}} \\ \xleftarrow{\text{전개}} \end{array} \underbrace{(x+1)}_{\text{인수}} \underbrace{(x+5)}_{\text{인수}}$$

❖ Factorization

- **Expansion** is the modification of an expression in the form of a product into the form of a sum
- Expressing it in the form of a monomial sum

인수분해 공식

$$(1) x^2 + 2xy + y^2 = (x + y)^2$$

$$(2) x^2 - 2xy + y^2 = (x - y)^2$$

$$(3) x^2 - y^2 = (x + y)(x - y)$$

$$(4) x^2 + (a + b)x + ab = (x + a)(x + b)$$

Data -> pre-processing -> Training -> classification

Training model training .

- For reference, an expression in the square of a polynomial or an expression that is multiplied by a constant, i.e., an expression that can be expressed in the form of $(ax + b)^2$ or $k(ax + b)^2$ (where k is a constant) is called a perfect square expression

❖ Factorization

- Use the Sympy library to implement in Python as follows

In [12]:

```
# 파이썬 SymPy의 expand, factor, Symbol을 호출하고 기호변수 x를 선언  
합니다
```

```
from sympy import expand, factor, Symbol  
x = Symbol('x')
```

In [13]:

```
# expand()는 수식을 (x + 1) x (x + 5)로 전개합니다  
expand((x + 1) * (x + 5))
```

x^2+6x+5

❖ Factorization

In [14]:

factor()는 인수분해하는 함수로, $x^2 + 6x + 5$ 를 인수분해합니다

```
factor(x**2 + 6*x + 5)
```

$(x+1)(x+5)$

❖ Factorization

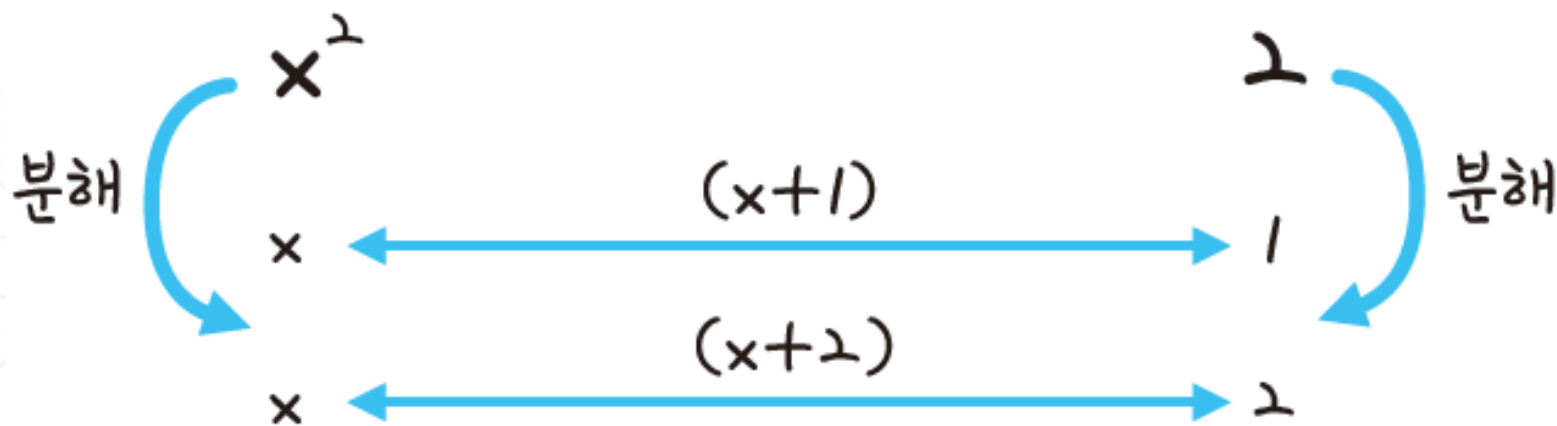
연습 문제

- (1) $x^3y - xy^3$ 을 인수분해하세요.
- (2) $x^2 + 3x + 2$ 를 인수분해하세요.

