

# PROOF OF STORAGE

## SHOW ME YOUR MEMORY

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@rssh1

github/rssh



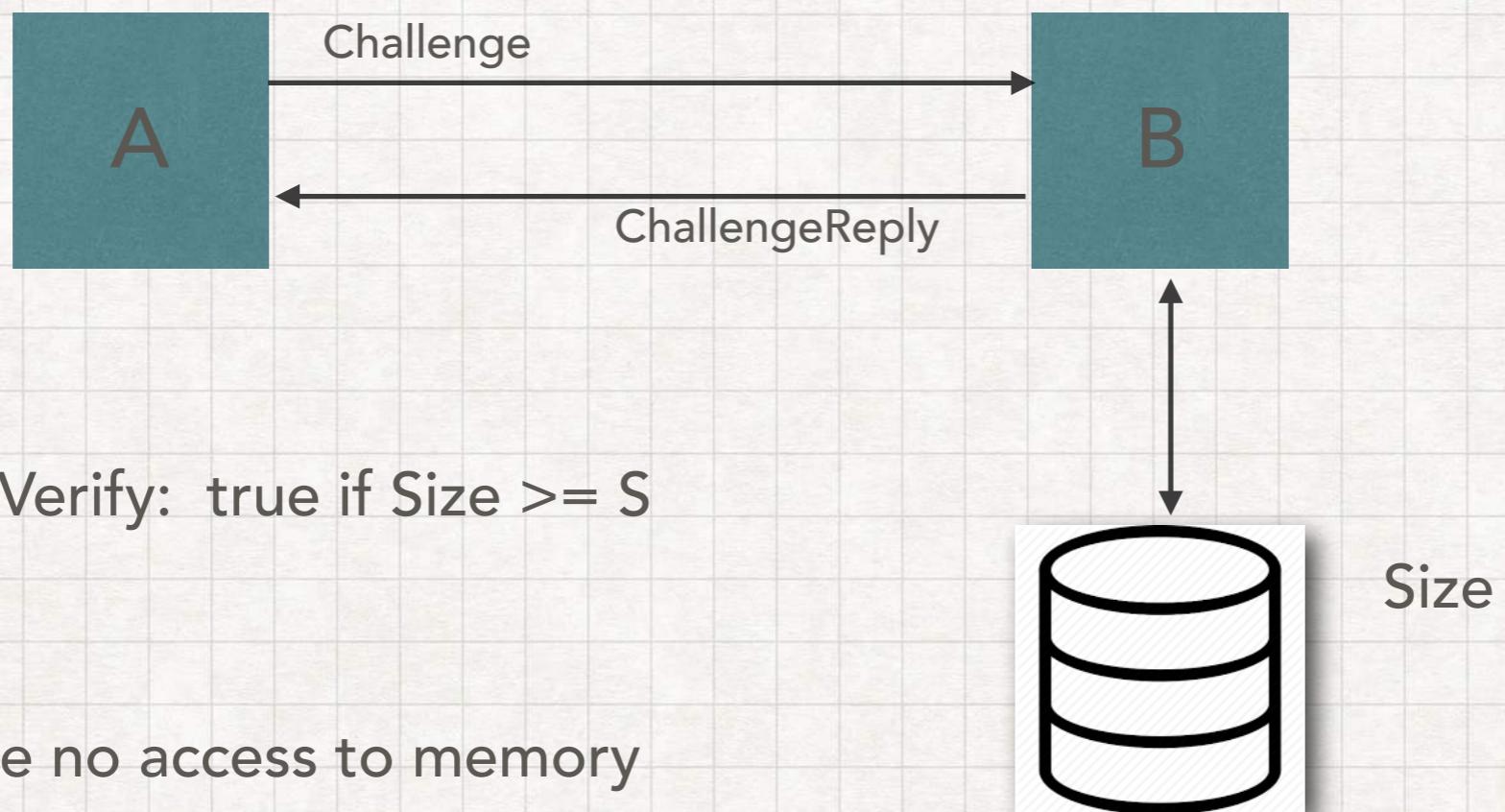
garuda.ai

<http://garuda.ai>

# PROOF OF STORAGE

HOW TO PROVE THAT I HAVE MEMORY ?

Interactive protocol

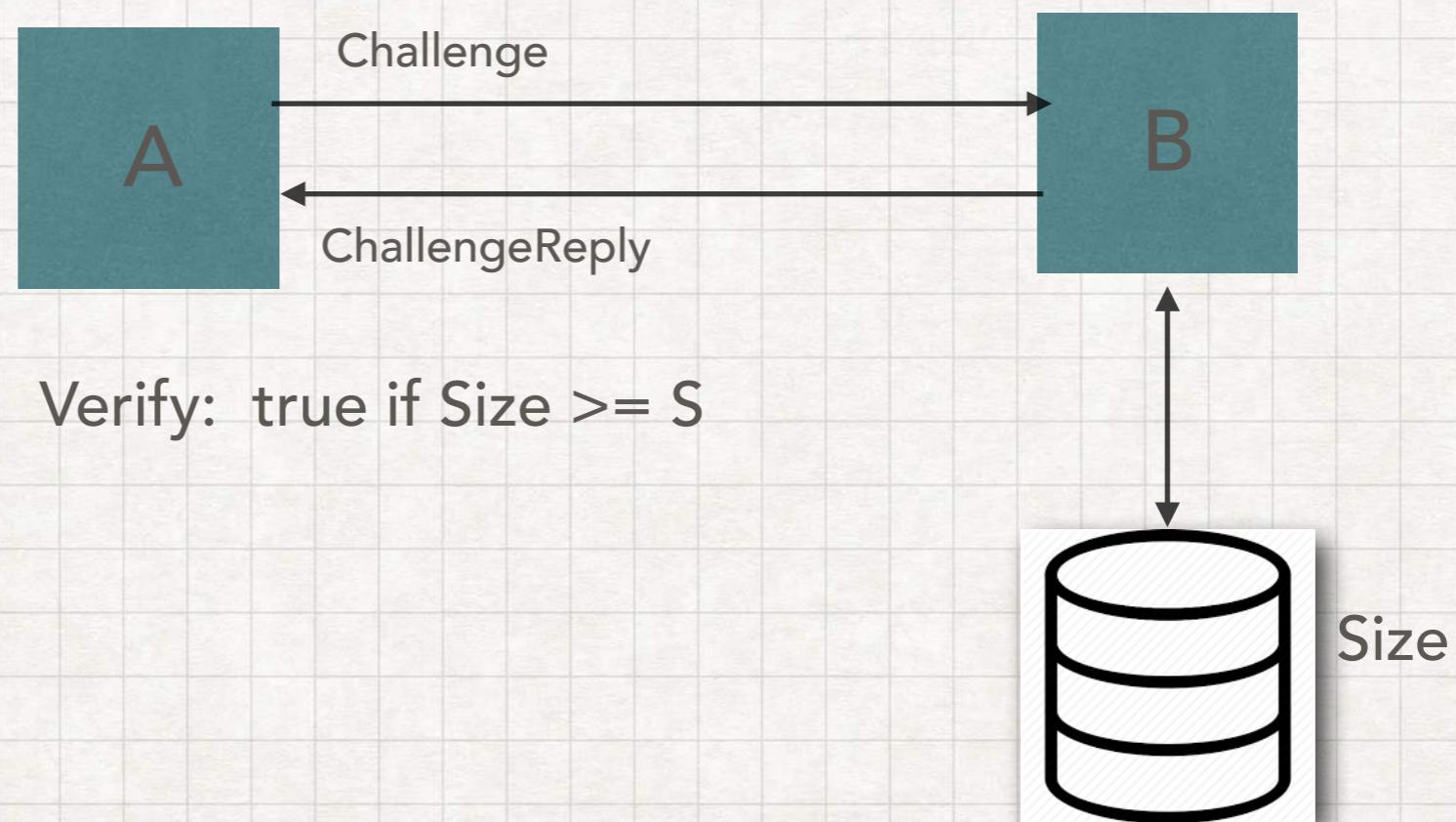


# PROOF OF STORAGE

## HOW TO PROVE THAT I HAVE MEMORY ?

?

- Ideas



# PROOF OF STORAGE

## HOW TO PROVE THAT I HAVE MEMORY ?



- Ideas

Strictly Incorrect, but useful:

- Inversion of one-way function.
- Meet in the middle collision attack

Base solution:

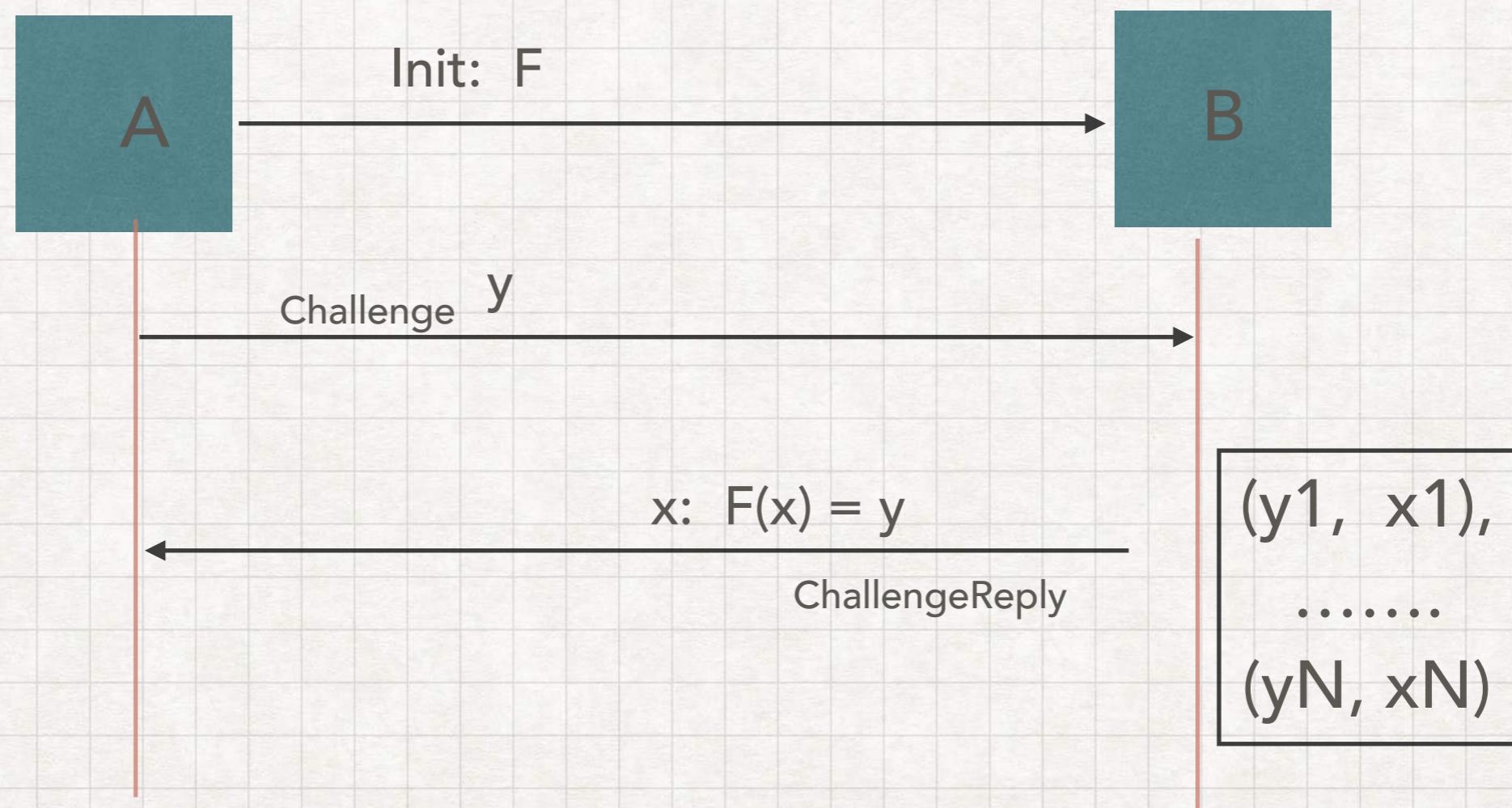
- Pebbling game.

# MEMORY HARD ALGORITHM (?)

## INVERSION OF ONE-WAY FUNCTION

- Init: one-way function F
- Challenge: y
- Reply:  $x: F(x) = y$

*Incorrect*



# MEMORY HARD ALGORITHM (?)

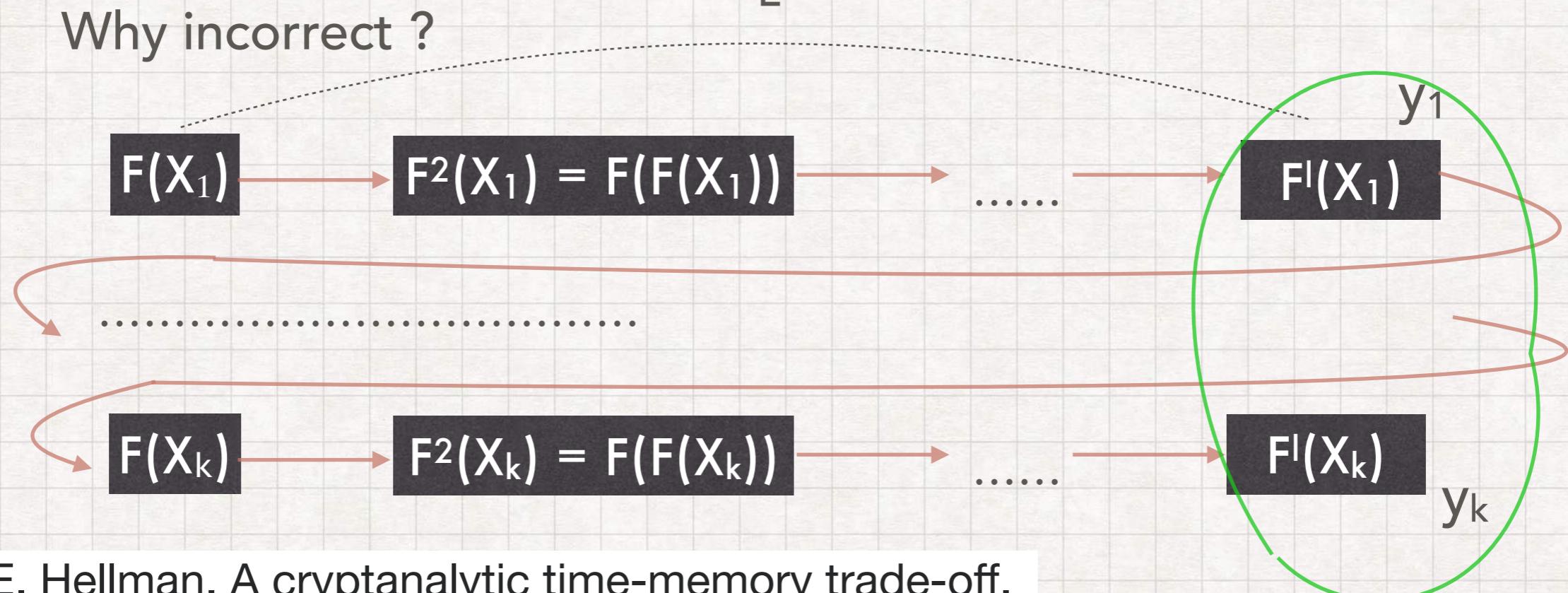
## INVERSION OF ONE-WAY FUNCTION

- Init: one-way function  $F$
- Challenge:  $y$
- Reply:  $x: F(x) = y$

Hellman Construction

$$N \Rightarrow N^{\frac{2}{3}}$$

Why incorrect ?



