Big Bang Nucleosynthesis

Riya Singh
Animesh Singh
Mekhala Paranjpe
Deepak Singh
Mayank Jain

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1 Introduction

1.1 Big Bang Theory

The Big Bang theory is the prevailing cosmological model for the universe from the earliest known periods through its subsequent large-scale evolution. The model describes how the universe expanded from a very high-density and high-temperature state, and offers a comprehensive explanation for a broad range of phenomena, including the abundance of light elements, the cosmic microwave background (CMB), large scale structure and Hubble's law. If the known laws of physics are extrapolated to the highest density regime, the result is a singularity which is typically associated with the Big Bang. Physicists are undecided whether this means the universe began from a singularity, or that current knowledge is insufficient to describe the universe at that time. Detailed measurements of the expansion rate of the universe place the Big Bang at around 13.8 billion years ago, which is thus considered the age of the universe. After the initial expansion, the universe cooled sufficiently to allow the formation of subatomic particles, and later simple atoms. Giant clouds of these primordial elements later coalesced through gravity in halos of dark matter, eventually forming the stars and galaxies visible today.

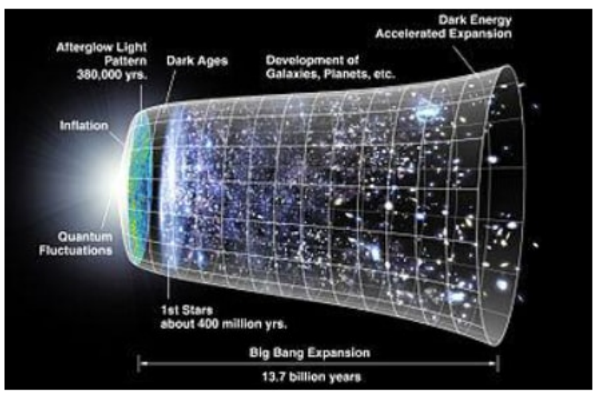

Figure 1: Timeline of the metric expansion of space

American astronomer Edwin Hubble observed that the distances to faraway galaxies were strongly correlated with their redshifts. This was interpreted to mean that all distant galaxies and clusters are receding away from our vantage point with an apparent velocity proportional to their distance: that is, the farther they are, the faster they move away from us, regardless of direction. Assuming the Copernican principle (that the Earth is not the centre of the universe), the only remaining interpretation is that all observable regions of the universe are receding from all others. Since we know that the distance between galaxies increases today, it must mean that in the past galaxies were closer together. The continuous expansion of the universe implies that the universe was denser and hotter in the past. The Big Bang theory offers a comprehensive explanation for a broad range of observed phenomena, including the abundance of light elements, the CMB, large scale structure, and Hubble's Law. The framework for the Big Bang model relies on Albert Einstein's theory of general relativity and on simplifying assumptions such as homogeneity and isotropy of space. The governing equations were formulated by Alexander Friedman, and similar solutions were worked on by Willem de Sitter. Since then, astrophysicists have incorporated observational and theoretical additions into the Big Bang model, and its parametrization as the Lambda-CDM model serves as the framework for current investigations of theoretical cosmology. The Lambda-CDM model is the current "standard model" of Big Bang cosmology, consensus is that it is the simplest model that can account for the various measurements and observations relevant to cosmology.

1.2 Nucleosynthesis

1.2.1 Definition

Nucleosynthesis is the process that creates new atomic nuclei from pre-existing nucleons, primarily protons and neutrons. The first nuclei were formed about three minutes after the Big Bang, through the process called Big Bang nucleosynthesis. It was then that hydrogen, helium and lithium formed to become the content of the first stars, and this primeval process is responsible for the present hydrogen/helium ratio of the cosmos.

1.2.2 History of nucleosynthesis theory

The first ideas on nucleosynthesis were simply that the chemical elements were created at the beginning of the universe, but no rational physical scenario for this could be identified. Gradually it became clear that hydrogen and helium are much more abundant than any of the other elements. All the rest constitute less than 2 percent of the mass of the Solar System, and of other star systems as well. At the same time, it was clear that oxygen and carbon were the next two most common elements, and also that there was a general trend toward high abundance of the light elements, especially those composed of whole numbers of helium-4 nuclei.

Arthur Stanley Eddington first suggested in 1920, that stars obtain their energy by fusing hydrogen into helium and raised the possibility that the heavier elements may also form in stars. This idea was not generally accepted, as the nuclear mechanism was not understood. In the years immediately before World War II, Hans Bethe first elucidated those nuclear mechanisms by which hydrogen is fused into helium.

The goal of the theory of nucleosynthesis is to explain the vastly differing abundances of the chemical elements and their several isotopes from the perspective of natural processes. The primary stimulus to the development of this theory was the shape of a plot of the abundances versus the atomic number of the elements. Those abundances, when plotted on a graph as a function of atomic number, have a jagged saw tooth structure that varies by factors up to ten million. A very influential stimulus to nucleosynthesis research was an abundance table created by Hans Sues and Harold Urey that was based on the unfractionated abundances of the non-volatile elements found within unevolved meteorites. Such a graph of the abundances is displayed on a logarithmic scale below, where the dramatically jagged structure is visually suppressed by the many powers of ten spanned in the vertical scale of graph.

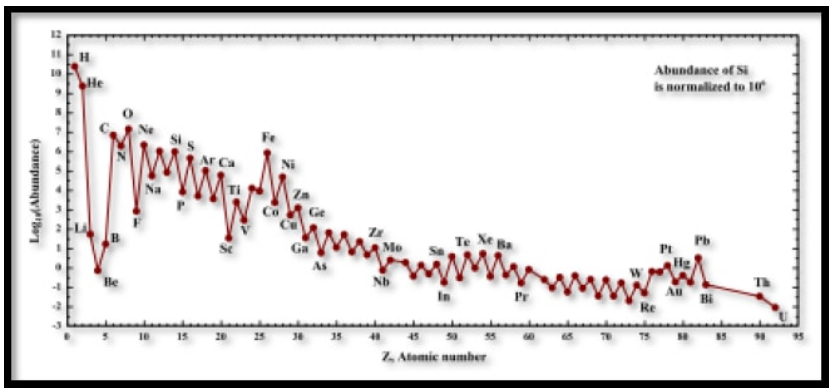

Figure 2: Abundance table created by Hans Sues and Harold Urey

1.3 Einstein theory

We all know about Einstein and about his amazing theories. The special theory of relativity came about when he was thinking in the patent office and thinking what would happen if we could move at the speed of light. Do we see standing waves? After his ground-breaking work on the special theory of relativity he set out to for the journey on the discovery of general theory of relativity.Most of us know that he realized that gravity and acceleration are the same things when he thought that people couldn't differentiate whether they are on the earth or on a satellite accelerating at g. But how did he realize about curvature of space due to acceleration is an interesting story not known by many. We know that the length of an object reduces along the direction of its travel and stays the same perpendicular to it. Einstein considered about this effect on a rotating disc. We know that the circumference of the disc will decrease as it is along the direction of motion but its radius remains constant which is a contradiction and this Einstein resolved by assuming that space bends due to acceleration. Now how is this related to big bang. Einstein assumed the universe to be stable but his equations predicted that this is impossible and the universe should either expand or contract. So, he added a constant to his equation to stabilize our universe Some people say that he committed this is one of the two major mistakes he committed in his life, the other being his unacceptance of the quantum theory. Hubble's observation that the universe is expanding proved that the universe is expanding and led to the idea that initially everything could have been at one place. But years later when dark matter were discovered people realized that the Einstein constant that had no explanation accidently had a physical implementation as dark matter. In a recently discovered work of his it was realized that Einstein had already predicted that if the universe was expanding then new matter must be created which is dark matter

1.4 Hubble's Law

When most people believed in an essentially static and unchanging universe, the question of whether or not it had a beginning was really one of metaphysics or theology. One could account for what was observed equally well on the theory that the universe had existed forever or on the theory that it was set in motion at some finite time in such a manner as to look as though it had existed forever. But in 1929, Edwin Hubble made the landmark observation that wherever you look, distant galaxies are moving rapidly away from us. In other words, the universe is expanding. Hubble's law is considered the first observational basis for the expansion of the universe and today serves as one of the pieces of evidence most often cited in support of the Big Bang model. The law is often expressed by the equation $v = H_o * D$, with H_o the constant of proportionality - Hubble constant - between the "proper distance" D to a galaxy, which can change over time, unlike the comoving distance, and its velocity v, i.e. the derivative of proper distance with respect to cosmological time coordinate. In the 1920s, when astronomers began to look at the spectra of stars in other galaxies, they found something most peculiar: there were the same characteristic sets of missing colors as for stars in our own galaxy, but they were all shifted by the same relative amount toward the red end of the spectrum(as a consequence of the Doppler effect of light) which means that nearly all of them are moving away from us. More surprising still was the finding that Hubble published in 1929: even the size of a galaxy's redshift is not random, but is directly proportional to the galaxy's distance from us. Or, in other words, the farther a galaxy is, the faster it is moving away! And that meant that the universe could not be static, as everyone previously had thought, it is in fact expanding. The current expansion of the universe means that at earlier times objects would have been closer together. In fact, it seemed that there was a time, about ten or twenty thousand million years ago, when they were all at exactly the same place and when, therefore, the density of the universe was infinite and the universe may have started with an explosion which is popularly known as the Big Bang.

2 Observations in support of Big Bang Nucleosynthesis (BBN)

2.1 Observation 1

Abundance of Helium in terms of mass fraction (Y_p)= 0.2477. This observation was made by M. Peimbert, V. Luridiana, A. Peimbert on the basis of new atomic physics computations of the recombination coefficients of He I and of the collisional excitation of the H I Balmer lines together with observations and photoionization models of metal-poor extragalactic H II regions.

This observation is a very strong verification of Big Bang Nucleosynthesis Theory. According to theory, during nucleosynthesis all the free neutrons are swept up and bound into nuclei. Once bound in this way, the strong interaction between the protons and neutrons stabilizes the neutrons preventing them from decaying. By the time that nucleosynthesis starts, the decay of free neutrons has shifted the p/n ratio to 87/13. Out of every 200 particles 26 neutrons will combine with 26 protons to form 13 helium nuclei. This leaves 148 protons, so the mass ratio of the nuclei produced is: (13*4)/(13*4+200-26*2)=26 percent. Thus, we see that even the crude calculation is in good argument with the observation. More refined calculations show that the exact proportions of the elements produced depend on the density of protons and neutrons and the expansion rate of the universe during the period of nucleosynthesis (both influence the rate at which particles can interact with each other). Helium comes out in the region of 24–25 percent, deuterium about 0.01 percent and lithium $10-7$ percent.

