LINEAR ALGEBRA (MA20105)

Class Test (2020)- II:

$\mathbb{R}$ will denote the field of real numbers

Answer all question!

1. Consider a subspace $H^* := \{(x, y, z) \in \mathbb{R}^3 : x + 3y + 5z = 0\}$ of $\mathbb{R}^3$. Which of the following is/are basis of H?

   (i) $\{(-9, 3, 0), (-5, 0, 1)\}$ (ii) $\{(-3, 1, 0), (-5, 0, 1)\}$ (iii) $\{(-3/2, 1/2, 0), (-5/4, 0, 1/4)\}$

   (iv) $\{(2, 3, 0), (0, 0, 5)\}$.

   Answer: $(i), (ii)$ and (iii).

2. Which one(s) of the follwing is (are) True?

   (i) If the columns of a matrix are dependent, so are the rows.

   (ii) The columns space of a 2 by 2 matrix is the same as iys row space.

   (iii) The column space of a 2 by 2 matrix has the same dimension as its row space.

   (iv) The columns of a matrix are a basis for the column space.

   Answer: (iii).

3. Which of the follwing is/are True?

   (i) If the columns of a matrix A are linearly independent, then $Ax = b$ has exactly one solution for every b.

   (ii) A 5 by 7 matrix never has linearly independent columns.

   Answer: (ii)

4. Suppose S is a five dimensional subspace of $\mathbb{R}^6$. Which of the following is/are true?

   (i) Every basis for S can be extended to a basis for $\mathbb{R}^6$ by adding one more vector.

   (ii) Every basis for $\mathbb{R}^6$ can be reduced to a basis for S by removing one vector.

   (iii) The orthogonal complement of S exists in $\mathbb{R}^6$.

   (iv) If the orthogonal complement of S exists in $\mathbb{R}^6$, then it is equal to the set theoretic complement of S in $\mathbb{R}^6$.

   Answer: (i) and (iii)

5. Let $Skew_4(\mathbb{R})$ be the space of all 4×4 skew symmetric matrices i.e. $A^t = -A$. Write down the dimension of $Skew_4(\mathbb{R})$.

   Answer: 6

6. Let A be an $n \times n$ real matrix. Then which of the following is/are False?

   (i) A and A^t have the same number of pivots.

   (ii) A and A^t have the same left null space.

   (iii) If the row space equals the column space then $A^t = A$

   (iv) If $A^t = -A$ then the row space of A equals the column space.

   Answer: (ii) and (iii).

7. Let A be a non-zero square matrix such that $A^3 = A$. Then which of the follwoing is/are true?

   (i) A must be idenitity.

   (ii) A^2 must be identity.

   (iii) A is invertible.

   (iv) None of the above.

   Answer: (iv)

8. Let $A = (a_{ij})$ be a 3×3 real matrix. Suppose that $A \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$. Then which one(s) of the following is (are) true?

   (i) $rank(A) = 3$; (ii) $det(A) = 0$; (iii) $rank(A) = 0$; (iv) $rank(A) < 3$.

   Answer: (ii) and (iv).

9. Let A be a 3×3 real matrix with the eigen values $1, -1, 0$. Then write down the $det(I + A^{100})$.

   (i) 1; (ii) -1; (iii) 4; (i) -4.

   Answer: (iii)

10. Let A be a 2×2 orthogonal matrix of trace and determinat one. Then the angle between Au and u ($u = (1, 0)^t$) is $------------$ degree. Fill in the blank.

    $(i) 15^o$; $(ii) 30^o$; $(iii) 45^o$ $(iv) 60^o$

    Answer: (iv)

11. Let V be an n-dimensional real vector space. If $T : V \to \mathbb{R}$ is a non-zero liear map then the dimension of the $Ker(T)$ is

    $(i) n$; $(ii) n - 1$; $(iii) 1$ $(iv) 0$

    Answer (ii)

12 Let P be the plane of vectors in $\mathbb{R}^4$ satisfying $x_1+x_2+x_3+x_4 = 0$. Which one(s) of the following is (are) basis of $P^{\perp}$?

$(i)\{(1,-1,1,-1)\}$; $(ii)\{(-1,1,-1,1)\}$; $(iii)\{(1,1,1,1)\}$ $(iv)\{(2,2,2,2)\}$

Answer: (iii) and (iv)

13 Let M_3 be the vector space of 3×3 matrices and $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1. \end{bmatrix}$.. Let $T : M_3 \to M_3$ be a linear map which is defined as $T(X) = AX - XA$. The dimension of the image of T is

$(i)4$; $(ii)6$; $(iii)8$ $(iv)9$

Answer: 4

14 Let U, V, W be subpaces of $\mathbb{R}^n$. Then which one(s) of the follwing statements is (are) false?

(i) If V is orthogonal to W, then $V^{\perp}$ is orthogonal to $W^{\perp}$.

(ii) U is orthogonal to V and V is orthogonal W makes U is orthogonal W.

(iii) $(V^{\perp})^{\perp} = V$.

(iv) None of the above.

Answer: (i) and (ii), (iv)

17 Let $\{\beta_1, \ldots \beta_r\}$ be an orthonormal set of vectors in $\mathbb{R}^n$ and $\alpha \in \mathbb{R}^n$. If c_i is the scalar component of α along β_i, $i = 1, 2, \ldots, r$. then which one(s) of the following is (are) False?

(i) $||\alpha||^2 = c_1^2 + c_2^2 + \cdots + c_r^2$;

(ii) $||\alpha||^2 > c_1^2 + c_2^2 + \cdots + c_r^2$

(iii) $||\alpha||^2 \geq c_1^2 + c_2^2 + \cdots + c_r^2$

(iv) None of the above.

Answer: $(i), (ii), (iv)$

18 Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map which defined as $T(x, y, z) = (3x + 5y + 2z, 2x + 3z)$. Then the matrix representation of T with respect to the standard bases of $\mathbb{R}^3$ and $\mathbb{R}^2$ is

(i) $\begin{bmatrix} 3 & 5 & 2 \\ 2 & 3 & 0 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 5 & 2 \\ 2 & 0 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} 3 & 2 \\ 5 & 0 \\ 2 & 3 \end{bmatrix}$ (iv) $\begin{bmatrix} 3 & 2 \\ 5 & 3 \\ 2 & 0 \end{bmatrix}$

Answer: (ii)

20 Let A be a 2×2 matrix of rank 1. If A is not diagonalisable, then which one(s) of the following is (are) true?

(i) A is nilpotent

(ii) A is not nilpotent

(iii) the characteristic polynomial of A is linear

(iv) A has a non-zero eigen value.

Answer (i)