Big Bang Nucleosynthesis

Riya Singh
Animesh Singh
Mekhala Paranjpe
Deepak Singh
Mayank Jain

February 2, 2017

1 Content

2 Introduction to Big Bang Theory

The Big Bang theory is the prevailing cosmological model for the universe from the earliest known periods through its subsequent large-scale evolution. The model describes how the universe expanded from a very high-density and high-temperature state, and offers a comprehensive explanation for a broad range of phenomena, including the abundance of light elements, the cosmic microwave background (CMB), large scale structure and Hubble's law. If the known laws of physics are extrapolated to the highest density regime, the result is a singularity which is typically associated with the Big Bang. Physicists are undecided whether this means the universe began from a singularity, or that current knowledge is insufficient to describe the universe at that time. Detailed measurements of the expansion rate of the universe place the Big Bang at around 13.8 billion years ago, which is thus considered the age of the universe. After the initial expansion, the universe cooled sufficiently to allow the formation of subatomic particles, and later simple atoms. Giant clouds of these primordial elements later coalesced through gravity in halos of dark matter, eventually forming the stars and galaxies visible today.

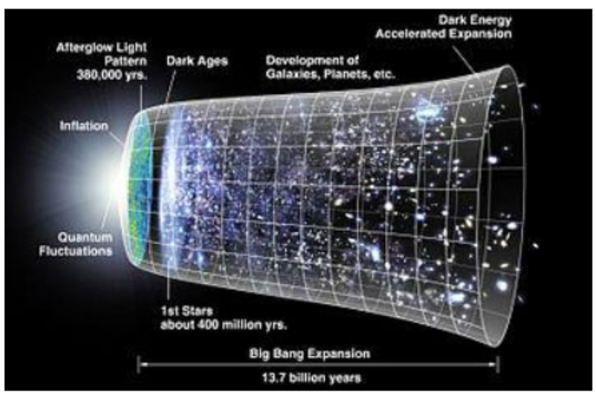


Figure 1: Timeline of the metric expansion of space

American astronomer Edwin Hubble observed that the distances to faraway galaxies were strongly correlated with their redshifts. This was interpreted to mean that all distant galaxies and clusters are receding away from our vantage point with an apparent velocity proportional to their distance: that is, the farther they are, the faster they move away from us, regardless of direction. Assuming the Copernican principle (that the Earth is not the centre of the universe), the only remaining interpretation is that all observable regions of the universe are receding from all others. Since we know that the distance between galaxies increases today, it must mean that in the past galaxies were closer together. The continuous expansion of the universe implies that the universe was denser and hotter in the past. The Big Bang theory offers a comprehensive explanation for a broad range of observed phenomena, including the abundance of light elements, the CMB, large scale structure, and Hubble's Law. The framework for the Big Bang model relies on Albert Einstein's theory of general relativity and on simplifying assumptions such as homogeneity and isotropy of space. The governing equations were formulated by Alexander Friedman, and similar solutions were worked on by Willem de Sitter. Since then, astrophysicists have incorporated observational and theoretical additions into the Big Bang model, and its parametrization as the Lambda-CDM model serves as the framework for current investigations of theoretical cosmology. The Lambda-CDM model is the current "standard model" of Big Bang cosmology, consensus is that it is the simplest model that can account for the various measurements and observations relevant to cosmology.

3 Nucleosynthesis

3.1 Defination

Nucleosynthesis is the process that creates new atomic nuclei from pre-existing nucleons, primarily protons and neutrons. The first nuclei were formed about three minutes after the Big Bang, through the process called Big Bang nucleosynthesis. It was then that hydrogen, helium and lithium formed to become the content of the first stars, and this primeval process is responsible for the present hydrogen/helium ratio of the cosmos.

3.2 History of nucleosynthesis theory

The first ideas on nucleosynthesis were simply that the chemical elements were created at the beginning of the universe, but no rational physical scenario for this could be identified. Gradually it became clear that hydrogen and helium are much more abundant than any of the other elements. All the rest constitute less than 2 percent of the mass of the Solar System, and of other star systems as well. At the same time, it was clear that oxygen and carbon were the next two most common elements, and also that there was a general trend toward high abundance of the light elements, especially those composed of whole numbers of helium-4 nuclei.

Arthur Stanley Eddington first suggested in 1920, that stars obtain their energy by fusing hydrogen into helium and raised the possibility that the heavier elements may also form in stars. This idea was not generally accepted, as the nuclear mechanism was not understood. In the years immediately before World War II, Hans Bethe first elucidated those nuclear mechanisms by which hydrogen is fused into helium.

