INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Department of Computer Science and Engineering

Algorithms-I (CS21003)

End-semester examination (Spring)

Date: Tue, Apr 25, 2017 *Students:* 94 *Time:* 2-5pm (AN)

Place: NR-121/B, NR-122/B, NR-221/B, NR-321/D, NR-322/D *Marks:* 85

Answer all questions as directed

Q1 (a) You are given a binary tree with non-negative node weights. Devise an algorithm to find a maximum weight path in the given binary tree, between any pair of nodes, the weight of a path in the binary tree being the sum of weights of the nodes in the path. (**Hint:** Use dynamic programming in conjunction with tree traversal) 10

(b) A weighted graph indicating (non-negative) tonnage capacities of roads between adjacent towns is given. An operator of trucks needs to know the maximum tonnage vehicle that can be plied between pairs of towns and the route to be followed (so that the truck can pass through all the roads along that route). Present an algorithm, along with its complexity, that will enable the truck operator to determine this. (**Hint:** Use dynamic programming) 10

(c) A weighted graph indicating (non-negative) distances of roads between adjacent towns is given. A town planner wishes to identify the shortest set of roads to connect the towns. Present an algorithm, along with its complexity, that will enable the town planner to determine this. 5

Q2 Compute the failure function for the string `a a b a a b a b b` using the Knuth-Morris-Pratt algorithm. 10

Q3 Answer any two parts.

(a) Construct a quadratic probing scheme, with justifications, to achieve total table coverage for a hash table with 41472 ($= 2^9 \times 9^2$) places. 5

(b) Establish the average time to search for a key in a hash table constructed through open addressing (such as linear/quadratic probing). 5

(c) Derive the amortised cost of inserting elements in a hash table where the initial size of the table is 1 and the size of the table is doubled everytime it gets filled and the elements are transferred to the new table. 5

Q4 (a) Depict the steps to insert 18 in the B-tree of figure 1. 5

(b) Depict the steps to delete 15 in the B-tree of figure 2. 5

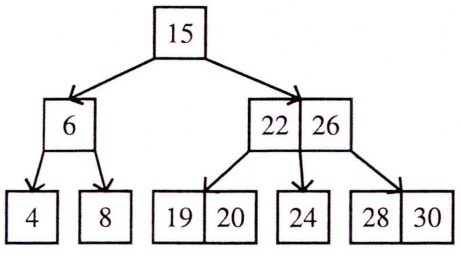

Figure 1: B-Tree for Q-4a

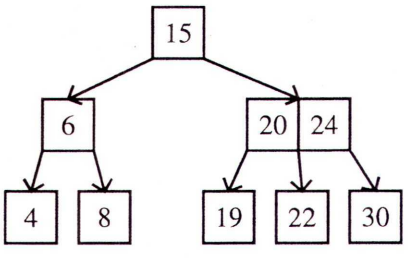

Figure 2: B-Tree for Q-4b

Q5 (a) With an example, explain the procedure to find a key in a skip list. 4

(b) Show that comparison based sorting techniques will require $\Omega(n \lg n)$ time for sorting n keys. 4

(c) Establish that the number of keys stored in an AVL tree is always exponential in the height of the tree. 4

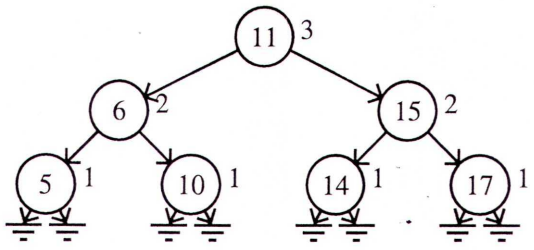

Figure 3: Red-black tree for Q-5d

(d) Depict the steps for deleting 14 from the red-black tree of figure 3. 4

(e) Present a non-deterministic algorithm for the vertex cover decision problem and illustrate the SAT problem that can be constructed from its run on the graph of figure 4. (**Hint:** The vertex cover decision problem is to check whether all edges of $G(V, E)$ can be covered by k vertices, $k \leq |V|$) 4

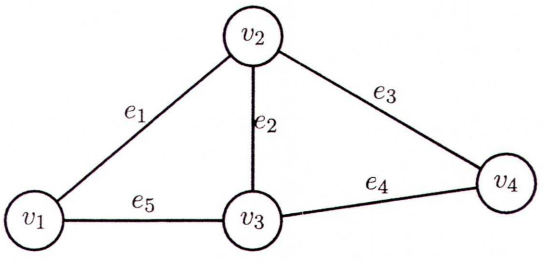

Figure 4: Graph for Q-5e

Q6 Your are given a circuit containing dc voltage sources, resistors and ideal diodes (0V forward drop). Present a scheme to arrive at the appropriate circuit equations (consistent with the the state of the diode, either conducting or in cutoff) to solve for the currents and voltages. Illustrate the working of your scheme on the circuit of figure 5 (**Hint:** Use branch and bound for forming the circuit equations) 10

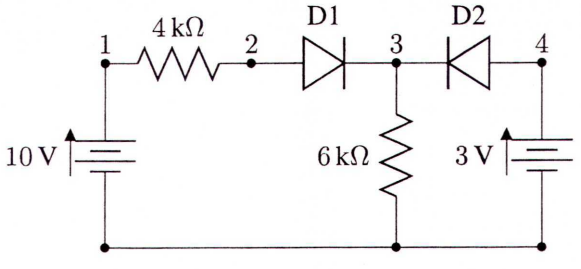

Figure 5: Diode circuit for Q-6