LINEAR ALGEBRA (MA20105)

Problems Sheet-12:

Notation: $\mathbb{F}$ will always denote a field and $\mathbb{F}^n := \mathbb{F} \times \cdots \times \mathbb{F}$ (n times).

$\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}$ will denote the field of reals, complexes, and rationals, respectively

Normal matrix related problems:

A matrix M is called self adjoint if $M = M^H$.

1. For each of the real symmetric matrices A, find a real orthogonal matrix P such that P^tAP is diagonal.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

2. Is a complex symmetric matrix self-adjoint? Is it normal?

3. For

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix},$$

there is a real orthogonal matrix P such that $P^tAP = D$ is diagonal. Find such a diagonal matrix D.

4. Let $V = \mathbb{C}^2$, with the standard inner product. Let T be the linear map on V which is represented in the standard ordered basis by the matrix

$$A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}.$$

Show that T is normal, and find an orthonormal basis for V, consisting of eigen vectors of T.

5. Give an example of a 2×2 matrix A such that A^2 is normal, but A is not normal.

6. Prove that T is normal if and only if $T = T_1 + iT_2$, where T_1 and T_2 are self-adjoint maps which commute.