Differential of Polynomial Functions

Differential of polynomial function

MATHEMATIC

❖ Average rate of change

- Differential is the calculation of the rate of change
- The average rate of change and the instantaneous rate of change are the rates of change that determine how much y changes for the amount of change x
- The instantaneous rate of change is also known as differential, and the instantaneous rate of change is called the instantaneous rate of change or the coefficient of differentiation
- In mathematics, the Δx symbol is used to express the amount of increase
- For example, assuming that x has varied from 1 to 5, if x has increased by 4, the symbol expresses it as $\Delta x = 4$

Differential of polynomial function

MATHEMATIC

❖ Average rate of change

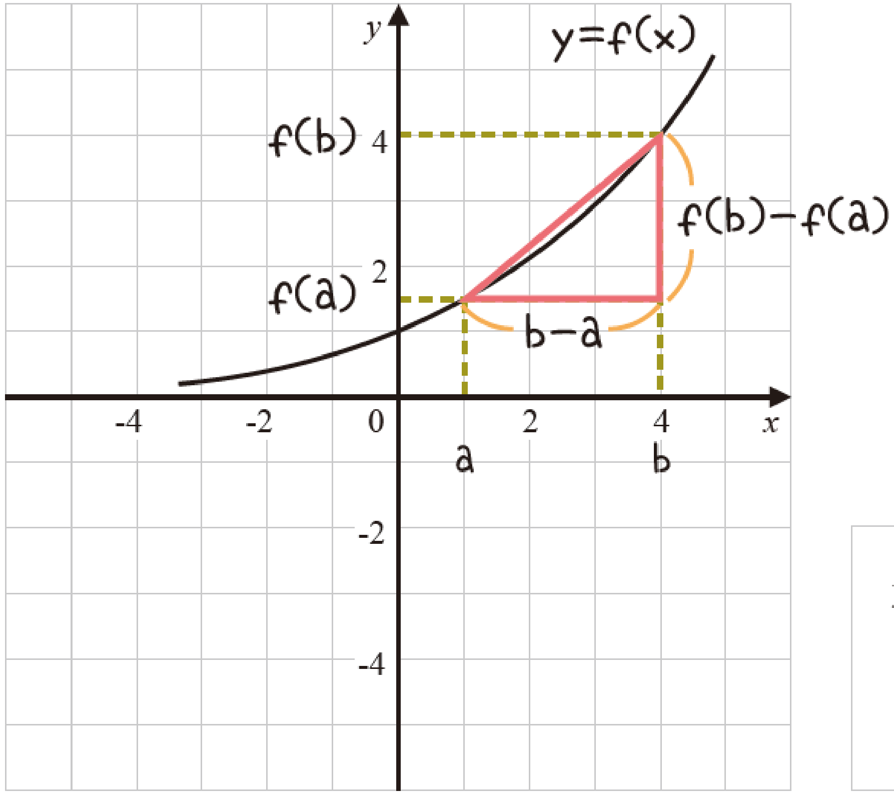
- Average rate of change means $\frac{y\text{의 증가량}}{x\text{의 증가량}}$ when function y=f(x) is present
- In mathematical terms, it is expressed in $\frac{\Delta y}{\Delta x}$
- If f(x) = x^2, the average rate of change when x increases from 1~5 can be expressed as follows
  - Increase in x : Δx = 5 - 1 = 4
  - Increase in y : Δy = f(5) - f(1)

$$\frac{\Delta y}{\Delta x} = \frac{f(5)-f(1)}{5-1} = \frac{25-1}{4} = 6$$

Differential of polynomial function

❖ Average rate of change

- If we generalize this, when x changes from a to b, the amount of change (increase) of x and y becomes $\Delta y = f(b)-f(a)$, $\Delta x = b - a$
- The average rate of change equation can be expressed as follows