문제 풀이

- (1) $x^3y - xy^3$ 을 공통인수 xy 로 묶어 주면 $xy(x^2 - y^2)$ 이 됩니다. 인수분해 공식 (3)을 적용하면 $xy(x + y)(x - y)$ 가 됩니다.
- (2) $x^2 + 3x + 2$ 에는 공통인수가 없으므로 식을 간단히 만들면 다음과 같습니다.

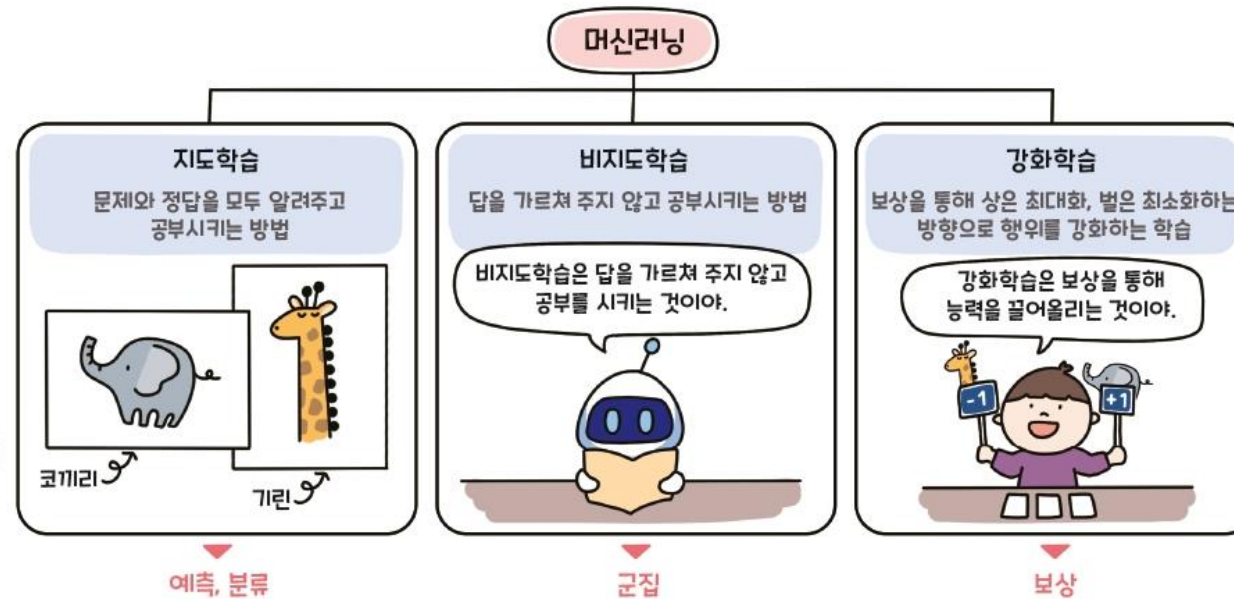
$x^2 + 3x + 2$ 식에서



❖ Categorization for AI training

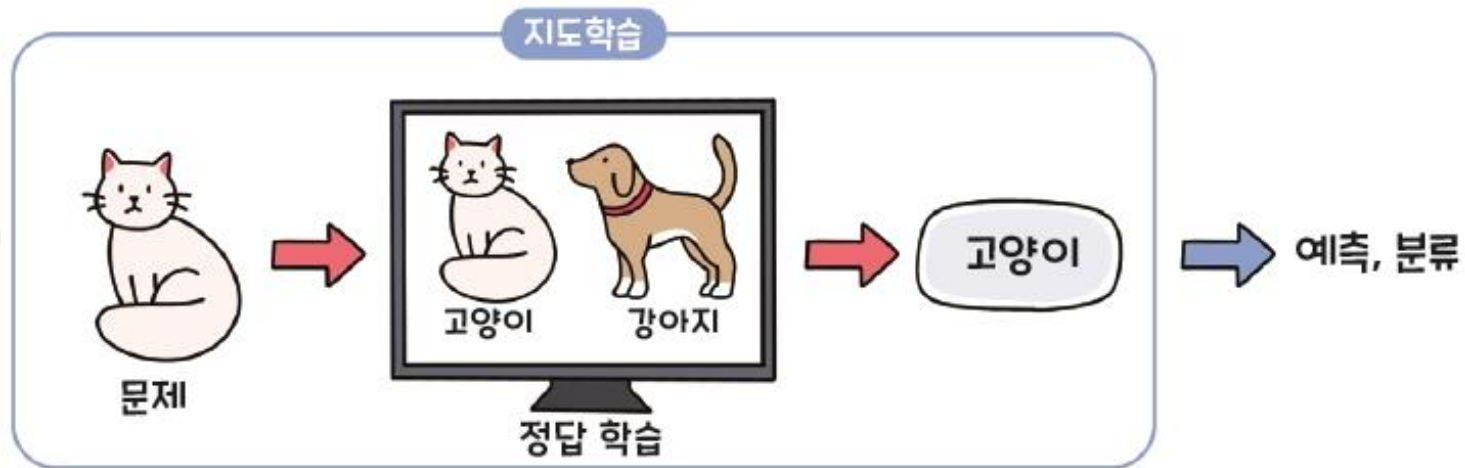
- Supervised learning : classification and regression = 가 가 .
- Unsupervised learning : clustering
- Reinforcement learning : use rewards for actions taken in the environment to conduct learning

