Source (Analog) → A/D → (01..) → Source coding → Channel Coding → Modulation → Analog signal / symbols → Channel

Receiver (Analog) ← D/A ← Source decoder ← Channel decoder ← Demodulation ← Channel

Source coding: Compress the data (Reduce redundancy)

Channel coding: - To overcome effects of noise and interference

- Add redundancy (say k bits added to get n bits)

$\Rightarrow \frac{k}{n} \rightarrow$ Rate of the code

Modulation:

Maps binary information to sequence of signal waveforms

ex: $0 \rightarrow s_0(t)$ $\quad 1 \rightarrow s_1(t)$

'b' bits are encoded to $M = 2^b$ symbols $\Rightarrow M$ signal waveforms

Channel can transfer R bits/s $\Rightarrow$ for b bits we need $\frac{b}{R}$ sec

Demodulation: mapping signals to bits and estimate the Tx symbol

- Performance metric: Probability of error (avg. bit error)

Random process: For WSS $R_x(\tau) \overset{\mathcal{F}}{\longleftrightarrow} S_x(f)$ → Power spectral density

Auto correlation

$R_{xy}(t_1,t_2) = E[x(t_1)y^*(t_2)]$; $R_x(t_1,t_2) = R_x(t_2 - t_1)$

$R_x(0) \rightarrow$ Power of signal

$x \rightarrow \boxed{H(f)} \rightarrow y$ $\quad E[y] = H(0) \cdot E[x]$

$S_y(f) = |H(f)|^2 S_x(f)$

$\updownarrow \mathcal{F}$

$R_{xx}(\tau) \rightarrow R_{xx}(\tau) * h(\tau) * h(-\tau)$

White process: $R_{xx}(\tau) = \frac{N_0}{2}\delta(\tau) \Leftrightarrow S_x(f) = \frac{N_0}{2}$

↳ ∞ power cannot exist physically.

Cyclostationary: $\mu_x(t+T) = \mu_x(t)$

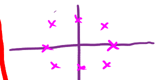
$R_x(t_1+T, t_2+T) = R_x(t_1, t_2)$

ex: $x(t) = \sum a_n g(t - nT)$, $\{a_n\} \rightarrow$ WSS

2. Phase modulation: (Phase shift keying)

$s_m(t) = Re\left[g(t) e^{j2\pi f_c t} \cdot e^{j\frac{2\pi}{M}(m-1)}\right]$

$= g(t)\cos\left(2\pi f_c t + \underbrace{\frac{2\pi}{M}(m-1)}_{\theta_m}\right)$



↳ This is DSB, we can use SSB also.

$\mathcal{E}_{avg} = \mathcal{E}_m = \frac{1}{2}\mathcal{E}_g$

3. Quadrature Amplitude Modulation (QAM)

$s_m(t) = Re\left[(A_{mi} + jA_{mq}) g(t) \cdot e^{j2\pi f_c t}\right]$

$= Re\left[r_m e^{j\theta_m} e^{j2\pi f_c t}\right]$

$= r_m \cos(2\pi f_c t + \theta_m)$

Combined Amplitude and phase mod.

M_1-level PAM $\quad M_2$-level PSK

$M_1 = 2^n$ $\quad M_2 = 2^m$

→ Total $m+n$ bits transmitted

Digital modulation:

mapping digital sequence to signals

1. Memoryless / Memory modulation

$\underbrace{010\ldots1}_{k}$ $\underbrace{111\ldots0}_{k}$ $\ldots$ → $\boxed{\text{Modulator}}$ → $s_m(t)$ $1 \le m \le 2^k$, $M = 2^k$

With memory: - current k + Past $(L-1)k$ bits are mapped to M messages (bLck)

- This represents $2^{(L-1)k}$ state Finite State Machine

$m_l = f_m(S_{l-1}, I_l)$ $\quad S_l = f_s(S_{l-1}, I_l)$

↳ current sequence

- This can be effectively represented by Markov chain

2. Linear / Non linear

orthogonal signaling:

$\langle s_m(t), s_n(t) \rangle = \begin{cases} \mathcal{E} & m = n \\ 0 & m \ne n \end{cases}$

- Signals are orthogonal, forms basis

$\phi_j = \frac{s_j(t)}{\sqrt{\mathcal{E}_j}}$

Frequency shift keying:

