LINEAR ALGEBRA (MA20105)

Problems Sheet:

Notation: $\mathbb{F} = (\mathbb{R}, \mathbb{C})$ will always denote a field and $\mathbb{F}^n := \mathbb{F} \times \cdots \times \mathbb{F}$ (n times).

$\mathbb{R}$ will denote the field of real numbers

$\mathbb{C}$ will denote the field of complex numbers

Eigen values, Eigen vectors, characteristic polynomial, diagonalization

1. Find the eigen values and eigen vectors of A thinking A as a matrix from $\mathbb{R}^2 \to \mathbb{R}^2$, also calculate the same while thinking them as a map from $\mathbb{C}^2 \to \mathbb{C}^2$.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, A = \begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix}, A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}.$$

2. Let $\mathbb{F}^n$ be a n-dimensional vector space over the field F. What is the characteristic polynomial for Identity map and zero map on $\mathbb{F}^n$.

3. Let A be an $n \times n$ triangular matrix over the field $\mathbb{F}$. Prove that the eigen values of A are the diagonal entries of $\mathbb{F}$.

4. Let T be a linear map on $\mathbb{R}^3$ which is represented in the standard ordered basis by the matrix $A =$

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}.$$

   Prove that A is diagonalizable by exhibiting a basis for $\mathbb{R}^3$, each of vector of which is a eigen vector of A.

5. Let $A =$

$$\begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}.$$

   Is A similar over $\mathbb{R}$ to a diagonal matrix? Is A similar over $\mathbb{C}$ to a diagonal matrix?