2.2 Observation 2

Abundance of Deuterium was determined by John M. O'Meara, David Tytler, David Kirkman, Nao Suzuki, Jason X. Prochaska, Dan Lubin Arthur M. Wolfe by analyzing SNR spectra of HS 0105+1619 in both low and high resolution. The low resolution spectra were obtained using the Kast double spectrograph on the Shane 3 meter telescope at Lick observatory. The high resolution spectra were obtained using the HIRES spectrograph on the Keck-I telescope. They found that the D/H ratio is log(D/H) = 4.48. Using this value baryon density was calculated and it was found that Ω_b*h*h = 0.0213.

Baryon density can be calculated using BBN and helium density mass fraction (which itself is obtained from BBN) and it was found that Ω_b*h*h = 0.02122.

Thus we can see that this observation is in accordance with the theory.

3 Open Problem

- Lithium Problem
- The horizon problem – How did the apparent large-scale homogeneity of the universe come about, when in the earliest moments of the expansion all parts of the universe would have been beyond the "horizon" of other parts and therefore unable to become so nearly the same in temperature and density?
- The flatness problem – How could the parameter called Ω have had a value so extraordinarily close to 1 in the early moments of the expansion (implying that the universe was then geometrically "flat") that even today Ω can't be much smaller than 0.01?
- The magnetic monopole problem – Why have no magnetic monopoles or other "topological defects" been detected anywhere, even though they are very likely to have been produced during the earliest moments of the universe as a result symmetry breaking between the strong and electroweak forces?
- Matter and antimatter forms when photons combined. So they should have been created in equal amount. Then why is there an asymmetry in their current amount?
- What is dark matter made up of?
- Does neutrino have mass? Can it be the constituent particle of dark matter?
- What physical principles produced an era of inflation? Why inflation stopped after about one hundred cycles of doubling the length scale of the universe?
- Why there are any inhomogeneities in the universe despite of inflation (stars/ galaxies/ clusters of galaxies in place of very thin gas of photons and atoms)? We see fluctuations of temperature in the early universe which can be related to these inhomogeneities but the origin of these fluctuations is itself unexplained
- The cosmological constant is the value of the energy density of the vacuum of space. How can such a thing be experimentally determined?
- Supersymmetry (SUSY) is a theory that links gravity with the other fundamental forces of nature by proposing a relationship between bosons and fermions. Can it explain presence of dark matter and how?
- Does graviton exist? What are effects of its properties on the Big Bang Nucleosynthesis?

- Where do the particles get their mass from and why do they have masses which is essential for the universe to be in the form which it is now?
- Why does there exist charge asymmetry within one generation of fundamental particles (leptons / quarks)?
- Does Higgs bosons have mass and if yes what changes need to be done in electroweak theory so that the infinite sum needed to understand the particle interactions converges?
- Computations of the cosmological constant, assuming the existence of Higgs fields, produce a result that is absurdly large. Computations of the cosmological constant, assuming the existence of Higgs fields, produce a result that is absurdly large. What modifications need to be made in the existing model to prevent this from happening?
- Spontaneous symmetry breaking of the gauge symmetry associated with the electro-weak force separates the electromagnetic and weak forces. What caused this breaking to occur?
- ARCADE, a mission to detect faint radio signals from ancient stars, detected a huge amount of radio noise—six times louder than scientists had predicted and this is known as Space Roar. What is the source of space roar and how could it be explained?
- Which fundamental forces caused the big bang to happen?
- What happened during the "first" plank time after the big bang?

3.1 Lithium Problem

Big-bang nucleosynthesis (BBN) theory, together with the precise WMAP cosmic baryon density, makes tight predictions for the abundances of the lightest elements. The missing lithium problem is centred around the earliest stages of the universe from about 10 seconds to 20 minutes after the Big Bang. The universe was super hot and it was expanding rapidly. This was the beginning of what's called the Photon Epoch. Only the lightest nuclei were formed during this time, including most of the helium in the universe, and small amounts of other light nuclides, like deuterium and lithium. According to Big bang nucleosynthesis the amount of lithium should be three time more than the current amount present in the universe. It is quickly becoming as well known as the solar neutrino problem which ruled for many years as a seed for the development of theoretical physics imagination. It eventually was solved with an old idea: that neutrinos can oscillate. A few minutes(4 m - 20 m) after the Big Bang, nuclei started forming first with the creation of the deuteron by neutron capture on proton, p(n,)d. The formation of deuterons is strongly dependent on the baryon-to-photon ratio in the Big Bang epoch, b = baryons=photons. After the deuteron bottleneck is surpassed, all other heavier elements are synthesized, and are also therefore strongly dependent on b. A series of reactions involving neutron, proton, deuteron, and helium captures allow elements up to lithium and beryllium to be created during the Big Bang.

Most favoured solution present right now is our assumption in Big Bang nucleosynthesis is that all of the nuclei are in thermodynamic equilibrium, and that their velocities conform to what's called the classical Maxwell-Boltzmann distribution. But the Maxwell-Boltzmann describes what happens in what is called an ideal gas. Real gases can behave differently, and this is what the researchers propose: that nuclei in the plasma of the early photon period of the universe behaved slightly differently than thought.

3.2 Space Roar

3.2.1 Discovery

In 2006 NASA's Goddard Space Fight Center sent a giant balloon called ARCADE(Absolute Radiometer for Cosmology, Astrophysics and Diffuse Emission) in search of radiation from early stars. It carried seven sensors that picked electromagnetic radiation like radio waves. It was sent above the earth's atmosphere so that no inference can occur due to atmosphere and thus the instrument can detect faint radio signals in space.But the instrument detected a 'boom' instead of faint signals from the early stars. The intensity was six times louder than anyone could have predicted. Therefore this blast of radio waves was termed as Space Roar. Other instruments were sent up before but they detected radiation from all the directions and as the space roar was uniform in all the directions they could not detect it

3.2.2 Theories

The following are some of the theories that try to explain it

- The Automaton theory(Digital Physics) - This theory tells us that the amount of matter in the early universe was far more than now and thus the powerful radio waves could be explained. Automaton is a mathematical game by John Conway.
- The radiation might be coming from the earliest star that did not have any dust and thus infrared radiation need not be hand in hand with the radio waves intensity.
- They may be emitted by extremely dim but powerful radio galaxies
- A new type of physics that combines theory of relativity and quantum mechanics is required that could explain this phenomenon that may be due to some undiscovered principle.
- Modified Newton Dynamics theory

3.3 What is dark matter made up of and what limits its amount?

3.3.1 Fundamental Particles

- Things we know exist - Neutrino : A light neutrino would be a type of dark matter known as hot dark matter, meaning that the particles have relativistic velocities for at least some fraction of the Universe's lifetime. In fact, hot dark matter does not have favourable properties for structure formation and if the neutrino has such a mass it is believed that it could at most contribute only part of the matter density, with some other form of dark matter also being required.
  Another possibility is that the neutrino could be very heavy, for example comparable to the proton mass. A heavy neutrino is an example of cold dark matter, meaning particles which have negligible velocities throughout the Universe's history. Having at least some cold dark matter is desirable for structure formation, but a heavy neutrino is much less desirable on particle physics grounds than a light one, and indeed is excluded by particle physics experiments unless the neutrino has unusual properties.

- Things we believe might exist: Particle physics theories (particularly those aiming at unification of fundamental forces) have a habit of throwing up all manner of new and as-yet- undiscovered particles, several of which are plausible dark matter candidates. Particle physicists associate a new companion particle to each of the particles we already know about. The lightest supersymmetric particle (LSP) eg. photino, or gravitino, or neutralino; is stable and is an excellent cold dark matter candidate. They are also sometimes known as WIMPs — Weakly Interacting Massive Particles.

3.3.2 Compact Objects

- Black Holes: A population of primordial black holes, meaning black holes formed early in the Universe's history rather than at a star's final death throes, would act like cold dark matter. However if they are made of baryons they must form before nucleosynthesis to avoid the nucleosynthesis bound. Baryons already in black holes by the time of nucleosynthesis don't count as baryons, as they are not available to participate in nuclei formation.

- MACHOs (MAssive Compact Halo Object) : MACHOs are unique amongst dark matter candidates in that they have actually been detected. They may be baryonic or non-baryonic. Brown dwarfs are a baryonic example.They were detected with the technique of gravitational microlensing in which we use the principle that when a MACHO passes in front of a star then its gravity bends the light, causing the star to appear brighter. The mass of these invisible objects is estimated as a little less than a solar mass. However, although present they appear to have insufficient density to completely explain the observations related to dark matter.

4 Introduction to Lithium Problem

The abundance of helium and hydrogen as calculated from the hot Big Bang Theory fits nicely with the observational data. Things gets ugly when we estimate the amount of lithium. The expected amount of lithium is approximately three times than that observed. This is referred to as the lithium problem.

Recently researchers from China have published a paper that may have solved the puzzle. They argued that the assumption of the Big Bang Theory that the universe was in thermodynamic equilibrium may be wrong and thus we cannot apply Maxwell-Boltzmann distribution as it applies for ideal gases only and we deal with real gases which behave differently.

The authors have used non-extensive statistics to solve the problem. This does not directly gives the abundance of lithium but it predicts the amount of beryllium that can then be used to calculate the amount of lithium.

5 Wilkinson Microwave Anisotropy Probe

The WMAP mission provided the first detailed full sky map of the microwave background radiation in the universe. The map produced is characterised as a map of the effective temperature of the microwave background radiation. WMAP was a spacecraft operating from 2001 to 2010 which measured temperature differences across the sky in the cosmic microwave background (CMB) - the radiation heat remaining from the Big Bang.

The WMAP is observed in five frequencies, permitting the measurement and subtraction of foreground contamination (from the Milky Way and extragalactic sources) of the CMB. The main emission mechanisms are synchrotron radiation free-free emission (dominating the lower frequencies), and astrophysical dust emissions (dominating the higher frequencies). The spectral properties of these emissions contribute different amounts to the five frequencies, thus permitting their identification and subtraction.

