Technology Arts Sciences
TH Köln

Modelling and simulation of continuous systems

Assignment 4

Tank model

Submitted to:

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Aim

1) To develop a simulation model of a system with two storage tanks and usage of a pump in the pipe between the two tanks and perform simulation studies.

   Also, to implement additional elements in our Model:

   - Non-linear Valves
   - Pump Speed Control
   - Pump reflow line, etc
2) To design at least one of the controllers for flow, level, temperature and to identify dynamic models for each control loop.
3) To Analyse models with linear system theory and to assign proper control parameters.

System description

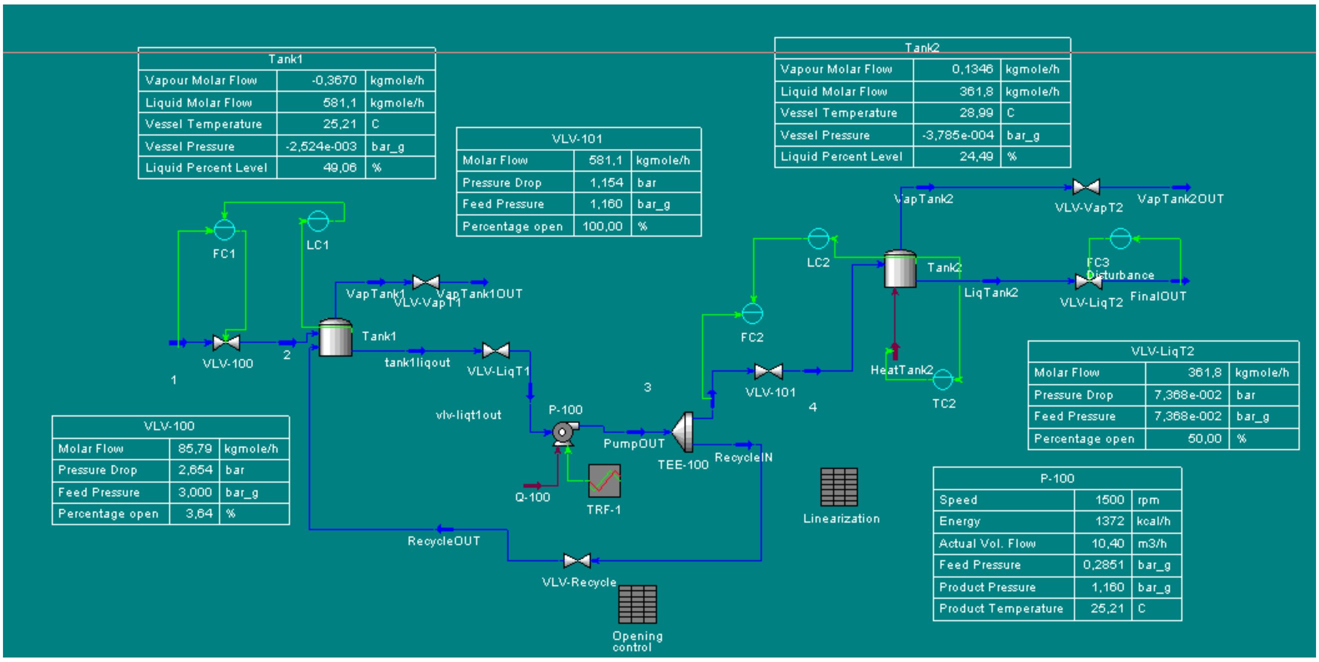


Figure 1 – Complete system model

In the figure above we have an overview of the full system model that we are going to design, analyse and discuss in this paper.

The system consists of two **tanks** (*Tank1* and *Tank2*) and a **centrifugal pump** (*P-100*) that is used in order to provide water to the second tank (*Tank2*) which is located higher than the first tank (*Tank1*). In more details:

- *Tank1* is used to store the water that needs to be provided into the second thank. It maintains a minimum re-circulation through the centrifugal pump and the vapor in it is released out via the valve *VLV-VapT1* to the atmosphere (0 barg).

  Since we need to supply water into the second tank continuously, we want to maintain a minimum level of 50% of liquid in this tank (*Tank1*). In order to fulfil this scope, we implement a **level controller** (*LC1*) and a **flow controller** (*FC1*) in cascade, with respect to *Tank1*, the valve *VLV-100* and the mass flow/inlet *1*.

- The centrifugal pump *P-100* provides the required pressure (head) and fluid flow to transfer the process fluid (water in our case) into the second tank.

  Since the amount of the fluid required to fill the second tank may vary, our pump has to work under different conditions. Therefore, the pump has to vary its pumping speed in order to provide more/less fluid. This is managed by the transfer function block *TRF-1* which takes into account the pumped flow *3*.

- *Tank2* is used to provide water for the final output of the overall system. In order to ensure water availability at any time, we want the level of liquid in this tank to be 50%.

  Moreover, we provide an energy flow (heat) to this tank in order to warm up the water in it. The temperature of the liquid the tank contains is controlled by a **temperature controller** (*TC2*).

- One more important point of such a structure is the reflow line. To avoid very high pressure against the valve *VLV-101* in case this is closed or partially closed, we need to pump the fluid back into the tank via the recycle line.

System design

For a better understanding of the design system control procedure, let us split the system in three parts:

1. First tank, *Tank1*
2. Pump, *P-100*
3. Second tank, *Tank2*

1. First tank, *Tank1*

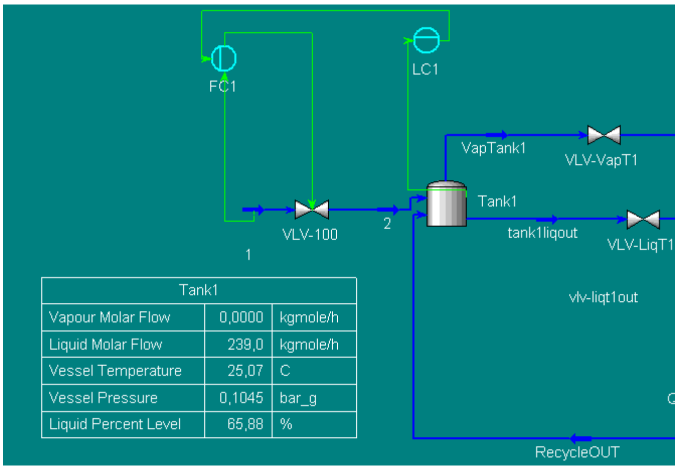


Figure 2 – *Tank1*

The design configurations of *Tank1* and *VLV*-100 are shown in the tables below:

Valve: *VLV-100*	
Type	Linear
Cv	30
Base elevation level relative to ground	0.000 m
Time Constant (First Order)	2 seconds

Table 1 – *VLV-100* design configurations

Tank: *Tank1*	
Volume	$10m^3$
Base elevation level relative to ground	2.000 m

Table 2 – *Tank1* design configurations

The idea is to use both level and flow controllers in a cascade fashion to establish a set point relative to the liquid percentage of the tank and increase/decrease the incoming flow accordingly.

More in detail, the level percent of the water inside the tank is read by a sensor. This level is then compared with the defined set point (50% in our case) of the level controller *LC1* (**PI controller**). When a difference between these two values (actual vs desired level percent) is detected, the flow controller *FC1* (**PI controller**) takes the difference between the current flow and the required flow as input. This input is used to generate an OP that is then used to manage the valve opening of *VLV*-100 in order to increase/decrease the incoming flow to the tank. The higher the flow the more the opening of the valve and vice versa.

Vapour Outlet Valve, *VLV-VapT1*

The vapour present in the tank is vented out to the atmosphere via the vapour outlet stream which contains a vapour control valve.

The control valve allows the air to pass through in and out of the tank. The inlet and outlet of this valve has only air composition (100%) to prevent any backflow of water into the tank.

The following configuration are made with the vapour stream:

Valve: *VLV-VapT1*	
Cv	50
Type	Linear

Table 3 – *VLV-VapT1* design configurations

Liquid Outlet Valve, *VLV-LiqT1*

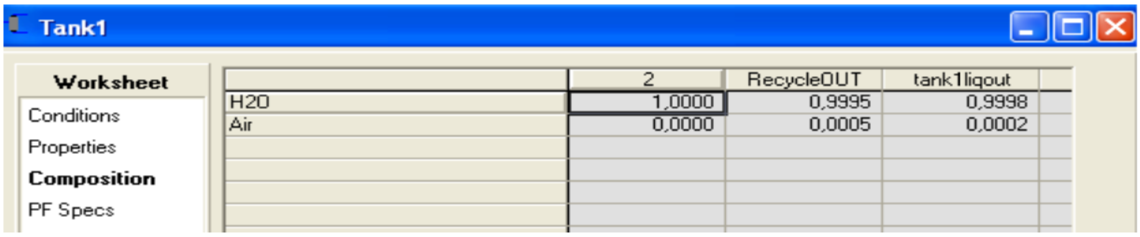
The output liquid (water) from the tank is then fed to the pump *P-100* through the outlet stream *tank1liqout*. This valve is fully open, and it controls the flow of water from the tank in order to pump the liquid.

The following configurations are made:

Valve: *VLV-LiqT1*	
Time Constant (First Order)	2 seconds
Cv	50
Type	Linear

Table 4 – *VLV-LiqT1* design configurations

Below is the composition of the fluid going into *Tank1*



	2	RecycleOUT	tank1liqout
H2O	1,0000	0,9995	0,9998
Air	0,0000	0,0005	0,0002

Figure 3 – Flow *2* composition

A. FC1 (inner loop) designing

The flow controller represents the inner loop of our cascade system control. It is faster than the outer loop, that consists of the level controller. This structure gives us the important advantage of compensating disturbances or nonlinearities quite quickly without effects in the level controller.

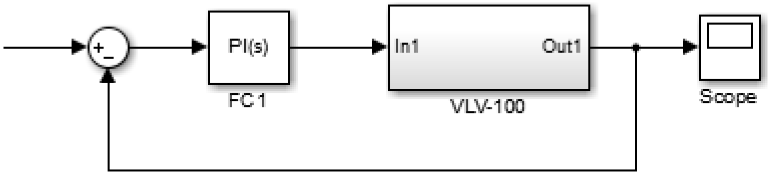


Figure 4 – Inner loop

We start designing the inner loop, consisting of the valve *VLV-100* and the flow controller *FC1*, by calculating the closed loop transfer function. As we recall, the flow controller is a PI controller in our case. Therefore, *FC1* transfer function:

$$G_F = K\left(1 + \frac{1}{T_i\ s}\right)$$

While, *VLV-100* transfer function:

$$G_V = \frac{V}{1 + T_V\ s}$$

Hence, the overall closed loop transfer function results to be:

$$G_{CL} = \frac{G_F\ G_V}{1 + G_F\ G_V} = \frac{K\ V\ T_i\ s + K\ V}{s^2\ T_i\ T_V + s(T_i + K\ V\ T_i) + K\ V}$$

Where *K* is the flow *controller gain*, *V* the valve gain (*plant gain*), T_i the flow controller time constant and T_V the valve time constant. In particular, since we are considering a linear valve, its gain *V* is constant throughout each operating point.

A first rough system tuning is possible by taking into consideration the overall transfer function previously derived and determining, for example, the *settling time*.

For this purpose, we consider $T_i = T_V = 2\ seconds$ as controller and plant time constant and the product between K and V to be equal to 1 (overall gain 1). Hence, $K = \frac{1}{V}$.

$$G_{CL} = \frac{T_i\ s + 1}{s^2\ T_i\ T_V + 2\ T_i\ s\ + 1}$$

Substituting the time constant values,

$$G_{CL} = \frac{2\,s + 1}{4\,s^2 + 4\,s\ + 1}$$

Or,

$$G_{CL} = \frac{0.5\,s + 0.25}{s^2 + s + 0.25}$$

A general second order system's closed loop transfer function is in the form:

$$TF_{CL} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where ω_n is the *natural frequency* and ζ the *damping factor*.

Comparing the two equations above, we can conclude that $2\zeta\omega_n = 1$ and $\omega_n^2 = 0.25$. Hence, $\zeta = 1$ and $\omega_n = 0.50$. We can now calculate the *settling time.*

$$t_s = -\frac{\ln(0.02)}{\zeta\omega_n} \approx \frac{3.9}{\zeta\omega_n} = 7.8\ seconds$$

(settling time to within 2%).

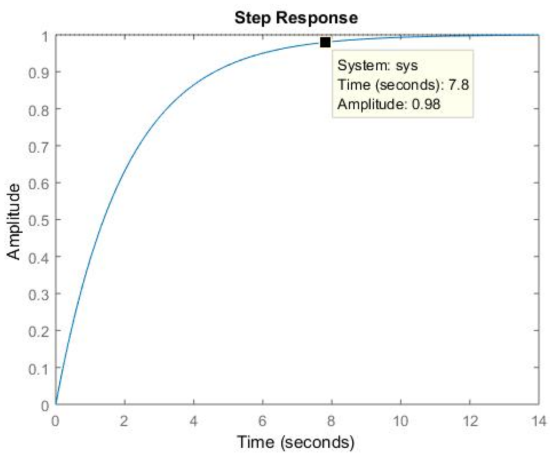


Figure 5 – *FC1* step response (settling time)

Settling time can be reduced by decreasing the controller time constant T_i or increasing the overall gain. This will result in a faster flow control loop.

B. LC1 (outer loop) designing

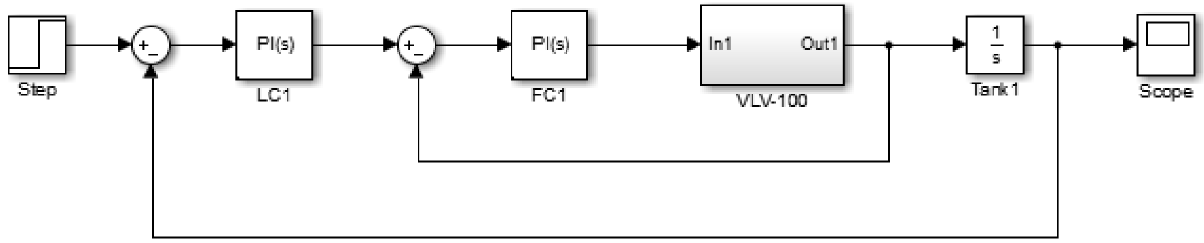


Figure 6 – Overall structure (Outer and inner loop)

As before, to design the level controller *LC1* (PI controller), we need to calculate the system closed loop transfer function. Since we know that the level controller is much slower than the flow controller, we can consider the flow controller loop (transfer function) as a unit block in the below structure. Basically, we can "ignore" the inner loop while considering the bigger loop.

