Dynamics of Mechanical Systems - Project

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February 2, 2015

Contents

0.1 Introduction

0.1.1 Project Description

The aim of this report is to outline the modelling and the analysis of a single span metallic truss bridge.
The figures below show a metallic bridge and its plane model.

Figure 1: Example of a truss bridge

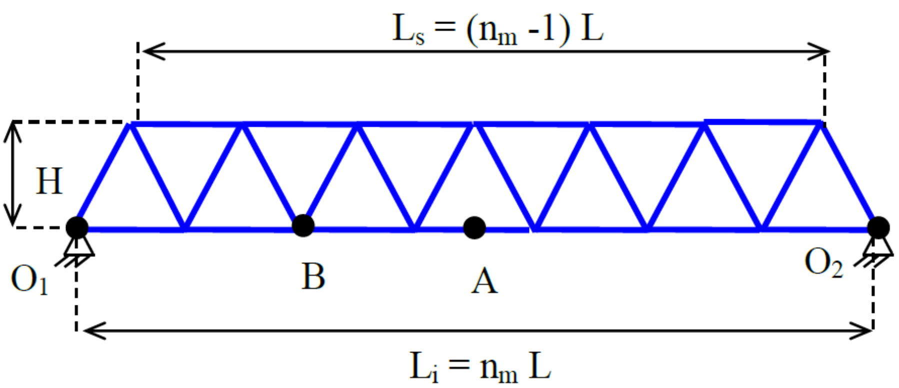


Figure 2: Plane model of a truss bridge

0.1.2 Project Scope

Various requests need to be fulfilled in order to complete the project. Here they are briefly described:

1. A FEM model for the bridge needs to be developed. The model has to be valid in the frequency range $f \in [0, 15]$ Hz.

2. Compute the system's natural frequencies and related modes of vibration in the frequency range $f \in [0, 15]$ Hz.

3. Compute the following frequency response functions (FRF) in the frequency range $f \in [0, 15]$ Hz with step $\Delta f = 0.01$ Hz:

   (a) Vertical displacement of point A produced by a vertical force on point A.

   (b) Vertical displacement of point B produced by a vertical force on point A.

   (c) Vertical acceleration of point A produced by a vertical force on point B.

   (d) Vertical acceleration of point B produced by a vertical force on point B.

4. Compute the bridge response due to a seismic motion of the ground:

   (a) The spectrum of the input displacements $yO1$, $yO2$.

   (b) The spectrum of the vertical displacements of points A and B.

   (c) The time histories of the vertical displacements of points A and B.

   (d) The spectrum and the time histories of the vertical accelerations of points A and B.

5. Considering the passage with constant speed V of a sequence of moving concentrated loads with distance of $26m$ from one another, discuss the possibility of producing a resonance condition in the bridge for the specified values of the train's speed V. To this end, consider an infinite sequence of moving load (approximation of a long train).

6. Either the 6.a or 6.b can be done:

   (a) Define a structural change that allows for a 20% increase of the first natural frequency of the bridge. To this aim, the total bridge mass must not increase more than 3%. It's not allowed to change the span length, to change the material, to add constraints. In case of variation of the beams cross-section, all of the inertial and elastic parameters (m, EA, EJ) must change according to the new cross-section dimension and shape, which has to be chosen among the standard metallic section provided in the tables of standardised geometric properties for beam sections made available on BeeP.

   (b) Define a structural change of the bridge constrains that allows for a 15% reduction of the maximum amplitude of vibration evaluated at point A when the bridge is subjected to the seismic excitation described at point 4).

0.1.3 Project Data

The bridge, as shown in figure 2, has two constraints:

1. Hinge in O_1.
2. Cart in O_2.

Moreover, it is necessary to analyse the vertical displacement of nodes A, B.

The data provided to describe the bridge is briefly summarised in the following table:

Geometric Data		
Length of base for one module L	$[m]$	10
Modules number n_m		7
Bottom chord total length L_i	$[m]$	70
top chord total length L_s	$[m]$	60
Bridge height H	$[m]$	2.8
Properties bottom chord		
Cross section type		IPE400
A	$[cm^2]$	84.46
J	$[cm^4]$	23130
Properties top chord		
Cross section type		IPE400
A	$[cm^2]$	84.46
J	$[cm^4]$	23130
Properties diagonal beams		
Cross section type		HEA160
A	$[cm^2]$	38.77
J	$[cm^4]$	1673
Material: Steel - Coefficients		
ρ	$[\frac{kg}{m^3}]$	7800
E	$[\frac{N}{m^2}]$	$2.06 \cdot 10^{11}$
α	$[s]$	0.2
β	$[s^{-1}]$	10^{-4}

Furthermore, it is assumed that the structural damping is proportional, under the proportional damping assumption.
Hence $\exists \alpha, \beta \in \mathbb{R} : [R] = \alpha[M] + \beta[K]$, where $[R]$ is the damping matrix, $[M]$ is the mass matrix and $[K]$ is the stiffness matrix.

Notice that the length of the diagonal beams L_d is not given. We can deduce some relationships:

$$H = L_d sin(\theta) \quad 5 = L_d cos(\theta)$$

Follows that:

$$\theta = tan^{-1}\left(\frac{H}{5}\right) \approx 29.25^{\circ} \quad L_d = \frac{5}{cos(\theta)} \approx 5.73m$$

Parameters also have to be converted to the appropriate dimensional units.

0.2 Project

0.2.1 FEM Model

In order to develop a convincing FEM model we have to thoroughly understand under which conditions a nodal section has to be inserted into the model. It is known that a nodal section must be inserted in the following situations:

1. Whenever there is a variation in the beam properties.
2. Intersection of 2 or more beams, with different axis direction.
3. Presence of a concentrated element (spring, mass, damper, force).
4. Whenever the displacement of a certain point has to be known.

Other rules may be applied, but one of the most important is the following:

- In case of statics loads the approximation of the system motion is the exact solution. Whilst for dynamic loads, in order to get an acceptable approximation of the actual displacement of the systems, each finite element has to work in the range of frequencies well below their first resonance (Quasi static range).

To ensure a good approximation of the solution, then the following has to be satisfied: $\omega_k \gg \Omega_{max}$, where ω_k is the first resonance of the finite element.
ω_k can be calculated from the following formula, which corresponds to the pinned-pinned case of a beam:

$$\omega_k = \left(\frac{\pi}{L_k}\right)^2 \sqrt{\frac{EJ_k}{m}}$$

In our particular problem $\Omega_{max} = 2\pi \cdot 15$, it follows that:

$$\left(\frac{\pi}{L_k}\right)^2 \sqrt{\frac{EJ_k}{m}} >> 2\pi \cdot 15$$

Hence:

$$L_k \ll \sqrt{\frac{\pi}{2 \cdot 15}\sqrt{\frac{EJ_k}{m}}} = L_{max}$$

How *small* should be L_k when compared to L_{max}? There are few rules, described in the appendix. Those rules are:

- *Half-Power*: L_k is approximated up to the frequency where the first resonance in ω_k should have half the power.
- *Derivative rule*: L_k is approximated up to the frequency where the slope of the FRF magnitude is approximately the value given by the user (e.g. -0.5).
- *Frequency Range*: L_k is approximated up to $k\Omega_{max}$, where k is a value given by the user.

Using the last rule, with $k = 5$, follows that $\omega_k = 5\Omega_{max}$. Since the properties of the beams change if we consider diagonal beams, it follows that we have $L_{k,i}, i = 1, 2$.
Specifically $L_{k,1} \approx 4.22m$, $L_{k,2} \approx 2.66m$, which are rounded down to
$L_{k,1} = \frac{10}{3}, L_{k,2} = 2$. In this way we also have nodal sections in A and B.

Once the length is found it is possible to set up the model by writing all of the parameters associated with the nodes and the beams on a file. This however may require a very long time if high precision is needed. For this reason I developed a matlab software, which given the position of the beams on a (x,y) reference system, their rotation with respect to the (x) axis, and lastly their parameters, is capable of automatically producing a FEM model when the frequency range and the accuracy are provided, which we discussed beforehand.

Moreover, this tool makes it possible to add nodes wherever we want in the beams. The development of this tool has undergone several steps:

1. Define what is the geometric model of a system and define what is a node.
2. For each beam calculate, accordingly to the parameters, how many nodes it needs to work in the range of frequencies provided by the user.
3. Join all the beams: remove all duplicated nodes (this may require to reduce precision of the geometric position of a node).
4. Add all the nodes that the user whishes to include, apply the constraints.
5. Write on file the FEM model.

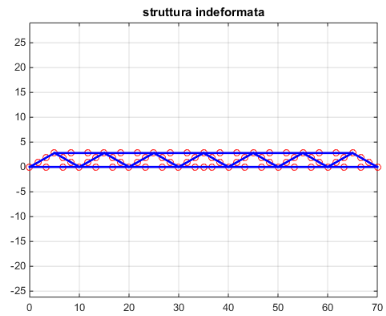


Figure 3: Fem Model

In this way it was possible to developed a FEM Model where the first frequency of the system lies over $100Hz$. The software produced a FEM Model with 70 nodes and 82 beams, as shown in figure **??** (the coefficient used in the tool was 7, to not have too many nodes).

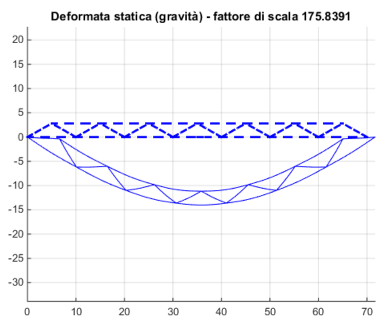


Figure 4: Static analysis: structure gravity deformation

Moreover, it is possible to check the static deformation due to gravity in order to make a first check of the validity of the model. The gravity deformation can be seen in figure **??**, which is scaled of a factor 175.8. Looking at the vertical displacement, we see that node A (which is node 32 in our model), is the one which presents maximum displacement being in the middle of the bridge, with a value of $y \approx -7.968 \cdot 10^{-2}m = -7.968cm$.

O_1 is a hinge and as such it's constrained in the x, y coordinates. In fact said hinge has 0 displacement for this position. Same check goes for O_2, which is a kart, and is therefore only constrained in the y direction having 0 displacement in that direction as expected.

