Data and Computer Communications

CHAPTER 6

Error Detection and Correction

Types of Errors

- An error occurs when a bit is altered between transmission and reception

  = bit error
  - Binary 1 is transmitted and binary 0 is received
  - Binary 0 is transmitted and binary 1 is received

Single bit errors

컨트롤 하기 쉽다

- Isolated error that alters one bit but does not affect nearby bits
- Can occur in the presence of white noise

Burst errors

데이터 레이트가 높을 때 더 영향이 크다

- Contiguous sequence of *B* bits in which the first and last bits and any number of intermediate bits are received in error
- Can be caused by impulse noise or by fading in a mobile wireless environment
- Effects of burst errors are greater at higher data rates

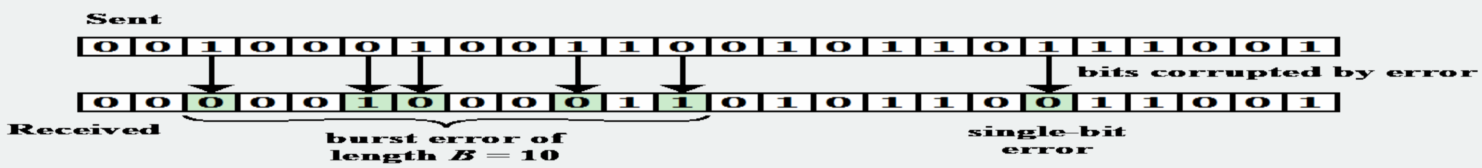


Figure 6.1 Burst and Single–Bit Errors

Error Detection

- Regardless of design you will have errors, resulting in the change of one or more bits in a transmitted frame

- Frames   단위로 데이터를 전송함.
  - Data transmitted as one or more contiguous sequences of bits

P_b
- Probability that a bit is received in error; also known as the bit error rate (BER)

P_1
- Probability that a frame arrives with no bit errors

P_2
- Probability that, with an error-detecting algorithm in use, a frame arrives with one or more undetected errors

pow(10, -31) 보다 작아야한다.

- The probability that a frame arrives with no bit errors decreases when the probability of a single bit error increases

- The probability that a frame arrives with no bit errors decreases with increasing frame length
  - The longer the frame, the more bits it has and the higher the probability that one of these is in error

