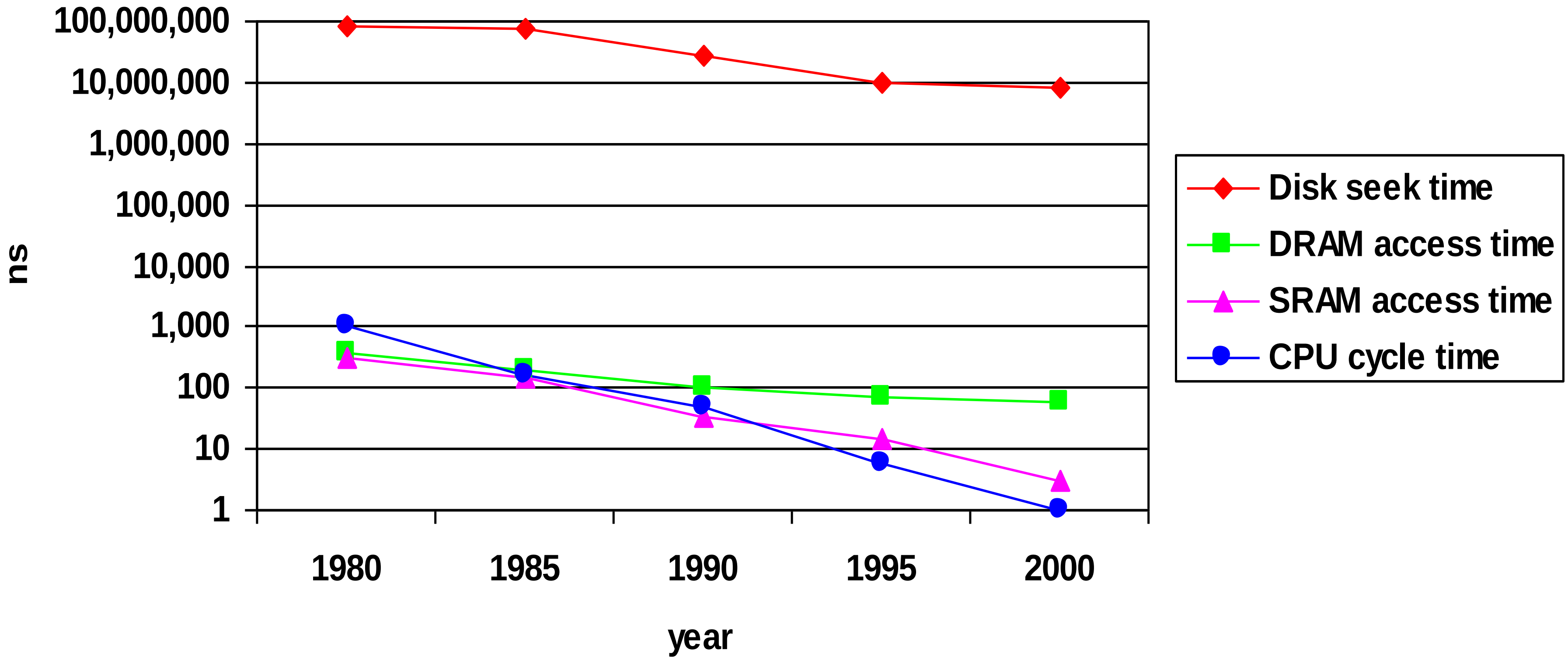
OPTIMIZE MEMORY ACCESS

Jo, Heeseung

The CPU-Memory Gap

The increasing gap between DRAM, disk, and CPU speeds



Principle of Locality (1)

Temporal locality

- Recently referenced items are likely to be referenced in the near future

Spatial locality

- Items with nearby addresses tend to be referenced close together in time

Principle of Locality (2)

Locality example:

```
sum = 0;
for (i = 0; i < n; i++)
    sum += a[i];
return sum;
```

Data

- Reference array elements in succession — Spatial locality
- Reference sum each iteration — Temporal locality

