LINEAR ALGEBRA (MA20105)

Problems Sheet-3:

Notation: $\mathbb{R}$ will always denote the field of real numbers and $\mathbb{R}^n := \mathbb{R} \times \cdots \times \mathbb{R}$ (n times).

Problems related to Linear maps/transformations:

1. Which of the following maps/functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ are linear map?

   (i) $T(x_1, x_2) = (1 + x_1, x_2)$;

   (ii) $T(x_1, x_2) = (x_2, x_1)$;

   (iii) $T(x_1, x_2) = (x_1^2, x_2)$;

   (iv) $T(x_1, x_2) = (sinx_1, x_2)$;

   (v) $T(x_1, x_2) = (x_1 - x_2, 0)$.

2. Find the range, rank, null space and nullity for the zero map and the identity map on a finite dimensional vector space V.

3. Describe the range and the null space for the differentiation map which is discussed in the class.

4. Describe explicitly (as in Exercise 1.) the linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that $T(e_1) = (a, b)$ and $T(e_2) = (c, d)$.

5. Let V be the vector space of all $n \times n$ matrices over the field F, and let B be a fixed $n \times n$ matrix. If $T(A) = AB - BA$, then check that T is a linear map from V into V.