Control of Linear Vibrations
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Equations of motion:

$$J\ddot{\theta} = c(t) - c_l(t) - f_m(\dot{\theta})$$
$$M\ddot{x} + C\dot{x} + Kx = F(t) - f_c(\dot{x})$$
$$\frac{D}{2}\theta = x$$

f_m describes the viscous friction of the motor, f_c describes the friction of the cart. The gearbox is assumed ideal.

Therefore $F(t)$ is the transmitted linear force from the motor, thus:

$$F(t)\frac{D}{2} = c_l(t) \Rightarrow F(t) = \frac{2}{D}\Big(c(t) - J\ddot{\theta} - f_m(\dot{\theta})\Big)$$

In the end we obtain:

$$\Big(M + \frac{4}{D^2}J\Big)\ddot{x} + C\dot{x} + Kx = \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x})$$

In case the gearbox is not assumed ideal, we have:

$$J\ddot{\theta} = \begin{cases} c(t) - c_l(t) - f_m(\dot{\theta}) & \text{in contact} \\ c(t) - f_m(\dot{\theta}) & \text{not in contact} \end{cases}$$

And

$$F(t) = \begin{cases} \frac{2}{D}c_l(t) & \text{in contact} \\ 0 & \text{otherwise} \end{cases}$$

3.1 Model1 - no BEMF,no disk inertia, no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D}, \quad \theta = \frac{2}{D}x$$
$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}\mathcal{L}\{v(t)\}$$

3.2 Model2 - no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x}, \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

3.3 Model 3 - no friction motor, backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x}), \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

3.4 Model 4

$$M\ddot{x} + C\dot{x} + Kx = F(t) - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x})$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

See introduction for gearbox modelling.

1. $\mathcal{L}\{\cdot\}$ Laplace transform.
2. J Disk inertia.
3. M Cart+load mass
4. C Spring damping.
5. K Spring stiffness.
6. $c(t)$ Torque.
7. D Disk diameter.
8. $f_c(t)$ friction applied to the cart.
9. $f_g(t)$ sliding friction applied to the teeth between the gearbox and the disk.
10. f_m friction of the motor
11. θ angle of the disk.
12. $v(t)$ tension applied to the motor.
13. R, L resistance and inductance of the motor
14. K_e backemf constant.

3.5 2 DOF - Model

To derive the equations of motion we can use the Lagrangian approach. Let T, V, D be the kinetic, potential and dissipated energy. Then:

$$
\begin{aligned}
T &= \frac{1}{2}\Big(M_1 + \frac{4}{D^2}J\Big)\dot{x_1}^2 + \frac{1}{2}M_2\dot{x_2}^2 \\
V &= \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2 \\
D &= \frac{1}{2}c_1\dot{x_1}^2 + \frac{1}{2}c_2(\dot{x_2} - \dot{x_1})^2
\end{aligned}
$$

Let Q be the external forces acting on the systems:

$$
\begin{aligned}
Q_1 &= \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x_1}) \\
Q_2 &= -f_c(\dot{x_2})
\end{aligned}
$$

The equations of motion are given by:

$$
\frac{d}{dt}\Big(\frac{\partial T}{\partial x_i}\Big) - \frac{\partial T}{\partial \dot{x}_i} + \frac{\partial V}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = Q_i
$$

$$
\begin{aligned}
\Big(M_1 + \frac{4}{D^2}J\Big)\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 &= k_2 x_2 + c_2\dot{x}_2 + \frac{2}{D}(c(t) - f_m(\dot{\theta})) - f_c(\dot{x}_1) \\
M_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 &= k_1 x_1 + c_1\dot{x}_1 - f_c(\dot{x}_2)
\end{aligned}
$$

Chapter 4

System Identification

4.1 Open vs Closed loop identification

In this experiment we had the necessity to choose whether to consider back-emf in the identification process or to completely ignore it.
As a matter of fact, ignoring it would mean to neglect a feedback component. But how much can it affect identification of other parameters?

