Ignorance is Bliss? Evaluating the Impact of Rank Non-Disclosure on High School Course Choice*

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Abstract

This research develops a dynamic model of high school course choice with learning to examine how students make decisions based on GPA and class rank feedback, which provide signals about unobserved individual and cohort abilities. Using detailed administrative data from a school district, the model is estimated using a sequential EM algorithm to account for unobserved heterogeneity in utility and state transitions. The results reveal that students face a trade-off: while Advanced Placement (AP) math courses require higher effort, they offer significant boosts to GPA and class rank. Counterfactual analysis and quasi-experimental evidence show that when rank information is withheld, students in the upper tail tend to overestimate their rank, leading to increased AP math enrollment, contrary to the intended goal of reducing course burden through rank non-disclosure. These findings have important implications for policymakers, as class rank plays a crucial role in university admissions and many schools are considering non-disclosure policies. The evidence that rank information can significantly influence high school course selection is particularly important, as these decisions have long-term consequences for students' educational and labor market outcomes.

***Keywords*:**
***JEL Codes*:**

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1 Introduction

Educational choices made during high school have lasting impacts on students' academic trajectories and labor market outcomes. In particular, advanced mathematics courses play a crucial role in shaping students' college enrollment, major selection, and success in STEM-related fields. Despite the high stakes, students often make these decisions with imperfect information. Academic rank, relative to peers, serves as a key signal for many students, influencing their self-perception and subsequent choices.

In states like Texas, where class rank is instrumental in determining access to public universities through policies like the Top 10% Plan, understanding students' responses to rank information is vital. Class rank disclosure can motivate or discourage students from pursuing challenging coursework, depending on their sensitivity to their relative standing. Several school districts have recently altered their rank disclosure policies to reduce the competitive pressures students face. This paper investigates the impact of such policy changes on students' course choices, focusing specifically on enrollment in advanced placement (AP) mathematics courses.

While higher academic rank is generally associated with improved outcomes, such as increased college enrollment and academic achievement (Murphy and Weinhardt, 2020; Bertoni and Nisticò, 2023; Denning, Murphy, and Weinhardt, 2023),[1], the mechanism through which rank influences student behavior is still debated. For instance, Murphy and Weinhardt (2020) show that ordinal rank during primary school has long-term impacts on secondary school performance, even independent of a student's underlying ability, with particularly strong effects on boys' subject choices, especially in mathematics. Bertoni and Nisticò (2023) highlight the complex trade-offs between peer ability and self-perception, showing that exposure to high-ability peers can lower a student's ordinal rank, thus reducing academic self-concept and influencing performance outcomes. Similarly, Denning, Murphy, and Weinhardt (2023) find that students with a higher academic rank in early grades are more likely to take AP courses, graduate from high school, attend college, and earn higher wages later in life.

While these studies demonstrate the significance of rank in shaping educational and life outcomes, there is considerable debate over whether rank information should be disclosed to students. Some research suggests that providing rank feedback improves performance, particularly for high-achieving students. For example, Azmat and Iriberri (2010) find that when students were given relative performance feedback in a high school setting, their grades improved significantly, with a 5% increase across the board. Goulas and Megalokonomou (2021) show that disclosing performance feedback enhances university enrollment and expected earnings for high-achieving students but discourages lower-performing students. Additionally, Brade, Himmler, and Jäckle (2022) demonstrate that feedback on being above average significantly boosts performance, indicating that positive feedback can correct students' underestimation of their relative abilities.

The central question this paper addresses is how changes in rank disclosure policies affect stu-

[1] Delaney and Devereux (2022) provides an excellent review of this literature.

dents' likelihood of enrolling in AP math courses. Specifically, we investigate whether withholding rank information encourages students to take more challenging courses by reducing competitive pressure, or whether it discourages students who might otherwise be motivated by their high rank. This research focuses on mathematics courses due to their critical role in shaping students' academic paths. Courses like Algebra II and Calculus often serve as prerequisites for college admissions and STEM-related fields, making them especially impactful on college attendance, major choice, and earnings (Rose and Betts, 2004; Aughinbaugh, 2012; Altonji, Blom, and Meghir, 2012; Tchuente, 2016; Todd and Wang, 2021).

To analyze these effects, we develop a dynamic structural model of high school course choice under uncertainty, building on the framework of Arcidiacono, Aucejo, Maurel, and Ransom (2025). The model allows students to update their beliefs about their individual ability based on the GPA feedback as well as update their beliefs about the average ability of their cohort based on class rank feedback. This setup naturally lends itself to exploring how withholding rank information influences course selection. The model is estimated using detailed administrative data from a large urban school district in Texas, covering student demographics, course selections, and grades from 2006 to 2016, when full rank disclosure was the norm.

Each year, a student can choose between not taking a rank-eligible math course, enrolling in a regular math course (e.g., Algebra I), opting for a weighted math course (e.g., Pre-AP Algebra I), or pursuing an accelerated math track (e.g., Algebra I and Geometry). The student's flow utility depends on their current rank and rank squared, capturing both linear and non-linear effects of rank on their course selection. At the start of each academic year, the student is characterized by three key state variables: the amount of math they have completed, their interim GPA, and their interim rank. These state variables determine the student's available choices and progression in math courses, which evolves deterministically based on their decisions. We model GPA production using a capital accumulation function, while rank production follows a cumulative distribution function (CDF) of GPA. The student's GPA is influenced by their unobserved individual ability, and their rank depends on the cohort's average unobserved ability, both of which are unknown at the outset.

Students are forward-looking, maximizing the discounted expected sum of their payoffs, with terminal utility based on their final high school rank. To estimate this model, we use the conditional choice probability (CCP) method (Hotz and Miller, 1993), which simplifies the dynamic optimization problem by expressing continuation values in terms of future choice probabilities. The estimation is further streamlined since leaving school is a terminal action. Unobserved heterogeneity in student preferences and GPA production is incorporated through a sequential EM algorithm. Following James (2011) and Arcidiacono and Miller (2011, 2019), we handle latent abilities by integrating them out and treating them as known during the maximization step, allowing for a computationally feasible approach to account for unobserved heterogeneity.

The model reveals several key insights. We find that students generally exhibit high disutility from enrolling in AP math unless they are on an accelerated track. For example, students who took Algebra I in middle school tend to prefer advanced courses like Pre-AP Geometry over regular Geometry in ninth grade. Additionally, we find that the utility of taking AP math increases with rank and accelerates with rank squared, suggesting that higher-ranked students gain more utility from enrolling in AP math courses, with the effect particularly strong for top performers. We also find that AP math courses tend to boost GPA. Regarding ability sorting, students who enroll in AP math consistently receive positive signals in terms of both GPA and rank.

We then simulate our model under a counterfactual scenario where students no longer have information on their interim class rank at the beginning of a grade level. The goal of the simulation is to understand how information affects course choices. We find that adding informational frictions would increase the AP math enrollment rate by 4, 3 and 7 percent in tenth, eleventh and twelfth grades, respectively. Without rank disclosure, students rely on GPA alone to form perceptions of their rank, leading to overestimation of their relative standing. This overestimation drives higher AP math enrollment among high-performing students.

To complement our model-based findings, we use quasi-experimental evidence from a school district that recently stopped rank disclosure. A difference-in-differences analysis confirms that students in the upper tail are more likely to enroll in AP math when rank information is withheld, with effects that grow over time. The alignment between our quasi-experimental findings and model predictions further validates our results.

The policy implications of this paper are significant. As more school districts experiment with rank non-disclosure policies, understanding their effects on student behavior is crucial for designing interventions that promote equitable educational outcomes.

This paper contributes to several strands of literature. First, it builds on the body of work in dynamic discrete choice models of education (Keane and Wolpin, 1997; Arcidiacono, 2004; Arcidiacono et al., 2025), modeling high school course choices while accounting for unobserved heterogeneity and learning about individual and cohort abilities. Second, it contributes to the literature on feedback in education (Azmat and Iriberri, 2010; Tran and Zeckhauser, 2012; Elsner and Isphording, 2017; Goulas and Megalokonomou, 2021; Floyd et al., 2024), providing empirical evidence on how rank disclosure affects student outcomes. Finally, it adds to the broader literature on feedback in dynamic contests (Genakos and Pagliero, 2012; Jiang et al., 2018; Lemus and Marshall, 2021), offering insights into how information frictions shape academic decision-making.

The remainder of the paper proceeds as follows. Section 2 details the related literature. Section 5 outlines our dynamic model of high school course choice, highlighting the role of imperfect information and belief updating. Section 4 describes our data sources. Sections 6 and 7 detail our identification strategy and estimation procedure. Section 8 reports the structural parameter estimates and model fit. Section 9 conducts counterfactual simulations of rank non-disclosure policies. Section 9.2 presents our quasi-experimental analysis and results. Section 10 discusses the implications of our findings and concludes.

2 Related Literature

This section reviews the relevant literature on dynamic discrete choice models in education, learning processes, methodological approaches, and the role of feedback in both educational and contest settings.

Dynamic Discrete Choice in Education: Dynamic discrete choice models have been extensively applied to educational decision-making processes. These models capture the sequential nature of educational choices and account for the uncertainty and learning that occurs over time.

A seminal contribution to this field was made by Keane and Wolpin (1997), who developed and estimated a dynamic structural model of schooling, work, and occupational choice decisions using data from the National Longitudinal Surveys of Labor Market Experience (NLSY). This work laid a strong foundation for subsequent research in the area, demonstrating the potential of dynamic structural models to provide insights into complex life-cycle decisions related to human capital accumulation.

Building on this foundation, Eckstein and Wolpin (1999) examined youth behavior in the context of education and employment choices. Their study focused on the factors influencing high school dropout decisions, incorporating heterogeneity in skills and preferences.

Arcidiacono (2004) made significant contributions by introducing ability sorting into the dynamic framework of college major choices. This work highlighted the importance of considering student heterogeneity and learning about one's abilities in educational decision-making.

More recently, Arcidiacono et al. (2025) extended this line of research by estimating a dynamic structural model where individuals face uncertainty about their academic ability and productivity, which respectively determine their schooling utility and wages. Their framework accounts for heterogeneity in college types and majors, as well as occupational search frictions and work hours. They find that removing informational frictions would increase the college graduation rate.

De Groote (2019) develop a dynamic model in which students choose the academic level of their program and their effort level. They find that encouraging underperforming students to switch to less academic programs reduces grade retention and dropout.

Learning: The process of learning plays a crucial role in both educational and economic decision-making. Several key studies have contributed to our understanding of learning dynamics in various contexts.

Miller (1984) applied learning models to job matching, demonstrating how individuals update their beliefs about job suitability over time. This work has important parallels in educational settings, where students learn about their abilities and preferences for different subjects or majors.

In the context of consumer choice, Erdem and Keane (1996) developed a model of learning under uncertainty. While focused on brand choice, their framework provides valuable insights into how individuals make decisions with imperfect information, a concept highly relevant to educational choices.

Arcidiacono (2004) incorporated learning about abilities into models of educational choice, showing how students update their beliefs about their aptitude for different fields of study based on their academic performance.

More recently, Arcidiacono et al. (2025) extended this line of research by estimating a dynamic structural model in which individuals learn about their academic ability and productivity, which respectively determine their schooling utility and wages.

Methodology: The estimation of dynamic discrete choice models often involves complex computational challenges. Several methodological approaches have been developed to address these issues. Hotz and Miller (1993) introduced the conditional choice probability (CCP) approach, which significantly reduced the computational burden of estimating dynamic discrete choice models. This method has been widely adopted in the education literature. Keane and Wolpin (1997) applied these techniques to career decisions while incorporating a finite mixture model to capture unobserved heterogeneity. Arcidiacono and Miller (2011) further advanced the methodology by developing techniques for estimating dynamic models with unobserved heterogeneity. Their approach combines the CCP method with the expectation-maximization (EM) algorithm, allowing for more flexible modeling of individual differences.

Role of Feedback in Education: Feedback plays a crucial role in shaping educational outcomes and student behavior. Several studies have examined the effects of different types of feedback in educational settings. Azmat and Iriberri (2010) investigated the impact of relative performance feedback on high school students. Their study found that providing students with information about their relative standing improved performance, particularly for those at the extremes of the achievement distribution. Tran and Zeckhauser (2012) examined the effects of rank feedback in a field experiment, finding that knowledge of one's rank can serve as an inherent incentive for improved performance. Floyd et al. (2024) explored the consequences of withholding grades, providing insights into how the absence of explicit feedback affects student behavior and subsequent employment outcomes. Elsner and Isphording (2017) studied the "big fish in a small pond" effect, showing how a student's relative rank within their school influences human capital investment decisions and long-term outcomes. Goulas and Megalokonomou (2021) investigated the short- and longer-term effects of providing students with feedback about their true academic standing. Their findings suggest that accurate self-knowledge can lead to more efficient educational investments.

Role of Feedback in Contests: The literature on feedback in contests provides valuable insights that can be applied to competitive educational settings. Genakos and Pagliero (2012) studied the effects of interim rank feedback in dynamic tournaments. Their findings suggest that relative performance information can influence risk-taking behavior and effort allocation in competitive environments. Lemus and Marshall (2021) examined feedback in dynamic innovation contests, providing insights into how information revelation affects participant strategies and contest outcomes. Jiang et al. (2018) analyzed the role of feedback in crowdsourcing contests, demonstrating how different feedback mechanisms can influence participation and performance in competitive settings.

In conclusion, this paper contributes to and extends several key strands of the above discussed strands of literature. Our work builds upon the foundational models of educational decision-making while focusing on the course choices of high school students. Additionally, we extend these models by incorporating a dual learning process where students update their beliefs about both individual and cohort ability based on GPA and rank signals, respectively, bridging the gap between the literature on learning with studies on the impact of relative performance feedback. Methodologically, our use of unobserved heterogeneity in both flow utility and the transition process, estimated via a sequential EM algorithm, builds on the computational advancements of Arcidiacono and Miller (2011). Finally, our counterfactual analysis examining the impact of removing rank signals on course choices contributes to the ongoing policy debate on the role of comparative feedback in educational settings. By integrating these various elements, our paper provides a comprehensive framework for understanding the complex dynamics of high school course selection, offering insights that can inform both future research and policy discussions in education.