x가 a에서 b로 변할 때 평균변화율

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$$

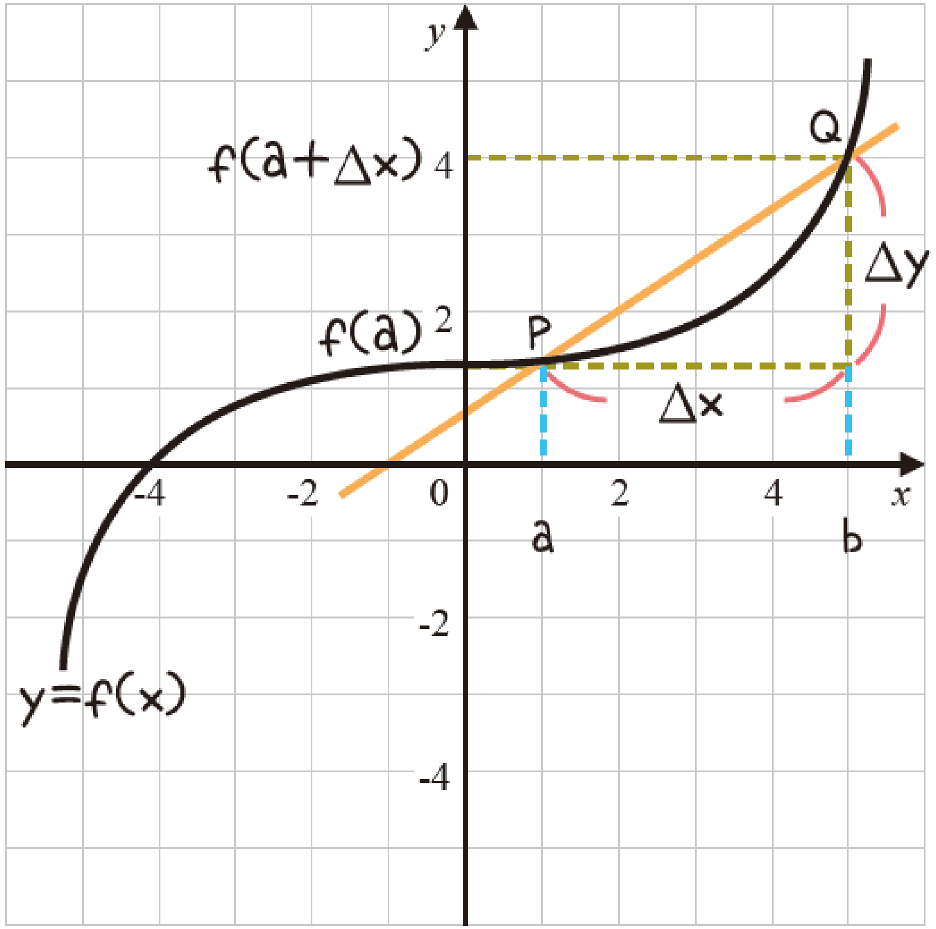
Differential of polynomial function

❖ Average rate of change

- The average rate of change can also be expressed in another way, which is expressed by replacing Δx with h
- If Δx is substituted with h in $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$, that is, if b - a = h, b = a + h

$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a} = \frac{f(a+h)-f(a)}{h}$$

Differential of polynomial function



두 정점을

지나는

직선의 기울기

Differential of polynomial function

❖ Average rate of change

- The slope at two fixed points (P and Q) is the average rate of change
- Python calculates the average rate of change as follows
- First, use the Symbols in the SymPy library to convert x into a symbolic variable, and define a function in fx
- After defining the function, use fx.subs (variable, substitution value) to obtain the y value
- Let's find the average rate of change of the two points (0, 4) and (2, 12) on the function $2x^2$ + 4x + 7

In [4]:

```
# 파이썬 NumPy 라이브러리를 호출합니다
from sympy import symbols
```

Differential of polynomial function

MATHEMATIC

❖ **Average rate of change**

```python
# 평균변화율을 구할 수 있는 함수를 정의합니다
def average(a,b):
    m = max(a,b)     # a, b의 최댓값
    n = min(a,b)     # a, b의 최솟값
    x = symbols('x') # 기호변수 x 선언

    fx = 2 * x ** 2 + 4 * x + 7 # 2x^2 + 4x + 7 함수 정의
    fb = fx.subs(x, m) # 함수에 m 대입
    fa = fx.subs(x, n) # 함수에 n 대입

    result = (fb - fa) / (b - a)
    return result
print(average(0,2))
```

```
8
```

❖ Average rate of change

연습 문제

함수 $f(x)=x^2$에 대해 x 값이 1에서 k까지 변할 때 평균변화율은 15입니다. 그렇다면 k 상수 값은 얼마일까요?

문제 풀이

$$\frac{\Delta y}{\Delta x}=\frac{f(k)-f(1)}{k-1}=\frac{k^2-1}{k-1}=\frac{(k+1)(k-1)}{k-1}=k+1=15 \text{ 이므로,}$$

$k=14$입니다.