The goal of the theory of nucleosynthesis is to explain the vastly differing abundances of the chemical elements and their several isotopes from the perspective of natural processes. The primary stimulus to the development of this theory was the shape of a plot of the abundances versus the atomic number of the elements. Those abundances, when plotted on a graph as a function of atomic number, have a jagged saw tooth structure that varies by factors up to ten million. A very influential stimulus to nucleosynthesis research was an abundance table created by Hans Sues and Harold Urey that was based on the unfractionated abundances of the non-volatile elements found within unevolved meteorites. Such a graph of the abundances is displayed on a logarithmic scale below, where the dramatically jagged structure is visually suppressed by the many powers of ten spanned in the vertical scale of graph.

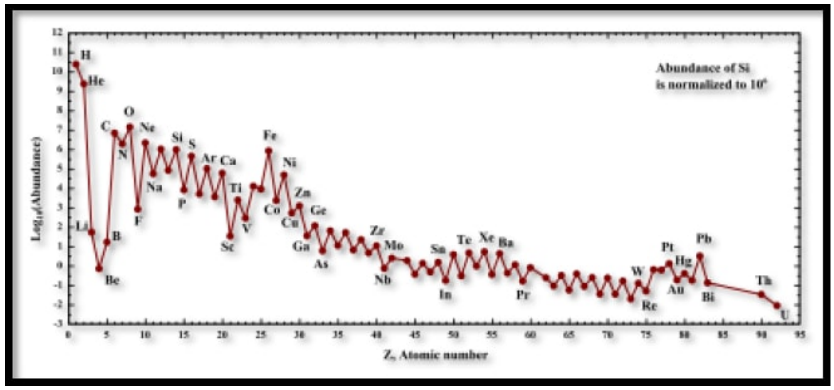


Figure 2: Abundance table created by Hans Sues and Harold Urey

4 Einstein theory of Big Bang

We all know about Einstein and about his amazing theories. The special theory of relativity came about when he was thinking in the patent office and thinking what would happen if we could move at the speed of light. Do we see standing waves? After his ground-breaking work on the special theory of relativity he set out to for the journey on the discovery of general theory of relativity.Most of us know that he realized that gravity and acceleration are the same things when he thought that people couldn't differentiate whether they are on the earth or on a satellite accelerating at g. But how did he realize about curvature of space due to acceleration is an interesting story not known by many. We know that the length of an object reduces along the direction of its travel and stays the same perpendicular to it. Einstein considered about this effect on a rotating disc. We know that the circumference of the disc will decrease as it is along the direction of motion but its radius remains constant which is a contradiction and this Einstein resolved by assuming that space bends due to acceleration. Now how is this related to big bang. Einstein assumed the universe to be stable but his equations predicted that this is impossible and the universe should either expand or contract. So, he added a constant to his equation to stabilize our universe Some people say that he committed this is one of the two major mistakes he committed in his life, the other being his unacceptance of the quantum theory. Hubble's observation that the universe is expanding proved that the universe is expanding and led to the idea that initially everything could have been at one place. But years later when dark matter were discovered people realized that the Einstein constant that had no explanation accidently had a physical implementation as dark matter. In a recently discovered work of his it was realized that Einstein had already predicted that if the universe was expanding then new matter must be created which is dark matter

5 Hubble's Law

When most people believed in an essentially static and unchanging universe, the question of whether or not it had a beginning was really one of metaphysics or theology. One could account for what was observed equally well on the theory that the universe had existed forever or on the theory that it was set in motion at some finite time in such a manner as to look as though it had existed forever. But in 1929, Edwin Hubble made the landmark observation that wherever you look, distant galaxies are moving rapidly away from us. In other words, the universe is expanding. Hubble's law is considered the first observational basis for the expansion of the universe and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. The law is often expressed by the equation $v = H_o * D$, with H_o the constant of proportionality - Hubble constant - between the "proper distance" D to a galaxy, which can change over time, unlike the comoving distance, and its velocity v, i.e. the derivative of proper distance with respect to cosmological time coordinate. In the 1920s, when astronomers began to look at the spectra of stars in other galaxies, they found something most peculiar: there were the same characteristic sets of missing colors as for stars in our own galaxy, but they were all shifted by the same relative amount toward the red end of the spectrum(as a consequence of the Doppler effect of light) which means that nearly all of them are moving away from us. More surprising still was the finding that Hubble published in 1929: even the size of a galaxy's redshift is not random, but is directly proportional to the galaxy's distance from us. Or, in other words, the farther a galaxy is, the faster it is moving away! And that meant that the universe could not be static, as everyone previously had thought, it is in fact expanding. The current expansion of the universe means that at earlier times objects would have been closer together. In fact, it seemed that there was a time, about ten or twenty thousand million years ago, when they were all at exactly the same place and when, therefore, the density of the universe was infinite and the universe may have started with an explosion which is popularly known as the Big Bang.

