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PHASE COMPENSATION OF RATE LIMITERS IN UNSTABLE AIRCRAFT

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Abstract

For a modern aerodynamically unstable fighter, like the JAS 39 Gripen, the flight control system typically provides 45° phase margin. Therefore rate limiting of control surfaces, which may cause large phase shifts, is an important issue. Software rate limiters are placed on the control servo commands in order to prevent the hydraulic servos from rate limiting. When a rate limiter is saturated, the phase shift drastically reduces the stability margins of the closed loop and increases the risk for pilot-induced oscillation (PIO). This paper describes a novel method for compensating the phase shift of a rate limiter. In contrast to earlier phase compensation methods, this method uses feedback instead of logic or feedforward. Open loop and closed loop properties of the method are discussed. The method gives a drastic improvement on stability margins and reduces PIO tendencies.

1. Introduction

In the past few years rate limiting has become an important issue in flight control system (FCS) design for modern fighter aircraft. Typically these aircraft are aerodynamically unstable and have very high demands on agility, precision, and flying qualities.

The hydraulic control surface servos have limited rates. In order not to rate saturate the hydraulic servos, software rate limiters are placed in the FCS immediately before the servo command outputs.

When a rate limiter is saturated using sinusoidal input, the output has a smaller amplitude and a significant phase shift compared to the input signal. Especially the phase shift is critical since it drastically reduces the stability margin of the closed loop. For an aerodynamically unstable aircraft the FCS may typically not provide more than 45° phase margin, and a rate limiter easily gives such phase shifts. A corresponding phase shift also affects the pilot (outer) loop and creates an effect corresponding to a time delay. This extra delay explains why a pilot-induced oscillation (PIO) is more likely to occur once the pilot has started to command with large and rapid stick inputs. PIO caused by rate limiting has been observed on many modern military and civil aircraft, and some of these PIO incidents have led to loss of control.

Before getting into the details of phase compensation of rate limiters, it may be interesting to first discuss why this approach is chosen in flight control systems. In general the control community deals with fully automatic systems, where the control law is completely specified. Here we instead have a human pilot as part of the control loop, and a pilot is not a linear time-invariant controller. Instead a pilot uses different control techniques, depending on the task, etc. This can be interpreted as different gains and bandwidths, different amounts of phase lead or lag, and sometimes bang-bang types of control. The pilot needs an aircraft response within roughly known limits in order to be able to control it properly. If the aircraft response satisfies such limits, stability and good performance is predicted when the pilot closes the loop.

Two very simple solutions which may prevent rate limiting are 1) reducing a gain, or 2) including an extra low pass filter, both on the stick input signal. But this would either reduce the aircraft response to a very low level, e.g., a JAS 39 Gripen flying like a Boeing 777, or make the aircraft uncontrollable by adding to much phase lag. This phase lag would be there also for small stick inputs, while the lag filter was inserted only as a protection against very large and rapid stick inputs.

Constrained predictive control is another possible control strategy which handles state and control constraints. However, these methods are computationally more expensive compared to ordinary flight control laws. They are also typically used in applications where set points are constant for longer periods of time. In order to make these methods work really well in a flight control system it would be necessary to also predict the pilot input, but this is not possible.

In Section 2 different rate limiters will be discussed. Earlier algorithms for phase compensation of rate limiters typically involve logical expressions, if-then-else constructions, where states and outputs of the algorithm are determined by conditions on the sign and magnitude of, e.g., the input rate, see [1], [2], and [3]. Other algorithms use feedforward, see [4]. The novel phase compensation methods described in this paper rely solely on feedback. The methods are inspired by anti-windup methods, see [5], which feed back the difference between the input and the output of a nonlinearity. The methods, which have been filed for patent rights, will be described in detail. In Section 3 the different rate limiters will be analysed.

It was found that stability results for rate limiters could always be explained by the describing function (DF) method, see [6] and [7]. This will be further discussed in Section 4. This section will also demonstrate the stability properties of conventional and compensated rate limiting on an aircraft, in this case the JAS 39 Gripen.

