Extra

INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
COMPUTER SCIENCE AND ENGINEERING DEPARTMENT

B.TECH. MID SEMESTER EXAMINATION, AUTUMN 2018-19

ALGORITHMS II (CS31005)

Full marks: 60 **Time: 2 hours**

1. Answer Question 1, Question 2, and any two from the rest.
2. All parts of a particular question should be answered together.
3. Credits will be given for neat and to-the-point answering.

1. Answer all questions. Each question has 2 marks. Write only the question numbers and the answers without any explanation, in the **1st page** of your answer-script. $(2 \times 10 = 20)$

   i) Write the time complexity of the best algorithm that can compute the value of $f(n)$ for a given value of n, where n is a positive integer, $f(1) = f(2) = 1$, and $f(n) = f(n-2) + n - 1$ if $n > 2$.

   ii) P is a set of n points on xy-plane. Write the time complexity for finding the smallest axis-parallel rectangle containing all points of P.

   iii) Write the asymptotic upper bound of the recurrence $T(n) = O(1)$ if $n \leq 4$ and $T(n) = 3T(n/2) + O(n)$ if $n > 4$.

   iv) P and Q are two convex polygons that lie on two different sides of a given straight line. How quickly can you compute the convex hull of $P \cup Q$? Just write the time complexity. Consider m and n as the respective number of vertices of P and Q.

   v) A planar graph is stored in a doubly connected edge list. Given the IDs of two half-edges, your task is to decide whether they describe a common face, as quickly as possible. What would be its time complexity?

   vi) Draw an example of 5 sites so that their Voronoi diagram has no vertex.

   vii) Draw a flow network with exactly 4 vertices in which Ford-Fulkerson algorithm takes $O(|f_{\max}|)$ time to compute the max-flow.

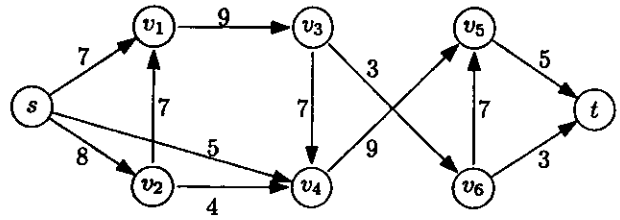
   viii) Write the worst-case time complexity to determine whether a bipartite graph admits perfect matching.

   ix) What is the expected value when a fair dice (six faces with values $1, 2, \ldots, 6$) is thrown at random?

   x) Write the expected time complexity to construct a binary search tree with n nodes using Las Vegas technique. The expected height of the tree should be $O(\log n)$.

2. Answer all questions. Each question has 5 marks. $(5 \times 4 = 20)$

   i) Demonstrate Edmonds-Karp algorithm to find the max flow in the following network.



ii) Let G be a flow network, f_1 a flow in G, and f_2 a flow in G_{f_1}. Prove that $f_1 \uparrow f_2$ is a flow in G and its value is $|f_1| + |f_2|$.

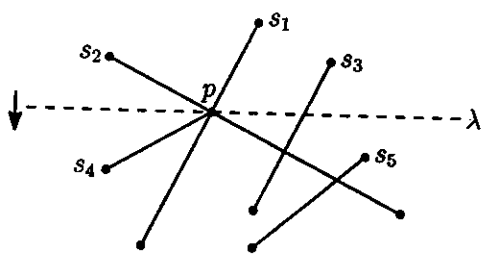
iii) Prove that the number of edges in the Voronoi diagram of n sites is $O(n)$.

iv) At most how many points can be placed inside a unit square so that the distance between any two points is at least unity? Prove your claim.

3. Given a set $P = \{p_1, p_2, \ldots, p_n\}$ containing n points on the 2D plane. Assume that all the $\binom{n}{2}$ point-pair distances in P are distinct. Suggest a deterministic algorithm to prepare a 1D array $A[1..n]$ in which $A[i] = j$ implies that out of all points in P, p_j is nearest to p_i. Explain its time complexity.

(7+3)

4. Consider the line segments s_1, s_2, s_3, s_4, s_5, as shown in the diagram, in the algorithm to compute their intersection points. Illustrate the data structures just before and just after the sweep line λ encounters the point p. The operations on these data structures at p should be clearly explained. Notice that p is the point of intersection of s_1 and s_2, and it is also an endpoint of s_4. (10)



5. Input is an array $A[1..n]$ and a number k. Suggest a Las Vegas algorithm to partition A into two sub-arrays B and C of lengths k and $n-k$ respectively, such that the difference d of the sum of the elements in B and the sum of those in C is maximum. Deduce its expected time complexity.

$(7+3)$

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