Differential of polynomial function

- **Derivative (Instantaneous rate of change)**
  - The derivative is the average rate of change as the increase in x approaches 0
  - Derivative are expressed as follows

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

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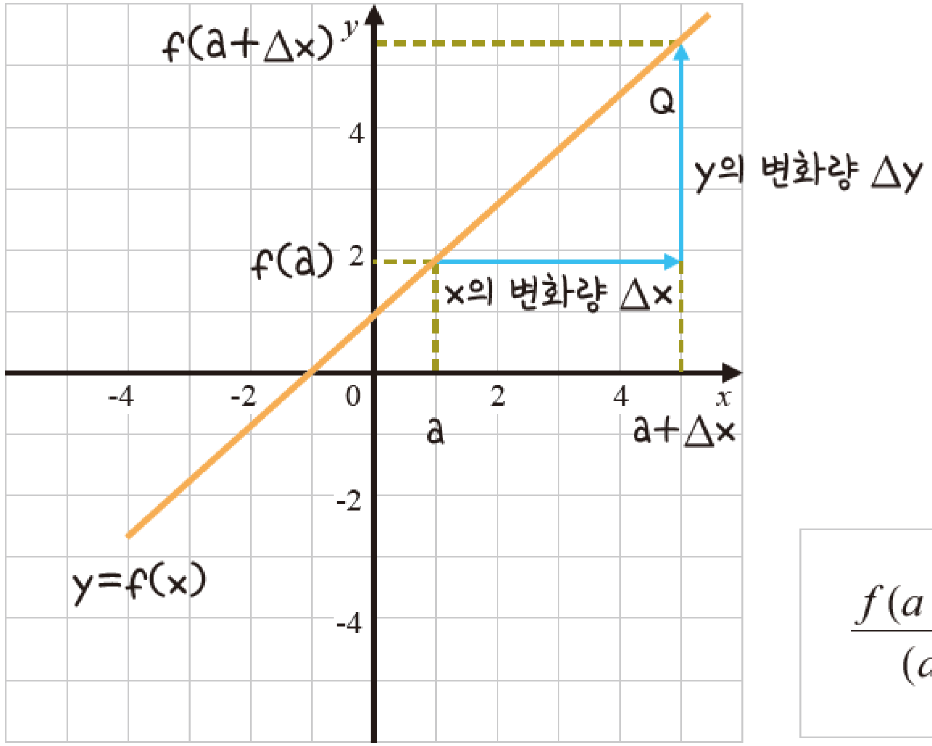
❖ Derivative (Instantaneous rate of change)

- Generally, if function y = f(x), the average rate of change when x changes from a to a + Δx is expressed as follows



$$\frac{f(a+\Delta x)-f(a)}{(a+\Delta x)-a}=\frac{f(a+\Delta x)-f(a)}{\Delta x}$$

Differential of polynomial function

❖ Derivative (Instantaneous rate of change)

- At this time, the derivative is that the amount of change (Δx) of x is sent close to 0
- That is, in the average rate of change when x changes from a to a + Δx, the limit value when $\Delta x \to 0$ is called the derivative or instantaneous rate of change at x = a, and is expressed as follows

$$f'(a)$$

- Since Δx is often expressed as h, f'(a) can be expressed as

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Differential of polynomial function

❖ Derivative (Instantaneous rate of change)

- When a +h = x, h → 0 can also be expressed as x → a

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

- The derivative f'(a) at x = a can be summarized as follows

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
$$= \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