The tank is considered as a capacitor (pure **integrator**) with an integration time T_t. This time constant depends on the volume of the tank and on the inflow rate into the tank and it can be calculated as follows:

$$T_t = \frac{Tank\ volume}{Max\ inflow\ rate} = \frac{10m^3}{45\,m^3/h} = 0.22h = 13.2\ min$$

We can now define *Tank1* transfer function

$$G_T = \frac{1}{T_t\ s}$$

And *LC1* (PI controller) transfer function:

$$G_{LC} = K_{LC}\left(1 + \frac{1}{T_{LC}\ s}\right)$$

As already mentioned, and as it can be seen from the previous calculation, since the time constant of the valve is significantly smaller than the time constant of the tank, we can consider the full system design structure as below:

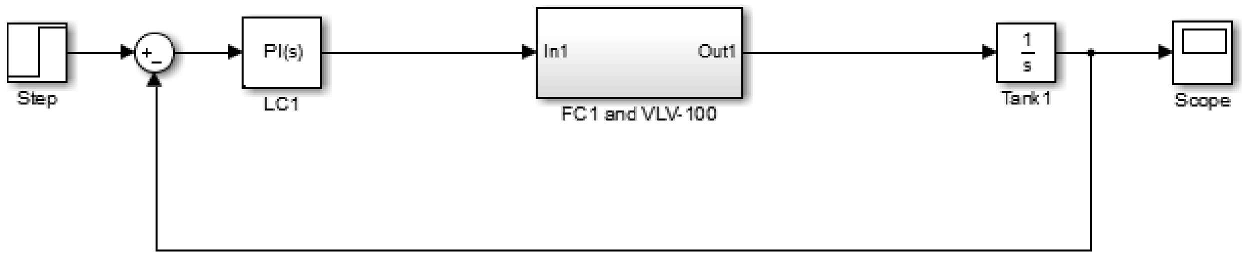


Figure 7 – Outer loop (the inner loop is "ignored")

The overall transfer function of the system above can be calculated as follows (the unit block represents the inner loop transfer function):

$$G_{Tank1} = \frac{G_{LC}\, G_T}{1 + G_{LC}\, G_T}$$

Where G_{LC} is the level controller transfer function and G_T the tank transfer function. Hence, G_{Tank1} in its expanded form:

$$G_{Tank1} = \frac{K_{LC}\left(1 + \frac{1}{T_{LC}\, s}\right)\frac{1}{T_t\, s}}{1 + K_{LC}\left(1 + \frac{1}{T_{LC}\, s}\right)\frac{1}{T_t\, s}}$$

$$G_{Tank1} = \frac{K_{LC}\, T_{LC}\, s + K_{LC}}{T_t\, T_{LC}\, s^2 + K_{LC}\, T_{LC}\, s + K_{LC}}$$

Where K_{LC} is the level controller gain, T_t the tank time constant previously calculated ($13.2\, min = 792\, seconds$) and T_{LC} is the level controller time constant.

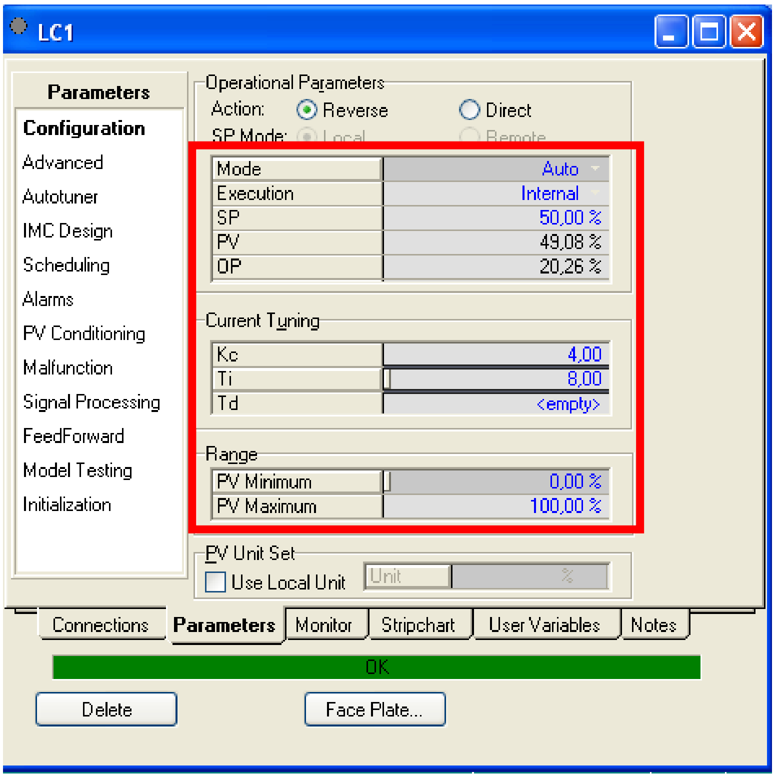


Figure 8 – *LC1* parameters configuration

As shown in the screenshot above, we have chosen $K_{LC} = 4$ and $T_{LC} = 8min$ (*480 seconds*) for our simulation. We can also see the *setpoint* set to *50%* (percentage level required). If we substitute all the values, we get

$$G_{Tank1} = \frac{1920\, s + 4}{380160\, s^2 + 1920\, s + 4}$$

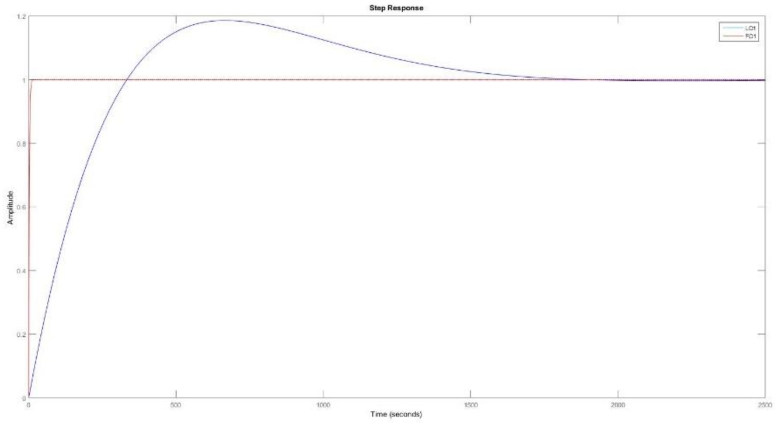


Figure 9 – *FC1* (red) and *LC1* (blue) step response comparison

From the plot above we can conclude that the settling time of the outer loop is much higher than the settling time of the inner loop. This was expected as we know that the inner loop, composed of the flow controller, is much faster than the level controller in terms of response to changes in the process system.

Linear system theory analysis (Eigenvalues, controllability and observability)

We now analyse the systems (inner and outer loop) based on eigenvalues, controllability and observability.

- We start with the inner loop composed of the flow controller *FC1* and the valve *VLV-100*.

Flow controller	**Valve**
FC1 gain $K = 2.222$	*VLV-100* gain $V = 0.45$
FC1 time constant $T_i = 2\ seconds$	*VLV-100* time constant $T_V = 2\ seconds$

$$G_{CL} = \frac{K\ V\ T_i\ s + K\ V}{s^2\ T_i\ T_V + s(T_i + K\ V\ T_i) + K\ V} = \frac{2\, s + 1}{4\, s^2 + 4\, s\ + 1}$$

Since $K\ V = 1$,

$$\frac{y}{u} = \frac{T_i\ \mathrm{s} + 1}{s^2\ T_i\ T_V + s\ 2\ T_i + 1}$$

$$y\ (s^2\ T_i\ T_V + 2\ T_i\ s + 1) = u(T_i\ s + 1)$$

Taking inverse Laplace on both sides,

$$\ddot{y}\, T_i\, T_V + 2\, T_i\, \dot{y} + y = T_i\, \dot{u} + u$$

Now, let $T_i\, T_V = a, 2\, T_i = b$, c = 1 and $T_i = d$. Thus, the equation becomes

$$a\ddot{y} + b\dot{y} + cy = d\dot{u} + cu$$

$$a\ddot{y} = d\dot{u} - b\dot{y} + c(u - y)$$

$$\ddot{y} = \frac{d}{a}\dot{u} - \frac{b}{a}\dot{y} + \frac{c}{a}(u - y)$$

Integrating,

$$\dot{y} = \frac{c}{a}\int (u - y) - \frac{b}{a}y + \frac{d}{a}u$$

Let $x_1 = y$ and $x_2 = \int (u - y)$

$$\dot{x}_1 = \frac{c}{a}x_2 - \frac{b}{a}x_1 + \frac{d}{a}u$$

$$\dot{x}_2 = u - y = u - x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -{}^{b}/_{a} & {}^{c}/_{a} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d\,/\,a \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Let us substitute the values, we get the following **State Space form**

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -{}^{2T_i}/_{T_iT_V} & {}^{1}/_{T_iT_V} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} {}^{T_i}/_{T_iT_V} \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0.25 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} u$$

Hence $A = \begin{bmatrix} -1 & 0.25 \\ -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $D = 0$.

Eigenvalues

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda + 1 & -0.25 \\ 1 & \lambda \end{vmatrix} = 0$$

$$\lambda^2 + \lambda + 0.25 = 0$$

$$\boldsymbol{\lambda_1, \lambda_2 = -0.5, -0.5}$$

- Now we analyse the outer loop, *LC1* controller

The general transfer function is

$$H(s) = \frac{y}{u} = \frac{K\,T_{LC}\,s + K}{s^2\,T_{LC}\,T_t + K\,T_{LC}\,s + K}$$

$$y(s^2\,T_{LC}\,T_t + K\,T_{LC}\,s + K) = u(K\,T_{LC}\,s + K)$$

Taking inverse Laplace on both sides,

$$\ddot{y}\,T_{LC}\,T_t + K\,T_{LC}\,\dot{y} + Ky = KT_{LC}\dot{u} + Ku$$

$$\ddot{y} = \frac{K}{T_t}\dot{u} - \frac{K}{T_t}\dot{y} + \frac{K}{T_{LC}\,T_t}(u - y)$$

Integrating,

$$\dot{y} = \frac{K}{T_{LC}\,T_t}\int (u - y) - \frac{K}{T_t}y + \frac{K}{T_t}u$$

Let $x_1 = y$ and $x_2 = \int (u - y)$

$$\dot{x}_1 = \frac{K}{T_{LC}\,T_t}x_2 - \frac{K}{T_t}x_1 + \frac{K}{T_t}u$$

$$\dot{x}_2 = u - y = u - x_1$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\dfrac{K}{T_t} & \dfrac{K}{T_{LC}\,T_t}\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}\dfrac{K}{T_t}\\ 1\end{bmatrix}u$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-0.005 & 0.00001\\ -1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}-\,0.005\\ 1\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Hence $A = \begin{bmatrix}-0.005 & 0.00001\\ -1 & 0\end{bmatrix}$, $B = \begin{bmatrix}-\,0.005\\ 1\end{bmatrix}$, $C = [1 \quad 0]$, and $D = 0$.

Eigenvalues

$$|\lambda I - A| = 0$$

$$\begin{vmatrix}\lambda + 0.005 & -0.00001\\ 1 & \lambda\end{vmatrix} = 0$$

$$\lambda^2 + 0.005\lambda + 0.00001 = 0$$

$$\boldsymbol{\lambda_1, \lambda_2 = -0.0025 + j, -0.0025 - j}$$

2. Pump, *P-100*

A centrifugal pump is a mechanical device designed to move a fluid by means of the transfer of rotational energy from one or more driven rotors, called impellers. Fluid enters the rapidly rotating impeller along its axis and is cast out by centrifugal force along its circumference through the impeller's vane tips. The action of the impeller increases the fluid's velocity and pressure and directs it towards the pump outlet. The pump casing is specially designed to constrict the fluid from the pump inlet, direct it into the impeller and then slow and control the fluid before discharge.

The impeller is the key component of a centrifugal pump. It consists of a series of curved vanes. These are normally sandwiched between two discs (an enclosed impeller).

Fluid enters the impeller at its axis (the "eye") and exits along the circumference between the vanes. The impeller, on the opposite side to the eye, is connected through a drive shaft to a motor and rotated at high speed (typically *500-5000rpm*). The rotational motion of the impeller accelerates the fluid out through the impeller vanes into the pump casing.

There are two basic designs of pump casing: volute and diffuser. The purpose in both designs is to translate the fluid flow into a controlled discharge at pressure.

The efficient operation of a centrifugal pump relies on the constant, high speed rotation of its impeller. With high viscosity feeds, centrifugal pumps become increasingly inefficient: there is greater resistance and a higher pressure is needed to maintain a specific flow rate. In general, centrifugal pumps are therefore suited to low pressure, high capacity, pumping applications of liquids with viscosities.

Pressure, friction and flow are three important characteristics of a pump system. Pressure is the driving force responsible for the movement of the fluid. Friction is the force that slows down fluid particles. Flow rate is the amount of volume that is displaced per unit time. Energy and head are two terms that are often used in pump systems. We use energy to describe the movement of liquids in pump systems because it is easier than any other method.

There are four forms of energy in pump systems: pressure, elevation, friction and velocity. Pressure is produced at the bottom of the reservoir because the liquid fills up the container completely and its weight produces a force that is distributed over a surface which is pressure. This type of pressure is called static pressure. Pressure energy is the energy that builds up when liquid or gas particles are moved slightly closer to each other

The steps to follow to select a centrifugal pump are:

1. Determine the **flow rate**: To size and select a centrifugal pump, first determine the flow rate. Selecting the right flow rate may be as simple as determining that it takes 100 gpm (6.3 L/s) to fill a tank in a reasonable amount of time or the flow rate may depend on the interaction between processes.

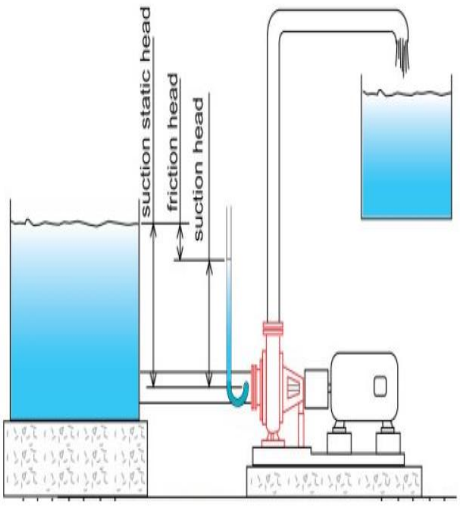
   The maximum capacity of our tank is $10m^3$and we need to fill our tank in 1 hour. So, the design flow is $10 * 1 = 10\, m^3/h$

2. Determine the **static head**: When a pump is used to displace a liquid to a higher level it is usually located at the low point or close to it. The head of the reservoir, which is called static head, will produce pressure on the pump that will have to be overcome once the pump is started.

Discharge Static Head = Suction head + Static Head

Suction Static Head = Level of Tank * Height of the Tank - Elevation of Pump from ground

Static Head = Height of 2nd tank - Elevation of Pump from ground



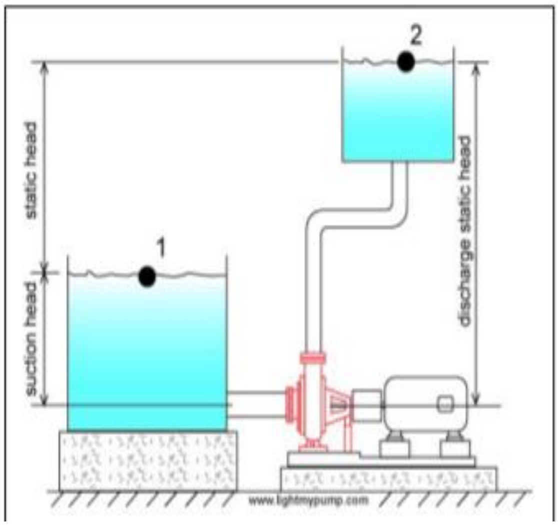


Figure 10 – Vessels-pump structure

3. Determine the **friction head**: The friction head depends on the flow rate, the pipe size and the pipe length.

4. Calculate the **total head**: The total head is the sum of the static head (remember that the static head can be positive or negative) and the friction head.