Martin E. Hellman. A cryptanalytic time-memory trade-off.

IEEE Transactions on Information Theory, 26(4):401–406, 1980

# MEMORY-HARD ALGORITHMS

MEET IN THE MIDDLE ATTACK

**Incorrect**

$$k = (k_1, k_2) \quad k \in 2^n, k_1 \in 2^{n/2}, k_2 \in 2^{n/2}$$

$$E(k, x) = E(k_1, E(k_2, x))$$

Many ciphers have such structure. (DES)

$$y = E(x) \quad \text{we know } x \& y \text{ for some case, let find } k$$

$$E^{-1}(k_1, y) = E(k_2, x)$$

enumeration  $k_1$

$$E^{-1}(k_1, y)$$

sort  
=

enumeration  $k_2$

$$E(k_2, x)$$

$2^{\frac{n}{2}+1}$  Memory

$2^{\frac{n}{2}+1}$  Time

# MEMORY-HARD ALGORITHM (?)

MEET IN THE MIDDLE ATTACK

Why incorrect ?

Pollard memoryless collision search.

Calculate  $E(E(\dots(x)\dots))$  and search collision at the same time.

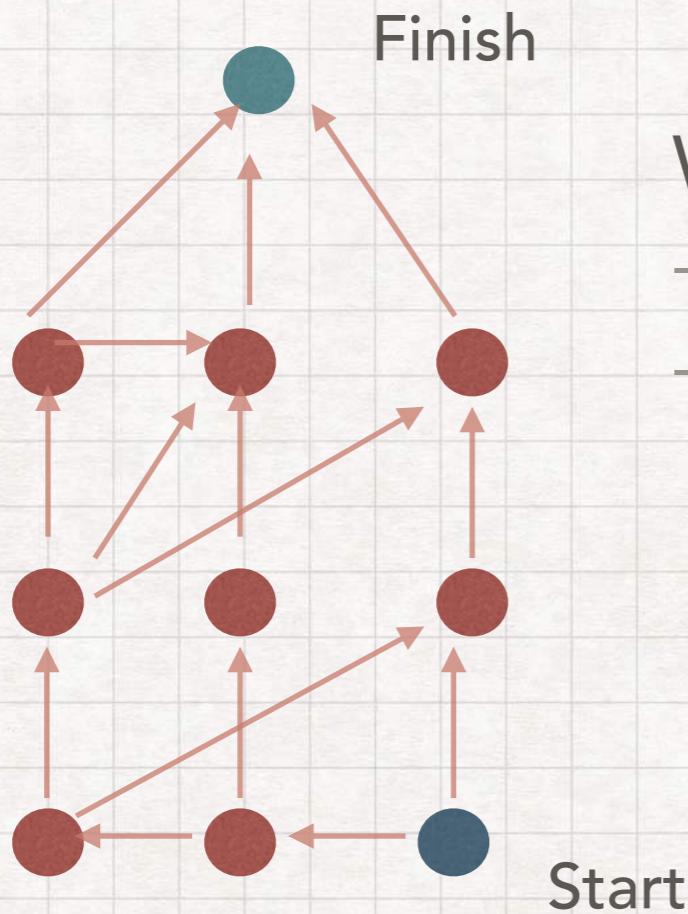
$$Time \times Memory = C$$

$$\boxed{\begin{array}{l} \text{enumeration } k_1 \\ E^{-1}(k_1, y) \end{array}} \stackrel{\text{sort}}{=} \boxed{\begin{array}{l} \text{enumeration } k_2 \\ E(k_2, x) \end{array}}$$

$$\boxed{\begin{array}{l} 2^{\frac{n}{2}+1} \\ Time \end{array}} \quad \boxed{\begin{array}{l} 2^{\frac{n}{2}+1} \\ Memory \end{array}}$$

# PEBBLE GAME

## MEMORY HARD ALGORITHM



We can pebble vertex, if

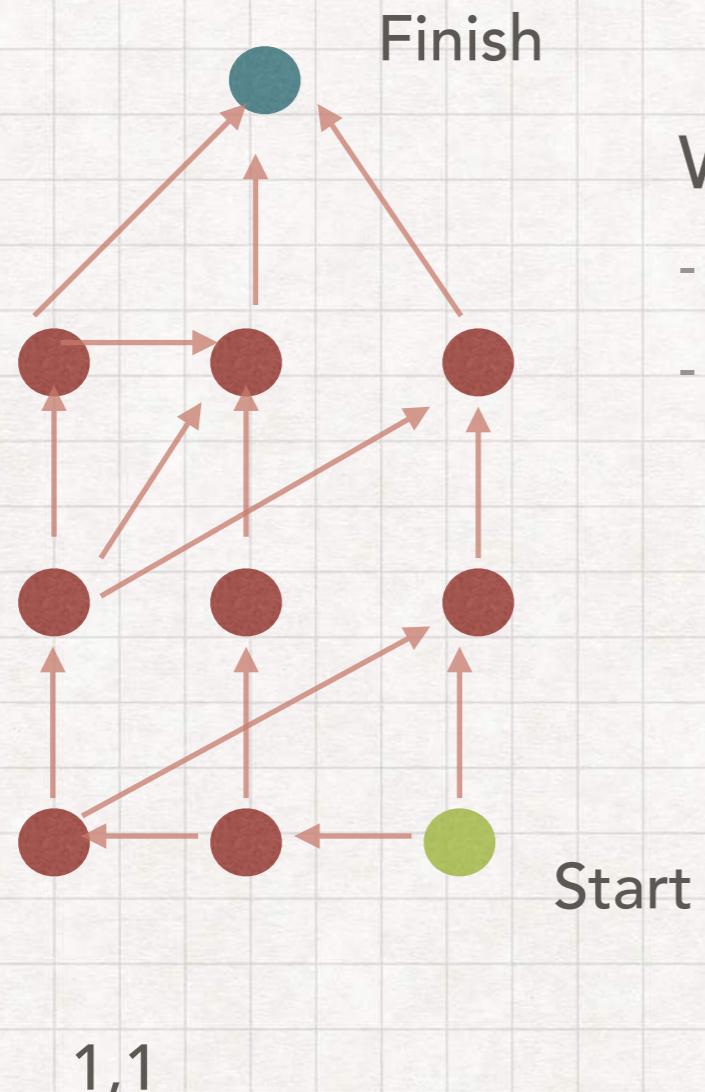
- this is start vertex
- all input vertex are pebbled.

Goal - pebble finish vertex

Use minimal number of pebbles

# PEBBLE GAME

## MEMORY HARD ALGORITHM



We can pebble vertex, if

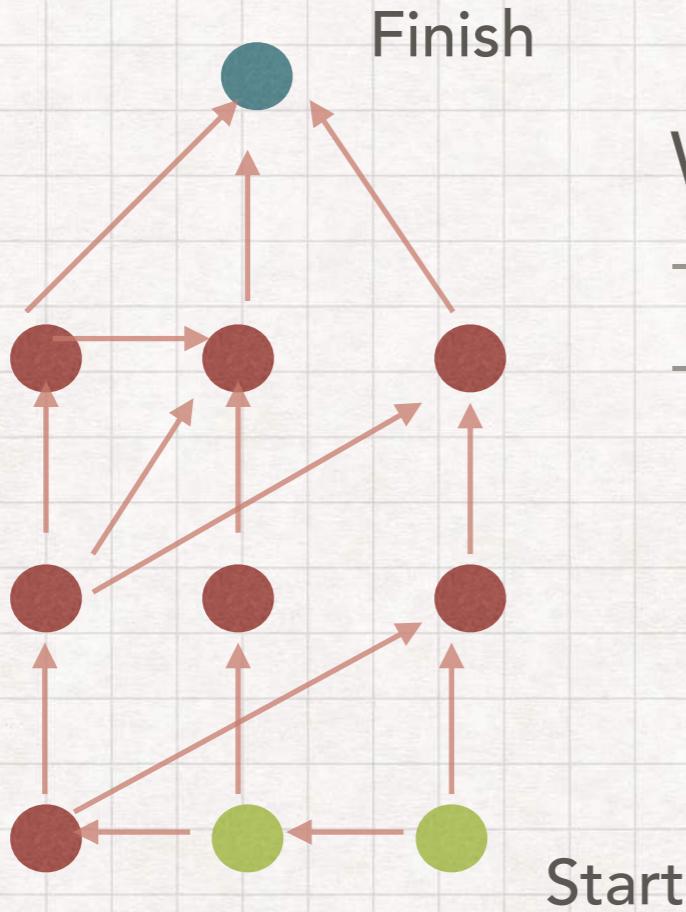
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Goal - pebble finish vertex

Use minimal number of pebbles

# PEBBLE GAME

## MEMORY HARD ALGORITHM



2,2

We can pebble vertex, if

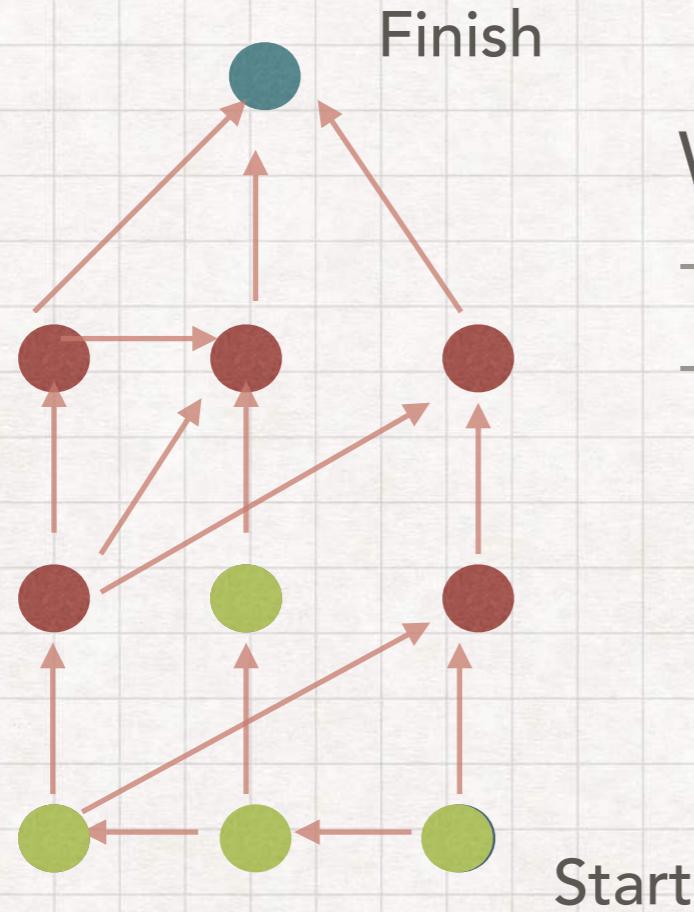
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- all input vertex are pebbled.

Goal - pebble finish vertex

Use minimal number of pebbles

# PEBBLE GAME

## MEMORY HARD ALGORITHM



We can pebble vertex, if

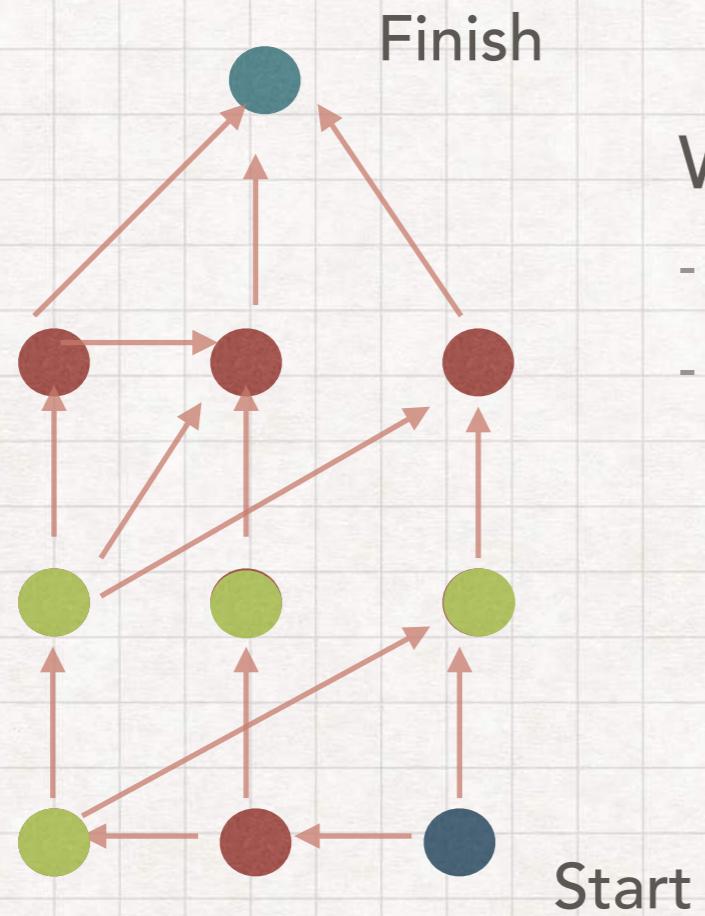
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Goal - pebble finish vertex

Use minimal number of pebbles

# PEBBLE GAME

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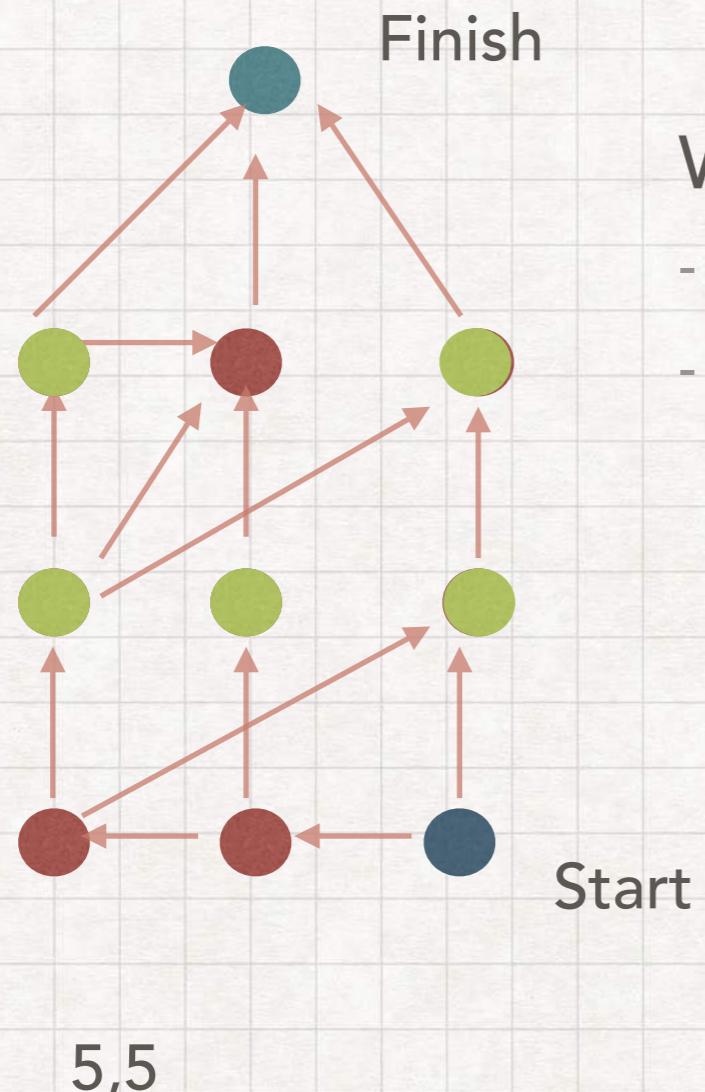
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# PEBBLE GAME

