Aab

End-Semester Examination, Autumn 2015-16
CS60007: Algorithm Design and Analysis

Full marks: 100

Time: 3 hours

1. Answer Question 1, Question 2, and any four from the rest.
2. All parts of a particular question should be answered together.
3. Credits will be given for neat and to-the-point answering.
4. Unnecessary / confusing words are liable to negative marking.

1. Write the answers without any explanation. $\boxed{2 \times 10 = \textbf{20 marks}}$

   (i) What's the asymptotic upper bound of $T(n) \leq T(3n/4) + \Theta(n)$?

   (ii) How to implement a priority queue when searching is also frequent?

   (iii) a, b, c, d are non-negative integers. $f(a, b, c, d) = g(a, b) \times g(c, d)$, where,

   $$g(a,b) = \begin{cases} b & \text{if } a = 0 \\ g(b,a) & \text{if } a > b \\ g(b \bmod a, a) & \text{otherwise.} \end{cases}$$

   What's the value of $f(4, 6, 5, 15)$?

   (iv) What's the time complexity of non-deterministic sorting algorithm?

   (v) What would be the worst-case time complexity of the "best possible algorithm" to compute the convex hull of n triangles?

   (vi) Let $G(V, E)$ be a flow network and f be a flow in G. Let $v \in V$. Let $\delta_0(s, v)$ = shortest path length of v from s in G, $\delta_f(s, v)$ = shortest path length of v from s in G_f. What is the relation between $\delta_0(s, v)$ and $\delta_f(s, v)$?

   (vii) What's the number of clauses in the 3-CNF of $(x_1 \vee \neg x_2)$?

   (viii) What's the expected time complexity of a Las Vegas algorithm for choosing an element x from a given list A so that x is larger than at least 90% elements of A? Assume that A contains n distinct elements.

   (ix) Among Halting Problem, Clique Decision Problem, Min-Cut Problem, Max-Cut Problem, which one is hardest and which one easiest?

   (x) Choose the correct option: Checking whether a given undirected graph $G(V, E)$ is k-colorable can be done in polynomial time if (i) $k \leq 2$ (ii) $k \leq 3$ (iii) $k \geq 0$ (iv) $k \geq 3$. [Note: G is k-colorable if its vertices can be colored by k colors such that no two adjacent vertices get the same color.]

2. A village consists of n cottages, which are all rectangular in shape with horizontal and vertical boundaries. The cottages are not of same size. So, for each cottage C_i, the coordinates (a_i, b_i) of its bottom-left corner and (c_i, d_i) of its top-right corner are given. The village has to be surrounded by a circular fencing of minimum perimeter so that all its cottages lie inside the fencing. Suggest a Las Vegas algorithm for this, and derive its expected time complexity. $\boxed{\textbf{20 marks}}$

3. What is meant by a flow network? Write its necessary and sufficient properties. Using these properties and the definition of magnitude $|f|$ of a flow f in the network, prove that for any cut (S, T) with $s \in S, t \in T$, we get $f(S, T) = |f|$. $\boxed{\textbf{15 marks}}$

4. State the maximum independent set problem (ISP) and the 3-SAT problem. Given that 3-SAT is $\mathcal{NP}$-complete, prove that ISP is also so. Provide a suitable example. **15 marks**

5. Given a set P of $n(> 16)$ 2D points. No two points have same x- or y-coordinate. The task is to partition P into four subsets A, B, C, D, such that the following conditions are true:

   i) x-coordinate of any point in $A \cup B$ is less than that of any point in $C \cup D$.

   ii) y-coordinate of any point in A is less than that of any point in B.

   iii) y-coordinate of any point in C is less than that of any point in D.

   iv) Each of A, B, C, D contains at least $\frac{n}{16}$ points.

   Suggest a Las Vegas algorithm for this and derive its expected runtime. **15 marks**

6. A large committee A consists of n members. It has to be divided into two sub-committees B and C so that every member belongs to exactly one sub-committee. For this, every pair of members is labeled as 'friendly' or 'unfriendly', depending on the mutual relationship of its two members. The "friendliness information" is made available in a binary matrix M of size $n \times n$, where $M[i][j] = 1$ if the ith and the jth members are friendly with each other; and $M[i][j] = 0$ otherwise. The sub-committees B and C have to be made in a way so as to remove the unfriendly pairs as far as possible. Can you design a deterministic polynomial-time algorithm for this? If yes, write it, and explain its time complexity. Otherwise, suggest a Monte Carlo algorithm, derive appropriate lower bounds on its probability and correctness, and explain its time complexity.

   Example: Let $A = \{a, b, c, d, e\}$ be a 5-member committee with $(a, d), (b, c), (b, e), (c, e)$ as unfriendly pairs. Then $(B = \{a, b, c\}, C = \{d, e\})$ is an optimum solution. Notice that in this solution, b and c, though mutually unfriendly, still remain in the same sub-committee B, but the other three unfriendly pairs no longer exist! **15 marks**

7. A library wants to purchase some books based on choices from its m members. For this, the librarian has prepared a master list L of n books, and has asked its members to submit their choices (minimum 1, maximum n) from L. After getting their choices, he decides to purchase as few books as possible so that at least one book is purchased from the choice of every member. Is this problem in $\mathcal{P}$ or not? Suggest an efficient deterministic algorithm if it is in $\mathcal{P}$, or a non-deterministic algorithm otherwise. Explain the time complexity.

   Is it $\mathcal{NP}$-complete? You can use any known $\mathcal{NP}$-complete problem for your proof. **15 marks**

8. There are n cities. All pairs of cities are not connected by train lines. So the government decides to check whether a tourist can visit all the cities in some sequence, if wishes to do so, without revisiting any city during his tour. You are a government consultant and have to fix your consultation fee depending on the difficulty level of the problem. If the solution can be found in polynomial time and in polynomial space, then you charge less, and charge high otherwise. What would be your policy for this case? Provide arguments with necessary proofs. **15 marks**

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