$s_m(t) = Re\left[s_{m_l} e^{j2\pi f_c t}\right]$; $1 \le m \le M$

$= \sqrt{\frac{2\mathcal{E}}{T}}\cos(2\pi f_c t + 2\pi m \Delta f t)$

Non linear modulation

$Re[\langle s_m(t), s_n(t)\rangle] = 0 \Leftrightarrow \Delta f = \frac{K}{2T}$

Channels: Wired, wireless, optical fibers, storage devices

- It introduces attenuation, amplitude & phase distortion

Mathematical Models:

1. Additive noise: simplest noise, due to system components (resistors, ss devices)

$s(t) \rightarrow (+) \rightarrow r(t) = s(t) + n(t)$

$n(t) \rightarrow$ Statistically Gaussian noise process

2. Linear filter channel: $r(t) = s(t) * c(t) + n(t)$

3. Linear Time variant channel: $r(t) = s(t) * c(\tau; t) + n(t)$

$= \int_0^\infty s(t-\tau) \cdot c(\tau; t)\, d\tau + n(t)$

At transmitter: Signals are transmitted every T_s sec

T_s: Signaling interval, symbol duration

$R_s = \frac{1}{T_s}$ symbols/sec are transmitted

↓

symbol rate / signaling rate

$T_b = \frac{T_s}{k} = \frac{T}{\log_2 M} \rightarrow$ effective bit duration

$R_b = k \cdot R_s = \log_2 M \cdot R_s \rightarrow$ bit rate

Avg. Signaling energy $(\mathcal{E}_{avg}) = \sum_{i=1}^{M} p_m \mathcal{E}_m$ → $P(m^{th}$ symbol$)$

- If equiprobable $\mathcal{E}_{avg} = \mathcal{E}_m$

and avg. bit rate $\mathcal{E}_b = \mathcal{E}_m / k$

Avg. Power sent by Tx $= \frac{\mathcal{E}_{b,avg}}{T_b} = R_b \mathcal{E}_{b,avg}$

Signaling with memory:

$m_l = f(S_{l-1}, I_l)$ $\quad S_l = f_s(S_{l-1}, I_l)$

NRZ-I → Change amplitude when bit changes

↳ differential encoding

$b_k = a_k \oplus b_{k-1}$

↳ input

Information signal is baseband but spectrum is at high freq.

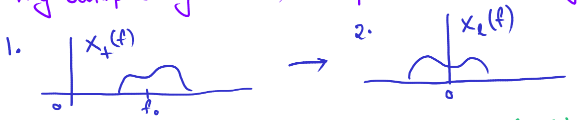
- Processing at high freq. is difficult and generally done at base band

If $x(t)$ is real then, $X(-f) = X^*(f) \Rightarrow |X(-f)| = |X(f)|$ and → even

$\angle X^*(f) = -\angle X(f)$ → odd

∴ All information is in +ve or -ve freq.

Any band pass signal → Complex base band signal



$X_+(f) = X(f) \cdot U(f)$

$\Rightarrow x_+(t) = x(t) * \left[\frac{j}{2\pi t} + \frac{1}{2}\delta(t)\right]$

$= \frac{1}{2}\left[x(t) + j\hat{x}(t)\right]$

Hilbert transform

$\frac{1}{2}X_l(f) = X_+(f + f_0) \cdot U(f + f_0)$

$\Rightarrow x_l(t) = 2 \cdot e^{-j2\pi f_0 t} x_+(t)$

$= \left[x(t) + j\hat{x}(t)\right] e^{-j2\pi f_0 t}$

$\Rightarrow x(t) = Re\left[x_l(t) \cdot e^{j2\pi f_0 t}\right]$

$x_l(t) \triangleq x_i(t) + j\, x_q(t) \Rightarrow x(t) = x_i(t)\cos(2\pi f_0 t) - x_q(t)\sin(2\pi f_0 t)$

↳ inphase $\quad$ ↳ quadrature phase

$x_i(t), x_q(t) \rightarrow$ lowpass signals ; $x_l(t), x_i(t), x_q(t)$ depends on f_0

Any bandpass signal can be written in terms of lowpass signals

$\mathcal{E}_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$; $\mathcal{E}_{x+} = \frac{1}{2}\mathcal{E}_x = \mathcal{E}_{x_l}$

Memoryless modulation methods:

$s_m \rightarrow$ signal to be transmitted

1. Phase Amplitude Modulation (PAM)

$s_m(t) = A_m\, p(t)$ $\quad 1 \le m \le M$

↳ pulse of duration T

$A_m = 2m - 1 - M \in \{\pm 1, \pm 3, \ldots, \pm(M-1)\}$

$\mathcal{E}_m = A_m^2 \mathcal{E}_p$; $\mathcal{E}_p$: Energy of pulse

$\mathcal{E}_{avg} = \frac{1}{M}\sum_{m=1}^{M} \mathcal{E}_m = \frac{(M^2-1)}{3}\mathcal{E}_p$

After modulation with carrier

$s_m(t) = Re\left[A_m g(t) e^{j2\pi f_c t}\right] \rightarrow \mathcal{E}_m = \frac{A_m^2}{2}\mathcal{E}_g$

Passband signal $= A_m \underbrace{g(t)}_{p(t)} \cos 2\pi f_c t$

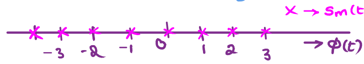
This is one dimensional and $\phi(t) = \frac{p(t)}{\sqrt{\mathcal{E}_p}}$

$\Rightarrow s_m(t) = A_m \sqrt{\mathcal{E}_p}\,\phi(t) \rightarrow$ base band PAM

$= A_m \sqrt{\frac{\mathcal{E}_g}{2}}\,\phi(t) \rightarrow$ band pass PAM (Amplitude Shift keying)

If $M = 2$ then $s_1(t) = -s_2(t) \rightarrow$ binary antipodal

$\times \rightarrow s_m(t)$



Constellation PAM

Signal space: Inner product $\langle x(t), y(t)\rangle = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt$; $\|x(t)\| = \sqrt{\mathcal{E}_x}$

$\phi_1(t), \phi_2(t) \ldots \phi_n(t) \rightarrow$ orthonormal basis signals $\Rightarrow \langle \phi_i(t), \phi_j(t)\rangle = \begin{cases} 0 & i \ne j \\ 1 & i = j \end{cases}$

Gram-Schmidt orthogonalization method:

1. $\phi_1(t) = \frac{s_1(t)}{\sqrt{\mathcal{E}_1}}$ $\quad$ 2. $\phi_2 = \frac{\gamma_2(t)}{\sqrt{\mathcal{E}_{\gamma_2}}}$; $\gamma_2(t) = s_2(t) - \langle s_2(t), \phi_1(t)\rangle \cdot \phi_1(t)$,

↳ error after projection

3. $\phi_3(t) = \frac{\gamma_3(t)}{\sqrt{\mathcal{E}_{\gamma_3}}}$; $\gamma_3(t) = s_3(t) - \sum_{i=1}^{2}\langle s_3(t), \phi_i(t)\rangle\, \phi_i(t)$ . . .

They are not unique, depends on order chosen

Optimal reciver for AWGN channel:

$r(t) = S_m(t) + n(t)$ $\quad S_m(t) \rightarrow \oplus \rightarrow r(t)$, $n(t)\uparrow$

optimal decision $\Rightarrow$ minimize $P_e = \mathbb{P}(\hat{m} \neq m)$

Q. why study AWGN?

Ans: 1. Isolate from other impairments and study its effects

2. Simple model and good for deepspace comm.

$\bar{r} \rightarrow r(t)$ in vector form $\Rightarrow \bar{r} = \bar{S}_m + \bar{n}$

$\Rightarrow P(n) = \frac{1}{(\sqrt{\pi N_0})^N} e^{-\frac{\|n\|^2}{N_0}}$ $\quad \mathcal{N}(0, \frac{N_0}{2})$

$S_m \rightarrow \boxed{\text{channel } P(r|S_m)} \rightarrow r$; Decision function $g(r) = \hat{m}$

$P(\text{correct} \mid r) = \mathbb{P}(\hat{m} \text{ sent} \mid r)$

$\Rightarrow P(\text{correct}) = \int P(r) \cdot \mathbb{P}(\hat{m} \text{ sent} \mid r)\, dr$

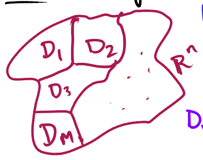
$\Rightarrow$ optimal rule $g(r) = \underset{m}{\operatorname{argmax}}\ P(m|r)$