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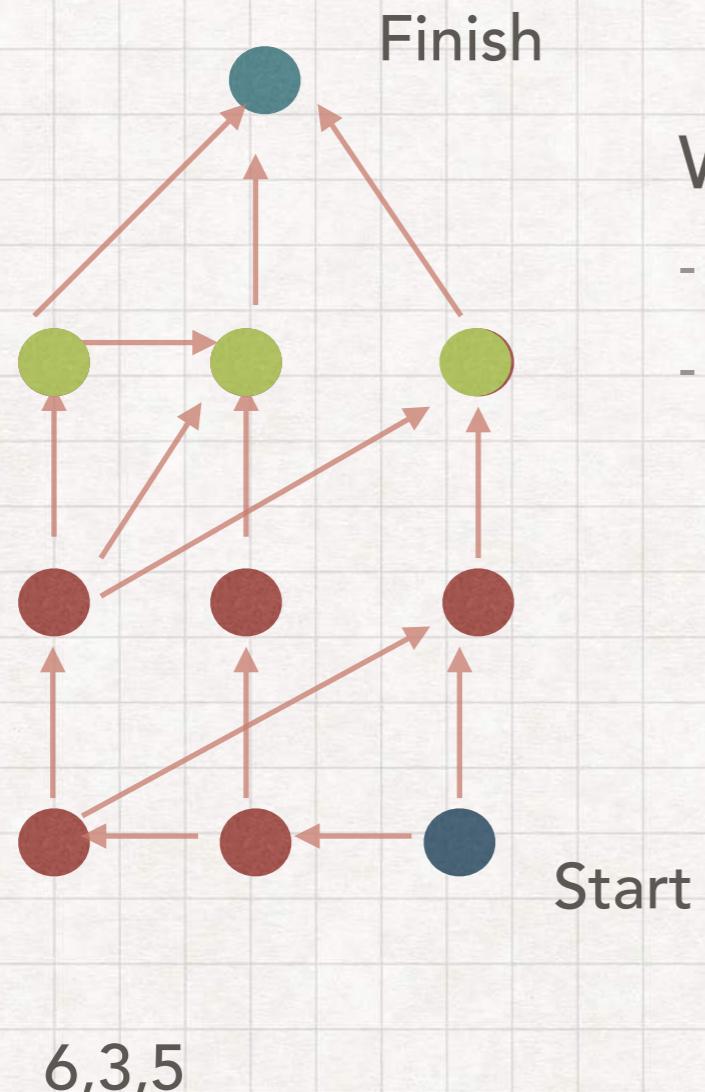
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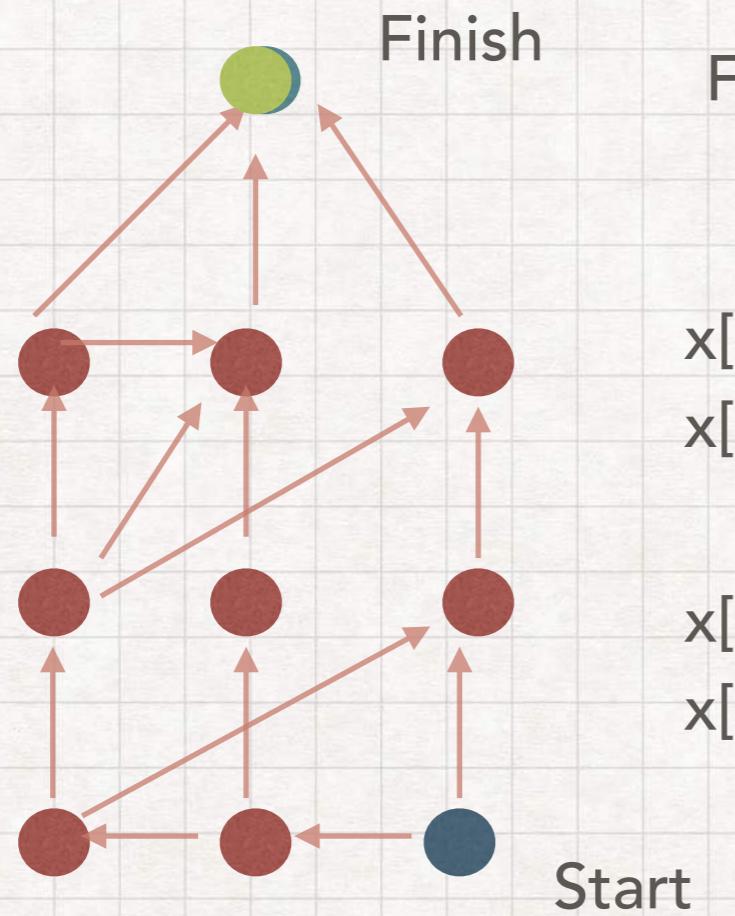
- this is start vertex
- all input vertex are pebbled.

Goal - pebble finish vertex

Use minimal number of pebbles

# PEBBLE GAME

## MEMORY HARD ALGORITHM



$$\text{Finish} = F(x[3,1], x[3,2], x[3,3])$$

$$x[3,1] = F(x[2,1]); \quad x[3,2] = F(x[3,1], x[2,1], x[2,2])$$

$$x[3,3] = F(x[2,1], x[2,3])$$

$$x[2,1] = F(x[1,1]); \quad x[2,2] = F(x[1,2])$$

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$$x[1,1] = F(x[1,2]); \quad x[1,2] = F(x[1,1])$$

$$x[1,1] = \text{Start}$$

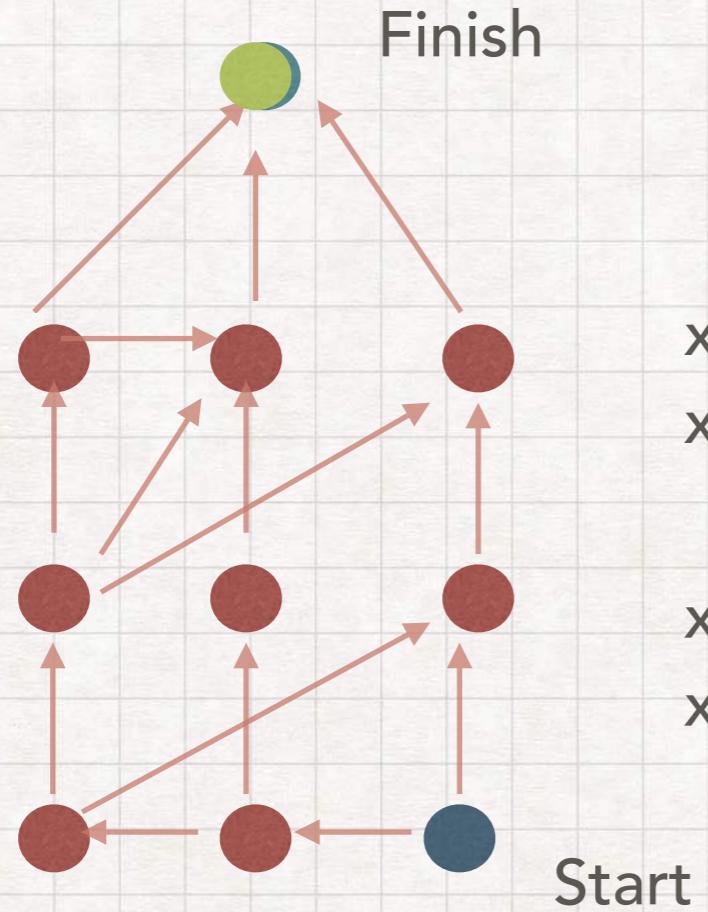
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



7,1,5

$$\text{Finish} = F(x[3,1], x[3,2], x[3,3])$$

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$$x[1,1] = F(x[1,2]); \quad x[1,2] = F(x[1,1])$$

$$x[1,1] = \text{Start}$$

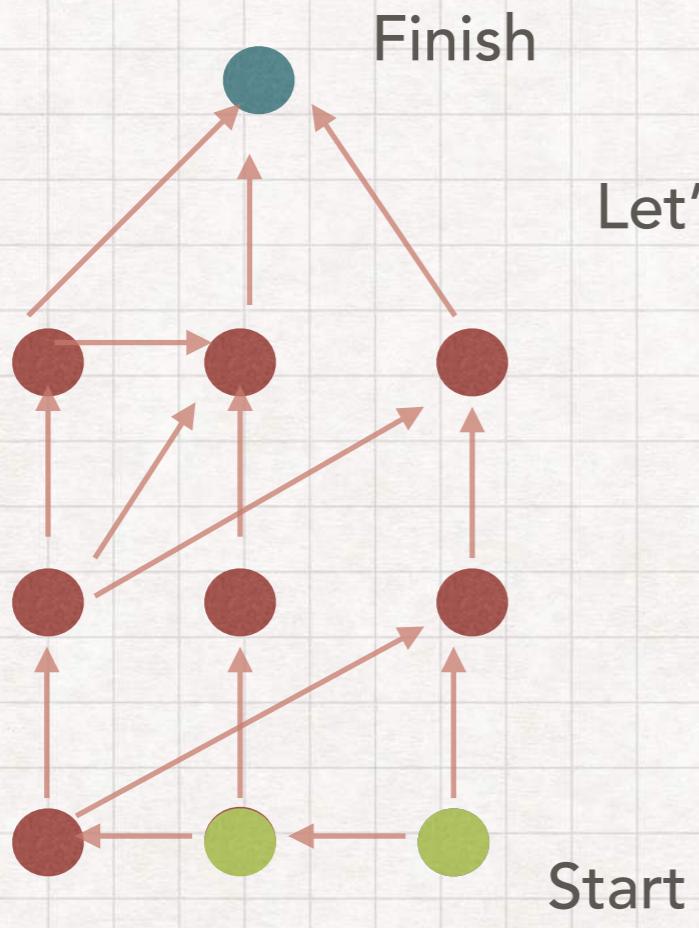
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

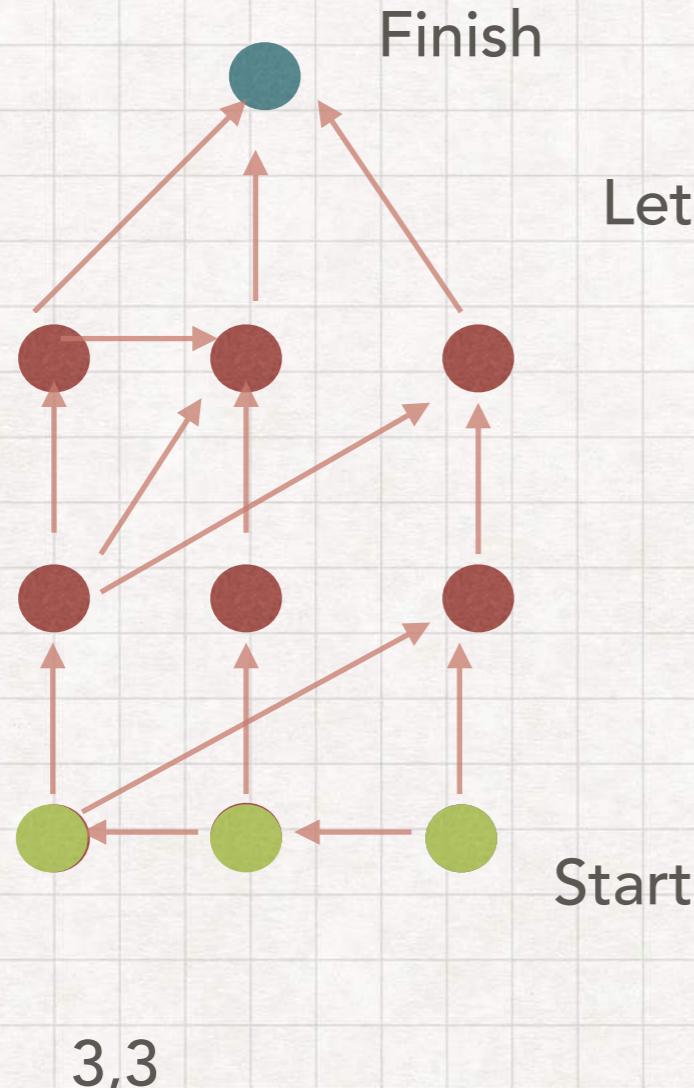
Moves = Time

Pebbles = Memory

2,2

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

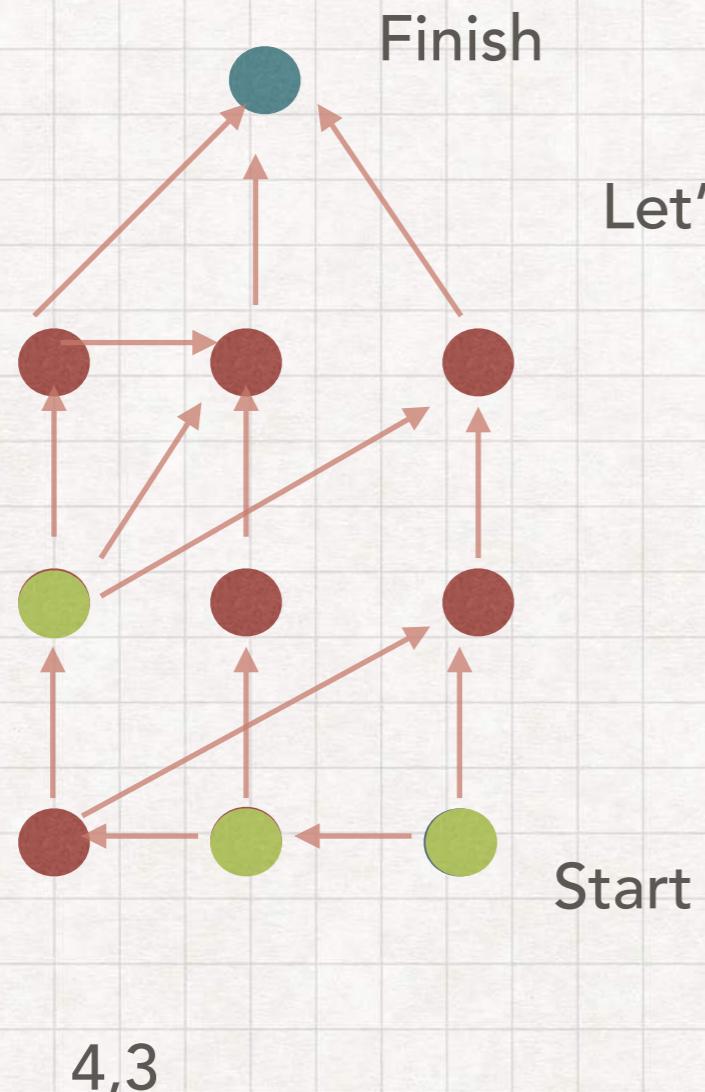
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