6 Observations in support of Big Bang Nucleosynthesis (BBN)

6.1 Observation 1

Abundance of Helium in terms of mass fraction (Y_p)= 0.2477. This observation was made by M. Peimbert, V. Luridiana, A. Peimbert on the basis of new atomic physics computations of the recombination coefficients of He I and of the collisional excitation of the H I Balmer lines together with observations and photoionization models of metal-poor extragalactic H II regions.

This observation is a very strong verification of Big Bang Nucleosynthesis Theory. According to theory, during nucleosynthesis all the free neutrons are swept up and bound into nuclei. Once bound in this way, the strong interaction between the protons and neutrons stabilizes the neutrons preventing them from decaying. By the time that nucleosynthesis starts, the decay of free neutrons has shifted the p/n ratio to 87/13. Out of every 200 particles 26 neutrons will combine with 26 protons to form 13 helium nuclei. This leaves 148 protons, so the mass ratio of the nuclei produced is: (13*4)/(13*4+200-26*2)=26 percent. Thus, we see that even the crude calculation is in good argument with the observation. More refined calculations show that the exact proportions of the elements produced depend on the density of protons and neutrons and the expansion rate of the universe during the period of nucleosynthesis (both influence the rate at which particles can interact with each other). Helium comes out in the region of 24–25 percent, deuterium about 0.01 percent and lithium $10-7$ percent.

6.2 Observation 2

Abundance of Deuterium was determined by John M. O'Meara, David Tytler, David Kirkman, Nao Suzuki, Jason X. Prochaska, Dan Lubin Arthur M. Wolfe by analyzing SNR spectra of HS 0105+1619 in both low and high resolution. The low resolution spectra were obtained using the Kast double spectrograph on the Shane 3 meter telescope at Lick observatory. The high resolution spectra were obtained using the HIRES spectrograph on the Keck-I telescope. They found that the D/H ratio is $\log(D/H) = 4.48$. Using this value baryon density was calculated and it was found that Ω_b*h*h = 0.0213.

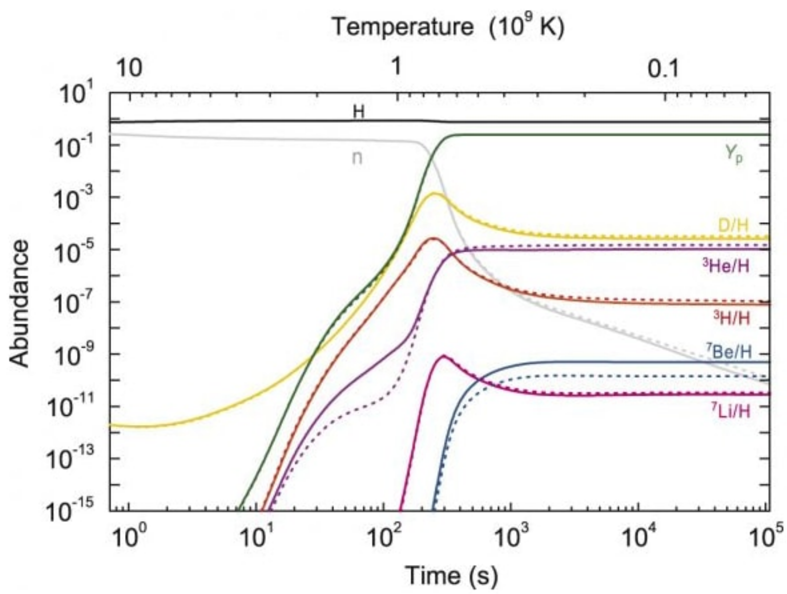
Baryon density can be calculated using BBN and helium density mass fraction (which itself is obtained from BBN) and it was found that Ω_b*h*h = 0.02122.