Total Head = Discharge static Head +Friction head.

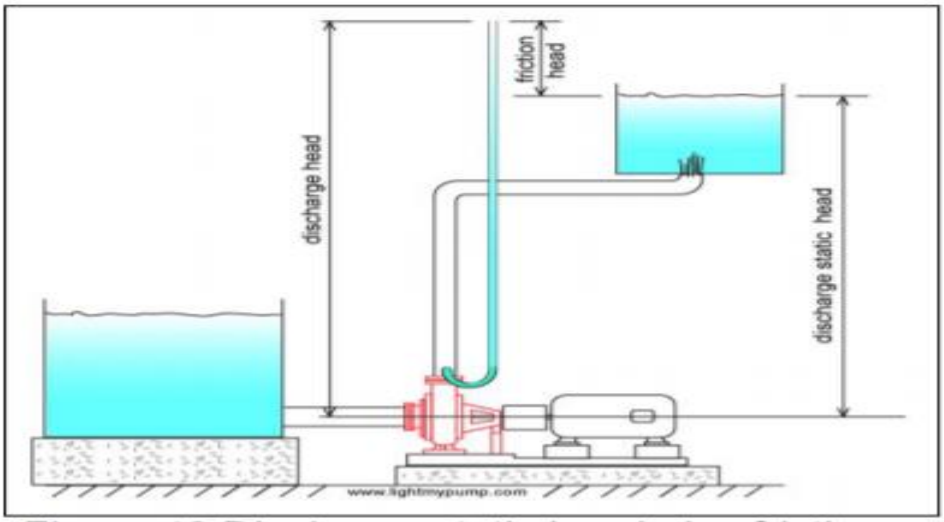


Figure 11 – Vessels-pump structure (2)

5. Select the pump: You can select the pump based on the pump manufacturer's catalogue information using the total head and flow required as well as suitability to the application.

6. **Best Efficiency Point**: The B.E.P is the operating point for design head and flow at which the pump will operate at its best possible efficiency which in turn will avoid any excess vibrations or noise and prolong the life of the pump. In our pump design, we will set the required efficiency to be 70%

The above figure gives us the specifications of the dynamics of our pump.

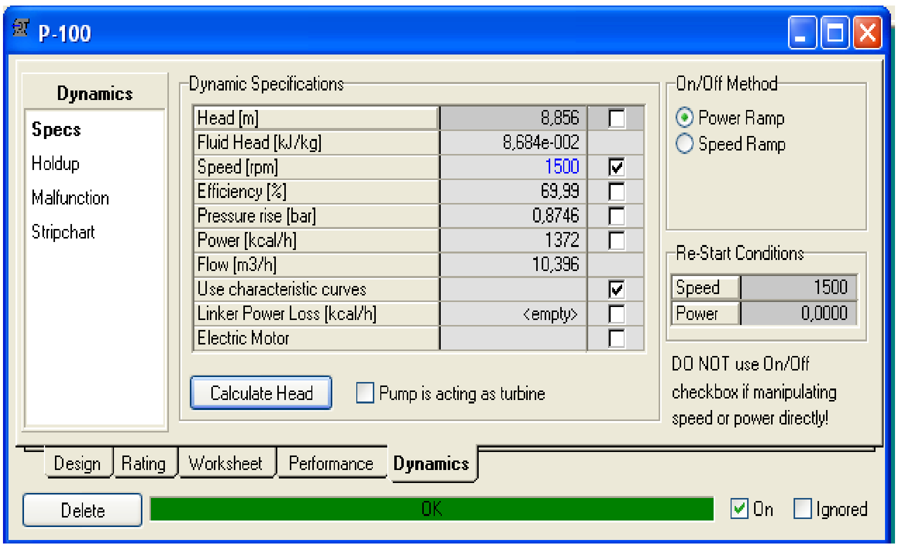


Figure 11 – Pump *P-100* dynamics specifications

The design configurations are used to generate the **pump curves** in UniSim

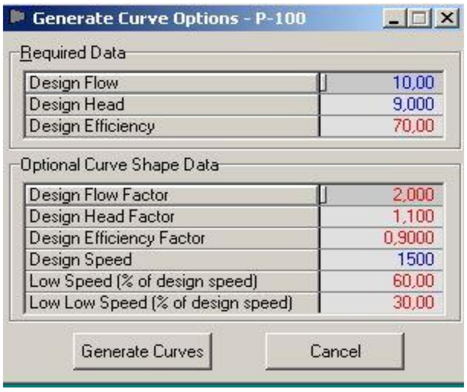


Figure 12 – Pump *P-100* generate curves options

The characteristic pump curves generated are shown below:

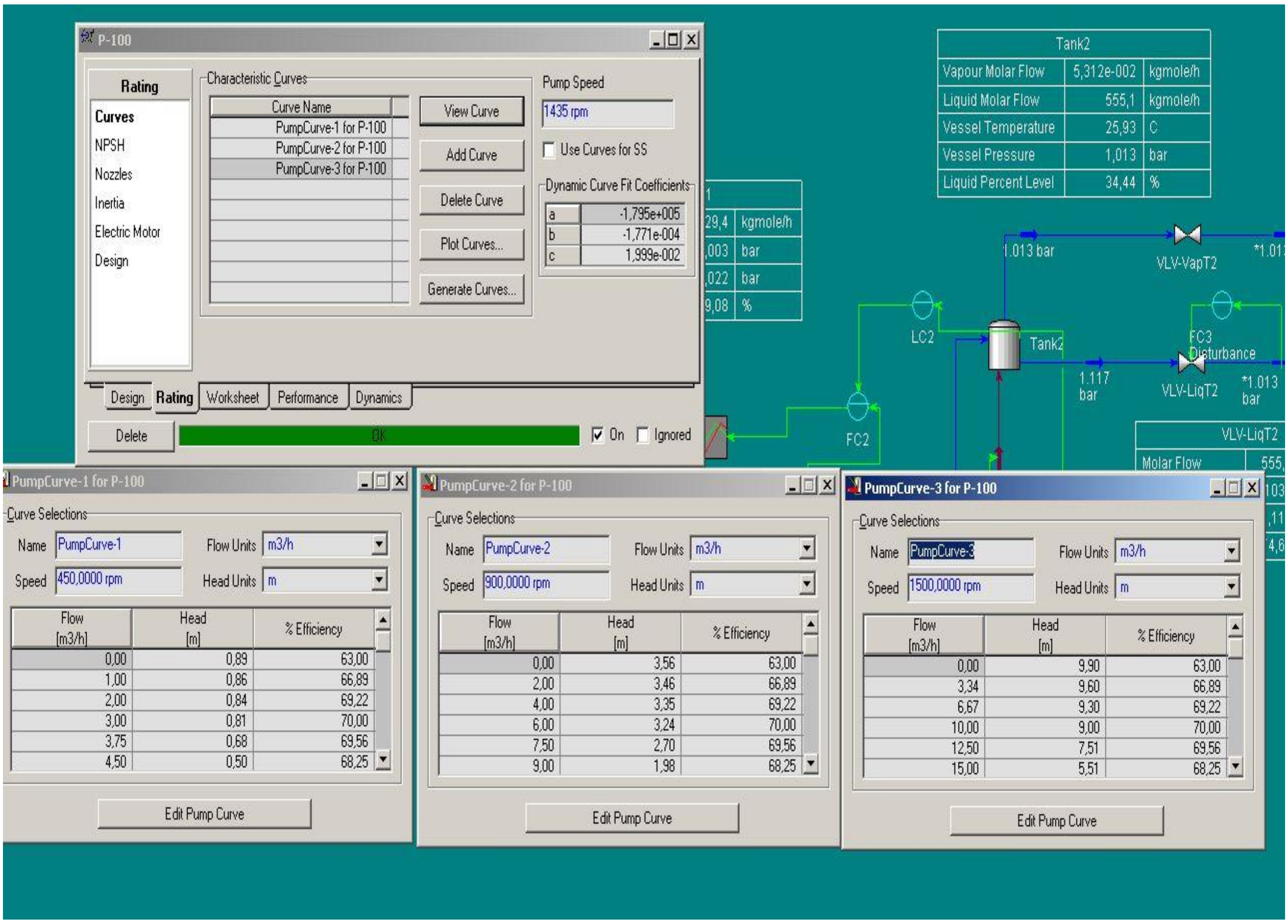


Figure 13 – Pump *P-100* characteristic curves. **450**, **900**, **1500rpm**

The below figure is the plot of the pump curves:

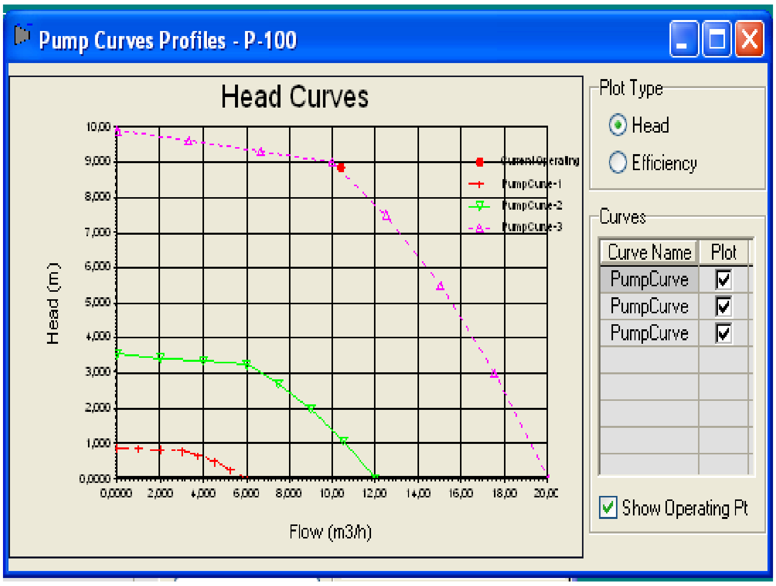


Figure 14 – Pump *P-100* characteristic curves plot

Pump speed control

The pump is designed to provide a design flow of $10\,m^3/h$ at speed of *1500rpm* with an efficiency of *70%*.

The flow requirement of *Tank2* decides the efficiency. If the flow requirement is less, then the efficiency will be reduced because we are using more energy to pump liquid at a lesser flow rate. Hence, it is important to reduce the speed of the pump based on the flow requirement so that we can improve the efficiency.

The pump system curve is given below:

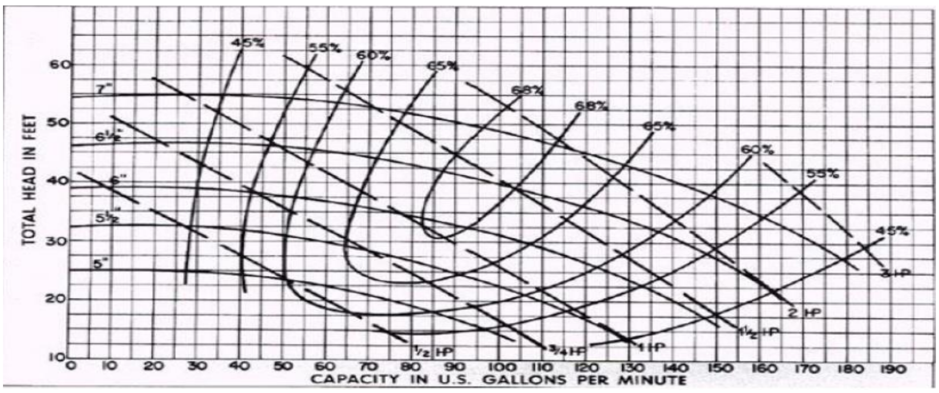


Figure 15 – Pump characteristic and efficiency curves

The efficiency curves of our pump P-*100* is shown below

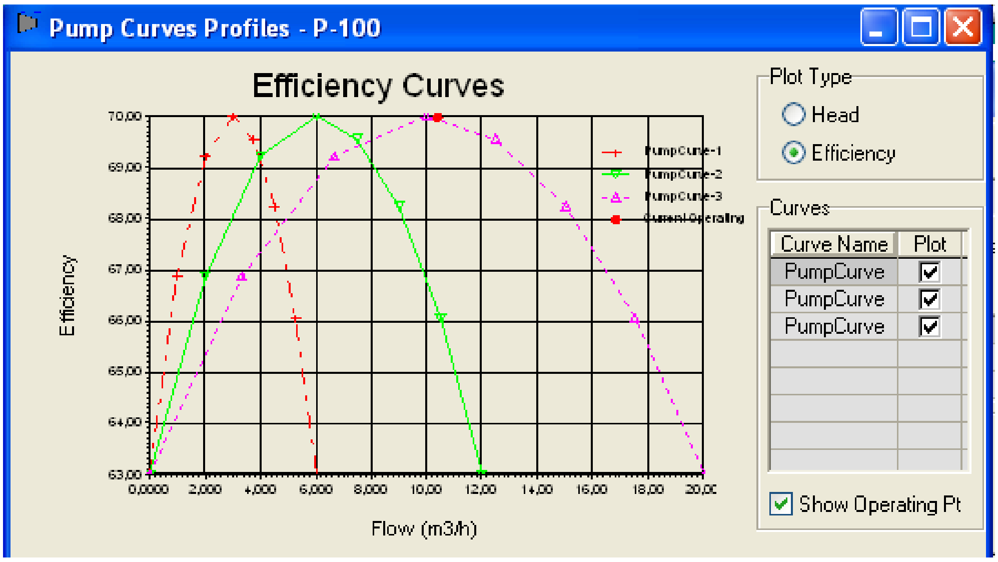


Figure 16 – Pump *P-100* efficiency vs flow (m^3/h) curves

We can observe that our pump works at its maximum efficiency at its maximum speed as designed. We also have to size the valve such that the system curve is in the efficient operating region of the pump.

So, in order to achieve the relationship between the flow requirement of the tank and the pump speed, we are connecting the *OP* of the flow controller *FC2* to be the *PV* of the transfer function block through spreadsheet. The transfer function block is used to make the pump dynamic.