Foreground contamination is removed in several ways. First subtract extant emission maps from the WMAP's measurements; second, use the components' known spectral values to identify them; third, simultaneously fit the position and spectral data of the emission, using extra data sets. Foreground contamination was reduced by using only the full-sky map portions with the least foreground contamination, whilst masking the remaining map portions

The main result of the mission is contained in the various oval maps of the CMB spectrum over the years. These oval images present the temperature distribution gained by the WMAP team from the observations by the telescope of the mission. Measured is the temperature obtained from a Planck's law interpretation of the microwave background. The oval map covers the whole sky. The results describe the state of the universe only some hundred thousand years after the Big Bang, which happened about 13.8 billion years ago. The microwave background is very homogeneous in temperature (the relative variations from the mean, which presently is still 2.7 kelvins, are only of the order of 5×10^{-5}.

The implication of WMAP as summarized in the mission report includes the following quote from the WMAP site:

- Light in WMAP picture is from 379000 years after Big Bang.
    1. 4 % baryonic matter, 23 % cold dark matter, 73 % dark energy.
    2. The data places new constraints on the Dark Energy. It seems more like a 'cosmological constant' than a negative-pressure energy field called 'quintesscence'. But quintessence is not ruled out.
    3. Fast moving neutrinos do not play any major role in the evolution of the structure of the universe. They would have prevented the early clumping of gas in the universe, delaying the emergence of the first stars, in conflict with the new WMAP data.
- Mapped the pattern of tiny fluctuations in the Cosmic Microwave Background(CMB) radiation (the oldest light in the universe) and produced the first fine-resolution (0.2 degree) full-sky map of the microwave sky.

- Universe is 13.7 billion years old, with a margin of error close to 1 %
- First stars ignited 200 million years after the Big Bang.
- Expansion rate (Hubble constant) value: H_0=71km/sec/Mpc, with a margin of error of about 5 %
- Nailed down the curvature of space to within 0.4%of 'flat' Euclidean
- Detected that the amplitude of the variations in density of the universe on big scales is slightly larger than on smaller scales. This, along with other results, supports 'inflation', the idea that the universe underwent a dramatic period of expansion, growing by more than a trillion trillion fold in less than a trillionth of a trillionth of a second. Tiny fluctuations were generated during this expansion that eventually grew to form galaxies.
- Determined that the distribution of these fluctuations follow a bell curve with the same properties across the sky, and that there are equal numbers of hot and cold spots in the map. The simplest version of the inflation idea predicted these properties and remarkably, WMAP's precision measurement of the properties of the fluctuations has confirmed these predictions, in detail.
- Mapped the polarization of the microwave radiation over the full sky and discovered that the universe was reionized earlier than previously believed. WMAP measured the polarization in the CMB it is possible to look at the amplitude of fluctuations of the density in the universe that produced the first galaxies.

5.1 Calculation of baryon asymmetry from WMAP experiments

One of the most significant contributions of WMAP has been to predict the baryon asymmetry, which can be quantified by the baryon-photon ratio or more favourably, the baryon-to-entropy ratio and agrees quite accurately with the value predicted by Standard BBN.
A parameter to characterize the baryon asymmetry, as mentioned above, could be the baryon-to-photon ratio, which can be determined as follows:
$\eta \equiv \frac{n_b - n_b^{'}}{n_\gamma} = \frac{n_B}{n_\gamma}$
where n_b = matter density $n_b^{'}$ = anti-matter density n_γ = radiation density Temperature anisotropies measured by WMAP robustly record acoustic oscillations of the (re)combining baryon-photon plasma within dark matter potential.

The figure above depicts the dependence of the acoustic peaks in the power spectrum of the CMB temperature anisotropy on the baryon-to-photon density ratio η. The value of n_B is determined mostly by the ratio of the amplitudes of the first and second peaks. $\eta = 6.1^{+0.3}_{0.2} * 10^{10}$ turns out to be value which

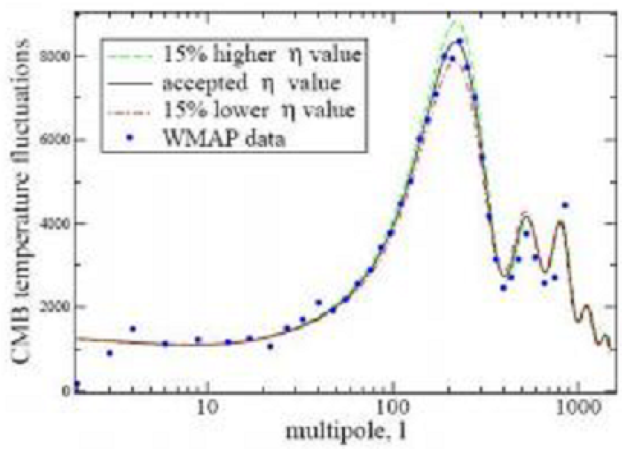

Figure 3: Determining η with the help of Temperature measurements

satisfies most of the measured data of WAMP mission. So, this value of η is widely accepted.
Since one can safely assume that there is no antimatter at present i.e. $n_b^{'} = 0$, this ratio is simply $\eta = n_b/n_\gamma$. The baryon-minus-antibaryon density per comoving volume is conserved, because every baryon either decays to another baryon or annihilates with an antibaryon, causing no net change in the number of quarks. But the photon density has not been the same consistently but only in recent times, as earlier heavy particles annihilated to produce only photons and not baryons. Therefore, this ratio is not a 'good' measure to indicate the baryon asymmetry. Instead, we choose the baryon-to-entropy density ratio, as the entropy density per comoving volume $S = a^3 s$ has remained fairly constant with time.
For calculation of entropy density from the photon number density, we have the following equation:
$s = g_{*s}\frac{2\pi^4}{45}(\frac{k_B T}{\hbar c})^3$,
where g_{*s} denotes the effective relativistic degrees of freedom, and is given by
$g_{*s} = \sum_{\alpha=bosons} g_\alpha (T_\alpha/T)^3 + 7/8 \sum_{\alpha=fermions} g_\alpha (T_\alpha/T)^3$
In the above relation, g_α gives the degrees of freedom for each species of bosons or fermions, which gives a direct relation between entropy density and photon density as
$\frac{s}{n_\gamma} = \frac{g_{*s}}{g_{*\gamma}} \frac{2\pi^2}{45\zeta(3)}$
For all practical purposes, the only current relativistic particles are photons and neutrinos for which g_s turns out to be 3.91, by substituting the corresponding

values of relativistic degrees of freedom, giving s = 7.04 n_γ and so the baryon-to entropy density ratio is
$\frac{n_B}{s} = \frac{1}{7.04}\frac{n_B}{n_\gamma} = \frac{1}{7.04}\eta = 8.7 \pm 0.3 * 10^{-11}$
which helps conclude that the asymmetry as found from this experiment is about 1 quark per million antiquarks.

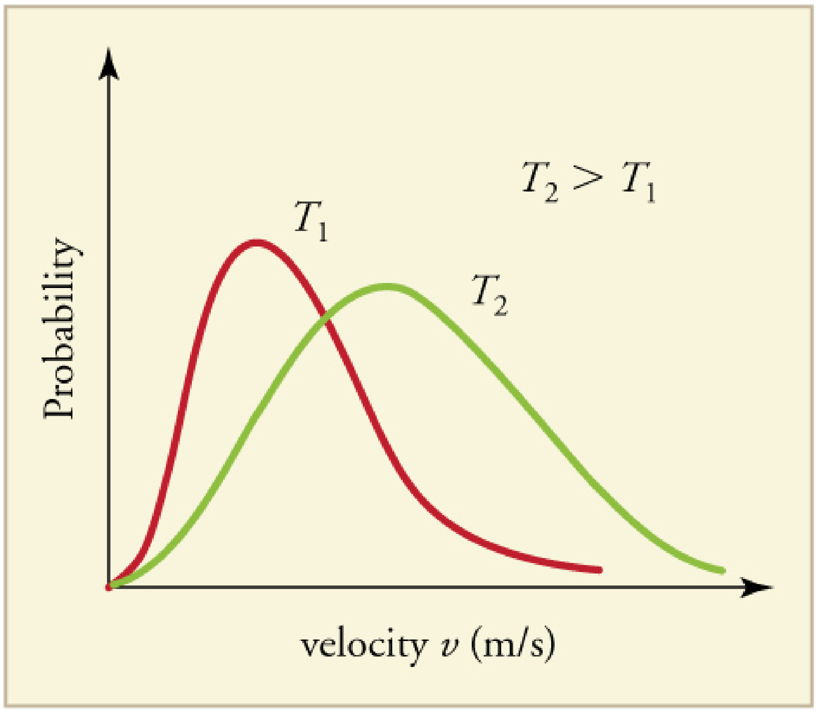

Figure 4: Plot of Maxwell Boltzmann Distribution

6 Maxwell Boltzmann Distribution

Maxwell Boltzmann distribution is mainly used to describe properties of molecules of ideal gas. The three-dimensional Maxwell–Boltzmann probability distribution is the function $f(v) = (m/2\pi k_B T)^{3/2} 4\pi v^2 exp(-mv^2/2k_B T)$

where

- f(v) is the fraction of molecules moving at velocity v to v + dv,
- m is the mass of the molecule,
- k_B is the Boltzmann constant, and
- T is the absolute temperature

6.1 Boltzmann Equation for Annihilation

According to the Boltzmann equation, Rate of change in abundance = Rate of particle production - Rate of particle annihilation.

For example, let a reaction where 1 + 2 $\longleftrightarrow$ 3 + 4

then $(1/a^3)d(n_1 a^3)/dt = \int(d^3p_1)/((2\pi)^3 2E_1) \int(d^3p_2)/((2\pi)^3 2E_2) \int(d^3p_3)/((2\pi)^3 2E_3)$ $\int(d^3p_4)/((2\pi)^3 2E_4) * A$

where $A = [(2*\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)|M|^2] * S$

where $S = [f_3 f_4(1 \pm f_1)(1 \pm f_2) - f_1 f_2(1 \pm f_3)(1 \pm f_4)]$

where $f_i = 1/(e^{(E_i - \mu_i(t))/T} \pm 1)$ is the occupation number according to Fermi-Dirac (FD, -)/Bose-Einstein (BE,+) distribution.