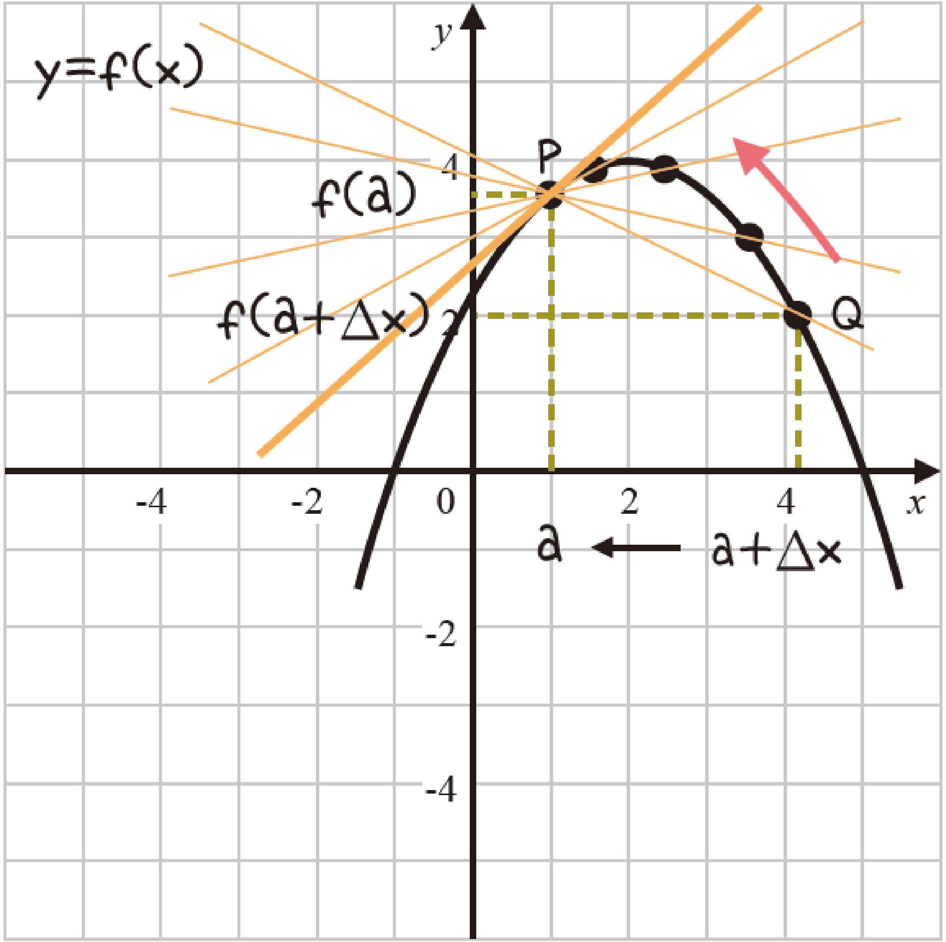
Differential of polynomial function

❖ Derivative (Instantaneous rate of change)

- Let's take a look at the geometric meaning of derivative
- Derivative f'(a) is the slope of the tangent line at x = a when there is function y = f(x)
- For example, given two points, P(a, f (a)), Q(a + Δx, f(a + Δx)), the slope of the straight line PG is defined as the average

  rate of change $\lim_{\Delta x \to 0} \frac{f(a+\Delta x)-f(a)}{\Delta x}$
- Δx → 0 means 'a + Δx approaches a'

Differential of polynomial function



Differential of polynomial function

❖ Derivative (Instantaneous rate of change)

- Find the slope at the point P on the line if the line turns counterclockwise and a + Δx approaches a
- This is the slope of the tangent line in (a, f(a))
- The geometric meaning of f'(a) can be summarized as follows

함수 $y=f(x)$가 있을 때 $(a, f(a))$에서 접선의 기울기

Differential of polynomial function

❖ Derivative (Instantaneous rate of change)

- In Python, you can easily differentiate function fx using a Derivative function
- The doit() function turns the object into a derivative

In [5]:

```
# 파이썬 NumPy 라이브러리를 호출합니다
from sympy import Derivative, symbols

# 평균변화율을 구할 수 있는 함수를 정의합니다
x = symbols('x') # x를 기호변수화
fx = 2 * x ** 2 + 4 * x + 7

# fprime라는 Derivative 클래스의 객체를 생성한 후
# subs() 메서드를 사용하여 x = 3에서의 미분계수 f′(3)을 구합니다
fprime = Derivative(fx, x).doit() # x에 대해 미분
n = fprime.subs({x: 3})
print("fx에서 x = 3에서의 순간변화율(미분계수는) ", n , "입니다")
```

Differential of polynomial function

MATHEMATIC

❖ **Derivative (Instantaneous rate of change)**

연습 문제

함수 $f(x)=x^2+3x$에 대해 x 값이 1에서 5로 변할 때 평균변화율과 $x=k$에서의 순간변화율이 같을 때 k 상수 값을 구하세요.

문제 풀이

x 값이 1에서 5까지 변할 때 평균변화율은 다음과 같습니다.

$$\frac{f(5)-f(1)}{5-1}=\frac{40-4}{5-1}=\frac{36}{4}=9$$

Differential of polynomial function

❖ **Derivative (Instantaneous rate of change)**

또 $x=k$에서 순간변화율은 다음과 같습니다.

$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$
$$=\lim_{h\to 0}\frac{\{(k+h)^2+3(k+h)\}-(k^2+3k)}{h}$$
$$=\lim_{h\to 0}\frac{k^2+2kh+h^2+3k+3h-k^2-3k}{h}$$
$$=\lim_{h\to 0}\frac{2kh+h^2+3h}{h}$$
$$=\lim_{h\to 0}\frac{h(2k+h+3)}{h}$$
$$=2k+h+3=9$$

따라서 $h=0$이고, $2k=6$이므로 $k=3$입니다.