From the FEM Model we can retrieve the $[M], [R], [K]$ matrices, which are of dimension $N \times N$, where $N = \#$ of nodes $\times$ indipendent coordinates, which is $N = 70 \times 3 = 210$ xindependent coordinates. From those we have to remove the constrained displacements, which are 3. Therefore the degrees of freedom are $n = 3N - p = 210 - 3 = 207$.

The matrices are symmetric, and can be splitted into 4 submatrices, like $[M]_{ff}, [M]_{fc}, [M]_{cf}, [M]_{cc}$, thanks to the division of the nodes from the constrained ones to those which are not constrainted $\underline{x} = (\underline{x}_f, \underline{x}_c)^T$.

Finally, also the matrix **IDB** can be retrieved from the software, which is the matrix that indexes the displacements (x, y, θ) of the nodes: $\mathbf{IDB}(32, 2)$ is the y displacement of node 32.

The total mass is $10990.4209Kg$.

0.2.2 System's natural frequencies and mode of vibrations

To compute the natural frequencies of a system we can make the $dmb_f em$ software or make a code to automatically calculate it. It's straightforward to implement:

```
[v,d]=eig(Mff\Kff);
[omega,I]=sort(sqrt(diag(d)));
freq=omega/2/pi;
```

It's worth to notice that the damping was not taken into account while calculating the natural frequencies, due to the fact that it doesn't affect the natural frequencies to enough an extent. Even so it can be considered by setting $\underline{z} = \underline{\dot{x}}$:

```
Anew = [zeros(nf,nf), eye(nf,nf); -Mff\Kff, -Mff\Rff];
shapes2,puls2]=eig(Anew);
[omega,I]=sort( diag(puls2) );
freq = unique(abs(imag(omega)/(2*pi)));
```

By using the tool the first natural frequencies in the range $[0, 15]Hz$ are:

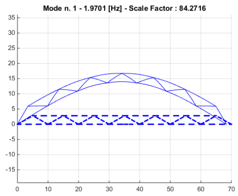
1. Frequency $1.9701Hz$, with relative mode of vibration displayed in figure 5a.
2. Frequency $6.8843Hz$ and mode of vibration displayed in figure 5b.
3. Frequency $12.2399Hz$ and mode of vibration displayed in figure 5c.
4. Frequency $14.3477Hz$ and mode of vibration displayed in figure 5d.
5. Frequency $14.3538Hz$ and mode of vibration displayed in figure 5e.

Since the modes of vibration have a sinusoidal shape, it's obvious that for lower frequencies the amplitude assumes the meaning of being a 'mean' value. Moreover, it's important to understand the effect of the constrained nodes on the natural frequencies.

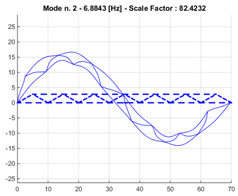
Having a kart on O_2 implies that the vibrations can extend on the x axis over the bridge length, thus reducing the amplitude of the oscillations.

Furthermore, adding more constraints to O_2 implies only that the bridge oscillates at higher frequencies, since constant modes (lower frequencies) are constrained.

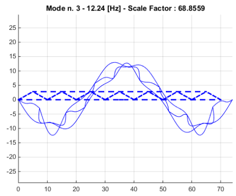
In the various figures we can also see the nodal points, which are points that have a 0 mode vibration when a force is applied on them, that mode of vibration is 0 for that point.



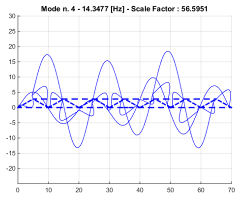
(a) 1-st mode



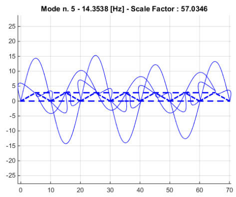
(b) 2-nd mode



(c) 3-rd mode



(d) 4-th mode



(e) 5-th mode

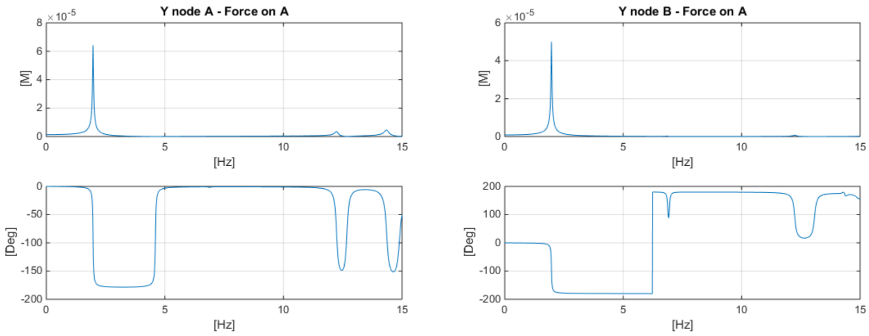
Figure 5: Modes of vibration for $f \in [0, 15]Hz$

0.2.3 Frequency responce functions

In this section we have to analyse the FRFs of the point A, B for the displacement and the acceleration.
To do so we have to calculate for each frequency the amplitude and the phase of the response, in order to obtain the bode plot of the transfer function, which is equal to sending an impulse of amplitude 1 as input.
A sample matlab code to do so is:

```
df = 0.01;
frequencies = (0:df:15)';
F= zeros(nf,1); F(NA,1) = 1; %NA is the idb(32,2) value
for i=1:size(frequencies,1);
    omega = frequencies(i)*2*pi;
    x = (-omega^2*Mff+j*omega*Rff+Kff)\F;
    xa = x(NA); %*-(omega)^2 to obtain acceleration
    xb = x(NB);
    moda(i) = abs(xa);
    modb(i) = abs(xb);
    fasa(i) = angle(xa);
    fasb(i) = angle(xb);
end
```

We can see the displacements in figure 6:

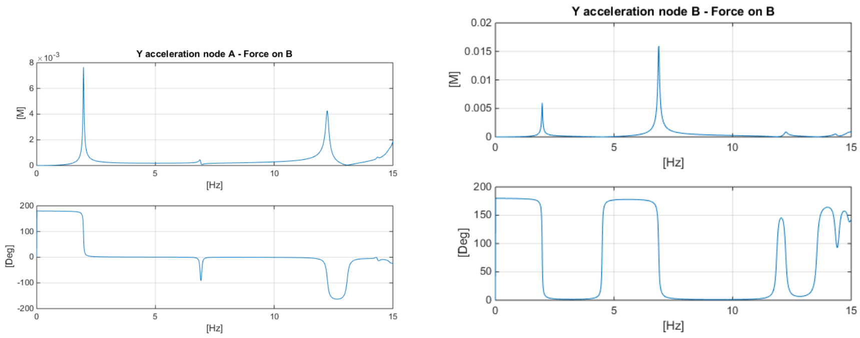


(a) y displacement of node A   (b) y displacement of node B

Figure 6: FRF y Displacement of nodes A, B when a vertical force is applied on A, for $f \in [0, 15]Hz$

It's important to consider that we applied a force on node A.
If we consider the displacement of node A it's trivial to notice that the second frequency is not excited, due to the fact that A is a nodal point for that natural frequency (figure 5b). For the last 3 frequencies node A is not a nodal point even though the modes of vibration associated to those frequencies have very small amplitude.
If we consider the displacement of node B for the second frequency we have that it's 0 because node A is a nodal point and we applied a force on A. For the third frequency node B is not a nodal point even if it's near to being one. As for the other two frequencies we have that it's nearly a nodal point, therefore its FRF is ≈ 0.

Now we consider the acceleration of node A, B when a force is applied on B:



(a) y acceleration of node A

(b) y acceleration of node B

Figure 7: FRF y acceleration of nodes A, B when a vertical force is applied on B, for $f \in [0, 15] Hz$

A is a nodal point only for the second frequency whilst for all the others, and especially for the third one, we see peaks as expected.
B is not a nodal point for the first two frequencies as we can see from the presence of two very high peaks. This happens because B is the point on which the force is applied. For the other two frequencies we have a near-nodal point situation and therefore its FRF is 0.

0.2.4 Earthquake analysis

In this section we have to analyse the effect of a seismic motion of the ground applied on the bridge, specifically on the constrained nodes O_1, O_2. A picture of the y displacement of those point is given in figure ??.

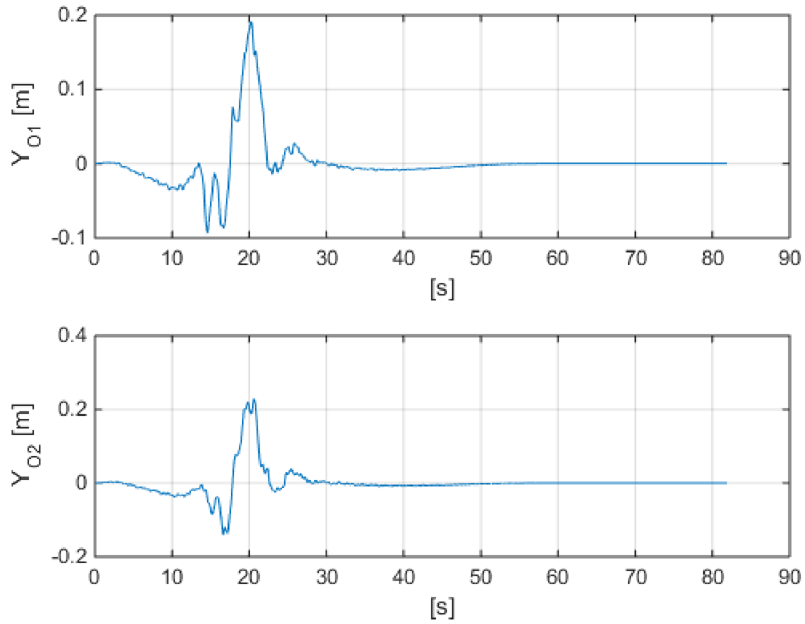
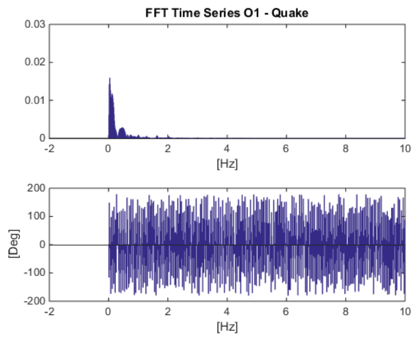
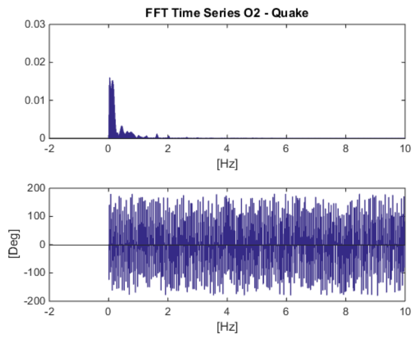


Figure 8: Earthquake y displacement of nodes O_1, O_2

If we consider the FFT (figure 9a, 9b) of these two signals, the frequency content of these signals is located in a neighbourhood of 0, for this reason we'll consider frequencies $f \in [0, 10]Hz$.

