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Deeplearningtest (Class test 1)

1.(a) Let $A \in R^{n \times n}$

then $A^T \in R^{n \times n}$ where A^T = transpose of A

$$\therefore \sum_{i=1}^{n}\sum_{j=1}^{n} (A^T)_{ij} = A_{ji}$$

$$\therefore \sum_{j=1}^{n}\sum_{i=1}^{n}\left[(A^TA)_{ij} = \sum_{k=1}^{n} (A^T)_{ik}(A)_{kj}\right]$$

$$\Rightarrow \sum_{\substack{i=1\\j=i}}^{n}\left[(A^TA)_{ii} = \sum_{k=1}^{n} (A^T)_{ik}(A)_{ki}\right]$$

$$\Rightarrow \sum_{\substack{i=1\\j=i}}^{n}\left[(A^TA)_{ii} = \sum_{k=1}^{n} (A_{ki})(A)_{ki} = \sum_{k=1}^{n}(A_{ki})^2\right]$$

$$\Rightarrow \sum_{i=1}^{n}\left[(A^TA)_{ii} = \sum_{k=1}^{n}(A_{ki})^2\right]$$

$$\Rightarrow \sum_{i=1}^{n}(A^TA)_{ii} = \sum_{i=1}^{n}\sum_{k=1}^{n}(A_{ki})^2$$

(Put j=k)

$$\Rightarrow \text{trace}(A^TA) = \sum_{i=1}^{n}\sum_{j=1}^{n}(A_{ji})^2$$

$\rightarrow$ $\text{trace}(A^T A) = \sum_{i=1}^{n}\sum_{j=1}^{n} (A_{ij})^2$ (Replacing i, j)

$\therefore$ Forbenius norm $\|A\|_F = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n} A_{ij}^2}$

$= \sqrt{\text{trace}(A^T A)}$

(b) Given $\frac{\partial AB}{\partial x} = \frac{\partial A}{\partial x} B + A \frac{\partial B}{\partial x}$

Let $X = A^{-1} A A^{-1}$

$\therefore \frac{\partial X}{\partial x} = \frac{\partial}{\partial x}(A^{-1} A A^{-1}) = (AA^{-1})\left(\frac{\partial A^{-1}}{\partial x}\right) + A^{-1} \frac{\partial}{\partial x}(AA^{-1})$

$= \left(\frac{\partial A^{-1}}{\partial x}\right) + A^{-1} \frac{\partial}{\partial x}(AA^{-1})$ $[\text{As } AA^{-1} = I]$

$= \frac{\partial A^{-1}}{\partial x} + A^{-1}\left[\frac{\partial A}{\partial x} A^{-1} + A \frac{\partial A^{-1}}{\partial x}\right]$

$= \frac{\partial A^{-1}}{\partial x} + A^{-1} \frac{\partial A}{\partial x} A^{-1} + A^{-1} A \frac{\partial A^{-1}}{\partial x}$

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$$\Rightarrow \frac{\partial X}{\partial x} = \frac{\partial (A^{-1} A A^{-1})}{\partial x} = \frac{\partial A^{-1}}{\partial x} + A^{-1}\frac{\partial A}{\partial x}A^{-1} + \frac{\partial A^{-1}}{\partial x}$$

$$\Rightarrow \cancel{\frac{\partial A^{-1}}{\partial x}} = \cancel{\frac{\partial A^{-1}}{\partial x}} + A^{-1}\frac{\partial A}{\partial x}A^{-1} + \frac{\partial A^{-1}}{\partial x}$$

$$\Rightarrow \frac{\partial A^{-1}}{\partial x} = -A^{-1}\left(\frac{\partial A}{\partial x}\right)A^{-1}$$

Hence proved.