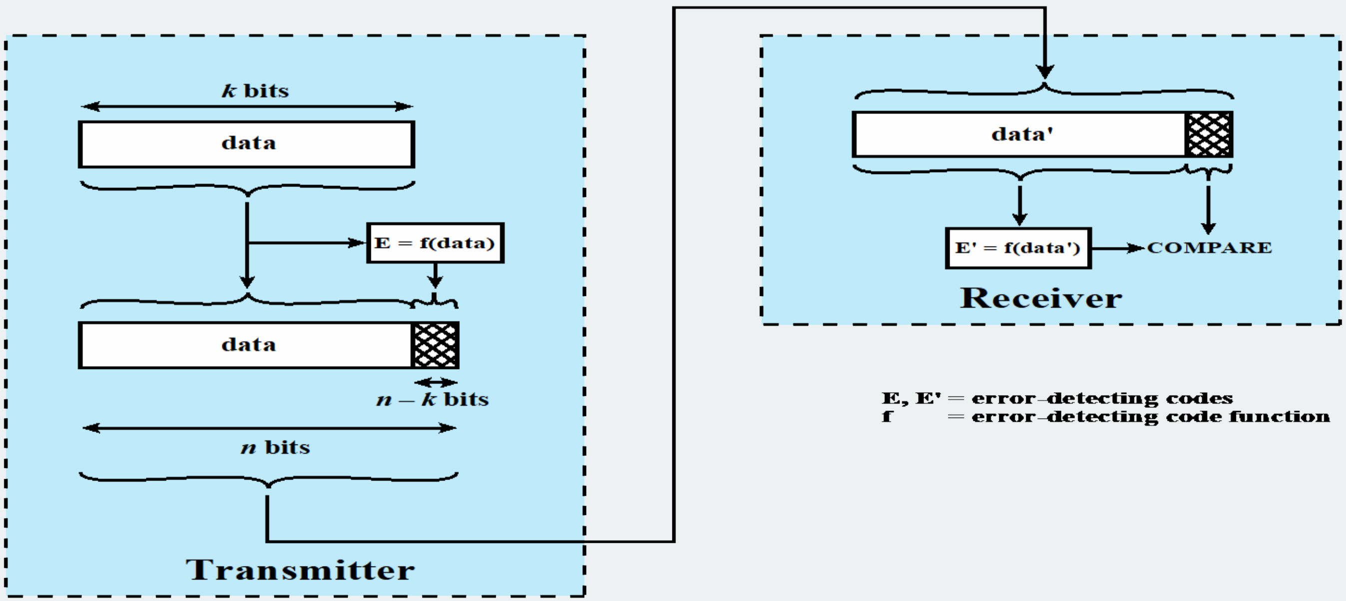


Figure 6.2 Error Detection Process

Parity Check

추가되는 비트는 단 1bit 이다

- The simplest error detecting scheme is to append a parity bit to the end of a block of data

- If any even number of bits are inverted due to error, an undetected error occurs

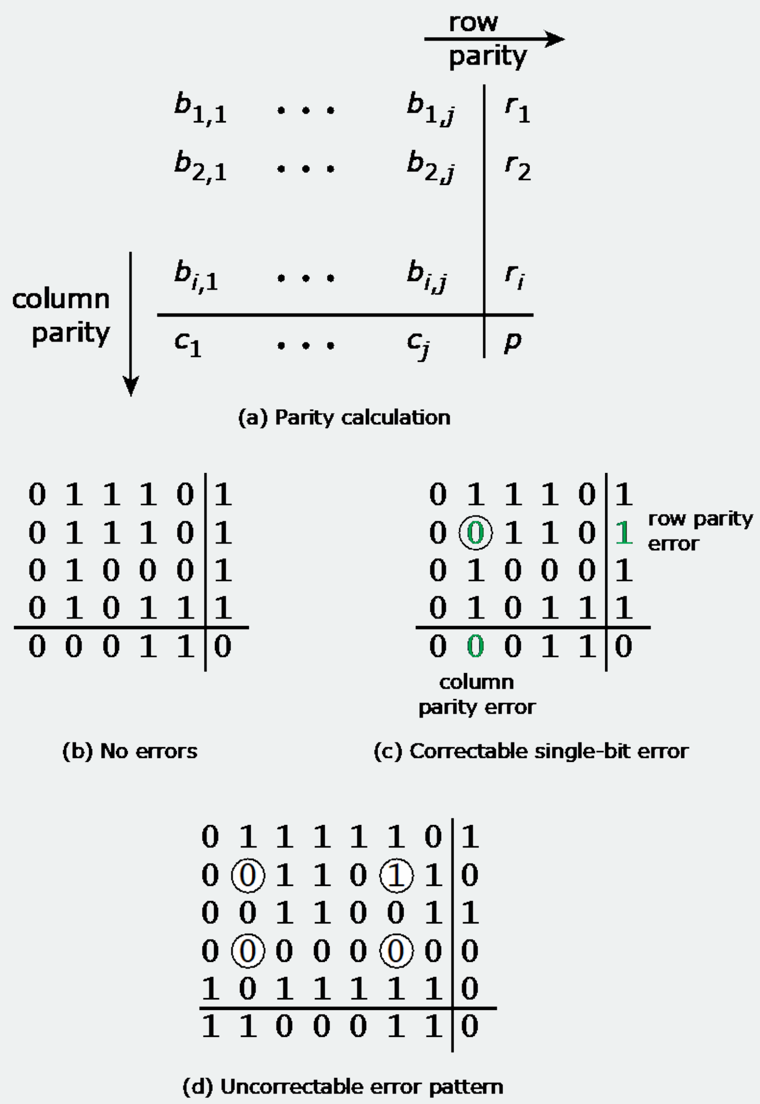


Figure 6.3 A Two-Dimensional Even Parity Scheme

The Internet Checksum

- Error detecting code used in many Internet standard protocols, including IP, TCP, and UDP
- Ones-complement operation
  - Replace 0 digits with 1 digits and 1 digits with 0 digits
- Ones-complement addition
  - The two numbers are treated as unsigned binary integers and added
  - If there is a carry out of the leftmost bit, add 1 to the sum (end-around carry)