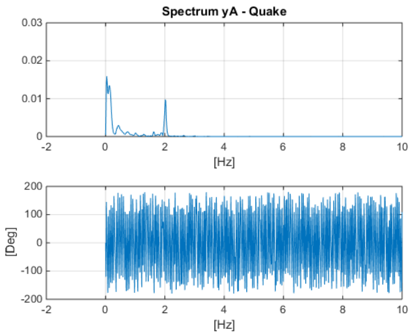


(a) FFT earthquake in O_1

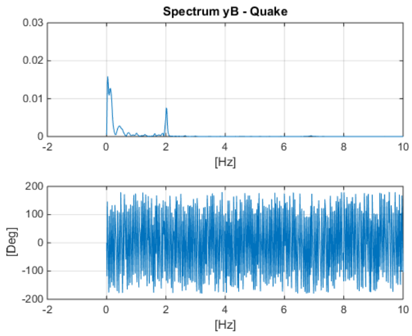


(b) FFT earthquake in O_2

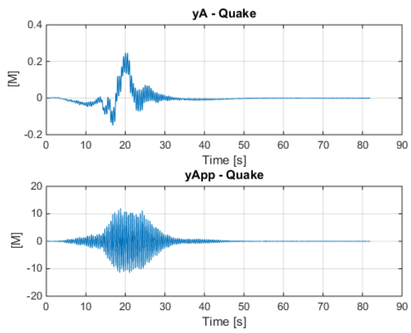
The frequencies of the earthquake are well below the first natural frequency of the system, the displacements of nodes A, B are:



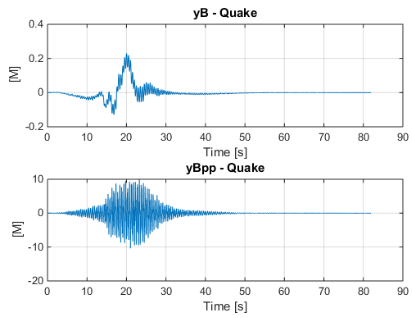
(a) FFT earthquake in A



(b) FFT earthquake in B



(a) FFT earthquake in A



(b) FFT earthquake in B

```
quake=load('sismaspost.txt');
plotQuake(quake);
t= (quake(:,1))';
y01 = quake(:,2);
y02 = quake(:,3);
N = size(quake,1);
T = quake(end,1);
fft01 = fft(y01);
fft02 = fft(y02);
df=1/T;
fmax = df*(N/2-1);
frequencies = (0:df:fmax)';

fftabs01(1) = abs(fft01(1))/N;
fftabs02(1) = abs(fft02(1))/N;
fftabs01(2:N/2) = abs(fft01(2:N/2))*2/N;
fftabs02(2:N/2) = abs(fft02(2:N/2))*2/N;
fftfas01(1:N/2) = angle(fft01(1:N/2));
fftfas02(1:N/2) = angle(fft02(1:N/2));
...
for i =1:length(frequencies)
        omega = frequencies(i)*2*pi;
        Xc01 = fftabs01(i)*exp(j*fftfas01(i));
        Xc02 = fftabs02(i)*exp(j*fftfas02(i));
        Xc = [0; Xc01; Xc02];
        Qc = (-omega^2*Mfc+j*omega*Rfc+Kfc)*Xc;
        x = (-Mff*omega^2 +j*omega*Rff+Kff)\(-Qc);

        xa = x(NA);
        xb = x(NB);
        xapp = x(NA)*(-omega^2);
        xbpp = x(NB)*(-omega^2);

        moda(i) = abs(xa);
        modb(i) = abs(xb);
        modapp(i) = abs(xapp);
        modbpp(i) = abs(xbpp);


        fasa(i) = angle(xa);
        fasb(i) = angle(xb);
        fasapp(i) = angle(xapp);
        fasbpp(i) = angle(xbpp);

        qr01 = qr01+fftabs01(i)*cos(omega*t+fftfas01(i));
        yA = yA+ moda(i)*cos(omega*t+fasa(i));
        yApp = yApp+ modapp(i)*cos(omega*t+fasapp(i));
        yB = yB+ modb(i)*cos(omega*t+fasb(i));
        yBpp = yBpp+ modbpp(i)*cos(omega*t+fasbpp(i));
    end
```

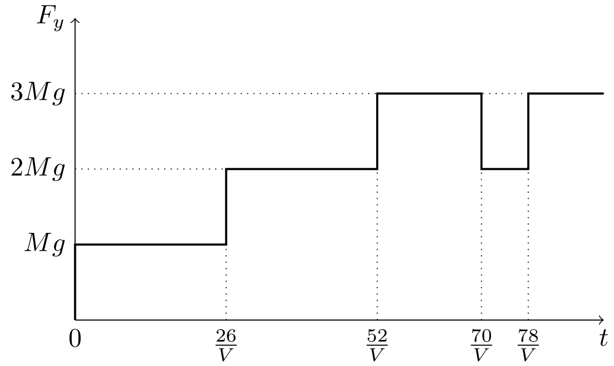
0.2.5 Long Train passage

The passage of a long train, with concentrated loads distancing $d = 26m$ from one another can be mathematically modelled. For simplicity we assume the train to be infinitely long.

The displacement of the k-eth load is given by the following formula:

$$x_k(t) = Vt - dk \quad k = 0, 1, \cdots$$

It's trivial to say that at any given time we have at most 3 loads on the bridge. As a matter of fact, when we have a load on $x = 0$, since $d = 26$, for $k = 2$ we obtain 52 while for $k = 3$ 78.
The only vertical force component acting on the bridge is due to gravity.



Notice that along the bridge the force is constant, whilst during the step we have an impulsive force acting either on O_1 or O_2. Therefore, it is worth to study the effects on nodes O_1 and O_2. To this end we set up some analytical formulae that characterize this impulsive force. To do so we use the Dirac Impulse $\delta(t)$. Then, on O_1 , considering that the period is given by the formula $0 = Vt - dk$,we have:

$$F_{y,O_1}(t) = \sum_{k=0}^{\infty} Mg\delta\big(t - kT\big), \quad T = \frac{d}{V}[s]$$

Similarly, for O_2, $70 = Vt - dk$:

$$F_{y,O_2}(t) = \sum_{k=0}^{\infty} Mg\delta\big(t - kT - \hat{T}\big), \quad \hat{T} = \frac{70}{V}$$

In practice, $F_{y,O_2}(t) = F_{y,O_1}(t - \hat{T})$, as it should be. Thus we can study only $F_{y,O_1}(t)$ and we start by considering its spectrum.

Consider the fourier series:

$$f(t) = \frac{a_0}{2} + \sum_{k=1}^{\infty} a_k cos(k\omega t) + b_k sin(k\omega t)$$

Where $f(t) = \sum_{k=0}^{\infty} Mg\delta\big(t - kT\big)$, $\omega = \frac{2\pi}{T}$. The coefficients a_k, b_k are given by:

$$a_k = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)cos(k\omega t)dt$$
$$b_k = \frac{2}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)sin(k\omega t)dt$$

Therefore $a_k = \frac{2Mg}{T}, b_k = 0$:

$$f(t) = \frac{Mg}{T} + \sum_{k=1}^{\infty} \frac{2Mg}{T} cos(k\omega t) = \frac{Mg}{T}\sum_{k=-\infty}^{\infty} cos(k\omega t) = \frac{Mg}{T}\sum_{k=-\infty}^{\infty} e^{jk\omega t}$$

Therefore the FFT of the signal has *phase* equal to 0 and amplitude that is given by $\frac{Mg}{T}$, every $k\omega = k\frac{2\pi}{T} = k\frac{2\pi V}{d} = k\frac{\pi V}{13}$. There might be resonance if $\exists k \in \mathbb{Z} : k\frac{\pi V}{13} \approx \omega_i$, where ω_i is a natural frequency of the system, $i = 1, \cdots, n_f$.

Consider now that $V \in [0, 150]\frac{Km}{h} \Rightarrow V \in [0, 41.67]\frac{m}{s}$, therefore:

$$k \overset{?}{\approx} \omega_i \frac{13}{\pi V}$$

To check if it is possible, consider the vector of natural frequencies of the system $\underline{\omega} = (\omega_1, \cdots \omega_n)^T$ and the vector of the inverse of velocities $\underline{V}^{-1} = (\frac{1}{V_1}, \cdots, \frac{1}{V_m})^T$, where n is the number of natural frequencies of the system, specifically the DOF, and m the number of velocities considered. We can construct a matrix $\mathbf{C} = \frac{13}{\pi}\underline{V}^{-1}\underline{\omega}^T$, where $C_{ij} = \frac{13\omega_j}{\pi V_i}$. We can impose a rule with a parameter $\varepsilon > 0$ chosen by us, to define whenever there might be the chance of a resonance:

$$min_{i,j}(|k\underline{1}\underline{1}^T - \mathbf{C}|) \leq \frac{13\varepsilon}{V\pi 2} = \overline{\varepsilon}$$

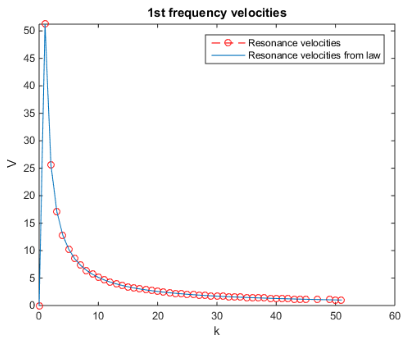
It's obvious to say that resonance happens, but for high values of k, for which there is an hyperbole that has the values of V interesting to us for elevated k.

