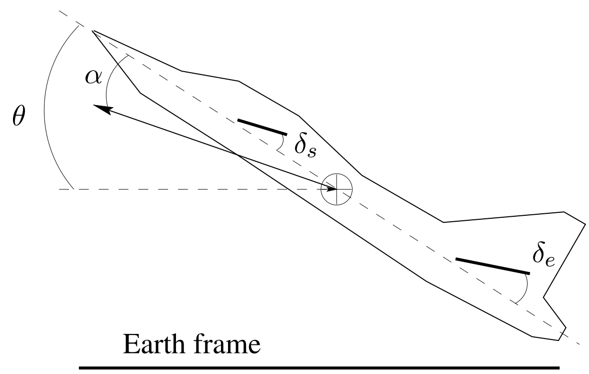
1.4 Homework 3

Simulation and control of the pitch angle dynamics of a JAS 39 Gripen is discussed in this homework, but first we give some background. The dynamics of an airplane is highly nonlinear. The control system is based on gain scheduling of speed and altitude to compensate for some of the nonlinearities. For the JAS 39 Gripen, linear models have been obtained for approximately 50 different operating conditions. The models are the result of extensive research including wind tunnel experiments. A linear controller is designed for each linear model, and a switching mechanism is used to switch between the different controllers. Many of the parameters in the models vary considerably within normal operation of the aircraft. Two extreme cases are "heavily loaded aircraft at low speed" and "unloaded aircraft at high speed". The case when the velocity of the aircraft is approximately equal to the speed of sound is also critical, because at higher velocities the aircraft is stable while for lower it is unstable. The velocity of an aircraft is specified by the Mach number $M = v/a$, which is the conventional velocity v normalized by the speed of sound a.

The following sketch of the JAS 39 Gripen illustrate the state variables of the system:



The direction in which the aircraft is pointing defines the pitch angle θ. The pitch angle does not necessarily equal the angle of the velocity direction of the aircraft (indicated by an arrow from the center of mass). The difference is the angle of attack α. JAS 39 Gripen has two surfaces for controlling the pitch angle: the spoiler and the elevator. Their corresponding angles are denoted δ_s and δ_e, respectively. The state-space model considered in the homework consists of the seven states

α	angle of attack
$q = \dot{\theta}$	pitch rate
θ	pitch angle
δ_e	elevator angle
δ_s	spoiler angle
x_e	internal elevator state
x_s	internal spoiler state

where the latter two states correspond to dynamics in the rudder servos. The control inputs are

u_e	elevator command
u_s	spoiler command

We consider a linear state-space model for normal load at $M = 0.6$ and altitude 1 km. This model has been released by SAAB Military Aircraft and discussed in a master thesis at Chalmers [1]. The model is given by

$$\dot{x} = Ax + Bu,$$

where

$$x = (\alpha, q, \theta, \delta_e, \delta_s, x_e, x_s)^T, \qquad u = (u_e, u_s)^T.$$

Download the Matlab files from the course homepage to your current directory and introduce the state-space model into the Matlab workspace by running

```
>> jasdata
```

1. [1p] Which are the dynamical modes of the JAS 39 Gripen state-space model, i.e., which are the eigenvalues of A? Is the model stable? Which eigenvalues correspond to the flight dynamics and which correspond to the rudder dynamics?

The aircraft is controlled by linear state feedback

$$u(t) = -Lx(t) + (K_f, K_f)^T u^f_{\text{pilot}}(t),$$

where the matrix L is derived using linear quadratic control theory and the scalar K_f is chosen such that the steady-state gain is correct. The internal elevator and spoiler states are not used in the controllers, so $L_{16} = L_{17} = L_{26} = L_{27} = 0$.[6] The feedback stabilizes the angle of attack α and the pitch rate q. The pitch angle θ is not stabilized (and not used in the controller since $L_{13} = L_{23} = 0$), but the control of this mode is left for the pilot. The signal u^f_{pilot} is the filtered pilot command signal:

$$u^f_{\text{pilot}} = \frac{1}{T_f s + 1} u_{\text{pilot}}.$$

Write

```
>> planemodel
```

to see a Simulink model of the aircraft. Match the blocks in the model with the equations above.