3 Background

Percent Plans: Following the elimination of race-conscious affirmative action policies, several states, including California, Texas, and Florida, adopted percent plans for college admissions, relying on high school grades and class rank. The Texas Ten Percent Plan, introduced in 1997, guarantees automatic admission to Texas public universities for students graduating in the top 10% of their high school class. Under this policy, eligible students can choose from any of the 35 public universities in Texas, including prestigious institutions like the University of Texas at Austin (UT Austin) and Texas AM University. High school class rank serves as the primary criterion for admission and is calculated by the district or school based on the most recent academic record, which could include the student's rank at the end of their 11th grade, mid-12th grade, or upon graduation.

In contrast, California's program guarantees admission to one of the eight undergraduate campuses in the University of California (UC) system for students graduating in the top 4% of their high school class. However, it does not ensure admission to the student's campus of choice. Eligibility is based on GPA in UC-required courses taken during 10th and 11th grades. Since highly selective campuses like UC Berkeley use additional criteria such as academic preparation, extracurriculars, and personal qualities, students must still compete for spots at these campuses. Florida's percent plan guarantees admission to the state university system for students in the top 20% of their class, based on district-determined class rank. Eligibility hinges on either a B average in required academic units or a combination of GPA and test scores. However, like California, Florida's plan only guarantees entry into the system, not into specific universities, which may have additional selection criteria.

Key differences exist among the percent plans in these states. Unlike Texas, which guarantees admission to flagship institutions, California and Florida merely promise access to the broader state university system. The methods for determining eligibility also differ, with Texas and Florida relying on GPA calculations at the district level, while California identifies juniors from specific coursework, and the UC system finalizes eligibility. Texas uniquely emphasizes class rank as the sole determinant for the Ten Percent Plan, making class rank the central criterion for guaranteed admission. This singular focus sets Texas apart from other states, ensuring that placement within one's high school cohort dictates eligibility for the state's premier universities. Additionally, Texas's flexible course and credit requirements make the plan more accessible to a wider range of students compared to California and Florida, further distinguishing its approach to college admissions.

Texas Top Ten Percent Plan The Texas Top Ten Percent Plan, also known as Texas House Bill 588, was implemented in 1998 to grant automatic admission to any public university in the state for students who rank within the top ten percent of their high school graduating class. The primary goal of this policy was to increase representation of students from under-performing high schools in selective universities without explicitly relying on race-based affirmative action. Given the economic and racial segregation in Texas public schools, policymakers expected that the plan would enhance diversity by using school-specific class rank as the primary criterion for eligibility.

While House Bill 588 formalized this automatic admission policy, it was not a radical departure from previous practices at the University of Texas (UT). Until 1993, UT allowed automatic admission for students in the top 10 percent of their class. In 1994, the university adjusted its admission criteria to include a combination of class rank and SAT scores, making the process more restrictive. House Bill 588, signed into law by Governor George W. Bush and passed by the 75th Texas Legislature in 1997, reverted to the previous approach by basing eligibility solely on class rank and eliminating the standardized test score requirement for automatic admission. This policy sought to broaden access to highly selective public institutions by enabling students to compete with peers within their own schools, thereby leveling the playing field for students from different educational backgrounds.

Research on the Texas Top Ten Percent Plan has shown varied impacts on college enrollment and long-term outcomes. Using regression discontinuity, Niu and Tienda (2010) finds that the policy increases flagship university enrollment for Hispanic students and those from schools with a high proportion of minority and economically disadvantaged students. Black et al. (2020) further demonstrate that the policy benefits highly ranked students from non-traditional feeder schools, boosting their college enrollment, graduation, and earnings, while students from traditional feeder schools, who lose access to flagship universities, do not experience declines in overall enrollment or long-term earnings. Similarly, Daugherty et al. (2014) show that guaranteed admission significantly increases flagship enrollment and persistence, especially among students in schools with high college-going rates, but has little effect on students from the most disadvantaged schools and results in a displacement from private universities rather than an increase in overall college enrollment.

Some studies suggest that the heterogeneous effects of the Top Ten Percent Plan can be attributed to strategic school choices aimed at maximizing class rank. Cullen et al. (2013) analyze students' transitions between 8th and 10th grades three years before and after the policy change and find that among students with both the motive and opportunity for strategic high school choice, at least 5% enroll in different high schools to enhance their chances of graduating in the top 10%. These students often opt for their neighborhood high schools instead of transferring to more competitive institutions, which, regardless of their own race, can displace minority students from the top 10% pool. Additionally, Cortes et al. (2014) provide evidence that families reacted strategically to this policy by relocating to neighborhoods with lower-performing schools, thereby increasing property values in those areas. This effect is most pronounced in schools that were very low-performing before the policy change. They also find that the strategic reactions were influenced by the availability of local schooling options, with the effects of the Top 10% Plan being weaker in areas with fewer school choices.

Concerns have been raised regarding students manipulating the system through strategic course selection to boost their class rank (The Texas Senate, 2001). Initially, the Texas Top Ten Percent Plan did not specify any required academic curriculum for eligibility, which led to worries that students were opting out of rigorous courses. In response, the Texas legislature passed an amendment in 2001 that increased curriculum requirements for eligibility (Office of House Bill Analysis, 2001). The amendment, effective for the graduating class of 2008 and beyond, introduced three curriculum options: the minimum graduation plan (which excluded college preparatory courses), the state-recommended graduation plan (which included college prep courses), and the advanced graduation plan. Under the new rules, students must complete either the recommended or advanced graduation plans to qualify for automatic admission under the Top Ten Percent Plan.

Additionally, school districts have the authority to determine how class ranks are calculated and disclosed, and in recent years, several districts have revised their rank calculation policies (Webb, 2019b; Dellinger, 2022; Potter, 2018; Donaldson, 2020; Rozen, 2022; Webb, 2019a). These changes often involve adjustments to the eligible courses and the weights assigned to them. Districts also have discretion over how rank information is shared with students. While state law mandates that rank information be disclosed to students within the top ten percent, some districts have proposed alternative methods of disclosure, such as providing rank information in quartiles or withholding specific rank details altogether. Table 1 summarizes some recent policy changes. One motivation behind these policy changes is the concern that an overemphasis on class rank encourages unhealthy competition among students, leading them to take courses primarily to boost their rank rather than selecting classes that align with their college and career goals. Similar concerns have prompted schools and parents in other states to consider adopting comparable rank disclosure policies (Balingit, 2015; Hui, 2022).

3.1 Math Course-taking in Texas

According to the current high school graduation requirements in Texas, students must complete Algebra I and Geometry. Within these courses, students have the option to choose between regular and weighted versions, such as Pre-AP Algebra I and Pre-AP Geometry. The recommended path

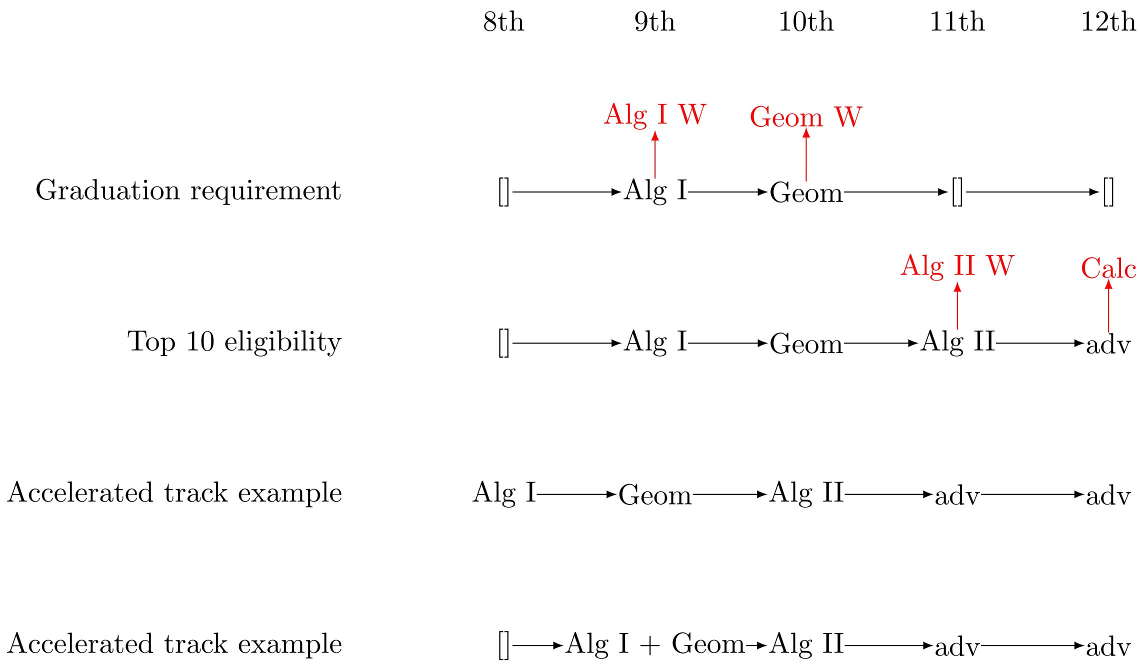
Table 1. Changes in Local Policies

District	Policy	Effective	Change
Dallas	Rank calculation	Class of 2021	Rank will now be based on 15 core courses in English Language Arts, Math, Science, and Social Studies. It was earlier based on any 12 courses including electives.
Frisco	Rank disclosure	Class of 2019	For students outside the top ten percent, rank will not be reported in their transcript. Students will still know their GPA. The lowest GPA in the top 10 percent, first quartile, second quartile and third quartile will be published in the Student Portal.
Clear Creek	Rank calculation	Class of 2027	Rank will be based on only core (non-elective) courses in English Language Arts, Math, Science, Social Studies, and World Languages and Culture.
Katy	Rank disclosure	Class of 2021	For students outside the top ten percent, only quartile rank will be reported in their transcript.

is to take Algebra I in the ninth grade and Geometry in the tenth grade. However, students who aspire to be eligible for consideration in the Top Ten Percent Plan need to go a step further and take Algebra II, as well as an advanced math course like Calculus. These advanced courses also offer weighted versions, such as Pre-AP Algebra II and AP Calculus.

Students have additional decisions to make regarding their math course selection. They can opt for the accelerated track, where they take Algebra I in middle school, Geometry in ninth grade, Algebra II in tenth grade, and advanced math courses in the final two years of high school. Another option to expedite their progress is to double up on math courses in the same year, such as taking both Algebra I and Geometry in the ninth grade. Figure 1 provides an overview of the various course sequences available to a student. It is important to note that math courses follow a sequential structure, with Algebra I serving as a prerequisite for Geometry, which is a prerequisite for Algebra II, and so on, culminating in Calculus.

Figure 1. Course map: horizontal and vertical tracking



4 Data

We use administrative data obtained from a large urban school district in Texas. The dataset includes academic records for 106,761 high school students and covers their demographic information (race, ethnicity, gender), course selections, course grades, GPA, and class rank. The data spans 32 schools and 304 graduating classes between 2006 and 2016. Additionally, we observe the high school-level courses credited to students during their middle school years (6th, 7th, and 8th grades) as well as during high school (9th through 12th grades).

4.1 Descriptive Statistics

Table 2 provides a summary of the academic and demographic characteristics of the 106,761 students in our dataset. Panel A presents the demographic breakdown of the student sample. On average, 25% of the students identify as Black, while 60% are Hispanic, reflecting the district's diverse student population. Female students make up 50% of the sample.

Panel B summarizes the math course-taking patterns. On average, students completed 3.28 math courses, with 0.71 of those being weighted (e.g., advanced or honors courses). Nearly all students completed Algebra I (100%) and Geometry (93%), while 80% completed Algebra II by the end of high school. However, more advanced courses see a sharp drop in completion rates, with 48% of students taking Pre-Calculus and only 14% completing Calculus.

Panel C reports the dropout rates by grade level. The dropout rate is zero by Grade 9 but increases steadily as students progress through high school. By Grade 10, 9% of students have

Table 2. Past academic and background characteristics by course choice

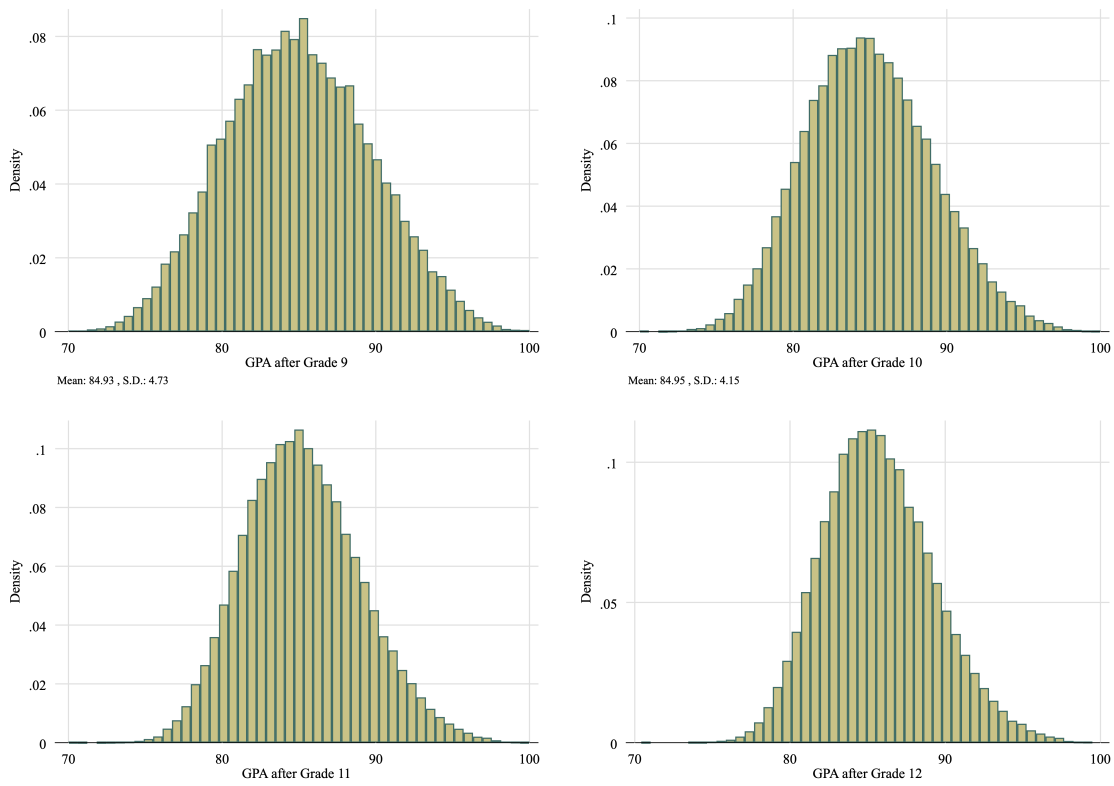
	(1) [Min,Max]	(2) Mean	(3) S.D.
Panel A: Demographics			
Black	[0,1]	0.25	(0.44)
Hispanic	[0,1]	0.60	(0.49)
Female	[0,1]	0.50	(0.50)
Panel B: Math Coursetaking			
Total Math courses	[0,5]	3.28	(0.88)
Total Weighted Math courses	[0,4]	0.71	(1.16)
Ever Completed Algebra I	[0,1]	1.00	(0.06)
Ever Completed Geometry	[0,1]	0.93	(0.25)
Ever Completed Algebra II	[0,1]	0.80	(0.40)
Ever Completed Pre-Calculus	[0,1]	0.48	(0.50)
Ever Completed Calculus	[0,1]	0.14	(0.34)
Panel C: Dropout			
Dropout by Grade 9	[0,0]	0.00	(0.00)
Dropout by Grade 10	[0,1]	0.09	(0.28)
Dropout by Grade 11	[0,1]	0.18	(0.38)
Dropout by Grade 12	[0,1]	0.26	(0.44)
Panel D: GPA			
GPA after Grade 9	[70,100]	84.93	(4.73)
GPA after Grade 10	[70,100]	84.95	(4.15)
GPA after Grade 11	[70,100]	85.22	(3.81)
GPA after Grade 12	[70,99]	85.70	(3.58)
Obs		106,761	

