**LINEAR ALGEBRA (MA20105)**

Problems Sheet-4:

Notation: $\mathbb{F} = (\mathbb{R}, \mathbb{C})$ will always denote a field and $\mathbb{F}^n := \mathbb{F} \times \cdots \times \mathbb{F}$ ($n$ times).

$\mathbb{R}$ will denote the field of real numbers

$\mathbb{C}$ will denote the field of complex numbers

**Problems related to Systems of Linear Equations:**

1. Are the following two system equations equivalent? If so, express each equation in each system as a linear combination of the equation in the other system.
   System - I:
   $x_1 - x_2 = 0$
   $2x_1 + x_2 = 0$;
   System -II:
   $3x_1 + x_2 = 0$
   $x_1 + x_2 = 0$.

2. If $A =$
$$\begin{bmatrix} 3 & -1 & 2 \\ 2 & 1 & 1 \\ 1 & -3 & 0 \end{bmatrix},$$
   find all solutions $AX = 0$ by row-reducing $A$.

3. If $A =$
$$\begin{bmatrix} 6 & -4 & 0 \\ 4 & -2 & 0 \\ -1 & 0 & 3 \end{bmatrix},$$
   find all solutions of $AX = 2X$ and all solutions of $AX = 3X$.

4. Prove that the following two matrices are not row-equivalent:
$$\begin{bmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{bmatrix}.$$

5. Give an example of a system of two linear equations in two unknown which has no solution.

6. Find all solutions of
   $2x_1 - 3x_2 - 7x_3 + 5x_4 + 2x_5 = -2$
   $x_1 - 2x_2 - 4x_3 + 3x_2 + x_5 = -2$
   $2x_1 \quad - 4x_3 + 2x_4 + x_5 = 3$
   $x_1 - 5x_2 - 7x_3 + 6x_4 + 2x_5 = -7$.

7. Let $A$ be as in Problem (2.). For which triples $(y_1, y_2, y_3)$ does the system $AX = Y$ have a solution?

8. Find two different $2 \times 2$ matrices $A$ such that $A^2 = 0$ but $A \neq 0$.

9. Let $A =$
$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}.$$
Find elementary matrices $E_1, E_2, \ldots, E_k$ such that $E_k \ldots E_1 A = I$.