Thus we can see that this observation is in accordance with the theory.

7 Observations which can't be explained by Big Bang Nucleosynthesis

Martin Asplund, David L. Lambert, Poul Erik Nissen,Francesca Primas, Verne V. Smith studied stars from the uvby- catalogue of Schuster Nissen and from the Li abundance survey of very metal-poor stars by Ryan et al. They found two observations which are incompatible with Big Bang Nucleosynthesis. WMAP-based analysis of the cosmic microwave background implies Ω_b*h*h = 0.0224 ,which in standard Big Bang nucleosynthesis corresponds to an abundance of log (^{7}Li) = 2.65 . This predicted abundance of ^{7}Li is about a factor of three larger than the measured abundance of lithium on the Spite plateau (The Spite plateau is a baseline in the abundance of lithium found in old stars orbiting the galactic halo.). Thus, the open question is – "How does one account for the factor of 3 between observation and prediction?" Their results suggest that there may be a 6Li plateau parallel to the Spite plateau for ^{7}Li with the implication that the major fraction of the ^{6}Li may have been synthesised prior to the onset of star formation in the Galaxy. The ^{6}Li abundance of the plateau exceeds by a large factor the ^{6}Li expected from a standard Big Bang. Thus, the open question is – "How does one account for the high abundance of ^{6}Li in some metal-poor stars?"

7.1 Modifications done in BBN theory to account for above problem



One assumption in Big Bang nucleosynthesis is that all of the nuclei are in thermodynamic equilibrium, and that their velocities conform to classical Maxwell-Boltzmann distribution. But the Maxwell-Boltzmann describes what happens in what is called an ideal gas. Suqing Hou and Jianjun He propose that nuclei in the plasma of the early photon period of the universe behaved slightly differently than thought (in the same way as real gases are different from ideal gases). They applied non-extensive statistics and got the value shown by dotted line. New values confirm to observation.

8 Open Problem

- Lithium Problem
- The horizon problem – How did the apparent large-scale homogeneity of the universe come about, when in the earliest moments of the expansion all parts of the universe would have been beyond the "horizon" of other parts and therefore unable to become so nearly the same in temperature and density?
- The flatness problem – How could the parameter called have had a value so extraordinarily close to 1 in the early moments of the expansion (implying that the universe was then geometrically "flat") that even today can't be much smaller than 0.01?
- The magnetic monopole problem – Why have no magnetic monopoles or other "topological defects" been detected anywhere, even though they are very likely to have been produced during the earliest moments of the universe as a result symmetry breaking between the strong and electroweak forces?
- Matter and antimatter forms when photons combined. So they should have been created in equal amount. Then why is there an asymmetry in their current amount?
- What is dark matter made up of?
- Does neutrino have mass? Can it be the constituent particle of dark matter?
- What physical principles produced an era of inflation? Why inflation stopped after about one hundred cycles of doubling the length scale of the universe?
- Why there are any inhomogeneities in the universe despite of inflation (stars/ galaxies/ clusters of galaxies in place of very thin gas of photons and atoms)? We see fluctuations of temperature in the early universe which can be related to these inhomogeneities but the origin of these fluctuations is itself unexplained
- The cosmological constant is the value of the energy density of the vacuum of space. How can such a thing be experimentally determined?
- Supersymmetry (SUSY) is a theory that links gravity with the other fundamental forces of nature by proposing a relationship between bosons and fermions. Can it explain presence of dark matter and how?
- Does graviton exist? What are effects of its properties on the Big Bang Nucleosynthesis?

- Where do the particles get their mass from and why do they have masses which is essential for the universe to be in the form which it is now?
- Why does there exist charge asymmetry within one generation of fundamental particles (leptons / quarks)?
- Does Higgs bosons have mass and if yes what changes need to be done in electroweak theory so that the infinite sum needed to understand the particle interactions converges?
- Computations of the cosmological constant, assuming the existence of Higgs fields, produce a result that is absurdly large. Computations of the cosmological constant, assuming the existence of Higgs fields, produce a result that is absurdly large. What modifications need to be made in the existing model to prevent this from happening?
- Spontaneous symmetry breaking of the gauge symmetry associated with the electro-weak force separates the electromagnetic and weak forces. What caused this breaking to occur?
- ARCADE, a mission to detect faint radio signals from ancient stars, detected a huge amount of radio noise—six times louder than scientists had predicted and this is known as Space Roar. What is the source of space roar and how could it be explained?
- Which fundamental forces caused the big bang to happen?
- What happened during the "first" plank time after the big bang?

