Indian Institute of Technology Kharagpur Class Test I: 2020-21

Date: 19 Jan. 2021 Subject No.: CS60010 Subject: Deep Learning

2. (a) (3 points) Suppose we have a cost function

$$J(\boldsymbol{\theta}) = \frac{1}{N} \sum_{i=1}^{N} \boldsymbol{\theta}^{T} \mathbf{x}^{(i)} + b \mathbf{y}^{(i)} + \frac{1}{2} \boldsymbol{\theta}^{T} \mathbf{A} \boldsymbol{\theta}$$

where $\boldsymbol{\theta} \in \mathbb{R}^d$ is the parameter vector $\mathbf{x}^{(i)} \in \mathbb{R}^d$, $y^{(i)} \in \mathbb{R}$, $\{\mathbf{x}^{(i)}, y^{(i)}\}$ are N training data points, $\mathbf{A} \in \mathbb{R}^{d \times d}$ is a symmetric matrix and $b \in \mathbb{R}$. We want to find parameters $\boldsymbol{\theta}$ using gradient descent. Find the vector of partial gradients of the cost function.

- (b) (1 point) Give the closed-form solution of θ from the above expression you found.
- (c) (4 points) Let λ and \mathbf{x} are respectively the eigenvalue and eigenvector of a square matrix \mathbf{A} . Prove that \mathbf{x} is also an eigenvector of \mathbf{A}^k where k is a positive integer. Also prove that λ^k is the eigenvalue of \mathbf{A}^k .