TUTORIAL SOLUTION: September 29, 2021

1. a. $P(\neg E,\neg S,\neg M,\neg B) = P(\neg E)\, P(\neg M)\, P(\neg S \mid \neg E,\neg M)\, P(\neg B \mid \neg M)$
$= (1 - 0.4) * (1 - 0.1) * (1 - 0.1) * (1 - 0.1) = 0.4374$

b. $P(B) = P(B \mid M)\, P(M) + P(B \mid \neg M)\, P(\neg M)$
$= 1.0 * 0.1 + 0.1 * (1 - 0.1) = 0.19$

c. $P(M|B) = P(B \mid M)\, P(M) / P(B)$
$= (1.0)(0.1) / (0.19) \cong 0.5263$

d. $P(M \mid S, B, E) = P(M, S, B, E) / P(S, B, E)$
$= P(M, S, B, E) / [\, P(M, S, B, E) + P(\neg M, S, B, E)\,]$

$P(M, S, B, E) = P(E)\, P(M)\, P(S \mid E,M)\, P(B \mid M) = 0.4 * 0.1 * 1.0 * 1.0 = 0.04$

$P(\neg M, S, B, E) = P(E)\, P(\neg M)\, P(S \mid E,\neg M)\, P(B \mid \neg M)$
$= 0.4 * 0.9 * 0.8 * 0.1 = 0.0288$

Therefore, $P(M \mid S, B, E) = 0.04 / [\, 0.04 + 0.0288\,] = 0.5814$

e. $P(E|M) = P(E) = 0.4$
In the absence of evidence on S, the node S d-separates E and M. Therefore, E and M are conditionally independent.

2. a.

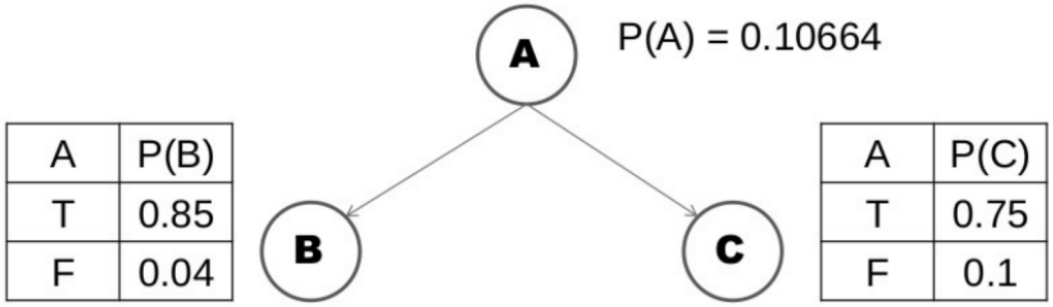
F	C	
T	T	0.1 * 0.8 = 0.08
T	F	0.1 * 0.2 = 0.02
F	T	0.9 * 0.3 = 0.27
F	F	0.9 * 0.7 = 0.63

b. No; because $P(C) = 0.35$ but $P(C \mid F) = 0.8$

c. $P(C) = 0.08 + 0.27 = 0.35$
$P(F|C) = P(F, C) / P(C) = 0.08 / 0.35 \cong 0.23$
$P(F|\neg C) = P(F, \neg C) / P(\neg C) = 0.02/0.65 \cong 0.03$

3. P(A | ¬B, C) = P(¬B, C | A) P(A) / P(¬B, C)
= P(¬B, C | A) P(A) / (P(¬B, C | A) P(A) + P(¬B, C |¬A) P(¬A))

Now, P (¬B, C | A) = P (¬B | A) P (C | A) = (0.15)(0.75) = 0.1125
P (¬B, C | ¬A) = P (¬B | ¬A) P (C | ¬A) = (0.96)(0.1) = 0.096

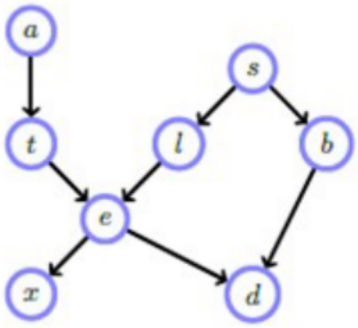


A	P(B)
T	0.85
F	0.04

A	P(C)
T	0.75
F	0.1

Therefore, P(A | ¬B, C)
= (0.1125 * 0.10664) / ((0.1125 * 0.10664) + (0.096* 0.89336))
= 0.1227

4.



x = Positive X-ray
d = Dyspnea (Shortness of breath)
e = Either Tuberculosis or Lung Cancer
t = Tuberculosis
l = Lung Cancer
b = Bronchitis
a = Visited Asia
s = Smoker

The table values are:

P(a) = 0.01 P(s) = 0.5

P(t|a) = 0.05 P(t|¬a) = 0.01

P(l|s) = 0.1 P(l|¬s) = 0.01

P(b|s) = 0.6 P(b|¬s) = 0.3

P(x|e) = 0.98 P(x|¬e) = 0.05

P(d|e,b) = 0.1 P(d|e,¬b) = 0.7

P(d|¬e,b) = 0.8 P(d|¬e,¬b) = 0.1

P(e|t,l) = 0 only if both t and l are false, and 1 otherwise.

P(¬s) = 0.5 P(¬a) = 0.99

P(d) = P(d,e,b) + P(d,¬e,b) + P(d,e,¬b) + P(d,¬e,¬b)

P(d,e,b) = P(d|e,b) P(e,b) = P(d|e,b) P(e) P(b)

P(e) = P(e,t,l) + P(e,¬t,l) + P(e,t,¬l) + P(e,¬t,¬l)

P(e,t,l) = P(e|t,l) P(t,l) = P(e|t,l) P(t) P(l)

P(t) = P(t|a) P(a) + P(t|¬a) P(¬a) = 0.05 * 0.01 + 0.01 * 0.99 = 0.0104

P(¬t) = 1 – P(t) = 1 – 0.0104 = 0.9896

P(l) = P(l|s) P(s) + P(l|¬s) P(¬s) = 0.1 * 0.5 + 0.01 * 0.5 = 0.055

P(¬l) = 1 – P(l) = 1 – 0.055 = 0.945

P(e) = P(e|t,l) P(t) P(l) + P(e|¬t,l) P(¬t) P(l) +
P(e|t,¬l) P(t) P(¬l) + P(e|¬t,¬l) P(¬t) P(¬l)

= 1 * P(t) P(l) + 1 * P(¬t) P(l) + 1 * P(t) P(¬l) + 0 * P(¬t) P(¬l)

= P(t) P(l) + P(¬t) P(l) + P(t) P(¬l) = 1 – P(¬t) P(¬l)
{Since, P(t) P(l) + P(¬t) P(l) + P(t) P(¬l) + P(¬t) P(¬l) = 1}

= 1 – 0.9896 * 0.945 = 0.064828

P(¬e) = 1 – 0.064828 = 0.935172

P(b) = P(b|s) P(s) + P(b|¬s) P(¬s) = 0.6 * 0.5 + 0.3 * 0.5 = 0.45

P(¬b) = 1 – P(b) = 1 – 0.45 = 0.55

P(d) = P(d|e,b) P(e) P(b) + P(d|¬e,b) P(¬e) P(b) +
P(d|e,¬b) P(e) P(¬b) + P(d|¬e,¬b) P(¬e) P(¬b)

= 0.1 * 0.064828 * 0.45 + 0.8 * 0.935172 * 0.45 +
0.7 * 0.064828 * 0.55 + 0.1 * 0.935172 * 0.55

= 0.41597242