The below figures are the configurations of our Transfer function block:

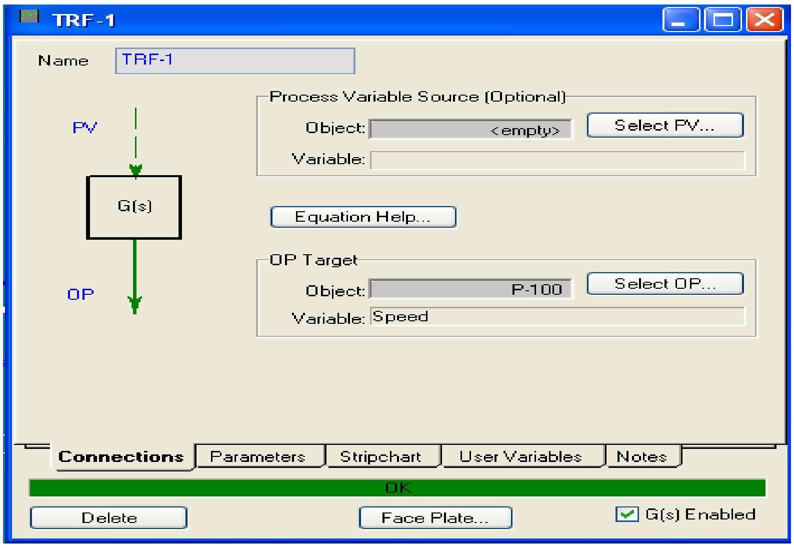


Figure 17 – *TRF-1* block connections

We have set our pump speed working range to be between *450* and *1500rpm*.

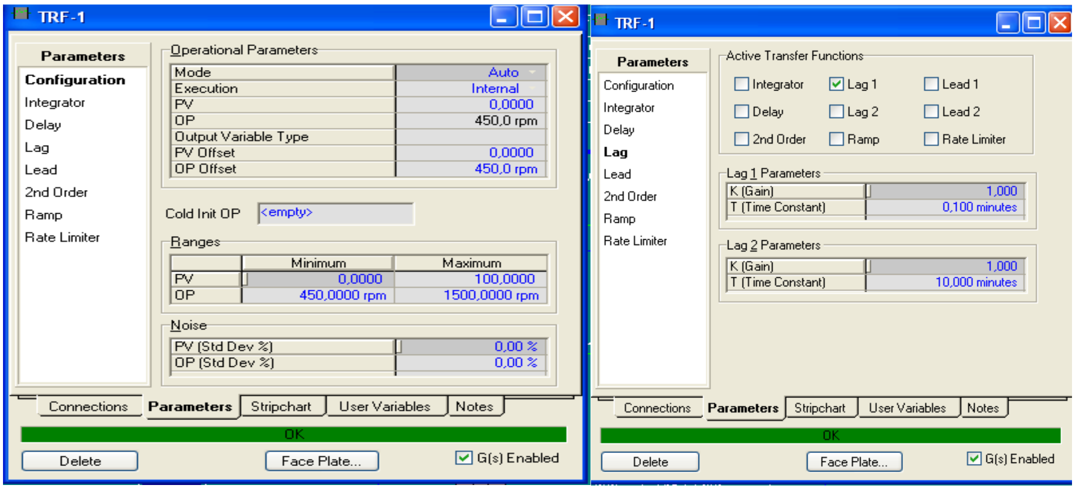


Figure 18 – *TRF-1* block parameters

The below figures show the connection between the transfer function block and the OP of the Flow controller FC2

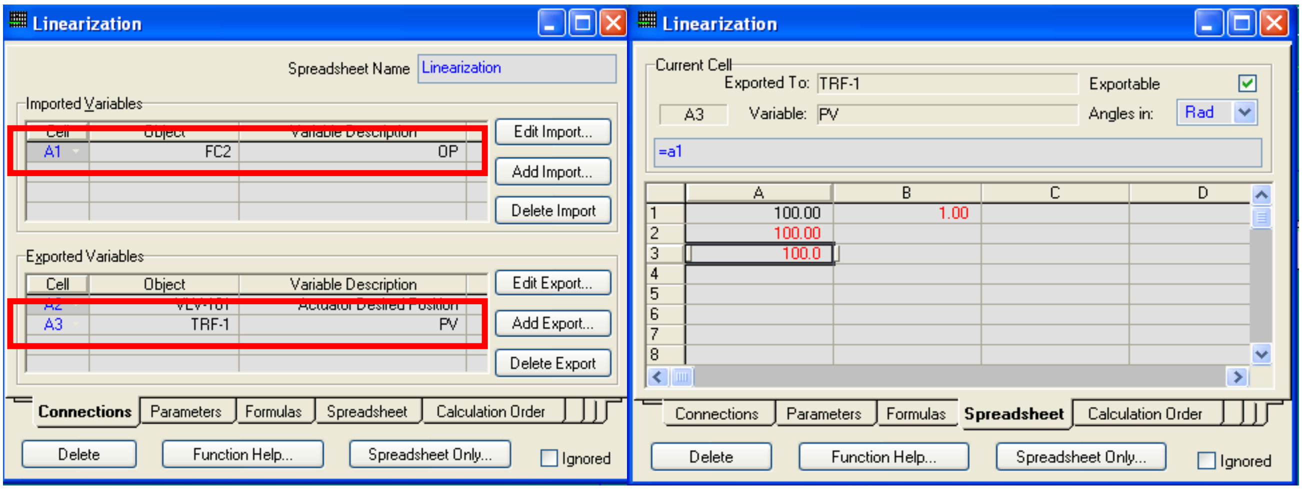


Figure 20 – *Spreadsheet* configuration for *TFR-1* block

The below table is to show that the speed of the pump changes according to the *OP* of the *FC2* and the flow at the *VLV-101* and recycle valve works as expected.

FC2 op	Pump Speed	Flow	VLV-101	Recycle flow
100 %	1500 rpm	10500 KJ / hr	10500 KJ / hr	0 KJ / hr
80%	1290 rpm	6553 KJ / hr	6553 KJ / hr	0 KJ / hr
60%	1080 rpm	3703 KJ / hr	2886 KJ / hr	817 KJ / hr
40%	870 rpm	2053 KJ / hr	40.18 KJ / hr	1998 KJ / hr
20%	660 rpm	1768 KJ / hr	16.83 KJ / hr	1751 KJ / hr
0%	450 rpm	1729 KJ / hr	-0.22 KJ / hr	1729 KJ / hr

Table 5 – Pump Speed-Flow table

Relation between *VLV-Recycle* and *VLV-101* (Opening control spreadsheet)

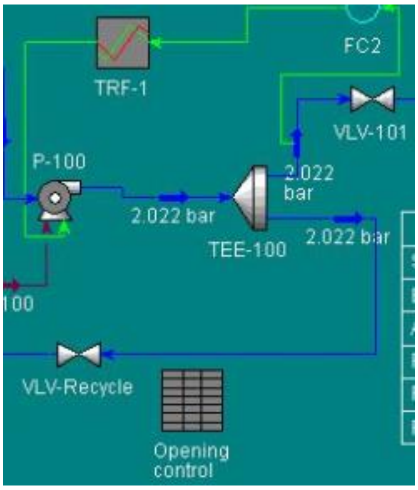


Figure 21 – *VLV-Recycle* valve

Recirculation Control Valve (VLV-Recycle):

The recirculation valve is used to maintain minimum flow at discharge of the pump in order to prevent pump from mechanical damages. Hence the recycle outlet stream (*RecycleOUT*) is fed back to the tank1.

The following configurations are used for valve design:

Valve: *VLV-Recycle*	
Time Constant (First Order)	1 second
Cv	10
Type	Linear

Table 6 – *VL* design configurations

Recycle valves are usually used to maintain some limited flow through a pump to avoid cavitation under dead-head or low flow conditions. Some plants just install a recycle line with a restriction orifice in it that is sized to pass the minimum flow to avoid pump cavitation and let it recycle all the time.

We have controlled the opening and closing of the recycle valve *VLV-Recycle* as according to the opening of the *VLV-101* which is in turn controlled by the controller *FC2*.

We have assumed the relation between the *VLV-Recycle* and *VLV-101* as according to the table given below

VLV-101	VLV-Recycle
0	69
10	49
20	32
30	19
40	9
50	3
60	1
70	0
80	0
90	0
100	0

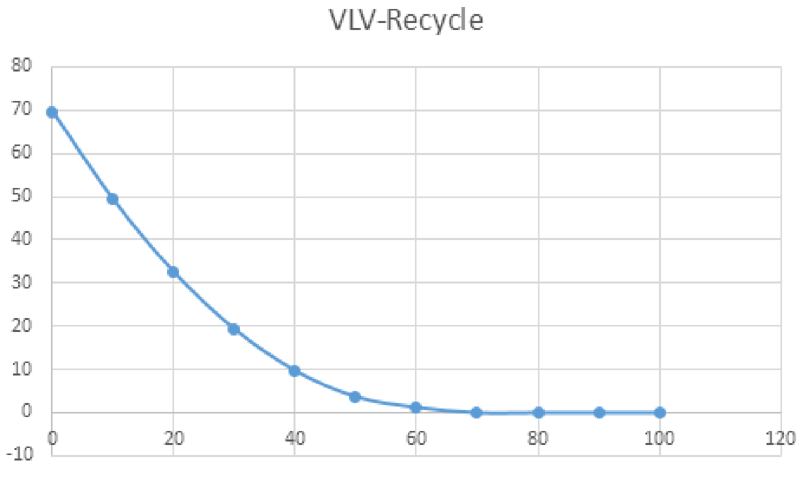


Figure 22 – *VLV-101* vs *VLV-Recyle* table and plot

We plotted opening of *VLV-Recycle* to the opening of VLV-101 and got the relation curve as show in the plot on the right side of the picture above.

Plot of *VLV-Recycle* opening with respect t to *VLV-101* opening. We arrive with the equation:

$$y = 0.0179x^2 - 2.2143x + 69.643$$

We implemented the relation into the spreadsheet:

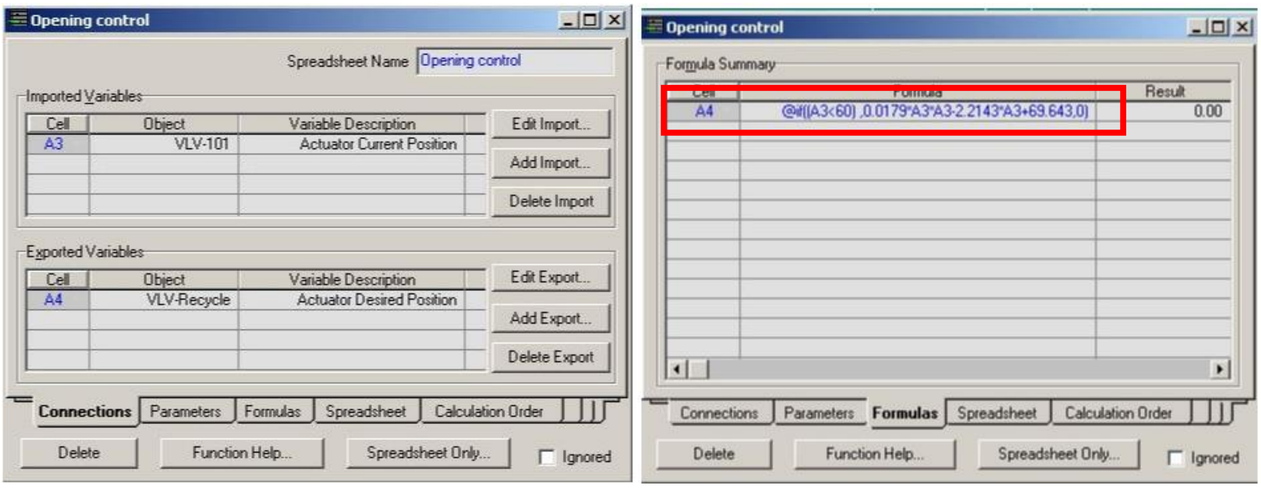


Figure 23 – *Spreadsheet* configuration for *VLV-101* and *VLV-Recycle*

Implementation of the relation between *VLV-101* and *VLV-Recycle*

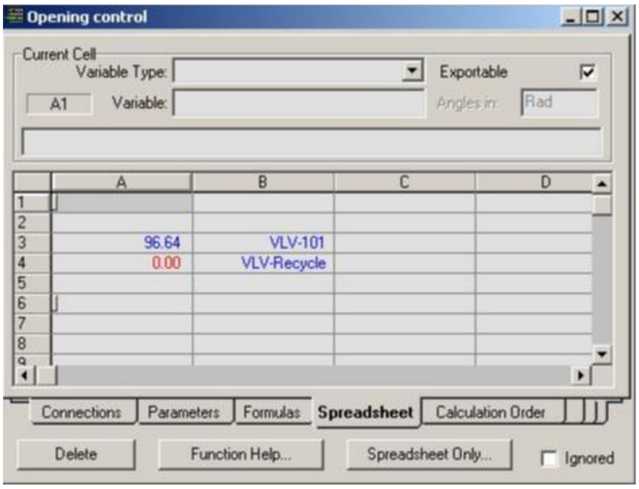


Figure 24 – *Spreadsheet* configuration for *VLV-101* and *VLV-Recycle* (2)

The values of opening of *VLV-101* and *VLV-Recycle*

It can be seen from the above picture that when our *VLV-101* is *96.6* percent open the recycle valve is completely closed and thus there is almost maximum mass flow through *VLV-101* and this prevents the pump from cavitation. Also, if we take the valve opening of *VLV-101* to be less than *60* than *VLV-Recycle* comes into the play and helps in recycling the mass flow through it and thus preventing the pump from cavitation.

3. Second tank, Tank2

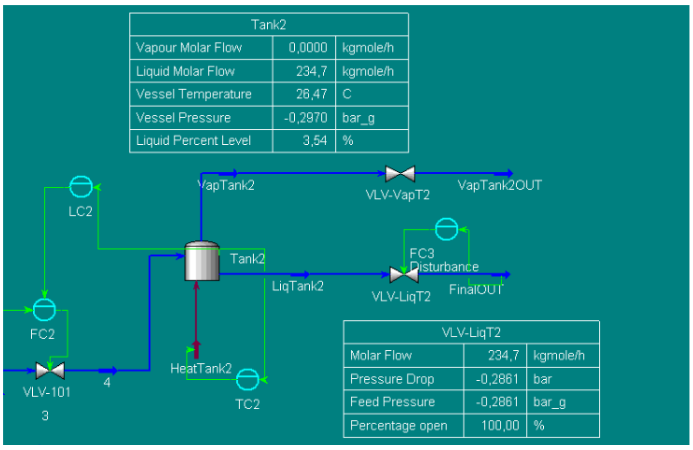


Figure 25 – *Tank2*

As mentioned above in our objectives, *Tank2* is used to provide water for the final output of the overall system. In order to ensure water availability at any time, we want the level of liquid in this tank to be *50%*.

The Design configuration for tank2 is given below:

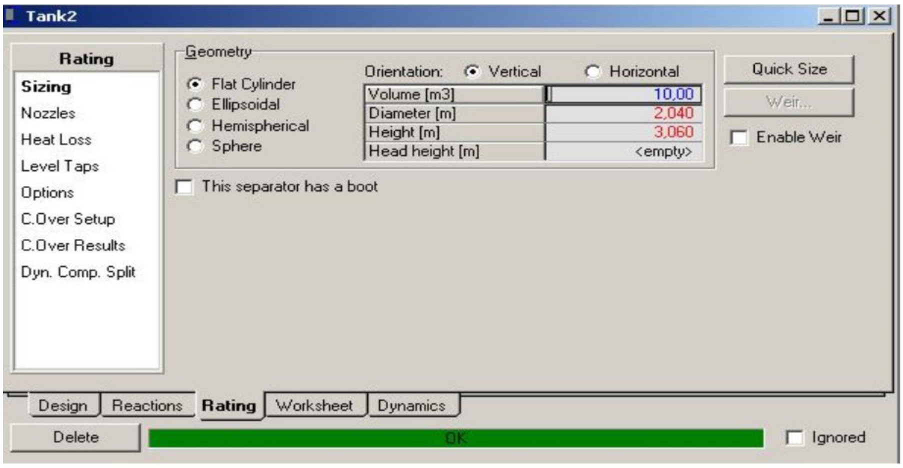


Figure 26 – *Tank2* sizing

We consider the tank to be a flat cylinder and the volume of the tank is specified as $10m^3$ as seen in figure. The total height of tank is $3.060m$ and diameter is $2.040m$.

