

Linear Algebra (MA20105)
Assignment-1
Instructor: Dr. H.P. Sarwar
Indian Institute of Technology Kharagpur

1. [2 marks] Consider the set of equations

$$ax + 2y = 0$$

$$2x + ay = 0.$$

These equations are certain to have the solution $x = y = 0$. For which value of a is there a whole line of solutions?

2. [4 marks] Choose a right-hand side which gives no solution and another right-hand side which give infinitely many solutions. What are two of those solutions?

$$3x + 2y = 10$$

$$6x + 4y = - - .$$

3. [5 marks] The matrix that rotates x - y plane by an angle θ is

$$A(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}.$$

Verify that $A(\theta_1)A(\theta_2) = A(\theta_1 + \theta_2)$ from the identities for $\cos(\theta_1 + \theta_2)$ and $\sin(\theta_1 + \theta_2)$. What is $A(\theta)A(-\theta)$?

4. [7 marks] Which three matrices E_{21}, E_{31}, E_{32} put A into an upper triangular form U ?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \text{ and } E_{21}E_{31}E_{32}A = U.$$

Multiply those E 's to get onematrix M that does elimination: $MA = U$.

5. [2 marks] On \mathbb{R}^2 , define

$$(x, y) + (x_1, y_1) = (x + x_1, 0)$$

$$c(x, y) = (cx, 0).$$

Is \mathbb{R}^2 , with these operations, a vector space over \mathbb{R} ? Provide reasons.