Instructions

- Reference instructions in sequence — Spatial locality
- Cycle through loop repeatedly — Temporal locality

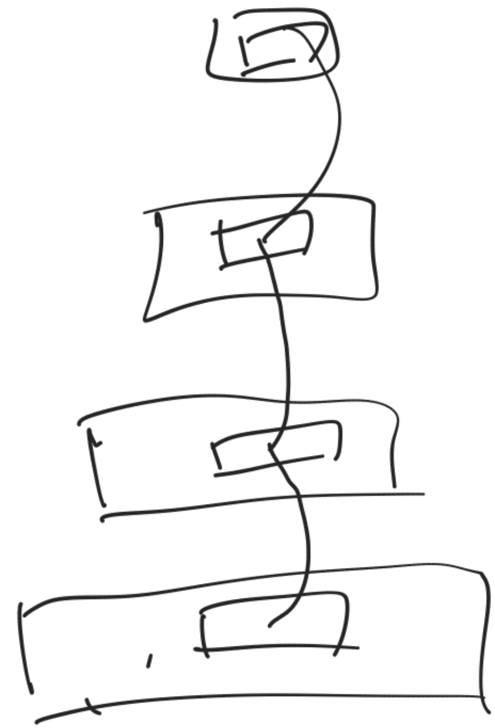
Principle of Locality (3)

How to exploit temporal locality?

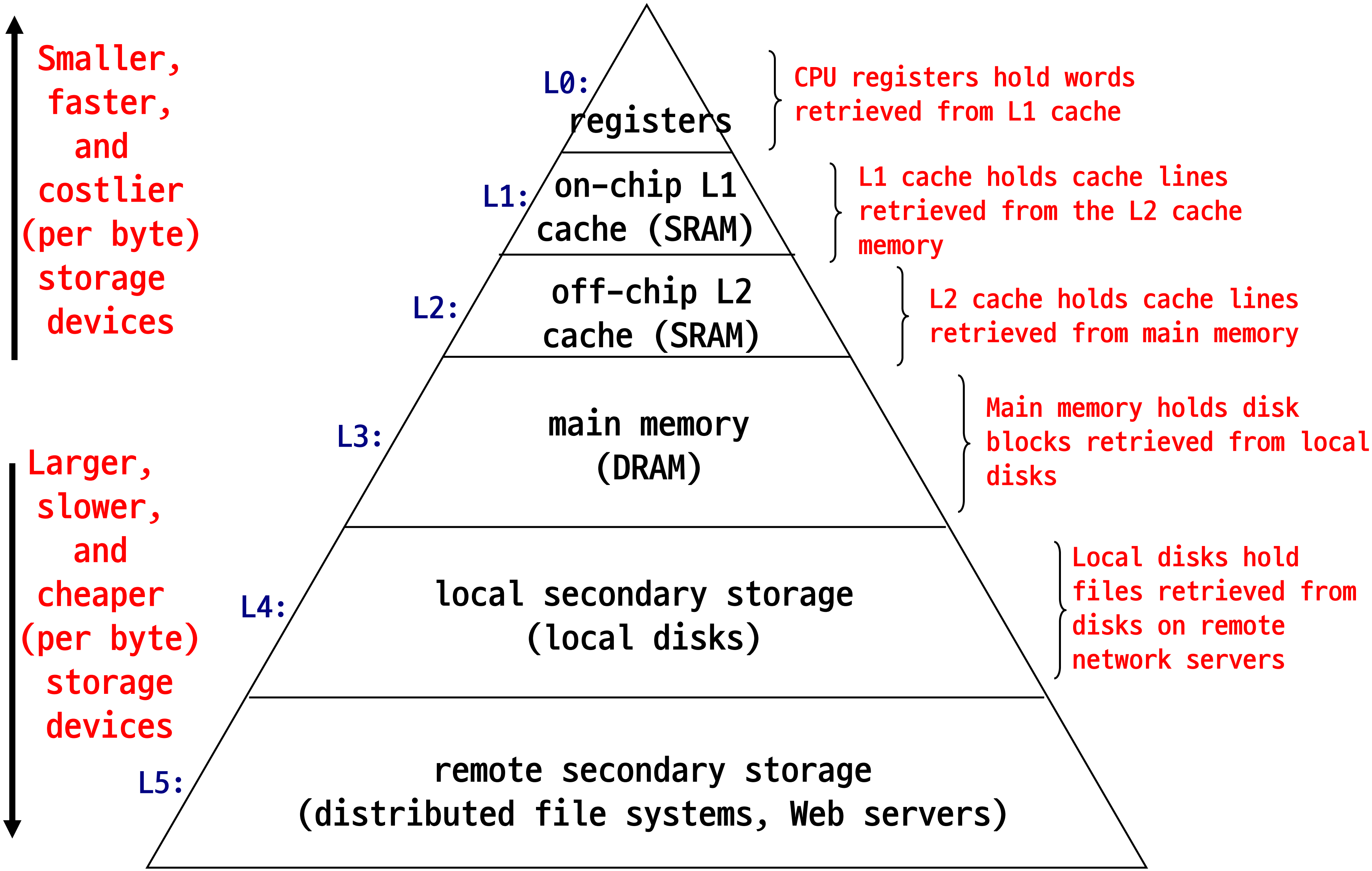
- Speed up data accesses by caching data in faster storage
- Caching in multiple levels: form a memory hierarchy:
  - The lower levels of the memory hierarchy tend to be slower, but larger and cheaper

