the scope of this book. One of the most interesting of these codes is the **Reed-Solomon code** used today for both detection and correction.

10.5 CHECKSUM

The last error detection method we discuss here is called the **checksum.** The checksum is used in the Internet by several protocols although not at the data link layer. However, we briefly discuss it here to complete our discussion on error checking.

Like linear and cyclic codes, the checksum is based on the concept of redundancy. Several protocols still use the checksum for error detection as we will see in future chapters, although the tendency is to replace it with a CRC. This means that the CRC is also used in layers other than the data link layer.

Idea

The concept of the checksum is not difficult. Let us illustrate it with a few examples.

Example 10.18

Suppose our data is a list of five 4-bit numbers that we want to send to a destination. In addition to sending these numbers, we send the sum of the numbers. For example, if the set of numbers is (7, 11, 12, 0, 6), we send (7, 11, 12, 0, 6, **36**), where 36 is the sum of the original numbers. The receiver adds the five numbers and compares the result with the sum. If the two are the same, the receiver assumes no error, accepts the five numbers, and discards the sum. Otherwise, there is an error somewhere and the data are not accepted.

Example 10.19

We can make the job of the receiver easier if we send the negative (complement) of the sum, called the *checksum*. In this case, we send (7, 11, 12, 0, 6, **−36**). The receiver can add all the numbers received (including the checksum). If the result is 0, it assumes no error; otherwise, there is an error.

One's Complement

The previous example has one major drawback. All of our data can be written as a 4-bit word (they are less than 15) except for the checksum. One solution is to use **one's complement** arithmetic. In this arithmetic, we can represent unsigned numbers between 0 and $2^n - 1$ using only n bits.† If the number has more than n bits, the extra leftmost bits need to be added to the n rightmost bits (wrapping). In one's complement arithmetic, a negative number can be represented by inverting all bits (changing a 0 to a 1 and a 1 to a 0). This is the same as subtracting the number from $2^n - 1$.

Example 10.20

How can we represent the number 21 in one's complement arithmetic using only four bits?

†Although one's complement can represent both positive and negative numbers, we are concerned only with unsigned representation here.

Solution

The number 21 in binary is **10101** (it needs five bits). We can wrap the leftmost bit and add it to the four rightmost bits. We have (0101 + **1**) = 0110 or 6.

Example 10.21

How can we represent the number −6 in one's complement arithmetic using only four bits?

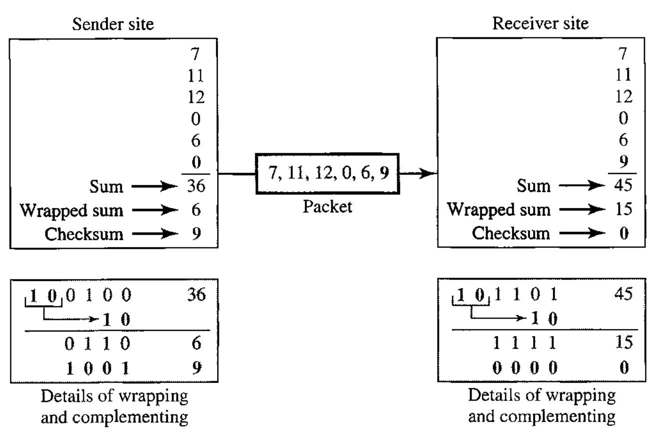
Solution

In one's complement arithmetic, the negative or complement of a number is found by inverting all bits. Positive 6 is 0110; negative 6 is 1001. If we consider only unsigned numbers, this is 9. In other words, the complement of 6 is 9. Another way to find the complement of a number in one's complement arithmetic is to subtract the number from $2^n - 1$ ($16 - 1$ in this case).

Example 10.22

Let us redo Exercise 10.19 using one's complement arithmetic. Figure 10.24 shows the process at the sender and at the receiver. The sender initializes the checksum to 0 and adds all data items and the checksum (the checksum is considered as one data item and is shown in color). The result is 36. However, 36 cannot be expressed in 4 bits. The extra two bits are wrapped and added with the sum to create the wrapped sum value 6. In the figure, we have shown the details in binary. The sum is then complemented, resulting in the checksum value 9 (15 − 6 = 9). The sender now sends six data items to the receiver including the checksum 9. The receiver follows the same procedure as the sender. It adds all data items (including the checksum); the result is 45. The sum is wrapped and becomes 15. The wrapped sum is complemented and becomes 0. Since the value of the checksum is 0, this means that the data is not corrupted. The receiver drops the checksum and keeps the other data items. If the checksum is not zero, the entire packet is dropped.