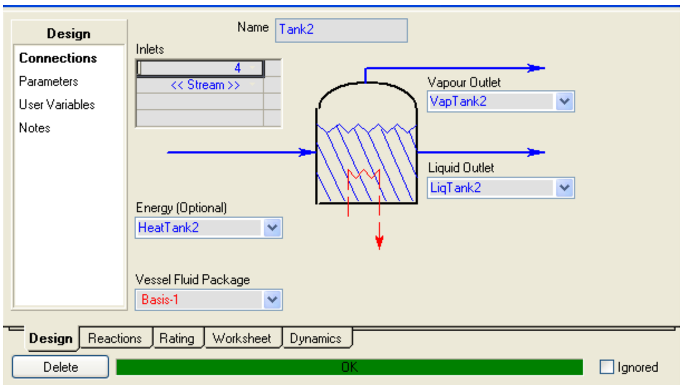


Figure 27 – *Tank2* connections

There are 3 nozzles in the tank. 1 input nozzle and 2 output nozzles. The input nozzle is connected to inlet fluid stream *4* through inlet control *VLV-101* while the output nozzles are connected to outlet stream to destination tank and vapour stream to the atmosphere respectively. Below is the values of the tank nozzles which is in a height of *5m* from the ground.

The *Tank2* continuously receive material from buffer tank and transfers the product to destination tank. The tank is kept at a height of *5m* from the ground.

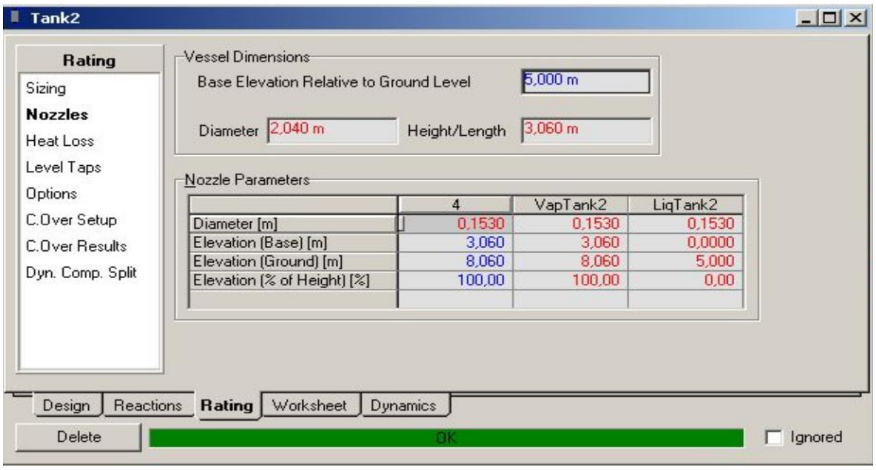


Figure 28 – *Tank2* nozzles

Inlet control Valve-101

The inlet control *VLV-101* which is responsible for delivering fluid to *Tank2* is connected to the outlet of the pump, the flow through this valve is controlled by the cascaded flow controller *FC2* and level controller *LC2*.

The control valve needs to be sized in a manner that the pump operates at its best efficient region or operating region. The following configurations are made to the inlet control *VLV-101*:

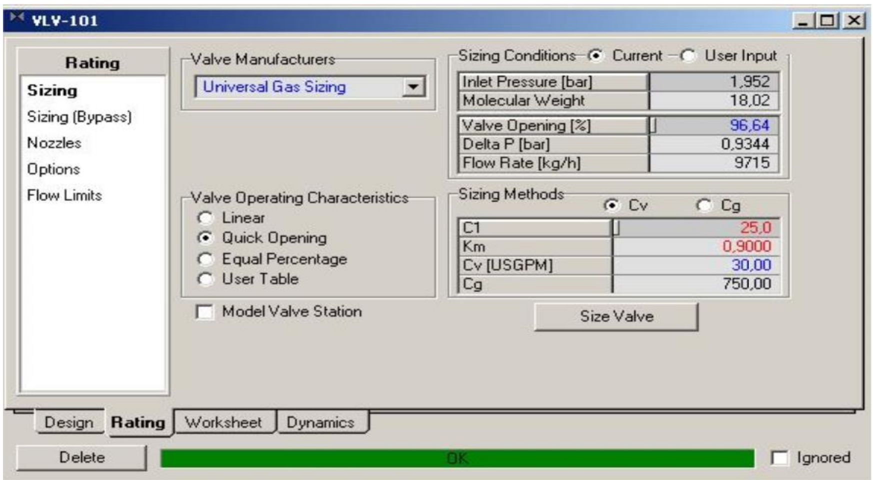


Figure 28 – *VLV-100* rating

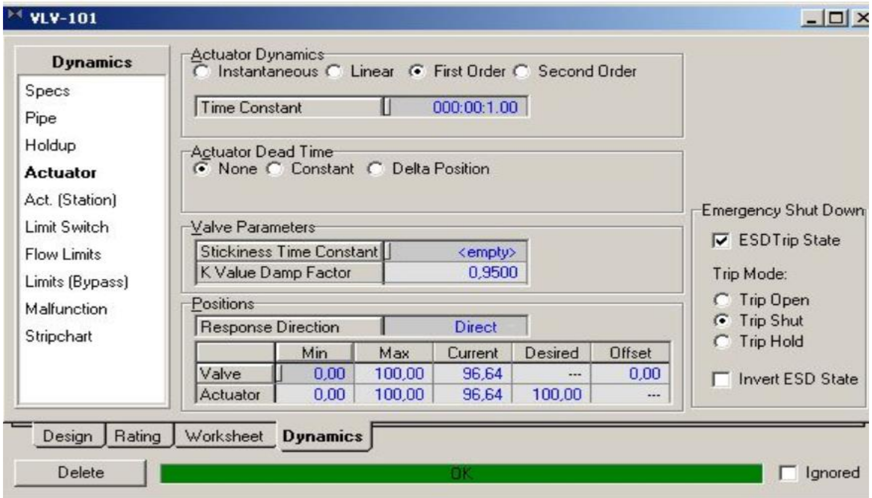


Figure 29– *VLV-100 actuator*

The *VLV-101* is made to be a *Quick Opening* valve having valve size $CV = 30$. Linearization of this valve has been explained below in detail.

Outlet Valve Liquid *Tank2* (*VLV-LiqT2*)

The configuration for the outlet valve for the tank 2 is given below:

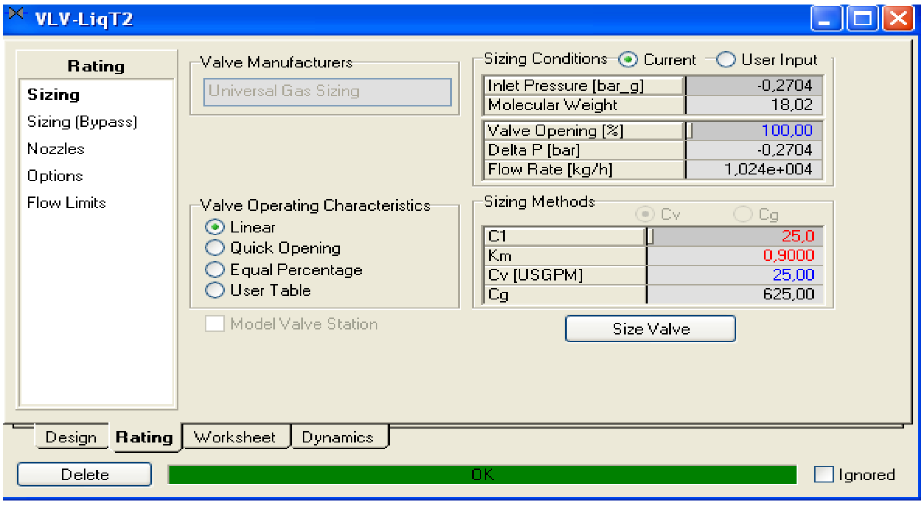


Figure 30 – *VLV-100 actuator*

We have kept the output valve opening for the *Tank2* to be *100%* to keep the maximum delivery of water. Also, we have tried to implement some disturbances by the application of flow controller *FC3*. Below figure shows the time constant values:

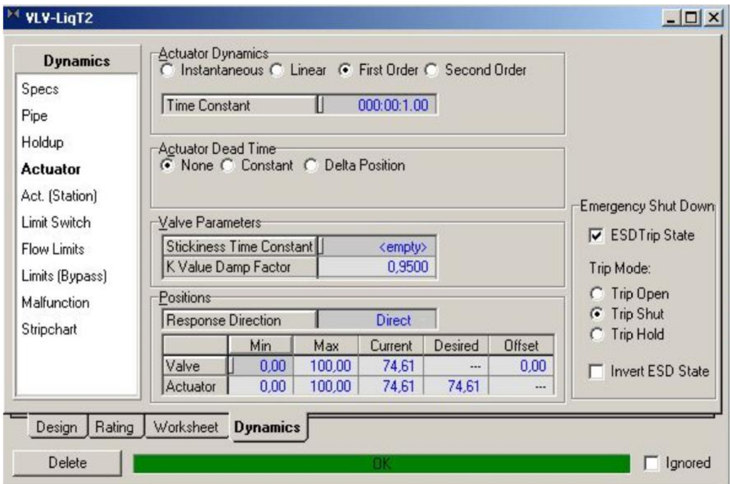


Figure 31 – *VLV-LiqT2 actuator*

Outlet Vapour Valve (*VLV-VapT2*)

The configuration of the valve for the vapour outlet by *Tank2* is given below:

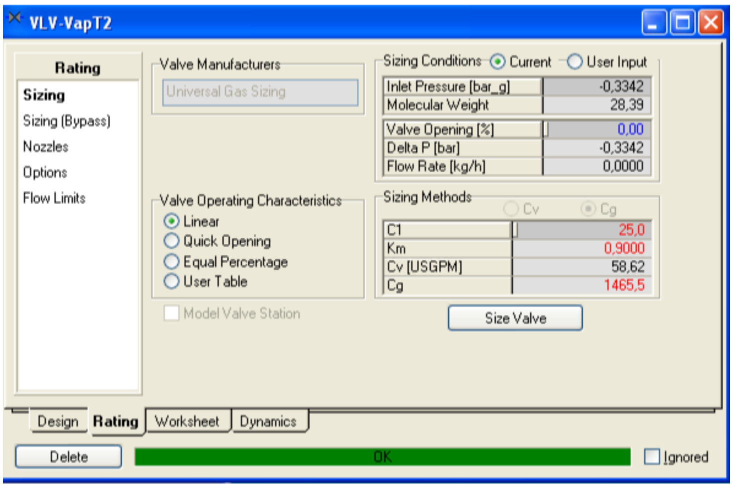


Figure 32 – *VLV-VapT2 sizing*

The control valve regulates the flow of air in and out of the tank. The inlet and outlet of this valve has only air composition to prevent any backflow of water into the tank2. This can be explained by the figure given below:

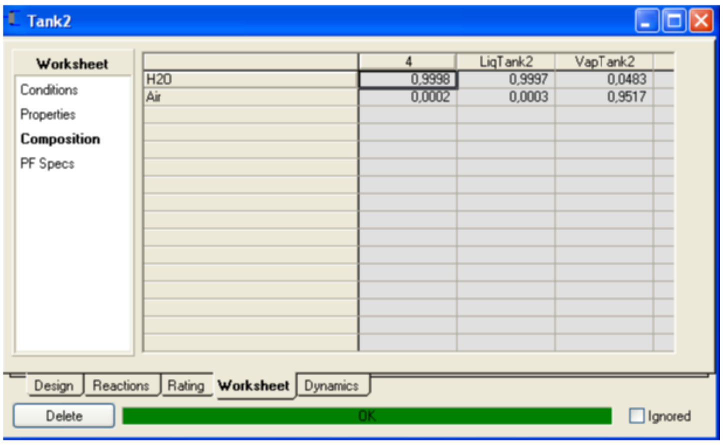


Figure 33 – *Tank2* composition

Here we can see for the *Tank2*: The outlet valve for the liquid in *Tank2* is approx. 99% this means there is almost no air in the liquid *Tank2* Valve

The outlet valve for the Vapour in *Tank2* is approx. 95%this means there is almost no water in the vapour *Tank2* Valve

Hence, we can conclude that, the inlet and outlet of this valve has only air composition to prevent any backflow of water into the *Tank2*.