Note: This table reports summary statistics for the data that is used to estimate our structural model. This is student-level data.
Data source: Administrative data from a large urban district in Texas

dropped out, with the rate rising to 18% by Grade 11 and 26% by Grade 12. This indicates a significant attrition rate, particularly in the later years of high school.

Panel D provides GPA statistics over time. Students' average GPA shows a steady increase from 84.93 after Grade 9 to 85.70 by the end of Grade 12. Meanwhile, the standard deviation of GPA decreases from 4.73 in Grade 9 to 3.58 in Grade 12, suggesting reduced variation in academic performance as students approach graduation. Figure 2 further illustrates the distribution of GPA by grade level, showing that GPAs are approximately normally distributed across each grade.

Figure 2. GPA Distribution by Grade Level



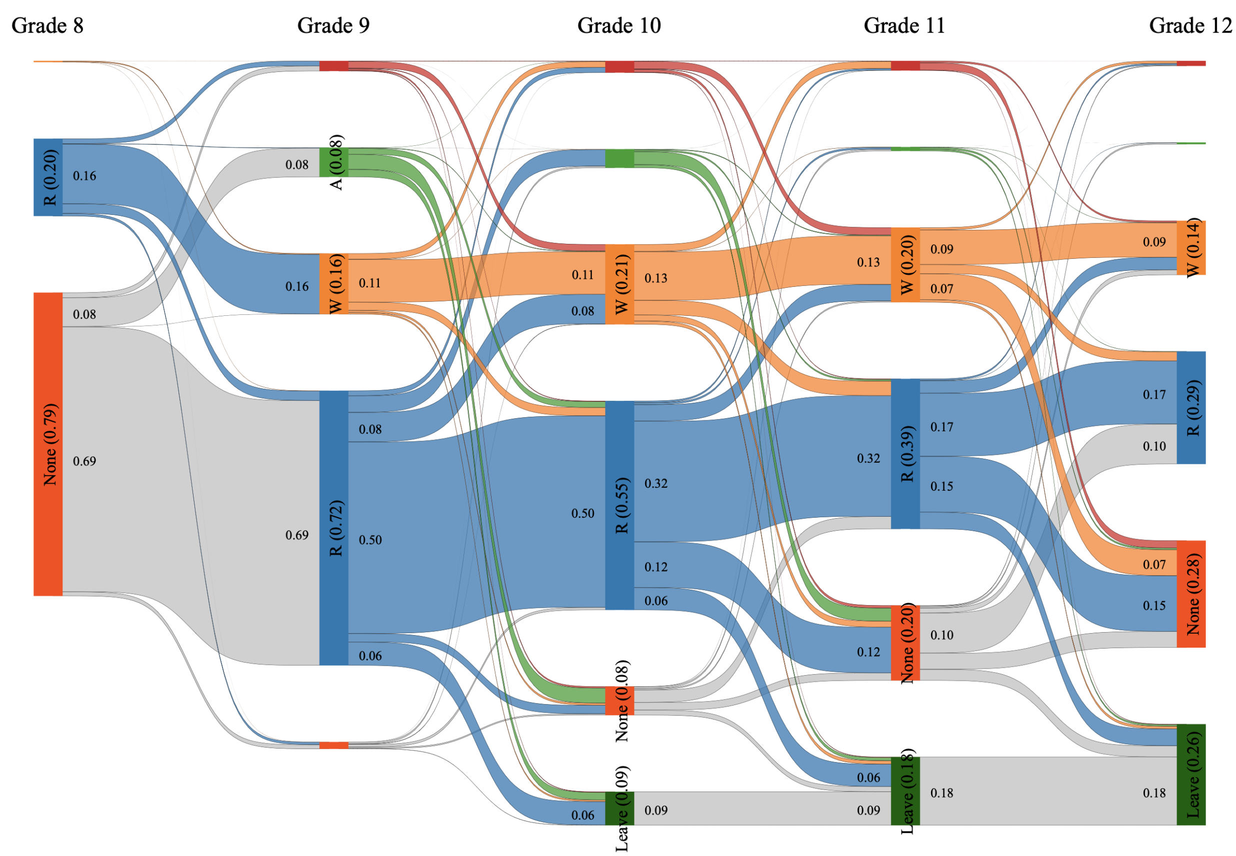
Note: The figure shows the distribution of GPAs for students after grades 9, 10, 11, and 12. GPA data for students after completing grades 9-12. Mean GPA and standard deviation are provided for each grade level. The distributions appear to be roughly normal, with slight changes in shape and central tendency across grade levels.
Data source: Administrative data from a large urban district in Texas

Course-taking Patterns: Figure 3 visualizes the math course progression of students from Grade 8 through Grade 12, highlighting various pathways that students take throughout their high school years. The diagram categorizes students into different groups based on their course-taking behavior: Regular (R), Weighted (W), Accelerated (A), None (no rank-eligible math courses), and Leave (students who exit the school system). The flow lines between these categories represent transitions between courses as students progress through high school, with the thickness of the lines corresponding to the proportion of students making each transition.

In Grade 8, a majority of students (79%) do not take any rank-eligible math courses. Most of these students transition into the regular math option (Algebra I) in Grade 9, while a small proportion pursue the accelerated math option (Algebra I + Geometry) in Grade 9. On the other hand, 20% of the students take Algebra I in Grade 8, and a majority of them subsequently enroll in the weighted math option (Geometry Pre-AP) in Grade 9. This early acceleration in middle school sets the foundation for these students to continue in more advanced math courses in high school.

In Grade 9, 72% of students take the regular math option, while 16% opt for weighted courses, and 8% pursue accelerated math options. Students enrolled in regular courses in Grade 9 tend to persist in the regular track in subsequent grades, although there is some movement into other tracks over time. The proportion of students taking regular math courses decreases gradually as students move to higher grades, while there is a notable increase in the proportion of students opting not to take any rank-eligible math courses as they approach the end of high school. Similarly, students who take the weighted option in Grade 9 are more likely to continue with the weighted course progression in the following years, but there is some leakage into regular and other course options.

Figure 3. Student Progression and Academic Pathway Flows Across Grades 8-12



Note: This Sankey diagram illustrates the academic pathways of students from Grade 8 through Grade 12. The width of each flow represents the proportion of students following that particular path. The main categories tracked appear are R ("Regular" courses), W ("Weighted" courses), A ("Accelerated" courses), "None" (which might indicate students not enrolled in specific rank-eligible courses), "Leave" (which indicates leaving school). The visualization demonstrates how students transition between these academic categories as they progress through high school grades.
Data source: Administrative data from a large urban district in Texas

Course-taking and Academic Standing Table 3 presents the summary statistics conditional on the math course chosen, where we distinguish math course options into five categories: leaving school, staying but not taking any (rank-eligible) math course, taking a regular (unweighted) math course (e.g., Geometry after completing Algebra I), taking a weighted math course (e.g., Pre-AP Geometry after Algebra I), or taking an accelerated course (e.g., taking both Geometry and Algebra II after Algebra I). Panel A displays the beginning-of-year GPA, rank, and math progression statistics. Students enrolled in weighted or accelerated math courses have, on average, higher GPAs and ranks at the start of the year compared to those taking regular courses. For instance, the average GPA for students taking weighted or accelerated courses is 86.92 and 86.17, respectively, compared to 84.73 for those in regular courses. Similarly, the average rank percentile is 0.61 and 0.56 for students in weighted and accelerated courses, respectively, compared to 0.49 for those in regular courses.

We also examine whether students are on the recommended math progression, which typically involves completing Algebra I in ninth grade, Geometry in tenth grade, Algebra II in eleventh grade, and an advanced course like Pre-Calculus in twelfth grade. Students who have accelerated their math progression (e.g., by completing Algebra I in middle school) are more likely to take weighted courses. On average, 71% of students enrolled in weighted courses have accelerated previously, compared to only 6% of those in regular courses.

Panel B provides residualized GPA and rank statistics, controlling for past academic performance and demographic characteristics. The negative coefficients indicate that students with lower residualized GPA and rank are more likely to leave school or take fewer rank-eligible courses, while those with higher residuals are more likely to enroll in weighted courses.

Panel C summarizes the end-of-year GPA and rank for each group, showing that weighted and accelerated course-takers continue to outperform their peers by the end of the year, with average GPAs of 86.49 and 85.50, respectively, compared to 84.81 for regular course-takers. Similarly, those who enroll in the weighted or accelerated courses have a higher rank – 0.58 and 0.52, respectively – at the end of the year compared to those who enroll in the regular math course – 0.49.

Overall, the table highlights that students who take weighted or accelerated courses have stronger academic backgrounds and are more likely to have deviated from the standard math progression. At the same time, these students go on to have a higher academic performance in the future as well. The results suggest that early acceleration and strong academic performance are correlated with more advanced course-taking in high school. These patterns may also reflect learning – students upon learning about higher-than-expected GPA and rank may increase their enrollment in rigorous coursework.

Table 3. Past and Future academic characteristics by course choice

	(1)	(2)	(3)	(4)	(5)	(6)
	Leave	None	Regular	Weighted	Accelerated	Total
Panel A: Beginning-of-Year GPA, Rank, and Math Progression						
GPA	82.81	84.61	84.73	86.92	86.17	85.03
	(4.30)	(4.09)	(3.98)	(4.10)	(5.10)	(4.28)
Rank	0.36	0.46	0.49	0.61	0.56	0.50
	(0.29)	(0.29)	(0.28)	(0.27)	(0.30)	(0.29)
Above the recommended level	0.17	0.18	0.06	0.71	0.21	0.20
	(0.38)	(0.38)	(0.24)	(0.45)	(0.40)	(0.40)
Panel B: Beginning-of-Year Residualized GPA and Rank						
GPA	-3.37^{***}	-2.16^{***}	-1.29^{***}		-0.15^{***}	
	(0.03)	(0.02)	(0.02)		(0.04)	
Rank	-3.37^{***}	-2.16^{***}	-1.29^{***}		-0.15^{***}	
	(0.03)	(0.02)	(0.02)		(0.04)	
Panel A: End-of-Year GPA and Rank						
GPA	-	84.63	84.81	86.49	85.50	85.17
	-	(3.96)	(4.04)	(4.05)	(4.89)	(4.15)
Rank	-	0.44	0.49	0.58	0.52	0.50
	-	(0.29)	(0.28)	(0.28)	(0.30)	(0.29)
Obs	37,683	145,652	230,602	75,380	25,384	514,701

Note: This table reports summary statistics for the data that is used to estimate our structural model. Standard deviations are listed directly below the mean (in parentheses) for each entry. This is student-grade-level pooled data.
Data source: Administrative data from a large urban district in Texas

5 Model

5.1 Overview

Motivated by the descriptive patterns in the data, we now turn to our model of high school course decisions. Individuals in each period from $t = 1$ to $t = T$ make a decision regarding their mathematics course-taking. A student's options include whether to leave school, stay in school but not take a (rank-eligible) mathematics course, take the regular mathematics course, take the weighted mathematics course, or take the accelerated mathematics course.

Students have imperfect information about their mathematics ability, which we denote by A_i. We assume that A_i is also unobserved to the econometrician and normally distributed, with mean zero and variance $\sigma^2_{g,a}$. Additionally, students have imperfect information about the average ability of their cohort, which we denote by R_{-i}. We assume that R_{-i} is also unobserved to the econometrician and normally distributed, with mean zero and variance $\sigma^2_{r,a}$. Beyond these individual and relative abilities that are initially unknown to the students, we also allow for unobserved (to the econometrician only) heterogeneity.

Students update their beliefs about A_i by receiving GPA signals that depend on their choices. These signals reveal information regarding their ability. Similarly, students update their beliefs about R_{-i} by receiving rank signals. These signals reveal information regarding the average ability of their cohort.

Students are assumed to be forward-looking and choose the sequence of actions yielding the highest value of expected lifetime utility. Hence, when making their course decisions, individuals consider the option value associated with the new information acquired on different choice paths.

We now detail the main elements of the model. We first discuss the grade and rank production functions. We then describe how individuals update their beliefs about their individual and relative abilities. Finally, we model the flow payoffs and the optimization problem the individuals face. Discussions of model identification and estimation are deferred to Sections 6 and 7, respectively.