Various tests were done on this code, especially to tests low frequency resonance. From these tests it results that the velocities that cause a resonance depend on the maximum velocity that can excit the i-eth natural frequency, which we will call $V_{max,i}$. The relationship is obviously hyperbolic, as previously stated. We can say from empirical tests that the k-eth velocity that excites the i-eth resonance frequency is of the form::

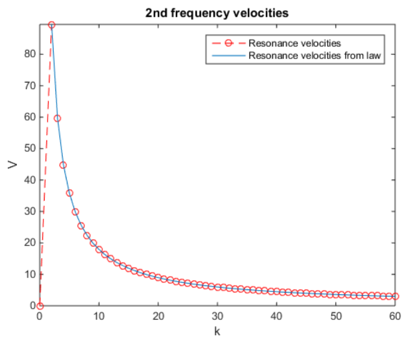
$$V_{res,k} = \frac{V_{max,i}}{1 + ki^{-1}}, \quad i = 1, 2, 3, \cdots \quad k = 0, 1, \cdots$$

From this a matrix can be built, where the k, i cell represents the k-eth velocity that excties the i-eth resonance frequency.

```
v=1:0.01:55;
v=1./v;
freqk=freq(1);
Cn = 26*v'*freqk';
kmax = ceil(max(Cn(:)));
 sCn=size(Cn);
for k=1:kmax
   eps = abs(ones(sCn)*k - Cn);
   if (min(eps(:)) < 1e-5 *13*45/pi)
       [row,col] = find(eps == min(eps(:)));
       disp(['K: ' num2str(k) '- Matches: '
       num2str(length(find(eps<1e-5*13*45/pi))) '
        - [MIN]  Diff: ' num2str(eps(row,col)) '
        - Velocity: ' num2str(1/v(row)) '
        - Frequency: ' num2str(freq(col))]);
   end
end
```



(a) 1-st frequency resonance velocities



(b) 2-nd frequency reosonance velocities

A sample output given by the execution of the previous code is the following (for the first 4 frequencies, with velocity ranging from 0 to 100 $\frac{m}{s}$):

```
K: 1- Matches: 19 - [MIN]  Diff: 6.6453e-05 - Velocity: 51.22 - Frequency: 1.9701
K: 2- Matches: 21 - [MIN]  Diff: 7.7904e-05 - Velocity: 89.5 - Frequency: 6.8843
K: 3- Matches: 10 - [MIN]  Diff: 0.00021836 - Velocity: 59.66 - Frequency: 6.8843
K: 4- Matches: 22 - [MIN]  Diff: 7.8548e-06 - Velocity: 79.56 - Frequency: 12.24
K: 5- Matches: 13 - [MIN]  Diff: 0.00011786 - Velocity: 74.61 - Frequency: 14.3477
K: 6- Matches: 9 - [MIN]  Diff: 1.1782e-05 - Velocity: 53.04 - Frequency: 12.24
K: 7- Matches: 6 - [MIN]  Diff: 0.00011841 - Velocity: 25.57 - Frequency: 6.8843
...(output omitted)
```

Modal approach verification

It is possible to come up with the same solution using the modal approach. Remember that to using the modal approach means applying a linear trasformation to our coordinate $x, x = \Phi q$, where q is the modal coordinate and Φ is the modal matrix containing the modes of vibration.
For beams the modal shape is $sin_k\left(\frac{k\pi x}{L}\right), k = 1, \cdots$, where L is the length of the beams' section considered and k the k-eth modal shape.
In our case $x = Vt$ and the section of the beam we have to consider is $L = 70m$. Moreover, since we have multiple trains, each one is $T = \frac{d}{V}$ seconds distant to the next train. Thus we obtain $sin_k\left(\frac{k\pi(Vt-iT)}{L}\right), i = 0, \cdots$.

Consider for simplicity just the first mode and the passage of 5 trains, then the generalized ecitation force is the following depicted in figure 13 (which is the sum of the component of each train): Since we considered only the first

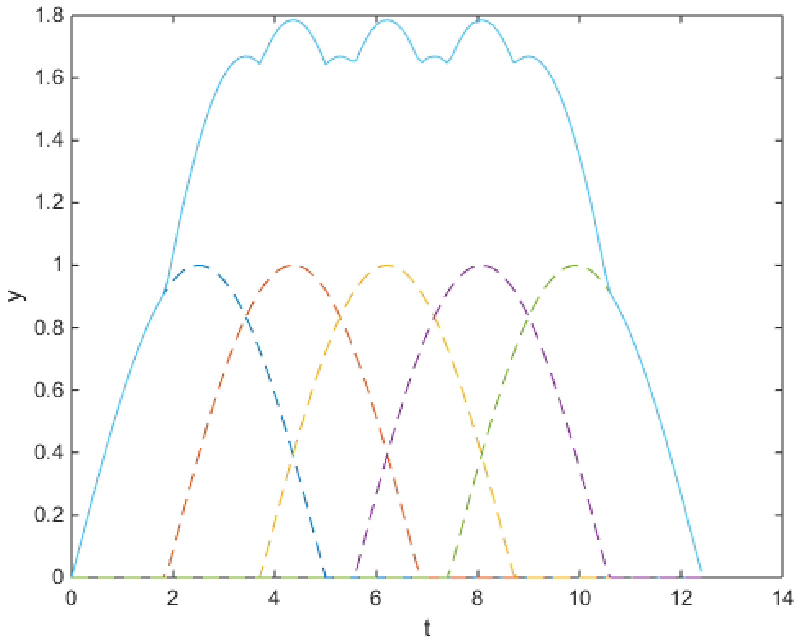
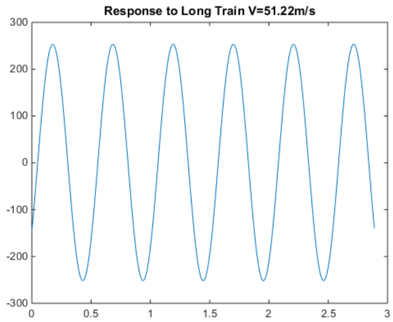


Figure 13: Trend of the first generalized periodic excitation forces with 5 trains

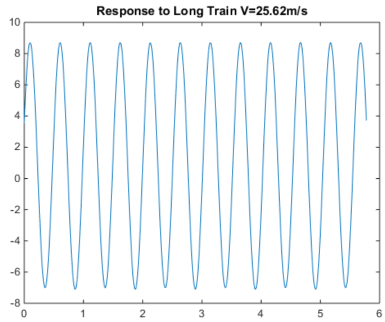
mode we obtain a concave parabola shape for each component. Moreover, each one lasts $Tmax = \frac{L}{V}$ and since we have n trains then $Tmax = \frac{L}{V} + (n-1)\frac{d}{V}$.

It's straightforward to see that each sine has an offset of $T = \frac{d}{V}$ one another, therefore the excitation happens every T seconds $\Rightarrow$ period T. If we take the the FFT of such force the armonics are $\frac{k2\pi}{T} = \frac{2k\pi V}{d}$, which is like considering d meters of the beam.
From this we obtain the same results previously reported.

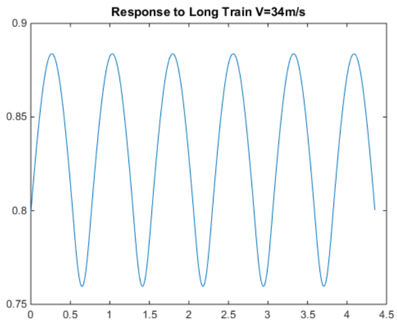
For completeness, it is shown for the first frequency what happens for $V = \{51.22, 25.6\}\frac{m}{s}$ which are resonance velocities and for $V = 34\frac{m}{s}$ which is not (notice that damping was not considered, and that images are not scaled, so check the amplitude):



(a) Resonance velocity $V = 51$m/s



(b) Resonance velocity $V = 25.6$m/s



(a) Not resonant velocity $V = 34$m/s

0.2.6 Structural change

In this part of the project we have to either make a structural change of the bridge in order to increase the first natural frequency by 20% or to make a structural change on the bridge constraints to reduce by 15% the maximum amplitude of vibration evaluated at point A when the bridge is subjected to the seismic excitation described at point 4.

Structural change of the bridge

The maximum increase in mass admitted is 3%. Therefore, since the starting mass is $10990.4209Kg$ the 3 % increase is $11320Kg$. To do so it's important to observe that increasing the height of the bridge leads to an increase of the first natural frequency of the system. This is due to the fact that the beams are not parallel to the vertical axis, therefore they try to make the system oscillate along the vertical direction, then increasing the frequency of the vibration.
Increasing the height by a factor of 1.3 leads to a change of mass that is under the 3%, and the new mass is $11183.6377Kg$. The new natural frequency is: $2.4987Hz$. It's trivial to observe though that from a certain point onwards the horizontal oscillations prevail, leading to a reduction of the natural frequency (just imagine increasing the height to ∞)

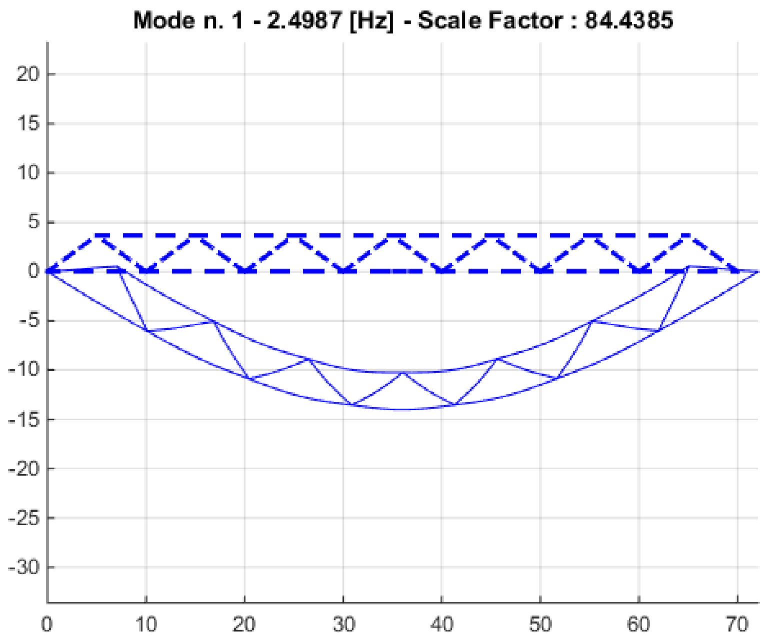
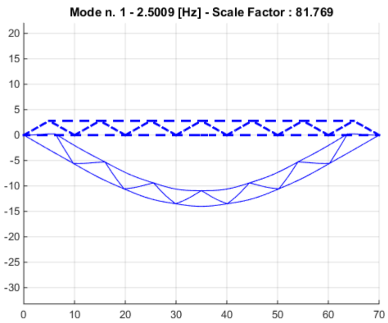