Inlet feed control valve (*VLV-101*) has been taken as a Quick Opening Valve

Since quick opening valve has nonlinear behaviour and to make the valve characteristics linear (Compensation of non-linearity) we have followed the below steps:

Nonlinear valve characteristics

Below is the quick opening valve *OP* vs *PV* characteristics are tabulated using UniSim and the graph is plotted

OP	PV	PV scaled to 100
0	0	0
10	4869	31.62098974
20	6886	44.72009352
30	8434	54.77334719
40	9739	63.24847383
50	10888	70.71048188
60	11928	77.46460579
70	12883	83.66670996
80	13773	89.44668139
90	14608	94.86946357
100	15398	100

Figure 34 – *OP* vs *PV VLV-101*

Below is Quick Opening characteristics (left side) and in order to compensate the non-linearity the curve has been inverted by interchanging the *PV* and *OP* and therefore scaling has been done to get a better fit. Hence *PV* has been scaled to *100* by dividing with Maximum *PV* and the following graphs has been plotted:

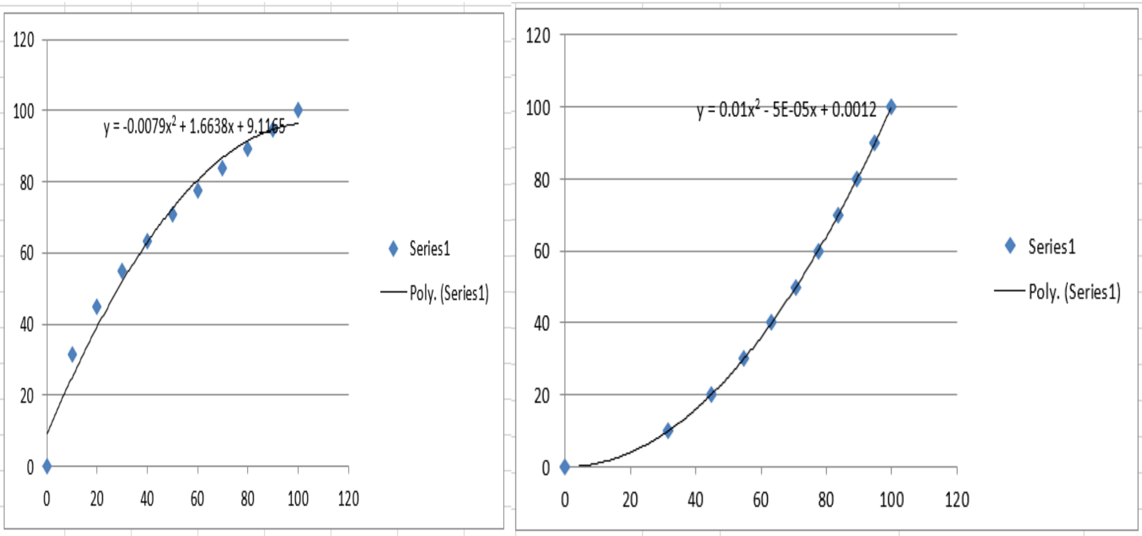


Figure 35 – *Quick Opening behaviour and the same curve inverted for compensating the nonlinearity*

Now the *OP* vs *PV* has been scaled to *1* and the graph has been plotted:

OP Scaled to 1(OP/100)	Pv scaled to 1(PV/100)
0	0
0.1	0.316209897
0.2	0.447200935
0.3	0.547733472
0.4	0.632484738
0.5	0.707104819
0.6	0.774646058
0.7	0.8366671
0.8	0.894466814
0.9	0.948694636
1	1

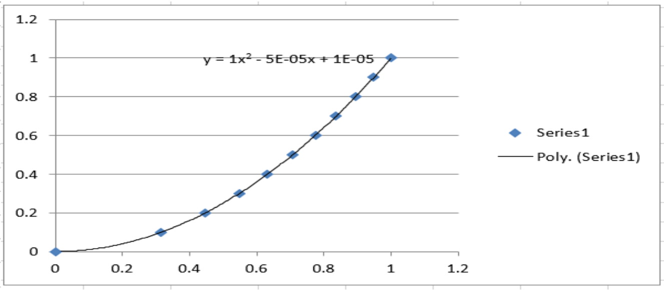


Figure 36 – *OP* vs *PV* scaled to *1*

Now, in UniSim, we use a spreadsheet by implementing the curve equation got above for linearization of the valve. In the below figure, A1 represents the *OP* of the flow controller and A2 represents the actuator's desired position of *VLV-101*.

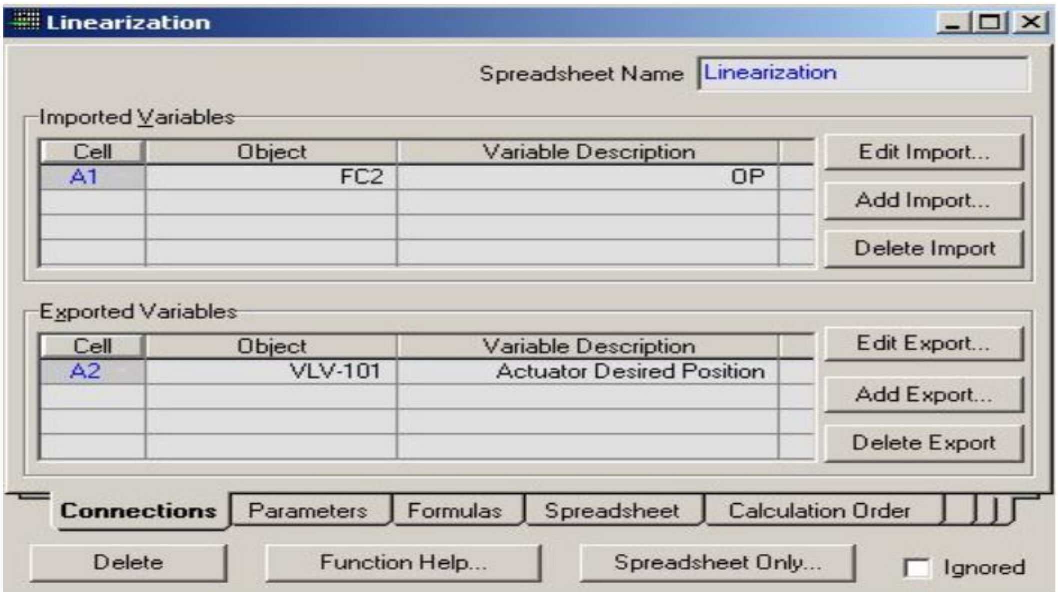


Figure 37 – Implementation via *spreadsheet*

Now the curve equation has been fed to the spreadsheet and rescaling is done i.e multiplied by 100 and the *OP* of the flow controller has been divided by 100 in order to obtain the linear behaviour of the Valve after the compensation of linearity.

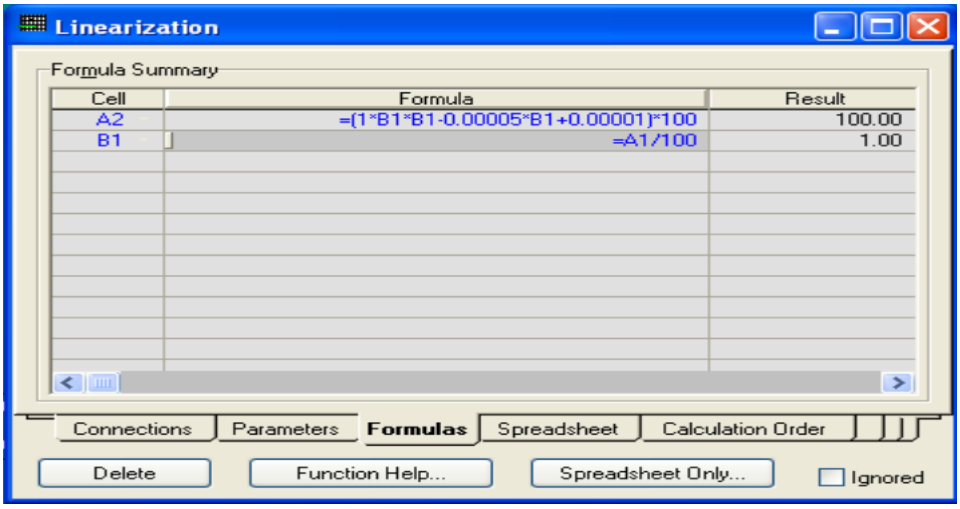


Figure 37 –*Spreadsheet* formula to compensate nonlinearity

Observations

The behaviour of the nonlinear valve before compensation and after compensation of non-linearity has been verified using strip-chart as shown below:

Nonlinear characteristics (Before Compensation):

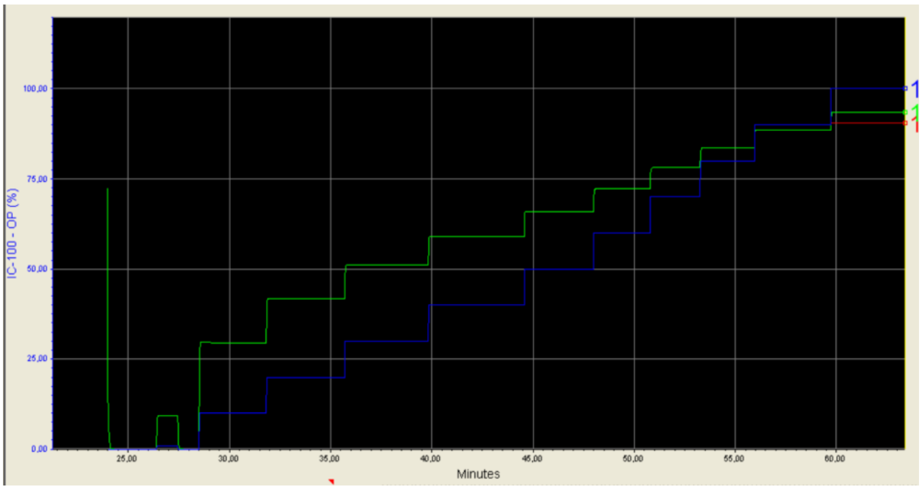


Figure 38 – *Stripchart* showing nonlinear behaviour

Linear characteristics (After Compensation):

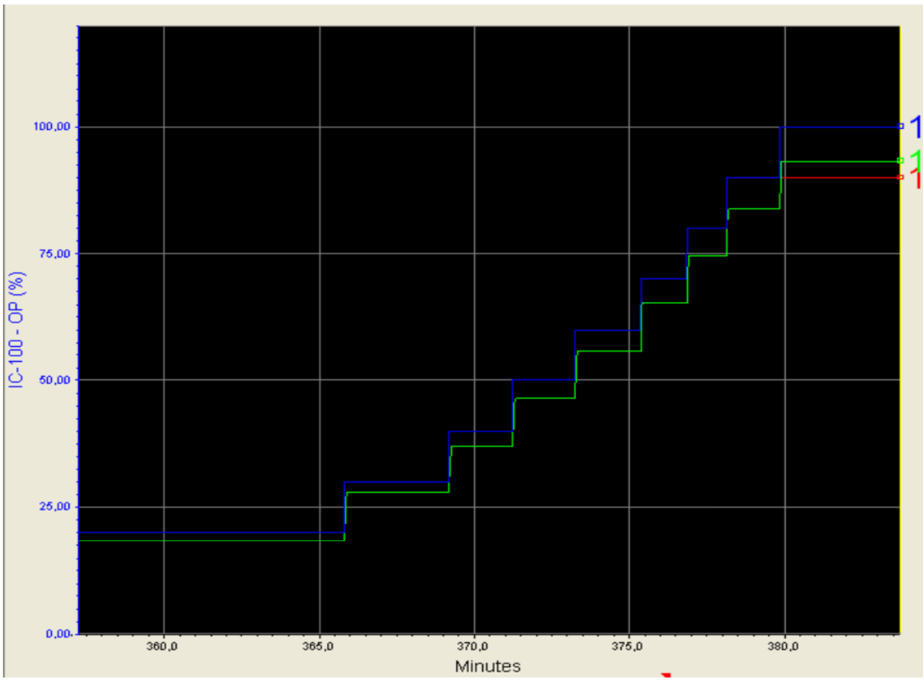


Figure 39 – *Stripchart* showing linear behaviour

Designing of FC2 flow controller (Inner Control loop)

To design the *FC2* controller, we need to calculate the closed loop transfer function of the Inner loop consisting of *VLV-101* and *FC2*:

$$\text{Transfer function of FC2: } G_{Fc2} = k_{F2}\left(1 + \frac{1}{T_i s}\right)$$

$$\text{Transfer function of Valve101: } G_{v2} = \frac{V_{v2}}{1+T_{v2}s}$$

Therefore, the closed loop transfer function of the inner control loop (Fc2-VLV101):

$$GInnerLoop = \frac{G_{Fc2}.G_v}{1+\ G_{Fc2}G_v} = \frac{(k_{F2}V_{v2})s+V_{v2}k_{F2}/T_i}{(T_{v2})s^2+(k_{F2}V_{v2}+1)s+V_{v2}k_{F2}/T_i} \qquad \text{---------- } 1$$

After performing simulation studies, we have taken our values for the respective parameters above as:

$$k_{F2} = 1\ , T_i = 2\ seconds$$

Since *VLV-101* is nonlinear, it has a nonlinear gain. Therefore, firstly, the Flow rates (PVs) for different OPs of the *FC2* controller are measured, as it is shown in Table above. And then, the gain of the valve which is a function of *OP* can be obtained by calculating the derivate of the *PV-OP* curve.

Hence the equation found for the quick- opening valve (Non-linear):

$$y = \frac{PV}{OP} = -0.0079(OP)^2 + 1.6638(OP) + 9.1165$$

$$\frac{dPV}{dOP} = -0.0158(OP) + 1.6638$$

Thus, after performing the non-linear gain compensation methods (shown previously) and by substituting different *OP* values in the above derivative equation, our valve gain becomes constant and has been found to be:

$$V_{v2} = 1.65, T_{v2} = 1\ second$$

Now substitute the above values in equation 1:

$$\boxed{GInnerLoop = \frac{3.3s+1.65}{2s^2+5.3s+1.65}}$$

Analysis of Linear System theory

We know the general state space equation from our linear system theory as:

$$\ddot{y}T_iT_{v2} + \dot{y}(T_i+KVT_i) + KVy = \dot{u}(KVT_i) + KVu$$

This is of the form, $a\ddot{y} + b\dot{y} + cy = d\dot{u} + cu$

Substituting the values:

$$\ddot{y}(2) + \dot{y}(2 + 3.3) + 1.65y = \dot{u}(3.3) + 1.65u$$

Where a = 2, b = 5.3, c = 1.65, d = 3.3

$$\dot{x}1 = \frac{c}{a}x_2 - \frac{b}{a}x_1 + \frac{d}{a}x_2 \longrightarrow \dot{x}1 = 0.825x_2 - 2.65x_1 + 1.65$$

$$\dot{x}2 = -x_1 + u$$

The state space model for the above transfer function is as follows,

$$\begin{pmatrix} \dot{x}1 \\ \dot{x}2 \end{pmatrix} = \begin{pmatrix} -2.65 & 0.825 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1.65 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigenvalues

To find eigen values, $|\lambda I - A| = 0$

We have, $|\lambda I - A| = \begin{pmatrix} \lambda + 2.65 & -0.825 \\ 1 & \lambda \end{pmatrix} = 0$

On calculating we get, $\lambda^2 + 2.65\lambda - 0.825 = 0$

$$\lambda_1, \lambda_2 = \frac{-2.65 \pm 1.929}{2} = -0.36, -2.285$$

As the poles/Eigen values are said to be negative and lies in negative half of s-plane, the system is said to be stable.

Controllability

$$S \sim |B\ AB| \neq 0$$

$$A = \begin{pmatrix} -2.65 & 0.825 \\ -1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 1.65 \\ 1 \end{pmatrix} \rightarrow AB = \begin{pmatrix} -3.5475 \\ -1.65 \end{pmatrix}$$

$$S = \begin{vmatrix} 1.65 & -3.5475 \\ 1 & -1.65 \end{vmatrix} = -2.7225 + 3.5475 = 0.825$$

Therefore $S \neq 0$ and rank of controllability matrix S is 2 and full, hence system is controllable.