x = data
 y = class label



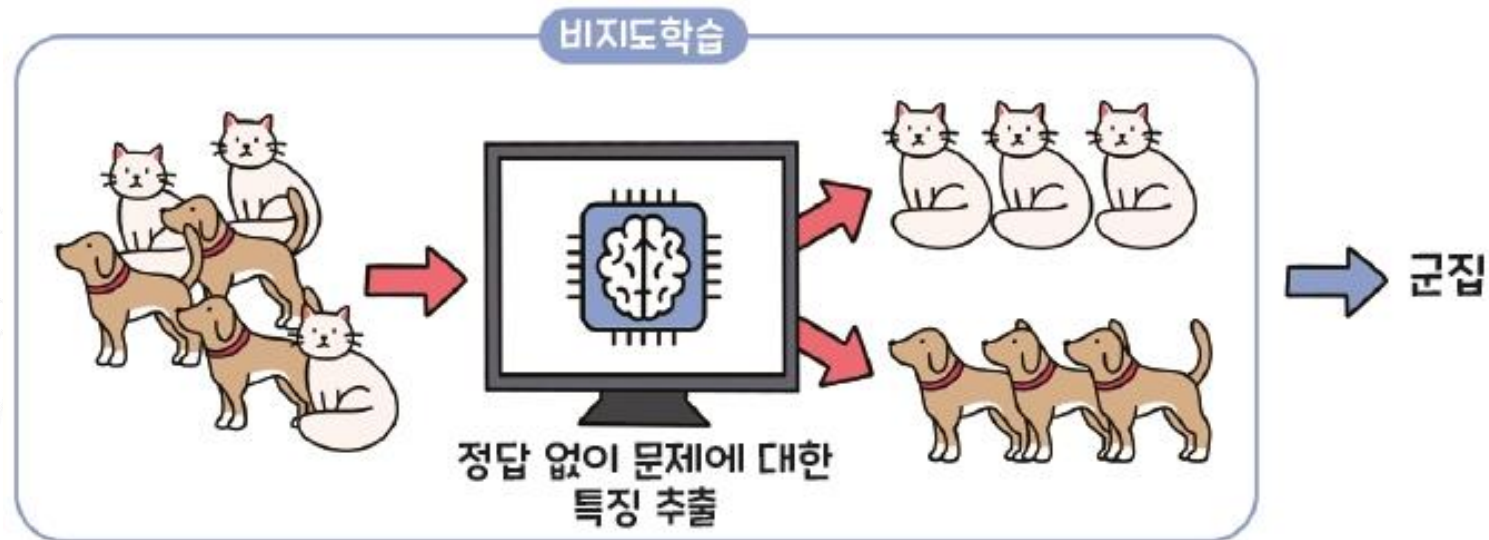
❖ Supervised learning

- Learning to predict the right answer to an unknown problem by learning questions and answers together
- The models used in supervised learning include prediction and classification



❖ Unsupervised learning

- A form of computer learning without the help
- Computer uses training data to find regularity between data



