

Assignment 4

1. Slowest to fastest

$$n!, 3^n, 4n^2, 20n, n^{2/3}, \log_2 n, \log_3 n, 2$$

2. a) $T(n) = 3 \cdot 2^n$

$$3 \cdot 2^x = 64 \cdot T(n) = 64 \cdot 3 \cdot 2^n$$

$$= 3 \cdot 2^n \cdot 2^6$$

$$= 3 \cdot 2^{n+6}$$

$$\Rightarrow \boxed{x = n+6}$$

b) $T(n) = n^2$

$$x^2 = 64 \cdot n^2$$

$$\sqrt{x^2} = \sqrt{64n^2}$$

$$\boxed{x = 8n}$$

c) $T(n) = 8n$

$$\frac{8x}{8} = \frac{64 \cdot 8n}{8}$$

$$\boxed{x = 64n}$$

3. n) $x = 100 \cdot n$

$$\boxed{x = 100n}$$

$$n^2) \sqrt{x^2} = \sqrt{100n^2}$$

$$\boxed{x = 10n}$$

$$n^3) \sqrt[3]{x^3} = \sqrt[3]{100n^3}$$

$$\boxed{x = \sqrt[3]{100} n}$$

$$2^n) 2^x = 100 \cdot 2^n$$

$$= 2^{\log_2 100} \cdot 2^n$$

$$\boxed{x = \log_2 100 + n}$$

4a.) $f(n) = \log n^2, g(n) = \log n + 5$

$$= 2 \cdot \log n$$

Ignoring coefficients and non dominant terms, both functions have the same growth of $\log n$, so $\boxed{f(n) = \Theta(g(n))}$

$$4b.) f(n) = \sqrt{n} = n^{1/2}, g(n) = \log n^2 = 2 \cdot \log n$$

$f(n)$ grows at a faster rate than $g(n)$, $\boxed{f(n) = \Omega(g(n))}$

$$4c.) f(n) = \log 2^n = n \cdot \log 2, g(n) = \log n$$

$f(n)$ grows at a faster rate than $g(n)$, $\boxed{f(n) = \Omega(g(n))}$

$$4d.) f(n) = n, g(n) = \log n$$

Same as 4c, $f(n)$ grows faster $\boxed{f(n) = \Omega(g(n))}$

$$4e.) f(n) = n \log n + n, g(n) = \log n$$

$f(n)$ grows faster than $g(n)$, $\boxed{f(n) = \Omega(g(n))}$

$$4f.) f(n) = \log n^2 = 2 \cdot \log n, g(n) = (\log n)^2 = \log n \cdot \log n$$

$f(n)$ grows slower than $g(n)$, $\boxed{f(n) = O(g(n))}$

$$4g.) f(n) = 10, g(n) = \log 10$$

Both $f(n)$ & $g(n)$ run at constant time, $\boxed{f(n) = \Theta(g(n))}$

$$4h.) f(n) = 2^n, g(n) = 10n^2$$

$f(n)$ is exponential time compared to $g(n)$ in quadratic, $\boxed{f(n) = \Omega(g(n))}$

$$4i.) f(n) = 2^n, g(n) = n \log n$$

$f(n)$ is exponential time compared to $g(n)$ in linearithmic, $\boxed{f(n) = \Omega(g(n))}$

$$4j.) f(n) = 2^n, g(n) = 3^n$$

Both $f(n)$ & $g(n)$ run at exponential time, $\boxed{f(n) = \Theta(g(n))}$

$$4k.) f(n) = 2^n, g(n) = n^n$$

$g(n)$ grows much faster compared to $f(n)$, $\boxed{f(n) = O(g(n))}$

5. a.) $\Theta(1)$

b.) $\Theta(n)$

c.) $\Theta(n^2)$

d.) $\Theta(n^2)$

e.) $\Theta(n \log n)$

f.) $\Theta(n \log n)$

g.) $\Theta(n^3 \log n)$

h.) $\Theta(n^2)$

i.) $\Theta(n)$