How to exploit spatial locality?

- Larger cache line size
  - Cache nearby data together



Memory Hierarchy

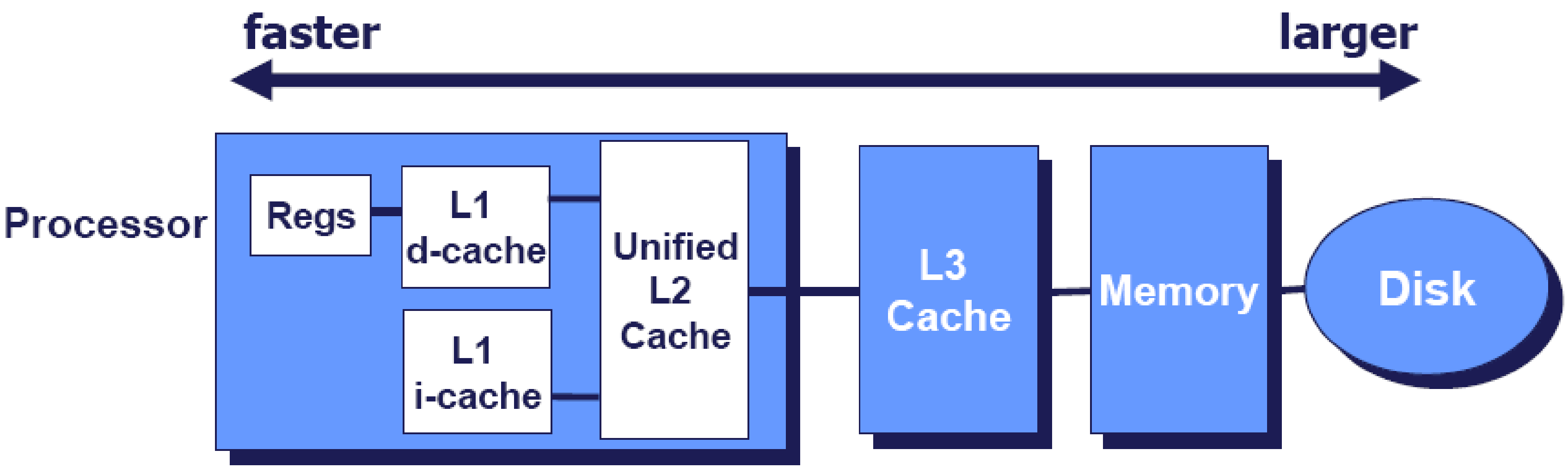


Caching (1)