MAP $\leftarrow \quad = \underset{m}{\operatorname{argmax}}\ \mathbb{P}(S_m|r)$

$\Rightarrow \hat{m} = \underset{m}{\operatorname{argmax}}\ \underbrace{\mathbb{P}(r|S_m)}_{\text{Likelihood}} \cdot \underbrace{P(S_m)}_{\text{Prior}}$

Decision regions:

ML, MAP detectors partition $\mathbb{R}^n$ into M regions $D_1, D_2 \cdots D_M$



For MAP detector

$D_m = \{\bar{r} \in \mathbb{R}^n : \mathbb{P}(m|\bar{r}) > \mathbb{P}(m'|\bar{r}),\ m \neq m'\}$

The error prob. $P_e = \sum_{m=1}^{M} P_m \cdot P_{e|m} = \sum_{m=1}^{M} P_m \cdot \mathbb{P}(r \notin D_m | S_m \text{ sent})$

Symbol error prob.

$P_{e|m} = \int_{D_m} P(r|S_m)\, dr$

bit error: $P_b < P_e \leq k P_b$

Preprocessing at receiver:

$S_m(t) \rightarrow \boxed{\text{Channel}} \xrightarrow{r} \boxed{G(r)} \xrightarrow{\rho} \boxed{\text{Detector}} \rightarrow \hat{m}$

$\hat{m} = \underset{m}{\operatorname{argmax}}\ P_m P(r, \rho \mid S_m)$

$= \underset{m}{\operatorname{argmax}}\ P_m\, P(r|S_m) \cdot P(\rho|S_m)$

$= \underbrace{\underset{m}{\operatorname{argmax}}\ P_m\, P(r|S_m)}_{\text{MAP}} \rightarrow$ Same w/o Processing

Optimal detection for Vector AWGN channel:

By MAP: $\hat{m} = \underset{m}{\operatorname{argmax}}\ P_m\, P_n(\bar{r} - S_m)$

$= \underset{m}{\operatorname{argmax}}\ \log P_m - \frac{\|\bar{r} - S_m\|^2}{N_0}$

$= \underset{m}{\operatorname{argmax}} \left(\underbrace{\frac{N_0}{2} \ln P_m - \frac{1}{2}\mathcal{E}_m}_{\eta_m (\text{bias})} + r \cdot S_m \right)$

If equiprobable then 1. $\hat{m} = \|\bar{r} - \bar{S}_m\|$ (nearest neighbor)

2. The Decision boundary is the perpendicular bisector (for two Points)

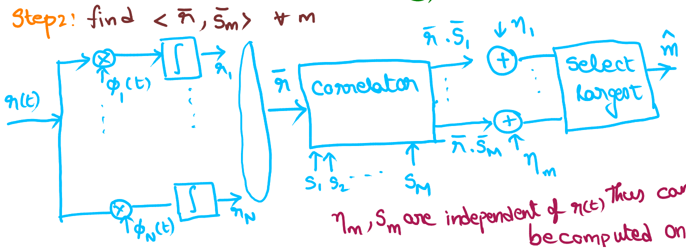
- If not equiprobable then it is line $\perp$ to joining Points but not bisector.

Implementation of optimal receiver:

1. Correlation receiver: receiver has access to $r(t)$ not $\bar{r}$

Step1: Derive $\bar{r} \Rightarrow r_j = \int_{-\infty}^{\infty} r(t)\, \phi_j(t)\, dt$ $\rightarrow$ correlation with $\phi_j(t)$

Step2: find $\langle \bar{r}, \bar{S}_m \rangle\ \forall m$



η_m, S_m are independent of $r(t)$ Thus can be computed once and store.