Partial sum	0001 F203 F204
Partial sum	F204 F4F5 1E6F9
Carry	E6F9 1 E6FA
Partial sum	E6FA F6F7 1DDF1
Carry	DDF1 1 DDF2
Ones complement of the result	220D

(a) Checksum calculation by sender

Partial sum	0001 F203 F204
Partial sum	F204 F4F5 1E6F9
Carry	E6F9 1 E6FA
Partial sum	E6FA F6F7 1DDF1
Carry	DDF1 1 DDF2
Partial sum	DDF2 220D FFFF

(b) Checksum verification by receiver

Figure 6.4 Example of Internet Checksum

Cyclic Redundancy Check (CRC)

- One of the most common and powerful error-detecting codes
- Given a k bit block of bits, the transmitter generates an $(n - k)$bit frame check sequence (FCS) which is exactly divisible by some predetermined number
- Receiver divides the incoming frame by that number
  - If there is no remainder, assume there is no error

CRC Process

- Modulo 2 arithmetic
  - Uses binary addition with no carries
- Polynomials
  - Express all values as polynomials in a dummy variable X, with binary coefficients
  - Coefficients correspond to the bits in the binary number
- Digital logic
  - Dividing circuit consisting of XOR gates and a shift register
  - Shift register is a string of 1-bit storage devices
  - Each device has an output line, which indicates the value currently stored, and an input line
  - At discrete time instants, known as clock times, the value in the storage device is replaced by the value indicated by its input line
  - The entire register is clocked simultaneously, causing a 1-bit shift along the entire register

generate polynomial

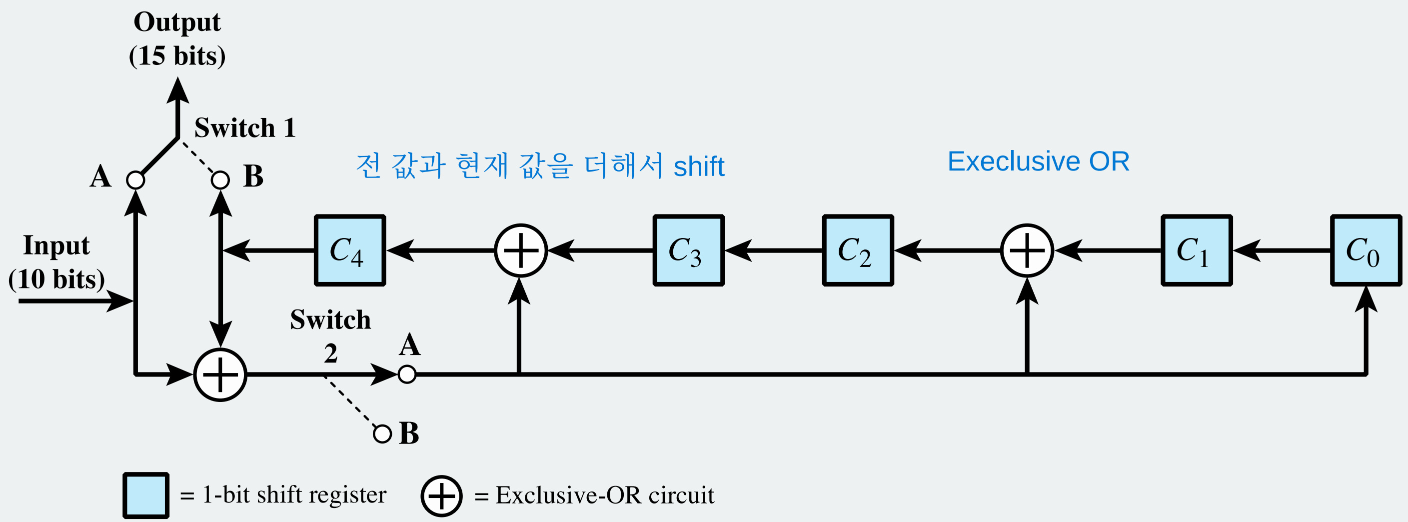
= 1 1 0 1 0 1

$$Q(X) = X^9 + X^8 + X^6 + X^4 + X^2 + X$$

$$P(X) \rightarrow X^5 + X^4 + X^2 + 1 \;\big/\; X^{14} + X^{12} + X^8 + X^7 + X^5 \leftarrow X^5 D(X)$$