5.2 Timing:

The model is designed to capture the progression of high school students through different grade levels, denoted by $t = 1, \ldots, T$, where $T = 4$. Each grade level represents a specific point in time during a student's academic journey. The timing of the grade levels is as follows: $t = 1$ corresponds to the first year of high school (aka ninth grade or freshman year), $t = 2$ corresponds to the second year of high school (aka tenth grade or sophomore year), $t = 3$ corresponds to the third year of high school (aka eleventh grade or junior year), and $t = 4$ corresponds to the fourth year of high school (aka twelfth grade or senior year).

5.3 Choice Variable:

In each grade level $t \in 1, \ldots, T$, student i makes a decision regarding their math course-taking, denoted by $a_{it} \in 0, \ldots, 4$ corresponding to leave, no math course, recommended course, weighted course, accelerated course. In other words, they can leave school, stay in school but not take a (rank-eligible) mathematics course, take the regular mathematics course, take the weighted mathematics course, or take the accelerated mathematics course. Mathematics in high school is taken in progression. We define the math pre-requisite level, represented by $\ell_{it} \in \mathcal{L}$, which indicates the highest math course student i has taken before reaching grade level t, and therefore determines their eligibility to take certain courses in the subsequent grade levels. The set $\mathcal{L}$ consists of six elements: $0, 1, 2, 3, 4, 5$ corresponding to the pre-requisite levels of none, Algebra I, Geometry, Algebra II, Pre-Calculus, Calculus, respectively.

For example, if $\ell_{it} = 0$, it means student i has not taken any math course before reaching grade level t, and they are eligible to take Algebra I in the current grade level (t), or in future grade levels. If $\ell_{it} = 1$, it indicates that the student has completed Algebra I and is now eligible to take Geometry. Similarly, if $\ell_{it} = 2$, the student has completed Geometry and can proceed to Algebra II, and so on. Hence, based on their pre-requisite level ℓ_{it}, student i makes a math course decision $a_{it} \in \mathcal{A}\ell$ where $\mathcal{A}\ell$ represents the choice set available to a student if their pre-requisite level in grade t is ℓ, i.e., $\ell_{it} = \ell$. The choice sets are summarized in Table 4. Alternatives 1 and 2 mean leave school and stay in school but not take any (rank-eligible) math course, irrespective of the math pre-requisite level. Someone who has not taken any high school mathematics course yet ($\ell = 1$) can choose from: Leave, No math course, Algebra I, Algebra I Pre-AP, and Geometry. Someone who has completed Algebra I can choose from: Leave, No math course, Geometry, Geometry Pre-AP, and Algebra II. Someone who has completed Geometry can choose from: Leave, No course, Algebra II, Algebra II Pre-AP, and Pre-Calculus. Someone who has completed Algebra II can choose from: Leave, No course, Pre-Calculus, Pre-Calculus Pre-AP, and Calculus AP. Note that Calculus is only available in the weighted (AP) version. Someone who has completed Pre-Calculus can choose from: Leave, No course, and Calculus AP. Note that there is no math course more advanced than Calculus.

Table 4. State-specific Choice set

ℓ \ $\mathcal{A}_\ell$	Regular	Weighted	Accelerated
None	Algebra I	Algebra I (Pre-AP)	Geometry
Algebra I	Geometry	Geometry (Pre-AP)	Algebra II
Geometry	Algebra II	Algebra II (Pre-AP)	Pre-Calculus
Algebra II	Pre-Calculus	Pre-Calculus (Pre-AP)	-
Pre-Calculus	-	Calculus (AP)	-

Note: ℓ refers to the math pre-requisite level attained before making the course choice and $\mathcal{A}_\ell$ refers to the set of alternatives available to someone who is at pre-requisite level ℓ. Alternatives 1 and 2 mean leave school and stay in school but not take any math course, irrespective of the math pre-requisite level, so they are omitted from the table to save space.

5.4 Flow Utility

The flow payoff for each math course decision is influenced by three primary factors. First, a student's incentives are shaped by their observed, time-varying ability as measured by their class rank, denoted as $r_{it} \in [0, 1]$. This rank is based on the student's high school class percentile, where a higher value indicates a better rank, reflecting the student's relative performance compared to peers in the cohorts (i.e., individuals who graduate from the same school in the same year).

Second, there is a time-varying, unobserved utility component specific to course a, denoted as $\epsilon_{it}(a)$. This component captures individual-level idiosyncratic preferences for each course that vary over time. For example, changes in personal interests or external circumstances could influence a student's inclination toward particular math courses at different stages.

Third, there is an unobserved, time-invariant utility component, $\beta_k(a)$, which depends on their unobserved type k. This component captures persistent preferences for a given course option based on the student's type, which could reflect characteristics such as intrinsic ability, motivation, or external constraints that do not change over time. This unobserved heterogeneity allows for variation in students' preferences that are not captured by observable characteristics and persist across different grade levels.

In addition to these three factors, utility also depends on whether the student has completed the recommended math course before entering that grade level, represented as $\ell_{it} > \bar{\ell}_t$. Here, ℓ_{it} indicates the highest math course completed by student i before grade t and $\bar{\ell}_t$ is the recommended level of math proficiency for grade t. If a student is above this recommended level, they may derive additional utility from feeling more prepared or confident in tackling advanced coursework. Finally, the student's demographic information X_i can also influence their utility from different course options.

The utility from leaving school, i.e., $a = 0$, is normalized to zero. In grade level $t \in \{1, \ldots, T\}$, the flow payoff for student i from choice $a \in \{1, 2, 3, 4\}$ is given by:

$$u_t(a, \ell_{it}, r_{it}, \epsilon_{it}; \beta) = \beta_{a,0} + \beta_{a,\ell}(\ell_{it} > \bar{\ell}_t) + \beta_{a,r,1} r_{it} + \beta_{a,r,2} r_{it}^2 + \beta_{a,x} X_i + \sum_k \beta_{k,a}(k_i = k) + \epsilon_{it}(a). \quad (1)$$

Here, $\beta_{a,0}$ represents the base utility level of choosing course a while $\beta_{a,\ell}$ captures how this base utility differs for those above the recommended math level. This specification allows the utility to vary by whether the student has accelerated beyond the standard math progression. The recommended levels are: no math course required before entering Grade 9, Algebra I before Grade 10, Geometry before Grade 11, and Algebra II before Grade 12, i.e., $\bar{\ell}_t = t - 1$. Additionally, the terms $\beta_{a,r,1}$ and $\beta_{a,r,2}$ capture the linear and quadratic effects of class rank on utility. The inclusion of both terms allows the relationship between class rank and utility to be nonlinear, reflecting potential diminishing or increasing returns to rank as students move up in the distribution.

5.5 Terminal Utility

At the end of high school, student has no more math course choices to make but they now have their final class rank r_{iT+1} that shapes their terminal utility:

$$u_{T+1}(r_{iT+1}; \beta) = \beta_{r,1} r_{iT+1} + \beta_{r,2} r_{iT+1}^2. \quad (2)$$

5.6 Rank Production and Learning

In this subsection, we delve into the process by which students form and update their beliefs about their academic rank within their cohort. We introduce a model that captures the relationship between a student's GPA and their rank, while accounting for the uncertainty surrounding the average ability of their peers. By receiving noisy signals in the form of realized GPA and rank outcomes, students engage in Bayesian updating to refine their beliefs about the cohort's ability distribution. This learning process generates endogenous heterogeneity in perceived ability, even among students with identical initial priors, highlighting the role of heterogeneous experiences in shaping academic decisions.

Central to a student's uncertainty about their future rank is their lack of knowledge about the average ability of their cohort. While students understand that their rank depends on their own GPA, they are also aware that it is influenced by the GPA distribution of their peers, which is unobserved. To capture this relationship, we propose the following model:

$$r_{it} = \Phi(\alpha_g g_{it} - R_{-i} + \eta_{r,it}) \quad (3)$$

where r_{it} represents the rank of student i in grade level t, g_{it} is their GPA before entering grade level t, and

where $R_{-i} \sim \mathcal{N}(\tilde{\mu}_r, \tilde{\sigma}_r^2)$ denotes the average ability of the cohort relative to student i. The cohort's ability distribution is assumed to be normal with mean $\tilde{\mu}_r$ and variance $\tilde{\sigma}_r^2$, both of which are unknown to the student and the econometrician. The parameter α_g captures the influence of the student's GPA on their rank, while $\eta_{r,it} sim \mathcal{N}(0, \gamma_r^2)$ is an idiosyncratic error term. The function $\Phi(\cdot)$ represents the cumulative distribution function (cdf) of the standard normal distribution.

But like we said before, the student does not know the true distribution $\tilde{\mu}_r, \tilde{\sigma}_r^2$. Students are unsure about the cohort's relative ability. We assume that students are rational and update their beliefs in a Bayesian fashion. Students receive rank signals but before they receive any information We assume that student has a prior belief about the cohort ability which is normally distributed with mean zero and variance σ_r^2.

In the absence of perfect information about the cohort's ability distribution, students are assumed to hold prior beliefs and update them rationally as they receive new information. Initially, students' prior beliefs about the cohort's ability are normally distributed with mean zero and variance σ_r^2. Throughout their high school years, students observe their realized GPA and rank outcomes, which serve as noisy signals about the true cohort ability. The signal received by student i in grade level t is given by:

$$s_{r,it} \equiv R_{-i} - \eta_{r,it} = \alpha_g g_{it} - \Phi^{-1}(r_{it}). \tag{4}$$

This signal is the difference between the student's realized GPA and the inverse of the standard normal cdf evaluated at their realized rank. Intuitively, it captures the discrepancy between the student's actual rank and the rank they would expect given their GPA, providing information about the cohort's ability.

Students incorporate these signals into their beliefs using Bayesian updating. The updated beliefs about the cohort's ability distribution are characterized by the following equations:

$$\sigma_{r,t+1}^2 = \left(\frac{1}{\sigma_{r,t}^2} + \frac{1}{\gamma_r^2} \right)^{-1} \tag{5a}$$

$$\mu_{r,t+1} = \left(\frac{1}{\sigma_{r,t}^2} + \frac{1}{\gamma_r^2} \right)^{-1} \left(\frac{1}{\sigma_{r,t}^2} \mu_{r,it} + \frac{1}{\gamma_r^2} s_{r,it} \right) \tag{5b}$$

where $\sigma_{r,t}^2$ represents the perception error variance, which decreases as the student accumulates more information. The magnitude of the update to the mean belief, $\mu_{r,t}$, depends on the signal's accuracy, with more precise signals (i.e., smaller γ_r^2) leading to larger updates. Notably, when a student's realized rank is lower than their expected rank, their belief about the mean of the cohort's ability distribution shifts upward.

A key insight from this learning model is that the signals received by students, $s_{r,it}$, are inherently heterogeneous due to the randomness in the realized GPA and rank outcomes. As a result, even students who start with identical prior beliefs about the cohort's ability will develop different perceived abilities over time. This endogenous heterogeneity in beliefs, driven by heterogeneous experiences, offers a compelling explanation for the observed variation in course choices that goes beyond the traditional assumption of inherent preference differences. By incorporating rank production and learning into our model, we capture the dynamic process by which students form and update their beliefs about their academic standing within their cohort.

5.7 GPA Production and Learning

We now turn our attention to the evolution of a student's GPA and the role of learning in shaping their beliefs about their own ability. We propose a model that captures the relationship between a student's course decisions, past academic performance, and unobserved heterogeneity in determining their GPA. Alongside this production process, we introduce a Bayesian learning mechanism that allows students to update their beliefs about their own ability based on the noisy signals they receive in the form of realized GPA outcomes.

We model the evolution of a student's GPA as a function of their course decision (a_{it}), the highest math course taken before the current grade level (ℓ_{it}), their GPA in the previous grade level (g_{it}), and an idiosyncratic shock ($\eta_{it}(a)$). Additionally, we incorporate unobserved heterogeneity in the form of discrete types (k_i) that are known to the student but unobserved by the econometrician, as well as a time-invariant ability component (A_i) that is unobserved to both the student and the econometrician. Formally, the GPA production function is given by:

$$\begin{aligned} g_{it+1}(a, \ell_{it}, g_{it}, \eta_{it}; \alpha) &= \alpha_{a,0} + \alpha_{a,\ell}(\ell_{it} > \bar{\ell}_t) + \alpha_{a,g} g_{it} + \sum_k \alpha_{a,k}(k_i = k) + A_i + \eta_{g,it}(a) \\ &= f_g\left(a, \ell_{it}, g_{it}, k_i; \theta_g\right) + A_i(a) + \eta_{g,it}(a) \end{aligned} \tag{6}$$

where $A_i(a) \sim \mathcal{N}(\tilde{\mu}_{g,a}, \tilde{\sigma}_{g,a})$ represents the time-invariant ability of student i that is unobserved to the student and the econometrician. GPA also depends on $\beta_{a,k}$ that is the unobserved heterogeneity for student of type k that is known to the student but not the econometrician. We assume that the idiosyncratic shocks, $\eta_{g,it}(a)$, are mutually independent and distributed $\mathcal{N}(0, \gamma_g(a)^2)$, and are also independent from the other state variables. This specification allows for level shifts in GPA based on course alternatives and grade level through $\alpha_{a,0}$. Through $\alpha_{a,g}$, it allows for variation in student's GPA at the time of entering grade level $t+1$ by their GPA at the time of entering grade level t.

