Mid-semester Examination: CS60007: Algorithms Design and Analysis

Department of Computer Science and Engineering, IIT Kharagpur

LTP 4-0-0: Credits 4: Time 2 hours: Marks 100: September 25, 2012: 2-4 pm

There is limited choice.

1. Show that the cardinality of the *maximum independent set* of vertices of a connected bipartite undirected graph $G(V, E)$ can be computed in polynomial time. A subset $A \subseteq V$ is an independent set if $\{u, v\}$ does not belong to E for any $u \in A$ and $v \in A$. Analyse the running time complexity of your algorithm. [10 marks]

2. Given a connected undirected graph $G(V, E)$, let the *minimum cardinality* vertex cover for G be $C^* \subseteq V$. Show that we can compute a vertex cover of size no more than $2|C^*|$ for G in polynomial time. [8+7 marks]

3. State the definition of $D(G)$, the *discrepancy* for a hypergraph (set system) $G(V, E)$. Show that the discrepancy of an n-vertex r-uniform hypergraph is $O(\sqrt{n \log n})$, if r is a constant with respect to n. Determine whether an upper bound for $D(G)$ can be a constant which is independent of n when we restrict the set of hyperedges such that $|E| < 2^{r-2}$. [7+12+6 marks]

4. Formulate the *minimum weighted vertex cover* problem for a graph $G(V, E)$ with vertex weights w_i for vertex i, $1 \leq i \leq |V|$, as an *integer linear program.* If OPT is the weight of the minimum weighted vertex cover, and OPT^* is the minimum value of the objective function for the *relaxation* of the integer linear program, then show that $OPT \geq OPT^*$. [10+5 marks]

5. Precisely state the definitions of (i) a *flow network* $G(V, E, c)$, where c is the capacity function defining the capacity $c(u, v)$ for the edge $(u, v) \in E$, (ii) a *valid flow function* f defining $f(u, v)$ for edge $(u, v) \in E$, (iii) the *residual network* $G_f(V, E_f)$ for a valid flow function f, and (iv) an *augmenting path* p in G_f. Show that $|E_f| \leq 2|E|$. [5+8+7+5+5 marks]

6. Consider the usual bin packing problem with unit sized bins and n items to be packed with non-zero sizes a_i, $1 \leq i \leq n$. Suppose each $a_i > 0.5$. Can we compute the optimal bin packing scheme in polynomial time in such a case? Explain. [10 marks]

7. Demonstrate an instance of the set covering problem where the greedy heuristic yields the minimum cardinality set cover. Demonstrate an instance where the number of sets in the minimum weighted set cover is larger than the cardinality of the minimum cardinality set cover. These instances must have at least six elements in the universe. Generalize your construction for instances where there are n elements in the universe, where n is a natural number. [8+7+5+5 marks]