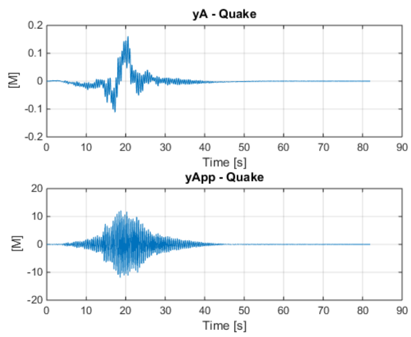
Figure 16: New natural frequency

Structural change of the constraints

To increase the natural frequency by simply changing the constraint, it's trivial to observe that hindering the bridge to elongate on the horizontal direction leads to an automatic increase of the natural frequency (remember that the longer an object, the lower the natural frequencies $\Rightarrow$ if the bridge doesn't elongate, then we increase the frequencies). This can be done by replacing the cart on O_2 with a pin.
This leads to an increase of the first frequency up to $2.5Hz$, and the reduction of the maximum amplitude of the quake is of about 35%.
In fact in point 4 the maximum amplitude was $0.24727m$, whilst now it's $0.16095m$.
This is simple to understand since the quake has no frequency content in $2.5Hz$.
The same reasoning can be applied to point $6.a$.



(a) New natural frequency



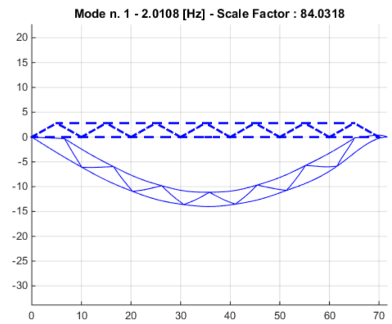
(b) y displacement of Node A after the structural change

0.2.7 Previous version of the assignment

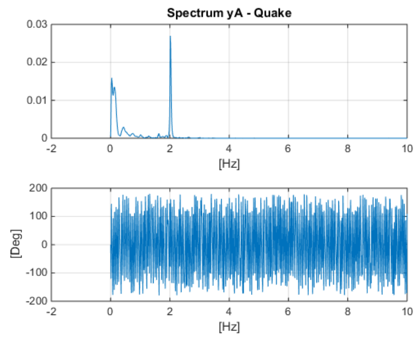
The first request for the project was to reduce the maximum amplitude of vibration of node A when subjected to an earthquake of about 20%, with a maximum increase of the bridge's mass of 5%, which ends up being 11540Kg. The difference is that it was modelled as a kart, with slider constraint and this meant that the first natural frequency was in the range of frequencies of the earthquake frequency content (see FRF of A in figure 18b). This led to the maximum vibration of A being of A being $0.2867cm$.
The first approach was to try changing the bridge's beams.
From the natural frequencies formula there is a direct evidence that reducing J and increasing m decreases the natural frequencies. Remember that $m = \rho A$, so we have a linear mass density.

To do so the bottom and top beams were replaced with a beam type **IPE 220**, which has larger section and lower inertia.
This leads to a bridge of mass $11657Kg$ but with a natural frequency of $1.9817Hz$.This, however, was not enough, since it reduced by $4cm$ the maximum amplitude (about 15%).

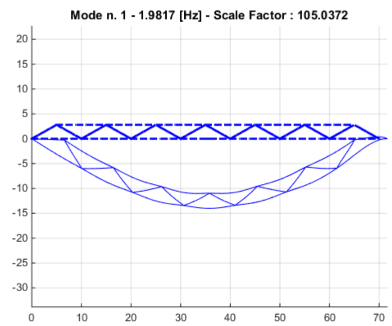
To improve the bridge the diagonal beams were modified. Some beams were replaced with the **HEB 120**, which have lower area and inertia (the inertia in particular is much lower, about 1 order of magnitude, while the area changed by 10%). This led to a bridge with acceptable mass, but instead of decreasing, the natural frequency rose. This is due to diagonal beams vibrating along the y axis. Higher frequencies are thus preferred. Some **HEB 140** beams were then added, these have larger mass and about the same inertia as **HEB 120**. The bridge mass then was $11519Kg$ and the first natural frequency was $1.968Hz$. Again, the maximum reduction of amplitude was of about $4.5-5cm$, which again is reasonable since it's the same first frequency of the hinge,kart constraint.



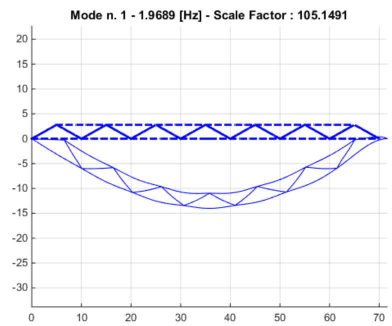
(a) First natural frequency with hinge,slider constraints



(b) FFT node A when subjected to an earthquake



(a) First natural frequency after first change



(b) First natural frequency after all the changes

0.3 Appendix

In the following sections I'll consider the analysis of the following function:

$$g(s) = \frac{k\omega_0^2}{s^2 + 2\xi\omega_0 s + \omega_0^2} \quad s \in \mathbb{C}, k \in \mathbb{R}, 0 \leq \xi \leq 1, \omega_0 = 2\pi f_0 \geq 0$$

By setting $s = j2\pi f$ we obtain:

$$T(f) = g(j2\pi f) = \frac{k}{\left(1 - \frac{f^2}{f_0^2} + j\frac{2\xi f}{f_0}\right)}$$

The modulus of T is given by:

$$|T(f)| = \frac{|k|}{\sqrt{(1 - \frac{f^2}{f_0^2})^2 + (\frac{2\xi f}{f_0})^2}}$$

.

0.3.1 Half-Power Rule

This rule determines when the resonance peak's power drops by $\frac{1}{2}$. For this reason we consider a resonance peak with damping coefficient ξ equal to 0 and $k = 1$.
Therefore the problem is to find the values of f such that:

$$|T(f)| = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The following steps are trivial:

$$2|T(f)|^2 = 1 \Rightarrow 2T(f)^2 = 1$$

We obtain:

$$2 = \left(1 - \frac{f^2}{f_0^2}\right)^2 = 1 + \frac{f^4}{f_0^4} - 2\frac{f^2}{f_0^2}$$

By setting $z = f^2$ and by multiplying for f_0^4:

$$z^2 - 2zf_0^2 - f_0^4 = 0$$

Then:

$$z_{1,2} = \frac{2f_0^2 \pm \sqrt{4f_0^4 + 4f_0^4}}{2} = f_0^2(1 \pm \sqrt{2})$$

$$\Rightarrow f = \pm\sqrt{f_0^2(1 \pm \sqrt{2})}$$

0.3.2 Derivative Rule

This method determines when the slope of the resonance peak is nearly the value given by the user, which we will call m. For simplicity we'll assume $\xi = 0$.

$$\frac{d}{df}|T(f)| = m$$
$$\frac{d}{df}\frac{|k|}{\sqrt{(1-\frac{f^2}{f_0^2})^2}} = m$$

Therefore:

$$\frac{d}{df}|T(f)| = |k|\frac{-\frac{d}{df}\sqrt{(1-\frac{f^2}{f_0^2})^2}}{(1-\frac{f^2}{f_0^2})^2} = |k|\frac{-\big(-\frac{2f}{f_0^2}\big)2\big(1-\frac{f^2}{f_0^2}\big)\frac{1}{2}\frac{1}{(1-\frac{f^2}{f_0^2})^2}}{(1-\frac{f^2}{f_0^2})^2}$$
$$= \frac{2|k|}{f_0^2}\frac{f(1-\frac{f^2}{f_0^2})}{((1-\frac{f^2}{f_0^2})^2)^{\frac{3}{2}}} = \frac{2|k|}{f_0^2}\frac{f(1-\frac{f^2}{f_0^2})}{|(1-\frac{f^2}{f_0^2})|^3} = m$$

We can do the semplification as long as we remember that we need to consider the sign, but $T(f) = T(-f)$, therefore we can proceed and use this rule thereafter:

$$\frac{2|k|}{f_0^2}\frac{f}{(1-\frac{f^2}{f_0^2})^2} = m$$

By letting $\frac{2|k|}{mf_0^2} = \hat{k}$:

$$1 + \frac{f^4}{f_0^4} - 2\frac{f^2}{f_0^2} = \hat{k}f$$
$$f^4 - 2f_0^2f^2 - \hat{k}f_0^4f + f_0^4 = 0$$

From this point onward an approximation method is used, but matlab provides function for finding roots of a polynomial.
Since we seek for a solution around f_0, a Taylor expansion in f_0 might be suitable:

$$W(f) = f^4 - 2f_0^2f^2 - \hat{k}f_0^4f + f_0^4$$
$$W(f_0) = -\hat{k}f_0^5$$
$$W'(f_0) = -\hat{k}f_0^4$$
$$W''(f_0) = 8f_0^2$$
$$W'''(f_0) = 24f_0$$

So:

$$W(f) \approx -\hat{k}f_0^5 - \hat{k}f_0^4(f-f_0) + 4f_0^2(f-f_0)^2 + 4f_0(f-f_0)^3$$

$$0 = -\hat{k}f_0^5 - \hat{k}f_0^4(f-f_0) + 4f_0^2(f-f_0)^2 + 4f_0(f-f_0)^3$$
$$0 = -\hat{k}f_0^3f + 4f_0(f^2+f_0^2-2f_0f) + 4(f^3 - f_0^3 + 3f_0^2f - 3f_0f^2)$$
$$0 = f(-\hat{k}f_0^3 + 4f^2 + 4f_0^2 - 8f_0f) = f(-\hat{k}f_0^3 + 4(f-f_0)^2)$$

Finally:

$$-\hat{k}f_0^3 + 4(f - f_0)^2 = 0 \Rightarrow f = f_0 \pm \frac{1}{2}\sqrt{\hat{k}f_0^3}$$

Since $T(-f) = T(f)$ also $-f$ is solution.