Differential of polynomial function

❖ Derivative

- The derivative is a function f'(x) obtained by differentiating the function f(x)
- Since we learned the differential coefficient, it would not be difficult to repeat the process of obtaining the differential (f'(a)) when x = a and obtaining the differential (f'(b)) when x = b
- For example, let's find f(1), f(2), and f(3) from the function f(x) = x^2 + x

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$$f'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

Differential of polynomial function

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❖ Derivative

- Derivative can be calculated as functions to reduce the hassle of calculating each time
- It would be convenient to apply the expression f(x) as follows

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

- Finding the derivative f'(x) of function f(x) is called 'differentiating function f(x) with respect to x', and its calculation method is called differentiation

Differential of polynomial function

❖ Derivative

- Here are a number of different notation of derivatives

$$y', f'(x), \frac{dy}{dx}, \frac{d}{dx}f(x)$$

- The reason why there are many symbols that express derivatives is because many people have studied derivatives

표현 방법	y′	$\frac{\triangle y}{\triangle x}$
읽는 방법	와이 프라임	디와이 디엑스
의미	y를 미분(y의 순간변화율)	y를 x에 대해 미분(x에 대한 y의 순간변화율)
수학자	뉴턴	라이프니츠
연구	한 물체의 운동 방향 연구	특정한 두 변수의 관계 연구

Differential of polynomial function

❖ Derivative

- For reference, in $\frac{d}{dx}f(x)$, d is a symbol that means change(Δ), and the change is sent as a very small unit of 0
- In Python, the Derivative class can be used to easily obtain the derivative f'(x)

In [6]:

```
# 파이썬 SymPy 라이브러리를 호출합니다
from sympy import Derivative, symbols

# x 변수와 함수 fx를 정의합니다
x = symbols('x') # x를 기호변수화
fx = 2 * x ** 2 + 4 * x + 7

Derivative(fx, x).doit()
```

Differential of polynomial function

❖ **Derivative**

Out [6]:

4x+4

연습 문제

함수 $f(x) = 2x^2 - 1$의 도함수를 구하세요. 또 이 도함수를 이용하여 $f(x)$에서 $x =$ 6에서의 미분계수를 구하세요.

Differential of polynomial function

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❖ **Derivative**

문제 풀이

함수 $f(x) = 2x^2 - 1$의 도함수는 다음과 같습니다.

$$
\begin{aligned}
f'(x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{\{2(x+h)^2 - 1\} - (2x^2 - 1)}{h} \\
&= \lim_{h \to 0} (4x + 2h) = 4x
\end{aligned}
$$

Differential of polynomial function

❖ Derivative

- Derivative f'(6) = 4 × 6 = 24 in function f(x)
- At this time, if $f(x) = 2x^2 - 1$ is differentiated, it can be expressed as $f'(x) = 4x$

In [7]:

```
from sympy import Derivative, symbols
x = symbols('x')
fx = 2 * x ** 2 -1
Derivative(fx, x).doit()
```

Out [7]:

```
4x
```

Differential of polynomial function

❖ **Continuous and differentiability of function**

- A function can be differentiated at x = a means that there is a derivative at x = a
- This means that the following formula is established

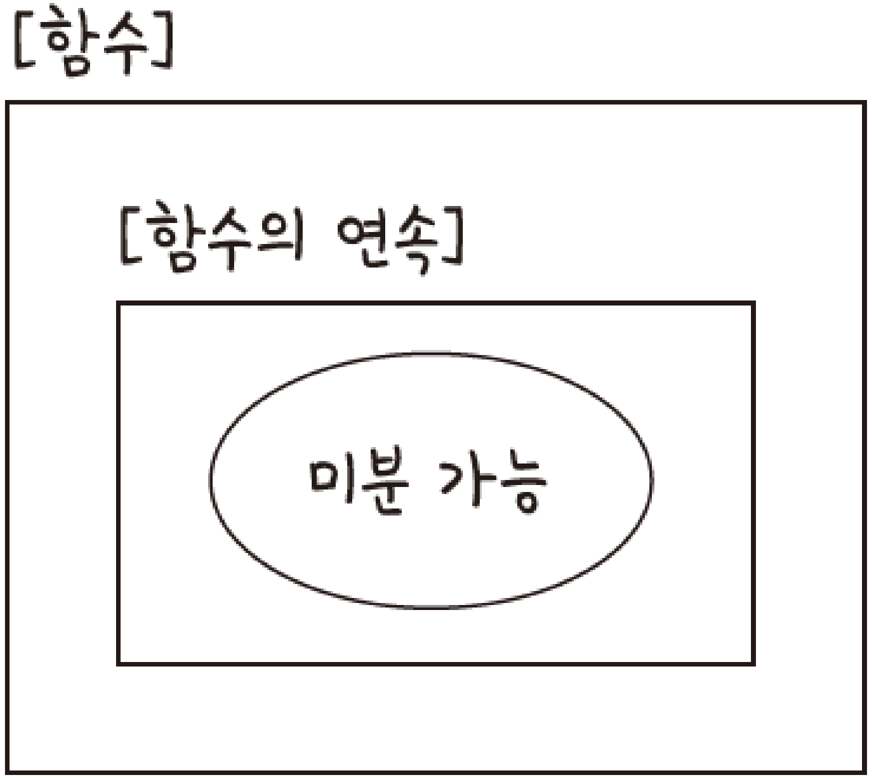
$$f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