Interactions are included in the right-hand side of the Boltzmann equation. From the formulae for S, we see that the rate of producing species 1 is proportional to the occupation numbers of species 3 and 4, f_3 and f_4. Similarly the loss term is proportional to $f_1 * f_2$. The $1 \pm f$ terms represent the phenomena of Bose enhancement and Pauli blocking. According to Bose Enhancement if particles of type 1 already exist, a reaction producing more such particles is more likely to occur if 1 is a boson. So, plus sign in this term is for boson. On the other hand, Pauli Blocking states that if particles of type 1 already exist, a reaction producing more such particles is less likely to occur if 1 is a fermion. So, minus sign in the term is for fermion. Here, we are not taking into account the momentum dependence of f , but all the occupation numbers depend on the corresponding momentum. Term 'A' consisting of the Dirac delta functions enforce energy and momentum conservation. The factors of 2π are the result of moving from discrete Kronecker deltas to the continuous Dirac version. The energies here are related to the momenta via $E = (p^2 + m^2)^{1/2}$. M represents amplitude and is determined from the fundamental physics in question. In almost all cases of interest, this amplitude is reversible i.e. identical for 1 + 2 $\rightarrow$ 3 + 4 and 3 + 4 $\rightarrow$ 1 + 2. Thus, the Boltzmann Equation for Annihilation is completely reversible. Since, S and A depends on the momenta of particles involved, to find the total number of interactions, we must sum over all momenta. That is the reason of presence of integrals in the equation that. The factors of $(2\pi)^3$ represent the volume of one unit of phase space; we want to sum over all such units. Relativistically, the phase space integrals should really be four-dimensional, over the three components of momentum and one of energy. However, these are constrained to lie on the 3-sphere fixed by $E^2 = p^2 + m^2$. In equations,

$\int d^3p \int_0^\infty dE\delta(E^2 - p^2 - m^2) = \int d^3p \int_0^\infty dE \frac{\delta(E-(p^2+m^2)^{1/2})}{2E}$

Performing the integral over E with the delta function yields the factor of 2E. In the absence of interactions, the left-hand side of above equation says that the density times the scale factor cubed is conserved. This reflects the nature of the expanding universe: as the comoving grid expands, the volume of a region containing a fixed number of particles grows as a^3. Therefore, the physical number density of these particles falls off as a^{-3}.

For Big Bang Nucleosynthesis, our interest is in systems at temperatures much smaller than E - μ. In this limit, the exponential in the Bose-Einstein or Fermi-Dirac distribution is large and dwarfs the $\pm$ 1 in the denominator. Thus, the distributions become

$f(E) \to e^{\mu/T} e^{-E/T}$

and the Pauli blocking/Bose enhancement factors in the Boltzmann equation can be neglected. Also due to energy conservation, $E_1 + E_2 = E_3 + E_4$. Thus, S becomes

$e^{-(E_1+E_2)/T}[e^{(\mu_3+\mu_4)/T} - e^{(\mu_1+\mu_2)/T}]$

Since,

$n_i = g_i e^{\mu_i/T} \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}$

where g_i is the degeneracy of the species. We define the species-dependent equilibrium number density as

$n_i^{(0)} \equiv g_i \int \frac{d^3p}{(2\pi)^3} e^{-E_i/T}$

if $m_i >> T$, $n_i^{(0)} = g_i (\frac{m_i T}{2\pi})^{3/2} e^{-m_i/T}$

if $m_i << T$, $n_i^{(0)} = g_i \frac{T^3}{\pi^2}$

So, $e^{\mu_i/T} = n_i/n_i^{(0)}$

So, S $= e^{-(E_1+E_2)/T}[\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}]$

We define thermally averaged cross section as

$< \sigma\nu > \equiv \frac{n_1^{(0)}}{n_2^{(0)}} \int (d^3p_1)/((2\pi)^3 2E_1) \int (d^3p_2)/((2\pi)^3 2E_2) \int (d^3p_3)/((2\pi)^3 2E_3) \int (d^3p_4)/((2\pi)^3 2E_4)\ e^{-(E_1+E_2)/T} * [(2*\pi)^4 \delta^3(p_1 + p_2 - p_3 - p_4)\delta(E_1 + E_2 - E_3 - E_4)|M|^2]$

So, the Boltzmann equation becomes

$(1/a^3)d(n_1 a^3)/dt = n_1^{(0)} n_2^{(0)} < \sigma\nu > [\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} - \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}]$

The left-hand side is of order n_1/t or, since the typical cosmological time is H^{-1},$n_1 H$. The right-hand side is of order $n_1 n_2 < \sigma\nu >$. Therefore, if the reaction rate $n2 < \sigma\nu >$ is much larger than the expansion rate, then the terms on the right side will be much larger than the one on the left.

The only way to maintain equality then is for the individual terms on the right to cancel. Thus, when reaction rates are large,

$\frac{n_3 n_4}{n_3^{(0)} n_4^{(0)}} = \frac{n_1 n_2}{n_1^{(0)} n_2^{(0)}}$

6.2 Boltzmann Equation for Annihilation in Big Bang Nucleosynthesis

The first simplification is that essentially no elements heavier than helium are produced at appreciable levels. So the only nuclei that need to be traced are hydrogen and helium, and their isotopes: deuterium, tritium, and ^{3}He. The second simplification is that the physics splits up neatly into two parts since above T ≈ 0.1 MeV, no light nuclei form: only free protons and neutrons exist. Therefore, we first solve for the neutron/proton ratio and then use this abundance as input for the synthesis of helium and isotopes such as deuterium.

Both of these simplifications — no heavy elements at all and only n/p above 0.01 MeV — rely on the physical fact that, at high temperatures, comparable to nuclear binding energies, any time a nucleus is produced in a reaction, it is destroyed by a high-energy photon.To understand this let's consider this equation applied to deuterium production, $n + p \to D + \gamma$. Since photons have $n_\gamma = n_\gamma^{(0)}$ the equilibrium condition becomes

$\frac{n_D}{n_n n_p} = \frac{n_D^{(0)}}{n_n^{(0)} n_p^{(0)}}$

By solving the integral for $n_i^{(0)}$, we get

$\frac{n_D}{n_n n_p} = (3/4)(\frac{2\pi m_D}{m_n m_p T})^{3/2} e^{[m_n + m_p - m_D]/T}$

Now, $m_n + m_p - m_D = B_D$ where $B_D = 2.22 MeV$ is the binding energy of deuterium.

So, $\frac{n_D}{n_n n_p} = (3/4)(\frac{4\pi}{m_p T})^{3/2} e^{[B_D]/T}$

Further simplification due to the fact that both the neutron and proton density are proportional to the baryon density, gives

$\frac{n_D}{n_b} \approx \eta_b (\frac{T}{m_p})^{3/2} e^{[B_D]/T}$

where η_b is baryon-to-photon ratio. As long as B_D/T is not too large, the prefactor dominates this expression. As we have seen that WMAP experiment predicts baryon-to-photon ratio is $6.1^{+0.3}_{0.2} * 10^{10}$. The small baryon to photon ratio thus inhibits nuclei production until the temperature drops well beneath the nuclear binding energy. At temperatures above 0.1 MeV, then, virtually all baryons are in the form of neutrons and protons.

6.3 Calculation of Helium Density

Here the basic assumption is particles in the beginning of universe behaved like an ideal gas. As already seen according to Maxwell Boltzmann distribution no. density N is proportional to m$^{3/2}$ exp (-mc^2/$k_B T$)

Since constant of proportionality is same for each particle species, so

$N_n/N_p = (m_n/m_p)^{3/2} exp[-(m_n - m_p) * c^2/k_B T]$

Since (m_n/m_p) is approximately equal to 1 and $k_B T >> (m_n - m_p)c^2$, so

$N_n \approx N_p$

When temperature is large, following two reactions proceed rapidly

n + $\nu_e \longleftrightarrow p + e^-$
n + $e^+ \longleftrightarrow p + \bar{\nu}_e$
and the neutrons and protons are in thermal equilibrium.

But when temperature lowers to such an extent such that $k_B T \sim 0.8MeV$, rate of above reaction is lowered very much and abundance of protons and neutrons don't change due to above 2 reactions. Since $(m_n - m_p) * c^2 = 1.3MeV$, $N_n/N_p = (m_n/m_p)^{3/2} exp(-1.3MeV/0.8MeV) \approx 0.1965$

After this change in abundance is due to decay of free neutrons to protons. But all neutrons are not as free neutrons due to below reactions
$p + n \rightarrow D$
$D + p \rightarrow {}^3He$
$D + D \rightarrow {}^4He$
But, as we have already seen, if temperature is high enough such that $k_B T > 0.1MeV$ above reactions do not affect neutron and proton density.
Half life of neutron decay to proton is 614s and $k_B T = 0.1MeV$ when t = 400s.
So $N_n/N_p = 0.1965 * exp(-400ln2/614) = 0.125$

Since abundance of He and H_2 much greater than other elements, so we can simply assume that all matter is in the form of H_2 and He only.
Since 2 neutrons form 1 He, so $X_{He} = N_{n/2}/(N_n + N_p)$ which implies $Y_{He} = 4 * N_{n/2}/(N_n + N_p) \approx 0.22$
Above value is pretty close to observed value of 0.24

6.4 Calculation of Deuterium Density

6.4.1 Deuterium abundance

Deuterium is in some ways the opposite of He^4; in that while He^4 is very stable and often difficult to destroy, deuterium is marginally stable and easy to destroy. The temperatures, time and densities were sufficient to combine a substantial fraction of the deuterium nuclei to form He^4 but insufficient to carry the process further using He^4 in the next fusion step. BBN did not convert all of the deuterium in the universe to He^4 due to the expansion that cooled the universe and reduced the density, and so cut the conversion short before it could proceed any further. One consequence of this is that, unlike He^4, the amount of deuterium is very sensitive to initial conditions. The denser the initial universe was, the more deuterium would be converted to He^4 before time ran out, and less deuterium would remain.

There are no known post-Big Bang processes which can produce significant amounts of deuterium. Hence observations about deuterium abundance suggest that the universe is not infinitely old, which is in accordance with the Big Bang theory.

6.4.2 Deuterium Synthesis

Synthesis of elements heavier than hydrogen has to start with deuterium formation denoted by the reaction

n + p $\longrightarrow D+\gamma$
The binding energy of deuterium is B$_D$ = 2.22 MeV. Naively one expects deuterium synthesis to begin when the temperature falls to kT $\sim B_D$. However the baryon-to-photon ratio is $\eta\sim 5\times10^{-10}$, so the exponential tail of the blackbody distribution can still dissociate deuterium even when kT is significantly below B_D.