❖ Unsupervised learning

- Unlike supervised learning, which identified the relationship between x (input data) and y (labels in supervised learning),
- Unsupervised learning identifies the relationship between x by itself
- In other words, the difference between y (label)
 - Clustering is a model used in unsupervised learning

구분	지도학습	비지도학습
필요한 데이터 종류	x (학습 데이터), y (레이블)	x (학습 데이터)

❖ Reinforcement learning

- Learning to be rewarded for what you've done
- How computers learn to choose the best behavior for a given state

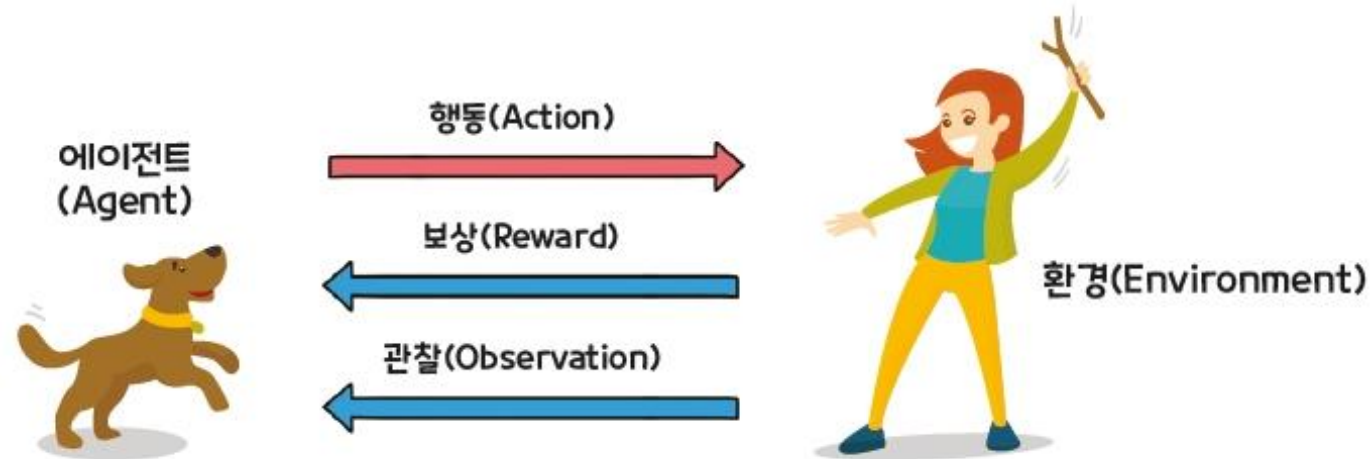


❖ Reinforcement learning

- Agent: Subject to act in a given problem situation
- State : Current situation
- Action: Options that the player can take
- Rewards: Benefits that follow when a player does something
- Environment: means the problem itself
- Observation : Information about the collected by the agent