Figure 10.24



Internet Checksum

Traditionally, the Internet has been using a 16-bit checksum. The sender calculates the checksum by following these steps.

Sender site:

1. **The message is divided into 16-bit words.**
2. **The value of the checksum word is set to 0.**
3. **All words including the checksum are added using one's complement addition.**
4. **The sum is complemented and becomes the checksum.**
5. **The checksum is sent with the data.**

The receiver uses the following steps for error detection.

Receiver site:

1. **The message (including checksum) is divided into 16-bit words.**
2. **All words are added using one's complement addition.**
3. **The sum is complemented and becomes the new checksum.**
4. **If the value of checksum is 0, the message is accepted; otherwise, it is rejected.**

The nature of the checksum (treating words as numbers and adding and complementing them) is well-suited for software implementation. Short programs can be written to calculate the checksum at the receiver site or to check the validity of the message at the receiver site.

Example 10.23

Let us calculate the checksum for a text of 8 characters ("Forouzan"). The text needs to be divided into 2-byte (16-bit) words. We use ASCII (see Appendix A) to change each byte to a 2-digit hexadecimal number. For example, F is represented as 0x46 and o is represented as 0x6F. Figure 10.25 shows how the checksum is calculated at the sender and receiver sites. In part a of the figure, the value of partial sum for the first column is 0x36. We keep the rightmost digit (6) and insert the

Figure 10.25

1	0	1	3		Carries
4	6	6	F		(Fo)
	7	2	6	F	(ro)
	7	5	7	A	(uz)
	6	1	6	E	(an)
	0	0	0	0	Checksum (initial)
	8	F	C	6	Sum (partial)
				1	
	8	F	C	7	Sum
	7	0	3	8	Checksum (to send)

a. Checksum at the sender site

1	0	1	3		Carries
	4	6	6	F	(Fo)
	7	2	6	F	(ro)
	7	5	7	A	(uz)
	6	1	6	E	(an)
	7	0	3	8	Checksum (received)
	F	F	F	E	Sum (partial)
				1	
	F	F	F	F	Sum
	0	0	0	0	Checksum (new)

b. Checksum at the receiver site

leftmost dight (3) as the carry in the second column. The process is repeated for each column. Hexadecimal numbers are reviewed in Appendix B.

Note that if there is any corruption, the checksum recalculated by the receiver is not all 0s. We leave this an exercise.

Performance

The traditional checksum uses a small number of bits (16) to detect errors in a message of any size (sometimes thousands of bits). However, it is not as strong as the CRC in error-checking capability. For example, if the value of one word is incremented and the value of another word is decremented by the same amount, the two errors cannot be detected because the sum and checksum remain the same. Also if the values of several words are incremented but the total change is a multiple of 65535, the sum and the checksum does not change, which means the errors are not detected. Fletcher and Adler have proposed some weighted checksums, in which each word is multiplied by a number (its weight) that is related to its position in the text. This will eliminate the first problem we mentioned. However, the tendency in the Internet, particularly in designing new protocols, is to replace the checksum with a CRC.

10.6 RECOMMENDED READING

For more details about subjects discussed in this chapter, we recommend the following books. The items in brackets [. . .] refer to the reference list at the end of the text.

Books

Several excellent book are devoted to error coding. Among them we recommend [Ham80], [Zar02], [Ror96], and [SWE04].

RFCs

A discussion of the use of the checksum in the Internet can be found in RFC 1141.

10.7 KEY TERMS

block code
burst error
check bit
checksum
codeword
convolution code
cyclic code
cyclic redundancy check (CRC)
dataword
error
error correction
error detection
forward error correction
generator polynomial
Hamming code
Hamming distance
interference
linear block code
minimum Hamming distance
modular arithmetic