

2. Give

$$J(\theta) = \frac{1}{N} \sum_{i=1}^N (\theta^T x^{(i)} + b y^{(i)}) + \frac{1}{2} \theta^T A \theta$$

$$(a) \quad \frac{\partial J(\theta)}{\partial \theta} = \frac{1}{N} \sum_{i=1}^N \frac{\partial}{\partial \theta} (\theta^T x^{(i)} + b y^{(i)}) + \frac{1}{2} \frac{\partial}{\partial \theta} (\theta^T A \theta)$$

$$= \frac{1}{N} \sum_{i=1}^N (x^{(i)}) + \frac{1}{2} (2A)$$

$$\boxed{\frac{\partial J(\theta)}{\partial \theta} = \frac{1}{N} \sum_{i=1}^N x^{(i)} + A}$$

(b) Closed form

$$\frac{\partial J(\theta)}{\partial \theta}$$

(c) Given $Ax = \lambda x$

($\lambda = \text{scalar}$)

Multiply A ,

$$A^2 x = A \lambda x = \lambda A x = \lambda^2 x$$

Multiply A ,

$$A^3 x = A \lambda^2 x = \lambda^2 A x \\ = \lambda^3 x$$

\vdots
 k times

$$A^k x = \lambda^k x$$

$\therefore x$ is eigen vector for A^k

Also

$$A^k x = \lambda^k x$$

where

λ^k is eigen value

for eigen vector x

for A^k