8.1 Lithium Problem

Big-bang nucleosynthesis (BBN) theory, together with the precise WMAP cosmic baryon density, makes tight predictions for the abundances of the lightest elements. The missing lithium problem is centred around the earliest stages of the universe from about 10 seconds to 20 minutes after the Big Bang. The universe was super hot and it was expanding rapidly. This was the beginning of what's called the Photon Epoch. Only the lightest nuclei were formed during this time, including most of the helium in the universe, and small amounts of other light nuclides, like deuterium and lithium. According to Big bang nucleosynthesis the amount of lithium should be three time more than the current amount present in the universe. It is quickly becoming as well known as the solar neutrino problem which ruled for many years as a seed for the development of theoretical physics imagination. It eventually was solved with an old idea: that neutrinos can oscillate. A few minutes(4 m - 20 m) after the Big Bang, nuclei started forming first with the creation of the deuteron by neutron capture on proton, p(n,)d. The formation of deuterons is strongly dependent on the baryon-to-photon ratio in the Big Bang epoch, b = baryons=photons. After the deuteron bottleneck is surpassed, all other heavier elements are synthesized, and are also therefore strongly dependent on b. A series of reactions involving neutron, proton, deuteron, and helium captures allow elements up to lithium and beryllium to be created during the Big Bang.

Most favoured solution present right now is our assumption in Big Bang nucleosynthesis is that all of the nuclei are in thermodynamic equilibrium, and that their velocities conform to what's called the classical Maxwell-Boltzmann distribution. But the Maxwell-Boltzmann describes what happens in what is called an ideal gas. Real gases can behave differently, and this is what the researchers propose: that nuclei in the plasma of the early photon period of the universe behaved slightly differently than thought.

8.2 Space Roar

8.2.1 Discovery

In 2006 NASA's Goddard Space Fight Center sent a giant ballon called ARCADE(Absolute Radiometer for Cosmology, Astrophysics and Diffuse Emission) in search of radiation from early stars. It carried seven sensors that picked electromagnetic radiation like radio waves. It was sent above the earth's atmosphere so that no inference can occur due to atmosphere and thus the instrument can detect faint radio signals in space.But the instrument detected a 'boom' instead of faint signals from the early stars. The intensity was six times louder than anyone could have predicted. Therefore this blast of radio waves was termed as Space Roar. Other instruments were sent up before but they detected radiation from all the directions and as the space roar was uniform in all the directions they could not detect it

8.2.2 Theories

The following are some of the theories that try to explain it

- The Automaton theory(Digital Physics) - This theory tells us that the amount of matter in the early universe was far more than now and thus the powerful radio waves could be explained. Automaton is a mathematical game by John Conway.
- The radiation might be coming from the earliest star that did not have any dust and thus infrared radiation need not be hand in hand with the radio waves intensity.
- They may be emitted by extremely dim but powerful radio galaxies
- A new type of physics that combines theory of relativity and quantum mechanics is required that could explain this phenomenon that may be due to some undiscovered principle.
- Modified Newton Dynamics theory

8.3 What is dark matter made up of and what limits its amount?

Scientists propose 2 types of constituents

8.3.1 Fundamental Particles

- Things we know exist - Neutrino : A light neutrino would be a type of dark matter known as hot dark matter, meaning that the particles have relativistic velocities for at least some fraction of the Universe's lifetime. In fact, hot dark matter does not have favourable properties for structure formation and if the neutrino has such a mass it is believed that it could at most contribute only part of the matter density, with some other form of dark matter also being required.

  Another possibility is that the neutrino could be very heavy, for example comparable to the proton mass. A heavy neutrino is an example of cold dark matter, meaning particles which have negligible velocities throughout the Universe's history. Having at least some cold dark matter is desirable for structure formation, but a heavy neutrino is much less desirable on particle physics grounds than a light one, and indeed is excluded by particle physics experiments unless the neutrino has unusual properties.

- Things we believe might exist: Particle physics theories (particularly those aiming at unification of fundamental forces) have a habit of throwing up all manner of new and as-yet- undiscovered particles, several of which are plausible dark matter candidates. Particle physicists associate a new companion particle to each of the particles we already know about. The lightest supersymmetric particle (LSP) eg. photino, or gravitino, or neutralino; is stable and is an excellent cold dark matter candidate. They are also sometimes known as WIMPs — Weakly Interacting Massive Particles.