$$\begin{array}{l}
X^{14} + X^{13} + X^{11} + X^9 \\
\hline
X^{13} + X^{12} + X^{11} + X^9 + X^8 \\
X^{13} + X^{12} + X^{10} + X^8 \\
\hline
X^{11} + X^{10} + X^9 + X^7 \\
X^{11} + X^{10} + X^8 + X^6 \\
\hline
X^9 + X^8 + X^7 + X^6 + X^5 \\
X^9 + X^8 + X^6 + X^4 \\
\hline
X^7 + X^5 + X^4 \\
X^7 + X^6 + X^4 + X^2 \\
\hline
X^6 + X^5 + X^2 \\
X^6 + X^5 + X^3 + X \\
\hline
X^3 + X^2 + X \leftarrow R(X)
\end{array}$$

Figure 6.5 Example of Polynomial Division



(a) Shift-register implementation

	C_4	C_3	C_2	C_1	C_0	$C_4 \approx C_3 \approx I$	$C_4 \approx C_1 \approx I$	$C_4 \approx I$	I = input
Initial	0	0	0	0	0	1	1	1	1
Step 1	1	0	1	0	1	1	1	1	0
Step 2	1	1	1	1	1	1	1	0	1
Step 3	1	1	1	1	0	0	0	1	0
Step 4	0	1	0	0	1	1	0	0	0
Step 5	1	0	0	1	0	1	0	1	0
Step 6	1	0	0	0	1	0	0	0	1
Step 7	0	0	0	1	0	1	0	1	1
Step 8	1	0	0	0	1	1	1	1	0
Step 9	1	0	1	1	1	0	1	0	1
Step 10	0	1	1	1	0				

Message to be sent

(b) Example with input of 1010001101

Figure 6.6 Circuit with Shift Registers for Dividing by the Polynomial $X^5 + X^4 + X^2 + 1$

