Some Math Rendering Tests

Not very strenuous, but here we go!

Binomial R.V. (n, p):

- p$(X = k) = \binom{n}{k} p^n (1-p)^{n-k}$ where $k \in [0, \infty)$

But for now we aren't going to prove anything about these limits.

Let X be a binomial random value with parameters (n, p).

- What is $E[X]$?
  - $E[X] = \sum_{k=0}^{n} p(X = k)k = \sum_{k=0}^{n} \binom{n}{k} p^k q^{n-k} k$ where $q = 1 - p$.
    - $\binom{n}{i} = \frac{n \times (n-1) \times \ldots \times (n-i+1)}{i \times (i-1) \times \ldots \times (1)}$
    - Important identity: $i\binom{n}{i} = n\binom{n-1}{i-1}$
  - Using the identity:

$$E[X] = \sum_{i=0}^{n} i \binom{n}{i} p^i q^{n-i} = \sum_{i=1}^{n} n \binom{n-1}{i-1} p^i q^{n-i}$$

  - Rewrite as $E[X] = np \sum_{i=0}^{n} \binom{n-1}{i-1} p^{(i-1)} q(n-1) - (i-1)$
  - Substitute $j = i - 1$ to get:

$$E[X] = np \sum_{j=0}^{n-1} \binom{n-1}{j} p^j q^{(n-1)-j} = np(p+q)^{n-1} = np$$

    - Remember $\sum_{j=0}^{n} \binom{n}{j} p^j q^{n-j} = (p+q)^n = 1^n = 1$
  - Alternate solution:
    - $E_i = \{i\text{th coin heads}\}$
    - $E_1, E_2, ..., E_n$
    - $\forall i.p(E_i) = p$
    - $E[\#\text{ events that happen}] = E[\sum_{i=1}^{n} 1_{E_i}] = \sum_{i=1}^{n} E[1_{E_i}] = np$

Let X be binomial (n, p) and fix $k \geq 1$. What is $E[X^k]$?

- Recall identity: $i\binom{n}{i} = n\binom{n-1}{i-1}$
- Generally, $E[X^k]$ can be rewritten as:

$$\sum_{i=0}^{n} i \binom{n}{i} p^i (1-p)^{n-i} i^{k-1}$$

- Identity gives:

$$E[X^k] = np \sum_{i=1}^{n} \binom{n-1}{i-1} p^{i-1} (1-p)^{n-1} i^{k-1} =$$

- Variance of binomial is $npq = np(1-p)$.