Control of Linear Vibrations

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Academic Year 2015/2016

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Chapter 1

Model

Equations of motion:

$$J\ddot{\theta} = c(t) - c_l(t) - f_m(\dot{\theta})$$
$$M\ddot{x} + C\dot{x} + Kx = F(t) - f_c(\dot{x})$$
$$\frac{D}{2}\theta = x$$

f_m describes the viscous friction of the motor, f_c describes the friction of the cart. The gearbox is assumed ideal.

Therefore $F(t)$ is the transmitted linear force from the motor, thus:

$$F(t)\frac{D}{2} = c_l(t) \Rightarrow F(t) = \frac{2}{D}\Big(c(t) - J\ddot{\theta} - f_m(\dot{\theta})\Big)$$

In the end we obtain:

$$\Big(M + \frac{4}{D^2}J\Big)\ddot{x} + C\dot{x} + Kx = \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x})$$

In case the gearbox is not assumed ideal, we have:

$$J\ddot{\theta} = \begin{cases} c(t) - c_l(t) - f_m(\dot{\theta}) & \text{in contact} \\ c(t) - f_m(\dot{\theta}) & \text{not in contact} \end{cases}$$

And

$$F(t) = \begin{cases} \frac{2}{D}c_l(t) & \text{in contact} \\ 0 & \text{otherwise} \end{cases}$$

1.1 Model1 - no BEMF,no disk inertia, no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D}, \quad \theta = \frac{2}{D}x$$
$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}\mathcal{L}\{v(t)\}$$

1.2 Model2 - no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x}, \quad \theta = \frac{2}{D}x$$
$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

1.3 Model 3 - no friction motor, backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x}), \quad \theta = \frac{2}{D}x$$
$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

1.4 Model 4

$$M\ddot{x} + C\dot{x} + Kx = F(t) - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x})$$
$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

See introduction for gearbox modelling.

1. $\mathcal{L}\{\cdot\}$ Laplace transform.
2. J Disk inertia.
3. M Cart+load mass
4. C Spring damping.
5. K Spring stiffness.
6. $c(t)$ Torque.
7. D Disk diameter.
8. $f_c(t)$ friction applied to the cart.
9. $f_g(t)$ sliding friction applied to the teeth between the gearbox and the disk.
10. f_m friction of the motor
11. θ angle of the disk.
12. $v(t)$ tension applied to the motor.
13. R, L resistance and inductance of the motor
14. K_e backemf constant.

1.5 2 DOF - Model

To derive the equations of motion we can use the Lagrangian approach. Let T, V, D be the kinetic, potential and dissipated energy. Then:

$$T = \frac{1}{2}\Big(M_1 + \frac{4}{D^2}J\Big)\dot{x_1}^2 + \frac{1}{2}M_2\dot{x_2}^2$$
$$V = \frac{1}{2}k_1x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2$$
$$D = \frac{1}{2}c_1\dot{x_1}^2 + \frac{1}{2}c_2(\dot{x_2} - \dot{x_1})^2$$

Let Q be the external forces acting on the systems:

$$Q_1 = \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x_1})$$
$$Q_2 = -f_c(\dot{x_2})$$

The equations of motion are given by:

$$\frac{d}{dt}\Big(\frac{\partial T}{\partial x_i}\Big) - \frac{\partial T}{\partial \dot{x}_i} + \frac{\partial V}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = Q_i$$

$$\Big(M_1 + \frac{4}{D^2}J\Big)\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 = k_2x_2 + c_2\dot{x}_2 + \frac{2}{D}(c(t) - f_m(\dot{\theta})) - f_c(\dot{x}_1)$$
$$M_2\ddot{x}_2 + c_2\dot{x}_2 + k_2x_2 = k_1x_1 + c_1\dot{x}_1 - f_c(\dot{x}_2)$$