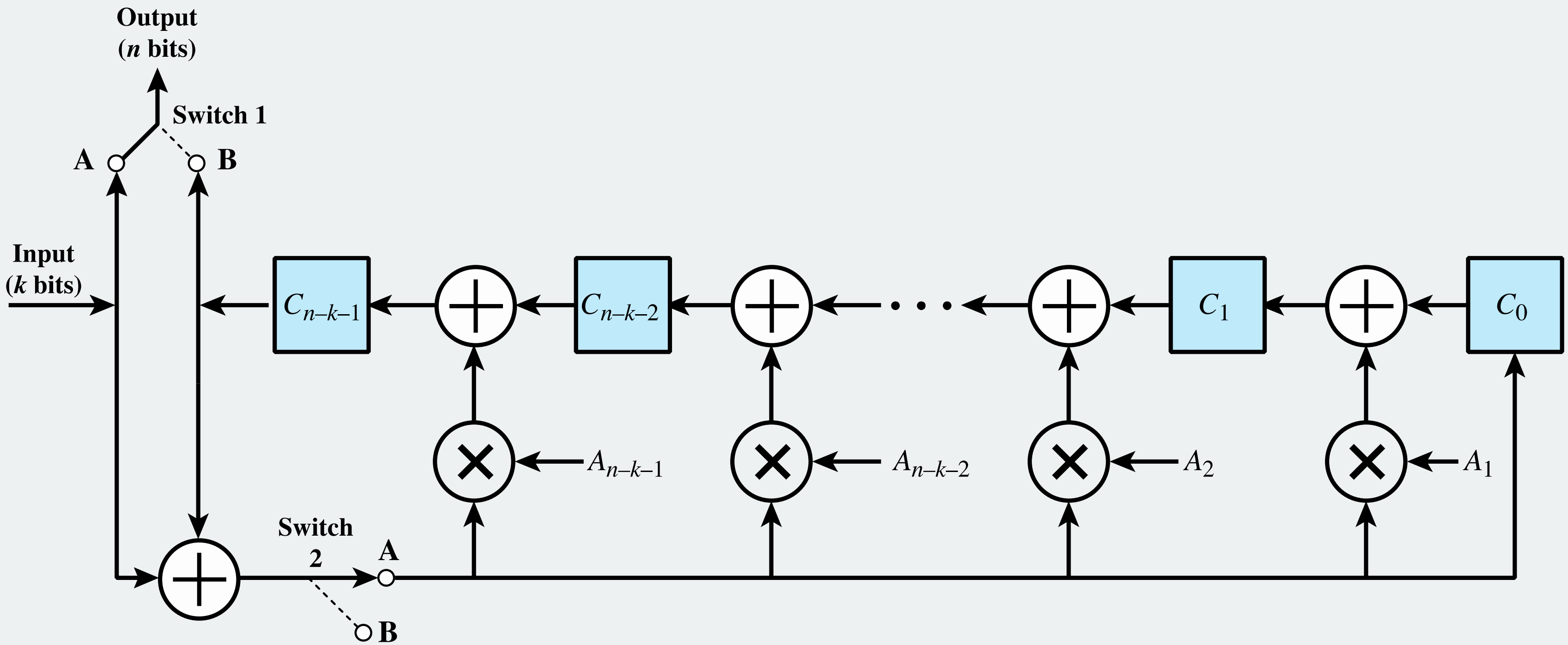


Figure 6.7 General CRC Architecture to Implement Divisor
$(1 + A_1X + A_2X^2 + \ldots + A_{n-k-1}X^{n-k-1} + X^{n-k})$

Forward Error Correction

= Channel Coding

- Correction of detected errors usually requires data blocks to be retransmitted
- Not appropriate for wireless applications:
  - The bit error rate (BER) on a wireless link can be quite high, which would result in a large number of retransmissions
  - Propagation delay is very long compared to the transmission time of a single frame
- Need to correct errors on basis of bits received

Codeword
• On the transmission end each k-bit block of data is mapped into an n-bit block ($n > k$) using a forward error correction (FEC) encoder

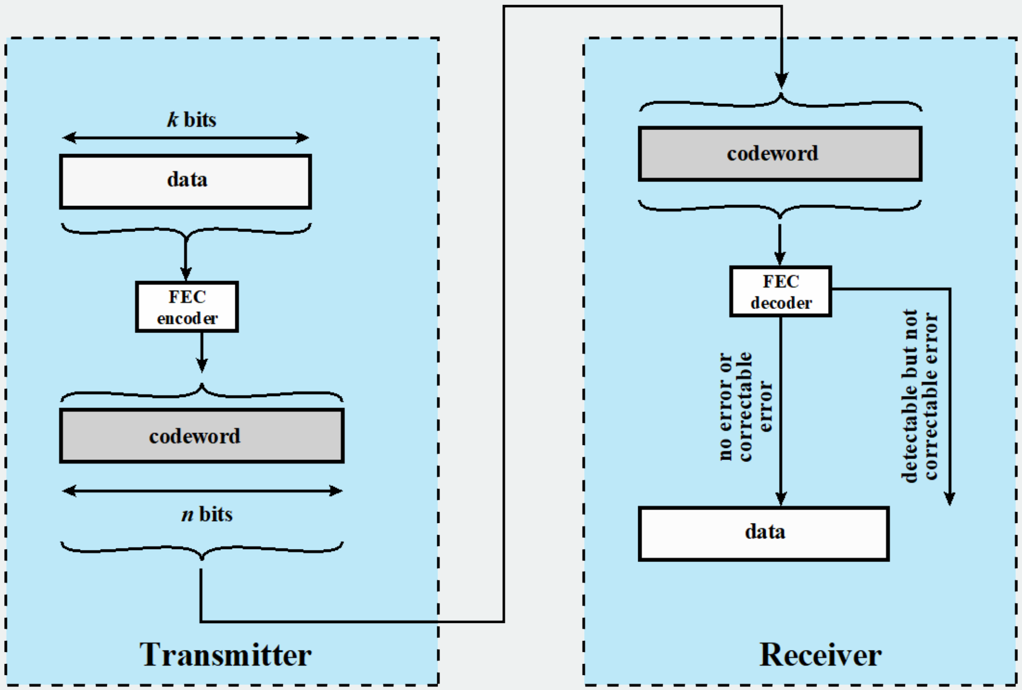


Figure 6.8 Error Correction Process

Block Code Principles

- Hamming distance
  - $d(v_1, v_2)$ between two n–bit binary sequences v_1 and v_2 is the number of bits in which v_1 and v_2 disagree
- Redundancy of the code
  - The ratio of redundant bits to data bits $(n-k)/k$
- Code rate
  - The ratio of data bits to total bits k/n
  - Is a measure of how much additional bandwidth is required to carry data at the same data rate as without the code

