

## Some Math Rendering Tests

Not very strenuous, but here we go!

Binomial R.V. ( $n, p$ ):

- $p(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$  where  $k \in [0, n]$

But for now we aren't going to prove anything about these limits.

Let  $X$  be a binomial random value with parameters  $(n, p)$ .

- What is  $E[X]$ ?
  - $E[X] = \sum_{k=0}^n p(X = k)k = \sum_{k=0}^n \binom{n}{k} p^k (1-p)^{n-k} k$  where  $q = 1 - p$ .
  - \*  $\binom{n}{i} = \frac{n \times (n-1) \times \dots \times (n-i+1)}{i \times (i-1) \times \dots \times (1)}$
  - \* Important identity:  $i \binom{n}{i} = n \binom{n-1}{i-1}$
  - Using the identity:

$$E[X] = \sum_{i=0}^n i \binom{n}{i} p^i q^{n-i} = \sum_{i=1}^n n \binom{n-1}{i-1} p^i q^{n-i}$$

- Rewrite as  $E[X] = np \sum_{i=0}^n \binom{n-1}{i-1} p^{i-1} q^{n-i} = np \sum_{i=0}^n \binom{n-1}{i-1} p^{i-1} q^{(n-1)-(i-1)}$
- Substitute  $j = i - 1$  to get:

$$E[X] = np \sum_{j=0}^{n-1} \binom{n-1}{j} p^j q^{(n-1)-j} = np(p+q)^{n-1} = np$$

- \* Remember  $\sum_{j=0}^n \binom{n}{j} p^j q^{n-j} = (p+q)^n = 1^n = 1$
- Alternate solution:
  - \*  $E_i = \{i\text{th coin heads}\}$
  - \*  $E_1, E_2, \dots, E_n$
  - \*  $\forall i. p(E_i) = p$
  - \*  $E[\# \text{ events that happen}] = E[\sum_{i=1}^n 1_{E_i}] = \sum_{i=1}^n E[1_{E_i}] = np$

Let  $X$  be binomial  $(n, p)$  and fix  $k \geq 1$ . What is  $E[X^k]$ ?

- Recall identity:  $i \binom{n}{i} = n \binom{n-1}{i-1}$
- Generally,  $E[X^k]$  can be rewritten as:

$$\sum_{i=0}^n i \binom{n}{i} p^i (1-p)^{n-i} i^{k-1}$$

- Identity gives:

$$E[X^k] = np \sum_{i=1}^n \binom{n-1}{i-1} p^{i-1} (1-p)^{n-1} i^{k-1} =$$

- Variance of binomial is  $npq = np(1-p)$ .