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18CS10062

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SHORT TEST

1. Given $\alpha^t\beta = 0$ $\{\alpha$ & β are orthogonal$\}$
$\beta^t\alpha = 0$

$\|\alpha-\beta\|^2 + \|\alpha+\beta\|^2$

$\Rightarrow (\alpha-\beta)^t(\alpha-\beta) + (\alpha+\beta)^t(\alpha+\beta)$

$\Rightarrow (\alpha^t-\beta^t)(\alpha-\beta) + (\alpha^t+\beta^t)(\alpha+\beta)$

$\Rightarrow \alpha^t\alpha - \cancel{\alpha^t\beta} - \cancel{\beta^t\alpha} + \beta\beta^t + \alpha^t\alpha + \cancel{\alpha^t\beta} + \cancel{\beta^t\alpha} + \beta\beta^t$

$\Rightarrow \alpha^t\alpha + \beta\beta^t + \alpha^t\alpha + \beta\beta^t$

$\Rightarrow \|\alpha\|^2 + \|\beta\|^2 + \|\alpha\|^2 + \|\beta\|^2$

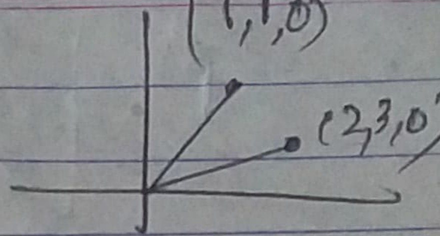
$\Rightarrow 2\|\alpha\|^2 + 2\|\beta\|^2$

$\Rightarrow 2(\|\alpha\|^2 + \|\beta\|^2)$

$\therefore \|\alpha-\beta\|^2 + \|\alpha+\beta\|^2 = 2(\|\alpha\|^2 + \|\beta\|^2)$

Hence verified.

2. $W = L\left\{\begin{pmatrix}1\\1\\0\end{pmatrix}, \begin{pmatrix}2\\3\\0\end{pmatrix}\right\}$



W is a plane in R^2

$$b = (11, 13, 17), \quad A = \overset{3\times 2}{\begin{bmatrix}1 & 2\\1 & 3\\0 & 0\end{bmatrix}}$$

Projection of b on the plane represented by W

$$= \left(\frac{A^t b}{A^t A}\right) A$$

$$= A(A^tA)^{-1}A^tb$$

$$= \underset{3\times 2}{\begin{bmatrix}1 & 2\\1 & 3\\0 & 0\end{bmatrix}} \underset{2\times 2}{\begin{bmatrix}13 & -5\\-5 & 2\end{bmatrix}} \underset{2\times 3}{\begin{bmatrix}1 & 1 & 0\\2 & 3 & 0\end{bmatrix}} \underset{3\times 1}{\begin{bmatrix}11\\13\\17\end{bmatrix}}$$

$$(A^tA) = \underset{2\times 3}{\begin{bmatrix}1 & 1 & 0\\2 & 3 & 0\end{bmatrix}} \underset{3\times 2}{\begin{bmatrix}1 & 2\\1 & 3\\0 & 0\end{bmatrix}} = \begin{bmatrix}1+1 & 2+3\\2+3 & 4+9\end{bmatrix}$$

$$= \begin{bmatrix}2 & 5\\5 & 13\end{bmatrix}$$

$$(A^tA)^{-1} = \frac{1}{26-25}\begin{bmatrix}13 & -5\\-5 & 2\end{bmatrix}^t = \begin{bmatrix}13 & -5\\-5 & 2\end{bmatrix}$$

$\therefore$ Projection (p) $= \underset{3\times 2}{\begin{bmatrix}13-10 & -5+4\\13-15 & -5+6\\0+0 & 0+0\end{bmatrix}} \underset{2\times 1}{\begin{bmatrix}11+13+0\\22+39+0\end{bmatrix}}$

$$\begin{array}{cc} 3\times 2 & 2\times 1 \end{array}$$

$$\therefore \vec{p} = \begin{bmatrix} 3 & -1 \\ -2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 24 \\ 61 \end{bmatrix}$$

5 $\cancel{61}$ 48

$$= \begin{bmatrix} 72 - 61 \\ -48 + 61 \\ 0 + 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix}$$

$\therefore$ Projection point $\vec{p}$ onto W = $\begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix}$

Projection matrix $P = A(A^tA)^{-1}A^t$

$$= \underset{3\times 2}{\begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 0 & 0 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 13 & -5 \\ -5 & 2 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 13-10 & -5+4 \\ 13-15 & -5+6 \\ 0+0 & 0+0 \end{bmatrix} \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix}}$$

3×2

$$= \underset{3\times 2}{\begin{bmatrix} 3 & -1 \\ -2 & 1 \\ 0 & 0 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 1 & 1 & 0 \\ 2 & 3 & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 3-2 & 3-3 & 0 \\ -2+2 & -2+3 & 0 \\ 0+0 & 0+0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Verification

$Qb = p$

$$\underset{3\times3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 11 \\ 13 \\ 17 \end{bmatrix}} = \begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11+0+0 \\ 0+13+0 \\ 0+0+0 \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ 13 \\ 0 \end{bmatrix}$$

Hence verified