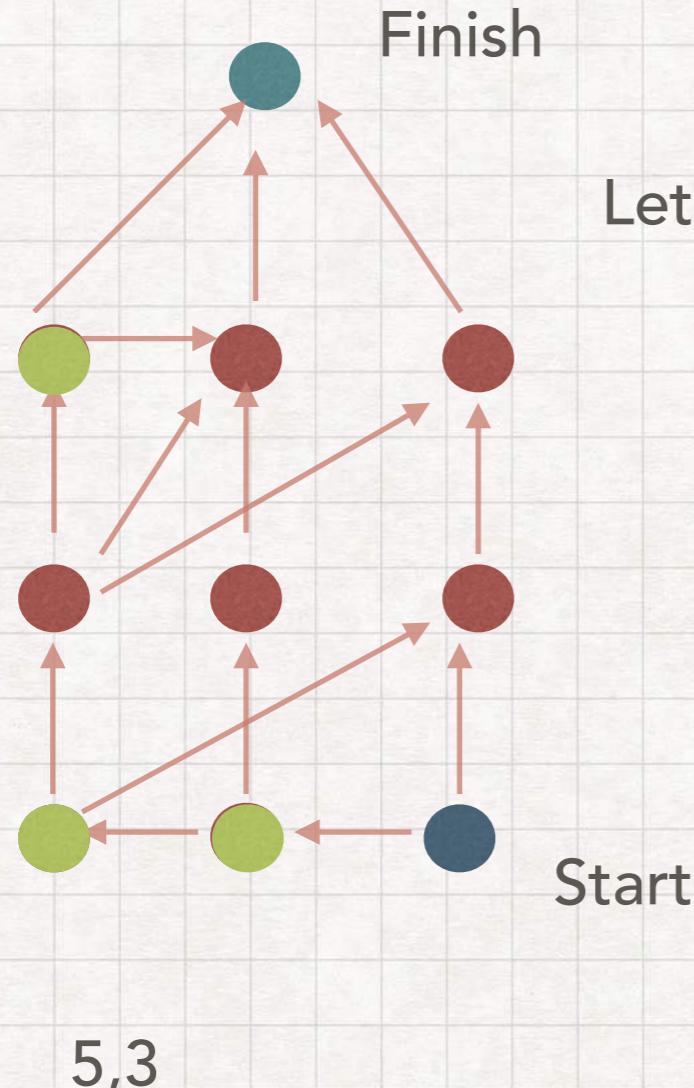
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

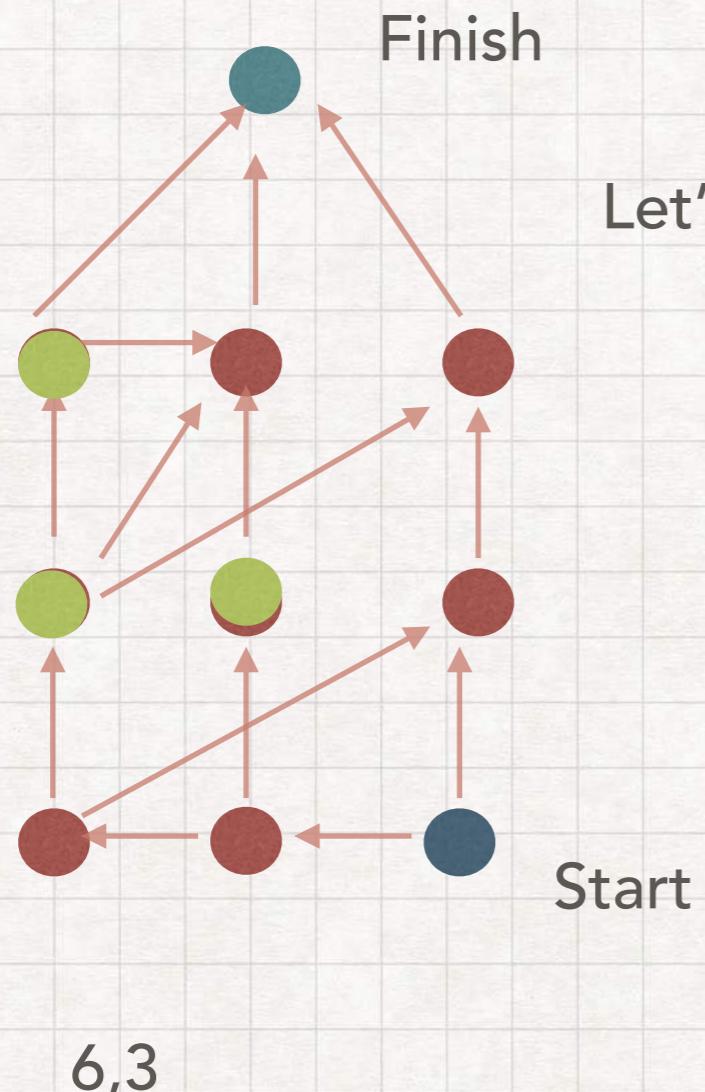
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

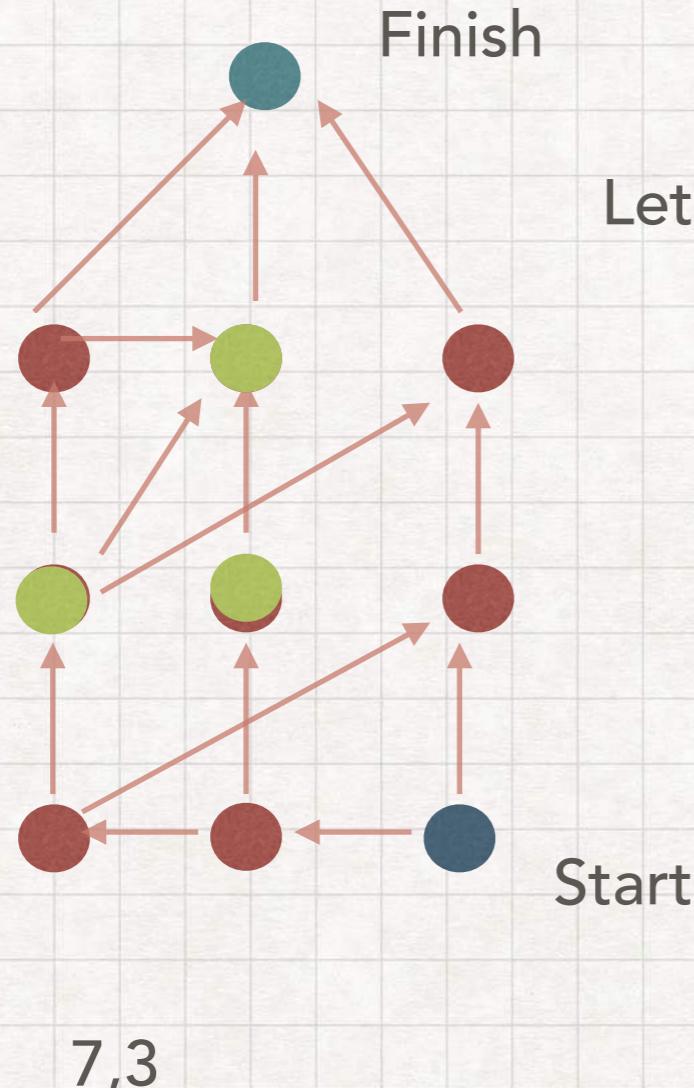
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

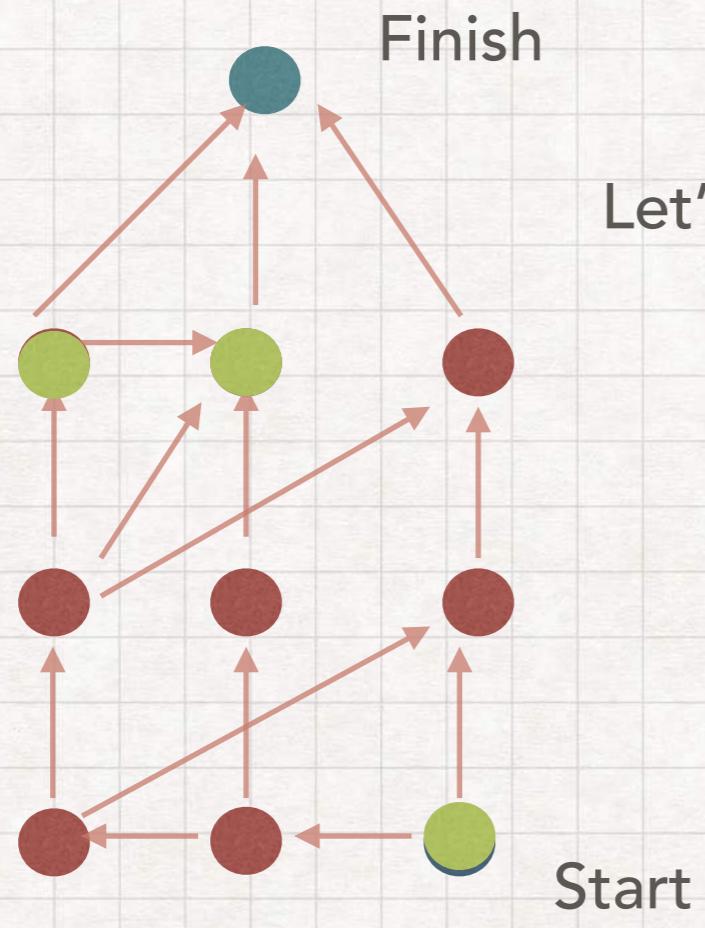
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