As with the cohort's ability distribution, students do not have perfect knowledge of their own ability $A_i(a)$ and the associated parameters $\tilde{\mu}_{g,a}(a)$ and $\tilde{\sigma}_{g,a}(a)$. Instead, they hold prior beliefs about their ability, which are assumed to be normally distributed with mean zero and variance $\sigma^2_{g,a}(a)$. As students progress through high school, they observe their realized GPA outcomes, which serve as noisy signals about their true ability. The signal received by a type-k student i in grade level t from course decision a is given by:

$$s_{g,it}(a, k) \equiv A_i(a) + \eta_{g,it-1}(a) = g_{it} - f_g\left(a, \ell_{it-1}, g_{it-1}, k; \theta_g\right). \tag{7}$$

This signal represents the difference between the student's realized GPA and their expected GPA based on their observable characteristics and course decision. Students incorporate these signals into their beliefs using Bayesian updating, with the updated beliefs characterized by the following equations:

$$\sigma^2_{g,it+1}(a) = \left(\frac{1}{\sigma^2_g(a)} + \frac{\sum_{\tau=0}^{t}(a_{i\tau} = a)}{\gamma^2_g(a)} \right)^{-1} \tag{8a}$$

$$\mu_{g,it+1}(a,k) = \left(\frac{1}{\sigma^2_g(a)} + \frac{\sum_{\tau=0}^{t}(a_{it} = a)}{\gamma^2_g(a)} \right)^{-1} \left(\frac{1}{\sigma^2_{gt}(a)} \mu_g(a) + \frac{1}{\gamma^2_g(a)} \sum_{\tau=0}^{t+1} s_{g,i\tau}(a,k)(a_{i\tau} = a) \right) \tag{8b}$$

where $\sigma^2_{g,t}(a)$ represents the perception error variance, which decreases as the student accumulates more signals. The magnitude of the update to the mean belief, $\mu_{g,it}(a,k)$, depends on the precision of the signals, with more precise signals (i.e., smaller $\gamma^2_g(a)$) leading to larger updates. When a student's realized GPA is higher than their expected GPA, their belief about their own ability shifts upward.

As with the learning process for cohort ability, the heterogeneity in the signals received by students, $s_{g,it}(a,k)$, leads to endogenous differences in perceived ability across individuals, even when they start with the same priors and make the same course decisions. This feature of the model highlights the importance of heterogeneous experiences in shaping students' beliefs about their own ability, rather than relying solely on inherent preference differences to explain variation in course choices.

The GPA production and learning mechanism, in conjunction with the rank production and learning process described in the previous subsection, provides a comprehensive framework for understanding how students form and update their beliefs about their academic abilities and standing within their cohort. By incorporating these learning processes into our model, we can better capture the dynamic nature of student decision-making and the role of uncertainty and information in shaping educational outcomes.

5.8 The Optimization Problem

We now consider the dynamic optimization problem faced by students as they make their course decisions throughout high school. We assume that students are forward-looking and choose the sequence of courses that maximizes the expected present value of their lifetime utility, taking into account the uncertainty surrounding their future preferences and ability signals.

Formally, the student's objective is to choose a sequence of course decisions $(a_{it})_{t=1\ldots T}$ that maximizes the discounted sum of expected payoffs:

$$E \left[\sum_{t=1}^{T} \delta^{t-1} \sum_{a} \left(u_t(a, \ell_{it}, r_{it}; \beta) + \epsilon_{it}(a) \right) 1 \{ a_{it} = a \} \right] \tag{9}$$

where $\delta \in (0,1)$ is the discount factor, $u_t(\cdot)$ represents the flow utility derived from course decision a in grade level t, and $\epsilon_{it}(a)$ captures idiosyncratic preference shocks. The expectation is taken with respect to the distribution of future preference shocks, and ability signals, conditional on the student's information set at each decision point.

To characterize the optimal course sequence, we introduce the ex-ante value function $V_t(\ell_{it}, r_{it}, k)$, which represents the expected discounted sum of current and future payoffs at the beginning of grade level t, before the realization of the idiosyncratic preference shock. The conditional value function $v_t(a, \ell_{it}, r_{it}, k; \beta)$, which represents the value of choosing course a in grade level t given the state variables and the realized preference shock, can be expressed as:

$$v_t\left(a, \ell_{it}, r_{it}, k; \beta\right) = u_t\left(a, \ell_{it}, r_{it}, k; \beta\right) + \delta E_t\left[V_{t+1}\left(\ell', r', k\right) \mid \ell_{it}, r_{it}, a_{it} = a\right] \tag{10}$$

where $E_t[\cdot \mid \cdot]$ denotes the expectation conditional on the student's information set at the beginning of grade level t, which includes the sequence of ability signals received up to period $t-1$.

Assuming that the preference shocks $\epsilon_{it}(a)$ are independently and identically distributed according to a Type 1 extreme value distribution, the conditional value function for $t < T$ can be written as the following weighted log-sum formula:

$$v_t\left(a, \ell_{it}, r_{it}, k; \beta\right) = u_t\left(a, \ell_{it}, r_{it}, k; \beta\right) + \delta E\left[\ln \sum_{a'} \left(\exp(v(a', \ell', r', k; \beta)\right) \mid \ell_{it}, r_{it}, a_{it} = a, k\right] + \delta\Gamma \text{ if } t < T \tag{11a}$$

$$= u_t\left(a, \ell_{it}, r_{it}, k; \beta\right) + \delta E\left[u_{t+1}(r'; \beta) \mid \ell_{it}, r_{it}, a_{it} = a, k\right] \text{ if } t = T \tag{11b}$$

where Γ denotes Euler's constant.

To estimate the model, we leverage the conditional choice probability (ccp) inversion method proposed by Hotz and Miller (1993). By exploiting the fact that leaving school ($a = 1$) is a terminal action, we can rewrite the conditional value function as:

$$v_t\left(a, \ell_{it}, r_{it}, p_{it+1}, k; \beta\right) = u_t\left(a, \ell_{it}, r_{it}, k; \beta\right) + \delta E\left[\ln \sum_{a'} \left(\exp(\log p_{it+1}(a', \ell', r') - \log p_{it+1}(1, \ell', r')\right) \mid \ell_{it}, r_{it}, a_{it} = a \right. \tag{12a}$$

$$= u_t\left(a, \ell_{it}, r_{it}, k; \beta\right) + \delta E\left[u_{t+1}(r'; \beta) \mid \ell_{it}, r_{it}, a_{it} = a, k\right] \text{ if } t = T \tag{12b}$$

The ccp inversion approach allows us to express the continuation value in terms of future choice probabilities, which can be estimated from the data. This reformulation simplifies the dynamic optimization problem and facilitates the estimation of the structural parameters governing students' preferences and beliefs.

Table 5. Mathematical Notations

Symbol	Description	Main equations of reference
a_{it}	Course choice of student i in period t	(1) , (6)
ℓ_{it}	Math prerequisite level of student i in period t	(1), (6)
r_{it}	Class rank of student i in period t	(1), (3), (9)
$\epsilon_{it}(a)$	Idiosyncratic preference shock for course a	(1), (9)
β	Parameters in utility function	(1), (9)
g_{it}	GPA of student i in period t	(3), (6)
R_{-i}	Average ability of cohort relative to student i	(3), (4)
$\eta_{r,it}$	Idiosyncratic error in rank production	(3), (4)
α_g	Parameter in rank production function	(3), (4)
σ_r^2	Variance of prior belief about cohort ability	(5)
γ_r^2	Variance of rank signal noise	(5)
$\mu_{r,t}$	Mean of posterior belief about cohort ability	(5)
$A_i(a)$	Time-invariant ability of student i for course a	(6)
$\eta_{g,it}(a)$	Idiosyncratic shock in GPA production	(6)
$\sigma_g^2(a)$	Variance of prior belief about individual ability	(8)
$\gamma_g^2(a)$	Variance of GPA signal noise	(8)
$\mu_{g,it}(a)$	Mean of posterior belief about individual ability	(8)
V_t	Value function	(10), (11)
v_t	Conditional value function	(10), (11), (12)
p_{it}	Conditional choice probability	(12)
$q_i(k)$	Posterior probability of being type k	(22)
$\pi_{k\|x}$	Type probability conditional on initial state	(23)

6 Identification

This section discusses the identification of the key components of our dynamic discrete choice model, including unobserved heterogeneity, conditional choice probabilities, conditional value functions, flow utilities, GPA and rank production functions, and unobserved abilities. We highlight the assumptions and variations in the data that allow us to separately identify these components and estimate the structural parameters of interest.

6.1 Unobserved Heterogeneity

One of the primary challenges in identifying the model parameters is the presence of unobserved student preferences that are correlated over time. To address this issue, we incorporate permanent unobserved heterogeneity following Keane and Wolpin (1997), allowing for K distinct unobserved types of students, each characterized by unique type-specific components that shift the intercepts of the GPA transition and flow utility functions. The identification of the distribution of unobserved heterogeneity relies on the dynamic choices of observationally equivalent students over time (Arcidiacono et al., 2025). Intuitively, if some students consistently make choices that deviate from what we would expect based on their observed characteristics and past outcomes, this suggests the presence of unobserved factors driving their behavior. The type proportions and associated type-specific parameters are estimated by comparing the choice probabilities and outcome distributions of these students. To separately identify permanent type-specific preferences from initially unknown unobserved abilities, we assume that the unobserved types are discrete, while the unobserved abilities are continuous and normally distributed. The discreteness of types generates "lumpy" variation in choices and outcomes that is distinct from the smooth variation in abilities.

However, the unobserved types and abilities may be correlated conditional on observed outcomes. We rely on functional form assumptions of discrete types and normally distributed abilities, as well as the exclusion restriction that flow utilities depend only on observed rank, to disentangle their effects.

6.2 Conditional Choice Probabilities

Conditional on the observed state variables, which include students' observed characteristics and past choices, the choice probabilities can be expressed as a finite mixture of type-specific conditional choice probabilities, with mixture weights corresponding to the type proportions identified in the previous step. Identification of type-specific conditional choice probabilities in dynamic discrete choice models with unobserved heterogeneity requires that the observed state variables generate sufficient variation in choices across types, such that the matrix of conditional choice probabilities is full rank (Kasahara and Shimotsu, 2009; Hu and Shum, 2012). Intuitively, the observed variation in choices across states must be "rich enough" to distinguish between the different unobserved types.

6.3 Conditional Value Functions

Once the type-specific conditional choice probabilities have been identified, the conditional value functions associated with each choice alternative can be identified using standard arguments from the dynamic discrete choice literature (Hotz and Miller, 1993; Arcidiacono and Miller, 2011). This requires assuming that the idiosyncratic preference shocks follow a Type 1 extreme value distribution, allowing us to invert the conditional choice probabilities to obtain the conditional value functions. The scale and location of the conditional value functions are not separately identified, so we normalize the flow utility for one reference alternative (e.g., dropping out of school) to zero. To simplify the computation of the conditional value functions, we follow Arcidiacono and Miller (2011) and Arcidiacono and Miller (2019) and express the future value terms in the Bellman equation in terms of a few period-ahead conditional choice probabilities and flow utilities, rather than solving the full dynamic programming problem. This is possible due to the linearity of the flow utilities in the parameters and the additive separability of the preference shocks, allowing us to write the expected maximum of the conditional value functions as a function of the logarithms of the conditional choice probabilities (the "log-sum formula").

6.4 Flow Utilities

The flow utility parameters are identified from the variation in choices across students with different observed characteristics and choice histories. The key identifying assumption is that the observed variation in choices reflects differences in the flow utilities associated with each choice alternative, conditional on the expected future value terms that depend on ability beliefs and the transition probabilities of the observed states. The flow utility parameters are estimated by matching the observed choice probabilities to the model's predicted choice probabilities, which are functions of the flow utilities and the conditional value functions. To ensure identification of the scale of the flow utilities, we normalize the flow utility of one reference alternative (e.g., dropping out) to zero and fix the scale of the Type 1 extreme value shocks to 1. Given these normalizations, the remaining flow utility parameters are identified from the differences in the log odds of choosing each alternative relative to the reference alternative, for different values of the observed states.

6.5 GPA and Rank Production Functions

The parameters of the GPA and rank production functions are identified from the variation in observed GPA and rank outcomes across students with different observed characteristics, choice histories, and unobserved types. A key challenge is the potential for sample selection bias, as GPA and rank outcomes for each course are only observed for students who choose to take that course. The observed variation in outcomes may thus reflect not only the causal effects of observed characteristics and choices on performance but also the unobserved preferences and abilities driving course selection. To address this issue, we make two key assumptions. First, we assume that the unobserved types are independent of the observed characteristics and past choices, conditional on the student's unobserved abilities. This allows us to express the expected outcome for each course as a function of the student's observed characteristics, past choices, unobserved abilities, and a course-specific "match effect" that depends on the student's unobserved type. Second, we assume that the unobserved abilities enter the GPA and rank production functions additively and are independent of the other inputs, conditional on the unobserved type. This allows us to write the expected outcomes as the sum of a type-specific component that depends on the observed inputs and a type-specific mean ability term. Given these assumptions, the parameters of the GPA and rank production functions can be identified from the variation in outcomes within and between unobserved types, controlling for students' observed characteristics and past choices. The type-specific components are identified from the variation in outcomes across students with different observed inputs who are predicted to belong to the same unobserved type based on their choice histories. The mean ability terms are identified from the average differences in outcomes across unobserved types, controlling for the observed inputs.

6.6 Unobserved Abilities

The means and variances of the unobserved ability distributions are identified from the observed persistence in GPA and rank outcomes over time, which depends on the true values of the unobserved abilities and the signal noise. Intuitively, if students who perform well in one period tend to perform well in subsequent periods, this suggests that their performance is driven by persistent unobserved abilities rather than idiosyncratic shocks. The ability variances are identified from the degree of persistence in outcomes, with higher variances implying greater persistence, all else equal. The parameters of the learning process are identified from the observed changes in outcomes and choices over time as students accumulate more signals of their abilities. The speed of learning depends on the signal-to-noise ratios of the GPA and rank signals, which are identified from the relative magnitudes of the ability variances and the signal noise variances. The initial ability beliefs are identified from the variation in outcomes and choices in the first period, before any signals have been received.

7 Estimation

In this section, we describe the estimation method for the structural model parameters. We begin by presenting the estimation procedure without accounting for unobserved heterogeneity among students, as this simplifies the exposition and allows for a clearer understanding of the core steps. We then extend the estimation method to incorporate unobserved heterogeneity, which captures important differences across students that are not directly observable in the data.

7.1 GPA and Rank Production and Learning Parameters

We use Expectation-Maximization (EM) algorithms to estimate the parameters θ_g, $\sigma_g(a)$, θ_r and σ_r. The EM algorithm for the GPA production parameters alternates between two steps until reaching convergence:

E-step: We update the posterior ability distribution using all observed GPA data and the GPA production parameters from the previous iteration. Equation 8 provides the Bayesian updating formulas that update the posterior ability mean and variance. At each iteration h, we update the population variance of the ability distribution:

$$(\sigma_g(a)^{(h)})^2 = \frac{1}{N}\sum_{i=1}^{N}(\sigma_{g,i}(a)^{(h)})^2 + (\mu_{g,i}(a)^{(h)})^2 \tag{13}$$

where N represents the sample size, $\mu_{g,i}(a)^{(h)}$ denotes the posterior ability mean, and $\sigma_{g,i}(a)^{(h)}$ denotes the posterior ability standard deviation at the start of the E-step.