- In the end, we can conclude that differentiability = derivative = the limit of the average rate of change

Differential of polynomial function

❖ **Continuous and differentiability of function**

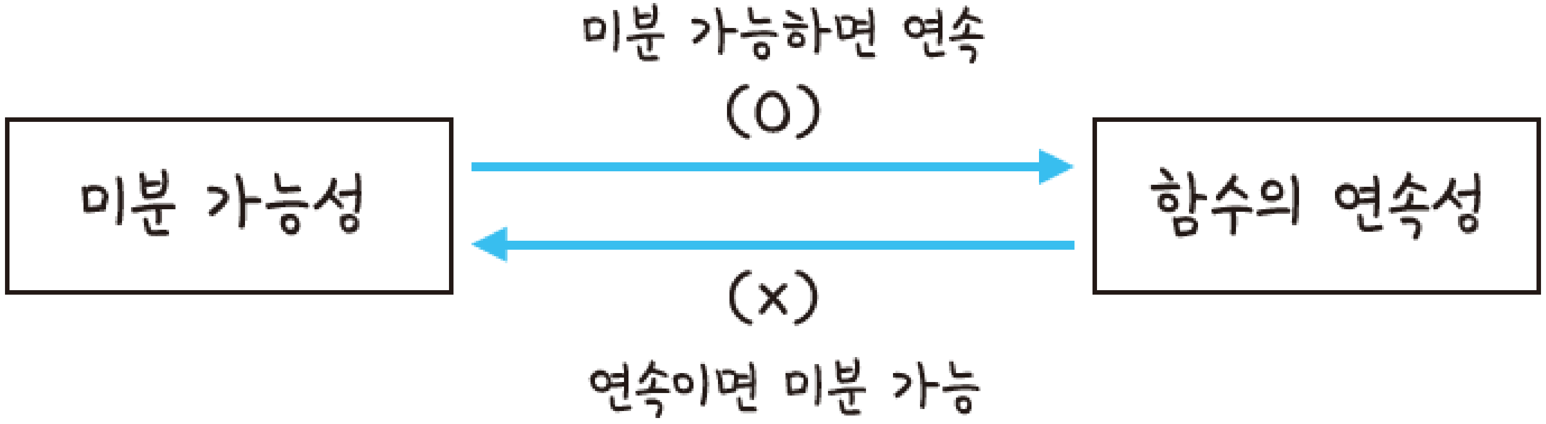
- Can be defined as continuous if differentiable



Differential of polynomial function

❖ Continuous and differentiability of function

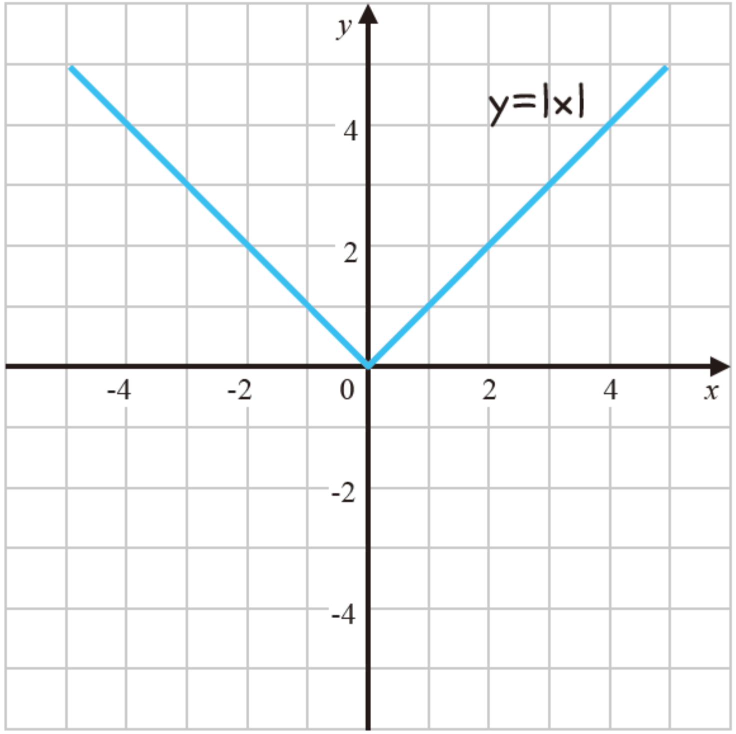
- Continuity of the function said that f(x) satisfies the $\lim_{x \to a} f(x) = f(a)$ when it is continuous at x = a
- If you think about this in connection with the possibility of differential, it is as follows
  - (1) Continuous if differentiable
  - (2) Continuous is not always differentiable



