INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Department of Computer Science and Engineering

Algorithms-I (CS21003)

Mid-semester examination (Spring 2017)

Date: Thu, Feb 16, 2017 *Total students:* 102 *Time:* 2-4pm (AN)

Place: NR121, NR122, NR221, NR321, NR322 *Marks:* 50

1. (a) Consider functions from reals to reals; show that the big-Oh relation between such functions is a partial order relation. 5

   (b) Draw the graphs for the following two functions and establish how they related with respect to big-Oh: $\frac{x}{2}(2 + \cos x)$ and $\frac{x}{2}(1 + \sin x)$; use the graph paper given overleaf and attach with your answer script, writing your name and roll number on it. 5

   (c) Graphically depict the relationship between Θ, Ω and ω between functions and justify with respect to their respective definitions. 5

2. Assume that you are given n integer keys in an array.

   (a) Explain the working of the quick select algorithm to find the k^{th} ranked element, with an example. 5

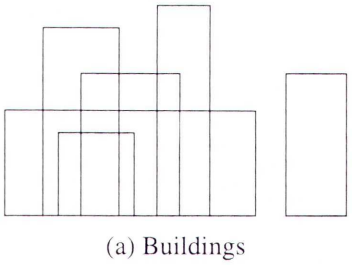
   (b) Do a complexity analysis of the quick select algorithm with the assumption that keys are uniformly distributed. 5

   (c) You are asked to determine the median on the given elements using the median-of-medians algorithm, but grouping the elements in sets of three elements; present an analysis of this scheme to establish whether it will perform as well as grouping the elements in sets of five elements. 5

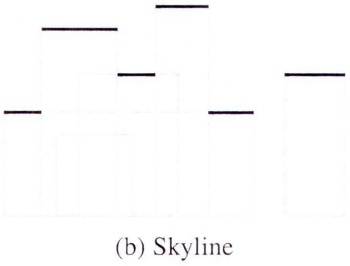
3. (a) You are given an array of n elements, sorted in ascending order, but circularly shifted through an unknown number of positions. For example, $\{35, 42, 5, 12, 23, 26\}$ is a sorted array that has been circularly shifted (right by two positions). You are required to (*i*) give an efficient algorithm to find the position of the largest element in a circularly shifted array and (*ii*) also form the recurrence equation to capture the running time of your algorithm and derive its solution. 5

   (b) You are given an $n \times n$ board B where $n = 2^d, d \geq 1$, with one missing cell (of size 1×1) at a known location. You are required to (*i*) develop an algorithm to fill the board using `L` shaped tiles (a 2×2 square with one cell, of size 1×1, missing) and (*ii*) also form the recurrence equation to capture the running time of your algorithm and derive its solution. (**Hint:** B can be cut in four parts and an `L` shaped tile can be formed at the common corner tiles of the other three parts where no tile is missing.) 5

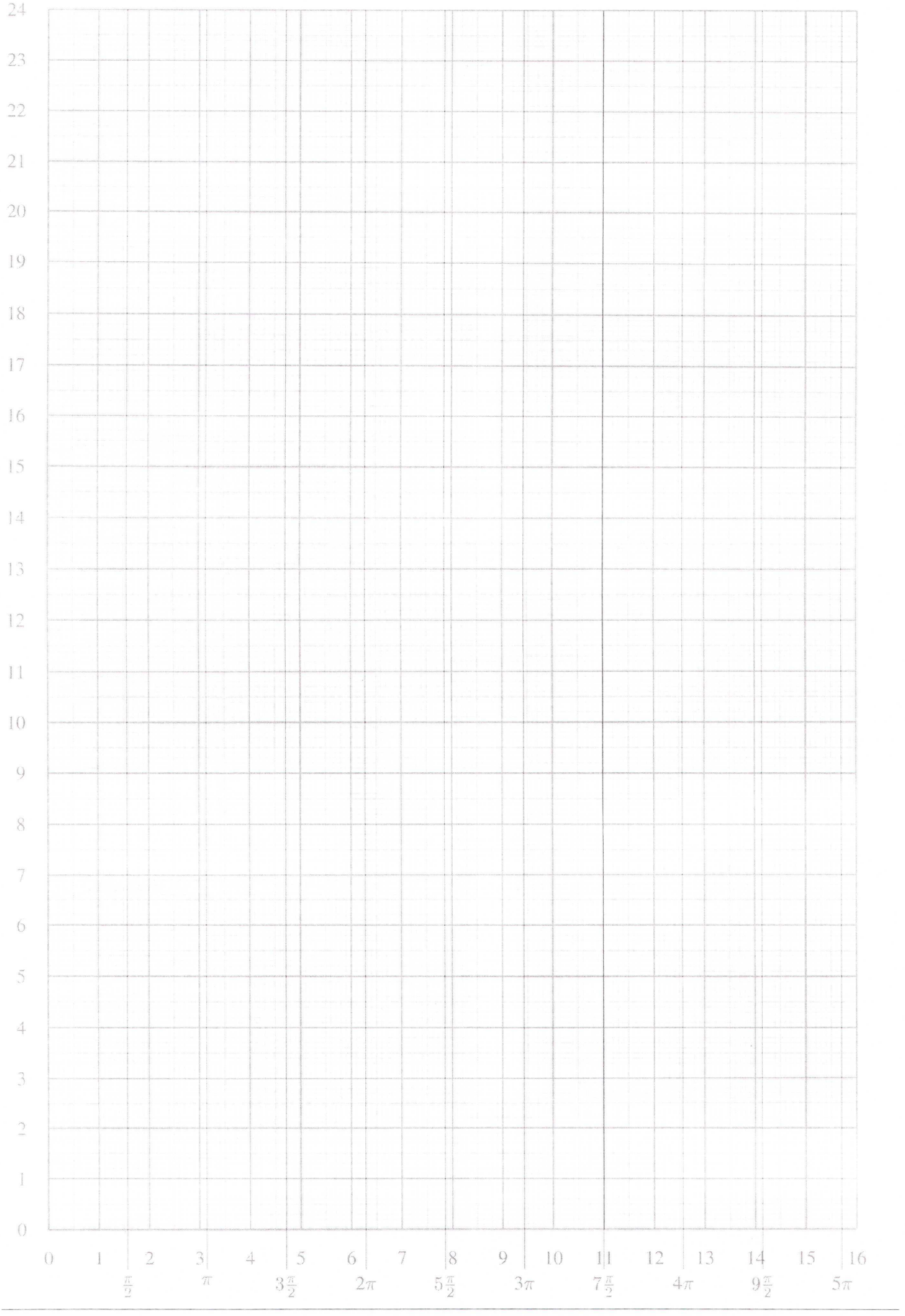
   (c) A village has rectangular shaped buildings with a flat roof; each one is represented as a triple $\langle l_i, r_i, h_i \rangle$ where l_i and r_i represent the leftmost and rightmost x-coordinates of the building and h_i represents the height. Information of the n buildings in the village is available in an array of n such triples. You are required to (*i*) give an efficient algorithm to compute the skyline, eliminating hidden lines, and represent as a sequence of pairs $\langle x_i, h_i \rangle$ in non-descending order of the x_i's and (*ii*) also analyse it time complexity. A sample set of buildings and the corresponding skyline is shown in the adjoining figure. (**Hint:** Use the principle of divide and conquer through merging, as in merge sort.) 10



(a) Buildings



(b) Skyline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