Consider for example the following 2-nd order system, such as the system considered in the experiment:

$$G(s) = \frac{1}{Ms^2 + cs + k}$$

First consider a feedback loop with a constant ρ on the feedback. Thus the closed loop transfer function is:

$$T(s) = \frac{G(s)}{1 + \rho G(s)} = \frac{1}{Ms^2 + cs + (k + \rho)}$$

The effect of ρ is to change the length of the poles, i.e. their absolute value, since for polynomial with real coefficients the zero-degree coefficient is the product of all roots.
Just compare with $s^2 + 2\xi\omega_0 s + \omega_0^2$, it's easy to see that $\omega_0^2 = \frac{k+\rho}{M}$.

In our case back-emf acts on the velocity of the cart, so if we have a feedback loop on the position, on the feedback we have γs, and the closed loop transfer function is:

$$T(s) = \frac{1}{Ms^2 + cs + k + \gamma s}$$

So what is the effect of γs? Again, if we compare with $s^2 + 2\xi\omega_0 s + \omega_0^2$ we have:

$$c + \gamma = 2\xi\omega_0$$

Where ξ has a strict relationship with the angle formed between the real negative axis and the imaginary axis of a pole, θ :

$$\theta = \arctan\left(\frac{\sqrt{1-\xi^2}}{\xi}\right)$$

So the effect of γs is to rotate the poles, but to which extent is this effect negligible?

From data we are mainly dealing with values of $\xi \in (0, 0.5)$, so we can approximate the value of θ:

$$\theta \approx \arctan\left(\frac{1-\frac{\xi^2}{2}}{\xi}\right) = \arctan\left(\frac{1}{\xi} - \frac{\xi}{2}\right) \approx \arctan\left(\frac{1}{\xi}\right)$$

Notice that in the last step we made use of the fact that $\frac{1}{\xi} \gg \frac{\xi}{2}$. Check figure 4.1 to compare the approximation.
Then, how much does θ change for a small variation of ξ?

$$\frac{d\theta}{d\xi} = \frac{1}{1+\frac{1}{\xi^2}} = 1 - \frac{1}{\xi^2+1}$$

For $\xi < 0.5$ the change is almost linear, as seen from figure 4.1 and from the fact that $\frac{d\theta}{d\xi} \approx 1$ for $\xi < 0.5$.

In our case $\xi = \frac{c+\gamma}{2\omega_0} = \frac{c}{2\omega_0} + \frac{\gamma}{2\omega_0}$, so the contribution of the backemf is $\frac{\gamma}{2\omega_0}$.

From the motor datasheet $\gamma \ll 1$ and from experiments ω_0 is always greater than $10\frac{rad}{sec}$, therefore the contribution is small, less than 1 and since the contribution to θ is linear with proportion ~ 1 also the change in θ is less than 1 degree, therefore backemf can be ignored and open-loop identification can be applied.

4.2 White box identification

4.3 Black box identification

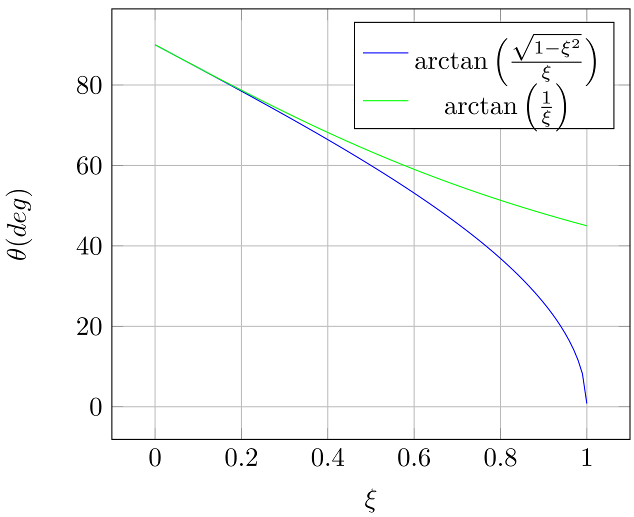

Figure 4.1: Comparison of the approximated value of θ with the real one

Chapter 5

Control of 1 Degree of Freedom

Chapter 6

Control of 2 Degree of Freedom

Chapter 7

Control of 3 Degree of Freedom

Chapter 8

Conclusions

Chapter 9

Appendix