The novel phase compensation methods are now used in production software for the JAS 39 Gripen FCS, i.e., all conventional rate limiters have been replaced by phase compensated rate limiters. More results on the tests and evaluations of the phase compensated rate limiters, and the resulting flying qualities in the JAS 39 Gripen are found in [8] and [9] respectively. Finally, conclusions are given in Section 5.

2. Rate limiters

This section will discuss different rate limiter algorithms. Unless otherwise stated the rate limit $r = 1$ (unit/s). A simple continuous time model of a conventional rate limiter is given in Figure 1 a. This model illustrates that the rate limiter output y always tries to be equal to the input u. The gain K must be chosen sufficiently high compared to the frequencies in u. For the sinusoidal input $u(t) = a\sin\omega t$, rate limiting occurs if

$$\rho = \frac{r}{a\omega} < 1, \quad (1)$$

where ρ is a nondimensional parameter, and r the rate limit. For $\rho \geq 1$ there is no rate limiting. In the sequel rate limiter elements are denoted by the symbol in Figure 1 b.

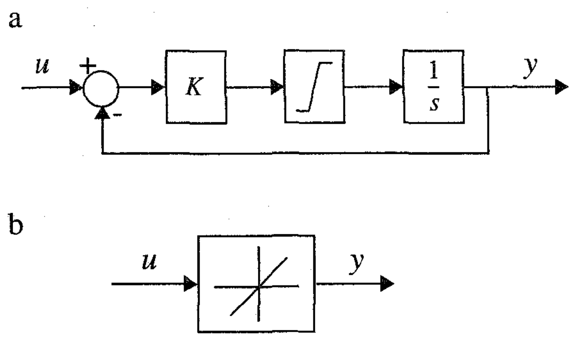

Figure 1. a) A simple model of a rate limiter element.
b) Rate limiter symbol.

Earlier phase compensation methods

The idea of avoiding or compensating for rate limitations is not new, see [1]-[4]. In [4] two algorithms, which both reduce a gain, are evaluated. One of the algorithms reduce a gain as a function of input frequency, the other as a function of input rate. By reducing a gain, rate limiting is avoided. These algorithms are intended for use in the pilot command path of the control law and were developed at NASA after a PIO incident during the first Space Shuttle runway landing.

In [1]-[3] phase compensation is obtained essentially by a three step procedure, differentiate - limit - integrate, see Figure 2. Thus we will always have

$$\operatorname{sgn}\left(\frac{dy}{dt}\right) = \operatorname{sgn}\left(\frac{du}{dt}\right), \quad (2)$$

i.e., the output y will always have the same direction as the input u, in other words perfect phase compensation. However, after rate limiting we will usually have $y \neq u$ since the limiter has discarded information. In order to make $y = u$, logical conditions on, e.g., the input rate du/dt, are used to select other inputs to the integrator. This could be denoted "input recovery", and the method will then resemble Figure 1 a. It is often possible to construct inputs u which give undesirable outputs y due to "unforeseen" effects of the logic. The methods are also noise sensitive, since they may rapidly shift between phase compensation and input recovery, see [1] and [3].

Rate limiters with feedback

The inspiration to the novel methods came from conventional anti-windup methods, see [5], which among other things have the property of increasing or advancing the phase of a transfer function around the nonlinearity. In anti-windup an error signal, the difference between the input and output of a nonlinearity, is fed back in order to adjust, or stabilize, some of the controller states. The first attempt was to feed back error signals to an integrator. A drawback with this approach was that the integrator had usually built up an unacceptable bias when rate limiting ceased.

Instead a stable low pass filter was used, see Figure 3. When the rate of the input signal u is greater than the rate limit r, the feedback signal becomes negative and reduces the input signal to the rate limiter. If the input reverses direction the output will almost immediately reverse direction too, i.e., less phase shift is obtained. When the input u

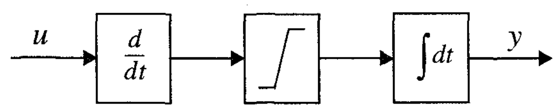

Figure 2. A simple phase compensation method.