M-step: Using the posterior ability distribution from the E-step, we maximize the expected complete log-likelihood of the GPA data. Let $\varphi_{g,i}(\cdot)$ represent the pdf of the posterior ability distribution at iteration h. We maximize:

$$\begin{aligned}\mathbb{E}\left(L_{g,it}(a)^{(h)}\right) &= \int \log\left(\mathcal{L}_i(g_{it+1} \mid a, \ell_{it}, g_{it}, A_i(a))\right) \varphi_{g,i}^{(h)}(A_i(a))dA_i(a)\\ &= -\frac{1}{2}\log(2\pi\gamma_g(a)^2) - \frac{1}{2\gamma_g(a)^2}\left(\left(g_{it+1} - f_g(a, \ell_{it}, g_{it}; \theta_g^{(h)}) - \mu_{g,i}(a)^{(h)}\right)^2 + (\sigma_{g,i}(a)^{(h)})^2\right)\end{aligned} \tag{14}$$

where the normality assumptions on the idiosyncratic shocks and unobserved ability lead to the second equality. We update the parameters θ_g by solving:

$$\max_{\theta_g}\sum_{i,t,a}(a_{it} = a)\mathbb{E}\left(L_{g,it}(a)^{(h)}\right) \tag{15}$$

We follow a similar two-step EM procedure to estimate the rank production parameters:

E-step: Using observed rank data and rank production parameters from the previous iteration, we update the posterior cohort ability distribution. Equation 5 provides the Bayesian updating formulas that update the posterior ability mean and variance. At each iteration h, we update the population variance:

$$(\sigma_r^{(h)})^2 = \frac{1}{N}\sum_{i=1}^{N}(\sigma_{r,i}^{(h)})^2 + (\mu_{r,i}^{(h)})^2 \tag{16}$$

where $\mu_{r,i}^{(h)}$ and $\sigma_{r,i}^{(h)}$ represent the posterior cohort ability mean and standard deviation at the start of the E-step.

M-step: Using the posterior cohort ability distribution from the E-step, we maximize the expected complete log-likelihood of the rank data. Let $\varphi_{r,i}(\cdot)$ represent the pdf of the posterior ability distribution at iteration h. We maximize:

In the M-step, Given the posterior ability distribution obtained at the E-step, we maximize the expected complete log-likelihood of the rank data. Namely, at the M-step of each iteration h of the EM estimation, denoting by $\varphi_{r,i}(\cdot)$ the pdf of the posterior ability distribution computed at the E-step, we maximize the expected complete log-likelihood:

$$\begin{aligned} &\int \sum_t \log\left(\mathcal{L}_i(r_{it} \mid g_{it}, R_{-i})\right)\varphi_{r,i}(R_{-i})dR_{-i} \\ &= -\frac{1}{2}\log(2\pi\gamma_r^2) - \frac{1}{2\gamma_r^2}\left((\Phi^{-1}(r_{i\tau}) - \alpha_g g_{i\tau} + \mu_{r,i}^{(h)})^2 + (\sigma_{r,i}^{(h)})^2\right) \end{aligned} \tag{17}$$

We update the parameters (γ_r, α_g) by solving:

$$\max_{\gamma_r}\sum_{i,\tau}\left[-\frac{1}{2}\log(2\pi\gamma_r^2) - \frac{1}{2\gamma_r^2}\left(((\Phi^{-1}(r_{i\tau}) - \alpha_g g_{i\tau} - \mu_{r,i}^{(h)})^2 + (\sigma_{r,i}^{(h)})^2\right)\right] \tag{18}$$

7.2 Utility Parameters

After estimating the GPA and rank production parameters, we focus on estimating the utility parameters θ_u that govern students' course choices.

To begin, we construct an initial guess of the conditional choice probabilities (CCPs) $p^{(1)}(a \mid \ell, r) = \Pr(a_{it} = a \mid \ell_{it} = \ell, r_{it} = r)$. We do this by grouping observations into bins based on discrete math level history $\ell_{it} \in \{0, \ldots, 5\}$ and discretized rank $r_{it} \in \{0.01, 0.02, \ldots, 0.99, 1.00\}$. For each (ℓ, r) bin, we compute the empirical frequency that students in that bin choose each course a. This gives us a non-parametric estimate of the CCPs.

Given $p^{(h)}$ and $\theta_u^{(h)}$, we solve for $\theta_u^{(h)}$ by maximizing:

$$\max_{\theta_u^{(h)}}\sum_{i,t,a}(a_{it} = a)L_{u,it}(a)^{(h)} \tag{19}$$

where

$$L_{u,it}(a)^{(h)} = \log\left[\frac{\exp\left(v_t\left(a, \ell_{it}, r_{it}, p_{it+1}^{(h)}; \theta_u^{(h)}, \hat{\theta}_g, \hat{\sigma}_g(a), \hat{\theta}_r, \hat{\sigma}_r\right)\right)}{\sum_a \exp\left(v_t\left(a, \ell_{it}, r_{it}, p_{it+1}^{(h)}; \theta_u^{(h)}, \hat{\theta}_g, \hat{\sigma}_g(a), \hat{\theta}_r, \hat{\sigma}_r\right)\right)}\right] \tag{20}$$

Then we update $p^{(h+1)}$

$$p_{it}(a, \ell, r)^{(h+1)} = \frac{\exp\left(v_t\left(a, \ell, r, p_{it+1}(\ell', r')^{(h)}; \theta_u^{(h+1)}, \hat{\theta}_g, \hat{\sigma}_g(a), \hat{\theta}_r, \hat{\sigma}_r\right)\right)}{\sum_a \exp\left(v_t\left(a, \ell, r, p_{it+1}(\ell', r')^{(h)}; \theta_u^{(h+1)}, \hat{\theta}_g, \hat{\sigma}_g(a), \hat{\theta}_r, \hat{\sigma}_r\right)\right)} \tag{21}$$

We continue these two steps until convergence.

7.3 Estimation With Unobserved Heterogeneity

After accounting for unobserved heterogeneity, we introduce additional parameters: α_k, β_k, and θ_k. These parameters capture the unobserved heterogeneity entering the utility, the unobserved heterogeneity entering the GPA production, and the probability of being unobserved type k conditional on the initial condition x_{i1}, respectively. At the start of iteration $(h+1)$, we first update the posterior ability distributions using all observed outcomes and course choice data, the production and learning parameters, and the Bayesian updating formulas for the posterior ability mean and covariance. With the obtained pdf of the posterior ability distribution, we construct the type-specific log-likelihood associated with the transition processes and the utility. We then calculate the posterior probability of being type k as follows:

$$q_i(k)^{(h)} = \frac{\pi_{k|x_{i1}}^{(h)} \exp\left[\sum_{t,a}(a_{it} = a)\mathbb{E}(L_{g,it}(a,k)^{(h)})\right] \exp\left[\sum_{t,a}(a_{it} = a)L_{u,it}(a,k)^{(h)}\right]}{\sum_k \pi_{k|x_{i1}}^{(h)} \exp\left[\sum_{t,a}(a_{it} = a)\mathbb{E}(L_{g,it}(a,k)^{(h)})\right] \exp\left[\sum_{t,a}(a_{it} = a)L_{u,it}(a,k)^{(h)}\right]} \tag{22}$$

where $\pi_{k|x_{i1}}^{(h)}$ denotes the probability of a student i being unobserved type k conditional on their initial conditions x_{i1}:

$$\pi_{k|x_{i1}}^{(h)} = \frac{\exp(x_{i1}\theta_k^{(h)})}{\sum_k \exp(x_{i1}\theta_k^{(h)})}. \tag{23}$$

We proceed to update the parameters governing the probability of student i being type k conditional on their initial conditions x_{i1}:

$$\theta_k^{(h+1)} = \arg max_{\theta_k} \sum_{i,k} q_i(k)^{(h)} \log\left(\pi_{k|x_{i1}}(\theta_k)\right) \tag{24}$$

Next, we update the population variance of the ability distribution:

$$(\sigma_g(a)^{(h+1)})^2 = \frac{1}{N}\sum_i \left((\sigma_{g,i}(a)^{(h)})^2 + \sum_k q_i(k)^{(h)}(\mu_{g,i}(a,k)^{(h)})^2\right) \tag{25}$$

where N is the sample size. We then update the parameters governing the GPA transition process:

$$\theta_g^{(h+1)} = \arg max_{\theta_g} \sum_{i,k} q_i(k)^{(h)} \sum_{t,a} (a_{it} = a) \mathbb{E}\left(L_{g,it}(a,k)^{(h)}\right) \tag{26}$$

followed by updating the parameters governing the utility:

$$\theta_u^{(h+1)} = \arg max_{\theta_u} \sum_{i,k} q_i(k)^{(h)} \sum_{t,a} (a_{it} = a) L_{u,it}(a,k)^{(h)} \tag{27}$$

Finally, we update the type-specific conditional choice probabilities. We repeat these steps until convergence is achieved. We summarize the estimation steps in 1.

Algorithm 1 Sequential E-M Algorithm

Require: Initial guesses for parameters and CCPs
Ensure: Convergence of parameters and CCPs
  Initialize iteration counter $h \leftarrow 1$
  while not converged **do**
    Construct the posterior probability of being type k using Equation (22), denoted by $q_i(k)^{(h)}$
    Update the parameters governing the probability of being type k $\theta_k^{(h+1)} \leftarrow \arg\max_{\theta_k} \sum_{i,k} q_i(k)^{(h)} \log\left(\pi_{k|x_{i1}}(\theta_k)\right)$
    Update population variance: $(\sigma_g(a)^{(h)})^2 \leftarrow \frac{1}{N}\sum_{i=1}^{N}(\sigma_{g,i}(a)^{(h)})^2 + \sum_k q_i(k)^{(h)}(\mu_{g,i}(a,k)^{(h)})^2$
    Update the GPA production parameters $\theta_g^{(h+1)} \leftarrow \arg\max_{\theta_g} \sum_{i,k} q_i(k)^{(h)} \sum_{t,a}(a_{it} = a)\,\mathbb{E}\left(L_{g,it}(a,k)^{(h)}\right)$
    Update the flow utility parameters $\theta_u^{(h+1)} \leftarrow \arg\max_{\theta_u} \sum q_i(k)^{(h)} \sum (a_{it} = a)\, L_{u,it}(a,k)^{(h)}$
    Update the CCPs $p_{it}(a,\ell,r,k)^{(h+1)} \leftarrow \frac{\exp\left(v_t\left(a,\ell,r,k,p_{it+1}(\ell',r',k)^{(h)};\theta_u^{(h+1)},\theta_g^{(h+1)},\sigma_g(a)^{(h+1)},\hat{\theta}_r,\hat{\sigma}_r\right)\right)}{\sum_a \exp\left(v_t\left(a,\ell,r,k,p_{it+1}(\ell',r',k)^{(h)};\theta_u^{(h+1)},\theta_g^{(h+1)},\sigma_g(a)^{(h+1)},\hat{\theta}_r,\hat{\sigma}_r\right)\right)}$
    $h \leftarrow h + 1$
  end while=0

8 Model Estimates

In this section, we present the estimated parameters of our dynamic discrete choice model of high school course-taking. We focus on the key components of the model, including the flow utility parameters, rank production and learning parameters, and GPA production and learning parameters. We also assess the model's performance in capturing observed choice patterns using a forward simulation approach.

8.1 Flow Utility Parameters

Table 1 reports the estimated flow utility parameters for each course option: None, Regular, Weighted, and Accelerated. The constant term is highest for the Regular course (-0.96) and lowest for the Weighted course (-7.15), suggesting that weighted courses may involve a higher cost of effort for students. The Accelerated dummy variable, indicating whether a student is above the

Table 6. Flow Utility Parameters

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	None		Regular		Weighted		Accelerated	
	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
Constant	-2.86	(0.03)	-0.96	(0.03)	-7.15	(0.05)	-2.55	(0.03)
Accelerated	1.16	(0.02)	-1.33	(0.02)	0.77	(0.02)	-0.21	(0.03)
Past Rank	3.49	(0.10)	-1.16	(0.09)	0.25	(0.10)	-3.44	(0.11)
Past Rank squared	-2.81	(0.11)	1.58	(0.10)	1.09	(0.11)	4.35	(0.12)
Black	-0.16	(0.02)	-0.00	(0.01)	0.08	(0.02)	-0.17	(0.02)
Hispanic	-0.24	(0.03)	0.16	(0.02)	0.06	(0.03)	-0.37	(0.03)
Female	-0.13	(0.02)	0.27	(0.02)	0.26	(0.02)	-0.17	(0.03)
Type 2	-0.85	(0.02)	0.13	(0.02)	5.16	(0.05)	0.25	(0.02)

*Note:*This table presents estimated coefficients and standard errors for various student characteristics' effects on course placement across different alternatives (None, Regular, Weighted, Accelerated). The dependent variable indicates which alternative is chosen. Coefficients are reported for variables including accelerated status, past rank, race/ethnicity, gender, and student type. Standard errors are shown in parentheses.
Data source: Administrative data from a large urban district in Texas

recommended math level for their grade, has a positive effect on the utility of choosing Weighted courses (0.77) but a negative effect on Regular (-1.33) and Accelerated (-0.21) courses. This suggests that accelerated students may prefer weighted courses to maintain their advanced standing while avoiding the additional challenge of accelerated courses or the repetition of regular courses.

Demographic factors also play a role in shaping students' course preferences. Being Black or Hispanic is negatively associated with the utility of choosing Accelerated courses, while being female is positively associated with the utility of choosing Regular and Weighted courses. The model accounts for unobserved student heterogeneity by incorporating two types, with Type 2 students exhibiting a significantly higher utility for Weighted courses (5.16) compared to other options, indicating systematic differences in motivation and preferences.