The frequency distribution of photons in a blackbody distribution is

$$\frac{dN}{dv}\alpha\frac{v^2}{e^{h\nu/kT}-1}\sim v^2e^{-h\nu/kT}$$

where the last approximation is for $h\nu/kT >> 1$. Synthesis of deuterium actually begins when kT $\sim B_D/\ln\,\eta$= 0.1 MeV, at time = 2 minutes.

The decay time for free neutrons is approximately 900 sec,so in two minutes a small but non-negligible fraction of the neutrons left over from 'freeze-out' have decayed. the neutron-proton ratio at the time of deuterium synthesis is n/p $\sim 1/7$. The amount of deuterium comes out to be about 27 parts per million of H-atoms from the equations.

There is one approach that may, in time, lead to a more accurate estimate of the primordial deuterium abundance. Sargent *et al* (1980) have identified the 'Lymann only' absorption line systems observed in quasars of high red shift (Lynds 1971) with primordial hydrogen clouds consisting of unprocessed and therefore unastrated material. In suitable objects, it should be possible to make a search for deuterium lines and thereby obtain interesting limits for primordial deuterium direct.

6.5 Calculation of Lithium Density

6.5.1 Lithium abundance

The abundance of lithium is 5×10^{-9} in meteorites but in stellar atmosphere it keeps decaying with the age of the star and depth of the convective zone as it is destroyed by hydrogen burning at low temperatures(2×10^6). So we can easily observe that its abundance in young stars must be of the same order as that in the meteorites and thi has been experimentally verified.

But about 100 times the cosmic amount of lithium has been found in cool stars that are old (eg. WZ Cas and WX Cyg) which can be explained by the synthesis of Be^7 which acts as its precursor through He^3. This process along with cosmic rays can lead to an enrichment of Li^7 over the pre-galactic value but the real reason is still unknown.

Recently scientists have discovered lithium lines in spectra of dwarf stars with temperatures of 5000-6000K. The fact that is interesting is:

- Amount of lithium is higher than expected.
- The amount is nearly independent of temperatures and is constant in the stars.

This means that the depletion of lithium in the convective zone of these stars has not been effective. They concluded that the amount of lithium is 5×10^{-9} though it is just a lower limit.

6.5.2 Lithium Synthesis

It is very difficult to study the abundance of lithium as Li^6 and Li^7 are produced by the interactions of cosmic rays with interstellar medium each second. Li^6, $Be^9$and Be^{11} can be studied by taking into account this factor. It has been shown by experiments and observation that the production of $Li^6$and Li^7 due to cosmic spallation should be in the ratio 1:2 but in the universe it has been observed to be 1:12. Therefore the amount of lithium (Li^7) available is more than can be produced by spallations alone. Hence we can naturally conclude that some of that has been produced at the time of the Big Bang, though the stars also play an important role in this and Li^7 can be produced in the stellar atmosphere also.

It is known that Li^6 cannot be produced easily in the stars nor in the Big Bang since its nucleus is fragile. Thus the amount of Li remains almost constant and the abundance is governed by the equation

$$\frac{Li^6}{H} = \frac{\rho\sigma f}{\omega}(1 - e^{-\omega t})$$

ω-Astration factor, t-Age of the galaxy, f-cosmic ray flux, ρ-Interstellar density of nitrogen, carbon and oxygen

Similarly the abundance equation for Li can be written as

$$\frac{Li^7}{H} = \frac{\rho\sigma f}{\omega}(1 - e^{-\omega t}) + Li^7_{BigBang}e^{-\omega t}$$

By fitting the equation, $\omega t<1$. Thus we see that Li^7/H ratio has increased since Big Bang. However the uncertainties themselves are of the order of 2-3. But this is just setting the lower limit.

6.6 Calculation of H density

As already seen, by the standard BBN model using the Maxwell distribution, the only major contributor to the relative abundances is He with an experimentally verified value of 0.2477, and the leftover protons of light nuclides go on to form hydrogen. Therefore the predicted hydrogen abundance value is around 0.75 which agrees to fair extent with the experimentally observed abundance. This value cannot be predicted directly from thermonuclear rates, since H atoms are not formed by fusion reaction.

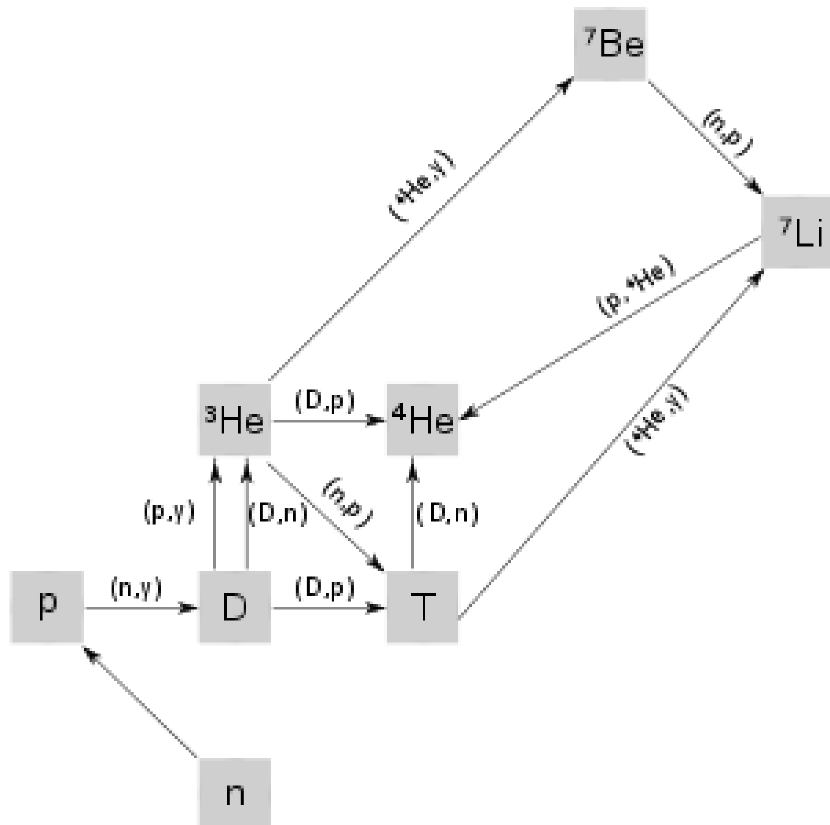

Figure 5: Sequence of formation of elements during Big Bang Nucleosynthesis

6.7 Calculation of Baryon Density from CMB data

We know that CMB is a fingerprint of the universe and we can read all the parameters off this chart. The most important thing that it tells us is that our universe is flat and not curved by more than 1The best way to measure baryon density is to study the Big Bang and how the nucleosynthesis occurred. During this process, in addition to helium (4He), a few other elements are produced: deuterium, helum-3, and lithium-7 (denoted as D, 3He, and 7Li). The amount of those isotopes produced depends on the density of baryons, i.e. on Ω_λ

It's very common in Physics that you can't directly measure a quantity. Instead you construct a mathematical model to describe your experimental results, in which the quantity you're trying to determine is an adjustable parameter. Then you pass your results to a big computer that adjusts the parameter to get the best fit to experiment. The result is your measurement, but it's an indirect measurement. Pretty much everything measured by the LHC is actually a best fit obtained from a mathematical model.

This is how the proportion of baryonic matter etc are determined. A minimal description of the universe requires six parameters:

1. physical baryon density
2. physical dark matter density
3. dark energy density
4. scalar spectral index
5. curvature fluctuation amplitude
6. reionization optical depth

Any satellite does not calculate the density of matter directly. Mathematical models have been built by physicists to explain observation. As there are 6 parameters and we know how the output should look we just input the observation and our model and the computer itselt gives the best fit.

The values are always revised as more accurate data becomes available but it doesn't results in any significant change in our models.

The most used model is the ΛCDM (Lambda cold dark matter) or Lambda-CDM model. It is the standard model of Big Bang cosmology because it is the simplest model that provides fairly accurate results.

The abundances of these elements can be measured quite accurately, and the latest result on the baryon density is

1. BBN : 0.0467 (Error 0.0026)
2. WMAP : 0.0463 (Error 0.0024)
3. Planck : 0.0486 (Error 0.0007)

7 Tsallis nonextensive statistics

7.1 Introduction

Statistical mechanics assumes that energy is an extensive variable, meaning that the total energy of the system is proportional to the system size; similarly the entropy is also supposed to be extensive.

This might be justified due to the short-range nature of the interactions which hold matter together. But if one deals with long-range interactions, most prominently gravity; one can then find that entropy is not extensive.

In classical statistics, to calculate the average values of some quantities, such as the energy of the system, the number of molecules, the volume it occupies, etc, one searches for the probability distribution which maximizes the entropy, subject to the constraint that it gives the right average values of those quantities. Tsallis proposed to replace the usual (BG) entropy with a new, non-extensive quantity, called the Tsallis entropy, and maximize that, subject to the same usual constraints. There is actually a whole infinite family of Tsallis entropies, indexed by a real-valued parameter q, which quantifies the degree of departure from extensivity (one gets the usual entropy back again when q = 1). In many circumstances, the classical results of statistical mechanics can be translated into the new theory. The importance of these families of entropies is that, when applied to ordinary statistical mechanics, they give rise to probabilities that follow power laws instead of the exponential laws of the standard case. In most cases where Tsallis formalism is adopted, the non-extensive parameter q is taken to be constant and close to the value for which ordinary statistical mechanics is obtained (i.e. q = 1). Some works have also probed sizable deviations of the non-extensive parameter q from the unity to explain a variety of phenomena in several areas of science. Recent studies on the temperature fluctuations of the cosmic microwave background (CMB) radiation have shown that a modified Planck distribution based on Tsallis statistics adequately describes the CMB temperature fluctuations measured by WMAP with q = 1.045 (+/-) 0.005.

7.2 Mathematical Formulation

Statistical systems in equilibrium are described by the Boltzmann-Gibbs entropy,

$$S_{BG} = -k_B \sum p_i ln p_i$$

where k_B is the Boltzmann constant, and p_i is the probability of the i-th microstate.

In the non-extensive statistics, one replaces the traditional entropy by the following one:

$$S_q = k_B \frac{1 - \sum p_i^q}{q - 1}$$

where q is a measure of non-extensivity. It is always a real number. For q = 1, $S_q = S_{BG}$.