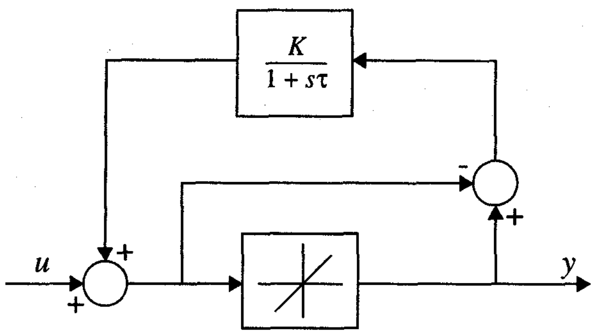

Figure 3. Rate limiter with feedback.

has a rate smaller than r, the feedback signal decays to zero. The low pass filter gives less phase compensation compared to an integrator, but on the other hand it does not give any bias either.

It was found that the circuit in Figure 3 is not a good solution in the presence of high frequency signals. The reason is that high frequency components of the input signal u almost blocks the circuit for low frequencies. For high frequencies the circuit does not provide phase compensation and it is not necessary either. Thus the problem would be solved if only the low frequency components of u were phase compensated. A number of different rate limiter circuits were tested, and the circuit in Figure 4 was finally selected. Since the low frequency components of u are limited by the first rate limiter (with phase compensation), only the high frequency components of u are limited in the second (uncompensated) rate limiter. Note that both rate limiters have equal limits.

The compensated rate limiters are protected by patent rights (Figure 3) or have been filed for patent rights (Figure 4).

3. Analysis and simulation of rate limiters

The properties of the different rate limiter circuits were studied by means of analysing the magnitude and phase properties of the describing function (DF), see [7], and by carrying out simulations for different input signals.

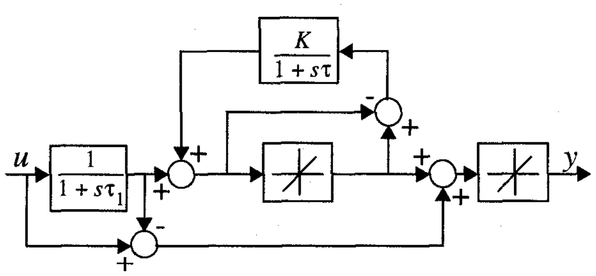

Figure 4. Rate limiter with feedback and bypass.

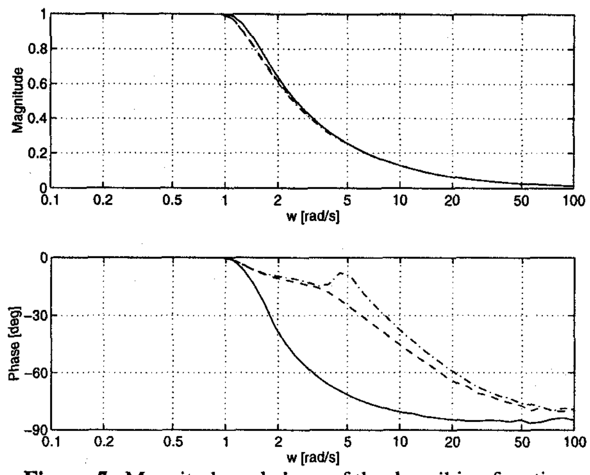

Figure 5. Magnitude and phase of the describing function $Y_N(1, \omega)$ for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

Since rate limiters are dynamic nonlinearities, the describing functions Y_N depend on both amplitude and frequency of the input signal. The describing function for a nonlinearity N with input $u(t) = C\sin\omega t$ and output $y(t)$ is given by

$$Y_N(C, \omega) = \frac{b_1 + ia_1}{C} = \frac{c_1 e^{i\phi}}{C} \tag{3}$$

where the Fourier coefficients a_1 and b_1 are given by

$$\begin{aligned} a_1 &= \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} y(t)\cos(\omega t)\,dt \\ b_1 &= \frac{\omega}{\pi}\int_0^{\frac{2\pi}{\omega}} y(t)\sin(\omega t)\,dt \end{aligned} \tag{4}$$

and where $c_1 = \sqrt{a_1^2 + b_1^2}$ and $\phi = \operatorname{atan}(a_1/b_1)$ are the magnitude and phase respectively of the DF. The Fourier coefficients of the DF's were calculated using numerical integration.

