$x(t) \rightarrow \boxed{h(t)} \rightarrow y(t) = x(t) * h(t)$; h(t) wireless channel

$h(t) = \sum_{i=0}^{L-1} a_i \delta(t-\tau_i) \rightarrow$ due to scattering

$y(t) = Re\{\sum_{i=0}^{L-1} s_b(t-\tau_i)\, a_i e^{-j2\pi f_c \tau_i} \cdot e^{j2\pi f_c t}\}$

complex base band representation of y(t)

Narrow band assumption: $f_m < \frac{1}{\tau_i} \Rightarrow s_b(t-\tau_i) \approx s_b(t)$

$\Rightarrow y_b(t) = s_b(t) \cdot \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}$ → complex fading coefficient (h)

fading: variation of power w.r.t time

$h = X + jY = a e^{j\phi}$

$X, Y \sim \mathcal{N}(0, 1/2)$; $f_A(a) = 2a e^{-a^2}$; $\phi \sim unif(-\pi,\pi)$

- To get knowledge of 'h' it must be measured atleast once every coherence time
- measuring/estimating h is called channel estimation
- Estimation is done using pilot symbol transmission

T_c < channel estimation time ← fast fading

T_c > channel estimation time ← slow fading

generally $\sigma_\tau << T_c$

$T_c \uparrow$		
	flat fading, slow fading	Freq. selective fading, slow fading
	flat fading, fast fading	Freq. selective fading, fast fading
		→ σ_τ

CDMA multiuser downlink:

0, 1, ... k → users

$s_0, s_1, \dots s_k$ → Information symbols

$c_0, c_1, \dots c_k$ → codes of users

Tx signal: $x(m) = \sum_{i=0}^{k} s_i c_i(m)$

Rx signal: $y(m) = \sum_{i=0}^{L-1} h_0(i)\, x(m-i) + n(m)$

at user 0: $r(0) = \frac{1}{N}\sum_{m=0}^{N} y(m)\, c_0(m)$

⇒ r(0) = Signal + Multi path interference + Multi user interference + noise

$r = \sum_m w^* r(m)$ → Rake receiver

→ SINR = $\frac{N P_0 \|h_0\|^2}{\sum_{i=0}^{k}\|h_0\|^2 P_i - P_0 \sum_{i=1}^{L-1}\frac{|h_i|^4}{\|h_0\|^2} + \sigma_n^2}$

for uplink also

	wireline	wireless
SNR:	$\frac{P^2}{\sigma_n^2}$	$\frac{P^2 a^2}{\sigma_n^2}$; a - random variable
	BER $= Q(\sqrt{SNR}) \approx e^{-SNR}$	avg. BER $= \frac{1}{2}\left(1-\sqrt{\frac{SNR}{SNR+2}}\right) \approx \frac{1}{2\,SNR}$

Deep fade → Noise power > signal power

$\mathbb{P}$(deep fade) ≈ $\frac{1}{SNR}$

TDMA → [u_1 | u_2 | u_3] → t   used in 2G GSM

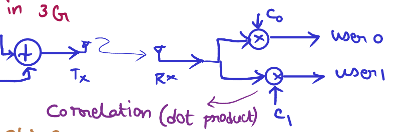
FDMA → [u_1 | u_2 | u_3] → f   used in 1G

CDMA - used in 3G

$u_0 \rightarrow a_0[1\,1\,1\,1]$ (c_0)

$u_1 \rightarrow a_1[1\,-1\,1\,-1]$

4 chips ← c_1



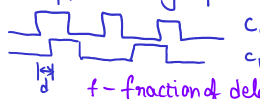
Code → collection of chips

N → # chips in a code ; Time duration of chip (T_c) = $\frac{T}{N}$

(or) spread factor ⇒ B.W = N×f → spread spectrum

- # orthogonal codes possible of length N are N

Asynchronous CDMA:
- happens during uplink



f - fraction of delay

$f \sim unif(0,1)$, $1-f \sim unif(0,1)$

$r_{01}^a = f\, r_{01}(-1) + (1-f)\, r_{01}(0)$

SINR = $\frac{\|h_0\|^2 P_0 N}{\frac{2}{3}\sum_{i=0}^{k}\|h_i\|^2 P_k - \frac{2}{3}P_0\sum_{d=0}^{L-1}\frac{|h_0(d)|^4}{\|h\|^2} + \sigma_n^2}$