We may think of q as a biasing parameter: q < 1 privileges rare events, while q > 1 privileges common events. Since, p < 1, p raised to a power q < 1 yields a value larger than p, and the relative increase p^q/p is a decreasing function of p, i.e., values of p closer to 0 (rare events) are benefited. Correspondingly, for q > 1, values of p closer to 1 (common events) are privileged. Therefore, the Boltzmann-Gibbs (BG) theory (i.e., q = 1) is the unbiased statistics. A concrete consequence of this is that the BG formalism yields exponential equilibrium distributions, whereas nonextensive statistics yields (asymptotic) power-law distributions. For q < 1, high energy states are more probable than in the extensive case. On the other hand, for q > 1 high energy states are less probable than in the extensive case, and there is a cutoff beyond which no states exist. Since the BG exponential is recovered as a limiting case, nonextensive statistics is a generalization, not an alternative.

The non-extensive description of the Maxwell-Boltzmann distribution corresponds to the substitution $f(E) - > f_q(E)$ where

$$f(E) = e^{-\frac{E}{k_B T}}$$

$$f_q(E) = \left[1 - \frac{q-1}{k_B T} E\right]$$

Where E is the relative energy of particles and T is the temperature. Thus we see that f(E) tends to $f_q(E)$ as q tends to one.

With this new statistics, the reaction rate becomes

$$R_{ij} = \frac{N_i N_j}{1 - \delta_{ij}} I_q$$

where N_i = no. of nuclei of species i. The rate integral, I_q, is given by

$$I_q = \int_0^{E_{max}} dE S(E) M_q(E, T)$$

where S(E) is astrophysical S-factor replacing the total cross section σ(E) to take into account the Coulomb repulsion between charged reactants. It is given by

$$S(E) \equiv \frac{E}{exp(-2\pi\eta)} \sigma(E)$$

where

$$\eta \equiv \frac{Z_1 Z_2}{4\pi\epsilon_o \bar{h}\nu}$$

where ν is the magnitude of the relative incident velocity. M_q is modified Gamow energy distribution given by

$$M_q = A_q \left(1 - \frac{q-1}{k_B T} E\right)^{\frac{1}{q-1}} e^{\frac{-b}{\sqrt{E}}}$$

where $b = 0.9898 Z_i Z_j \sqrt{A} MeV^{0.5}$ with Z_i the i-th nuclide charge and A is the reduced mass in amu And E_{max} = infinity for 0 < q < 1 and Emax = kBT/(1 -

q) for 1 < q < infinity, and E in MeV units. A(q,T) is a normalization constant which depends on the temperature and the non-extensive parameter q.

We can rewrite $f_q i$ as $[f_q^{(i)} = \left(1 - \frac{q-1}{k_B T} E_i\right)^{\frac{1}{q-1}} \; e^{-(\frac{E_i}{kT})}$

The two-particle energy distribution is $f_q^{(12)} = f_q^{(1)} * f_q^{(2)}$. So it reduces to

$$f_q^{(i)} = e^{\frac{1}{q-1}\left[ln\left(\left(1-\frac{q-1}{k_B T}E_1\right)\left(1-\frac{q-1}{k_B T}E_2\right)\right)\right]}$$

Since $E_i = m_i * v_i^2 = 2$, and thus, E1 + E2 = $\frac{\mu * v^2}{2} + \frac{M * V^2}{2}$, where μ is the reduced mass of the two particles, $M = m_1 + m_2$, v is the relative velocity, and V the center of mass velocity, the product inside the natural logarithm can be reduced to

$$\begin{aligned} 1 - \frac{q-1}{kT}\Big(\frac{\mu * v^2}{2} + \frac{M * V^2}{2}\Big) + \frac{q-1}{kT}^2\Big(\frac{\mu * v^2}{2} + \frac{M * V^2}{2}\Big) \\ = \Big(1 - \frac{q-1}{kT}\frac{\mu * v^2}{2}\Big)\Big(1 - \frac{q-1}{kT}\frac{M * v^2}{2}\Big) \end{aligned}$$

Thus, the two-body distribution factorizes into a product of relative and center of mass parts

$$f_q^{rel}(v,T) = A_{rel}(q,T)\Big(1 - \frac{q-1}{kT}\frac{\mu * v^2}{2}\Big)^{\frac{1}{q-1}}$$

$$f_q^{cm}(v,T) = A_{cm}(q,T)\Big(1 - \frac{q-1}{kT}\frac{M * v^2}{2}\Big)^{\frac{1}{q-1}}$$

We know that

$$\int d^3 v V f_q^{12}(v,V,T) = 1$$

Because the distribution factorizes, the unit normalization can be achieved for the relative and c.m. distributions separately. The distribution needed in the reaction rate formula is, therefore,

$$f_q(v,T) = \int d^3 v V f_q^{12}(v,V,T) = f_q^{rel}(v,T)$$

7.3 Application in Astronomy

Polytropic Equilibrium Solutions to the Vlasov-Poisson Equations - The first physical application of the non-extensive thermostatistical formalism was related to the study of maximum entropy solutions to the Vlasov-Poisson equations describing self gravitating N-body systems like galaxies. The maximization of the standard Boltzmann-Gibbs entropy under the constraints imposed by mass and energy conservation lead to the isothermal sphere distribution, which has infinite mass and energy. It was shown that the extremalization of the non extensive q-entropy under the same constraints leads to the stellar polytropic sphere distributions which, for a certain range of the q parameter, are endowed with finite mass and energy, as physically expected. This constituted the first clue suggesting that the generalized thermostatistical formalism based on S_q may be of some relevance for the study of systems exhibiting non extensive thermodynamical properties due to long range interactions.

Solar neutrino problem - The solar plasma is believed to produce large amounts of neutrinos through a variety of mechanisms such as the proton-proton chain. The calculation done using the Solar Standard Model (SSM) results in a neutrino flux over the Earth, which is roughly the double of what is measured. This is sometimes referred to as the neutrino problem or the neutrino enigma. It has recently been verified that neutrino oscillations do seem to exist, which would account for part of the deficit. But it is not at all clear that it would account for the entire discrepancy. Quarati and collaborators argue that part of it, could be due to the fact that BG statistical mechanics is used within the SSM. The solar plasma involves turbulence, long-range interactions, all of them phenomena that could easily defy the applicability of the BG formalism. Then they show in great detail how the modification of the "tail" of the energy distribution could considerably modify the neutrino flux to be expected. Consequently, small departures from q = 1 (e.g., —q 1— of the order of 0.1 or even less) would be enough to produce as much as 50 % difference in the flux. This is due to the fact that most of the neutrino flux occurs at what is called the Gamow peak. This peak occurs at energies quite above the temperature, i.e., at energies in the tail of the distribution.

Cosmic rays - The flux of cosmic rays arriving on Earth is a quantity whose measured range is among the widest experimentally known. This distribution refers to a range of energies which also is impressive. As shown in figure, this distribution is very far from exponential. This basically indicates that no BG thermal equilibrium is achieved, but some other state, characterized in fact by a power law. If the distribution is analyzed with more detail, one verifies that two power-law regimes are involved, separated by what is called the "knee" (slightly below 10^{16} ev). At very high energies, the power-law seems to be interrupted by what is called the "ankle" (close to 10^{19} ev). Seeing such wide ranges of both fluxes and energies, one would guess that a variety of complex intra- and inter-galactic phenomena are involved. However, from a phenomenological viewpoint, the overall results amount to something quite simple. Indeed, by solving the following differential equation, a quite remarkable agreement is obtained. $\frac{dp_i}{dE_i} = -b' p_i^{q'} - bp_i^q$
When $(q\prime, q)$ = (1,2) it precisely corresponds to the differential equation which enabled Planck, in guessing black-body radiation distribution.
$q' = 1$ and arbitrary q is a simple particular instance of the Bernoulli equation and has a simple explicit solution.
Finally, the generic case $q > q\prime > 1$ also has an explicit solution in terms of two hypergeometric functions and produces the above mentioned quite good

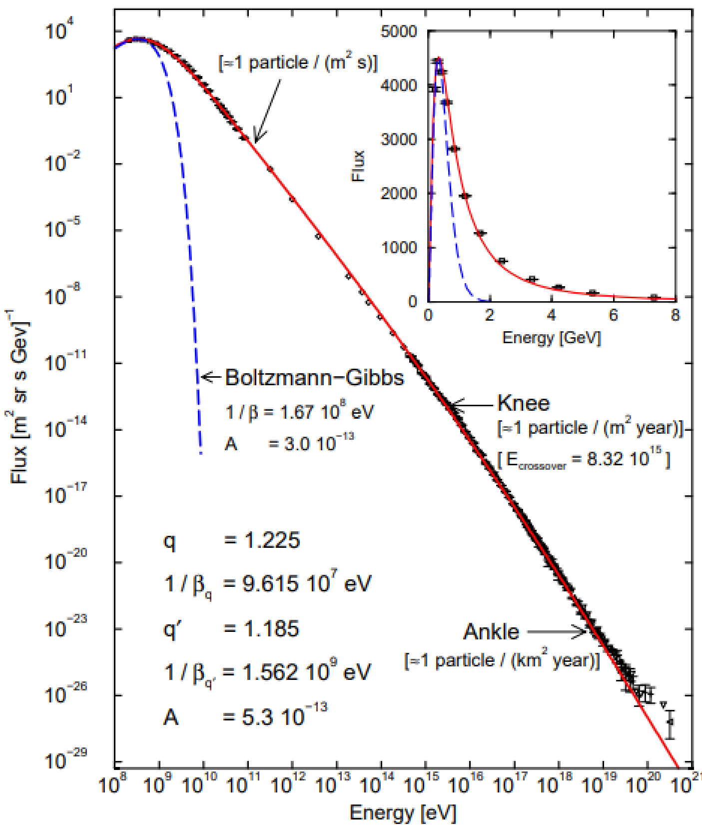

Figure 6: Flux v/s energy for cosmic rays

agreement with the observed fluxes. Indeed, if we assume $0 < b' << b$ and $q\prime < q$, the distribution makes a crossover from a power-law characterized by q at low energies to a power-law characterized by $q\prime$ at high energies, which is exactly what the cosmic rays exhibit to a quite good approximation.

Peculiar velocities of galaxy clusters - The COBE (Cosmic Background Explorer) satellite measured the peculiar velocities (difference of velocity with regard to the average expansion of the universe) of some clusters of spiral galaxies. A distribution was found which exhibits a cutoff around 500 Km/s. The Princeton astrophysical group analyzed the distribution of velocities within four different cosmological models but none of these attempts succeeded in reproducing the observed cutoff. By assuming within non-extensive statistical mechanics, an extremely simplified model, the empirical velocity distribution was quite satisfactorily matched. Only two fitting parameters were used, namely the scale of velocities and q ≈ 0.23.