Differential of polynomial function

❖ Continuous and differentiability of function

- The function y = |x| is continuous, but this is a representative non-differentiable example
- x > When 0 is y = x, the slope of the tangent line is 1 (because the slope of y = x itself is 1)
- When x < 0, y = -x, so the slope of the tangent line is -1, but when x = 0, it is impossible to differentiate because the slope of the tangent line cannot be accurately obtained



Differential of polynomial function

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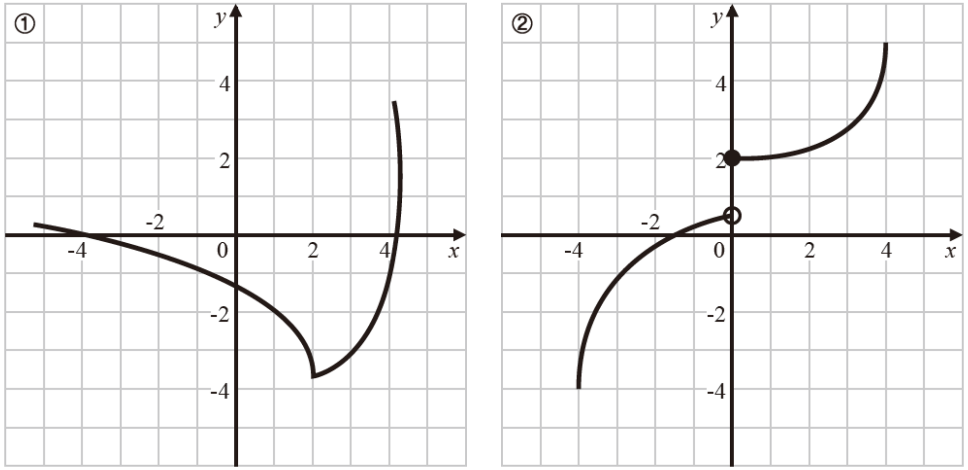
❖ Continuous and differentiability of function

- There are two main types that cannot be differentiated
  - Differential is not possible when the graph is sharp or discontinuous, such as ① and ②
  - In other words, if you think of it as a graph, a continuity is a series, and a discontinuity is a broken one
  - Differentiable is a smooth connection in the graph, meaning the slope of the tangent at that point
  - Differential is not possible when the graph is broken or pointed



Differential of polynomial function

❖ **Continuous and differentiability of function**

연습 문제

(1) 함수 $f(x) = x^2$이 $x = 2$에서 미분 가능할까요?

(2) 함수 $f(x) = |x|$가 $x = 1$에서 미분 가능할까요?

Differential of polynomial function

❖ Continuous and differentiability of function

문제 풀이

(1) $f'(2) = \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} = \frac{4 + 4h + h^2 - 4}{h} = \frac{(4+h)h}{h} = 4$

따라서 $f'(2)$가 존재하므로 함수 $f(x)$는 $x = 2$에서 미분 가능합니다.

(2) $f'(1) = \lim_{h \to 0} \frac{|1+h| - |1|}{h} = \lim_{h \to 0} \frac{|h|}{h}$

$$\lim_{h \to 0-} \frac{|h|}{h} = \lim_{h \to 0-} \frac{-h}{h} = -1, \quad \lim_{h \to 0+} \frac{|h|}{h} = \lim_{h \to 0+} \frac{h}{h} = 1$$

즉, 좌극한과 우극한의 값이 다르므로 $f'(1)$은 존재하지 않습니다. 따라서 미분 불가능합니다.