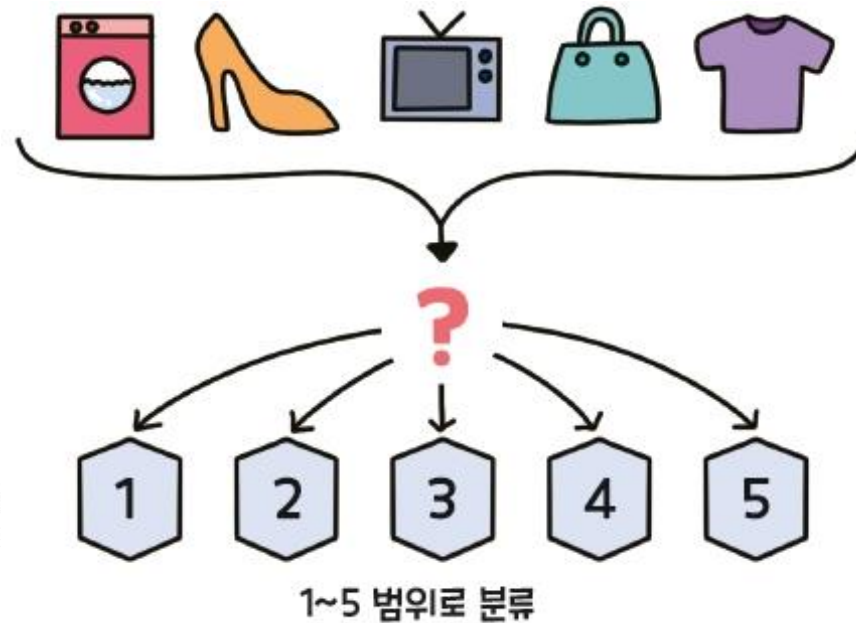
❖ Reinforcement learning

- Depending on the behavior chosen by the agent in a given environment, you are rewarded if the behavior is the right choice, and punished if the behavior is the wrong choice
- Reinforcement learning allows the agent to keep an eye on the status and learn (behavior) toward higher rewards



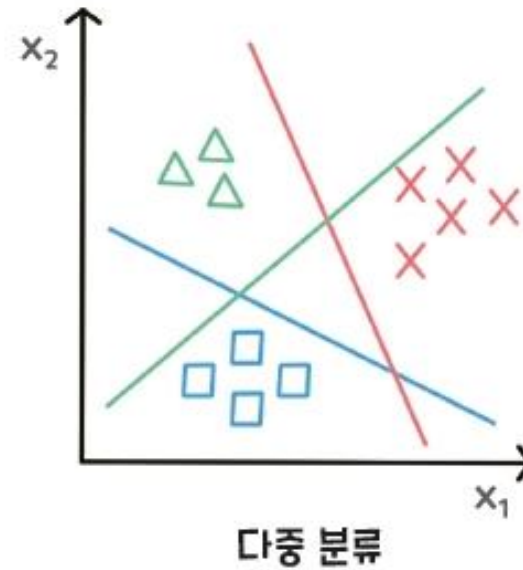
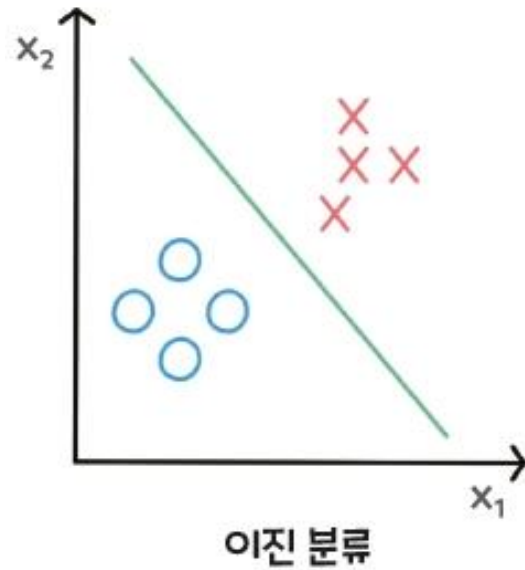
❖ Classification

- A technique for learning labeled data, classifying data with similar properties, and finding out which group the newly entered data