The utility from taking a weighted math course increases with rank, while the utility from taking either a regular or accelerated course decreases with rank, with coefficients of -1.16 and -3.44, respectively. Additionally, utility is positively associated with rank squared across all course types, with coefficients of 1.09 for the weighted course, 1.58 for the regular course, and 4.35 for the accelerated course. This suggests that higher-ranked students derive greater utility from enrolling in weighted courses, but less utility from regular and accelerated courses, though the positive rank squared coefficients indicate that for all course types, utility increases more rapidly as rank moves higher, particularly for accelerated courses.

8.2 Terminal Utility Parameters

The terminal utility parameters for final high school class rank and rank squared are 13.8 and -13.3, respectively. Students derive more utility from achieving a higher rank, but the marginal utility decreases as they move closer to the top of the rank distribution.

8.3 Rank Production and Learning Parameters

The rank production function parameter estimates reveal that a one standard deviation increase in GPA leads to a 1.78-unit increase in rank, underscoring the strong positive association between individual achievement and relative position within the cohort. The standard deviation of the noise in the rank production function is estimated to be 0.34, with a signal-to-noise ratio of 0.097, suggesting that rank signals are relatively noisy and may not provide highly precise information about a student's true relative math ability.

8.4 GPA Production and Learning Parameters

Table 7. GPA Production and Learning Parameters

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	None		Regular		Weighted		Accelerated	
	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
Constant	-0.10	(0.01)	-0.14	(0.00)	-0.11	(0.02)	-0.01	(0.01)
Accelerated	-0.19	(0.00)	-0.04	(0.01)	0.13	(0.00)	0.01	(0.01)
Past GPA	0.81	(0.00)	0.78	(0.00)	0.83	(0.00)	0.75	(0.01)
Black	0.11	(0.00)	0.14	(0.00)	0.10	(0.00)	0.12	(0.01)
Hispanic	-0.01	(0.01)	0.00	(0.01)	-0.09	(0.01)	-0.10	(0.02)
Female	0.02	(0.01)	0.01	(0.00)	-0.05	(0.01)	-0.09	(0.01)
Type 2	0.29	(0.00)	0.29	(0.00)	0.05	(0.02)	0.02	(0.01)
Noise S.D.	0.43	(0.00)	0.66	(0.00)	0.57	(0.00)	0.79	(0.01)
Signal-to-Noise Ratio	0.05		0.02		0.10		0.23	

*Note:*This table presents estimated coefficients and standard errors for various student characteristics' effects on future GPA across different alternatives (None, Regular, Weighted, Accelerated). The dependent variable indicates future GPA. Coefficients are reported for variables including accelerated status, past GPA, race/ethnicity, gender, and student type. Standard errors are shown in parentheses.
Data source: Administrative data from a large urban district in Texas

Table 2 presents the estimated GPA production function parameters for each course option. Having taken an accelerated course in the previous period has a positive effect on GPA for students who subsequently enroll in a Weighted course (0.13) but a slightly negative effect for those who take a Regular course (-0.04). Past GPA emerges as a strong predictor of current GPA across all course options, with coefficients ranging from 0.75 for Accelerated to 0.83 for Weighted courses.

The estimates also reveal notable patterns across demographic groups and unobserved student types. Hispanic and female students exhibit lower GPAs in Weighted and Accelerated courses compared to their counterparts. Type 2 students, on average, have a higher GPA across all course options, aligning with their higher utility from taking Weighted courses and suggesting that they possess unobserved characteristics contributing to superior academic performance.

The standard deviation of the noise in the GPA production function varies across course options, with the Accelerated course exhibiting the highest noise standard deviation (0.79). The signal-to-noise ratio, measuring the informativeness of GPA signals, is highest for the Accelerated course

(0.23) and lowest for the Regular course (0.02).

8.5 Model Fit

Table 8

	(1) Grade 10	(2) Grade 11	(3) Grade 12
Leave	0.00	-0.04	-0.02
None	0.10	-0.01	-0.07
Regular	-0.05	0.07	0.07
Weighted	-0.02	-0.02	0.02
Accelerated	-0.03	0.00	0.00

Note: Model frequencies are constructed using 10 simulations of the structural model for each individual included in the estimation.

To assess the model's performance, we employ a forward simulation approach, generating model-predicted choice probabilities and comparing them to actual choice probabilities in the data. The simulation process involves drawing individual and relative ability vectors, assigning unobserved types, generating preference shocks, and updating the state space based on the estimated structural parameters.

Table 8 presents a comparison of the model-predicted and actual choice probabilities by grade level and choice alternative, demonstrating that the model captures the choice patterns in the data remarkably well across all grades and alternatives.

8.6 Ability Sorting

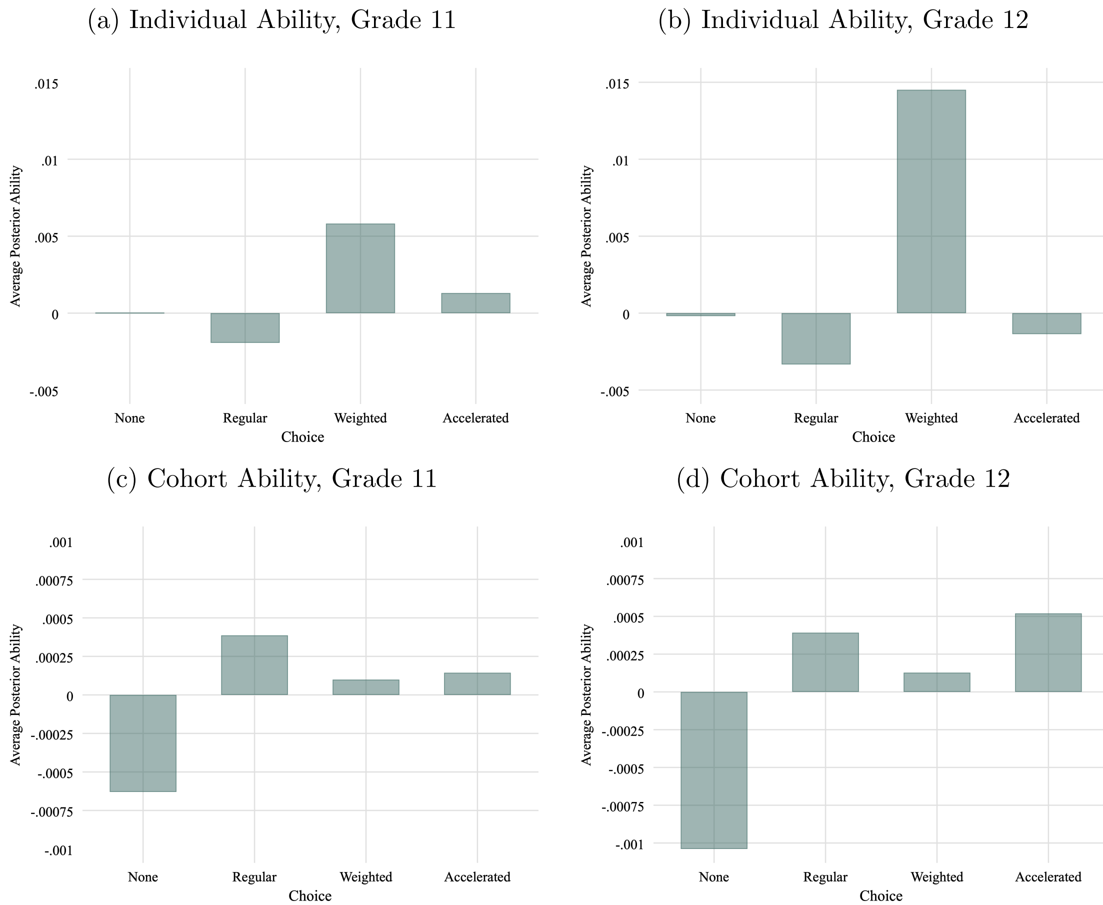
To better understand the patterns of ability sorting, we examine the posterior means of unobserved individual and cohort abilities at the time of the students' last high school enrollment. These results, depicted in Figure 4, were obtained through the forward simulation process outlined in the previous section.

Two key patterns emerge from Figure 4. First, students opting for weighted math courses tend to have received strong positive signals about their individual abilities. Second, students who remain in school but avoid rank-eligible math courses have received negative signals about their cohort's abilities.

Figures 4 (a) and (b) display the posterior means of individual ability for each course choice in grades 11 and 12, respectively. Students who select weighted courses show the highest posterior means of individual ability in both grades, with values of 0.006 in grade 11 and 0.015 in grade 12. These values indicate that these students have received positive feedback regarding their own academic capabilities.

Figures 4 (c) and (d) show the posterior means of cohort ability for each course choice, controlling for individual ability. Students who stay in school but do not take a rank-eligible math course

Figure 4. Ability Sorting



Note: This figure displays the average posterior abilities after last period of high school for different choice paths in the baseline model. Abilities are reported in standard deviation units. These figures are constructed using 10 simulations of the baseline model for each individual included in the estimation. Cohort abilities are plotted after residualizing over individual abilities.

have the lowest residualized posterior means of cohort ability, with values of -0.0006 in grade 11 and -0.001 in grade 12. This suggests that these students have received negative signals about the abilities of their peers.

Overall, these findings indicate a clear sorting pattern based on ability. Students who receive positive signals about their own abilities tend to enroll in weighted courses, while those receiving negative signals about their cohort's abilities tend to opt out of rank-eligible math courses.

9 Implications of Rank Non-Disclosure

In this section, we begin by using the structural parameter estimates and learning dynamics to examine the role of information regarding a student's own abilities in the counterfactual scenario where students are not informed about their rank. This analysis allows us to evaluate how the absence of rank information influences students' decision-making. We then complement these model-based predictions with model-free evidence to assess the impact of rank non-disclosure on course choices, providing additional empirical validation to our findings.

9.1 Counterfactual Policy Simulations

Table 9. Course enrollment frequencies in baseline and counterfactual: heterogeneity by clas rank

	(1) Grade 10	(2) Grade 11	(3) Grade 12
Panel A: Below 25th percentile			
Leave	-0.33	0.03	0.04
None	0.12	0.04	0.04
Regular	0.03	-0.02	-0.02
Weighted	0.06	-0.04	-0.12
Accelerated	0.02	0.03	0.05
Panel B: 25th to 50th percentile			
Leave	-0.33	0.02	0.02
None	0.08	0.02	0.03
Regular	0.02	-0.01	0.00
Weighted	0.04	-0.03	-0.05
Accelerated	0.03	0.01	0.01
Panel C: 50th to 75th percentile			
Leave	-0.37	-0.01	-0.01
None	0.04	0.00	-0.02
Regular	0.02	0.00	0.00
Weighted	0.04	0.01	0.03
Accelerated	0.03	0.01	0.02
Panel D: Above 75th percentile			
Leave	-0.44	-0.02	-0.04
None	0.05	-0.02	-0.09
Regular	0.00	-0.02	-0.02
Weighted	0.04	0.03	0.07
Accelerated	-0.01	0.03	0.04

Note: Model frequencies are constructed using 10 simulations of the structural model for each individual included in the estimation. Counterfactual frequencies use 10 simulations of each counterfactual model. "No disclosure" refers to our counterfactual where individuals have no information about their class rank.

Description: To simulate the counterfactual scenario, we consider three main sources of uncertainty: individual preference shocks, idiosyncratic shocks in the GPA and rank functions, and abilities. In addition, students are assumed to be unaware of their rank realization, although they continue to know their GPA realization. The absence of rank information has two implications for the model: (1) the student's perceived rank enters the flow utility, and (2) without rank signals, there is no further updating of beliefs about cohort abilities, so the belief about future rank is based on the student's initial prior.

We solve the model backwards to obtain the counterfactual choice probabilities and then forward simulate to obtain the distribution of choices and the average abilities across different choice paths. The results of these simulations are presented in Table 9.

Analysis: Table 9 displays the changes in choice probabilities under the rank non-disclosure policy, relative to the baseline scenario where students receive information about their rank. The results are disaggregated by grade level (10, 11, and 12) and by the student's initial rank percentile (below 25th, 25th to 50th, 50th to 75th, and above 75th).

Several key patterns emerge from the counterfactual simulations. First, across all rank percentiles, the probability of leaving school decreases substantially in grade 10 under the rank non-disclosure policy. This suggests that withholding rank information may encourage students to stay in school, particularly in the early years of high school.

Second, the impact of rank non-disclosure on course choice probabilities varies by initial rank percentile. For students below the 50th percentile (Panels A and B), the policy leads to a decrease in the probability of choosing weighted courses, particularly in grades 11 and 12. In contrast, for students above the 50th percentile (Panels C and D), the policy increases the probability of choosing weighted courses, especially in grade 12.

Third, the counterfactual simulations reveal heterogeneous effects of rank non-disclosure on the probability of choosing accelerated courses. While the policy increases the likelihood of choosing accelerated courses for students below the 50th percentile, it has mixed effects for students above the 50th percentile, with a decrease in grade 10 and an increase in grades 11 and 12.

These findings suggest that the impact of withholding rank information on students' course choices depends on their initial rank percentile. Students with lower initial ranks may be discouraged from taking more challenging courses in the absence of rank signals, while students with higher initial ranks may be more inclined to enroll in weighted and accelerated courses.

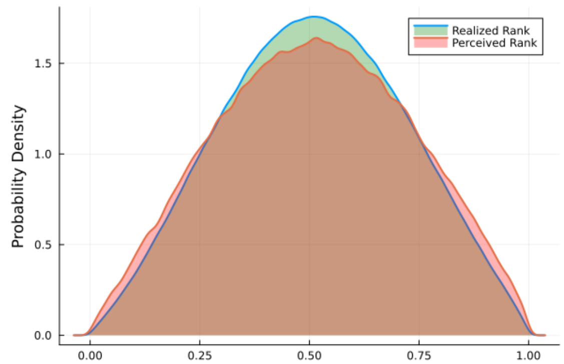
The counterfactual policy simulations provide valuable insights into the potential consequences of rank non-disclosure on students' educational decisions. The results highlight the importance of considering heterogeneous effects across different student subgroups when evaluating the impact of information provision policies.

