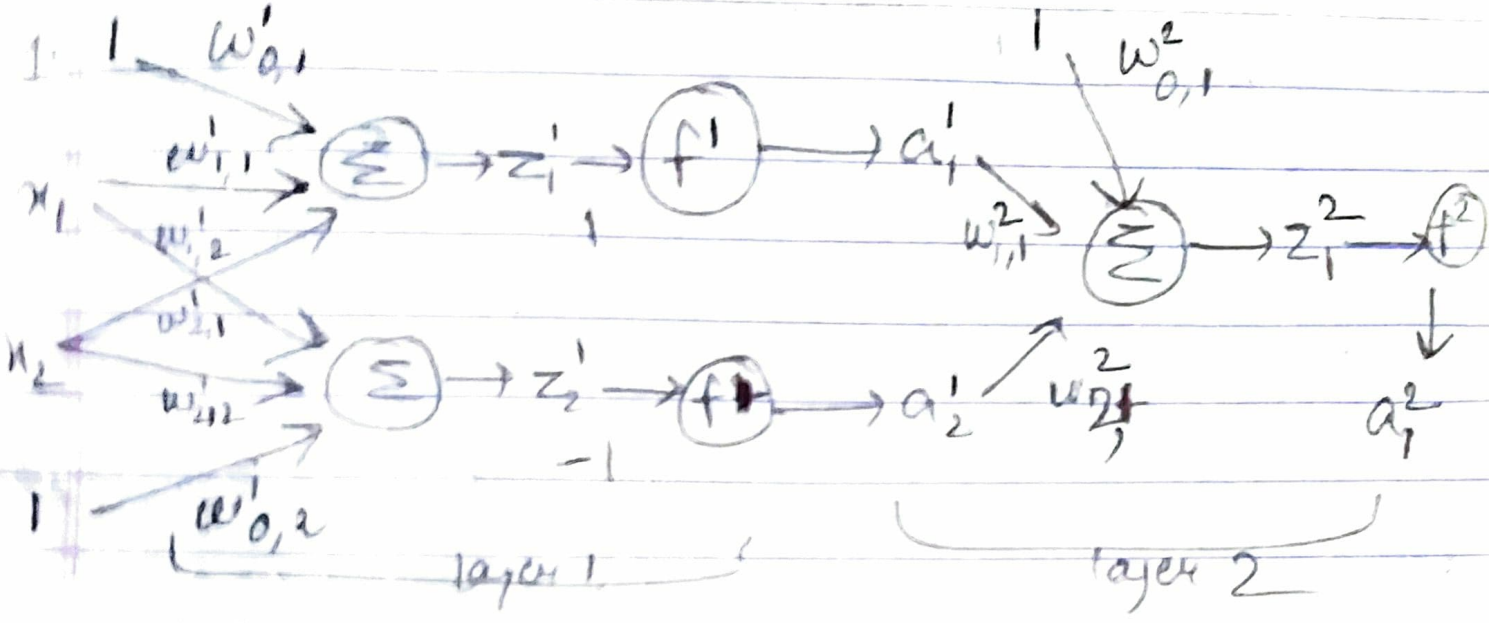
IRCS10062

9/2/2021

[CS60010] Class Test 2 → PART B



$$x^{(1)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad y^{(1)} = 0$$

$x_1 = 2, \quad x_2 = 1$

(1)

In layer 1,

$$z^1_1 = w^1_{0,1} + (w^1_{1,1})(x_1) + (w^1_{2,1})(x_2)$$

$$= 0 + (1)(2) + (-1)(1)$$

$$= 0 + 2 + (-1) = 1$$

$$z^1_2 = w^1_{0,2} + (w^1_{1,2})(x_1) + (w^1_{2,2})(x_2)$$

$$= 0 + (-1)(2) + (1)(1)$$

$$= 0 + (-2) + 1 = -1$$

a^1_1 ~~$f^1(z^1_1) = \max(0, z^1_1)$~~

$$a_1^1 = f'(z_1^1) = f'(1) = \max(0,1) = \underline{1}$$

(f' is Relu function)

$$a_2^1 = f'(z_2^1) = f'(-1) = \max(0,-1) = \underline{0}$$

In layer 2,

$$z_1^2 = w_{0,1}^2 + (a_1^1)(w_{1,1}^2) + (a_2^1)(w_{2,1}^2)$$

$$= 0 + (1)(2) + (0)(2)$$

$$= 0 + 2 + 0 = \underline{2}$$

$$a_1^2 = f^2(z_1^2) = f^2(2) = \frac{1}{1+e^{-2}}$$

$$= 0.880797$$

$$= \underline{0.881}$$

$$y^{(1)} = a_1^2 = \underline{0.881}$$

(b) Cross entropy loss $= -\log(y^{(1)})$

$$= -\log_e(0.881)$$

$$= -0.055124$$

$$= \underline{-0.055}$$

(c) 

