Control of Linear Vibrations

Automation and Control Laboratory

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Contents

Chapter 1

Model

1.1 Model1 - no BEMF,no disk inertia, no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D}, \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e \frac{1}{2R + 2sL}\mathcal{L}\{v(t)\}$$

1.2 Model2 - no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 2\frac{J}{D}\ddot{x}, \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e \frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

1.3 Model 3 - no friction motor, backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 2\frac{J}{D}\ddot{x} - f_c(\dot{x}), \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e \frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

1.4 Model 4

$$M\ddot{x} + C\dot{x} + Kx = F(t) - f_c(\dot{x})$$

$$\mathcal{L}\{c(t)\} = 2K_e \frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

$$\ddot{\theta} = \begin{cases} \frac{2\ddot{x}}{D} - f_m(\dot{\theta}) - f_g(\dot{x}), & \text{gearbox in contact} \\ -f_m(\dot{\theta}), & \text{otherwise} \end{cases}$$

$$F(t) = \frac{2}{D}(c(t) - J\ddot{x}) \quad \text{if in contact, otherwise 0}$$

1. $\mathcal{L}\{\cdot\}$ Laplace transform.
2. J Disk inertia.
3. M Cart+load mass
4. C Spring damping.
5. K Spring stiffness.
6. $c(t)$ Torque.
7. D Disk diameter.
8. $f_c(t)$ friction applied to the cart.
9. $f_g(t)$ sliding friction applied to the teeth between the gearbox and the disk.
10. f_m friction of the motor
11. θ angle of the disk.
12. $v(t)$ tension applied to the motor.
13. R, L resistance and inductance of the motor
14. K_e backemf constant.

1.5 Encoder model

$$x = R\phi$$

$$y = \phi$$