- Codes are generated using Linear Shift Feedback Registers (LSFR)
- # 1's are 1 more than 0's ; correlation (dot product) = $\begin{cases} 1 & \text{if shift} = 0 \\ -\frac{1}{N} & \text{else}\end{cases}$
- $\frac{1}{2^{i+1}}$ runs of length 2^i

Near-far problem of CDMA

SINR = $\frac{P_0}{\frac{P_1}{N} + \frac{\sigma_n^2}{N}}$ u_0 u_1

$P_r \propto \frac{1}{d^2}$

Power transmitted by user must be regulated.

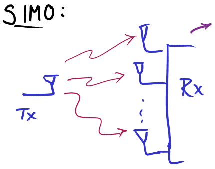
- Random spread sequence: 1. Each c(i) is ±1 w.p 1/2
2. c(i), c(j) are independent
3. $c_0(i)$, $c_1(i)$ are independent

$E[c_0(i)]=0$, $E[r_{00}(k)]=0$, $E[r_{01}(k)=0]$

$E[r_{00}^2(k)] = \frac{1}{N}$, $E[r_{01}^2(k)] = \frac{1}{N}$

$r_{ij}(k) = \frac{1}{N}\sum_m c_i(m)\, c_j(m+k)$

SIMO:



→ Improves BER

h_i - fading coefficient of link i

$\bar{y} = \bar{h} x + \bar{n}_{L\times1}$; $E[|n_i|^2] = \sigma_n^2$

Signal detection: linear combination of y_i's (Beam forming)

$\hat{x} = w^T \bar{y}$; $w^* = \frac{h}{\|h\|}$ → Maximal Ratio Combiner (MRC) (or) Spacial Matched filter

SNR = $\frac{\|h\|^2 P}{\sigma_n^2}$

avg. BER ≈ $\frac{1}{2^L}\cdot\frac{1}{(SNR)^L}\cdot {}^{2L-1}C_L \propto \frac{1}{(SNR)^L}$

$\mathbb{P}$(deep fade) ∝ $\frac{1}{(SNR)^L}$

Diversity order = $-\lim_{SNR\to\infty}\frac{\log(BER)}{\log(SNR)}$ → L SIMO System ; → ∞ for wireline

Multi user CDMA:

$x(n) = a_0 c_0(n) + a_1 c_1(n)$

$y(n) = x(n) + w(n)$

at user 0

$r_0 = \frac{1}{N}\sum_n y(n)\, c_0(n)$

$= \underbrace{\frac{1}{N}\sum_n a_0 c_0^2(n)}_{\text{desired signal}} + \underbrace{\frac{1}{N}\sum_n a_1 c_0(n) c_1(n)}_{\text{Interference}} + \underbrace{\frac{1}{N}\sum w(n) c_0(n)}_{\text{noise}}$ ($r_{01}(0)$)

⇒ SNR = $\frac{P_0}{\frac{P_1}{N}+\frac{\sigma_w^2}{N}} = N\left(\frac{P_0}{P_1+\sigma_w^2}\right)$ Spread gain

Multiple Input Multiple output (MIMO)

t - Tx antenna   n - Rx antenna
- spatial multiplexing

$\bar{y}_{n\times1} = H_{n\times t}\,\bar{x}_{t\times1} + \bar{n}_{n\times1}$
- Total nt channel coefficients
- Noise: $E[|n_i|^2] = \sigma_n^2$, $E[n_i n_j^*] = 0$

Receiver:

$\bar{y} = H\bar{x}+\bar{n}$; $\hat{x} = H^{-1}y - H^{-1}n$ (H is not a square matrix)

⇒ $\hat{x} = (H^T H)^{-1} H^T \bar{y}$ → MMSE estimator of $\|y - Hx\|^2$

Pseudo inverse ; Zero forcing receiver

(n-t+1) diversity ; amplifies noise if h is small ($\frac{n}{h}$ term)

Linear estimator: $\hat{x} = \bar{c}^T\bar{y}$

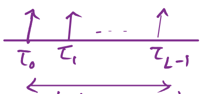
minimize over $\bar{c}$: $E[\|\hat{x} - x\|^2]$