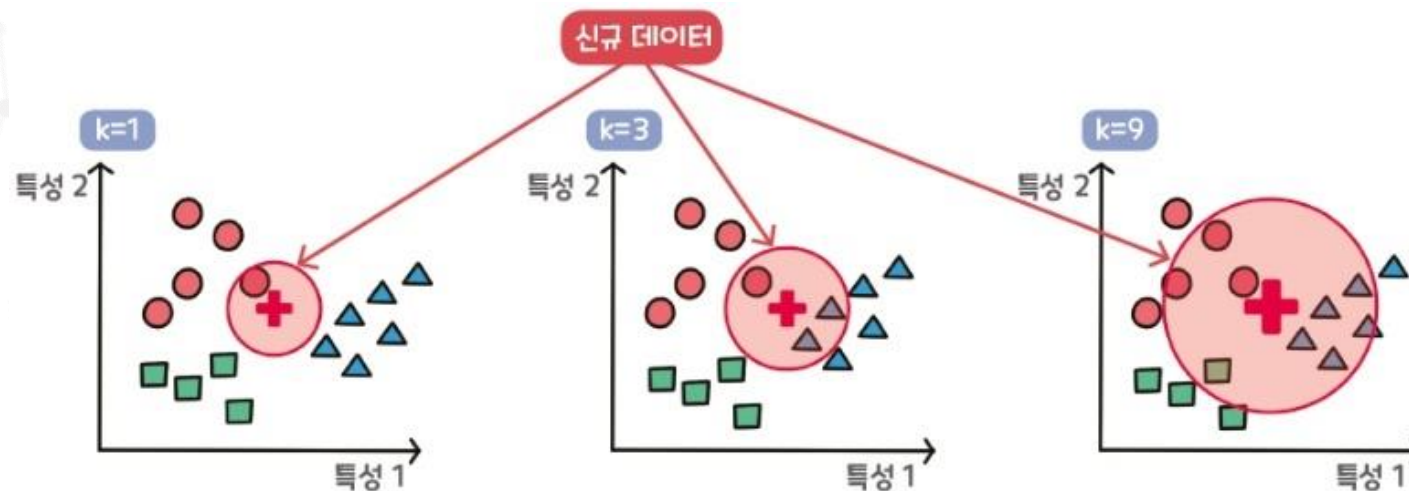
❖ Classification

- Binary classification : categorize data into 2 groups
- Multiclass classification : categorize data into 3 or more groups



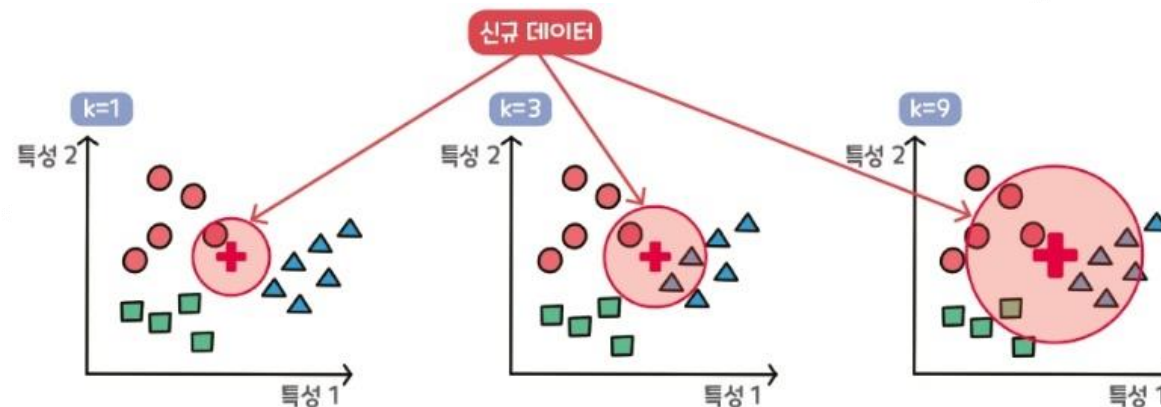
❖ Classification (Algorithm)

- K-neighbor nearest
 - Algorithms to classify which of the existing groups of data (K groups) belongs to when new data comes in
 - (Example) When new data is entered when $K=1$, new data is classified as a red circle, when $K=3$, and when $K=9$, it is classified as a blue triangle



❖ Classification (Algorithm)

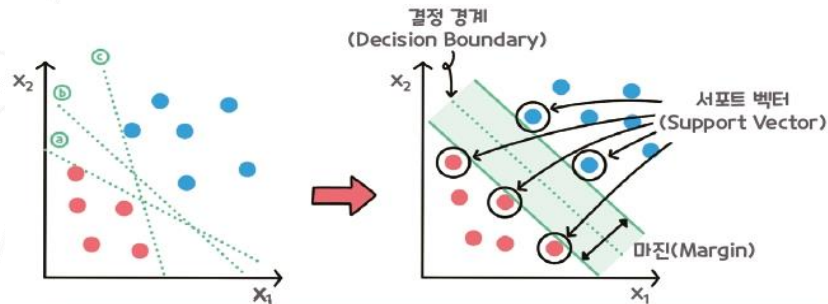
- K-neighbor nearest
 - KNNs are not significantly affected by the noise present in the learning data and are quite effective when the number of learning data is large
 - However, it is unclear which hyperparameters are suitable for analysis, so there is a disadvantage that researchers should randomly select according to each characteristic of the data



