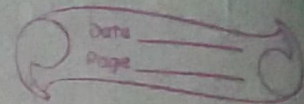


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Deep learning test (class test 1)

1. (2) Let $A \in \mathbb{R}^{n \times n}$

then $A^T \in \mathbb{R}^{n \times n}$ where $A^T = \text{transpose of } A$

$$\therefore \sum_{i=1}^n \sum_{j=1}^n (A^T)_{ij} = A_{ji}$$

$$\therefore \sum_{j=1}^n \sum_{i=1}^n (A^T A)_{ij} = \sum_{k=1}^n (A^T)_{ik} (A)_{kj}$$

$$\Rightarrow \sum_{i=1}^n (A^T A)_{ii} = \sum_{k=1}^n (A^T)_{ik} (A)_{ki}$$

$$\Rightarrow \sum_{i=1}^n \sum_{j=1}^n (A^T A)_{ii} = \sum_{k=1}^n (A_{ki}) (A)_{ki} = \sum_{k=1}^n (A_{ki})^2$$

$$\Rightarrow \sum_{i=1}^n (A^T A)_{ii} = \sum_{k=1}^n (A_{ki})^2$$

$$\Rightarrow \sum_{i=1}^n (A^T A)_{ii} = \sum_{i=1}^n \sum_{k=1}^n (A_{ki})^2$$

$$\Rightarrow \text{trace}(A^T A) = \sum_{i=1}^n \sum_{j=1}^n (A_{ji})^2 \quad (\text{Put } j=k)$$

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$$\rightarrow \text{trace}(A^T A) = \sum_{i=1}^n \sum_{j=1}^n (A_{ij})^2 \quad (\text{Replacing } i, j)$$

$$\therefore \text{Forbenius norm } \|A\|_F = \sqrt{\sum_{i=1}^n \sum_{j=1}^n A_{ij}^2}$$

$$= \sqrt{\text{trace}(A^T A)}$$

(b) Given $\frac{\partial AB}{\partial x} = \frac{\partial A}{\partial x} B + A \frac{\partial B}{\partial x}$

Sol- $X = A^{-1} A A^{-1}$

$$\therefore \frac{\partial X}{\partial x} = \frac{\partial (A^{-1} A A^{-1})}{\partial x} = (A A^{-1}) \left(\frac{\partial A^{-1}}{\partial x} \right) + A^{-1} \frac{\partial (A A^{-1})}{\partial x}$$

$$= \left(\frac{\partial A^{-1}}{\partial x} \right) + A^{-1} \frac{\partial (A A^{-1})}{\partial x} \quad [A A^{-1} = I]$$

$$= \frac{\partial A^{-1}}{\partial x} + A^{-1} \left[\frac{\partial A A^{-1}}{\partial x} + A \frac{\partial A^{-1}}{\partial x} \right]$$

$$= \frac{\partial A^{-1}}{\partial x} + A^{-1} \frac{\partial A A^{-1}}{\partial x} + A^{-1} A \frac{\partial A^{-1}}{\partial x}$$

$$\Rightarrow \frac{\partial X}{\partial x} = \frac{\partial (A^{-1}AA^{-1})}{\partial x} = \frac{\partial A^{-1}}{\partial x} + A^{-1} \frac{\partial A}{\partial x} A^{-1} + \frac{\partial A^{-1}}{\partial x}$$

$$\Rightarrow \cancel{\frac{\partial A^{-1}}{\partial x}} = \cancel{\frac{\partial A^{-1}}{\partial x}} + A^{-1} \frac{\partial A}{\partial x} A^{-1} + \cancel{\frac{\partial A^{-1}}{\partial x}}$$

$$\Rightarrow \frac{\partial A^{-1}}{\partial x} = -A^{-1} \left(\frac{\partial A}{\partial x} \right) A^{-1}$$

hence proved