INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
COMPUTER SCIENCE AND ENGINEERING DEPARTMENT

B.TECH. END SEMESTER EXAMINATION, AUTUMN 2012-13
ALGORITHMS II (CS31005)

Full marks: 100 **Time: 3 hours**

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1. Answer Question 1, and any four from the rest.
2. All parts of a particular question should be answered together.
3. Credits will be given for neat and to-the-point answering. Unnecessary or redundant words are liable to negative marking.

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1. Each answer should start in a new page and end in that page only. $(5 \times 8 = 40)$

   (a) Suggest an algorithm to build a min-heap from a BST. Mention its time complexity.

   (b) $A = \{a_1, a_2, \ldots, a_{2n}\}$ is a (unordered) set of $2n$ items. Weight of each item $a_i \in A$ is w_i. It is known that for each $a_i \in A$, there exists some item $a_j \in A$ such that $w_i + w_j = c$, where c is a given constant. The items in A have to be divided among n friends so that all friends carry two items each and share equal weight. Suggest an algorithm for this and explain its time complexity.

   (c) Consider the max-finding algorithm on A having n elements (randomly ordered):

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   1. max ← −∞
   2. for i ← 1 to n
   3.     if A[i] > max then
   4.         max ← A[i]
   5. return max
   ```

   Find the expected number of executions of Line 4.

   (d) A company has to engage its employee over a series of tasks $T_1, T_2, \ldots, T_n$, so that the employee remains busy to the maximum extent. The only condition is that the employee should not be given any task unless he finishes the previous task. Each task has a definite starting time and a definite finishing time, and two tasks can have overlapping time spans. Describe an algorithm to help the company.

   (e) Given a set of n points on the 2D plane, suggest an algorithm in brief to determine whether four or more points are co-circular (i.e., lie on the perimeter of same circle). Just write its time complexity (no explanation needed).

   (f) There are four problems: P_1, P_2, P_3, P_4. Following is know about them: $P_1 \in \mathcal{P}$, $P_2 \in \mathcal{NP}$, $P_1 \leqslant_P P_2$, $P_3 \leqslant_P P_4$, $P_4 \in \mathcal{NP}$-complete. Which one of them is easiest and which one is hardest? Explain.

   (g) Derive the 3-CNF expression for $\phi = (x_1 \leftrightarrow (x_2 \wedge \neg x_3))$.

   (h) Explain which one is harder between TSP and Vertex Cover Problem in the perspective of approximation algorithms.

2. Describe a Monte Carlo algorithm to find a min-cut with success probability $\Omega\left(1 - \left(1 - \frac{1}{\log n}\right)^2\right)$. Explain its time complexity. (10+5)

3. Suggest an efficient procedure to multiply two n-bit binary integers a and b. Describe an algorithm using this procedure to convert a decimal integer 10^n (n is a power of 2) into binary. Give a recurrence relation for the running time of the algorithm, and solve the recurrence. (10+5)

P.T.O.

4. Given a set S of n line segments on the 2D plane. Describe an algorithm to find whether there exists a 2D point that lies on more than two line segments of S. Explain its time complexity. (10+5)

5. P is a given set of n points in 2D. We have to find a set of circles S, such that:

   i) each point of P lies on the circumference of at least one circle of S and
   ii) the interior of no circle of S contains any point from P.

   The set S need not be minimum in size. Describe an algorithm to construct S and explain its time complexity. (12+3)

6. State the clique decision problem.
   Explain why it is $\mathcal{NP}$-complete.
   Now consider the problem P that asks to find the maximum-size set $X \subseteq V$ for an undirected graph $G(V, E)$ such that each edge $e \in E$ is incident on at most one vertex in X. Is P an $\mathcal{NP}$-complete problem? Justify. (2+5+8)

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