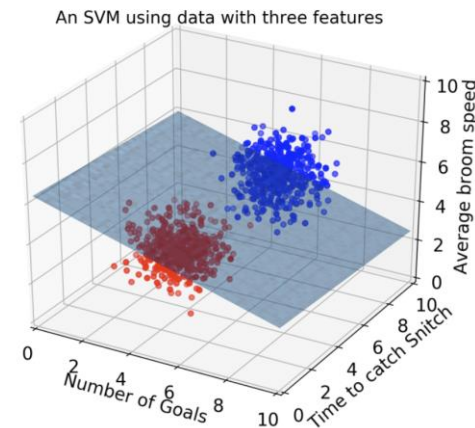
❖ Classification (Algorithm)

▪ Support vector machine

- Categorize data in the direction of maximizing margin, which means margin between two categories
- SVMs find and classify lines that maximize margins, so larger margins are more likely to be classified even if new data comes in
- SVM is easy to use and highly predictive
 - However, it takes time to build a model and the results are less descriptive



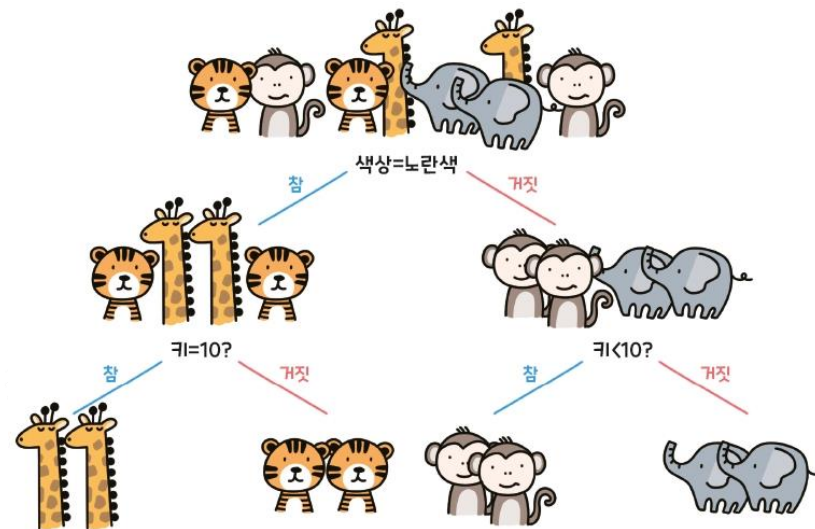
- 결정 경계(Decision Boundary) : 분류를 위한 기준선
- 서포트 벡터(Support Vector) : 결정 경계와 가장 가까운 위치에 있는 데이터
- 마진(Margin) : 결정 경계와 서포트 벡터 사이의 거리



❖ Classification (Algorithm)

▪ Decision tree

- An analysis method for classifying decision-making rules into tree forms
- It is called 'decision tree' because the method of starting from the upper node and expanding to the lower node according to the classification criteria resembles 'tree'

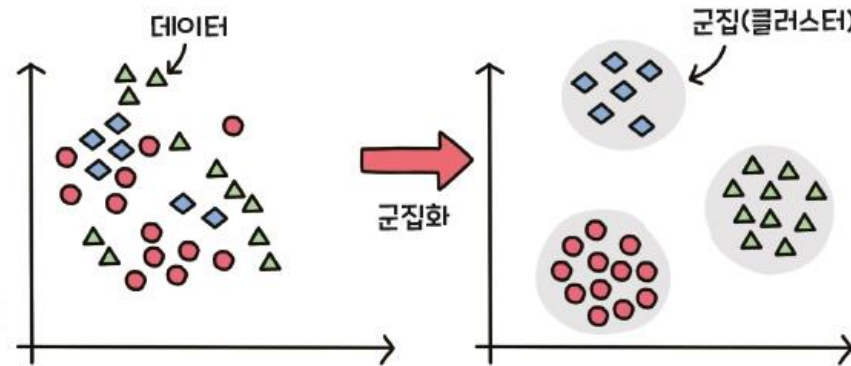


❖ Classification (Algorithm)

- Decision tree
 - Decision Tree is intuitive and easy to understand the analysis process
 - In the case of artificial neural networks, it is a black box model that is difficult to explain the analysis results, while decision trees can observe the analysis process with their eyes
 - Need for a clear explanation of the results