0.3.3 BeamLength function

```
function [L] = beamLength(...)
%
% Parameters is a vector with parameters P,A,J,E
%-> parameters = [P,A,J,E];
% Frequency denotes the frequency range to be approximated
% Approximation is a scale factor that multiplied by the frequency
% gives the approximation, by default is 1 decade (10)
%
% The formula is 2*pi*f = (pi/L)^2   * sqrt(E*J/M);
% Therefore L = sqrt(pi/(2*f)  * sqrt(E*J/M));

% switch(nargin)
%     case 2
%         approximation =10 ;
%     case 3 ;
%     otherwise
%         error('beamLength:TooManyInputs', ...
%         'requires at least 2 inputs');
% end

frequency = abs(frequency);
parameters = abs(parameters);
switch(approximationType)
    case ApproxType.HalfPower
        fmax = frequency*sqrt((1+sqrt(2)));
    case ApproxType.FreqRange
        fmax = frequency*approximationParam;
    case ApproxType.DerivativeRule
        a=frequency;
        k=approximationParam;
        r = roots([-k,0,3*a^2*k,0,-3*a^2*k,-4*a^2,k*a^6]);
        r = round(r*10000)/10000.0;
        fmax = r(real(r)>a & imag(r)==0);
    otherwise
        fmax = frequency*10;
end



M = parameters(1)*parameters(2);
sq1 = sqrt(parameters(4)*parameters(3)/M);
L = sqrt(pi*sq1/(2*fmax));


end
```

0.4 BuildStructure function

```
function [] = buildStructure(fileName, dampingCoefficients,
frequency, approximationType, approximationParam)
% Usage: buildstructure('file.txt',[a,b], ApproxType.DerivativeRule,
% 0.5);
%
%
%
    if nargin < 3 || nargin > 5
        error('buildStructure:TooManyInputs', ...
            'requires at least 3 inputs');
    end

    if (size(dampingCoefficients) ~= 2)
        error('buildStructure:WrongDampingCoefficients', ...
            'You need to provide atleast 2 parameters
            for the damping coefficients');
    end

    dampingCoefficients = abs(dampingCoefficients);
    frequency = abs(frequency);

    if ~exist('approximationType','var')
        approximationType = ApproxType.HalfPower;
    else
        if (approximationType == ApproxType.HalfPower)
            approximationParam=0;
        elseif (approximationType == ApproxType.FreqRange)
            if ~exist('approximationParam','var') ||
             ~isreal(approximationParam)
                disp('Not supplied an approximation param
                 for FreqRange or it is not a number.
                  A default one is provided (10)');
                approximationParam = 10;
            end
           % approximationParam = abs(approximationParam);
        elseif (approximationType == ApproxType.DerivativeRule)
           if ~exist('approximationParam','var') ||
           ~isreal(approximationParam)
                disp('Not supplied an approximation param
                for DerivativeRule or it is not a number.
                 A default one is provided (10)');
                approximationParam = 0.5;
            end
            approximationParam = abs(approximationParam);
        else
            error('buildStructure:WrongApproximationType', ...
                'Wrong approximation type');
        end
    end


    disp(['Damping Coefficients: ' num2str(dampingCoefficients(1))
    ' - ' num2str(dampingCoefficients(2))]);
    disp(['Approximation Type: ' char(approximationType)]);
    disp(['Approximation Parameter: ' num2str(approximationParam)]);

    beams = java.util.LinkedList;
    nodes = java.util.LinkedList;
```

```
    readStructure(fileName,beams.listIterator,nodes.listIterator);

    beams = unique(listToMatrix(beams),'rows');
    nodes = unique(listToMatrix(nodes),'rows');

    disp(['Read ' num2str(size(beams,1)) ' beams']);
    disp(['Read ' num2str(size(nodes,1)) ' nodes']);


    [nodesTree,beamsTree] = buildNodesStructure(beams,nodes,
    frequency,approximationType, approximationParam);
    writeStructure(fileName, nodesTree,beamsTree,
    dampingCoefficients);
end


function [nodesTree,beamsTree] = buildNodesStructure(beams,nodes,
frequency,approximationType, approximationParam)

    [nodesTree,beamsTree] = (placeNodes(beams,nodes,frequency,
    approximationType,approximationParam));
    nodesTree=round(nodesTree*100)/100.0;
    nodesTreeSize = size(nodesTree,1);

    for i=1:nodesTreeSize
       for j=i+1:nodesTreeSize
          if (nodesTree(i,2:3)==nodesTree(j,2:3))
              temp= nodesTree(j,1);
              nodesTree(j,1) = nodesTree(i,1);

              for w=1:size(beamsTree,1)
                for z=1:2
                    if (beamsTree(w,z+1)==temp)
                        beamsTree(w,z+1)=nodesTree(i,1);
                    end
                end
              end

          end
       end
    end

    nodesTree = unique(nodesTree,'rows');

    nodesTreeSize = size(nodesTree,1);
    for i=1:nodesTreeSize-1
       if (nodesTree(i+1,1) > nodesTree(i,1)+1)
          temp = nodesTree(i+1,1);
          n = nodesTree(i,1)+1;
          nodesTree(i+1,1) = n;
          for j=1:size(beamsTree,1)
              for z=1:2
                  if (beamsTree(j,z+1)== temp)
                      beamsTree(j,z+1) = n;
                  end
              end
          end

       end
    end

```

```
end

function [nodesTree,beamsTree] = placeNodes(beams,nodes,frequency,
approximationType,approximationParam)
    j=1;
    k=1;
    w=1;
    q=0;
    beamsTree = zeros(1,6);
    for z=1:size(beams,1)
        beam = beams(z,:);
        nlength = beamLength(beam(5:end), frequency,
        approximationType,approximationParam);
        nnodes = ceil(beam(4)/nlength);
        nlength = beam(4)/nnodes;
        nnodes=nnodes+1;
        bn = zeros(nnodes, 6);
        beam(3) = beam(3)*pi/180;
        q=0;
        for i=1:nnodes
          %  beam(1:3)
            bn(i,1) = w;
            bn(i,2) = beam(1)+nlength*(i-1)*cos(beam(3));
            bn(i,3) = beam(2)+nlength*(i-1)*sin(beam(3));
            bn(i,4:end) = [0,0,0];
            %bn


            beamsTree=placeNodesBeams(beamsTree,beam,j,i+q,nnodes
            ,w,k);
            %beamsTree(:,1:3)

            for p=1:size(nodes,1)
                if(nodes(p,1:2)==bn(i,2:3))
                    bn(i,4:end) = nodes(p,3:end);
                elseif ( i < nnodes)
                    angle = atan2((nodes(p,2)-beam(2)),(nodes(p,1)
                                -beam(1)));
                    r1 = beam(3)-angle;
                    if (r1 < 1e-10)
                        x = nodes(p,1);
                        y = nodes(p,2);
                        x0 = bn(i,2)+nlength*cos(beam(3));
                        y0 = bn(i,3)+nlength*sin(beam(3));
                        if ( ((x < x0 && y <= y0 )
                         || (x <= x0 && y < y0 ) )
                        && ( (bn(i,2) < x && bn(i,3) <=y)
                        ||  (bn(i,2) <= x && bn(i,3) <y)))
                            w=w+1;
                            q=q+1;
                            nnodes=nnodes+1;
                            bn = [bn; w, nodes(p,:)];
                            beamsTree=placeNodesBeams(beamsTree,beam
                            ,j,i+q,nnodes,w,k);
                        end
                    end
                end
            end

            w=w+1;
        end
        if j==1
```

```
            nodesTree = [ bn];
        else
            nodesTree = [nodesTree;bn];
        end
        j = j+nnodes-1;
        k=k+1;
    end
end

function [beamsTree] = placeNodesBeams(beamsTree,beam,j,i,nnodes,w,k)

    if i==1
        kn=nnodes-1;
        beamsTree= [beamsTree; k*ones(kn,1) zeros(kn,2)
         beam(5)*beam(6)*ones(kn,1) beam(8)*beam(6)*ones(kn,1)
          beam(8)*beam(7)*ones(kn,1)];
        if (j==1)
            beamsTree = beamsTree(2:end,:);
        end
        beamsTree(j+i-1, 2) =w;
        %5 = p, 6=A, 7=J, 8=E
    elseif i==nnodes
        beamsTree(j+i-2, 3) = w;
    else
        if (size(beamsTree,1) == j+i-2)
            kn=1;
            beamsTree= [beamsTree; k*ones(kn,1) zeros(kn,2)
            beam(5)*beam(6)*ones(kn,1) beam(8)*beam(6)*ones(kn,1)
            beam(8)*beam(7)*ones(kn,1)];
        end
            beamsTree(j+i-1, 2) =  w;
            beamsTree(j+i-2, 3) =  w;
    end
end



function [A] = listToMatrix(list)
    if(list.size()>0)
        A=zeros(list.size(), size(list.get(0),1));
        for i=1:list.size()
           A(i,:) = str2num(list.get(i-1))';
        end
    end
end
%---------------

function [] = readStructure(fileName, beams, nodes)
    fID = fopen(fileName);

    if fID ~= 1
         tline = fgetl(fID);
         while ischar(tline)
            strs = strsplit(tline,{'(',',',')'},
            'CollapseDelimiters',true);
            strs = strrep(strs, ' ', '');
            strs = strs(~cellfun('isempty',strs));
            parseLine(strs,beams,nodes);
            tline = fgetl(fID);
         end
    else
       error(['Cannot open the file: ' fileName]);
```

```
    end

    fclose(fID);
end

function [] = parseLine(line, beams,nodes)
    if size(line,2) > 0
        c = line(1,1);
        switch(c{:})
            case 'b'
                beams.add(line(2:end));
            case 'n'
                nodes.add(line(2:end));
            otherwise
                error(['Unidentified command found during the
                parsing of the structure file']);
        end
    end
end

function [] = writeStructure(fileName,nodesTree,
beamsTree, dampingCoefficients)
    fileName = strcat(fileName, '.inp');
    fID = fopen(fileName,'w');
    if fID ~= 1
        fprintf(fID, '*NODES\n');
        for i=1:size(nodesTree,1)
            fprintf(fID, '%d \t  %d %d %d \t %f \t %f\n',
            nodesTree(i,1), nodesTree(i,4),nodesTree(i,5),
            nodesTree(i,6), nodesTree(i,2),nodesTree(i,3));
        end
        fprintf(fID, '*ENDNODES\n');
        fprintf(fID, '*BEAMS\n');
        for i=1:size(beamsTree,1)
            fprintf(fID, '%d \t \t %d %d \t %f \t %f \t %f\n', i,
             beamsTree(i,2), beamsTree(i,3),beamsTree(i,4),
             beamsTree(i,5), beamsTree(i,6));
        end
        fprintf(fID, '*ENDBEAMS\n');
        fprintf(fID, '*DAMPING\n');
        fprintf(fID, '%f %f\n', dampingCoefficients(1),
         dampingCoefficients(2));
    else
       error(['Cannot open the file: ' fileName]);
    end
    fclose(fID);
end
```