8,3

# Let's play with 3 pebbles

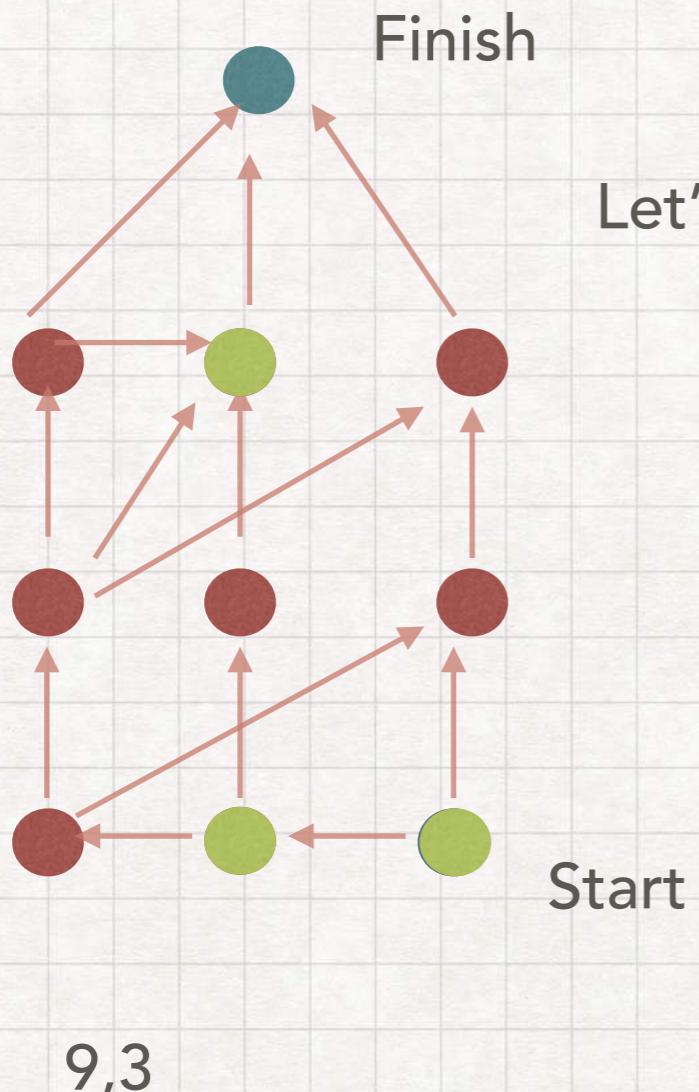
# Game = Computation Schema

# Moves = Time

# Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

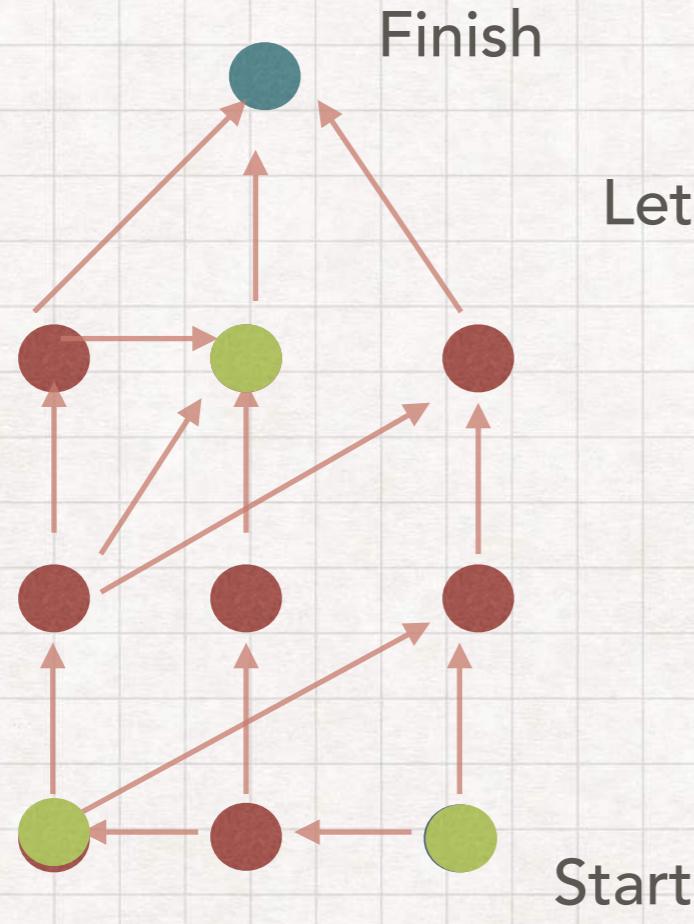
Game = Computation Schema

Moves = Time

Pebbles = Memory

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

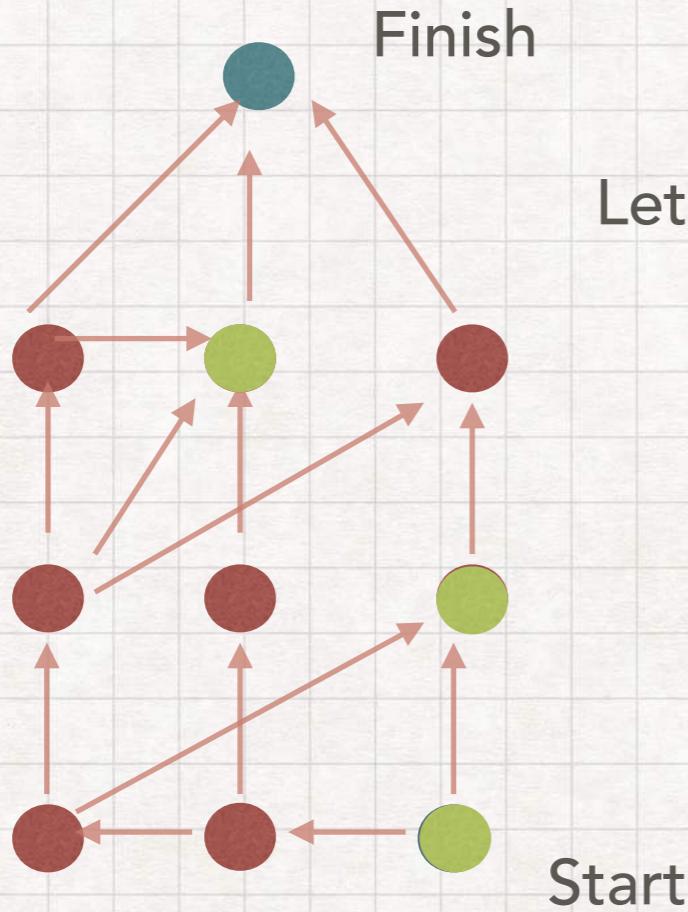
Moves = Time

Pebbles = Memory

10,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

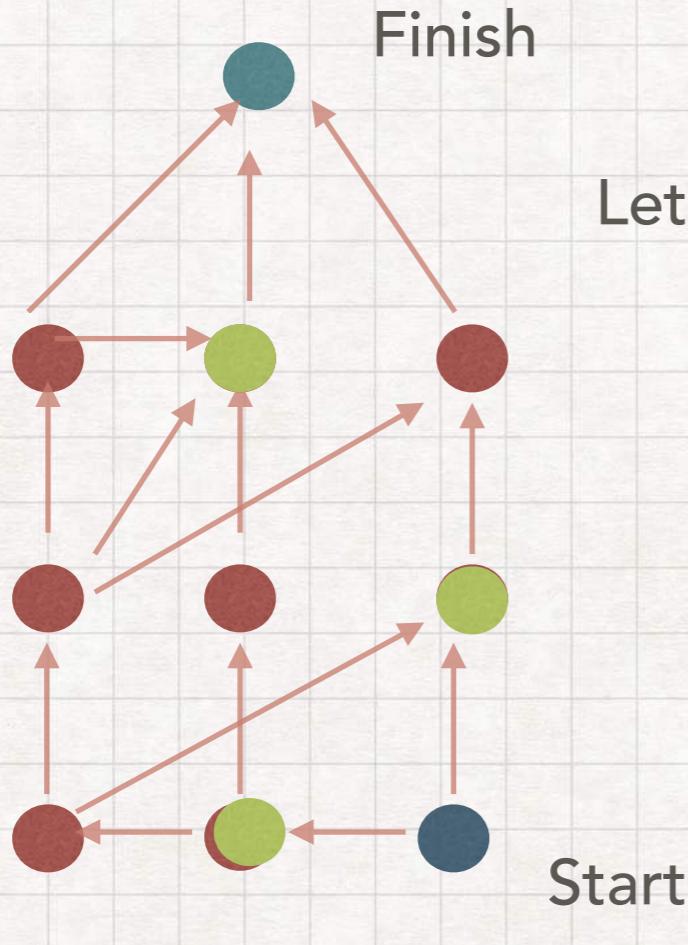
Moves = Time

Pebbles = Memory

11,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

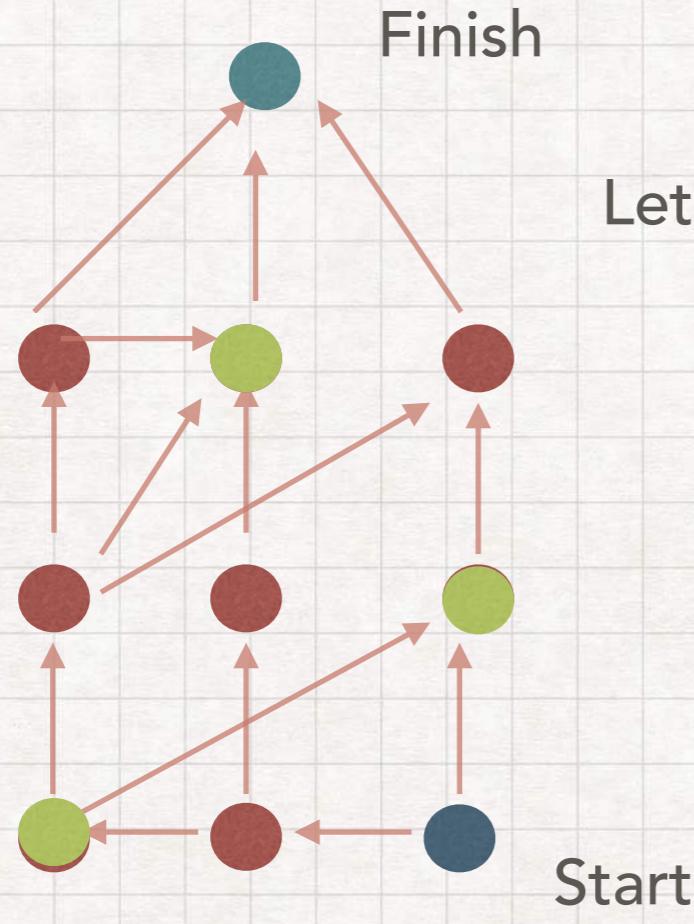
Moves = Time

Pebbles = Memory

12,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

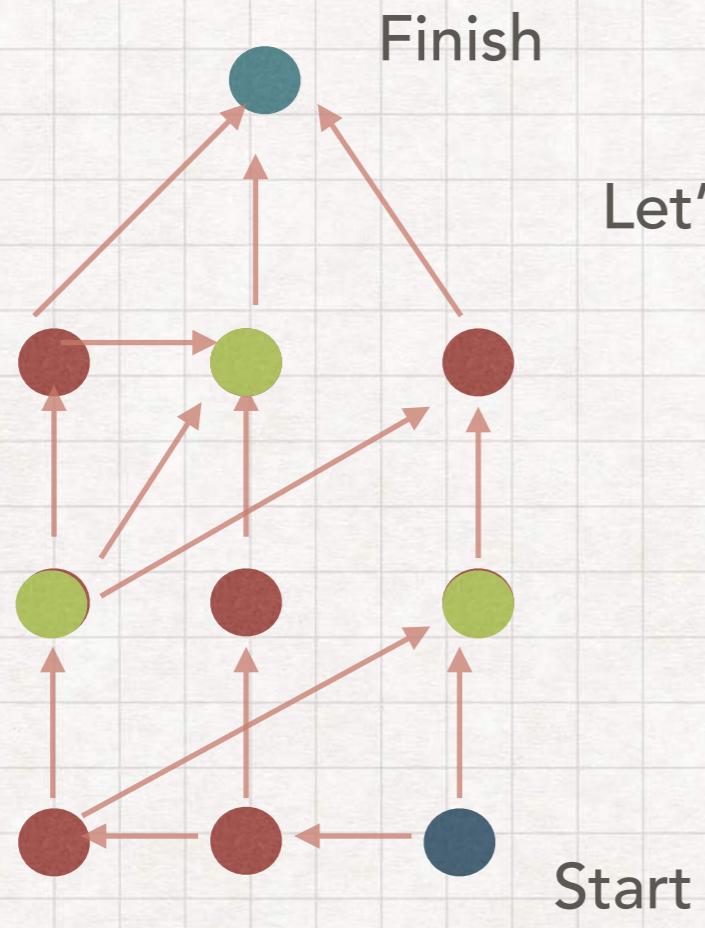
Moves = Time

Pebbles = Memory

12,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

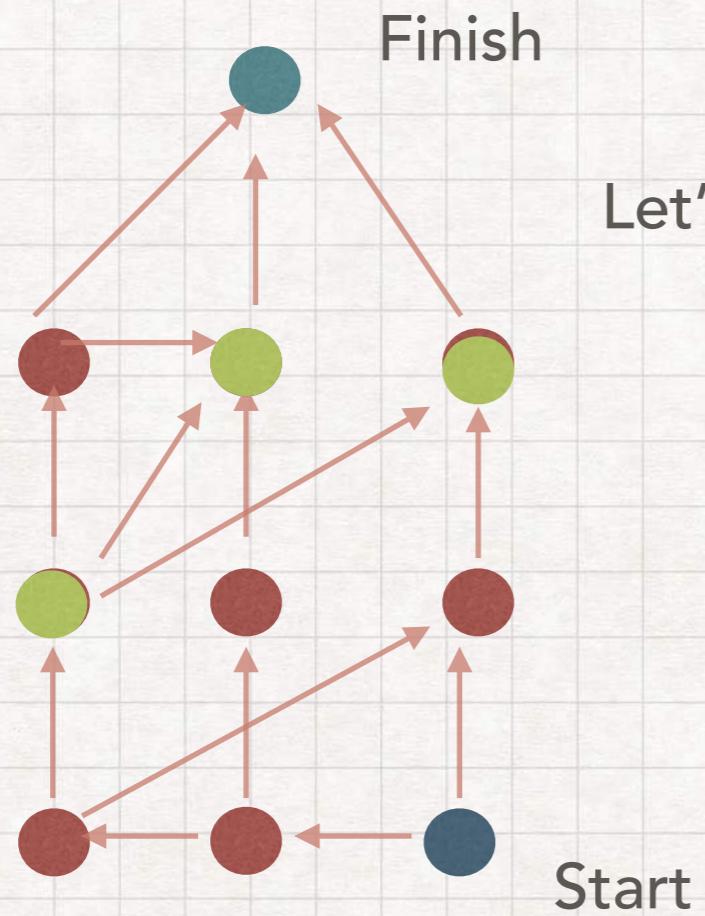
Moves = Time

Pebbles = Memory

13,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

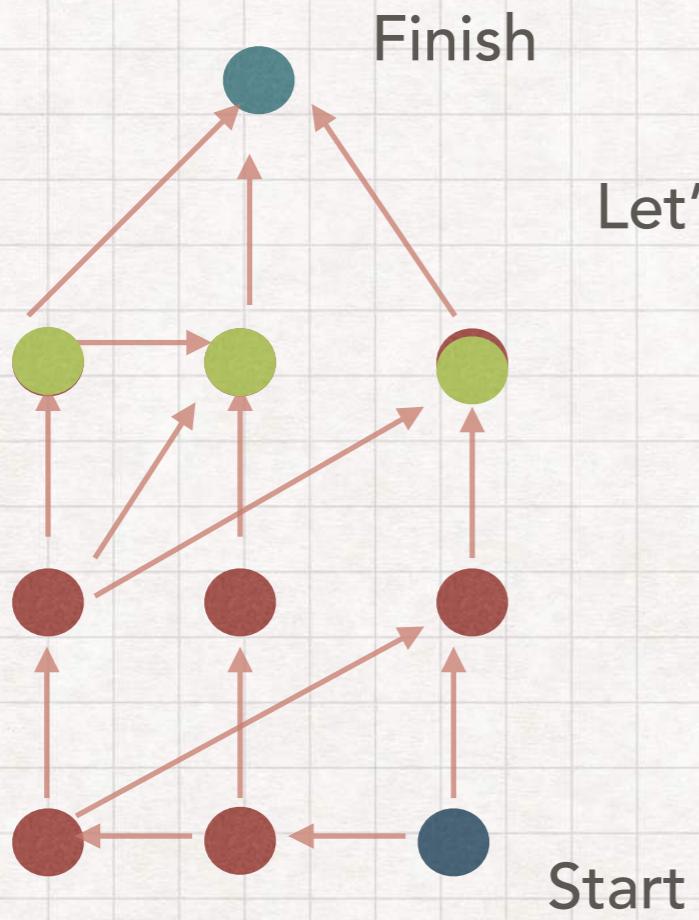
Moves = Time