In Figure 5 the magnitude and phase of different rate limiter DF's are shown as function of frequency for the input signal amplitude 1. Filter parameters are $K = 8$, $\tau = 1$ and $\tau_1 = 0.1$. In the figure it is shown that the feedback has almost no effect on the amplitude, while the phase is improved for compensated rate limiters, especially around 5 rad/s. The gain $K \approx 10$ is a good choice, while τ and τ_1 must be chosen with respect to, e.g., the bandwidth of the application, but at least $\tau_1 < \tau$ should hold.

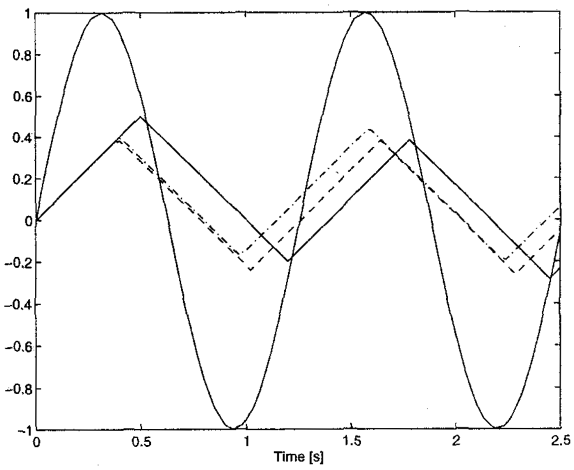

Figure 6. Responses to $u = \sin(5t)$ for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

In Figure 5 it is noted that the circuit with bypass, see Figure 4, has a better phase compensation compared to the first circuit, see Figure 3. The explanation is that there is phase advance in both the high frequency and low frequency paths, and this phase advance is preserved when the two signals are added.

In Figure 6 the responses to $u(t) = \sin(5t)$ for the different circuits are shown. The amplitude is about the same for all circuits, while the phase shift is smaller for the rate limiters with feedback.

The responses of the rate limiters to a step input are shown in the first part of Figure 7. The convergence rate of the responses from rate limiters with feedback depends on the time constant τ. It may seem as if it is better to stay as long as possible in the rate limit. But if the step input is followed by another large command in the opposite direction it may

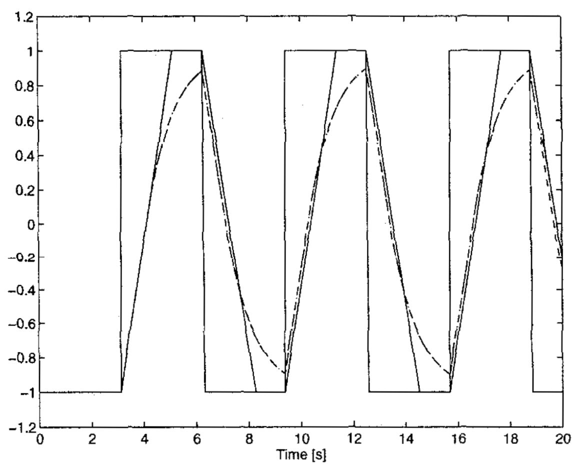

Figure 7. Response to square wave for a conventional rate limiter (solid), rate limiter with feedback (dashed) and rate limiter with feedback and bypass (dash-dotted).

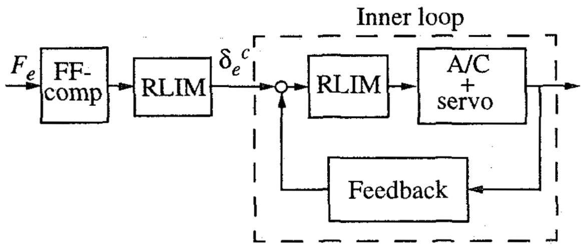

Figure 8. Simple block diagram of the stabilizing (inner) loop and the pilot command path.

be an advantage that the response is a bit slow. This is found when studying the response to a square wave, i.e., the rest of Figure 7. Due to the exponential decay of the feedback filter the output amplitude of the compensated rate limiter is reduced compared to the conventional rate limiter. Thus these rate limiters rather are equivalent to a smaller gain for square wave inputs. The phase is less important as long as the output reverses direction when the input does.