Matched filter receiver:

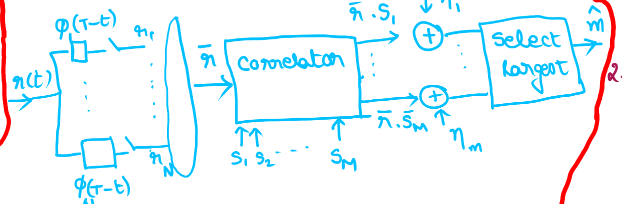
$r_x = \int r(t) \cdot x(t)\, dt$ $\rightarrow$ either $\phi(t)$ or $S_m(t)$

Define $h(t) = x(T - t)$, T is arbitary constant

$r(t) \rightarrow \boxed{h(t)} \rightarrow y(t)\ y(T)$ $\Rightarrow y(t) = \int r(\tau)\, h(t-\tau)\, d\tau$

$= \int r(\tau)\, x(T - t + \tau)\, d\tau$

$\Rightarrow y(T) = \int r(\tau)\, x(\tau)\, d\tau$ $\rightarrow$ output of correlator



SNR of matched filter: $y(t) = y_s(t) + v(t)$

after sampling: $y(T) = \int_{-\infty}^{\infty} H(f)\, S(f)\, e^{j2\pi f t}\, dt$

$\operatorname{Var}[v(T)] = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0}{2} \mathcal{E}_h$

$SNR = \frac{|y_s(T)|^2}{\operatorname{Var}[v(T)]} \leq \frac{\mathcal{E}_s \mathcal{E}_h}{\frac{N_0}{2}\mathcal{E}_h}$ $=$ if $\mathcal{E}_s = \alpha \mathcal{E}_h$

$\Rightarrow H(f) = \alpha S^*(f)\, e^{j2\pi f T} \longleftrightarrow h(t) = \alpha S(t - T)$

maximises SNR

Error correcting codes:

Channel codes

block codes

$M = 2^k$

k bits $\longrightarrow n$ bits

$n > k$, memoryless

Convolution codes

- Finite state machines

- can be represented by shift Registers of length K_k

- Represented using a) Tree diagram

b) Trellis diagram

c) State diagram

Conv. codes can be decoded using Trellis diagram by 'Viterbi Algorithm'

Information Theory:

Discrete source

1. Entropy: $H(S) = -\sum_{i=0}^{M-1} P_i \log P_i$ $\rightarrow$ uncertainity in S

$0 \leq H(S) \leq \log_2 M$ $\rightarrow$ uniform dist maximizes the entropy.

using Jensens inequality $E[\log x] \leq \log E[x]$

2. Joint entropy $H(X, Y) = \sum_{x \in P_x} \sum_{y \in P_y} P_{xy}(x, y) \log \frac{1}{P_{xy}(x, y)}$

$X \rightarrow \boxed{\text{Channel}} \rightarrow Y$ $\quad H(X, Y) \leq H(X) + H(Y)$

Similar to union bound

3. Conditional entropy: $H(Y|X) = \sum_i \sum_j p(x_i, y_j) \log \frac{1}{P(y_j|x_i)}$

uncertainity in y after observing x

$= \sum_{i=1}^{M_1} P(X = S_i)\, H(Y \mid X = S_i)$

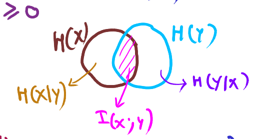
4. Mutual Information: $I(X; Y) = H(Y) - H(X|Y)$

Common uncertainity

$= H(X) - H(X|Y)$

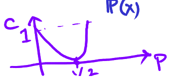
$= H(X, Y) - H(X) - H(Y)$

$I(X; Y) = I(Y; X) \geq 0$



5. Channel capacity: $C = \max_{P(X)} I(X; Y)$

ex: For BSC C



Continous distribution:

1. Differential entropy:

$h(X) = \int_{-\infty}^{\infty} f_X(x) \log_2 \frac{1}{f_X(x)}\, dx$

ex: $X \sim \mathcal{N}(\mu, \sigma^2)$ then

$h(X) = \frac{1}{2} \log(2\pi e \sigma^2)$

do not depend on μ

$h(X) < 0$ is possible

2. capacity of Gaussian channel

$C = \max_{f_X} I(X; Y)$

s.t $E[X^2] \leq P$ $\rightarrow$ Power constraint

$h(Y|X) = \frac{1}{2} \log(2\pi e \sigma^2)$

$\Rightarrow C = \max_{f_Y(y)} h(Y) - h(Y|X)$

s.t $E[Y^2] \leq P + \sigma^2$

$f_Y(y) \rightarrow$ Gaussian

$\Rightarrow C = \frac{1}{2} \log_2 (1 + SNR)$ bits/channel use

$\Rightarrow$ max. Rate $= (2 \cdot BW) \frac{1}{2} \log_2 (1 + SNR)$

$= B \log_2 (1 + SNR)$