0.5 Project Function

```
clc; clear all;
%load('bridgeStructure.txt_mkr.mat');
%load('bridgeStructure.New_mkr.mat');
%load('bridgeStructure.Newb_mkr.mat');
%load('bridgeStructure.springs_mkr.mat');
%load('bridgeNew.txt_mkr.mat');
load('bridgetestlast.txt_mkr.mat');
close all;
% Node O1: 1
% Node A: 32
% Node B: 13
% Node O2: 64
alpha=0.2;
beta=1e-4;
A = 50; %32
B = 20;%13;
O1 = 1;
O2 =101;%64;

NA = idb(A,2);
NB = idb(B,2);
NO1 = idb(O1,2);
NO2 = idb(O2,2);

n = size(idb,1)*3;
nc = 4;
nf = n-nc;

Mff = M(1:nf,1:nf);
Kff = K(1:nf,1:nf);
Rff = R(1:nf,1:nf);

Mfc = M(1:nf, nf+1:nf+nc);
Kfc = K(1:nf, nf+1:nf+nc);
Rfc = R(1:nf, nf+1:nf+nc);

Mcf = M(nf+1:nf+nc, 1:nf);
Kcf = K(nf+1:nf+nc, 1:nf);
Rcf = R(nf+1:nf+nc, 1:nf);
Mcc = M(nf+1:nf+nc, nf+1:nf+nc);
Kcc = K(nf+1:nf+nc, nf+1:nf+nc);
Rcc = R(nf+1:nf+nc, nf+1:nf+nc);
%%
[v,d]=eig(Mff\Kff);
[omega,I]=sort(sqrt(diag(d)));
freq=omega/2/pi;
sysomega = omega;

% Anew = [zeros(nf,nf), eye(nf,nf); -Mff\Kff, -Mff\Rff];
%  [shapes2,puls2]=eig(Anew);
%
%  [omega,I]=sort( diag(puls2) );
%  freq = unique(abs(imag(omega)/(2*pi)));
%%
project3Question(nf,NA,NB,Mff,Rff,Kff);

project4Question(Mff,Mfc,Rff,Rfc,Kff,Kfc,NA,NB);
```

0.6 Project3 Function

```
function [] = project3Question(nf,NA,NB,Mff,Rff,Kff)

    df = 0.01;
    frequencies = (0:df:15)';
    %----------
    F= zeros(nf,1); F(NA,1) = 1;
    for i=1:size(frequencies,1);
       omega = frequencies(i)*2*pi;
       x = (-omega^2*Mff+j*omega*Rff+Kff)\F;
       xa = x(NA);
       xb = x(NB);
       moda(i) = abs(xa);
       modb(i) = abs(xb);

       fasa(i) = angle(xa);
       fasb(i) = angle(xb);
    end

    figure;
    subplot 211; plot(frequencies, moda); grid;
    xlabel('[Hz]'); ylabel('[M]'); title('Y node A - Force on A');
    subplot 212; plot (frequencies, fasa*180/pi); grid;
    xlabel('[Hz]'); ylabel('[Deg]');

    figure;
    subplot 211; plot(frequencies, modb); grid;
     xlabel('[Hz]'); ylabel('[M]'); title('Y node B - Force on A');
    subplot 212; plot (frequencies, fasb*180/pi); grid;
     xlabel('[Hz]'); ylabel('[Deg]');

    %----------
    F= zeros(nf,1); F(NB,1) = 1;
    for i=1:size(frequencies,1);
       omega = frequencies(i)*2*pi;
       x = (-omega^2*Mff+j*omega*Rff+Kff)\F;
       xa = x(NA)*(-omega^2);
       xb = x(NB)*(-omega^2);
       moda(i) = abs(xa);
       modb(i) = abs(xb);

       fasa(i) = angle(xa);
       fasb(i) = angle(xb);
    end

    figure;
    subplot 211; plot(frequencies, moda); grid;
    xlabel('[Hz]'); ylabel('[M]');
     title('Y acceleration node A - Force on B');
    subplot 212; plot (frequencies, fasa*180/pi); grid;
    xlabel('[Hz]'); ylabel('[Deg]');

    figure;
    subplot 211; plot(frequencies, modb); grid;
    xlabel('[Hz]'); ylabel('[M]');
    title('Y acceleration node B - Force on B');
    subplot 212; plot (frequencies, fasb*180/pi); grid;
    xlabel('[Hz]'); ylabel('[Deg]');
end
```

0.7 Project 4

```
function [] = project4Question(Mff,Mfc,Rff,Rfc,Kff,Kfc,NA,NB)
    quake=load('sismaspost.txt');
    plotQuake(quake);
    t= (quake(:,1))';
    yO1 = quake(:,2);
    yO2 = quake(:,3);

    N = size(quake,1);
    T = quake(end,1);
    fftO1 = fft(yO1);
    fftO2 = fft(yO2);

    df=1/T;
    fmax = df*(N/2-1);

    frequencies = (0:df:fmax)';

    fftabsO1(1) = abs(fftO1(1))/N;
    fftabsO2(1) = abs(fftO2(1))/N;
    fftabsO1(2:N/2) = abs(fftO1(2:N/2))*2/N;
    fftabsO2(2:N/2) = abs(fftO2(2:N/2))*2/N;
    fftfasO1(1:N/2) = angle(fftO1(1:N/2));
    fftfasO2(1:N/2) = angle(fftO2(1:N/2));

    figure;
    subplot 211; bar(frequencies, fftabsO1); xlabel('[Hz]');
     title('FFT Time Series O1 - Quake');  axis([-2 10 0 0.03]);
    subplot 212; bar(frequencies, fftfasO1*180/pi); xlabel('[Hz]');
     ylabel('[Deg]');  axis([-2 10 -200 200]);


    figure;
    subplot 211; bar(frequencies, fftabsO2); xlabel('[Hz]');
     title('FFT Time Series O2 - Quake'); axis([-2 10 0 0.03]);
    subplot 212; bar(frequencies, fftfasO2*180/pi); xlabel('[Hz]');
     ylabel('[Deg]');  axis([-2 10 -200 200]);

    qrO1 = zeros(1,N);
    yA = zeros(1,N);
    yB = zeros(1,N);
    yApp = zeros(1,N);
    yBpp = zeros(1,N);
    for i =1:length(frequencies)
        omega = frequencies(i)*2*pi;
        XcO1 = fftabsO1(i)*exp(j*fftfasO1(i));
        XcO2 = fftabsO2(i)*exp(j*fftfasO2(i));
        Xc = [0; XcO1; XcO2;0];
        Qc = (-omega^2*Mfc+j*omega*Rfc+Kfc)*Xc;
        x = (-Mff*omega^2 +j*omega*Rff+Kff)\(-Qc);

        xa = x(NA);
        xb = x(NB);
        xapp = x(NA)*(-omega^2);
        xbpp = x(NB)*(-omega^2);

        moda(i) = abs(xa);
        modb(i) = abs(xb);
        modapp(i) = abs(xapp);
```

```
        modbpp(i) = abs(xbpp);


        fasa(i) = angle(xa);
        fasb(i) = angle(xb);
        fasapp(i) = angle(xapp);
        fasbpp(i) = angle(xbpp);

        qrO1 = qrO1+fftabsO1(i)*cos(omega*t+fftfasO1(i));
      %  if (frequencies(i) < 0.5)
            yA = yA+ moda(i)*cos(omega*t+fasa(i));
            yApp = yApp+ modapp(i)*cos(omega*t+fasapp(i));
            yB = yB+ modb(i)*cos(omega*t+fasb(i));
            yBpp = yBpp+ modbpp(i)*cos(omega*t+fasbpp(i));
       % end
    end

    figure;
    plot(t, qrO1); grid; xlabel('Time [s]'); ylabel('[M]');
    title('Quake in O1 rebuilt');

    figure;
    subplot 211; plot(frequencies, moda); grid; xlabel('[Hz]');
     title('Spectrum yA - Quake'); axis([-2 10 0 0.03]);
    subplot 212; plot(frequencies, fasa*180/pi); grid;
     xlabel('[Hz]');
    ylabel('[Deg]');  axis([-2 10 -200 200]);


    figure;
    subplot 211; plot(frequencies, modapp); grid; xlabel('[Hz]');
     title('Spectrum yApp - Quake'); axis([-2 10 0 2]);
    subplot 212; plot(frequencies, fasapp*180/pi); grid;
     xlabel('[Hz]');
     ylabel('[Deg]');   axis([-2 10 -200 200]);

    figure;


    subplot 211; plot(frequencies, modb); grid; xlabel('[Hz]');
    title('Spectrum yB - Quake'); axis([-2 10 0 0.03]);
    subplot 212; plot(frequencies, fasb*180/pi); grid;
     xlabel('[Hz]');
     ylabel('[Deg]');  axis([-2 10 -200 200]);

    figure;


    subplot 211; plot(frequencies, modbpp); grid; xlabel('[Hz]');
     title('Spectrum yBpp - Quake'); axis([-2 10 0 2]);
    subplot 212; plot(frequencies, fasbpp*180/pi); grid;
    xlabel('[Hz]');
    ylabel('[Deg]');  axis([-2 10 -200 200]);

    figure;
    subplot 211; plot(t, yA);grid;xlabel('Time [s]');
    ylabel('[M]'); title('yA - Quake');
    subplot 212; plot(t,yApp);grid;xlabel('Time [s]');
    ylabel('[M]'); title('yApp - Quake');

    figure;
    subplot 211; plot(t, yB);grid;xlabel('Time [s]');
```

```
     ylabel('[M]'); title('yB - Quake');
    subplot 212; plot(t,yBpp);grid;xlabel('Time [s]');
    ylabel('[M]'); title('yBpp - Quake');

    figure;
    subplot 211; plot(t,abs(yA-yB)); grid;xlabel('Time [s]');
     ylabel('[M]'); title('abs(yA-yB) - Quake');
    subplot 212; plot(t,abs(yApp-yBpp)); grid;xlabel('Time [s]');
    ylabel('[M]'); title('abs(yApp-yBpp) - Quake');
    disp(['Maximum yA: ' num2str(max(yA))]);
end
```