Cache

- A smaller, faster storage
- Improves the average access time
- Exploits both temporal and spatial locality

공간↓ 속도↑



Caching (2)

Cache performance metrics

- Average memory access time = $T_{hit} + R_{miss} * T_{miss}$
- Hit time (T_{hit})
  - Time to deliver a line in the cache to the processor
  - Includes time to determine whether the line is in the cache
  - 1 clock cycle for L1, 3 ~ 8 clock cycles for L2
- Miss rate (R_{miss})
  - Fraction of memory references not found in cache (misses/references)
  - 3 ~ 10% for L1, < 1% for L2
- Miss penalty (T_{miss})
  - Additional time required because of a miss
  - Typically 25 ~ 100 cycles for main memory

Caching (3)

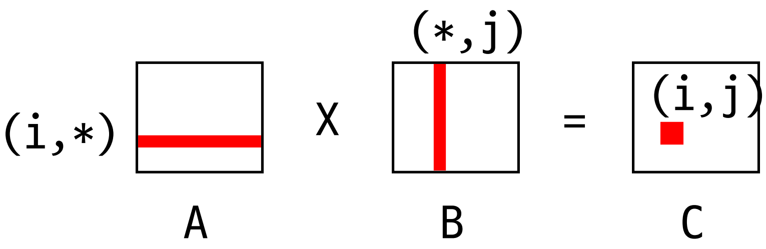
Cache design issues

- Cache size
  - 8KB ~ 64KB for L1
- Cache line size
  - Typically, 32B or 64B for L1
- Lookup
  - Fully associative
  - Set associative: 2-way, 4-way, 8-way, 16-way, etc.
  - Direct mapped
- Replacement
  - LRU (Least Recently Used)
  - FIFO (First-In First-Out), etc.

Matrix Multiplication (1)

Description

- Multiply N x N matrices
- $O(N^3)$ total operations



*Variable **sum** held in register*

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

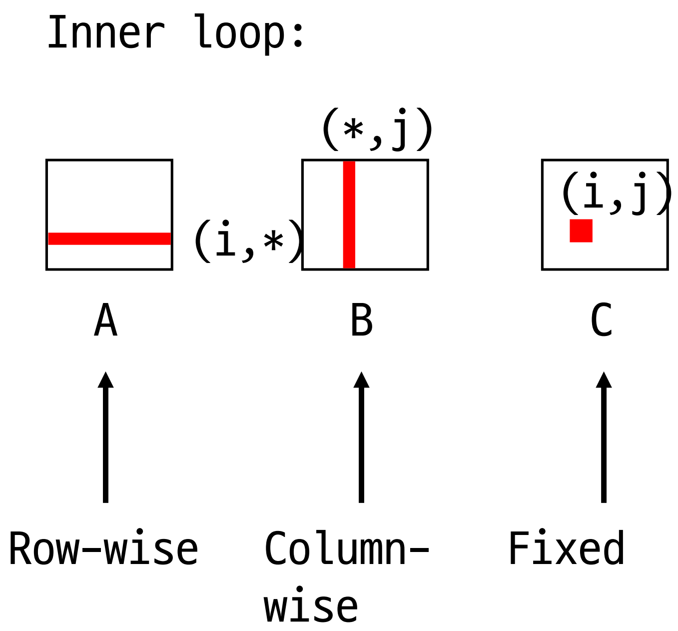
Assumptions

- Line size = 32B (big enough for 4 64-bit words)
- Matrix dimension (N) is very large
- Cache is not big enough to hold multiple rows

Matrix Multiplication (2)

Matrix multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```



Misses per Inner Loop Iteration:

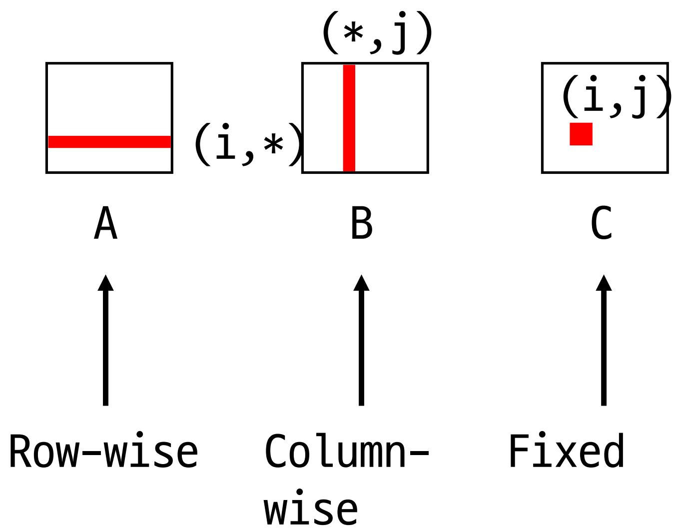
A	B	C
0.25	1.0	0.0

Matrix Multiplication (3)

Matrix multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

Inner loop:



Misses per Inner Loop Iteration:

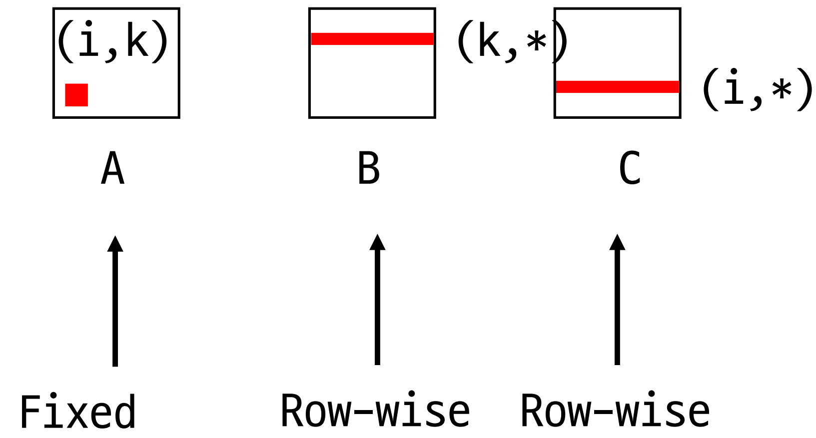
A	B	C
0.25	1.0	0.0

Matrix Multiplication (4)

Matrix multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



Misses per Inner Loop Iteration:

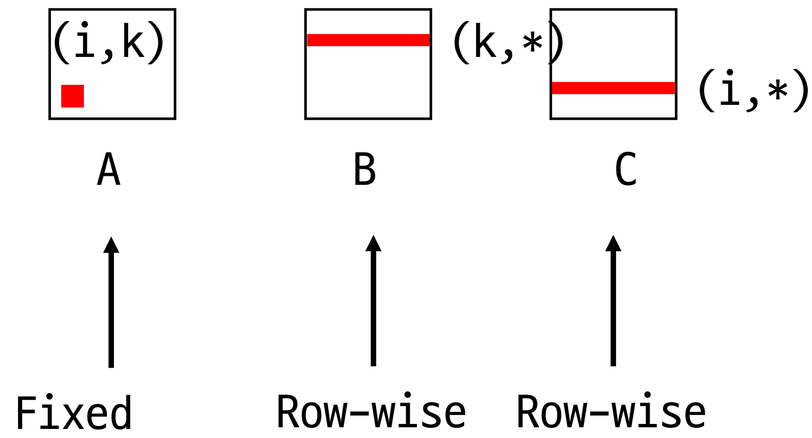
A	B	C
0.0	0.25	0.25

Matrix Multiplication (5)

Matrix multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

Inner loop:



Misses per Inner Loop Iteration:

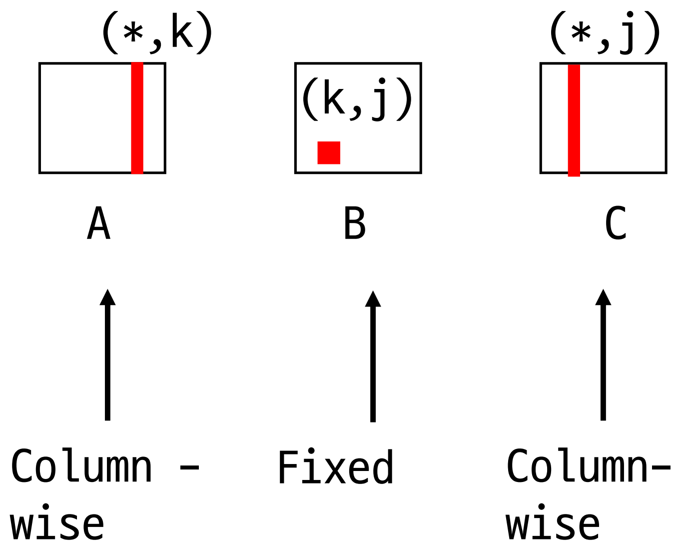
A	B	C
0.0	0.25	0.25

Matrix Multiplication (6)

Matrix multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per Inner Loop Iteration:

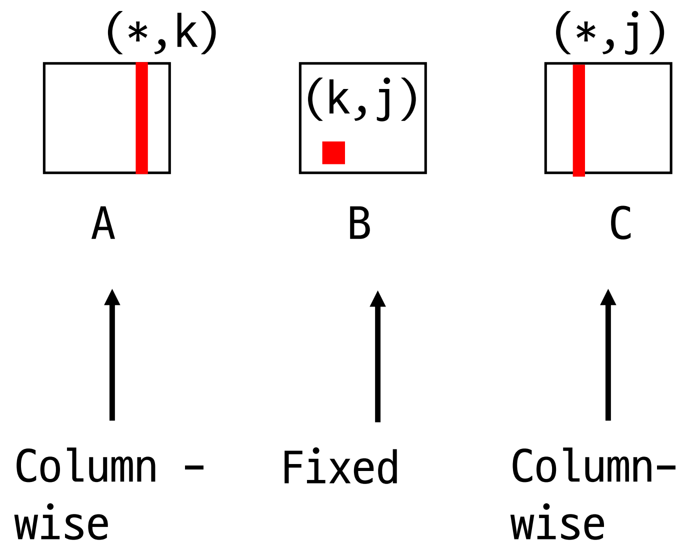
A	B	C
1.0	0.0	1.0

Matrix Multiplication (7)

Matrix multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

Inner loop:



Misses per Inner Loop Iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (8)

Summary

ijk (& jik):
- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * [k][j];
    c[i][j] = sum;
  }
}
```

A	B	C
0.25	1.0	0.0

kij (& ikj):
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * [k][j];
  }
}
```

