Control of Linear Vibrations

Automation and Control Laboratory

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Contents

Chapter 1

Team introduction

The team is composed by 3 people, all holding a B.Sci. in Engineering not obtained at Politecnico di Milano.

1. *Alessio Russo*: holds a B.Sci. degree in Computer Engineering, enrolled at the M.Sci. degree Automation and Control Engineering at Politecnico di Milano. Because of his interest in mathematics he prefers to deals with problems using precise and defined solutions. Currently he's also an ASP student, and his thesis will focus on the implementation of adaptive and robust controllers for the control of unmodelled dynamics of quadrotors, with the use of neural networks and L1 adaptive control techniques.

2. *Gianluca Savaia*:

3. *Alberto Ficicchia*:

Chapter 2

Experience introduction

Chapter 3

Models

Equations of motion:

$$
\begin{gathered}
J\ddot{\theta} = c(t) - c_l(t) - f_m(\dot{\theta}) \\
M\ddot{x} + C\dot{x} + Kx = F(t) - f_c(\dot{x}) \\
\frac{D}{2}\theta = x
\end{gathered}
$$

f_m describes the viscous friction of the motor, f_c describes the friction of the cart. The gearbox is assumed ideal.

Therefore $F(t)$ is the transmitted linear force from the motor, thus:

$$
F(t)\frac{D}{2} = c_l(t) \Rightarrow F(t) = \frac{2}{D}\Big(c(t) - J\ddot{\theta} - f_m(\dot{\theta})\Big)
$$

In the end we obtain:

$$
\Big(M + \frac{4}{D^2}J\Big)\ddot{x} + C\dot{x} + Kx = \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x})
$$

In case the gearbox is not assumed ideal, we have:

$$
J\ddot{\theta} = \begin{cases} c(t) - c_l(t) - f_m(\dot{\theta}) & \text{in contact} \\ c(t) - f_m(\dot{\theta}) & \text{not in contact} \end{cases}
$$

And

$$
F(t) = \begin{cases} \frac{2}{D}c_l(t) & \text{in contact} \\ 0 & \text{otherwise} \end{cases}
$$

3.1 Model1 - no BEMF,no disk inertia, no friction cart, no friction motor, no backlash

$$
M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D}, \quad \theta = \frac{2}{D}x
$$

$$
\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}\mathcal{L}\{v(t)\}
$$

3.2 Model2 - no friction cart, no friction motor, no backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x}, \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

3.3 Model 3 - no friction motor, backlash

$$M\ddot{x} + C\dot{x} + Kx = 2\frac{c(t)}{D} - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x}), \quad \theta = \frac{2}{D}x$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

3.4 Model 4

$$M\ddot{x} + C\dot{x} + Kx = F(t) - 4\frac{J}{D^2}\ddot{x} - f_c(\dot{x})$$

$$\mathcal{L}\{c(t)\} = 2K_e\frac{1}{2R + 2sL}(\mathcal{L}\{v(t)\} - 2K_e s\mathcal{L}\{\theta\})$$

See introduction for gearbox modelling.

1. $\mathcal{L}\{\cdot\}$ Laplace transform.
2. J Disk inertia.
3. M Cart+load mass
4. C Spring damping.
5. K Spring stiffness.
6. $c(t)$ Torque.
7. D Disk diameter.
8. $f_c(t)$ friction applied to the cart.
9. $f_g(t)$ sliding friction applied to the teeth between the gearbox and the disk.
10. f_m friction of the motor
11. θ angle of the disk.
12. $v(t)$ tension applied to the motor.
13. R, L resistance and inductance of the motor
14. K_e backemf constant.

3.5 2 DOF - Model

To derive the equations of motion we can use the Lagrangian approach. Let T, V, D be the kinetic, potential and dissipated energy. Then:

$$T = \frac{1}{2}\Big(M_1 + \frac{4}{D^2}J\Big)\dot{x_1}^2 + \frac{1}{2}M_2\dot{x_2}^2$$
$$V = \frac{1}{2}k_1 x_1^2 + \frac{1}{2}k_2(x_2 - x_1)^2$$
$$D = \frac{1}{2}c_1\dot{x_1}^2 + \frac{1}{2}c_2(\dot{x_2} - \dot{x_1})^2$$

Let Q be the external forces acting on the systems:

$$Q_1 = \frac{2}{D}c(t) - \frac{2}{D}f_m(\dot{\theta}) - f_c(\dot{x_1})$$
$$Q_2 = -f_c(\dot{x_2})$$

The equations of motion are given by:

$$\frac{d}{dt}\Big(\frac{\partial T}{\partial x_i}\Big) - \frac{\partial T}{\partial \dot{x}_i} + \frac{\partial V}{\partial x_i} + \frac{\partial D}{\partial \dot{x}_i} = Q_i$$

$$\Big(M_1 + \frac{4}{D^2}J\Big)\ddot{x}_1 + (c_1 + c_2)\dot{x}_1 + (k_1 + k_2)x_1 = k_2 x_2 + c_2\dot{x}_2 + \frac{2}{D}(c(t) - f_m(\dot{\theta})) - f_c(\dot{x}_1)$$
$$M_2\ddot{x}_2 + c_2\dot{x}_2 + k_2 x_2 = k_1 x_1 + c_1\dot{x}_1 - f_c(\dot{x}_2)$$

Chapter 4

System Identification

4.1 Open vs Closed loop identification

In this experiment we had the necessity to choose whether to consider back-emf in the identification process or to completely ignore it.
As a matter of fact, ignoring it would mean to neglect a feedback component. But how much can it affect identification of other parameters?