2. [1p] Choose a nominal design for the state feedback by typing

```
>> design1
```

Look at the L-matrix. Which states are used in the feedback loop? Which are the eigenvalues of $A-BL$? Why is there an eigenvalue close to the origin?

The pilot may be modeled as a PD-controller with a time delay of 0.3 s. Argue why this is a reasonable model. Let

$$u_{\text{pilot}} = K_p \frac{1 + T_d s}{1 + T_d/N s} e^{-0.3s}(\theta_{\text{ref}} - \theta),$$

where θ_{ref} corresponds to the desired pitch angle. Set $K_p = 0.2$, $T_d = 0.5$, and $N = 10$. Simulate the closed-loop system including the pilot. Check the magnitudes of the rudder angles (`plot(t,x(:,[4 5]))`). Why is it important that the rudder angles are not too large?

The PD-controller pilot model can be seen as a rational pilot. In an emergency situation, however, the pilot may panic and try to compensate the error $\theta_{\text{ref}} - \theta$ with maximal command signals.[7] This behavior may induce oscillations in the system. They are called pilot induced oscillations (PIO) and got considerable attention after the crash in Stockholm in 1993. Next we do a simplified analysis illustrating what happened.

[6] In reality there are no measurements of the rudder angles δ_e and δ_s. They are estimated using a Kalman filter.

[7] Imagine yourself in a balancing act. When you are in control, you can keep the balance using very small motions, but as soon as you are a little out of control, your movements tends to be very large. This is typical for systems with slow unstable dynamics, which cannot react fast enough to the control commands signals.

3. [1p] In order to analyze the PIO mode, we will replace the PD-controller pilot by a relay model. The pilot then gives maximal joystick commands based on the sign of θ. Such a "relay pilot" can be found in the Simulink pilot model library; run

```
>> pilotlib
```

Plot the Nyquist curve of the linear system from u_{pilot} to θ. This can be done by deleting the feedback path from θ, connecting an input and an output at appropriate places (inputs and outputs are found in the `simulink` libraries), saving the system to a new file and using the `linmod` and `nyquist` commands.

Change pilot in the plane model by deleting the PD-controller pilot and inserting the "relay pilot". The describing function for a relay is

$$N(A) = \frac{4D}{\pi A}$$

What is D for the "relay pilot"? Use describing function analysis to predict amplitude and frequency of a possible limit cycle. Simulate the system. How good is the prediction?

As you saw, the amplitude of the PIO is moderate. This is because the flight condition is high speed and high altitude, and thus not extreme. Let us anyway discuss ways to reduce PIO.

4. [1p] Use `design2` to change L and K_f to a faster design. Is the PIO amplitude decreased? Make the pilot filter faster by reducing the filter time constant to $T_f = 0.03$ (`design3`). Is the PIO amplitude decreased? Discuss the results using the describing function method and thus plot the Nyquist curves from $u_{\texttt{pilot}}$ to θ. Are there any drawbacks with the design that gives smallest PIO amplitude?

5. [1p] Suggest a control strategy for reducing PIO in JAS 39 Gripen, with minimal influence on the pilots ability to manually control the plane. Analyze the performance of your strategy and compare it to the previous two designs. It should outperform the previous ones.

Extra [0p] There are no rate limitations on the rudders in the discussed aircraft model. Rate limitations were, however, part of the problems with the JAS 39 Gripen control system. Introduce rate limitations as in the article [2], and investigate what happens to the limit cycle. Try to understand the idea of the (patented) nonlinear filter.

References

[1] Axelsson, L., *Reglerstudier av back-up regulator för JAS 39 Gripen*, MSc Thesis, CTH/SAAB Flygdivision, 1992.

[2] Rundquist, L., K. Ståhl-Gunnarsson, and J. Enhagen, "Rate Limiters with Phase Compensation in JAS 39 Gripen," European Control Conference, 1997.