4. Rate limiter properties in closed loop

In [6] it was found that the stability properties during rate limiting could always be predicted or explained by the DF method. Without disturbances the compensated rate limiters always had better stability properties in closed loop. But when high-frequency disturbances were added it was found that any of the three investigated rate limiters (Figure 1, Figure 3 and Figure 4) could be the better one or the worst one. It was all depending on the rest of the loop, i.e., the controller and the process. The measure of good and bad was the disturbance amplitude, and since all processes in the tests were unstable, it was only a matter of which amplitude that was required for destabilization.

In order to demonstrate the closed loop stability properties on a modern aerodynamically unstable aircraft using the different rate limiters, a linear model of the JAS 39 Gripen for a medium speed flight condition is used. In this section both inner loop stability properties and pilot in the loop stability properties will be discussed. The FCS typically consists of a stabilizing inner loop and a pilot command path, which is part of the "outer loop", see Figure 8. The pilot command is rate limited and the sum of the pilot command and the feedback command is also rate limited before it is fed to the control servos.

Inner loop stability

Let the linear open loop dynamics of the stabilizing inner loop, i.e., the servo, the aircraft, and the stabilizing feedback, have transfer function G. Y_N is the describing function of a rate limiter. Since Y_N depends on both amplitude C and frequency ω, it was chosen to plot the loop transfer

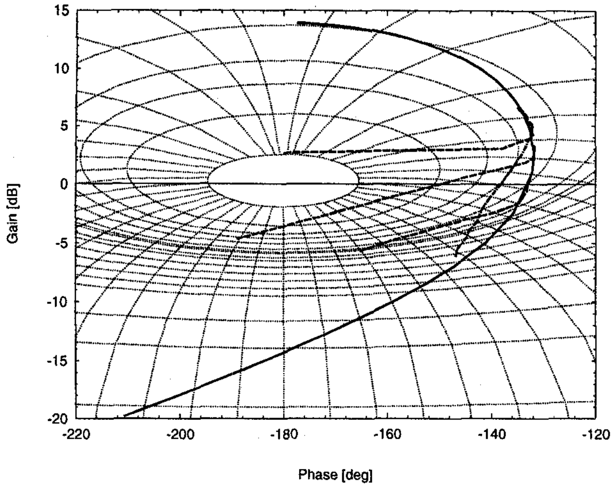

Figure 9. Nichols diagram for the linear transfer function G (solid), and the nonlinear loop transfers $Y_N G$ using a conventional rate limiter (dashed) for 3 different amplitudes, and rate limiter with feedback and bypass (dash-dotted). The upper dashed curve and the dash-dotted curve both correspond to the maximum input amplitude.

$Y_N(C, \omega) \cdot G(i\omega)$, using both a conventional rate limiter and a phase compensated rate limiter with bypass, in a Nichols diagram together with the linear transfer function G, see Figure 9. The loop transfers $Y_N G$ were calculated in closed loop. A sinusoidal input signal with fixed amplitude was applied at the input, δ_e^c, and the sensitivity functions, $S = (1 + Y_N G)^{-1}$, using rate limiters, were computed by numerical integration. Then the open loop transfer functions $Y_N G$ were computed and plotted in the Nichols diagram, see Figure 9.

A typical aerodynamically unstable aircraft has a phase margin of about 45° with FCS, when no rate limiter is saturated. The extensive phase shift from a saturated conventional rate limiter causes stability problems as the amplitude of the input is increased and the rate limiting onset frequency is decreased. The inner loop will be unstable if the sinusoidal input δ_e^c to the inner loop has maximum amplitude and the frequency is sufficiently high, see the upper dashed curve in Figure 9. If on the other hand phase compensated rate limiters with bypass are used, see Figure 4, then the phase shift is so small that there is no problem with the closed loop stability margins, see the dash-dotted curve in Figure 9.