A	B	C
0.0	0.25	0.25

miss ↓

jki (& kji):
- 2 loads, 1 store
- misses/iter = **2.0**

```
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

A	B	C
1.0	0.0	1.0

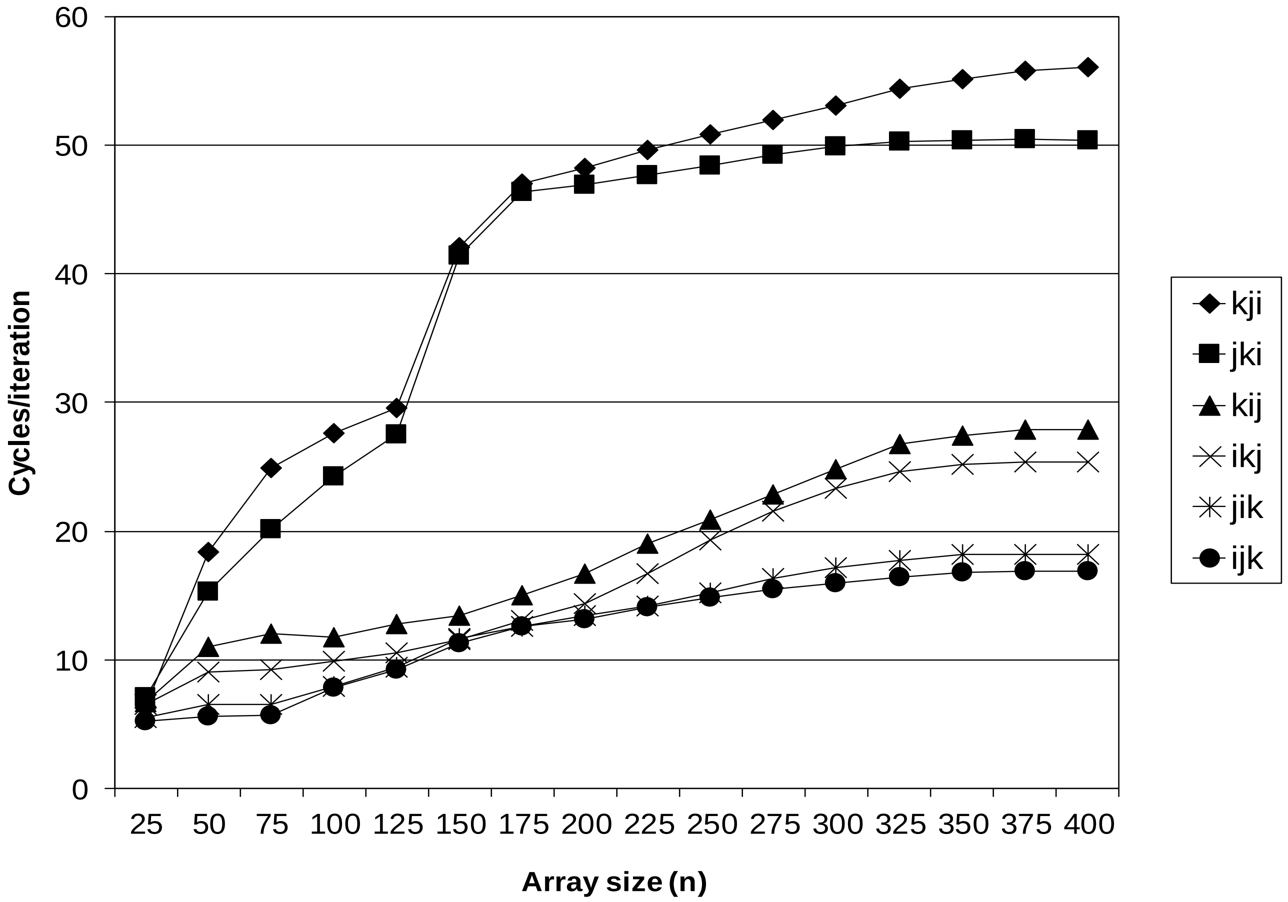
miss ↑

이론상 가장 빠른 것 : 2
실질적 〃 : 1

Matrix Multiplication (9)

Performance in Pentium

- Miss rates are helpful but not perfect predictors
  - Code scheduling matters, too



Blocked Matrix Multiplication (1)

Improving temporal locality by blocking

- "Block" means a sub-block within the matrix
- Example: N = 8, sub-block size = 4

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} X \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}$$

- Key idea: Sub-blocks (i.e., A_{xy}) can be treated just like scalars

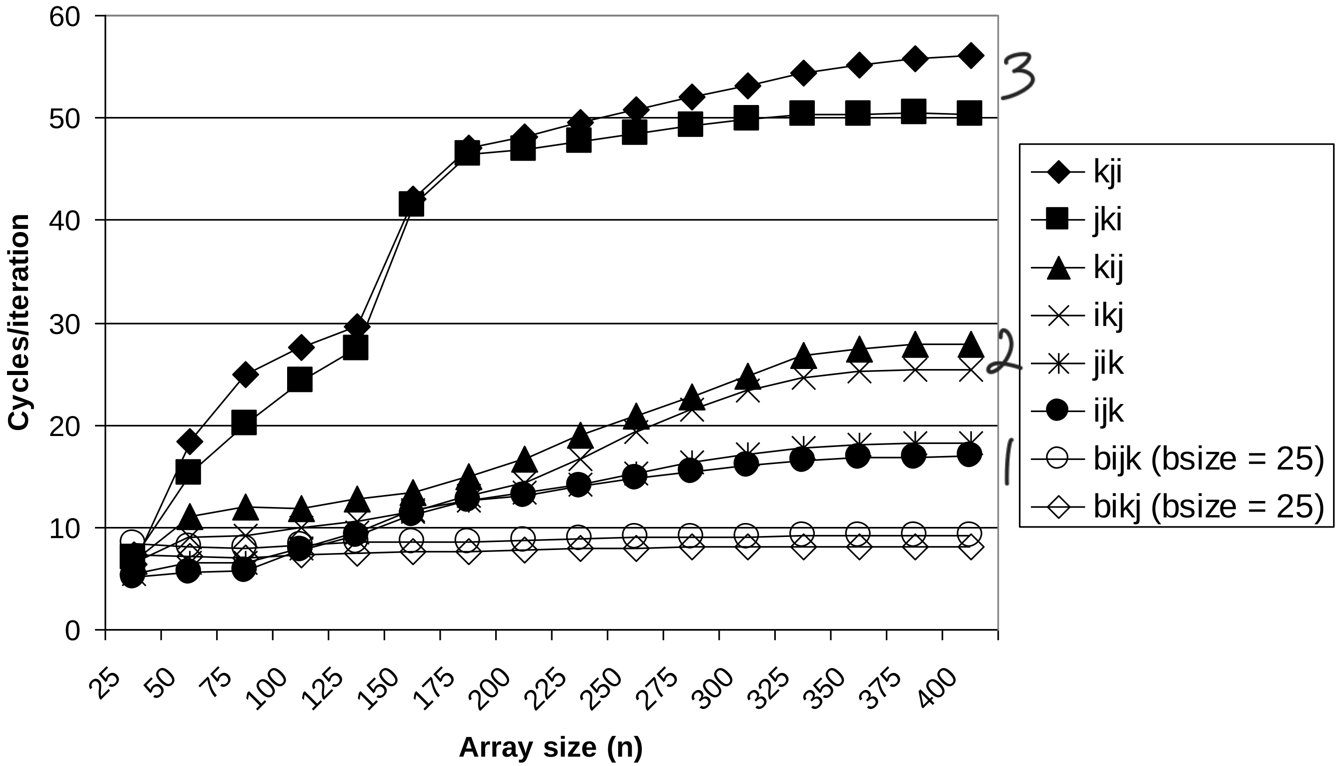
$$C_{11} = A_{11}B_{11} + A_{12}B_{21} \qquad C_{12} = A_{11}B_{12} + A_{12}B_{22}$$

$$C_{21} = A_{21}B_{11} + A_{22}B_{21} \qquad C_{22} = A_{21}B_{12} + A_{22}B_{22}$$

Blocked Matrix Multiplication (2)

Performance in Pentium

- Blocking improves performance by a factor of two over unblocked version (ijk and jik)



Observations

Programmer can optimize for cache performance

- How data structures are organized
- How data are accessed
  - Nested loop structure
  - Blocking is a general technique

All systems favor "cache friendly code"

- Getting absolute optimum performance is very platform specific
  - Cache sizes, line sizes, associativities, etc.
- Can get most of the advantage with generic code
  - Keep working set reasonably small (temporal locality)
  - Use small strides (spatial locality)

Finale

지식채널 적절한 기술

- https://www.youtube.com/watch?v=op4jznLfQQE