

Equation considered:

$$M\ddot{x} + C\dot{x} + Kx = F(t) - b\dot{x}$$

Where $b\dot{x}$ represents the Coloumb friction (basic situation), and:

$$b = \mu Mg$$

Let $b = M\hat{b}$, where $\hat{b} = \mu g$.

Move the friction to the left hand side of the equation, divide by K , and take the Laplace Transform of both sides:

$$X(s)(1 + \frac{C+b}{K}s + \frac{M}{K}s^2) = F(s)$$

We know that for a polynomial of the type:

$$\frac{s^2}{\omega_0^2} + \frac{2\xi}{\omega_0}s + 1$$

the period is (if $0 < \xi < 1$):

$$\omega_0\sqrt{1 - \xi^2}$$

Thus:

$$\omega_0 = \sqrt{\frac{K}{M}}$$

and

$$\frac{C+b}{K} = \frac{2\xi}{\sqrt{\frac{K}{M}}}$$

Then, ξ is given by:

$$\begin{aligned} 2\xi &= \frac{C+b}{K} \sqrt{\frac{K}{M}} \\ &= \frac{C + M\hat{b}}{\sqrt{KM}} \\ &= \frac{C}{\sqrt{KM}} + \hat{b}\sqrt{\frac{M}{K}} \end{aligned}$$

So for $M \rightarrow 0$ the damping of the spring "wins". The friction wins for $M \rightarrow \infty$. For $K = 800$ the second term is practically 0. Thus The pulsation is given by:

$$\sqrt{\frac{K}{M}} \sqrt{1 - (\frac{C^2}{4KM})} = \sqrt{\frac{K}{M} - \frac{C^2}{4M^2}}$$

Results: for $M \rightarrow \infty$ we have:

$$\sqrt{\frac{K}{M} - \frac{C^2}{4M^2}} \rightarrow \sqrt{\frac{K}{M}} = \omega_0$$