❖ Unsupervised learning

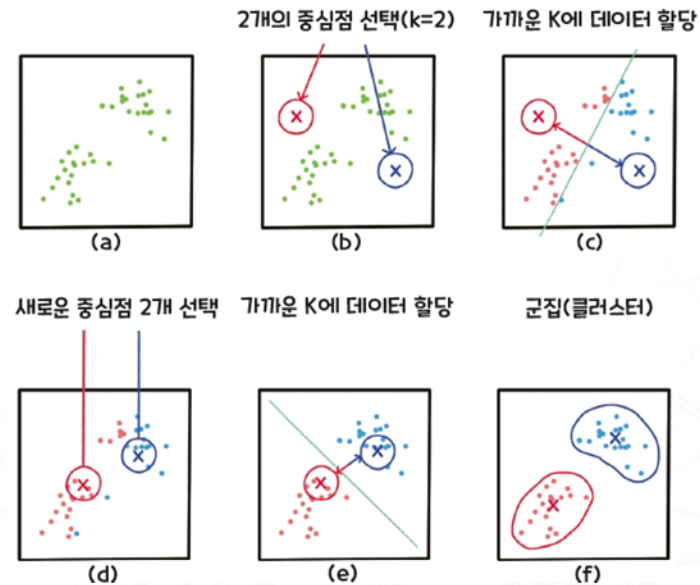
- Cluster
 - A group of data with similar characteristics
- Clustering
 - Classifying the data into clusters according to a similar degree when given the data
 - Various data are mixed together, but the clustering process groups similar data as shown in the graph on the right



❖ Unsupervised learning

▪ K-means clustering

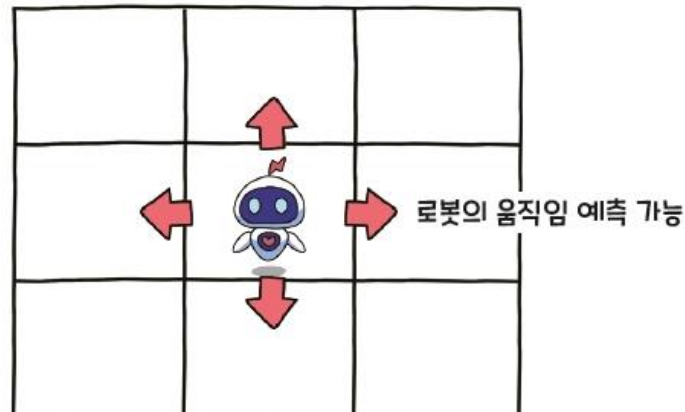
- 'K' is the number of groups to be grouped from the given data
- 'Means' means the average distance between the center of each cluster and the data
- The center of the cluster is called centroids



❖ Reinforcement learning

▪ Algorithm

- Model-based algorithms refer to the probability that an action in the current state will result in the next state
- Intuitive visibility of the robot's next state as it moves up, down, left, and right in a grid space
 - Model-based algorithms can predict changes in state according to behavior, resulting in optimal solutions



❖ 1-(1) 다음 연립방정식의 해를 python을 활용하여 구하여라

- $2x+3y=6$
- $3x+2y=12$

❖ 1-(2) 다음 연립방정식의 해를 python을 활용하여 구하여라

- $x+y+2z=9$
- $2x+4y-3z=1$
- $3x+6y-5z=0$

❖ Python을 활용하여 답을 구하여라

- 2-(1) 두 점 $(1, -2)$ $(3, 0)$ 을 지나는 직선의 방정식은?
- 2-(2) 두 점 $(3, 5)$ $(0, -4)$ 을 지나는 직선의 방정식은?
- 2-(3) 기울기가 3이고 점 $(5, 4)$ 를 지나는 직선의 방정식은?
- 2-(4) 기울기가 -2이고 점 $(3, 2)$ 를 지나는 직선의 방정식은?

❖ 다음 식을 python을 활용하여 인수분해 하여라

- $3xy - 6y^2 - x + 2y$
- $9x^2 - 4y^2 + 16y - 16$
- $x^2 + xy - x - 2y - 2$