Pebbles = Memory

14,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

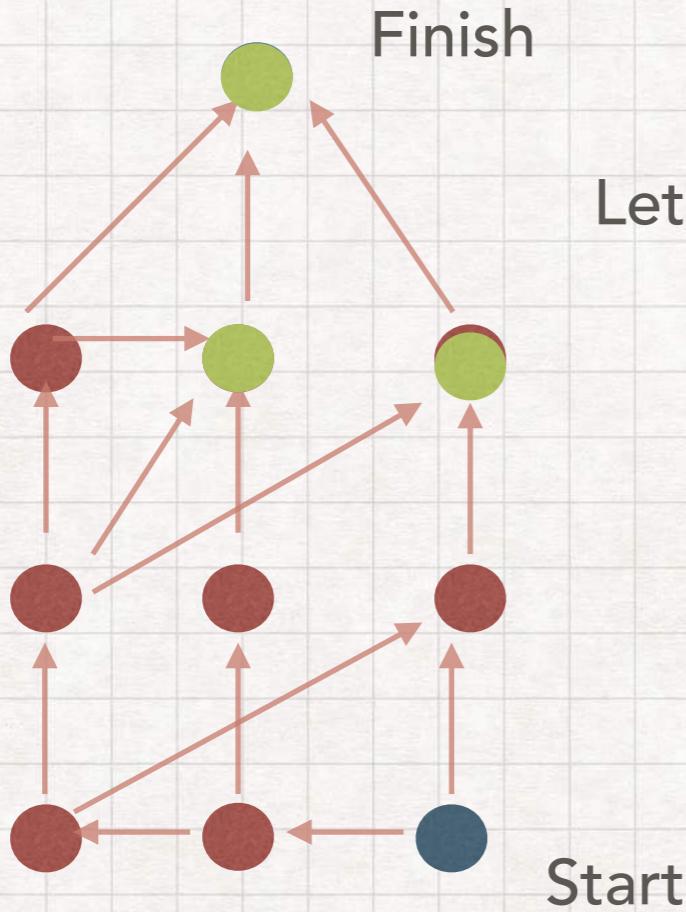
Moves = Time

Pebbles = Memory

15,3

# PEBBLE GAME

## MEMORY HARD ALGORITHM



Let's play with 3 pebbles

Game = Computation Schema

Moves = Time

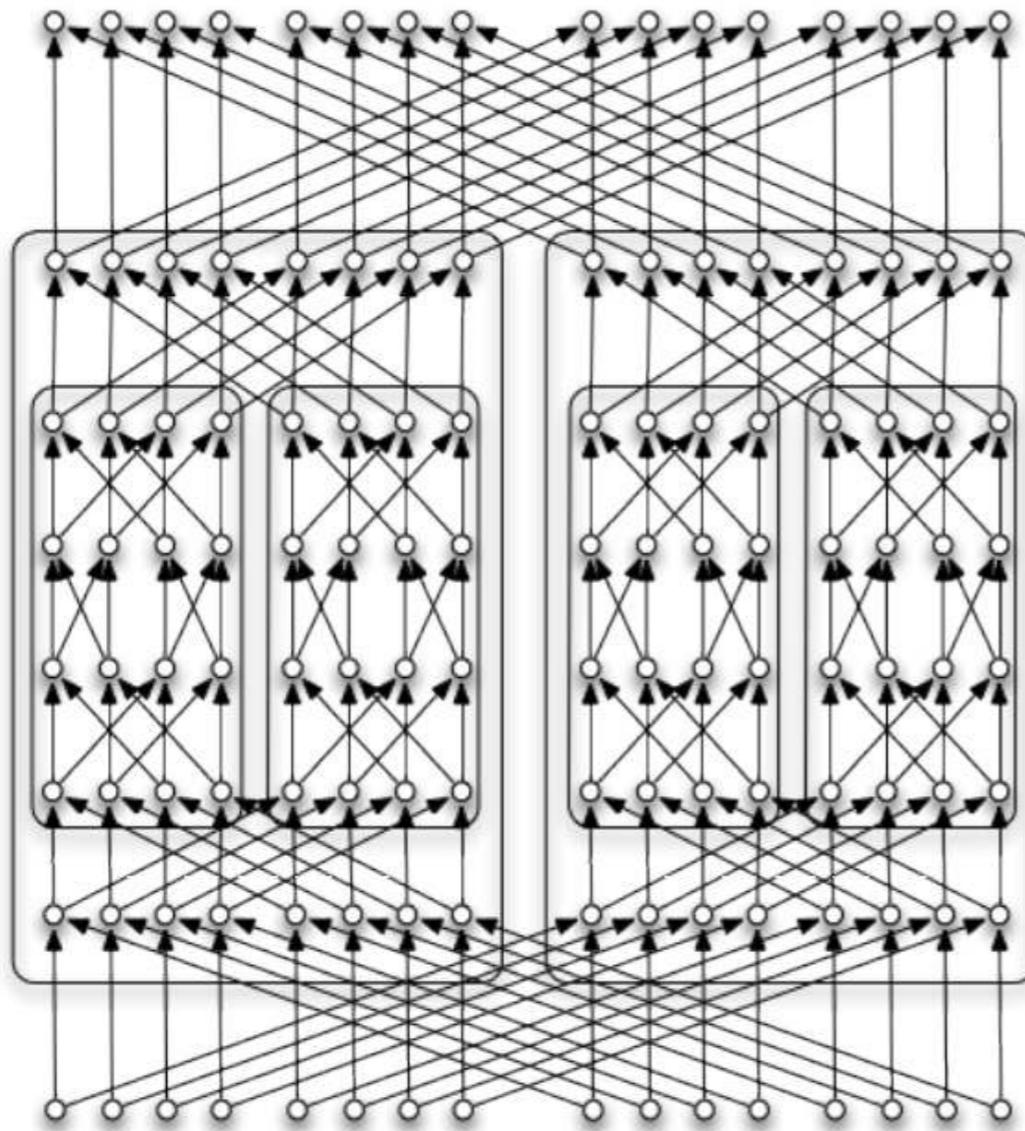
Pebbles = Memory

16,3

# PROOF OF STORAGE

## HARD TO PEBBLE GRAPH

Butterfly graph



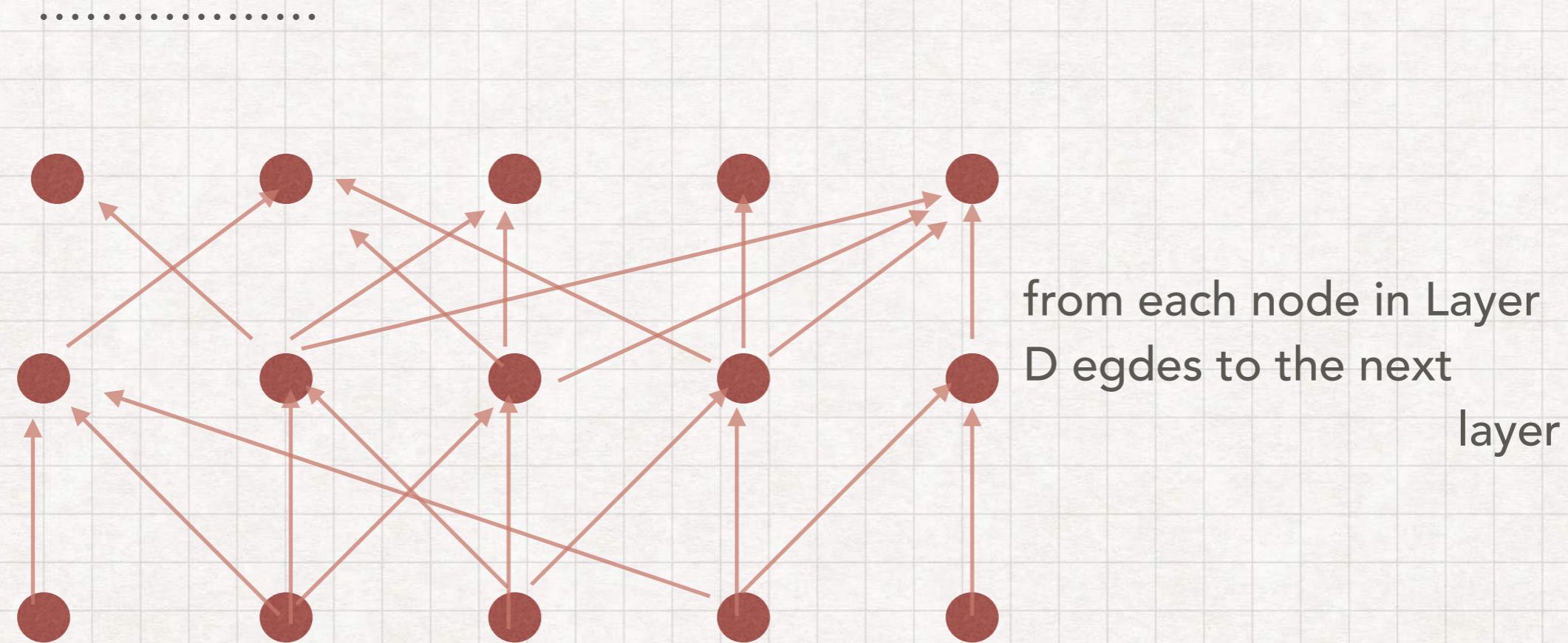
Idea:

- maximize time penalty  
for missing pebble.

$$\Delta T \approx e^{\Delta M}$$

# PROOF OF STORAGE

## PROBABILISTIC HARD TO PEBBLE GRAPH

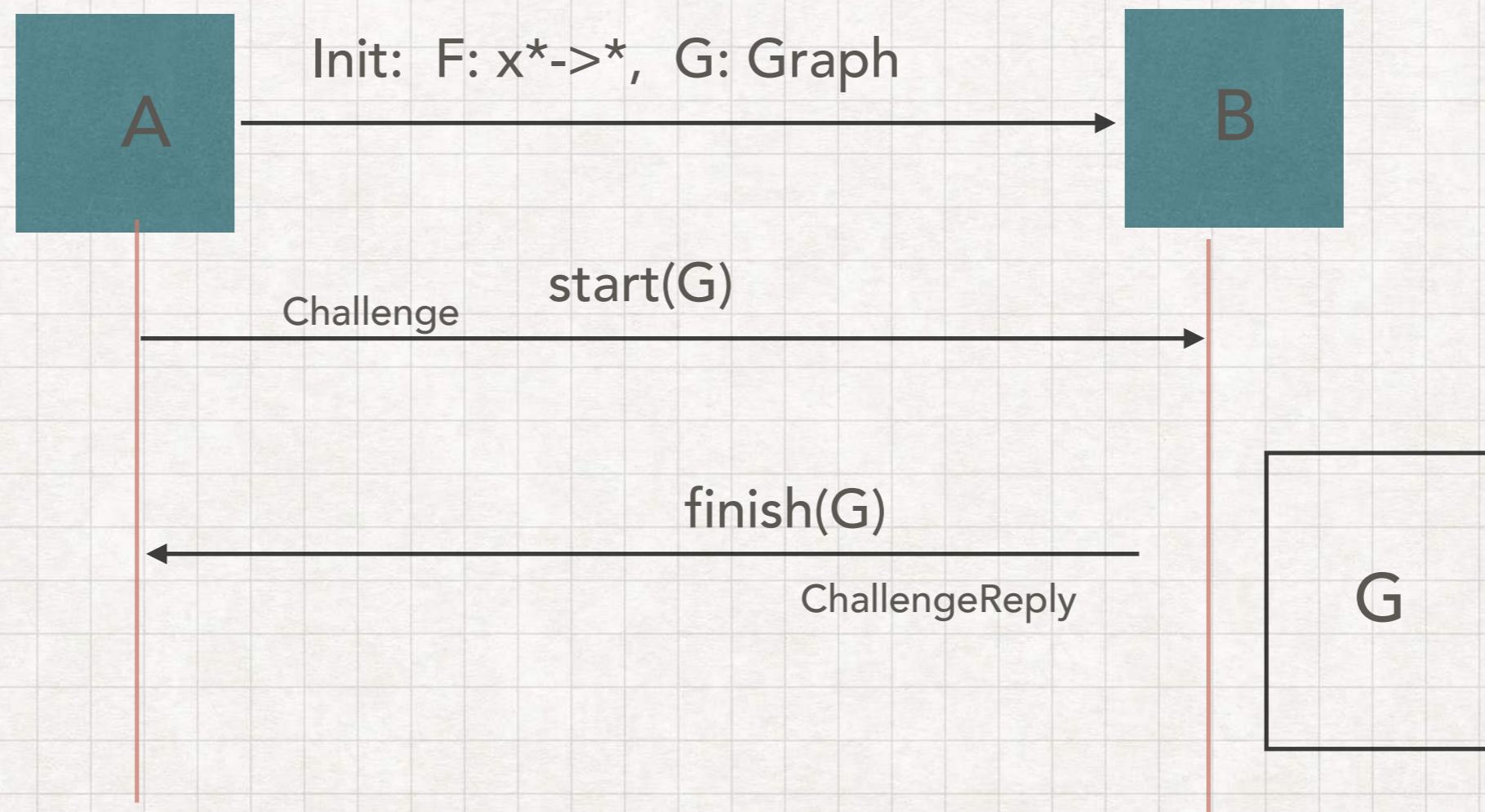


Randomly generated Chung expanders are hard to pebble  
Pinsker ( $D$  predescors for each sink)

# MEMORY HARD ALGORITHM (?)

## PEBBLING 'HARD TO PEBBLE' GRAPH

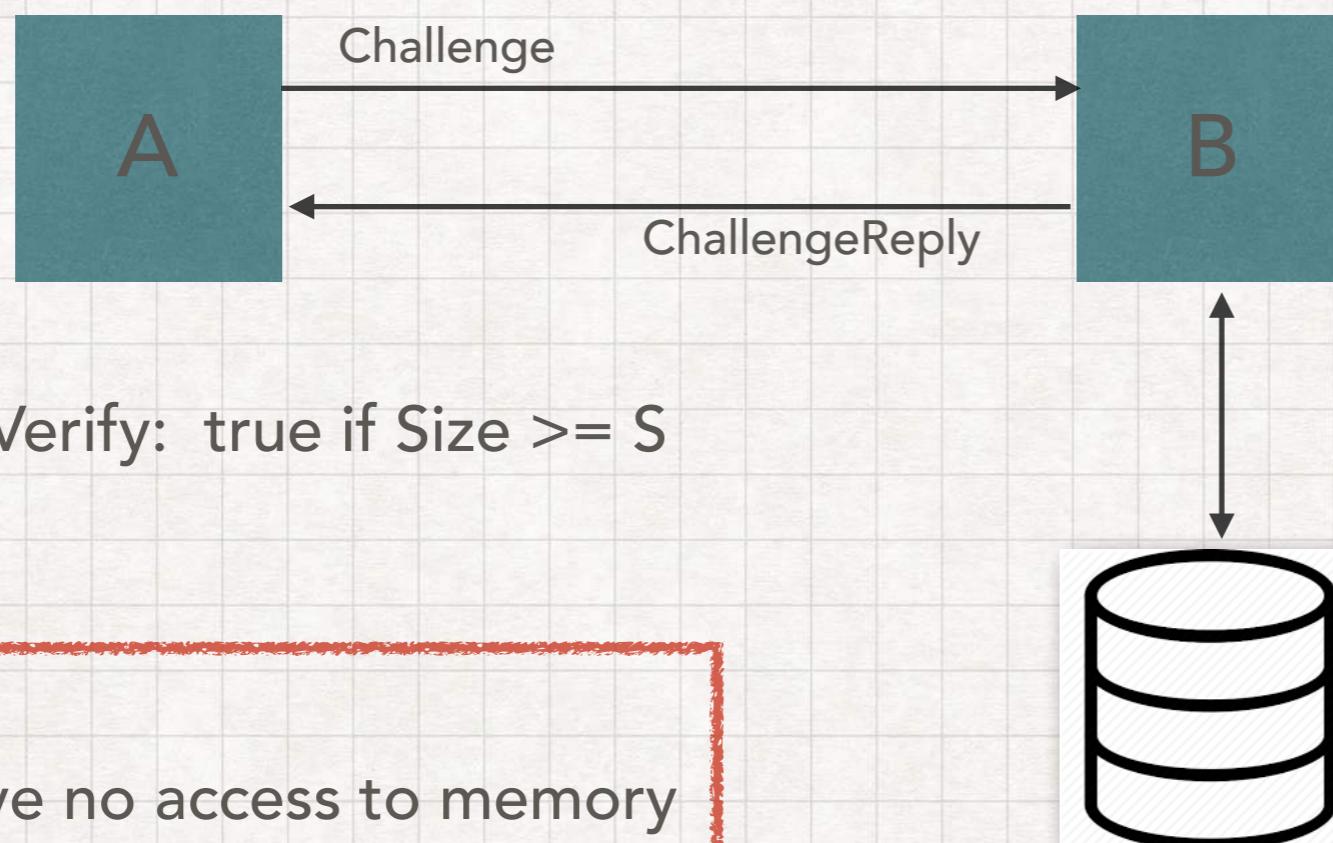
- Init: random function  $H$  and hard to pebble graph  $G$
- Challenge: values of pebbles at start-vertex
- Reply: value of pebbles at finish vertex



# PROOF OF STORAGE

HOW TO PROVE THAT I HAVE MEMORY ?