8.3.2 Compact Objects

- Black Holes: A population of primordial black holes, meaning black holes formed early in the Universe's history rather than at a star's final death throes, would act like cold dark matter. However if they are made of baryons they must form before nucleosynthesis to avoid the nucleosynthesis bound. Baryons already in black holes by the time of nucleosynthesis don't count as baryons, as they are not available to participate in nuclei formation.

- MACHOs (MAssive Compact Halo Object) : MACHOs are unique amongst dark matter candidates in that they have actually been detected. They may be baryonic or non-baryonic. Brown dwarfs are a baryonic example.They were detected with the technique of gravitational microlensing in which we use the principle that when a MACHO passes in front of a star then its gravity bends the light, causing the star to appear brighter.

The mass of these invisible objects is estimated as a little less than a solar mass. However, although present they appear to have insufficient density to completely explain the observations related to dark matter.

9 Approaching Lithium Problem

During the initial satges of the Big Bang besides nucleosynthesis, nuclear reactions were also taking place due to the high temperatures. The abundance of helium and hydrogen as calculated from the hot Big Bang Theory fits nicely with the observational data. Things gets ugly when we estimate the amount of lithium. The expected amount of lithium is three times than that observed. This is referred to as the lithium problem.

Recently researchers from China have published a paper that may have solved the puzzle. They argued that the assumption of the Big Bang Theory that the universe was in thermodynamic equilibrium may be wrong and thus we cannot apply Maxwell-Boltzmann distribution as it applies for ideal gases only and we deal with real gases which behave differently.

The authors have used non-extensive statistics to solve the problem. This does not directly gives the abundance of lithium but it predicts the amount of beryllium that can then be used to calculate the amount of lithium.

9.1 Maxwell Boltzmann Distribution

Maxwell Boltzmann distribution is mainly used to describe properties of molecules of ideal gas. It gives the number of molecules moving between velocities v and v+dv. If we assume that one dimensional velocities are independent of eachother it reduces to $dN/N = (m/2\pi k_B T)^{1/2} * exp(-mv^2/2k_B T)dV$

where

- dN/N is the fraction of molecules moving at velocity v to v + dv,
- m is the mass of the molecule,
- kb is the Boltzmann constant, and
- T is the absolute temperature

9.1.1 Initial conditions and Maxwell Boltzmann Distribution

In the starting matters were in thermal equilibrium. At that stage, interaction rate (Γ) was greater than expansion rate (H) where expansion rate at a temperature T,

$\Gamma_{int}(T) = n(T) < \sigma|v|\sigma >^T$

and

H $\approx$T$^2/M_p$

if Γ_{int} turns out to be equal to aT^n for all n > 2, then

$N_{int} = \int_t^\infty T_{int}(t')dt' = \Gamma(H)|_t/(n-2) < 1$

This implies that a particle interacts less than once after the time $\Gamma = H$

Any interaction mediated by a massless gauge boson provides $\sigma \sim \alpha_X^2$ with s $\sim E^2$ and $\alpha_X = g_X^2/4\pi$ and this implies that

