Mid-semester Examination: CS31005: Algorithms II

Department of Computer Science and Engineering, IIT Kharagpur

LTP 3-1-0: Credits 4: Time 2 hours: 2-4 pm: Marks 100

There is limited choice.

(Dated: September 17, 2010)

1. Show that the *maximum cardinality clique* for an undirected graph $G(V, E)$ can be computed in polynomial time, where the complement graph $G^c(V, E^c)$ is cycle-free. Determine whether this computation is also possible in linear time. [Clearly, $E \cap E^c = \phi$ and $|E \cup E^c| = \binom{n}{2}$. A clique K is a subset ($K \subset V$), such that each pair of vertices in K is connected by an edge in $G(V, E)$.] [10+5 marks]

2. Suppose we perform random bicoluring of the n vertices of an n-vertex r-uniform hypergraph $G(V, E)$. Each vertex gets either colour, 0 or 1, with equal probability.

   (a) Based on the number $|E|$ of hyperedges, estimate an upper bound on the probability that the random bicolouring does not properly bicolour some hyperedge.

   (b) If $r = \log_2 n$ then determine an upper bound on the number $|E|$ of hyperedges so that G is bicolourable, that is, each hyperedge can be properly bicoloured in some bicolouring of the vertices.

   (c) Can this bound give rise to a Las Vegas bicoloring algorithm for G with polynomial expected running time? If not, further modify your upper bound on the number of hyperedges in order to do so.

   (d) What happens to the upper bound in (c) above when $r = (\log_2 n)^k$, where $k > 1$ is a positive natural number?

   (e) What happens to the upper bound in (d) above when $k = \log_2 n$?

   [5+3+5+4+3 marks]

3. We know that the rightmost two points must be connected in the *Bitonic Travelling Salesman's Tour*, where n points in the plane are given as input such that no two points have the same x-coordinate and no three points are collinear. We wish to find the minimum Euclidean length tour covering each of the n points exactly once with the added restriction

that the tour is *bitonic*. A bitonic tour goes from the leftmost point to the rightmost point without turning left through a set F of *forward-path* points, and then goes from the righmost point to the leftmost point, right to left, without turning to the right, through a set B of *backward-path* points. Here $F \cap B = \phi$ and $|F \cup B| = n - 2$. Show that the sets B_{opt} and F_{opt} defining the minimum length bitonic tour can be determined in polynomial time. [15 marks]

4. Define *discrepancy* for a hypergraph $G(V, E)$ and show that the discrepancy of an n-vertex r-uniform hypergraph is $O(\sqrt{n \log n})$ if r is a constant with respect to asymptotic n. What happens to this bound when $r = \log_2 n$? [3+8+4 marks]

5. State the primal relaxed linear program for the problem of minimizing the total weight in the minimum weighted set covering problem. Construct the dual linear program and state the relationship between OPT, OPT_p^* and OPT_d^*, where OPT is the weight of the minimum weighted set cover, OPT_p^* is the minimum objective function value of the primal relaxed linear program and OPT_d^* is the maximum objective function value for the dual linear program. We wish to derive a suitable approximation ratio for the weight of the computed set cover using the greedy set cover heuristic, with respect to the minimum weighted set cover. In order to do so, why do we need a condition stronger than that in the *weak duality* result? How are feasible (presumably non-optimal) solutions of the primal and dual relaxation linear programs used for proving the logarithmic approximation ratio? [5+5+5+5 marks]

6. In the *longest upsequence* problem for an unsorted sequence of n numbers, design the data structures necessary for performing the entire computation in $O(n \log n)$ time. [10 marks]

7. Explain the main principles of the method of *conditional expectations* in the derandomizing process where a deterministic decision making step replaces a random step. You may illustrate using an example. [15 marks]

8. Estimate the cardinality/size of the $\frac{1}{r}$-net created by the *greedy method* for a set system with n vertices and m hyperedges? In contrast, the randomized sampling method generates an answer which is a $\frac{1}{r}$-net with high probability. Derive the expression for the expected size of such an $\frac{1}{r}$-net in terms of the number n of vertices, the number m of hyperedges and r. Is it possible to design a Las Vegas algorithm in this context? [7+5+3 marks]