Observability

$$S \sim \begin{vmatrix} C \\ CA \end{vmatrix} \neq 0$$

$$C = (1 \quad 0)\ A = \begin{pmatrix} -2.65 & 0.825 \\ -1 & 0 \end{pmatrix} \rightarrow CA = (-2.65 \quad 0.825)$$

$$S = \begin{vmatrix} 1 & 0 \\ -2.65 & 0.825 \end{vmatrix} = 0.825 \neq 0$$

Hence rank of observability matrix is also 2 and the system is observable

We have verified the same results in MATLAB by plotting step response of the inner loop (*FC2* and *VLV-101*) transfer function:

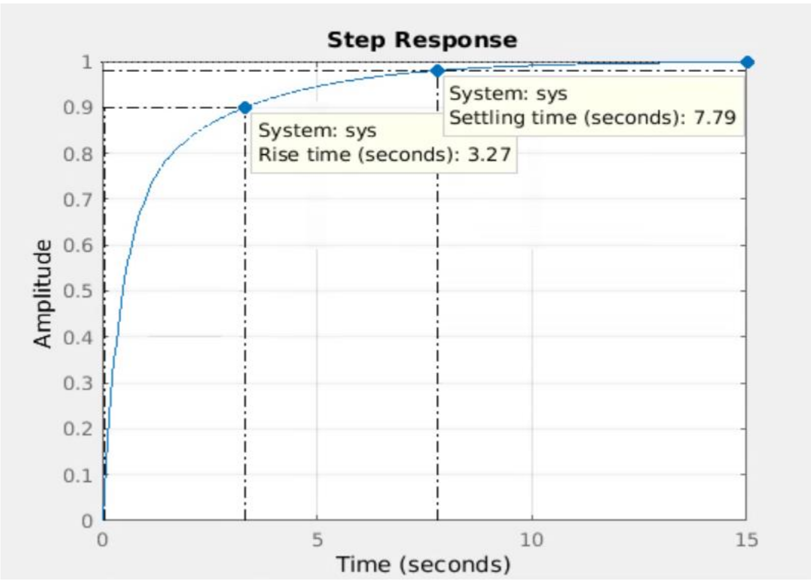


Figure 40 – Step response *inner loop*

Designing of LC2 level controller (Outer Control loop)

To design the *LC2* controller, which is a **PI controller**, we need to calculate the closed loop transfer function of a loop which contains the mentioned inner loop, *Tank2* and the *LC2*. Therefore, firstly, *Tank2* should be modelled. Since a storage tank is behaving as a capacitor, it can be modelled by an integrator as it is shown in formula (2)

The time constant of the tank depends on the volume of the tank and the inflow rate, so the Tank1 time constant is calculated via formula (3).

In order to find the maximum flow rate, the *OP* of the *FC2* controller should be equal to *100* which can happen when *Tank2* is empty.

$$\text{Transfer function of Tank2: } G_{T2} = \frac{1}{T_t S} \quad \text{-------- 2}$$

$$T_t(\boldsymbol{Tank}) = \frac{Volume\ of\ Tank}{Maximum\ Flow\ Rate} = \frac{10m^3}{\text{Open valve1 to 100\%,when tank1 is empty}}$$

$$T_t = \frac{10m^3}{\frac{10.5m^3}{hour}} = 0.95\ hour = 57\ minutes = 3420\ seconds$$

$$\text{Transfer function of LC2: } G_{Lc2} = k_{L2}\left(1 + \frac{1}{T_{L2}s}\right) \quad \text{----------- 3}$$

Therefore, the closed loop transfer function of the outer control loop (LC2 and Tank2-VLV101):

$$G\text{Outer}Loop = \frac{G_{Lc2}.G_{T2}}{1 + G_{Lc2}G_{T2}} = \frac{K_{L2}T_{L2}\text{S} + K_{L2}}{(T_tT_{L2})s^2 + (K_{L2}T_{L2})\text{S} + K_{L2}} \quad \text{------------ 4}$$

We assume the following parameters:

$K_{L2} = 4$, $T_{L2} = 8\ mins = 480\ seconds,\ T_t = 3420\ seconds$

$$\boxed{G\text{Outer}Loop = \frac{1920s+4}{1641600s^2+1920s+4}}$$

Analysis of Linear System Theory:

We know the state space equation from our linear system theory as:

$$\ddot{y}T_{L2}T_t + \dot{y}(T_{L2} + K_{L2}T_{L2}) + K_{L2}y = \dot{u}(K_{L2}T_{L2}) + K_{L2}u$$

This is of the form a$\ddot{y} + b\dot{y} + cy = d\dot{u} + cu$

Substituting the values: $\ddot{y}(1641600) + \dot{y}(2400) + 4y = \dot{u}(1920) + 4u$

Where a = 1641600, b = 2400, c = 4, d = 1920

$$\dot{x1} = \frac{c}{a}x_2 - \frac{b}{a}\ x_1 + \frac{d}{a}x_2$$

$$\dot{x1} = 0.000002x_2\text{-}0.00146x_1 + 0.001169$$

$$\dot{x2} = -x_1 + u$$

The state space model for the above transfer function is as follows,

$$\begin{pmatrix} \dot{x1} \\ \dot{x2} \end{pmatrix} = \begin{pmatrix} -0.00146 & 0.000002 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0.001169 \\ 1 \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Eigenvalues

To find eigen values, $|\lambda I - A| = 0$

We have, $|\lambda I - A| = \begin{pmatrix} \lambda + 0.00146 & 0.000002 \\ 1 & \lambda \end{pmatrix} = 0$

On calculating we get, $\lambda^2 + 0.00146\lambda - 0.000002 = 0$

$$\lambda_1, \lambda_2 = \frac{-0.00146 \pm 0.00241i}{2} = -0.0007 \pm 0.0012i$$

Controllability:

$$S \sim |B\ AB| \neq 0$$

$$A = \begin{pmatrix} -0.00146 & 0.000002 \\ -1 & 0 \end{pmatrix} \quad B = \begin{pmatrix} 0.001169 \\ 1 \end{pmatrix} \rightarrow AB = \begin{pmatrix} 0.0000003 \\ -0.001169 \end{pmatrix}$$

$$S = \begin{vmatrix} 0.001169 & 0.0000003 \\ 1 & -0.001169 \end{vmatrix} = -0.0000013 - 0.0000003 = -0.0000016$$

Therefore $S \neq 0$ *and rank of controllability matrix* S is 2 and full, hence system is controllable.

Observability:

$$Q \sim \begin{vmatrix} C \\ CA \end{vmatrix} \neq 0$$

$$C = \begin{pmatrix} 1 & 0 \end{pmatrix} A = \begin{pmatrix} -0.00146 & 0.000002 \\ -1 & 0 \end{pmatrix} \rightarrow CA = \begin{pmatrix} -0.00146 & 0.000002 \end{pmatrix}$$

$$Q = \begin{vmatrix} 1 & 0 \\ -0.00146 & 0.000002 \end{vmatrix} = 0.000002 \neq 0$$

Hence rank of observability matrix is also 2 and the system is observable.

Temperature controller, *TC*

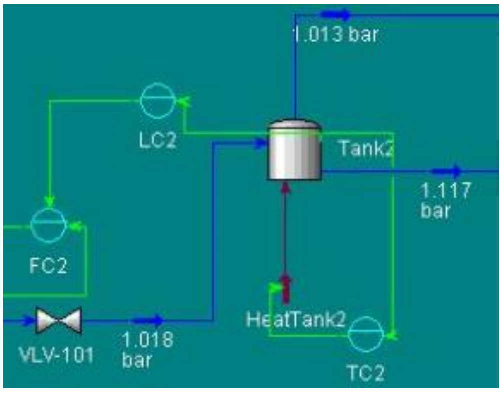


Figure 41 – *TC*

A temperature control loop has to be designed and implemented which will regulate the flow of heating fluid around the tank and maintain the temperature of process fluid at desired setpoint.

Temperature controller is in industrial processes since it is important and useful to control the temperature so that they are kept on or close to specifies values (setpoints).

Energy flow, *HeatTank2*

HeatTank2 is the external energy source that is supplied to tank in order to heat the it up. In order to check if the water temperature is maintained at the desired setpoint a temperature controller *TC2* is used with setpoint *35* degree Celsius.

The following expression is used to calculate the energy required to heat the full tank

$$Q = m * c * \Delta T$$

Where,

Q = Heat Energy in kJ

m = total mass of process fluid

c = specific heat capacity of process fluid

ΔT = the amount of temperature rise required.

m = Total volume of tank * Density of fluid.

In our case the process fluid is water, hence the mass for the water is

Mass of water, m = $10m^3 * \frac{1000kg}{m^3}$ = 10000kg

c = specific heat capacity of water = 4.19 KJ/Kg.DegC

ΔT = Required temperature - normal temperature of process fluid = 35 - 25 = 10 deg.C

Q = m ∗ c ∗ ΔT

Q=10000kg*4.19 KJ/Kg.DegC*10 deg.C= 419,000kJ

Thus, we set the range of heat energy supplied to heat the tank 2 from 0 to 119Kcal/h

When we set this energy range, we were getting a longer duration to heat the *Tank2*, so we Increased the energy upper bound range from 419.000kJ to 2.000.000 kJ (47801e+05Kcal/h)

Designing of temperature controller

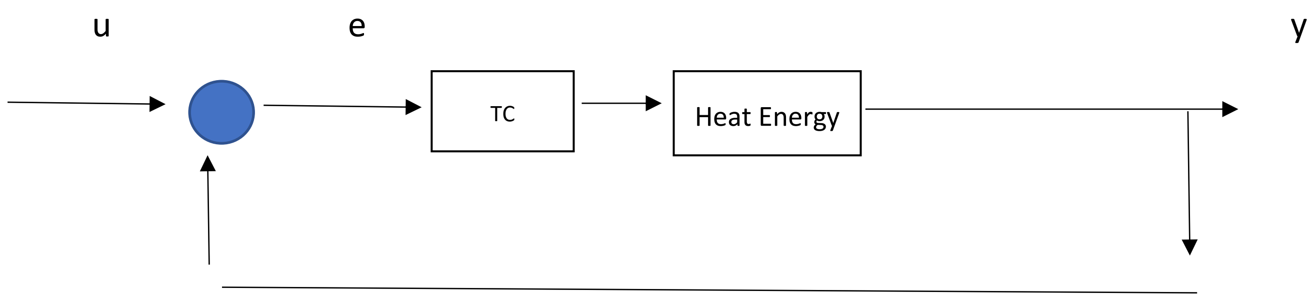


Figure 42 – *Temperature controller structure*

As you can see from the block diagram that the *TC* is connected to the *HeatTank2* (which is our energy source) that is connected directly to our *Tank2*. In this closed loop system, the heat energy supplied along with the tank is considered to be the plant that is controlled by the TC.

To find the transfer function of the plant (that is heat energy tank in our case), this plant behaves as a controller valve

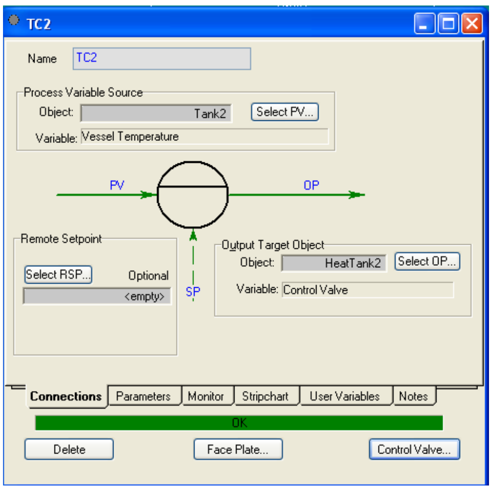


Figure 42 – *TC* connections

HeatTank2 as Control Valve. Hence, the Transfer function of the *HeatTank2*:

$$P = \frac{K_H}{1 + T_H s}$$

Transfer function of the *TC*:

$$TC = K_T \left(1 + \frac{1}{T_i s}\right)$$

By finding out the overall transfer function of this closed loop system we arrive at the following equation. The overall gain K_H K_T of this closed loop system is considered 1

$$T = \frac{T_i s}{(1+T_H S)T_i S+T_i S+1}$$

Now we looked at the strip chart of our *TC2* and we arrived with the following observation:

Level (%)	Time (minutes)
10%	4.208
50%	11.46
90%	16.16

Table 7 – Level percent vs Time

For finding out the transfer function of our temperature controller we take the time constant from the *50%* case, *11 minutes*.

Now specifying all the parameter in the transfer function as calculated above we get the following result:

$$T_i = 5 * 60\ sec = 300sec$$

And $T_H = 11 * 60sec = 660sec$

$$T = \frac{T_i s}{(1 + T_H S)T_i S + T_i S + 1}$$

$$T = \frac{300s}{(1 + 660s)300s + 300s + 1}$$

$$T = \frac{300s}{198000s^2 + 600s + 1}$$

Stripchart observations

– At 10% Level of the Tank

In the above figure we can see that the time to reach *35* degree Celsius when the tank is filled with *10%* of water is about *4.208 minutes*.

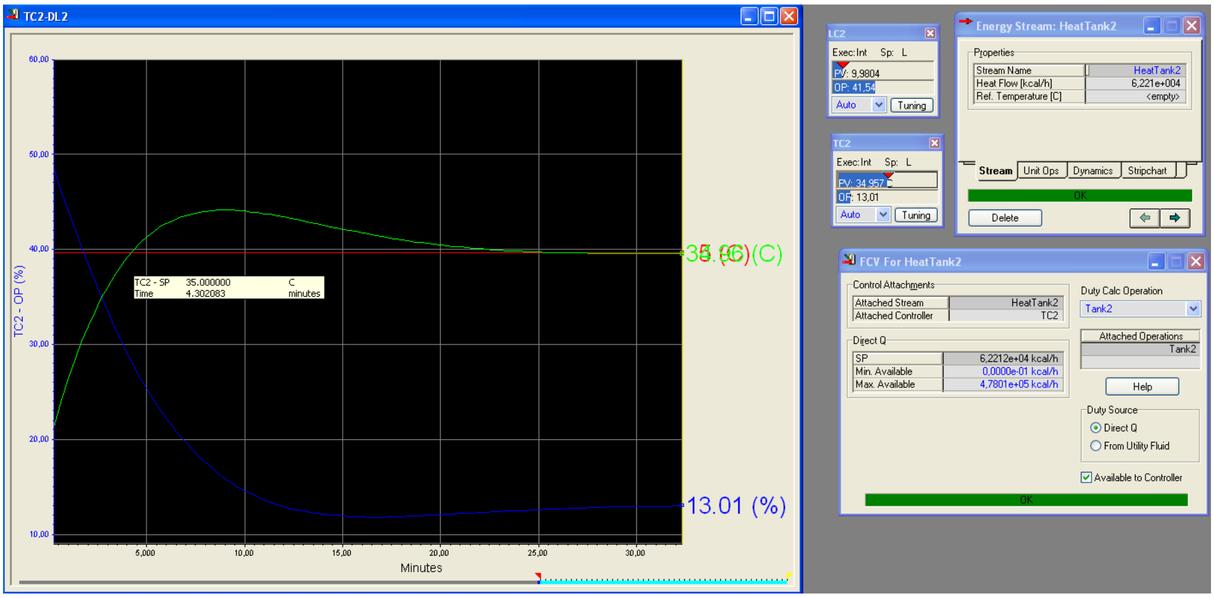


Figure 43 – *TC2* stripchart *Tank2* 10%

– At 50% Level of the Tank

In the above figure we can see that the rise time to reach 35 degree Celsius when the tank is filled with 50% of water is about 11.187 minutes.

