Sebastian, Ryan
ID#: 10933506

Assignment 4

1. Slowest to fastest

$n!,\ 3^n,\ 4n^2,\ 20n,\ n^{2/3},\ \log_2 n,\ \log_3 n,\ 2$

2. a) $T(n) = 3 \cdot 2^n$

$3 \cdot 2^x = 64 \cdot T(n) = 64 \cdot 3 \cdot 2^n$

$= 3 \cdot 2^n \cdot 2^6$

$= 3 \cdot 2^{n+6}$

$\Rightarrow \boxed{x = n+6}$

b) $T(n) = n^2$

$x^2 = 64 \cdot n^2$

$\sqrt{x^2} = \sqrt{64n^2}$

$\boxed{x = 8n}$

c) $T(n) = 8n$

$\frac{8x}{8} = \frac{64 \cdot 8n}{8}$

$\boxed{x = 64n}$

3. n) $x = 100 \cdot n$

$\boxed{x = 100n}$

n^2) $\sqrt{x^2} = \sqrt{100n^2}$

$\boxed{x = 10n}$

n^3) $\sqrt[3]{x^3} = \sqrt[3]{100n^3}$

$\boxed{x = \sqrt[3]{100}\, n}$

2^n) $2^x = 100 \cdot 2^n$

$= 2^{\log_2 100} \cdot 2^n$

$\boxed{x = \log_2 100 + n}$

4a.) $f(n) = \log n^2,\ g(n) = \log n + 5$

$= 2 \cdot \log n$

Ignoring coefficients and non dominant terms, both functions have the same growth of $\log n$, so $\boxed{f(n) = \Theta(g(n))}$

4b.) $f(n) = \sqrt{n}$, $g(n) = \log n^2$
$= n^{1/2}$ $\quad = 2 \cdot \log n$
$f(n)$ grows at a faster rate than $g(n)$, $\boxed{f(n) = O(g(n))}$

4c.) $f(n) = \log 2^n$, $g(n) = \log n$
$= n \cdot \log 2$
$= n$
$f(n)$ grows at a faster rate than $g(n)$, $\boxed{f(n) = O(g(n))}$

4d.) $f(n) = n$, $g(n) = \log n$
Same as 4c, $f(n)$ grows faster, $\boxed{f(n) = O(g(n))}$

4e.) $f(n) = n\log n + n$, $g(n) = \log n$
$= n\log n$
$f(n)$ grows faster than $g(n)$, $\boxed{f(n) = O(g(n))}$

4f.) $f(n) = \log n^2$, $g(n) = (\log n)^2$
$= 2 \cdot \log n$ $\quad = \log n \cdot \log n$
$= \log n$
$f(n)$ grows slower than $g(n)$, $\boxed{f(n) = \Omega(g(n))}$

4g.) $f(n) = 10$, $g(n) = \log 10$
Both $f(n)$ & $g(n)$ run at constant time, $\boxed{f(n) = \Theta(g(n))}$

4h.) $f(n) = 2^n$, $g(n) = 10n^2$
$f(n)$ is exponential time compared to $g(n)$ in quadratic, $\boxed{f(n) = O(g(n))}$

4i.) $f(n) = 2^n$, $g(n) = n\log n$
$f(n)$ is exponential time compared to $g(n)$ in linearithmic, $\boxed{f(n) = O(g(n))}$

4j.) $f(n) = 2^n$, $g(n) = 3^n$
Both $f(n)$ & $g(n)$ run at exponential time, $\boxed{f(n) = \Theta(g(n))}$

4k.) $f(n) = 2^n$, $g(n) = n^n$
$g(n)$ grows much faster compared to $f(n)$, $\boxed{f(n) = \Omega(g(n))}$

5. a.) $\Theta(1)$
b.) $\Theta(n)$
c.) $\Theta(n^2)$
d.) $\Theta(n^2)$
e.) $\Theta(n\log n)$
f.) $\Theta(n\log n)$
g.) $\Theta(n^3\log n)$
h.) $\Theta(n^2)$
i.) $\Theta(n)$