$c^* = R_{yy}^{-1} R_{yx}$ $E[\bar{x}\bar{y}^T] = R_{xy} = R_{yx}^T$

$= P_d H^T (P_d H H^T + \sigma_n^2 I)^{-1}\bar{y}$

$(H^T H)^{-1} H^T \bar{y}$ at low SNR (zero forcing) ; at high SNR $\frac{P_d}{\sigma_n^2} H^T \bar{y}$ (matched filter)

$|h(\tau)|^2 = \sum_{i=0}^{L-1}|a_i|^2\,\delta(\tau-\tau_i)$



max delay spread = $\tau_{L-1} - \tau_0$

RMS delay spread (σ_τ)

weighted power

$\sigma_\tau = \sqrt{\frac{\sum_{i=0}^{L-1}|a_i|^2(\tau-\bar{\tau}_i)^2}{\sum_i |a_i|^2}}$; $\bar\tau = \frac{\sum_i |a_i|^2\tau_i}{\sum_i|a_i|^2}$ ↳ Total power

- avg. Power = $\mathbb{E}[|h(\tau)|^2] = \bar\phi(\tau)$

$f(\tau) = \frac{\bar\phi(\tau)}{\int_0^\infty\bar\phi(\tau)\,d\tau}$; $\bar\tau = \int_0^\infty \tau f(\tau)\,d\tau$; $\sigma_\tau^2 = \int_0^\infty(\tau-\bar\tau)^2 f(\tau)\,d\tau$

Delay spread ≈ 1-3 μs outdoor
≈ 10-50 ns indoor

Advantages of CDMA

→ Jammer margin
disruptive user with high power P_I cause interference $\frac{P_I}{N}$ (suppress Jammer power)

→ Graceful degradation

SNR = $\frac{P_0}{\frac{P_1}{N}+\frac{P_2}{N}+\dots+\frac{\sigma_n^2}{N}}$

⇒ Interference by new user is distributed among other users

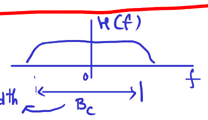
→ universal frequency reuse
- Same frequencies can be used in all cells
- Interference due to adjacent cell is $\frac{P_I}{N}$

→ Multipath diversity

$y(m) = h(0)x(m) + h(1)x(m-1) + \dots + h(L-1)x(m-L+1)$; $x(m) = s_0 c_0(m)$

ISI ⇒ freq. selective

$H(f) = \mathcal{F}(h(t)) \rightarrow$



coherence bandwidth

$B_c \approx \frac{1}{2\sigma_\tau}$ B_s - symbol Bandwidth

$B_s > B_c$ ⇒ frequency selective distortion

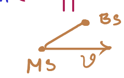
⇕

$\sigma_\tau >> T_{symbol}$ ⇒ Inter symbol Interference (ISI)

($2B_c << B_s$)

Change in frequency due to motion ← doppler effect

$h_b(t) = \sum_{i=0}^{L-1} a_i e^{-j2\pi f_c \tau_i}\, e^{j2\pi f_d t}$



↳ time varying channel ; time varying phase ; $\tau_i(t) = \tau_i - \frac{v\cos\theta}{c}t$

$f_d = f_c\frac{v\cos\theta}{c}$

mobility → doppler → Time selective channel.

Coherence Time $T_c = \frac{1}{4 f_d}$ (approx time channel is constant)

Doppler spread $B_d = 2f_d$ $T_c \approx$ ms

$r(0) = \frac{1}{N}\sum_m y(m)\, c_0(m) = s_0 h(0) + \tilde n(0)$

⋮

$r(k) = \frac{1}{N}\sum_m y(m)\, c_0(m-k) = s_0 h(k) + \tilde n(k)$ ⇒ $\bar r = \bar h s_0 + \bar n$ (receiver diversity)

- By combining $r = \sum \bar{w}^* \bar{r}$ → RAKE receiver

for maximizing SNR we get $\bar{w}^* = \frac{\bar h}{\|h\|}$

SNR = $\frac{\|h\|^2 P_0}{\sigma_n^2}\cdot N$; BER ~ $\frac{1}{(SNR)^L}$

RAKE receiver extracts multipath diversity by coherent combination of multipath components