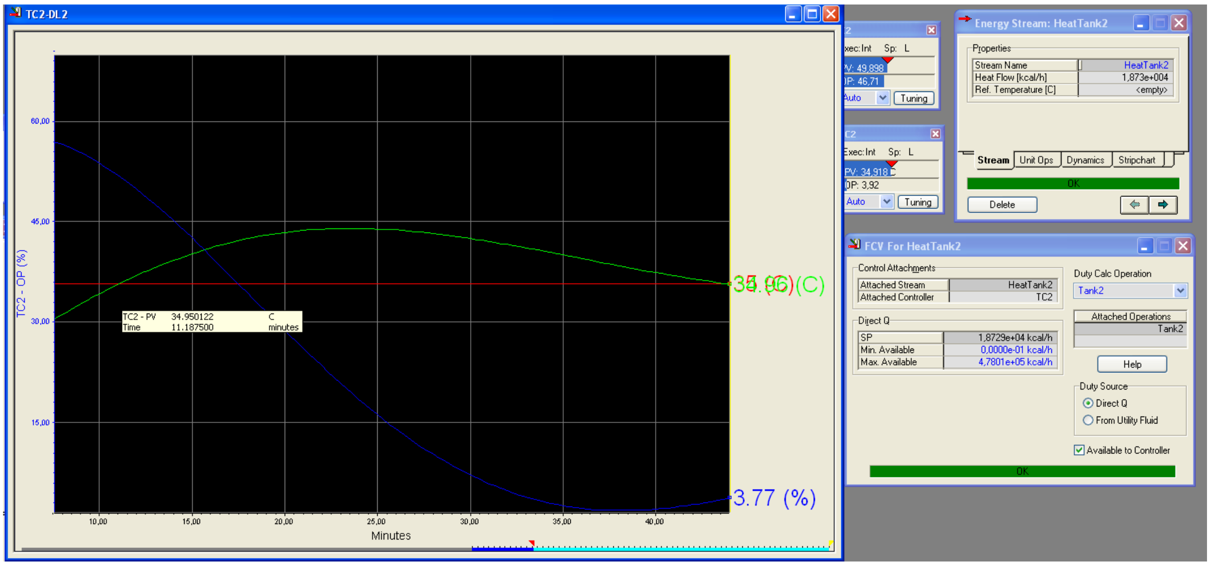


Figure 44 – *TC2* stripchart *Tank2* 50%

– At 90% Level of the Tank

In the above figure we can see that the rise time to reach 35 degree Celsius when the tank is filled with 90% of water is about 16.17 minutes.

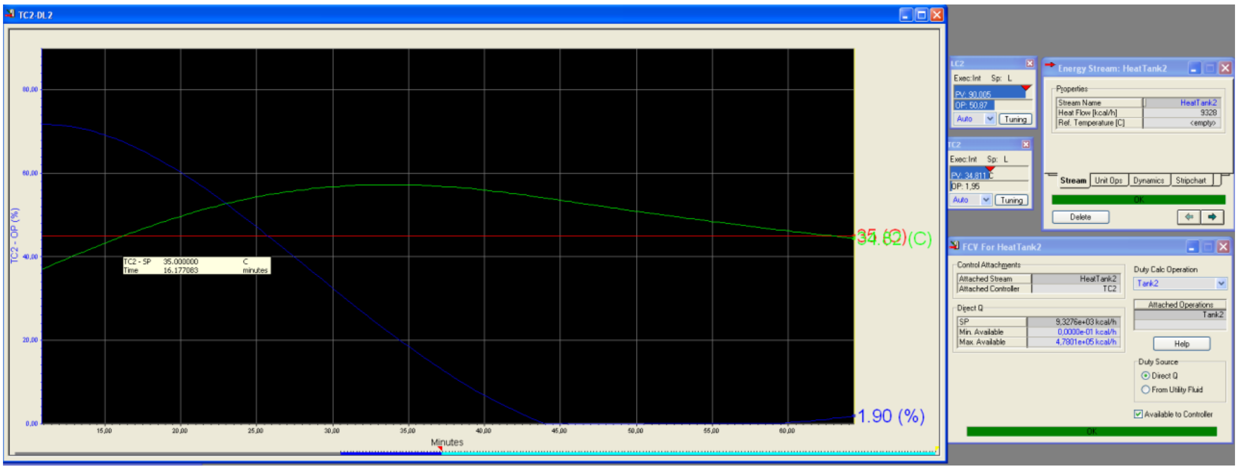


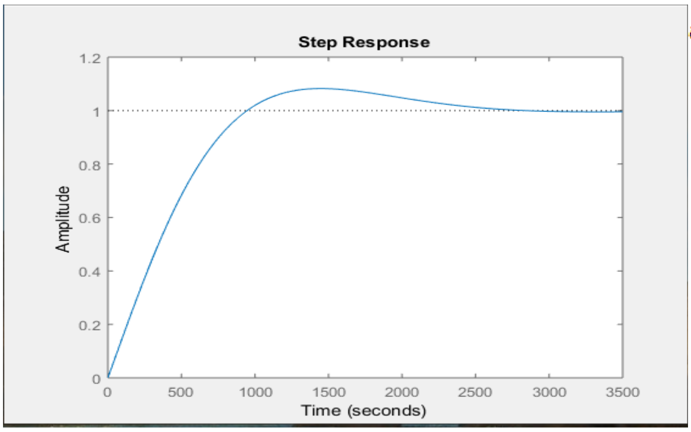
Figure 445– *TC2* stripchart *Tank2* 90%

Matlab Implementation

By implementing the overall transfer function of the close loop system consisting of temperature controller in the Matlab

$$T = \frac{300s}{198000s^2 + 600s + 1}$$

We get the following result, step response plot of vlosed loop system with Temperature controller




```
ans =

        RiseTime: 688.7757
    SettlingTime: 2.3600e+03
     SettlingMin: 0.9036
     SettlingMax: 1.0824
       Overshoot: 8.2424
      Undershoot: 0
            Peak: 1.0824
        PeakTime: 1.4589e+03
```

Figure 45 – Step response and stepinfo of *TC2-HeatEnergy2*

So from plotting the transfer function in the Matlab we again infer that the rise time for reaching the set point temperature of *35* degree Celsius is *688sec = 12 minutes* (approx.) along with some overshoot. The final settling time to reach back to its specified set point (*35* degree Celsius) is around *39* minutes.

CONTROL LOOP BETWEEN FC3 DISTURBANE CONTROLLER AND VLV-LIQT2

VLV-LiqT2

We have liquid outlet tank 2 as specified in the figure below:

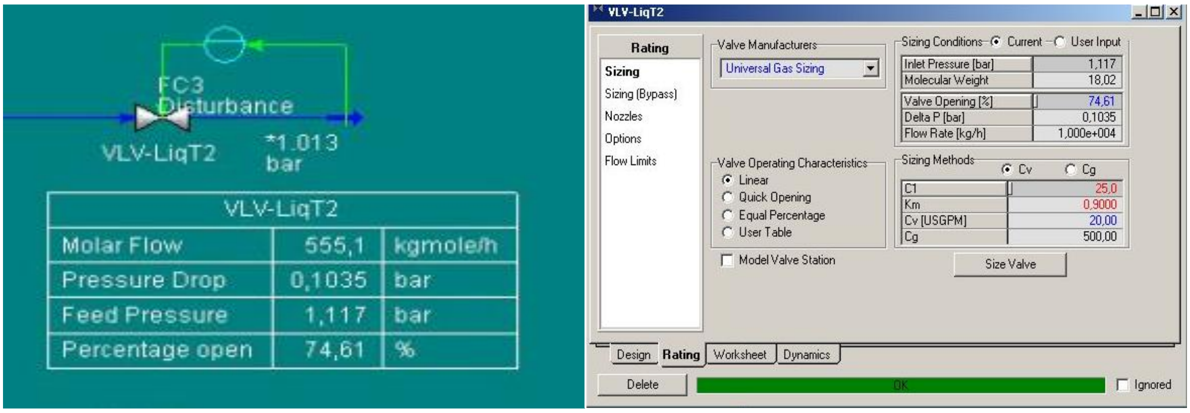


Figure 46 – *VLV-LiqT2*

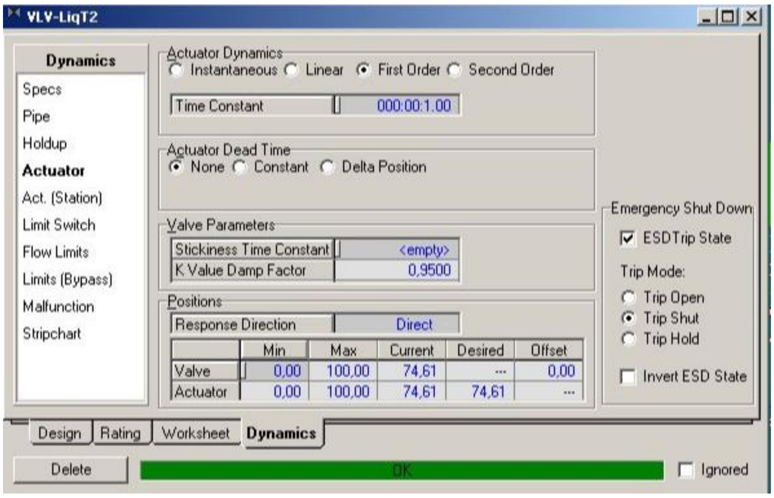


Figure 46 – *VLV-LiqT2 actuator*

ANALYSIS WITH DISTURBANCE CONTROLLER

The block diagram of the vlv-LiqT2 and FC3 can be specified as follows:

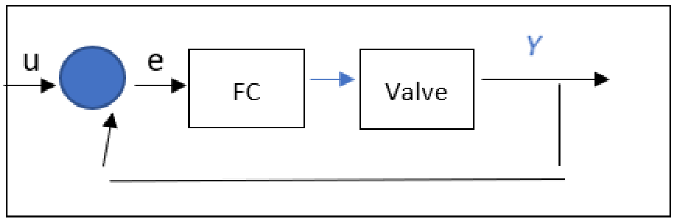


Figure 47 – System structure *FC3 Disturbance*

The valve considered in this loop is a linear valve and hence it will offer a constant gain.

FC3 Disturbance is a controller that is used to control the opening the *VLV-LiqT2*.

The transfer function of the above block diagram is given as:

$$FC = K_f(1 + \frac{1}{T_f S})$$

$$Valve = \frac{K_v}{1 + T_v * s}$$

e = u - y

$$y = e * FC * Valve$$

$$y = FC * Valve * (u - y)$$

$$y = FC * Valve * u - FC * Valve * y$$

$$y(1 + FC * Valve) = FC * Valve * u$$

$$\frac{y}{u} = \frac{FC * Valve * u}{(1 + FC * Valve)}$$

$$\frac{y}{u} = \frac{K_f\left(1 + \frac{1}{T_f S}\right)(\frac{K_v}{1 + T_v * s})}{\left(1 + K_f(1 + \frac{1}{T_f S}) * \frac{K_v}{1 + T_v * s}\right)}$$

$$\frac{y}{u} = \frac{K_f\, K_v\, T_f\, s + K_f\, K_v}{s^2\, T_f\, T_v + s\left(T_f + K_f\, K_v\, T_f\right) + K_f\, K_v}$$

***FC3* parameters**

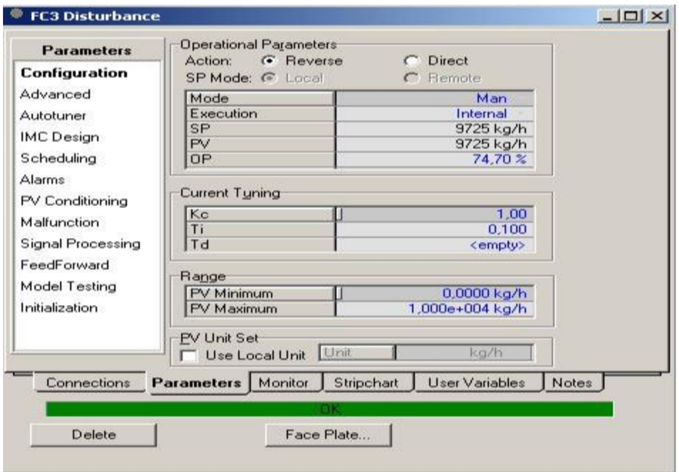


Figure 48 – *FC3 design configurations*

We consider the overall gain of our system as

$$K_v * K_f = 1$$

Also, the time constant T_v for the valve *VLV-LiqT2 = 1 sec*, and the time constant T_f for the controller *FC3 Disturbance* is *0.1 sec.*

From equation (10) given above:

$$\frac{y}{u} = \frac{K_f\ K_v\ T_f\ s + K_f\ K_v}{s^2\ T_f\ T_v + s(T_f + K_f\ K_v\ T_f) + K_f\ K_v}$$

Substituting the value of the parameters in the above equation we get:

$$\frac{y}{u} = \frac{1 + 0.1 * s}{0.1 * s^2 + 0.2 * s + 1}$$

Analysis of Linear System Theory:

Since $K_f * K_v = 1$,

$$\frac{y}{u} = \frac{T_f\ \mathrm{s} + 1}{s^2\ \mathrm{T_f}\ T_V + s\ 2\ T_f + 1}$$

$$y(s^2\ T_f\ T_V + 2\ T_f\ s + 1) = u(T_f\ s + 1)$$

Taking inverse Laplace on both sides,

$$\ddot{y}\ T_f\ T_V + 2\ T_f\ \dot{y} + y = T_f\ \dot{u} + u$$

Now, let $T_f\ T_V = a, 2T_f = b$, c = 1 and $T_f = d$. Thus, the equation becomes

$$a\ddot{y} + b\dot{y} + cy = d\dot{u} + cu$$

$$a\ddot{y} = d\dot{u} - b\dot{y} + c(u - y)$$

$$\ddot{y} = \frac{d}{a}\dot{u} - \frac{b}{a}\dot{y} + \frac{c}{a}(u - y)$$

Integrating,

$$\dot{y} = \frac{c}{a}\int (u - y) - \frac{b}{a}y + \frac{d}{a}u$$

Let $x_1 = y$ and $x_2 = \int (u - y)$

$$\dot{x}_1 = \frac{c}{a}x_2 - \frac{b}{a}x_1 + \frac{d}{a}u$$

$$\dot{x}_2 = u - y = u - x_1$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -{}^{b}/_{a} & {}^{c}/_{a} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} d\ /\ a \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Let’s substitute the values,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -{}^{2T_i}/_{T_iT_V} & {}^{1}/_{T_iT_V} \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} T_i\ /\ T_iT_V \\ 1 \end{bmatrix} u$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -2 & 10 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

Hence $A = \begin{bmatrix} -2 & -10 \\ 1 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$ $and\ D = 0$

Eigenvalues

$$|\lambda I - A| = 0$$

$$\begin{vmatrix} \lambda + 2 & -10 \\ 1 & \lambda \end{vmatrix} = 0$$

$$\lambda^2 + 2\lambda + 10 = 0$$

$$\lambda_1, \lambda_2 = 1+3i,\ 1-3i$$

Controllability

$$s = |B \quad AB|$$

$AB = \begin{bmatrix} -2 & 10 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -1 \end{bmatrix}$

$$s = \begin{vmatrix} 1 & 8 \\ 1 & -1 \end{vmatrix} \neq 0$$

Hence the system is controllable.

Observability

$$s = \begin{vmatrix} C \\ CA \end{vmatrix}$$

$$CA = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} -2 & 10 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & -10 \end{bmatrix}$$

$$s = \begin{bmatrix} 1 & 0 \\ -2 & -10 \end{bmatrix} = -8\ \neq 0$$

Hence our system is observable