Indian Institute of Technology Kharagpur
Class Test I: 2020-21

Date: 19 Jan. 2021 **Subject No.: CS60010** **Subject: Deep Learning**

1. (a) (4 points) The trace of a square matrix $\mathbf{A} \in \mathbb{R}^{n \times n}$ is defined as the sum of the its diagonal entries, or $\text{tr}\mathbf{A} = \sum_{i=1}^{n} \mathbf{A}_{ii}$. Prove the following fact.

$$||\mathbf{A}||_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} \mathbf{A}_{ij}^2} = \sqrt{\text{tr}(\mathbf{A}^T\mathbf{A})}$$

where, $||\mathbf{A}||_F$, is the Frobenius norm. Show all your steps.

(b) (3 points) Show that for a matrix $\mathbf{A}$ and vector $\mathbf{x}$, $\frac{\partial}{\partial \mathbf{x}}(\mathbf{A}^{-1}) = -\mathbf{A}^{-1}\big(\frac{\partial}{\partial \mathbf{x}}\mathbf{A}\big)\mathbf{A}^{-1}$. **Hint**: Use the fact that for any two matrices $\mathbf{A}$ and $\mathbf{B}$, $\frac{\partial \mathbf{AB}}{\partial \mathbf{x}} = \frac{\partial \mathbf{A}}{\partial \mathbf{x}}\mathbf{B} + \mathbf{A}\frac{\partial \mathbf{B}}{\partial \mathbf{x}}$.