Figure 5 plots the distributions of the realized and perceived rank. Those above the median tend to overestimate their rank when they are not informed of it explaining why AP math enrollment increases among those above the median when a rank non-disclosure policy is implemented.

9.2 Quasi-Experimental Evidence

To complement our structural estimates, we rely on quasi-experimental evidence on the effect of rank non-disclosure on high school course choices. The objective of this analysis is to provide additional evidence - independent of our model's assumptions-on how information non-disclosure

Figure 5. Distribution of Realized and Perceived Rank



Note: This figure displays the probability density distributions of realized rank (green) and perceived rank (red). The x-axis represents the rank from 0 to 1, where 0 is the lowest rank and 1 is the highest. The y-axis shows the probability density. The distributions largely overlap, with the perceived rank distribution showing a slight upward bias in the right-tail compared to the realized rank distribution. This suggests that individuals tend to perceive their rank as somewhat higher than their actual position, particularly in the right tail.

impacts high school course choices. The introduction of a rank non-disclosure in a school district in Texas provides us an opportunity to observe course choices in schools with and without rank disclosure.

Description: Our analysis utilizes a comprehensive dataset encompassing 57 schools across three independent school districts in Texas, spanning nine school years and including a total of 142,279 students. The dataset is particularly valuable due to a recent policy change that occurred in District A, which implemented rank non-disclosure starting in year y. This policy change creates a clear distinction between treatment and control groups: students in District A from year y onward no longer received information about their class rank, while students in the other districts and in District A prior to year y continued to receive rank information. The dataset provides rich information on student demographics, course selections, GPA, and rank for each school year.

Table 10 presents summary statistics for key variables across the three districts, revealing notable differences in student composition and academic outcomes.

Panel A of Table 10 highlights demographic variations among the districts. District A has a notably lower proportion of Black and Hispanic students (11% and 15% respectively) compared to Districts B (24% and 72%) and C (24% and 67%). This demographic composition suggests that District A serves a more predominantly white or Asian student population.

Panel B reveals distinct patterns in math course-taking behavior across the three districts. Students in District A, on average, take more math courses (3.80) compared to those in District B (3.15) and C (3.39). More strikingly, the average number of weighted math courses taken in District A (1.98) is substantially higher than in Districts B (0.58) and C (0.97).

The data also show a clear trend in advanced math course completion rates. While almost all students across all districts complete Algebra I and over 90% complete Geometry, the completion rates for higher-level math courses are consistently higher in District A. For instance, 87%

of students in District A complete Algebra II, compared to 73% and 80% in Districts B and C respectively. The gap widens further for Pre-Calculus (63% in A vs. 27% in B and 49% in C) and Calculus (29% in A vs. 4% in B and 14% in C).

Panel C presents dropout rates by grade level, revealing lower attrition in District A across all grades. By grade 10, only 6% of students in District A have dropped out, compared to 9% and 11% in Districts B and C. This trend persists through grades 11 and 12, with District A maintaining lower dropout rates. By grade 12, the cumulative dropout rate in District A (14%) is less than half that of District B (33%) and substantially lower than District C (29%).

Panel D illustrates consistently higher average GPAs in District A across all grade levels. This trend aligns with the observations from previous panels, further supporting the notion that District A students generally perform better academically.

It's important to note that District A is considerably smaller (8,769 students) compared to Districts B (37,750 students) and C (95,760 students). The combination of smaller size, higher course-taking rates in advanced math, lower dropout rates, and higher GPAs indicates that District A is, on average, a higher-performing district.

While these differences are noteworthy and could potentially confound our analysis, we account for these factors by including them as controls in our difference-in-differences approach to estimate the average treatment effect of the rank non-disclosure policy.

Table 10. Past academic and background characteristics by course choice

	(1)	(2)	(3)
	District A	District B	District C
Panel A: Demographics			
Black	0.11	0.24	0.24
Hispanic	0.15	0.72	0.67
Female	0.49	0.49	0.50
Panel B: Math Coursetaking			
Total Math courses	3.80	3.15	3.39
Total Weighted Math courses	1.98	0.58	0.97
Ever Completed Algebra I	1.00	0.99	1.00
Ever Completed Geometry	0.95	0.92	0.92
Ever Completed Algebra II	0.87	0.73	0.80
Ever Completed Pre-Calculus	0.63	0.27	0.49
Ever Completed Calculus	0.29	0.04	0.14
Panel C: Dropout			
Dropout by Grade 9	-	-	-
Dropout by Grade 10	0.06	0.09	0.11
Dropout by Grade 11	0.11	0.18	0.21
Dropout by Grade 12	0.14	0.33	0.29
Panel D: GPA			
GPA after Grade 9	91.07	87.47	85.07
GPA after Grade 10	91.29	87.58	85.33
GPA after Grade 11	91.24	87.80	85.67
GPA after Grade 12	91.16	87.90	86.24
Obs	8,769	37,750	95,760

Note: This table reports summary statistics for the data that is used to estimate our structural model. This is student-level data.
Data source: Administrative data from a large urban district in Texas

Analysis: Our study investigates the impact of a rank non-disclosure policy implemented in District A in year y^*. We employ a difference-in-differences approach to estimate the average treatment effect on the treated (ATT) of this policy on students' likelihood of enrolling in weighted math courses. The treatment group is defined based on two key conditions: enrollment in District A and placement below the 90th percentile. This definition accounts for the state mandate requiring disclosure of rank information to students in the top 10th percentile, irrespective of local policies.

To capture the timing of policy exposure, we introduce the variable $Post_{itsy} = y^* - (t - 9)$, indicating whether a student entered grade level t after the policy implementation. Our analysis utilizes the difference-in-differences approach developed by Callaway and Sant'Anna (2021), which effectively accounts for the staggered nature of treatment exposure across academic years. This method is particularly suitable for our context, where students were exposed to the rank non-disclosure policy at different stages of their academic journey.

To enhance the comparability of our treatment and control groups, we impose two key sample restrictions. First, we limit the sample to students at or above the 75th percentile. This allows us to compare students who are similar in terms of academic performance, with those below the 90th percentile in the treatment group and those above in the control group. Second, we restrict the sample to students who have completed at most the recommended level of math. This addresses the systematic differences observed in District A, which appears to be higher-performing overall. These restrictions ensure that we are comparing similar high-performing students and controlling for District A's apparent higher overall performance. In our analysis, we also control for student demographics and cluster standard errors at the school level.

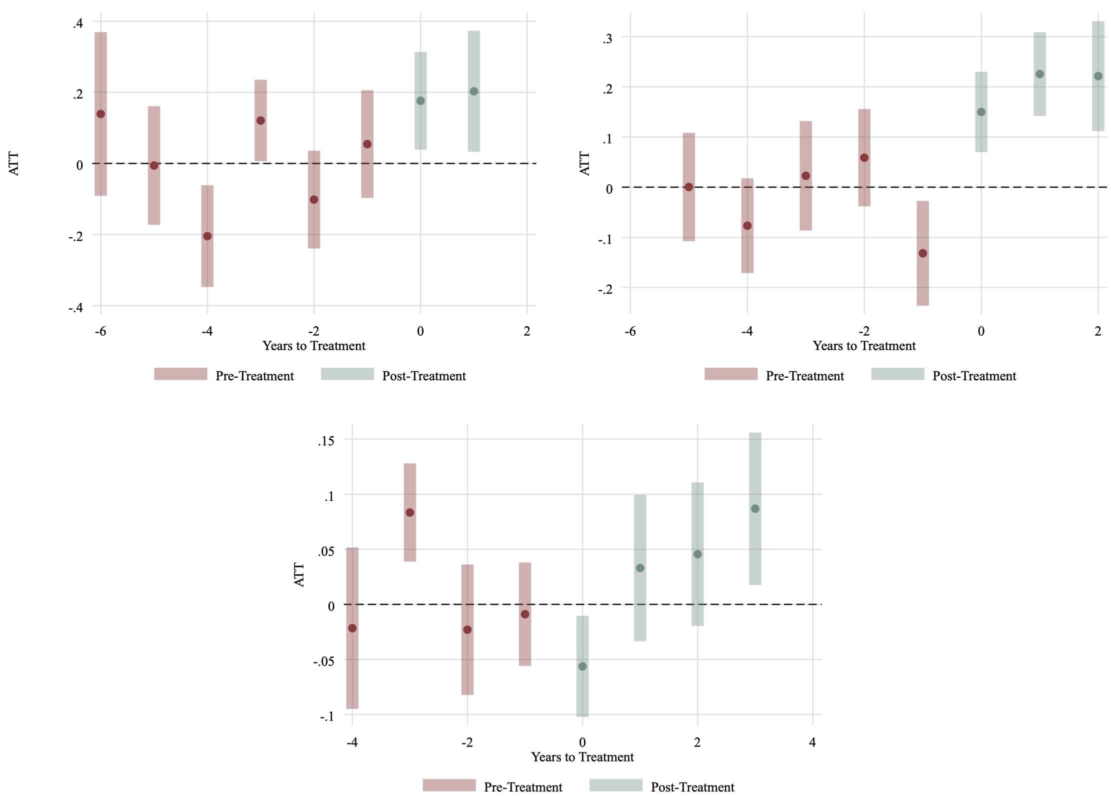
Figure 6 presents event study plots for grades 10, 11, and 12, illustrating the policy's impact on weighted math course enrollment. For grade 10 (Figure 6 (a)), we observe two post-treatment years, with the second cohort receiving full treatment (exposed from ninth grade). The results show a significant positive ATT in both post-treatment years, with no substantial difference between ninth and tenth-grade exposure. This similarity is likely because rank information is first received at the start of tenth grade, so exposure to the policy in ninth grade does not significantly alter the student's information set.

The grade 11 analysis (Figure 6 (b)) displays three post-treatment years, revealing a positive and significant ATT for all cohorts exposed to the policy. For grade 12 (Figure 6 (c)), we see four post-treatment years, with a significant positive ATT only for the fourth cohort that received full treatment starting from ninth grade. Interestingly, the ATT is close to zero or even negative for cohorts with shorter exposure periods.

Across all grades, we observe that pre-treatment trends are generally stable and close to zero, supporting the parallel trends assumption crucial for the validity of our difference-in-differences approach. However, we note a notable exception of a positive trend three years pre-implementation for grades 10 and 12, which warrants further investigation.

Table 11 summarizes the ATT for full treatment cohorts across grades, revealing substantial effects. Tenth graders are 20 percentage points more likely to take weighted math when they do

Figure 6. Average Treatment Effects Over Time for Three Different Cohorts



Note: These graphs show the average treatment effects (ATT) over time for three different cohorts. The x-axis represents years relative to treatment, with 0 being the year of treatment. The y-axis shows the magnitude of the treatment effect. Red bars and points represent pre-treatment periods, while green bars and points represent post-treatment periods. The vertical bars indicate 95% confidence intervals.
Data: Administrative data from Independent School Districts in Texas

not know their rank compared to when they do. The effect is even larger for eleventh graders, with a 22 percentage point increase, while twelfth graders show a 9 percentage point increase. These results suggest that the rank non-disclosure policy has a substantial positive impact on students' propensity to enroll in weighted math courses, with the effect varying by grade level and duration of exposure. The diminishing effect in grade 12 may indicate a ceiling effect or a shift in student priorities as they approach graduation.

These results are consistent with the model predicts and suggest that removing potential discouragement effects associated with rank disclosure may encourage students to pursue more rigorous coursework.

Table 11. Average Treatment Effect on Weighed Math Enrollment

	(1) Grade 10	(2) Grade 11	(3) Grade 12
ATT	0.20^{**}	0.22^{***}	0.09^{**}
	(0.09)	(0.06)	(0.04)
Mean	0.40	0.30	0.17
Obs	35,717	46,455	56,541

Note: This table reports the Average Treatment Effect (ATT) on weight math enrollment for grades 10, 11, and 12. Standard errors are reported in parentheses. $***p < 0.01, **p < 0.05, *p < 0.1$. Mean represents the average enrollment rate for each grade.
Data source: Administrative data from Independent School Districts in Texas

10 Conclusion

This paper investigates the impact of class rank disclosure policies on high school students' course choices, with a focus on math course-taking. We develop and estimate a dynamic structural model of course choice under imperfect information, incorporating Bayesian belief updating about students' own abilities and their cohort's average ability based on GPA and class rank signals. Our model captures the dynamic nature of student decision-making and the role of uncertainty and information in shaping educational outcomes.

The estimated model aligns closely with descriptive patterns in the data and reveals the significant role of rank information in shaping students' course choices. Counterfactual policy simulations demonstrate that removing rank information leads to increases in advanced course-taking among high-achieving students, consistent with reduced discouragement effects. The impact of rank non-disclosure varies by students' initial rank, with those above the 50th percentile being more inclined to enroll in weighted and accelerated courses in the absence of rank information.

To validate our structural results, we conduct a quasi-experimental analysis exploiting a natural experiment in rank disclosure policies across multiple school districts. Using a generalized difference-in-differences design, we find large positive effects of rank non-disclosure on the likelihood of taking advanced math courses, with the effects growing over time. The distributional patterns of the quasi-experimental estimates align with our model's counterfactual predictions, providing an important external validation.

Our findings highlight the importance of considering the informational environment and its impact on students' beliefs and choices when designing educational policies. The results suggest that strategic manipulation of information provision, such as withholding rank information, can lead to meaningful changes in course-taking behavior and potentially improved long-term outcomes for students. Policymakers and educators should carefully consider the potential consequences of altering the information available to students during their decision-making process.

This study contributes to the growing literature on the impact of relative performance feedback in educational settings. We innovate by modeling multi-dimensional Bayesian belief updating based on absolute and relative performance signals, bridging the gap between the learning and feedback literatures. Methodologically, our use of unobserved heterogeneity in both flow utility and transitions, estimated via a sequential EM algorithm, pushes the frontier of modeling unobserved heterogeneity in dynamic structural models.

Future research could extend our analysis in several directions. Eliciting expectations about future GPA and rank could provide a richer understanding of the learning process. Comparing the outcomes for districts that have switched to different types of non-disclosure policies will provide more robust evidence and further test the model's predictions. Finally, linking high school course choices to long-term educational and labor market outcomes would enable a more comprehensive assessment of the welfare consequences of these policies.

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