Interactive protocol



Verify: true if  $\text{Size} \geq S$

A

- have no access to memory
- have no superior memory

Computation  
requires  $\text{Size} \geq S$

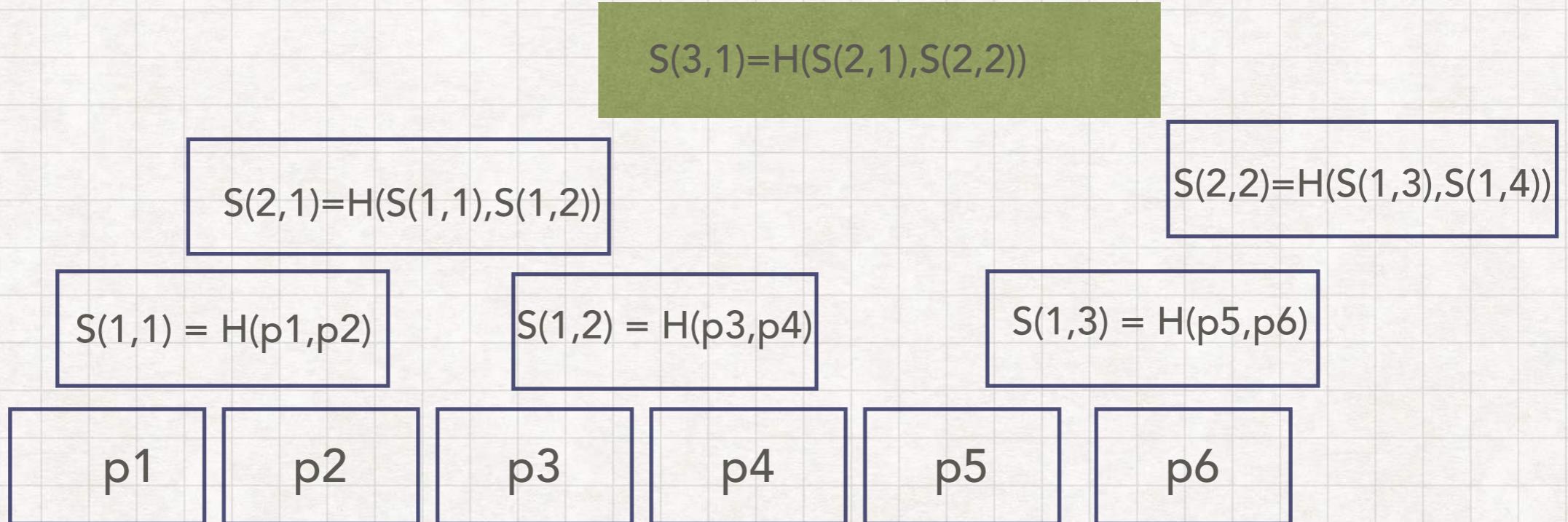
Size

# HOW TO PROVE COMPUTATION

## AUTHENTICATED DATA STRUCTURES

1. Authenticate log.
2. Open part of this log for inspection.

Merkle Tree



If you will change something in log, root hash of merkle tree will be other

## PROVE THAT I RUN GIVEN COMPUTATION SCHEME

1. A  $\Rightarrow$  B (send init values.)
2. B run scheme and calculate computation log Merkle tree root hash.
3. B  $\Rightarrow$  A (Merkle tree root hash)
4. A  $\Rightarrow$  B (Some parts of schema, i.e. coordinate in graph)
5. B rerun computation, recalculate Merkle tree
6. B  $\Rightarrow$  A
  1. : inputs and outputs of computation at specific vertexes.
  2. : hashes of this computation.
  3. : hashes, which allows A to recompute the Merkle tree root.
7. A :
  1. check that input and output at given vertexes are correct
  2. check, that Merkle tree root will be the same.

// It will be hard for A to build Merkle Tree for other computations with the same hash.

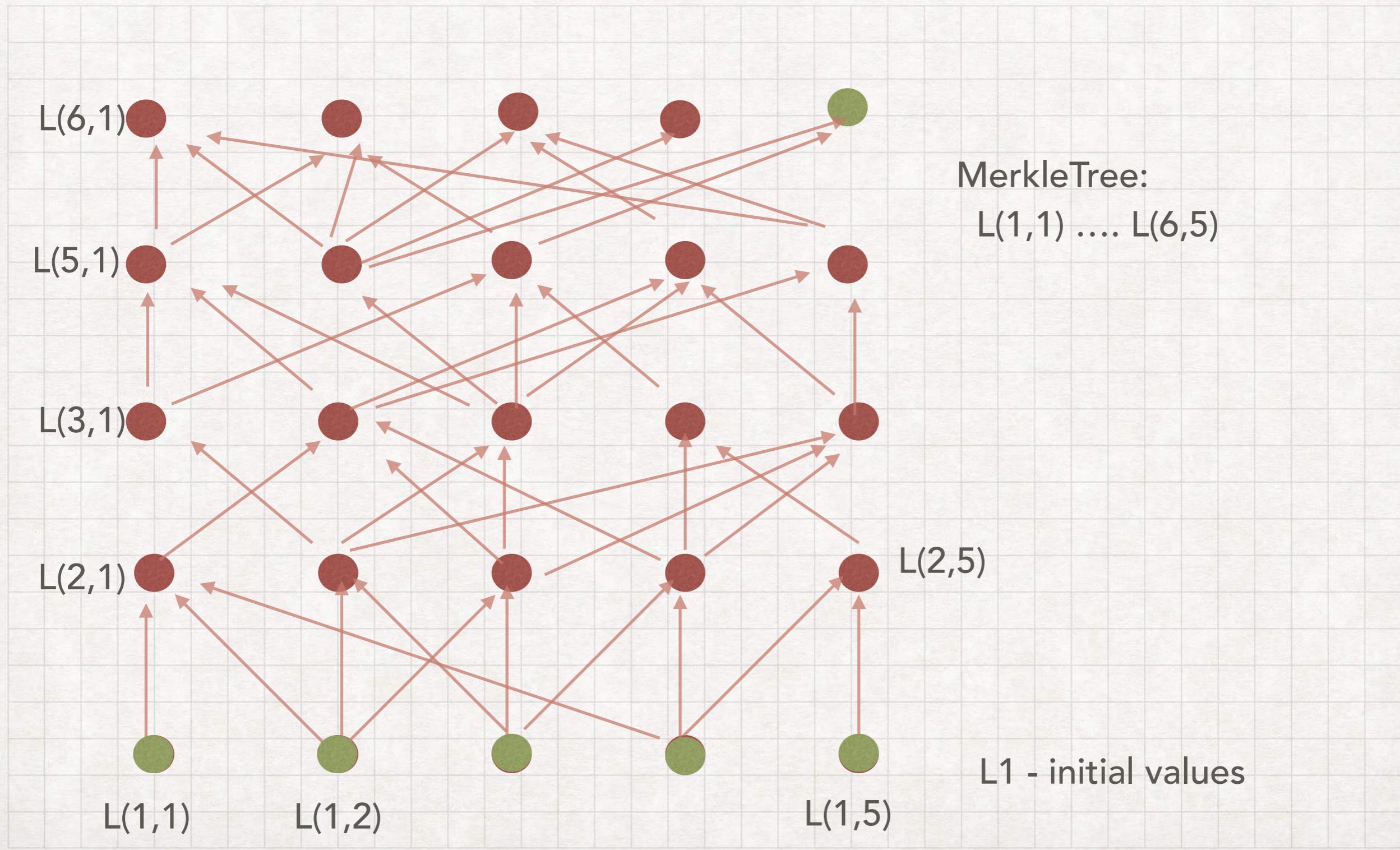
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# PROOF OF COMPUTATION OF GIVEN SCHEME

RN



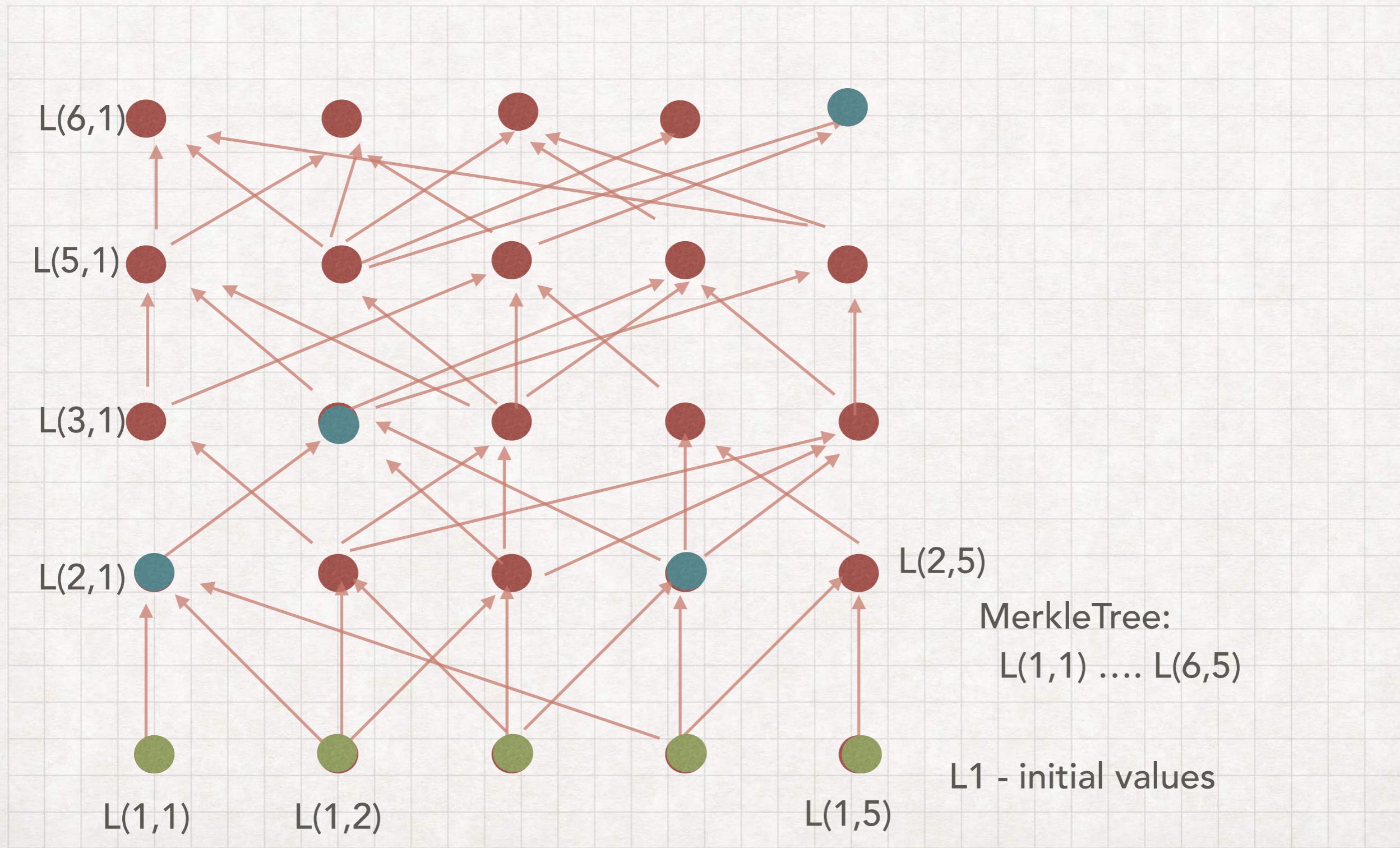
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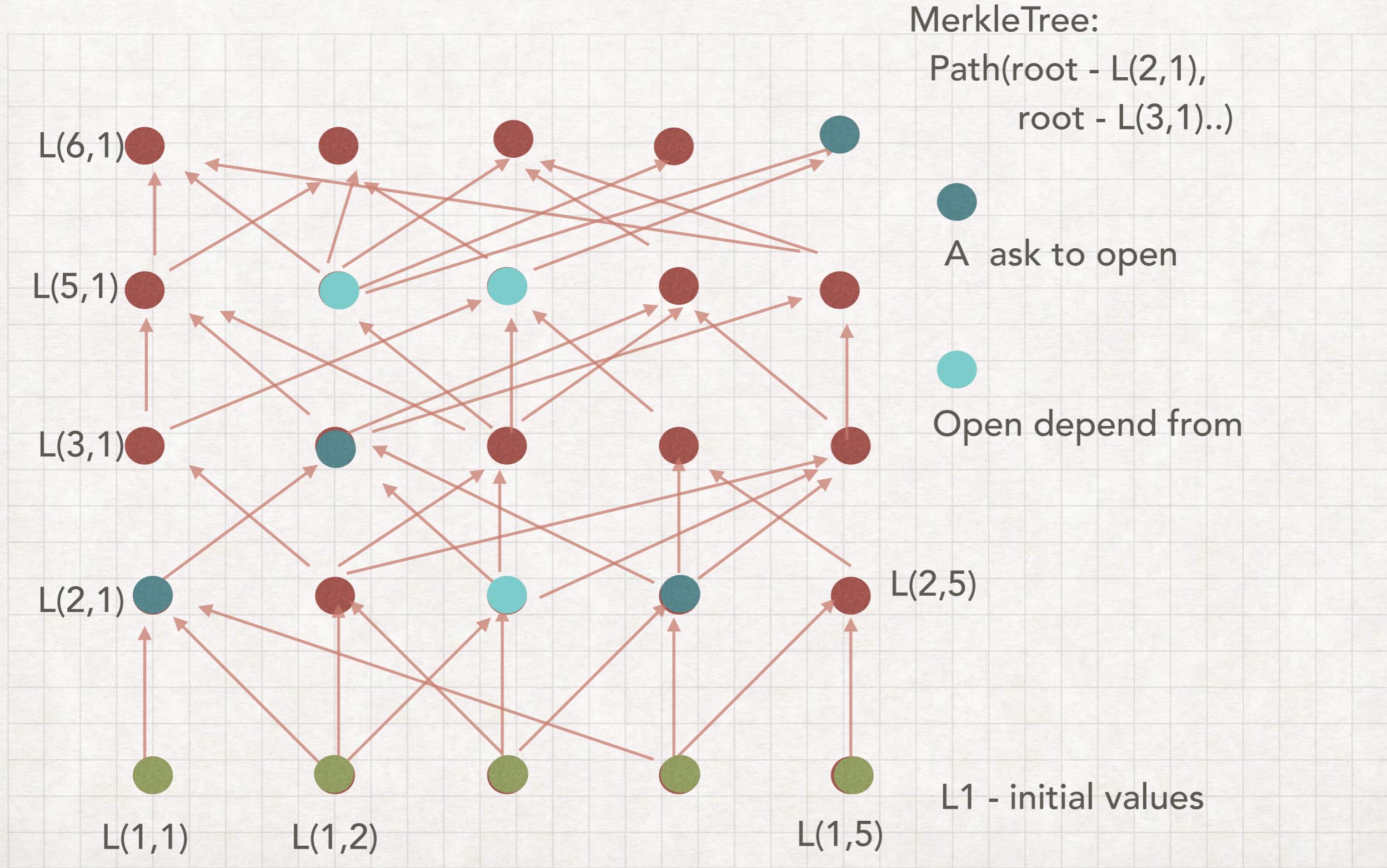
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# PROOF OF COMPUTATION OF GIVEN SCHEME

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## PROVE THAT I RUN GIVEN COMPUTATION SCHEME

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7. A :
  1. check that input and output at given vertexes are correct
  2. check, that Merkle tree root will be the same.

// Profit.