Differential of polynomial function

❖ Differentiation of polynomial function

- The following is the basic formula for differentiation used in polynomial functions
  - (1) If $f(x) = c$, $f'(x) = 0$ (where c is constant)
  - (2) If $f(x) = x^n$, $f'(x) = nx^{n-1}$(where n is a natural number)
  - (3) $\{cf(x)\}' = cf'(x)$ (where c is constant)
  - (4) $\{f(x) \pm g(x)\}' = f'(x) \pm g'(x)$
  - (5) $\{f(x)g(x)\}' = f'(x)g(x) + f(x)g'(x)$

Differential of polynomial function

❖ Differentiation of polynomial function

- Examples
  - (1) If $f(x) = c$, $f'(x) = 0$
    - If you differentiate the constant function, it’s zero
    - $Y = 10$ is differentiated, $y' = 0$
  - (2) If $f(x) = x^n$, $f'(x) = nx^{n-1}$
    - Differentiating $y = x^2$ gives $y' = 2x^1 = 2x$
    - This means that you can reduce the exponent by one after sending it forward
  - (3) $\{cf(x)\}' = cf'(x)$
    - If $y = 5x^3$ is differentiated, $y' = 5\times3x^2$ becomes $y' = 15x^2$

Differential of polynomial function

MATHEMATIC

❖ Differentiation of polynomial function

- (4) $\{f(x) \pm g(x)\}' = f'(x) \pm g'(x)$
  - In other words, if you expressed it as the sum of two functions that can be differentiated, you can differentiate each and connect the signs separately
  - For example, let's differentiate $y = 5x^2 + 2x$
  - First, if you differentiate $5x^2$, you get $5x^2 = 5\times2x = 10x$, and if you differentiate $2x$, you get 2
  - $y = 5x^2 + 2x$ is differentiated, $y' = 10x + 2$

Differential of polynomial function

❖ Differentiation of polynomial function

- (5) {f (x)g(x)}' = f'(x)g(x) + f(x)g’(x)
  - Differential of functions expressed as products

$$y = (4x^2+2)(2x-3)$$

(미분) (그대로) (미분)

$$y = (4\times 2x)(2x-3) + (4x^2+2)(2x)$$

Differential of polynomial function

❖ Differentiation of polynomial function

- For example, differentiating a function y = (5x + 2)(x^2 - 3) yields y' = 5(x^2 - 3) + (5x + 2)2x, as follows

$$y = (5x+2)(x^2-3)$$

(미분)

(그대로)

(미분)

$$y' = 5(x^2-3) + (5x+2)2x$$

Differential of polynomial function

- **Differentiation of polynomial function**
  - For reference, the differential formula for fraction is

$$y = \frac{f(x)}{g(x)} \rightarrow y' = \frac{= f'(x)g(x) - f(x)g'(x)}{\{g(x)\}^2}$$

Differential of polynomial function

❖ Differentiation of polynomial function

- Python can use SymPy to calculate differentials
- If you enter a function to be differentiated and a variable to be differentiated with the sym.diff argument, you can differentiate and output the result

In [8]:

```
# 파이썬 SymPy 라이브러리를 호출합니다
import sympy as sym

# x 변수를 선언하고 diff() 함수를 사용하여 미분을 계산합니다
x = sym.Symbol('x')
a = sym.diff((2*x**3 + 3*x**2 + x + 1),x)
print(a)
```

Out [8]:

```
6*x**2 + 6*x + 1
```

Differential of polynomial function

❖ Differentiation of polynomial function

연습 문제

(1) 함수 $y = (2x + 1)(3x^2 + 2x)$를 미분하세요.

(2) 함수 $y = (x - 1)(x^2 + 3x + 1)$을 미분하세요.

Differential of polynomial function

MATHEMATIC

❖ **Differentiation of polynomial function**

문제 풀이

$$
\begin{aligned}
(1)\ y' &= \{(2x+1)(3x^2+2x)\}' \\
&= (2x+1)'(3x^2+2x) + (2x+1)(3x^2+2x)' \\
&= 2(3x^2+2x) + (2x+1)(6x+2) \\
&= (6x^2+4x) + (12x^2+4x+6x+2) \\
&= 18x^2+14x+2
\end{aligned}
$$

Differential of polynomial function

❖ **Differentiation of polynomial function**

$$(2)\ y' = \{(x-1)(x^2+3x+1)\}'$$

$$= (x-1)'(x^2+3x+1) + (x-1)(x^2+3x+1)'$$

$$= 1(x^2+3x+1) + (x-1)(2x+3)$$

$$= (x^2+3x+1) + (2x^2+3x-2x-3)$$

$$= 3x^2+4x-2$$