

- Suppose a graph, in addition to capacities on edges, has a capacity on every vertex such that the total inflow (and therefore the total outflow from the node) cannot exceed the capacity of the vertex. The other constraints remain the same as in the standard maximum-flow problem. How will you find the maximum flow in this graph?

- Suppose all capacities in a graph are distinct. Will the max-flow be unique?

A company running a factory has to arrange for extra maintenance staff to work over vacation periods.

- There are K vacation periods in the year, numbered from 1 to K , with the i -th vacation period having d_i days.
- There are N maintenance staff available numbered from 1 to N , with the j -th maintenance being available for a total of c_j days over all vacation periods. In addition, for each vacation period i , each staff can only work on a subset of the d_i days (for example, if a vacation period spans over Friday, Saturday and Sunday, staff 1 may be available for only Friday and Saturday but not Sunday, staff 2 may be available for only Friday and Sunday but not Saturday, and staff 3 may be available for all days).

What is the maximum no. of vacation days that can be covered by the company with its available staff?

- Given an undirected graph, we want to find the smallest subset of edges that will disconnect the graph (the actual set of edges need to be found). Design an efficient algorithm for it.

- Find a maximum matching in a bipartite graph in $O(VE)$ time (assume $E = \Omega(V)$).