0.8 BridgeStructure tool file

```

(b,0,0,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,10,0,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,20,0,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,30,0,0,10,7800, 0.008446,0.0002313030,2.06e11)
(b,40,0,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,50,0,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,60,0,0,10,7800, 0.008446,0.00023130,2.06e11)


(b,5,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,15,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,25,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,35,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,45,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)
(b,55,2.8,0,10,7800, 0.008446,0.00023130,2.06e11)


(b,0,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,10,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,10,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,20,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,20,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,30,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,30,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,40,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,40,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,50,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,50,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,60,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,60,0,29.25,5.73,7800, 0.003877,0.00001673,2.06e11)
(b,70,0,150.75,5.73,7800, 0.003877,0.00001673,2.06e11)

(n,0,0,1,1,0)
(n,70,0,0,1,1)
(n,20,0,0,0,0)
(n,35,0,0,0,0)
```

0.9 FEM Model

```
*NODES
1      1 1 0      0.000000    0.000000
2      0 0 0      3.330000    0.000000
3      0 0 0      6.670000    0.000000
4      0 0 0      10.000000   0.000000
5      0 0 0      1.670000    0.930000
6      0 0 0      3.330000    1.870000
7      0 0 0      5.000000    2.800000
8      0 0 0      8.330000    2.800000
9      0 0 0      11.670000   2.800000
10     0 0 0      15.000000   2.800000
11     0 0 0      13.330000   0.000000
12     0 0 0      16.670000   0.000000
13     0 0 0      20.000000   0.000000
14     0 0 0      11.670000   0.930000
15     0 0 0      13.330000   1.870000
16     0 0 0      8.330000    0.930000
17     0 0 0      6.670000    1.870000
18     0 0 0      18.330000   2.800000
19     0 0 0      21.670000   2.800000
20     0 0 0      25.000000   2.800000
21     0 0 0      23.330000   0.000000
22     0 0 0      26.670000   0.000000
23     0 0 0      30.000000   0.000000
24     0 0 0      21.670000   0.930000
25     0 0 0      23.330000   1.870000
26     0 0 0      18.330000   0.930000
27     0 0 0      16.670000   1.870000
28     0 0 0      28.330000   2.800000
29     0 0 0      31.670000   2.800000
30     0 0 0      35.000000   2.800000
31     0 0 0      33.330000   0.000000
32     0 0 0      35.000000   0.000000
33     0 0 0      36.670000   0.000000
34     0 0 0      40.000000   0.000000
35     0 0 0      31.670000   0.930000
36     0 0 0      33.330000   1.870000
37     0 0 0      28.330000   0.930000
38     0 0 0      26.670000   1.870000
39     0 0 0      38.330000   2.800000
40     0 0 0      41.670000   2.800000
41     0 0 0      45.000000   2.800000
42     0 0 0      43.330000   0.000000
43     0 0 0      46.670000   0.000000
44     0 0 0      50.000000   0.000000
45     0 0 0      41.670000   0.930000
46     0 0 0      43.330000   1.870000
47     0 0 0      38.330000   0.930000
48     0 0 0      36.670000   1.870000
49     0 0 0      48.330000   2.800000
50     0 0 0      51.670000   2.800000
51     0 0 0      55.000000   2.800000
52     0 0 0      53.330000   0.000000
53     0 0 0      56.670000   0.000000
54     0 0 0      60.000000   0.000000
55     0 0 0      51.670000   0.930000
56     0 0 0      53.330000   1.870000
57     0 0 0      48.330000   0.930000
```

```
58    0 0 0      46.670000   1.870000
59    0 0 0      58.330000   2.800000
60    0 0 0      61.670000   2.800000
61    0 0 0      65.000000   2.800000
62    0 0 0      63.330000   0.000000
63    0 0 0      66.670000   0.000000
64    0 1 0      70.000000   0.000000
65    0 0 0      61.670000   0.930000
66    0 0 0      63.330000   1.870000
67    0 0 0      58.330000   0.930000
68    0 0 0      56.670000   1.870000
69    0 0 0      68.330000   0.930000
70    0 0 0      66.670000   1.870000
*ENDNODES
*BEAMS
1          1 2      65.878800   1739876000.000000   47647800.000000
2          2 3      65.878800   1739876000.000000   47647800.000000
3          3 4      65.878800   1739876000.000000   47647800.000000
4          1 5      30.240600   798662000.000000    3446380.000000
5          5 6      30.240600   798662000.000000    3446380.000000
6          6 7      30.240600   798662000.000000    3446380.000000
7          7 8      65.878800   1739876000.000000   47647800.000000
8          8 9      65.878800   1739876000.000000   47647800.000000
9          9 10     65.878800   1739876000.000000   47647800.000000
10         4 11     65.878800   1739876000.000000   47647800.000000
11         11 12    65.878800   1739876000.000000   47647800.000000
12         12 13    65.878800   1739876000.000000   47647800.000000
13         4 14     30.240600   798662000.000000    3446380.000000
14         14 15    30.240600   798662000.000000    3446380.000000
15         15 10    30.240600   798662000.000000    3446380.000000
16         4 16     30.240600   798662000.000000    3446380.000000
17         16 17    30.240600   798662000.000000    3446380.000000
18         17 7     30.240600   798662000.000000    3446380.000000
19         10 18    65.878800   1739876000.000000   47647800.000000
20         18 19    65.878800   1739876000.000000   47647800.000000
21         19 20    65.878800   1739876000.000000   47647800.000000
22         13 21    65.878800   1739876000.000000   47647800.000000
23         21 22    65.878800   1739876000.000000   47647800.000000
24         22 23    65.878800   1739876000.000000   47647800.000000
25         13 24    30.240600   798662000.000000    3446380.000000
26         24 25    30.240600   798662000.000000    3446380.000000
27         25 20    30.240600   798662000.000000    3446380.000000
28         13 26    30.240600   798662000.000000    3446380.000000
29         26 27    30.240600   798662000.000000    3446380.000000
30         27 10    30.240600   798662000.000000    3446380.000000
31         20 28    65.878800   1739876000.000000   47647800.000000
32         28 29    65.878800   1739876000.000000   47647800.000000
33         29 30    65.878800   1739876000.000000   47647800.000000
34         23 31    65.878800   1739876000.000000   47647800.000000
35         31 32    65.878800   1739876000.000000   47647800.000000
36         32 33    65.878800   1739876000.000000   47647800.000000
37         33 34    65.878800   1739876000.000000   47647800.000000
38         23 35    30.240600   798662000.000000    3446380.000000
39         35 36    30.240600   798662000.000000    3446380.000000
40         36 30    30.240600   798662000.000000    3446380.000000
41         23 37    30.240600   798662000.000000    3446380.000000
42         37 38    30.240600   798662000.000000    3446380.000000
43         38 20    30.240600   798662000.000000    3446380.000000
44         30 39    65.878800   1739876000.000000   47647800.000000
45         39 40    65.878800   1739876000.000000   47647800.000000
46         40 41    65.878800   1739876000.000000   47647800.000000
47         34 42    65.878800   1739876000.000000   47647800.000000
```

```
48        42 43   65.878800   1739876000.000000   47647800.000000
49        43 44   65.878800   1739876000.000000   47647800.000000
50        34 45   30.240600   798662000.000000    3446380.000000
51        45 46   30.240600   798662000.000000    3446380.000000
52        46 41   30.240600   798662000.000000    3446380.000000
53        34 47   30.240600   798662000.000000    3446380.000000
54        47 48   30.240600   798662000.000000    3446380.000000
55        48 30   30.240600   798662000.000000    3446380.000000
56        41 49   65.878800   1739876000.000000   47647800.000000
57        49 50   65.878800   1739876000.000000   47647800.000000
58        50 51   65.878800   1739876000.000000   47647800.000000
59        44 52   65.878800   1739876000.000000   47647800.000000
60        52 53   65.878800   1739876000.000000   47647800.000000
61        53 54   65.878800   1739876000.000000   47647800.000000
62        44 55   30.240600   798662000.000000    3446380.000000
63        55 56   30.240600   798662000.000000    3446380.000000
64        56 51   30.240600   798662000.000000    3446380.000000
65        44 57   30.240600   798662000.000000    3446380.000000
66        57 58   30.240600   798662000.000000    3446380.000000
67        58 41   30.240600   798662000.000000    3446380.000000
68        51 59   65.878800   1739876000.000000   47647800.000000
69        59 60   65.878800   1739876000.000000   47647800.000000
70        60 61   65.878800   1739876000.000000   47647800.000000
71        54 62   65.878800   1739876000.000000   47647800.000000
72        62 63   65.878800   1739876000.000000   47647800.000000
73        63 64   65.878800   1739876000.000000   47647800.000000
74        54 65   30.240600   798662000.000000    3446380.000000
75        65 66   30.240600   798662000.000000    3446380.000000
76        66 61   30.240600   798662000.000000    3446380.000000
77        54 67   30.240600   798662000.000000    3446380.000000
78        67 68   30.240600   798662000.000000    3446380.000000
79        68 51   30.240600   798662000.000000    3446380.000000
80        64 69   30.240600   798662000.000000    3446380.000000
81        69 70   30.240600   798662000.000000    3446380.000000
82        70 61   30.240600   798662000.000000    3446380.000000
*ENDBEAMS
*DAMPING
0.200000 0.000100
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