Block Code Principles

- Block code example (k = 2, n=5)
  - Data : 00, 01, 10, 11
  - Codeword : 00000, 00111, 11001, 11110

- Hamming distance (d) = 3
- Correction : $\left\lfloor \frac{d-1}{2} \right\rfloor$
- Detection : $d-1$

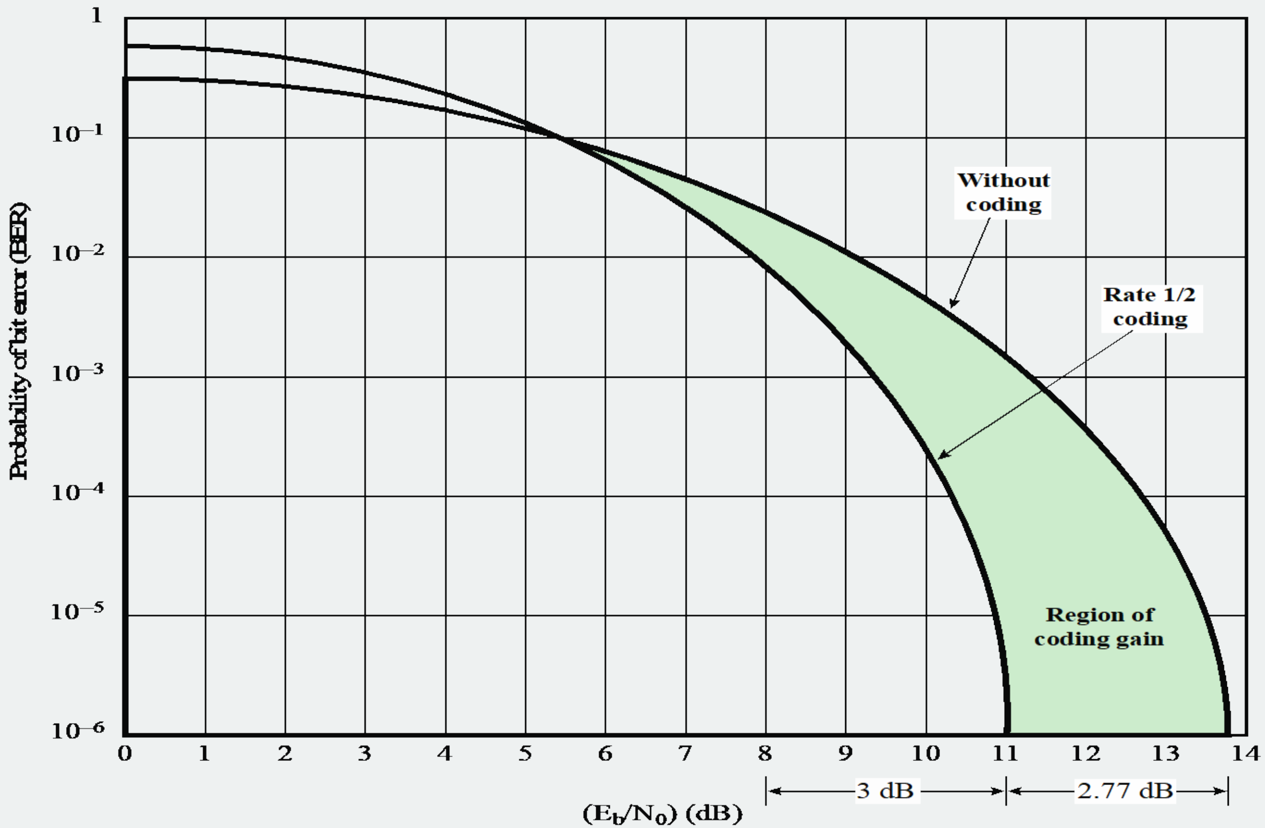


Figure 6.9 How Coding Improves System Performance