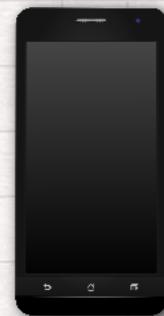
# PROOF OF STORAGE

## PROFIT



\*\*\*\*\*

ACIC resistant hashes



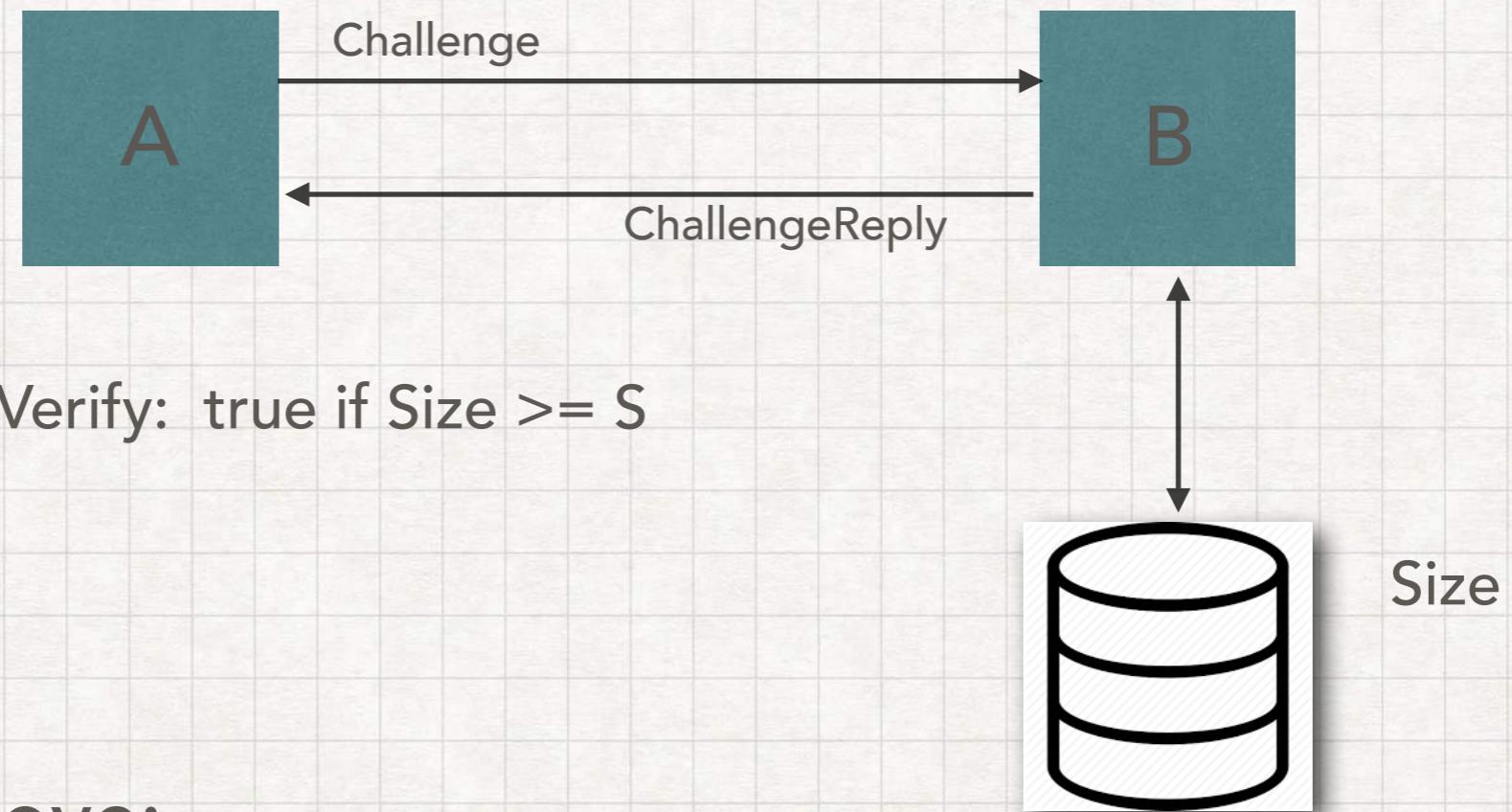
Proof of secure erasure



Decentralized storage.

# PROOF OF STORAGE

## DECENTRALIZED STORAGE



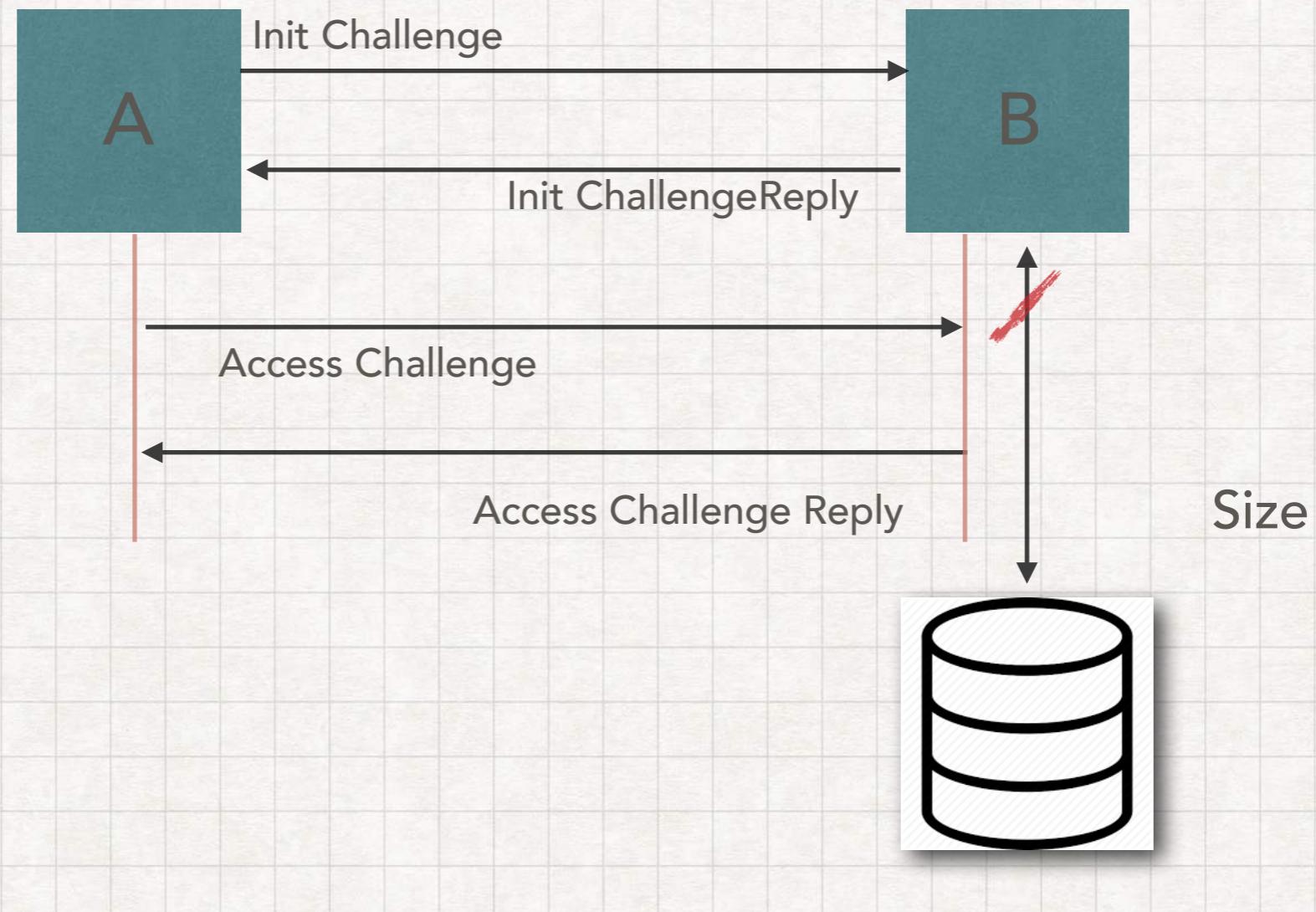
How to prove:

- B still have access to storage after init.
- B store something useful ....

# PROOF OF STORAGE

B STILL HAVE ACCESS TO STORAGE AFTER 1-ST CHECK.

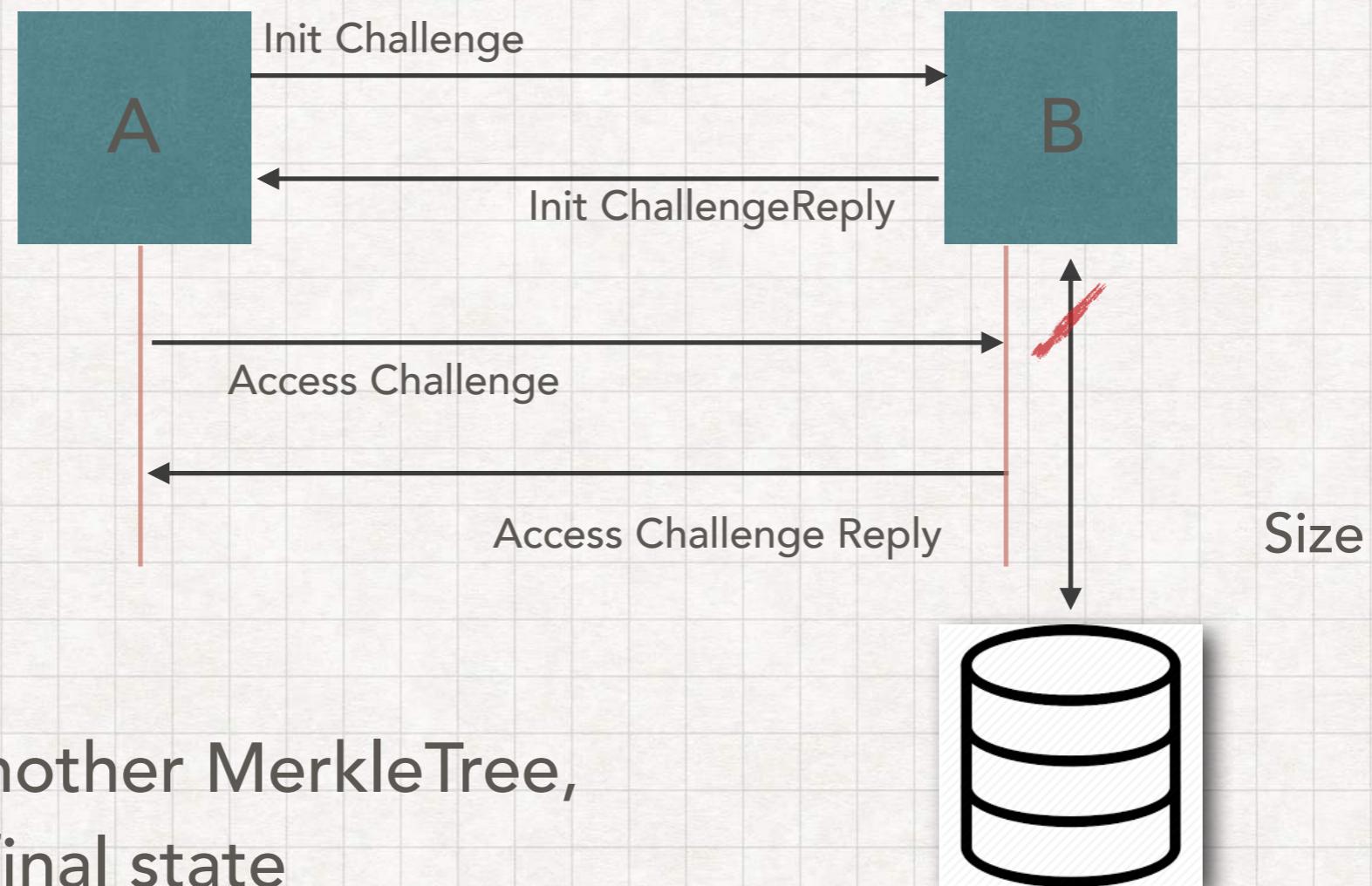
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# PROOF OF STORAGE

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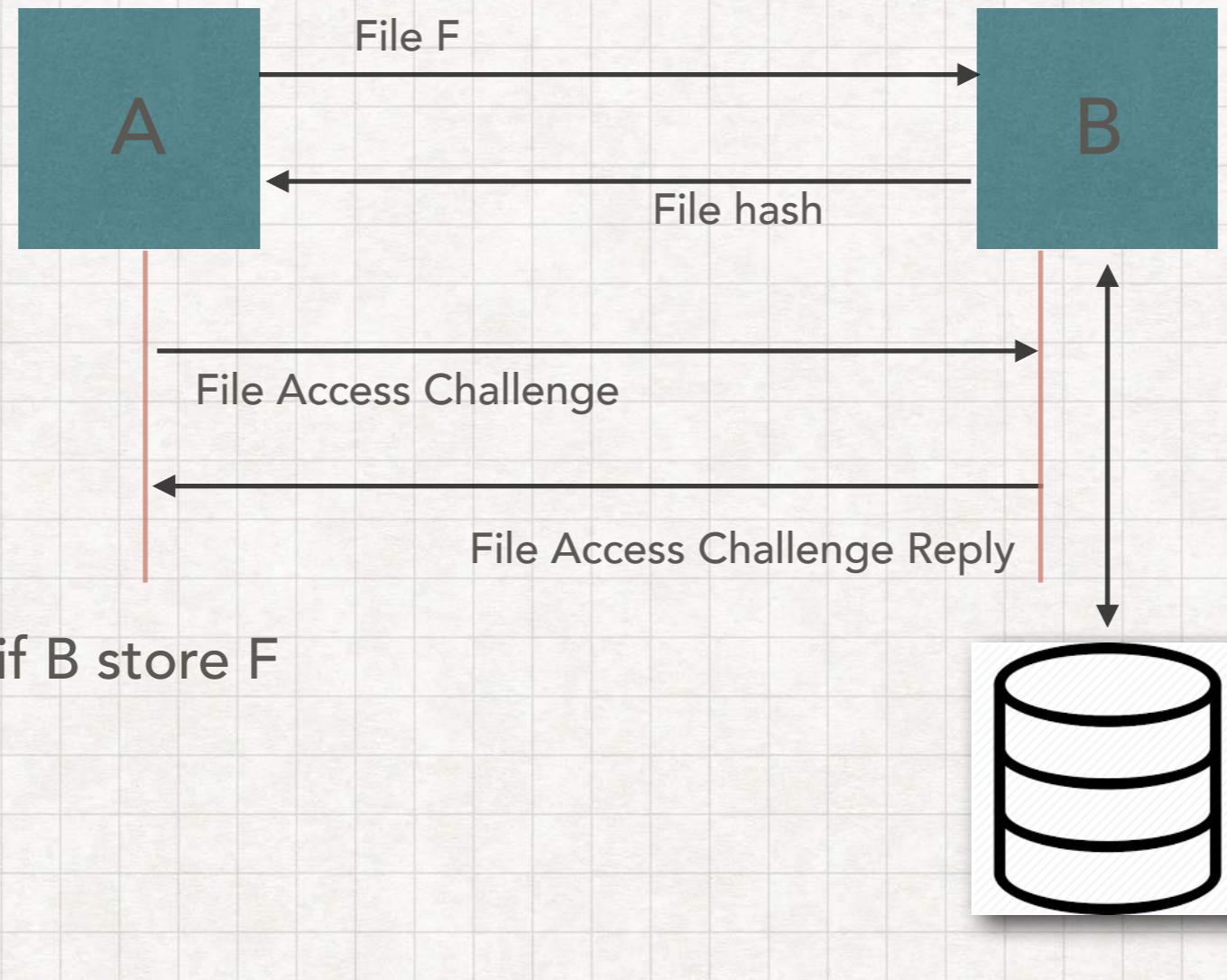
Verify: true if  $\text{Size} \geq S$



Idea — yet another MerkleTree,  
for final state

# PDP: PROVABLE DATA PROCESSION

STORE SOMETHING USEFUL



# PDP, POR, POST .....

## THIS WILL BE THE OTHER STORY

*Thanks for attention.*

[Q]

<ruslan@shevchenko.kiev.ua>

@rssh1

<https://github.com/rssh>

garuda.ai