Consider for example the following 2-nd order system, such as the system considered in the experiment:

$$G(s) = \frac{1}{Ms^2 + cs + k}$$

First consider a feedback loop with a constant gain ρ on the feedback. Thus the closed loop transfer function is:

$$T(s) = \frac{G(s)}{1 + \rho G(s)} = \frac{1}{Ms^2 + cs + (k + \rho)}$$

The effect of ρ is to change the length of the poles, i.e. their absolute value, since for polynomial with real coefficients the zero-degree coefficient is the product of all roots.
Just compare with $s^2 + 2\xi\omega_0 s + \omega_0^2$, it's easy to see that $\omega_0^2 = \frac{k+\rho}{M}$.

In our case back-emf acts on the velocity of the cart, so if we have a feedback loop on the position, on the feedback we have γs, and the closed loop transfer function is:

$$T(s) = \frac{1}{Ms^2 + cs + k + \gamma s}$$

So what is the effect of γs? Again, if we compare with $s^2 + 2\xi\omega_0 s + \omega_0^2$ we have:

$$c + \gamma = 2\xi\omega_0$$

Where ξ has a strict relationship with the angle formed between the real negative axis and a pole, θ :

$$\theta = \arctan\left(\frac{\sqrt{1 - \xi^2}}{\xi}\right)$$

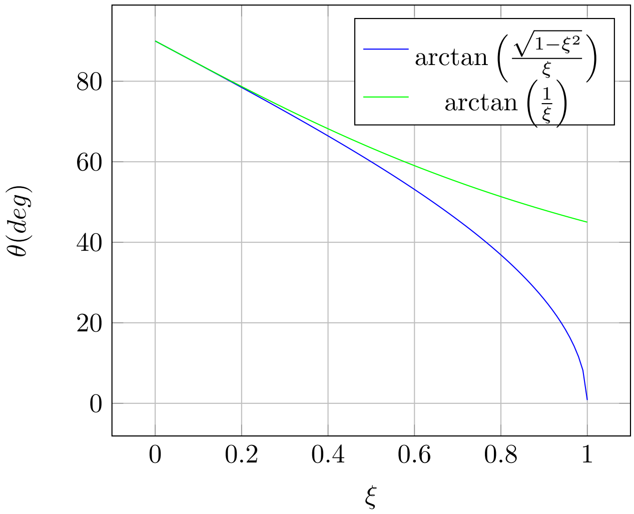

Figure 4.1: Comparison of the approximated value of θ with the real one

So the effect of γs is to rotate the poles, but to which extent is this effect negligible?

From data we are mainly dealing with values of $\xi \in (0, 0.5)$, so we can approximate the value of θ:

$$\theta \approx \arctan\left(\frac{1-\frac{\xi^2}{2}}{\xi}\right) = \arctan\left(\frac{1}{\xi} - \frac{\xi}{2}\right) \approx \arctan\left(\frac{1}{\xi}\right)$$

Notice that in the last step we made use of the fact that $\frac{1}{\xi} \gg \frac{\xi}{2}$. Check figure 4.1 to compare the approximation.
Then, how much does θ change for a small variation of ξ?

$$\frac{d\theta}{d\xi} = -\frac{1}{1+\xi^2} = -1 + \frac{\xi^2}{1+\xi^2}$$

For $\xi < 0.5$ the change is almost linear, as seen from figure 4.1. Moreover $\frac{d\theta}{d\xi} \approx -1$ for $0 < \xi < 0.5$, so the slope of the curve is almost -11.

In our case $\xi = \frac{c+\gamma}{2\omega_0} = \frac{c}{2\omega_0} + \frac{\gamma}{2\omega_0}$, so the contribution of the backemf is $\frac{\gamma}{2\omega_0}$.

From the motor datasheet $\gamma \ll 1$ and from experiments ω_0 is always greater than $10\frac{rad}{sec}$, therefore the contribution is small, less than 1 and since the contribution to θ is linear with proportion ~ -1 also the change in θ is less than 1 degree, therefore backemf can be ignored and open-loop identification can be applied.

4.2 White box identification

4.3 Black box identification

Chapter 5

Control of 1 Degree of Freedom

Chapter 6

Control of 2 Degree of Freedom

Chapter 7

Control of 3 Degree of Freedom

Chapter 8

Conclusions

Chapter 9

Appendix