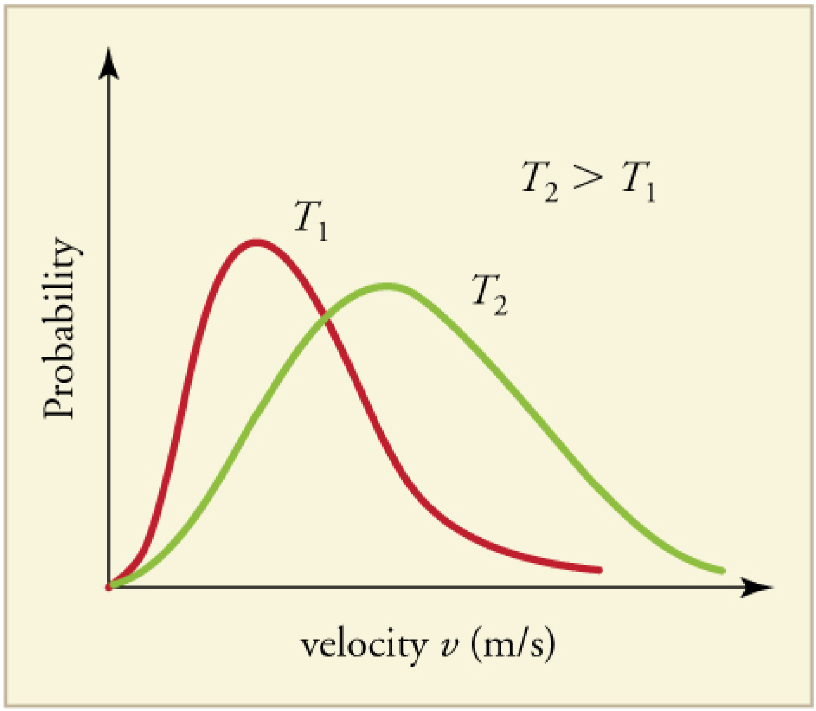


Figure 3: Plot of Maxwell Boltzmann Distribution

$\Gamma \sim n < \sigma v > \sim T^3 \alpha_X^2 / T^2 = \alpha_X^2 T$
and $\Gamma / H \sim \alpha_X^2 M_p / T$
so the equilibrium temperature whenever T $\leq \alpha_X^2 M_p$

Interactions mediated by any massive gauge boson provides $\sigma \sim G_X^2 s$ with $G_X \ \alpha_X / m_X^2$
This implies that
$\Gamma \sim T^3 G_X^2 T^2 = G_X^2 T^5$
and
$\Gamma / H \ G_X^2 M_p T^3$
and thus we get Equilibrium Temperature (T) $(G_X^2 M_p)^{-1/3} \leq T \leq m_X$
Also if T $\leq (G_X^2 M_p)^{-1/3} \ (m_X / 100GeV)^{4/3} MeV$, then Freeze out
But it is out-of-equilibrium phenomena which are more important to us as they provide the formation of light elements during Big Bang Nucleosynthesis, provide path of recombination of electrons and protons into hydrogen atoms and likely imply the production of dark matter.

Boltzmann equation condenses above physical process of equilibrium and particle production it yields at a given temperature. According to it, Rate of change in abundance + Rate of particle production - Rate of particle annihilation.

For example, let a reaction where A + B $\longleftrightarrow$ C + D

then $(1/a^3)d(n_A a^3)/dt = \int (d^3p_A)/((2\pi)^3 2E_A) \int (d^3p_B)/((2\pi)^3 2E_B) \int (d^3p_C)/((2\pi)^3 2E_C) \int (d^3p_D)/((2\pi)^3 2E_D) * A$

where $A = [(2*\pi)^4 \delta^3(p_A + p_B - p_C - p_D)\delta(E_A + E_B - E_C - E_D)|M|^2] * S$

where $S = [f_C f_D(1 \pm f_A)(1 \pm f_B) - f_A f_B(1 \pm f_C)(1 \pm f_D)]$

where $f_i = 1/e^{(E_i - \mu_i(t))/T \pm 1)}$

9.1.2 Big Bang Nucleosynthesis Abundance Determination

The abundance of various light nuclides formed during the BBN can be calculated by determining the thermonuclear reaction rates
$< \sigma v >= [8/\pi\mu(k_B T)^2]^{1/2} \int \sigma(E)Eexp(-E/k_B T)dE$
The expectation value of the nuclear cross section for that particular reaction is calculated using the Maxwell-Boltzmann distribution as a probability distribution function where v is the relative velocity of the reactants, μ is their reduced mass and σ is the mentioned cross section.

For deuterium formation, along with neutrons converting into protons through weak interactions, we have the sole reaction $p + n \rightarrow D + \gamma$

At higher temperature, weak interactions are dominant, but as soon as $k_B T$ drops below about 1MeV, the weak interactions become slower than the expansion of the universe, keeping the neutron-proton ratio essentially fixed.

The abundance of deuterium predicted by the above equation is of the order of 10 percent. Most of it is used in synthesis of further nuclides such as He.

The situation becomes more complicated for lithium since it can be obtained by a number of pathways, and the net reaction rate depend on all the reaction rates. The relative abundance of lithium is predicted by use of the above formula to be of the order of the order of 10^0.

Clearly, by the standard BBN model using the Maxwell distribution, the only major contributor to the relative abundances is He with an experimentally verified value of 0.2477, and the leftover protons of light nuclides go on to form hydrogen. This value cannot be predicted directly from thermonuclear rates, since there is no fusion reaction. Therefore the predicted hydrogen abundance value is around 0.75 which agrees to fair extent with the experimentally observed abundance.