Cosmology - Nonextensive statistical mechanics has also been applied to a variety of cosmological and general relativity problems including the cosmic background radiation in a RobertsonWalker universe, the dynamics of inflationary cosmologies, the universal density profile of dark halos, early universe phenomena (e.g., the primordial ^{4}He formation).

7.4 Application in Big Bang Nucleosynthesis

A standard BBN code was derived by Wagoner et al in 1967 for calculating abundances of elements formed during big bang. C. A. Bertulani, J. Fuqua and M. S. Hussein modified the code to incorporate the non-extensive statistics, 'q' into it. Cosmological parameter (η = $(6.203 \pm 0.137)10^{10}$) for the baryon-to-photon ratio and the neutron lifetime (880.3$\pm$1.1) s are given as input to code whose output are the plots for abundance of various elements with varying temperature.

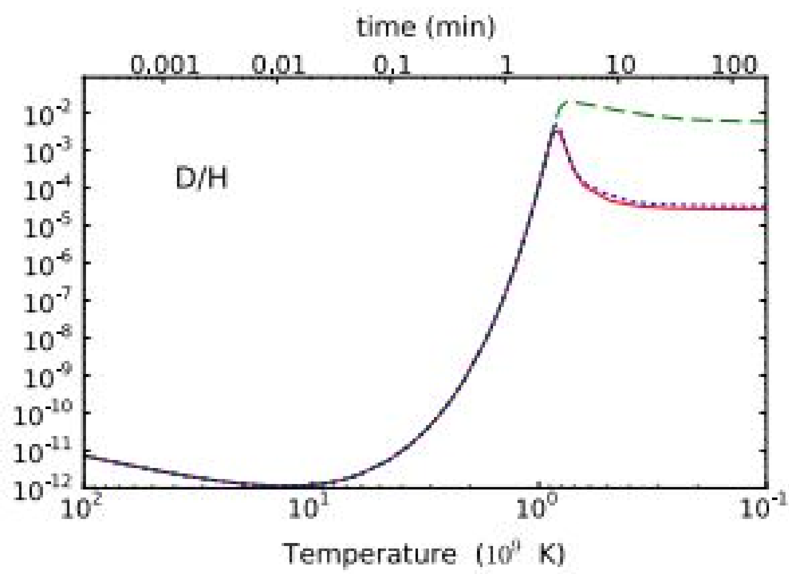

Figure 7: Deuterium Abundance

In Figure 7, the calculated deuterium abundance is shown. The solid curve is the result with the standard Maxwell distributions for the reaction rates. Using the non-extensive distributions yields the dotted line for q = 0.5 and the dashed line for q = 2. The deuterium abundances are only moderately modified due to the use of the non-extensive statistics for q = 0.5. Up to temperatures of the order of T_9 = 1, the abundance for D/H tends to agree for the extensive and nonextensive statistics. This is due to the fact that any deuterium that is formed is immediately destroyed (a situation known as the deuterium bottleneck). But, as the temperature decreases, the reaction rates for the p $\rightarrow$ d reaction are considerably enhanced for q = 2. This creates an unexpected overabundance of deuterons for the non-extensive statistics with q = 2. The present accepted average value of D/H = $2.82^{+0.20}_{0.19}10^{5}$. The predictions for the D/H ratio with the q = 2 statistics (D/H =5.7010^{-3}) are about a factor of 200 larger than those from the standard BBN model, clearly in disagreement with the observation.

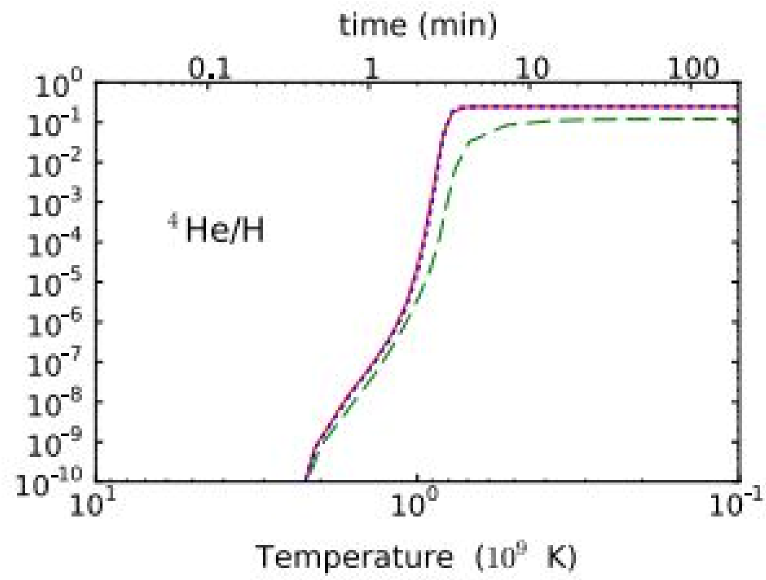

Figure 8: Helium Abundance

Now for helium, using the non-extensive distributions yields the dotted line for q = 0.5 and the dashed line for q = 2. Again, the predicted abundances for q = 2 deviate substantially from standard BBN results. This time only about half of ^{4}He is produced with the use of a non-extensive statistics with q = 2. The reason for this is the suppression of the reaction rates for the formation of ^{4}He with q = 2 through the charged particle reactions, t ^4He and $^3He \rightarrow ^4 He$

Non-extensive statistics for both q = 0.5 and q = 2 values also alter substantially the ^7Li abundance, as shown in Figure 9. For both values of the non-extensive parameter, q = 2 and q = 0.5, there is an overshooting in the production of ^7Li. The increase in the ^7Li abundance is more accentuated for q = 2. The lithium problem is associated with a smaller value of the observed ^7Li abundance as compared to the predictions of BBN. So, at the first glance it seems that incorporating Non-extensive statistics is worsening the problem.

The problem is that non-extensive statistics has been considered only for forward rates and the impact on reverse rates has not been considered. When the code is modified to take care of this, the predicted and observed abundances of D, ^4He and ^7Li fall into agreement (within 1σ uncertainty of observed data)

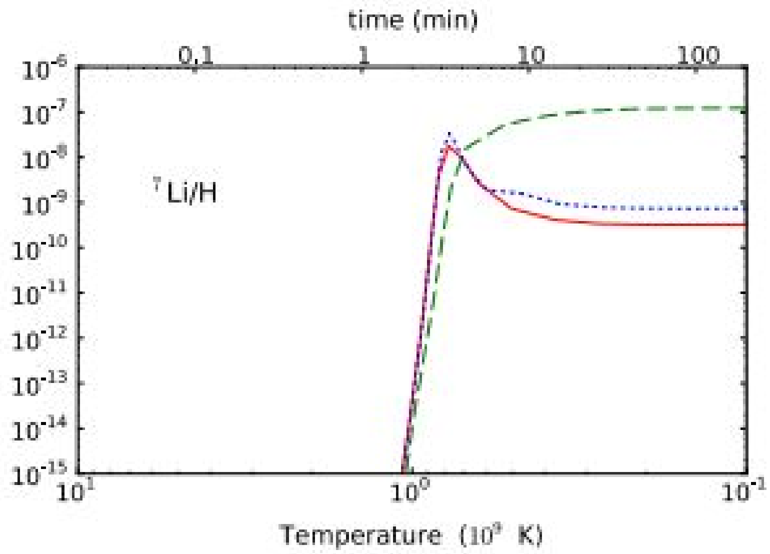

Figure 9: ^7Li Abundance

when a non-extensive velocity distribution with 1.069 < q < 1.082 is adopted.

When q > 1, rate for reactions $^3H \rightarrow^7 Li$ and $^3He \rightarrow^7 Be$ is decreased. The forward alpha-capture rates of these reactions decrease for q > 1 due to the decreased availability of high energy baryons relative to the MB (q = 1) distribution. On the other hand, the reverse photo-disintegration rates are independent of q due to our adoption of Planck's radiation law for the energy density of photons. As a result, the net production of ^7Li and ^7Be (which decays to ^7Li) decreases, giving rise to concordance between predicted and observed primordial abundances. (Value of ^7Li predicted by this model for above range of q is 1.621.90 while the value from observations is 1.58±0.31.

8 The Advent of Diffusion

Recent studies have shown diffusion of gas particles during the formation of the first structures in the early universe could have impacted the relative abundance of helium and hydrogen in the first galaxies. According to them, the changes in densities due to diffusion is not neglible compared to the nuclear theory predictions and thus they must be accounted for in the calculations of the densities of the different elements.

Pavel Medvedev, Sergey Sazonov and Marat Gilfanov proposed that as the first galaxies were forming, diffusion of gas could take place, effecting a change in the ratio of primordial helium to hydrogen. 'Galaxy formation begins with a contraction of dark matter, which is followed by an inflow of gas that is gravitationally attracted towards the centre of the future galaxy. We believe diffusion is possible in this flowing gas. As a consequence, particles of different masses move at different velocities. Suppose there is only hydrogen and helium in the gas. As helium is a heavier particle, it accretes faster than hydrogen, driven by the gravitational field of the forming galaxy. This means that when the galaxy is formed, the helium-to-hydrogen ratio in it is going to differ from that predicted by the nucleosynthesis theory,' says Sergey Sazonov of the IKI and MIPT.

The scientists examined particle diffusion in the gas during galaxy formation in the early universe. They estimated the changes in relative helium abundance, which could be induced by this phenomenon, for galaxies of different masses. Their research shows that these changes could be on the order of 0.01% or less in the case of diffusion in cold gas. However, if the gas was heated to several thousand degrees during the epoch when the first galaxies were formed (several hundred million years after the Big Bang), then diffusion-driven helium abundance changes could be on the order of 0.1%. One possible mechanism involving the preheating of gas is the transfer of energy from the first supernovae to the environment via cosmic rays. This was proposed in a recent paper by Sergey Sazonov and Rashid Sunyaev in the Monthly Notices of the Royal Astronomical Society.

Primordial helium abundance is normally inferred from the measurements of interstellar gas in the galaxies close to our own where star formation does not occur. Otherwise, scientists would not be looking at primordial elements, because the composition of the interstellar medium would have been enriched by the products of thermonuclear fusion in stars. The direct measurements of primordial helium abundance enable physicists to constrain cosmological parameters and test the Big Bang nucleosynthesis (BBN) theory. As stated above, cosmological parameters determine the state of the universe at any given time. That is why finding their precise values is one of the main objectives of cosmology.

