LINEAR ALGEBRA (MA20105)

Problems Sheet-2:

Notation: $\mathbb{R}$ will always denote the field real numbers and $\mathbb{R}^n := \mathbb{R} \times \cdots \times \mathbb{R}$ (n times).

Problems related to L.I., L.D., Bases and Dimension:

1. Prove that if two vectors are linearly dependent, then one of them is scalar multiple of the other.

2. Find three vectors in $\mathbb{R}^3$ which are linealy dependent, and are such that any two of them are linearly independent.

3. Let V be the vector space of all 2 matrices over the field $\mathbb{R}$. Let W_1 be the set of matrices of the form
$$\begin{bmatrix} a & -a \\ c & d, \end{bmatrix}$$
and let W_2 be the set of matrices of the form
$$\begin{bmatrix} a & b \\ -a & d \end{bmatrix}.$$
(i) Prove that W_1 and W_2 are subspaces of V.

(ii) Find the dimensions of W_1, W_2 $W_1 + W_2$, and $W_1 \cap W_2$.

4. Show that the vectors $\alpha_1 = (1, 0, -1)$, $\alpha_2 = (1, 2, 1)$ and $\alpha_3 = (0, -3, 2)$ form a basis for $\mathbb{R}^3$. Express each of the standdard basis vectors as linear combinations of α_1, α_2 and α_3.