9.1.3 Calculation of Helium Density

Here the basic assumption is particles in the beginning of universe behaved like an ideal gas As already seen according to Maxwell Boltzmann distribution no. density N is proportional to m$^{3/2}$ exp (-mc$^2/k_B T$)

Since constant of proportionality is same for each particle species, so $N_n/N_p = (m_n/m_p)^{3/2} exp[-(m_n - m_p) * c^2/k_B T]$

Since (m_n/m_p) is approximately equal to 1 and $k_B T >> (m_n - m_p)c^2$, so

$N_n \approx N_p$

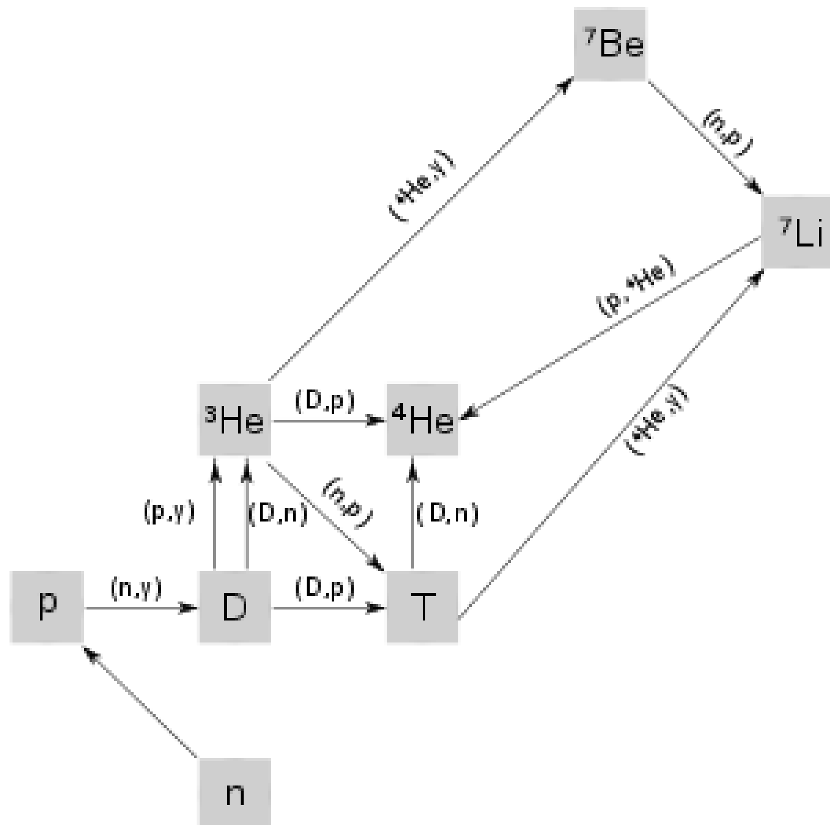


Figure 4: Sequence of formation of elements during Big Bang Nucleosynthesis

When temperature is large, following two reactions proceed rapidly and the neutrons and protons are in thermal equilibrium

n + $\nu_e \longleftrightarrow p + e^-$

n + $e^+ \longleftrightarrow p + \bar{\nu}_e$

But when temperature lowers to such an extent such that $k_B T \sim 0.8MeV$, rate of above reaction is lowered very much and abundance of protons and neutrons don't change due to above 2 reactions. Since $(m_n - m_p) * c^2 = 1.3MeV$,

$N_n/N_p = (m_n/m_p)^{3/2} exp(-1.3MeV/0.8MeV) \approx 0.1965$

After this change in abundance is due to decay of free neutrons to protons. But all neutrons are not as free neutrons due to below reactions

$p + n \rightarrow D$

$D + p \rightarrow {}^3He$

$D + D \rightarrow {}^4He$

But as long as temperature is high enough such that $k_B T > 0.1MeV$ above reactions does not affect neutron and proton density because destruction process is in opposite direction of above reactions and proceeeds at much higher rate.

Half life of neutron decay to proton is 614s and $k_BT = 0.1MeV$ when t = 400s.

So $N_n/N_p = 0.1965 * exp(-400ln2/614) = 0.125$

Since abundance of He and H_2 much greater than other elements, so we can simply assume that all matter is in the form of H_2 and He only.

Since 2 neutrons form 1 He, so $X_{He} = N_{n/2}/(N_n + N_p)$ which implies $Y_{He} = 4 * N_{n/2}/(N_n + N_p) \approx 0.22$

Above value is pretty close to observed value of 0.24