The diffusion-induced changes are comparable to the precision of current predictions of helium abundance. This means that the effect proposed by the authors could account for galactic helium content changes that are within the accuracy of observations. For this reason, any future predictions of higher accuracy that are based on measured data will have to take this effect into account.

9 Contributions due to the presence of BBN-CRs

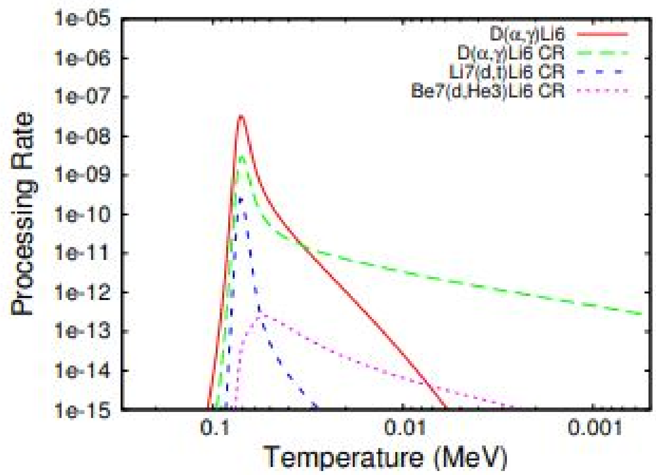

Figure 10: Processing rate of producing ^{6}Li as a function of temperature.

This theory attempts to explain the discrepancy in the predicted and observed data of ^{7}Li and ^{6}Li during the BBN, by including contributions from non-thermal cosmic rays (BBNCRs). These particles act as destroyers for ^{7}Li and could be a possible reason why the observed abundance is less by 3-5 times than predicted. These particles interact through exothermic as well as endothermic reactions. Some of the reactions directly associated with production and destruction of ^{7}Li and ^{6}Li are (The reactions which are involved in destruction of ^{6}Li are generally not considered, since we need to account for the excess observed amount of ^{6}Li):

Endothermic:

^{7}Be(d,^{3} He)^{6}Li - destroy ^{7}Be indeed, but less important than ^{7}Be(p, pα) ^{3}He, for D is much less than p; produce ^{6}Li

^{7}Li(p, n) ^{7}Be - transformation between ^{7}Li and ^{7}Be, important

Be(d, 2p) ^{7}Li - transformation between ^{7}Li and ^{7}Be, but less important than ^{7}Li(p, n) ^{7}Be

^{7}Li(d, t) ^{6}Li - destroy ^{7}Li; produce ^{6}Li

^{6}Li(t, np) ^{7}Li - destroy ^{6}Li

^{7}Li(p, pα)T - destroy ^{7}Li

^{7}Li(d, dα)T - destroy ^{7}Li

Exothermic:

^{4}He(d, γ) ^{6}Li - produce ^{6}Li

^{4}He(t, γ) ^{7}Li - produce ^{7}Li

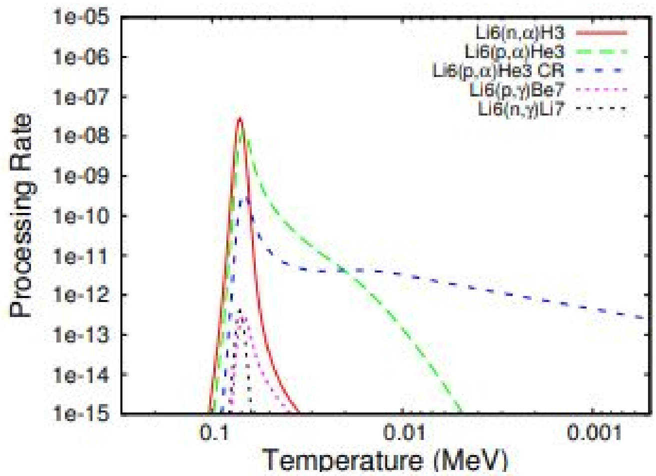

Figure 11: Processing rate of destroying ^{6}Li as a function of temperature.

^{7}Li(d, n)2^{4}He - destroy ^{7}Li, but less important than ^{7}Li(p, α) ^{4}He
T(t, γ) ^{6}He - produce ^{6}Li
^{3}He(t, γ) ^{6}Li - produce ^{6}Li
The endothermic reactions consist of BBNCRs with threshold energy below 3.337 MeV.

9.1 Calculation of abundances

We are still using Maxwell Boltzmann statistics with this theory. Thus the Boltzmann equation for variation of nuclide abundance is given as: $\frac{dYi}{dt} = -\sum_j Y_i \mathrm{Y}_j[\mathrm{ij}] + \sum_{k,l} Y_k \mathrm{Y}_l[\mathrm{kl}]$

where [ij] is the rate for destroying nuclide i and [kl] is the rate for creating i, $\mathrm{Y}_i = \mathrm{X}_i/\mathrm{A}_i$ is the abundance of nuclide i with X_i the mass fraction and A_i the mass number of nuclide i. The sum over j goes through all reactions to destroy nuclide i and the sum over k, l goes through all reactions to produce nuclide i. The rate [ij] is defined by
$[ij] = N_A r \langle ij \rangle$,

where N_A is the Avogadro's number, ρ is the baryon energy density, and $\langle ij \rangle = \langle \sigma v \rangle_{ij}$. Here σ is the cross section of the reaction $(ij \to kl)$,

and v is the relative velocity between i and j.

In the case of BBNCRs, this is given by
$\langle \sigma v \rangle_{12}(T, \alpha) = \frac{1}{K_3} \int_{-1}^{1} dcos\theta \frac{1}{K_1} \times \int_{-\infty}^{\infty} f_1(\mathrm{E}_1,\mathrm{T})\mathrm{dE}_1 \times \frac{1}{K_2} \int_{0.09}^{4} f_2(\mathrm{E}_2,\alpha) dE_2 \sigma(E_i) \mathrm{v}(\mathrm{E}_1, \mathrm{E}_2, \cos\theta)$

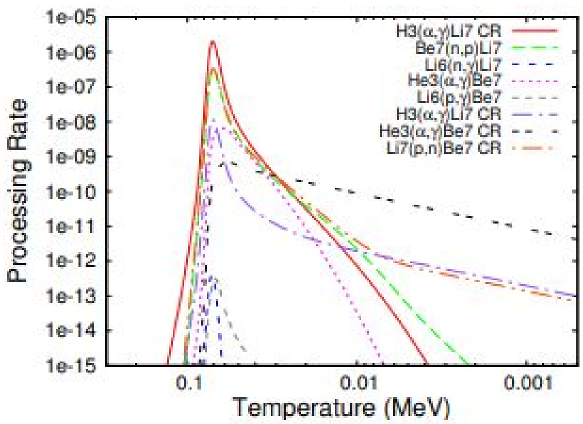

Figure 12: Processing rate of producing ^{7}Li as a function of temperature.

$$\alpha = \begin{cases} 2 \text{ or } 0 \text{ for } 0.09 \text{ MeV} < \mathrm{E}_2 < 2 \text{ MeV} \\ 4 \text{ for } 2 \text{ MeV} < \mathrm{E}_2 < 4 \text{ MeV} \end{cases}$$

Distribution of BBNCRs is power law with index α

$f_2(\mathrm{E}_2,\alpha) \propto {E_2}^{-\alpha}$

where E_2 is the energy of the BBNCR particle, with $\alpha = 2$ for power law case and $\alpha = 0$ for uniform distribution case.

9.2 Conclusions

These calculations were proven to make up for the deviation of observed abundance of ^{7}Li from the predicted abundance, however the newly predicted abundance of ^{6}Li has risen up by an order of magnitude, but is still far cry from the currently observed abundance. This has been accomplished without introducing new particles or applying Physics beyond the Standard BBN model.

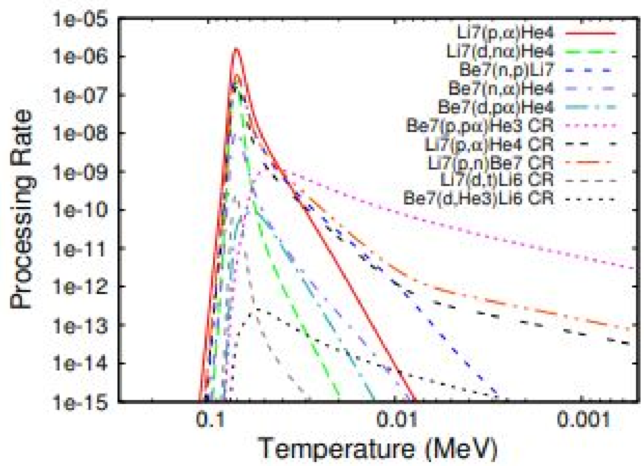

Figure 13: Processing rate of destroying ^{7}Li as a function of temperature.

10 Nuclear Theory

In the paper "One fewer solution to the cosmological lithium problem", the authors explores the way in which the greater amount of lithium could be explained by following conventional nuclear physics. In the paper, the amount of lithium that could be produced by atoms such as B_9 was studied.

Another theoretical paper(R. H. Cyburt and M. Pospelov, arXiv:0906.4373v1 [astroph] (2009).) explores the possibility of enhancing 7Be destruction through resonant reactions with p, d, t, 3He, α, leading to compound states in B_8, B_9,B_{10}, C_9, C_9, respectively. The paper concludes that, of the known excited states in these isotopes, only the 16.8MeV state in B_9 has the potential to significantly influence Be_7 destruction.

In the paper the correction amounted to only 8 percent increase in the predicted value of lithium which is far less than required as the amount of lithium is off by a factor of three. Thus it seems that conventional nuclear physics cannot resolve this discrepancy between the theoretical and observational value of lithium. Among other possibilities, the discrepancy could be due to new physics beyond the Standard Model of particle physics, errors in the observationally inferred primordial lithium abundance, or incomplete nuclear physics input for the BBN calculations. The paper addressed the last possibility and it failed.

11 Summary

We realised that the current theories are incapable of explaining the anomaly in lithium abundance. Some corrections were attempted using nuclear theory but they could not explain the huge difference. Diffusion theory, which takes into account the changes in abundance that can occur due to non-equilibrium diffusion is one theory which is in initial stage. The most promising one is the non-equilibrium Tsallis Statistics but whether they would be able to explain these observations or not and whether the present physics and maths possess enough tools to explain these observations is still a question.

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