INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR
COMPUTER SCIENCE AND ENGINEERING DEPARTMENT

B.TECH. END SEMESTER EXAMINATION, AUTUMN 2011-12

ALGORITHMS II (CS31005)

Full marks: 100 **Time: 3 hours**

NOTE:

(a) Answer Question 1 ($5 \times 8 = 40$ marks), Question 2 (20 marks), and any four (10 marks each) from the rest.

(b) All parts of a particular question should be answered together.

(c) Credits will be given for neat and to-the-point answering. Unnecessary or redundant words are liable to negative marking.

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1. Each answer should start in a new page and end in that page only. ($5 \times 8 = 40$)

(a) Suggest an algorithm to shuffle n distinct elements. Briefly explain its time complexity.

(b) Draw examples, each with 9 sites, for the following cases such that their Voronoi diagrams have: (i) no vertex; (ii) one vertex; (iii) four vertices; (iv) seven vertices.

(c) There are four problems: P_1, P_2, P_3, P_4. Following is know about them: $P_1 \in \mathcal{P} \cap \mathcal{NP}$; $P_2 \in \mathcal{NP}$ and $P_2 \notin \mathcal{P}$; $P_3 \in \mathcal{NP}$-hard and $P_3 \notin \mathcal{NP}$; $P_4 \in \mathcal{NP}$-complete. Which one of them is hardest and which one is easiest? Explain your opinion.

(d) S is a set of n line segments on the 2D plane. Let k be the number of intersection points among these line segments. Let p be an 'event point' while finding the intersection points among the line segments of P. Let $m(p)$ be the number of segments involved in the event point p; and let m be the sum of $m(p)$ over all event points. Then find an upper bound of m in terms of n and k.

(e) In a bipartite graph with n vertices, all vertices have identical degree. Then prove or disprove: "Its maximum matching has the maximum cardinality over the maximum matchings corresponding to all possible bipartite graphs with n vertices".

(f) P is a set of n points on the 2D plane; no four points in P are concyclic/co-circular. Let D_n be the smallest disc that contains all points of P. Let C_n be the boundary (circle) of D_n. Prove or disprove: "If there are three points lying on C_n, then with these three points as vertices, we never get any obtuse-angled triangle."

(g) State the MAX 3-SAT Problem. If the input certificate is randomly assigned in each trial, then find the expected number of trials that can produce its optimal solution.

(h) One day a genius proposes and proves a constant-factor, polynomial-time, deterministic approximation algorithm for TSP. Which greater fact does it prove? Prove it!

2. The max-flow in a flow network $G(V, E)$ is known, and it is $f_{\max}$. $G'(V', E')$ is another flow network with $V' = V$, $E' = E$, and the capacity of each edge $(u', v') \in E'$ is given by $c(u', v') = c(u, v) + f_{\max}$; note that, the vertices $u' \in V'$ and $v' \in V'$ correspond to the vertices $u \in V$ and $v \in V$. The following conditions are given:

(i) The source s' and the sink t' of G' correspond to s and t of G.
(ii) Excepting the source and the sink, the number of incoming edges at each vertex (of G or G') is same as the number of its outgoing edges.

Suggest a randomized algorithm to compute the max-flow in G'. Derive its time complexity and success probability.

3. Suggest a randomized algorithm to construct a search-efficient BST for a (ordered or unordered) list of n distinct keys. Explain its time complexity and the expected height of the BST.

4. A list $A = \{a_1, \ldots, a_n\}$ consists of n positive integers only. The problem is to prepare another list $A' = \{a'_1, \ldots, a'_n\}$ such that $|a'_i| = a_i$ for $i = 1, \ldots, n$, and the sum of the elements of A' is 0. Is the problem in $\mathcal{P}$? in $\mathcal{NP}$? $\mathcal{NP}$-complete? Justify.

5. State the Hamiltonian cycle problem. Given that the Hamiltonian cycle problem is $\mathcal{NP}$-complete, show that the following problem P is $\mathcal{NP}$-complete.
   Problem P: Given an undirected graph $G(V, E)$, $s \in V$, $t \in V$, and a number k, does there exist a path p from s to t with at least k edges so that all vertices in p are distinct?

6. Given an unordered set S containing $n\,(> 2)$ distinct integers, and given two appropriate positive integers i and j, suggest an algorithm—deterministic or randomized of Las Vegas type—to find a proper subset S' of S so that the smallest element of S' is larger than i elements of S and the largest element of S' is smaller than j elements of S. Explain its time complexity.

7. Let S be a set of n non-intersecting line segments in the 2D plane. Suggest a randomized scheme for constructing an efficient data structure resembling a binary search tree that can store the segments of S. Deduce its expected size.

8/11/11
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