Handling quality properties

Now the handling quality properties of the pilot loop are to be examined. Let the linear dynamics (FCS, servo, and aircraft dynamics) in the pilot loop, see Figure 8, have transfer function G_p. Also, let the corresponding nonlinear loop

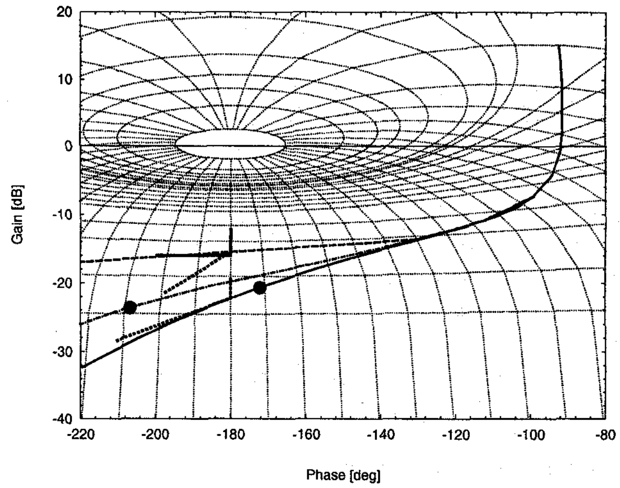

Figure 10. Evaluation against the Gibson handling quality criterion, see [10], for the linear "outer loop" G_p (solid), and the nonlinear "outer loop" $Y_N G_p$ using a conventional rate limiter (dashed), and rate limiter with feedback and bypass (dash-dotted), in both cases for maximum input amplitude. The 1 Hz points are marked with dots, and the 16 dB gain margin and the phase rate limits (-70°/Hz) are shown, except for the dash-dotted curve.

transfer be denoted $Y_N G_p$ for both a conventional rate limiter and a phase compensated rate limiter. In this case Y_N is a describing function which picks up the effects of both the rate limiters in Figure 8. The loop transfers $Y_N G_p$ are computed by numerical integration using sinusoidal inputs F_e, for both a conventional rate limiter (Figure 1) and a rate limiter with feedback and bypass (Figure 4). These loop transfers are plotted together with the linear transfer function G_p in a Nichols diagram, see Figure 10.

The transfer function G_p fulfils all parts of the Gibson handling quality criterion, see [10], i.e., 16 dB gain margin, phase rate at -180° is > -70°/Hz, and the frequency at -180° is > 1 Hz, see Figure 10. Thus PIO is not predicted as long as small stick inputs are used. If the input F_e has maximum amplitude and conventional rate limiters are used, the phase shift from the saturated rate limiters will increase the tendency for PIO according to the Gibson criterion. As can be seen from Figure 10 (the dashed curve) all three parts in the Gibson criterion will be violated. If instead phase compensated rate limiters with bypass are used, the phase shift from the saturated rate limiters are smaller and the PIO tendency will drastically decrease. However, two of the parts in the Gibson criterion are still violated but not as much as when conventional rate limiters are used.

5. Conclusions

This paper has described feedback-based phase compensation of rate limiters. The methods have been analysed both in open and closed loop and they give a significant reduction of the phase shift. The describing function method is a reliable method for stability analysis of these problems. If high-frequency disturbances are present, the compensated rate limiters are, however, not always better than a conventional rate limiter.

Phase compensation of rate limiters has been one of the keys in obtaining good and safe performance and handling qualities of the JAS 39 Gripen. Extensive analysis, simulations and flight tests have been carried out to ensure that rate limiting is no threat to the good handling qualities of the JAS 39 Gripen. The FCS editions yield high authority, accuracy, and good predictability for both small and large stick inputs and are now qualified for production use.

Using the novel methods above, the pilot will not get the full response during rate limiting but, quoting one of the JAS 39 Gripen test pilots, "you get a natural reduction of the response and you are never out of control". The reason is that when the pilot moves the stick in the other direction, the aircraft responds to his command without noticeable time delay.

It should be pointed out that rate limiting should not take place during normal tasks. But a pilot is capable of making surprisingly fast stick movements during special circumstances. Instead of reducing the forward gain in order to avoid rate limiting, it is now possible to use phase compensated rate limiters as a safety against fast stick input, but still keeping feedforward gains relatively high, which preserves a high agility of the aircraft.

6. Acknowledgements

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