$$L = -\log(a_1^2)$$

$$\frac{\partial L}{\partial a_1^2} = -\frac{1}{a_1^2}$$

$$a_1^2 = f^2(z_1^2) = \frac{1}{1+e^{-z_1^2}}$$

$$\frac{\partial a_1^2}{\partial z_1^2} = f^2(z_1^2)\left(1 - f^2(z_1^2)\right)$$

$$\frac{\partial L}{\partial z_1^2} = \frac{\partial a_1^2}{\partial z_1^2} \times \frac{\partial L}{\partial a_1^2} = -\frac{f^2(z_1^2)\left(1-f^2(z_1^2)\right)}{a_1^2}$$

$$z^2_{1\times 1} = \underset{1\times 2}{(w^2)^t}\ \underset{2\times 1}{a^1} + \underset{2\times 1}{w_0^2} \quad ; \quad \frac{\partial L}{\partial z^2} = \frac{\partial L}{\partial z_1^2}_{1\times 1}$$

$$\Rightarrow z_1^2 = \begin{bmatrix} w_{1,1}^2 & w_{2,1}^2 \end{bmatrix}\begin{bmatrix} a_1^1 \\ a_2^1 \end{bmatrix} + \begin{bmatrix} w_{0,1}^2 \\ w_{2,1}^2 \end{bmatrix} \qquad = -0.119$$

$$\frac{\partial z^2}{\partial a^1} = \underset{2\times 1}{w^2} \quad , \quad \frac{\partial z^2}{\partial w^2} = \underset{2\times 1}{a^1}$$

$$\therefore \frac{\partial L}{\partial w^2} = \underset{2\times 1}{\left(\frac{\partial z^2}{\partial w^2}\right)}\underset{1\times 1}{\left(\frac{\partial L}{\partial z^2}\right)} = (a^1)\left(\frac{\partial L}{\partial z_1^2}\right)$$

$$= \begin{bmatrix} a_1^1 \\ a_2^1 \end{bmatrix}\left[-\frac{f^2(z_1^2)\left(1-f^2(z_1^2)\right)}{a_1^2}\right]$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}\left[-\frac{(0.881)(1-0.881)}{0.881}\right]$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}[-0.119] = \begin{bmatrix} -0.119 \\ 0 \end{bmatrix}$$

$$\therefore \frac{\partial L}{\partial w^2} = \begin{bmatrix} \frac{\partial L}{\partial w^2_{1,1}} \\ \frac{\partial L}{\partial w^2_{2,1}} \end{bmatrix} = \begin{bmatrix} -0.119 \\ 0 \end{bmatrix}$$

$$\frac{\partial L}{\partial a^1} = \frac{\partial z^2}{\partial a^1} \cdot \frac{\partial L}{\partial z^2} = \underset{2\times 1}{(w^2)} \underset{1\times 1}{\left(\frac{\partial L}{\partial z^2}\right)}$$

$$= \begin{bmatrix} 2 \\ 2 \end{bmatrix} \begin{bmatrix} -0.119 \end{bmatrix}$$

$$= \begin{bmatrix} -0.238 \\ -0.238 \end{bmatrix}$$

$$\frac{\partial a^1}{\partial z^1} = \frac{\partial f^1(z^1)}{\partial z^1} = \begin{cases} 1 & z^1 \geq 0 \\ 0 & z < 0 \end{cases}$$

$$\frac{\partial a^1}{\partial z^1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}_{2\times 1}, \quad \frac{\partial L}{\partial a^1} = \begin{bmatrix} -0.238 \\ -0.238 \end{bmatrix}_{2\times 1}$$

$$\frac{\partial L}{\partial z^1} = \underset{2\times 1}{\left(\frac{\partial a^1}{\partial z^1}\right)} \underset{1\times 2}{\left(\frac{\partial L}{\partial a^1}\right)} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} -0.238 & -0.238 \end{bmatrix}$$

$$= \begin{bmatrix} -0.238 \\ 0 \end{bmatrix}$$

elementwise

$$z^1 = w^1 a^0 + w_0^1$$

$$\frac{\partial z^1}{\partial w^1} = a^0 \quad 2\times 1, \qquad \frac{\partial z^1}{\partial a^0} = w^1$$

$$\frac{\partial L}{\partial w^1} = \underset{2\times 1}{\frac{\partial z^1}{\partial w^1}} \underset{2\times 1}{\left(\frac{\partial L}{\partial z^1}\right)^t} = (a^0)\begin{bmatrix} -0.238 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \end{bmatrix}\begin{bmatrix} -0.238 \\ 0 \end{bmatrix}^t$$

$$= \begin{bmatrix} -0.476 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial L}{w_0^1} = \frac{\partial z^1}{\partial w_0^1}\frac{\partial L}{\partial z^1} = \begin{bmatrix} -0.238 \\ 0 \end{bmatrix}$$

$$\frac{\partial L}{w_0^2} = \frac{\partial z^2}{\partial w_0^2}\frac{\partial L}{\partial z^2} = [-0.119]$$

$$\frac{\partial L}{\partial w_{1,2}^1}$$